DOCUMENT RESUME

ED 436 348 SE 061 525

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TITLE	Discourse in Small Groups in an Algebra 1 Class.
PUB DATE	1998-04-00
NOTE	30p.
PUB TYPE	Reports - Research (143)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	*Algebra; Classroom Communication; Communication (Thought Transfer); *Cooperative Learning; *Discourse Modes; Group Discussion; High Schools; Mathematics Instruction

ABSTRACT

The purpose of this study was to examine the discourse in two different group-work contexts in a diverse Algebra 1 class to better understand what and how students working in small groups learn through talking about mathematics. The researcher gathered data in four categories: participant observation, interviews, collection of artifacts, and reflections. Results show that the cooperative groups discussed the theme in the problems, but they took longer to get the problems done due to checking and double checking and then checking with the teacher. All of the checking meant doing more mathematical talk with each group member repeating the solution him or herself. (ASK)

Discourse in Small Groups in an Algebra 1 Class

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DISCOURSE IN SMALL GROUPS IN AN ALGEBRA I CLASS

The national reform effort in mathematics education calls for a constructivist approach to teaching mathematics. Recently developed curricula, including program used in this study, are based on the theory that students construct their own understanding of mathematics and that teachers and materials can be prepared to better serve students in helping them to develop this understanding. These materials generally include problems designed for small group work so students can talk about their work as they are doing it.

The literature on cultural diversity discusses the importance of and differences in language use for different cultural groups. In a study of two high school Algebra 1 classrooms, in which a reform effort program was in use, Brenner (1995) reported that limited English proficient students were not participating in the mathematical communication advocated by the mathematics reform movement; although, they were engaged in working with the mathematical content and they were learning algebra.

Recent articles have urged researchers to study classrooms in which teachers are attempting to help students develop their own understanding through the social negotiation of meaning as they work together and talk with each other and the teacher (Cobb, 1996, Lerner 1996). However, very little research has been done in relation to language use in mathematics classrooms at any level, let alone high school. Very few researchers in mathematics education work as regular teachers in classrooms and report on their attempts to work with diverse groups of children in that setting. Lampert (1985,), Ball (1996), Parker (1993) and Romagno (1994) are exceptions. Lampert at the middle school and Ball in a third grade classroom have focused their work on generating thoughtful whole group discussions. Parker at the fifth grade level and Romagno in a ninth grade basic mathematics class, team taught with the regular teacher and focused on developing alternative curricula and teaching methods to engage students in thinking more deeply about mathematics. There are not long term studies of high school classrooms with the researcher as teacher, nor has any study besides Brenner's focused on the discourse that occurs between students and teacher and between students as they work in small groups on mathematics.

METHODS:

The purpose of this study was to examine the discourse in two different group-work contexts in a diverse Algebra 1 class to better understand what and how students working in small groups learn through talking about mathematics. This paper addresses the question:

- How do students use talk to build understanding of mathematics?
- What can we learn about their understanding of mathematics based on their talk?

Because this paper is part of a larger study addressing what actually happens in a classroom when students work in small groups I used an ethnographic approach. I arranged to teach an Algebra 1 class from October to June, in an inner-city school with a mixed population of Asian, African-American, Latino, and White students. The class was using the College Preparatory Mathematics: Change from Within (CPM) Mathematics 1 materials, a replacement course for Algebra 1 designed to enhance students' problem solving, reasoning, and communication skills and to use the other areas of mathematics, geometry, graphing and functions, probability and statistics, as a basis for understanding algebra.

As the teacher, fully responsible for all aspects of the class including attendance, grading, and talking with parents, I could be an insider, but because I was only teaching one class I would still be an outsider in the some important respects. While I did not live through a regular teacher's long and demanding five-period teaching day, my other full time responsibilities did not allow me any more preparation time for teaching than a regular teacher would have for such a class, so it was easy to stay with my plan of using the materials as recommended, not supplementing or changing them significantly. Using the materials as they were written was important to my goal of working in a classroom situation that might be considered close to a normal class where I would experience many of the same pressures and dilemmas as a regular teacher (Ball and Lampert).

The class was organized into groups of four or sometimes three. Students changed groups at the end of each unit. During the first semester assignment to the groups was totally random, based on the draw of a playing card which was immediately recorded to avoid the temptation of trades. During the second half of the year students could chose a partner, then the partners would draw to see who else they would work with. The curriculum was designed for use with small groups so included a variety of problems including developmental explorations, investigations, connecting problems, and applications as well as routine practice problems and exercises.

DATA:

I gathered data in the four categories described by Eisenhart (1988): participant observation, interviews, collection of artifacts, and reflections including "emergent interpretations, insights, feelings, and the reactive effects that occur as the work proceeds." (p.106) To supplement my observation as a participant, I used an audio-tape recorder during class in two ways. To record teacher-student exchanges I wore a small tape recorder and an external microphone as I moved around the classroom from group to group. I also recorded groups of students by placing a tape recorder with an external microphone in the center of a group of three to four students on one of the student's desks. A year after I taught the class I returned to interview fourteen of the students.

Artifacts gathered included written work and records: all the assignments the students did throughout the year, my grade-book records, their mathematics grades in previous and following courses, and information from the counseling office. For background information and to corroborate my observations I interviewed the following people: the teacher whose class I was teaching, the other teachers whom the students had for math the next year, three bilingual education teachers, and one counselor.

The analysis was based on the transcription of 38 classroom tapes and 13 interview tapes. The transcripts include discourse markings to show pitch, pauses, volume, breathing, drawn out words, openings, interruptions, and splices.[1]

ANALYSIS:

Group talk in six groups: samples from the transcripts.

In analyzing the transcripts of students working in their groups I found four working relationships which I describe as, Cooperative, Competitive, Businesslike, and Teacher/Student. In the cooperative groups participants worked through problems together, each participant building on what another said, thereby contributing to a mutual understanding of their solution. But in the competitive and businesslike groups each member attempted to do the problem on his own first,[2] often talking to himself while working through the problem.

The competitive group often discussed their solutions when they had completed a problem, at least partly to demonstrate that they knew how to do the problem. They often had used different approaches, and through these discussions or arguments had the opportunity to see the problem

[1]Appendix A contains a guide to the discourse markings.

[2]"His" is the appropriate pronoun here because in the groups I analyzed it was always boys.

from different perspectives as well as the chance to articulate and explain or defend their own approach. In the businesslike group, on the other hand, when individuals finished problems they went on to the next one, possibly pausing to check answers. Only when someone needed directions or asked the others what to do, would someone give a set of step-by-step directions on what to do, and when the recipient indicated he got it, the group would go on with the work.

In the teacher/student groups the participant who worked in the role of student used questions to elicit explanations and often restated or paraphrased what the "teacher" explained. So while one the student had the opportunity to clarify his thinking by explaining, the other formulated questions, and would try out rephrasings to make sure of getting it the explanation straight.

Examples representing the four working relationships follow. A clearer picture of the four working relationships will evolve through discussion of the examples.

Group A: Kao Pou, Ngoan, and Kou Sio worked in parallel but together in the sense that they worked on the same problems and kept track of each other. It was clear that Kao Pou was the strongest mathematician in the group, and he led the group in that he always moved to the next problem first, while Kou Sio sometimes struggled to keep up. As Kou Sio and Ngoan worked, "thought out-loud," and discussed problems, Kao Pou kept track of what they were doing.

In the example Ngoan is talking aloud while working alone on problem EF-98 (a) and Kou Sio is looking on. The problem did not include an illustration of the triangles; they have drawn them. Kao Pou has completed (a) is starting on part (b).

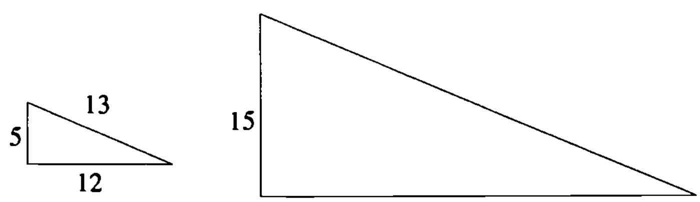
EF-98. Suppose a right triangle with sides of lengths of 5, 12, and 13 centimeters is similar to a right triangle whose shortest side is 15 centimeters long.

a) What is the perimeter of the larger triangle? 90 cm

b) What is the ratio of the perimeter of the smaller triangle to the perimeter of the larger triangle? 1:3]

c) How does the ratio in part (b) above compare to the ratio of the lengths of corresponding sides of the triangles? They're the same.]

d) What is the ratio of the area of the smaller triangle to the area of the larger triangle? 1:9]



Ngoan is finding the sides of the larger triangle by **adding** 10 to the sides of the smaller triangle which had sides 5, 12, and 13, a not uncommon incorrect effort to create a similar triangle, but an unsettling one for Ngoan to be making at this point in the unit. Kou Sio has expressed his confusion about this approach a few lines earlier, but is now going along with Ngoan. Kao Pou has overheard the conversation and injects the idea that they need to **multiply.** Kao Pou did not need to be asked for help because he is aware of what Ngoan is doing, and when Ngoan acknowledges hearing Kao Pou and Kou Sio picks up on multiplying, Kao Pou goes back to reading the next problem to himself. Only then Ngoan realizes "I think I did it wrong," and Kou Sio exclaims that he does not understand, thus, indirectly, requesting Kao Pou's renewed attention. So Kao Pou explains more completely what to do (without getting into why), then again goes ahead to parts (b) and (c). This group cooperates in the sense that they are very helpful, but it is clear that Ngoan and Kao Pou are working separately on the problems with Kou Sio following Ngoan.

Ngoan: Oh I think it's goin to be twenty-two and twenty-three . . . .
.22, . . 22 . . 15 . . 23

Kou: Twenty-three.

Ngoan: We **got** that.

Kao: We got to triple it huh? This is thirty-six.

Ngoan: What is thirty-six?

Kou: Triple it. Times it by three. . . huh?

Kao: [reading] "What is the perimeter of . .

Ngoan: I think I did it wrong . . . .

Kou: I don get this m·a ·n . . . .

Kao: What? How'd I get that? Times it , times this by three 'cause you gotta times this by three to get that . . see three times five's fifteen? That's the smallest side, in this triangle. And then we time all the rest a these by three . . . . .

Kao: [goes on reading the next problem] "What is the ratio of the sides of the smaller triangle?" . . . . O·oone - third.

Kao: [reading] "What is the ratio of b, compare to the corresponding sides of both triangles." Corresponding sides . . [mumble, mumble]

Ngoan: You take, you take this one time dis one?

Kao: What?

Kou: You take 5 times 15?

Kao: Yeah, No five times **thuree**.

Ngoan: to get this one

Kao: What. All right. We get, we have to times this by three to get fifteen.
See an' then . . so to get this we have to times by three too.

Ngoan: ↑ ↓ *Ohhh* . .

Ngoan does not pick up on Kao Pou's first or second attempts to explain that he should multiply. It is only after Ngoan asks the question himself that he sees what to do. In a later example Kou Sio did not understand the way the others set up a ratio because he did not automatically see ten cents as equivalent to the decimal .1. This time both Kao Pou and Ngoan "explain," but their hurried "explanations" are flatly stated facts. The problem is a set of exercises in writing ratios. Part a) asks for the ratio of one dime to one quarter.

Ngoan:	One dime to/	
Kou:	"One dime to one quater."	Reading
Kao:	Point-one to point-two-f·i·ve. Okay	
Ngoan:	What you got?	
Kou:	One dime to one quarter	Repeating, thinking
Kao:	Tw·e·nt·y f·i·ve. (b) . twelve feet. [to himself]	
Kou:	Be twenty-five, huh?	
Kao:	What? .. Point-one to point-twenty-five/	Kao Pou assumes every-body knows .1 = .10
Kou:	One to one twenty-five?	
Kao:	Yeah	
Kou:	I put ten to one, to to to to twenty-five.	But Kou Sio is using 10/25
Kao:	What?	
Kou:	ten to to to . .	Kou often stutters when he addresses Kao Pou
Kao:	I put one . Ten is the. . it's the same.	
Ngoan:	Ten *is a one with a point there.*	And that's all there is to that.
Kao:	(b) *is . . twelve feet* ______ *to nine yards . .*	Kao Pou and Ngoan are on the next problem. Kou Sio is still wondering.
Kou:	Ten over twenty-five?	

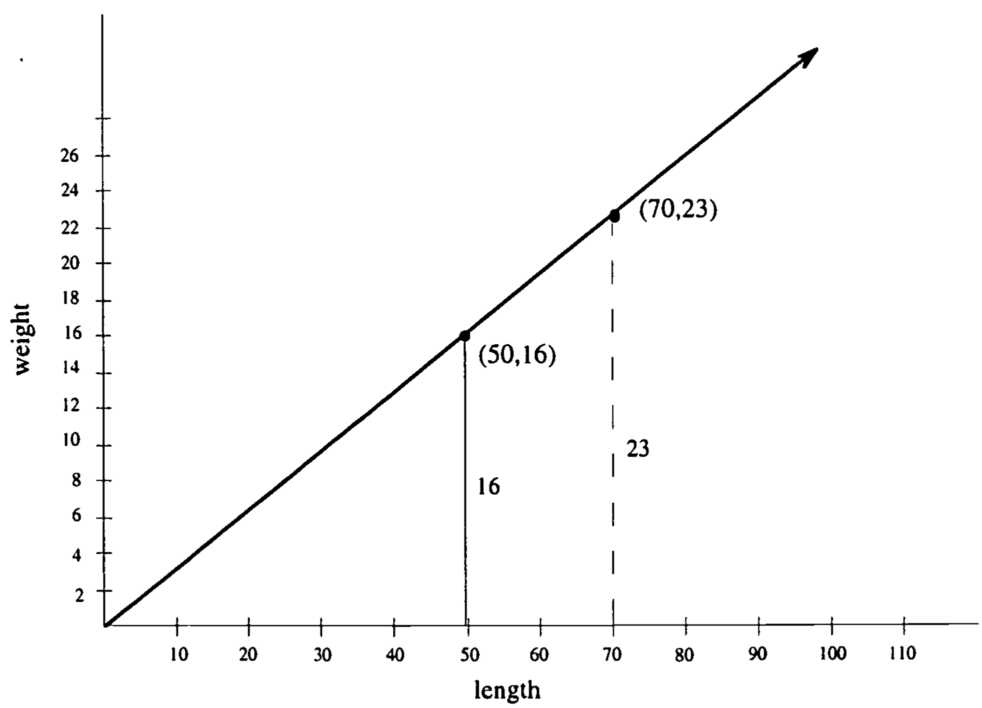
The Group A tape was the first group tape I transcribed. While I was disappointed in the procedural orientation of their explanations, I was impressed with the respect the students had for each other and the *Businesslike* efficiency with which they focused on completing the tasks at hand. There was virtually no off-task talk. I also wondered how much effect the problems themselves had on the way the students worked together. Group A had been working on a routine set of problems and exercises that were assigned as practice late in the unit, the kind of problems students should be able to do on their own. So Group A's approach to working these problems seemed appropriate for the type of problem, but it also turned out to be Group A's style of working in general.

Group B: The first time I listened to Group B's tape I thought I was in a completely different world. When the boys in Group A discussed the problems they leaned toward each other and talked quietly. It was sometimes very difficult to hear them on the tape. There was no problem hearing the boys in Group B. When they talked they often leaned back and talked loudly, requiring occasional reminders to use group voices. Group B was recorded two days earlier, on the first day back from the winter break, working on a problem designed to relate graphing, similar triangles, and equations involving ratios and as a precursor to slope, a connecting problem.

When I left Group B with the tape recorder and the words, "Just see if you can ignore it," David turned to the group and in a clipped, sophisticated tone announced, "We're mature young adults." Jason followed in a more supercilious tone with, "Oh, David, what would you possibly h·a·ve for (b)? " Group B made a total of 43 lines of comments related to the tape recorder, but that was only 23% of the 184 lines they devoted to such subjects as other students, favorite teachers, getting the flu during finals, bent wallets, the color of this month's bus passes, and .. rubber silverware. The following sequences represent Group B at its best. Each group member had drawn a graph something like the one following the problem in order to answer part c), and they were now working on part d).

EF-68. A 70-foot length of wire weighs 23 pounds.

a) How much does 0 feet of wire weigh?
b) Draw a graph showing the weight of the wire (vertical axis) compared to the length of the wire (horizontal axis). Label the axes with appropriate units.
c) Use your graph to estimate the weight of a 50-foot length of wire.
d) Write an equation using equivalent ratios to find the weight of a 50-foot length of wire. Solve the equation.
e) Use your graph to estimate the weight of 80 feet of wire.
f) Write an equation using equivalent ratios to find the weight of a piece of wire that is 80 feet long.
g) How close were your estimates in parts (c) and (e) to the actual values you found in parts (d) and (f), respectively?



In spite of the fact that Josh has, on two occasions **told** the group exactly what to write for the equation neither David nor Jason have written Josh's equation. David has a different version of the equation (x/23 = 50/70) and becomes confused in the process of solving it when he looks at Josh's paper and sees that his equation is different. It was at first difficult to tell what Jason is doing because he says repeatedly, "20 equals 7" which he, obviously, does not mean literally. After eight lines of off-task banter led by Jason, Josh brings the group back to solving the equation.

Josh: Okay, now wait-a-minute. Where's my calculator? We need to figure out what . . twenty-three

David: Fifty over seventy . .

Josh: Times . . fifty equals one, one, five, ohh . one, one, five, oohh.

Jason: Sixteen over fifty, so far we have 16 over 50 and 23 over seventy. [At this point, working on his own as the others calculate.]

Josh: ↑ Huh, wow!

David: What's the answer, Joshua? Read the answer.

Jason: So, I think . .

Josh: Oooh.

Jason: I think 20 equals seven . . Lemme think I just think twenty equals 7.

Josh: **I just solved the equation!**

Jason: I think 20 equals seven/

Josh: ↑ **Haah**, x equals **sixteen**. You should know that.

David: Du·u·ude

Josh: Big Dufus.

Jason: I think twenty equals seventeen.

With the exceptions of the last four lines and the line where David asks, "What's the answer Joshua?" the boys are not talking to each other. Josh and David celebrate getting the same result they had estimated earlier on their graphs, but they don't talk with each other about doing the mathematics. In the meantime Jason had skipped over part d) and was continuing on his own exploration of the relationship on his graph, which turned out to yield some interesting and correct results. As he worked he repeated what he was thinking, but is consistently ignored in the conversation until Josh disagrees with his result that 20 feet of wire would weight seven pounds because he has made an equation to get the result 6.4 pounds.

Jason's result is from his graph and two points (50,16) and (70,23). For twenty feet there is a seven pound difference. In effect he is using the slope. It is interesting to follow the train of thought in Jason's comments as he works. In the following I have left out David and Josh during the time that they appeared to be ignoring Jason. I also skipped several off-task wanderings. But it is clear that, while he was not responding to Jason, Josh was not ignoring him entirely, because he set up an equation to solve Jason's problem, a problem that was not posed in the materials.

Jason: Sixteen over fifty, so far we have 16 over 50 and 23 over 70.

Jason: So, I think . .

Jason: I think 20 equals seven/

Jason: I think twenty equals seventeen.

Jason: I think twenty equals seventeen

Jason: No I think . I should . .

Jason: No I really did have this . . . .
Seven equals 20, I think . . I'm getting something______ No I think this is

Jason: Seven equals 20

Jason: No 'cause, you know I think this is ______ seven equals twenty

Jason: Twenty feet, I think twenty feet equals seven . pounds .

Josh: Equals eight pounds, er **six** pounds

Jason: Seven.

Josh: No equals . six point four pounds. Use [yawning] an equation, solve an equation Jason. Use . your . calc·u·la·tor .

Jason: I don't use the calculator.

From this point on the discussion includes all the contributions by Josh, David, and Jason.

Josh: x equals . . .

Jason: See 80 feet. [Chimes in background for 11:00] Eighty feet of wire. Or I could just keep on . . . . 75, 80 ...

David: 26 whoa. [Josh had made an earlier estimate of 25 from his graph]

Jason: It should be 26 too. [Both methods give 26]

Josh: Yeah, I know.

Josh: Dude, I am so smart. I even had the right equation to solve that one.

Jason: I just went up . . I just went I just went a little bit.

Josh: I can't go up on my graph though [Josh scaled his graph so it rises faster than Jason's and will go off the paper]

The group finally came back together when Josh decided to use an equation to check Jason's result (23/70 = x/20). When the group found the weight of eighty feet of wire, Josh and David used the equation while Jason extended his graph his way, adding 3.5 pounds to 23 to account for ten feet of wire beyond 70. Josh finally acknowledges Jason's method but points out that there is not room to go up on his own graph. Much later in a discussion between Josh and Jason, after Jason re-reads part (f), he agrees that he could set up an equation, but Jason used the ratio 16/50 as a basis and Josh used the original 23/70, and again at first, neither saw what the other was doing.

Jason, Josh, and David work in parallel on the problems, and each sticks with his own method, in spite of Josh's effort to convince them that his is right. Each insists on completing the problem his own way before acknowledging that another way might work, and they have no hesitation in pointing out other's errors and sometimes insulting them at the same time as in David's proclamation, "That's **wrong**, Josh!" and later Josh's comment, "It's an equation, David. Use yer berain."

The members of both Group A and Group B for the most part worked independently, in parallel. Josh and Jason worked on their own and David sometimes followed Josh, but not as much as Kou Sio followed Ngoan. But while members of Group A talked in order to help to solve the problems, members of Group B talked to each other in order to brag about their solutions. I labeled this group *Competitive* because of the proclivity of each member to solve the problem his own way and argue for that way before admitting that another way might also work. Another way in which group members competed was in trying to distract each other with off-task trivia talk.

Group C: This Group was recorded in April. Veronica and Jessica chose to work together, and when they drew a cards to determine the rest of the group Ngan and Monica joined them. Both Ngan and Monica were older students (11th and 12 grade), and Monica was often absent. The problem Veronica and Jessica were working on was an equation involving rational expressions. The steps to solving the equations were already provided. They had to fill in a reason for each step. Ngan had decided to skip that problem and was trying to do the next one on her own.

Veronica leads with the question, "What do we do here?

RT-27 Copy the following problem on your paper. Fill in each of the lines labeled (a) through (e) to explain how the equation to its right was obtained from the equation above it.

Solve the equation: $\frac{4}{x} + \frac{3}{2x} = \frac{11}{6}$

a) ______________________ $6x(\frac{4}{x} + \frac{3}{2x}) = 6x \cdot (\frac{11}{6})$

b) ______________________ $6x \cdot (\frac{4}{x}) + 6x \cdot (\frac{3}{2x}) = 6x \cdot (\frac{11}{6})$

c) ______________________ $24 + 9 = 11x$

d) ______________________ $33 = 11x$

e) ______________________ $3 = x$

Veronica Okay so what do we do here?

Ngan Two x... [to self on another problem]

Veronica Oh, you have to either multiply it by th•e.....

Jessica	Find the common denominator..... Okay
Veronica	Well, right here, right here, on b) is it, do you multiply by a common denominator?
Jessica	Umhmm.... Ohh!
Veronica	Right here.
Jessica	This is where they multiply from the common denominator. See they multiply it.
Veronica	No 'cause they're bringing it down, right here they're gonna multiply it, see.
Ngan	Ms, Ms Kysh
Veronica	And then right here a·dd this.
Jessica	.... No. Ms. Kysh. Siempre los aiuda.
Ngan	Ms Kysh........
Veronica	Multipl·y what?
Jessica	Multiply both, six x to both sides.
Veronica	Okay. And then, so. Oh so this side you come up with that and....
	A·andale..............................
Jessica	I think you're sposed to multiply both, six to both sides...............

But they have already multiplied both sides by 6x in step a). They are having trouble seeing that the Distributive Property is the reason in step b). Veronica has reason to be puzzled because it seems as though Jessica is just using different words for her step a) reason to fill in step b).

In this case the students participate in the discussion as equals even though Jessica is the better mathematician, and Veronica acknowledges this in a later statement. They will go around and around on this problem, consulting the teacher several times before they are both satisfied that they have filled in all the blanks correctly. The discussion will go back and forth for steps c) and d). Group C differs from the others in that they often ask each other questions and one will often pick up where the other leaves off. The next seven lines show how each girl builds on what on what the other has said, a totally different approach than the boys in Groups A or B used, a group with members who respond to each other, a *Cooperative* group.

Jessica	cee) . Solve, twenty-four plus/
Veronica	You solve twenty-four plus nine equals eleven x, equals thirty three.
Jessica	Equals eleven x?
Ngan	Yeah

Jessica No, but how do you, won't it just be add twenty-four plus nine to get that?

Ngan Yeah

Veronica Yeah, well that's why we wrote that. Solve twenty-four plus nine. equals x. . . . . . .

Ngan be•e) I need b). So what d'you times it by.. . . . . What'd you get for d)

Veronica Uhhhh, divide eleven x by thirty three . . . . and e) is you get the answer.

Ngan I hope this is right.. . . . .

Veronica What d'you divide this by?. . two?

Jessica Uhhm, two x.

Group D_1: Group D is really two groups of two. Hector and Hugo chose to work together as did Paul and Mike. Then they drew randomly as pairs, and they got each other. At the very beginning of their tape there is a brief exchange in which Hugo asks Paul what he got for the first problem and Paul accidentally gives his answer 30 ft. for the broken telephone pole, which is the second problem. After that the two pairs do not speak to each other. Their talk alternates and sometimes overlaps, but it is clear that neither pair is paying any attention to what the other is discussing. I have separated the dialogue for Group D_1 , Hector and Hugo from that of Group D_2, Paul and Mike. When either appears to be taking more than one turn in a row that is usually because dialogue between Mike and Paul has been removed. Mike and Paul's talk often served as a pause, or thinking time for Hugo and Hector and vice versa.

In the following example Hector and Hugo are working on the broken telephone pole problem which requires a good diagram and the use of the Pythagorean theorem. They had already solved the problem when this excerpt starts, but unfortunately they remembered that Paul's answer was 30 feet, and Paul is known as one of the best mathematicians in the class. Veinte y cinco (25) y doce (and 12) is treinta y siete (37), the correct answer, but they keep going back over the problem to make sure they are right because Paul is such a formidable force. Both Hugo and Hector are bilingual in Spanish and English, although Hector is taking an ESL class in addition to regular English. When I informed them that today was their day to be recorded they vowed to speak Spanish for the tape, but they often went back and forth between languages and sometimes forgot and spoke English.

RT-31. A careless construction worker drove a forklift into a telephone pole. The pole cracked (but was not severed) seven feet above the ground. The remainder of the pole fell as if hinged at the crack. The tip of the pole hit the ground 24 feet from its base. If an additional five feet of the pole extends into the ground to anchor it, how long should the replacement pole be? **[37 feet]**

Hugo That's veinticinco

Hector y doce

Hugo No sé

Hugo Lo hago/

Hugo	toma así, toma así, toma cinco Treinta y cinco
Hector	Treinta y siete
Hugo	¿Por qué no es cinco, no?
Hector	Sí, .es cinco más, y
Hugo	Hay que poner la, más, en que cinco más Treinta y siete
Hector	No,
Hugo	Veinte y cinco, ¿veinte y cinco mas qué? Más
Hector	Más cinco.
Hugo	Más siete. . Veinte y cinco, más siete, más cinco. . más siete, más cinco. . ¿Que te sale?
Hector	Hmm?
Hugo	¿Te salío?
Hector	Wait a minute.
Hugo	That's veinte y cinc·o
Hector	Yeah!
Hugo	¿Qué te salío?
Hector	Still got treinta y siete.
Hugo	Let's tell them.
Hector	It can't be treinta.
Hector	It's more complicated. ¿Sí ?
Hugo	Veinte cinco con más, y con más. They are wrong. Es treinta y siete.
Hector	Yeah. Esta aquí.
Hugo	Por nosotros.

They worked and reworked the problem together until they were completely convinced, then they asked the teacher to be absolutely sure, but they never told Paul and Mike. In this give-and-take they recognize each other as equal contributors and work as equals. They ask each other questions, finish each other's sentences, and indicate uncertainty or agreement. Neither is the dominant source of information or explanation. Groups C and D_1 clearly have a different way of working from groups A and B. The are cooperating by sharing their thinking, and they are doing the problems together by putting pieces together, rather than working individually then comparing.

Group D_2: is still different. Paul charges into the problems, talking out loud as he works. Mike works alongside him, usually a quarter to a half step behind struggling to keep up with the details, but occasionally jumping out in front with an idea about how to start. At the beginning of the Group D tape they start on the same problem as Group D_1, the broken telephone pole.

Mike:	Okay, so it looks like, whatchu got there twenty-four, . squ•ared, . plus seven squ•ared. . . .
Mike	Oh boy, did you get six-twenty five?
Paul	I go•t .
Paul	Yeah, you get six-twenty five, but in the equat/ in the problem it says . yeah, you square root that.
Mike	An' it's twenty-five.
Paul	and then it's thirty because it says in the problem that five more feet go into the ground. [the total length of the pole is actually 37ft. He forgot the other 7 feet above the ground]
Mike	Oh, okay.
Paul	So you add five, an' so at the end it's thirty.. . . .
Mike	So it, so this is fi•ve, right the•re.

Unfortunately for Hugo and Hector, Mike went along with Paul on this one. While Hector and Hugo were going back over their work, Paul and Mike worked through the Flat family's TV antenna problem, a long problem involving several three dimensional applications of the Pythagorean theorem and were starting on the routine exercises toward the end of the assignment as Hugo and Hector were starting the antenna problem.

In the following excerpt Paul is leader and teacher. Mike follows along doing and checking as he goes. He is not copying. Paul continues to describe what he is doing as he works, sometimes clearly, others mumbling, but always ready to stop to explain to Mike. When Paul or Mike appear to be taking two or more turns in a row Hugo and/or Hector were taped speaking in between those turns, and their time would be like a "think" time for Mike and Paul.

RT-35. Write one polynomial to represent each of the following sums and differences.

a) $(8x^2 + 5x + 7) - (3x^2 + 2x + 2)$
b) $(8x^2 + 5x + 7) - (3x^2 - 2x + 2)$
c) $(5x^2 + 14x + 3) - (2x^2 - 9x + 5)$

Mike	Where'd you get x squared?. . . . **Da•mn.**
Paul	Allright, so . fi•ve x squared plus three
Paul	You just have to like combine those.

Mike	Like subtract 'em
Paul	Yeah, you can subtract 'em.
Mike	So what is that, um, it's five x squared?
Paul	Yeah
Mike	Five x squared minus, plus, plus
Paul	For b) I got five x squared plus seven x plus five. 'Cause you have to remember that, if you a•dd, uh, if you subtract a negative then it's just like adding. ....
Mike	Plus five?
Paul	Yeah....
Mike	C'mere. Hold on, hold on. So when you subtract one it's just like adding it?
Paul	No. When you subtr•a•ct a **negative** it's just like adding the two numbers. They're just positives.
Mike	*Okay.*
Mike	So you got three x squared plus twenty-three plus.. two? [part c)]
Paul	Negative two.
Mike	So twenty-three___ yeah **x** ... plus, negative two?
Paul	**x** Yeah. Whoa.

In Group D_2 Paul is doing the problems on his own and Mike is working along together with him. Mike and Paul do talk about the problems with each other, usually with Paul in the role of teacher; although, that does not keep Mike from putting forth his own ideas on how to solve problems. There were several other students who played the role of teacher, and they often dominated their groups. Group D_2 represents the *Teacher/Student* style.

Group E was a mixed group by grade level, culture, and gender. Martha, who is the organizer in this group, is bilingual in English and Spanish. Carmen speaks English but prefers Spanish, and neither Richard nor Brian speak Spanish, but Richard understands it. Carmen and Martha often work together, outside of class as well as in class, and because Carmen finds it easier to work in Spanish, Martha often translates the problems as well as what the teacher says, and discusses them with Carmen in Spanish. Richard was in the ninth grade, Martha and Carmen were in the tenth, and Brian the eleventh. The group was working on a group test so they knew they had to finish the problems before the end of the class and that they had to agree on their solutions because only one of their papers would be selected for evaluation. In general the groups worked more efficiently on group tests, avoiding off-task topics. They also did more comparing of methods and results.

The group is working on the following problem.

The duty tax on an $800 motorcycle was $34.

a) Use this information to draw a graph to show the relationship between the cost of a motorcycle and the amount of duty tax.
b) Explain how to use your graph to estimate the amount of tax on a $1000 motorcycle.
c) Write and solve an equation to check the estimate from your graph.
d) Write and solve an equation to figure out what percent of the cost must be paid in duty tax.

Carmen: So how we gonna do for this. .. How we do the l·ines?...... [Carmen has drawn the graph]

Richard: I'm not ready yet........

Carmen: You have to put, thirty-four and eight-hundred like over here? You put eight-**hund** red you put that, thirty-four

Brian: *Yup.*

Carmen: Put correct position. Then you follow this one by the same one/

Brian: Yeah.

Carmen: thousand, so you give the same line, you go like forty-three the points.

Brian: Yeah. That'd be right, yeah.....

Martha: No, no seran los puntos, here...

Carmen: Miss Kysh .... [as is often the case when Martha corrects Carmen, Carmen decides to consult the teacher]

Martha: Is true...

Teacher: Yup.....

Carmen: For this one, I'm con/I don't know how to explain it, an' I don't understand this one right *here*

Teacher: [reading] Use the graph to estimate. Well did you dr·aw the graph now?

Carmen: Yeah.

Teacher: Did you dra·w the line ___ Like they did?

Carmen: Umhmm. What line? Oh

Richard: That one you got by/

Teacher: Okay, talk to the other people you're working with.....

The pauses below indicate silence during which the group members are working their own. Brian and Martha come to the same answer to part d) at about the same time. Brian works quietly on his own and helps only when asked. Carmen and Richard both follow along, but both insist on understanding and when they are not satisfied with the explanations of other group members do not hesitate to call in the teacher.

Carmen: So ahmm, so explaining this... Que tenemos? [part b) asks for an explanation]

Martha: Como esta, esta linea. Understand? Just the opposite, continua la linea hasta forty-three. Si.

Carmen: ¿Por que? ....

Martha: Draw the lines. okay [more explanation here but inaudible]...............
....

Martha: (b), (c), and (d) What percent of cost must be paid to duty tax?[reading problem]

Carmen: Que tiene que pagar, um is it, is this .. is the thirty bucks?

Martha: No, cause the tax is thirty-four [cannot hear M & C over other groups, discussion of problem continues M: talks about percent, C asks about percent]

Carmen: Forty-two out of .. eight-hundred? ....

Martha: I found what is the percent.

Richard: What percent is it?

Brian: Four-point-two-five percent.

Martha: Quatro-punto-veinte-cinco?

Richard: Um. Four-point-two five percent. ....

Martha is direct in letting Carmen know when she has made a mistake, sometimes starting by handing her an eraser, and she does not hesitate to help others. When Richard had skipped part of the problem, she pointed it out and helped him find the missing part, and when the group started the problem of matching equations to graphs, Martha assigned the jobs. Brian and Martha were the first to solve the problems independently. Martha's role in solving problems was not surprising because she always did very well on all her written assignments and individual tests, but her role of organizer was more interesting because she never volunteered in a class discussion and never asked the teacher a question. Of course she never had to ask questions because Carmen would ask all of her questions and more.

Martha Why don't we, each of us make two of these?

Richard Huh?

Martha Each of us make two, two of these?

Richard All right.

Martha I'll do the, one and two

Brian I'll do three and four

Martha You do five and six, and you do seven and eight.........

Elements of several styles are evident in Group E's discourse. Martha, like Paul, takes on the role of teacher, especially in relation to her work with Carmen, but also with Richard. Brian and Martha tend to work in parallel and check with each other in the *Businesslike* style of Group A. Also Brian, while he does not jump into the conversation very often, keeps an ear open and responds when something is amiss. For example he saved the group at the end of the tape when Carmen, who had not figured out how to match her equations and graphs asked the group next door what they got. Fortunately Brian disagreed with the answer because he recognized that the equation with x^2 had to go with the parabola.

In groups that were heterogeneous in relation to gender, culture, and grade level the four styles were evident, but often in combinations. Students who assumed the teacher role, Martha, Paul, and several others, did so fairly consistently, so the *Teacher/Student* style could show up among students who tended toward any of the other styles: *Businesslike, Cooperative,* or *Competitive.*

An analyses of talk within the groups

To analyze and compare the ways the groups used talk I refined the following categories starting with the transcripts of Groups A and B and working through the others in the order they are presented.

Talk while Working: talking through one's thinking while working on solving a problem or answering a question. This included three subcategories: Working Alone (WA), Work Display (WD), and Working Together (WT). The Working Alone category came to mean individual "thinking out-loud" which did not receive any response from other group members, while Work Display meant either that the speaker clearly intended to show off his thinking (as Josh did on several occasions in Group B when he announced that he had set up and solved an equation) or that the speaker was just letting others overhear his thinking so they could keep track or follow along. For Group A a "thinking out-loud" statement became Work Display when another group member responded to it. For example, when Ngoan is trying to figure out the sides of the larger similar triangle, he describes aloud what he is thinking. This think-talk allows Kou Sio to follow along, repeat, and question, but it also allows Kao Pou to overhear and to correct Ngoan's error. That left the Working Together category for all the give-and-take among group members that did not fit into the categories for Questioning, Repeating, Explaining, Reading, or Other.

Questioning: included asking a mathematical question (MQ), checking a solution or answer (CS, CA), and asking for help either directly as in "How did you do part b) or indirectly as in "I don't get this, man" (AH).

Repeating: Students repeated other's statements (or sometimes their own) to verify it or set them in their own minds (RV), to re-explain or to pass on an explanation (RE), or as a way of questioning an explanation by repeating it with the intonation of a question (RQ).

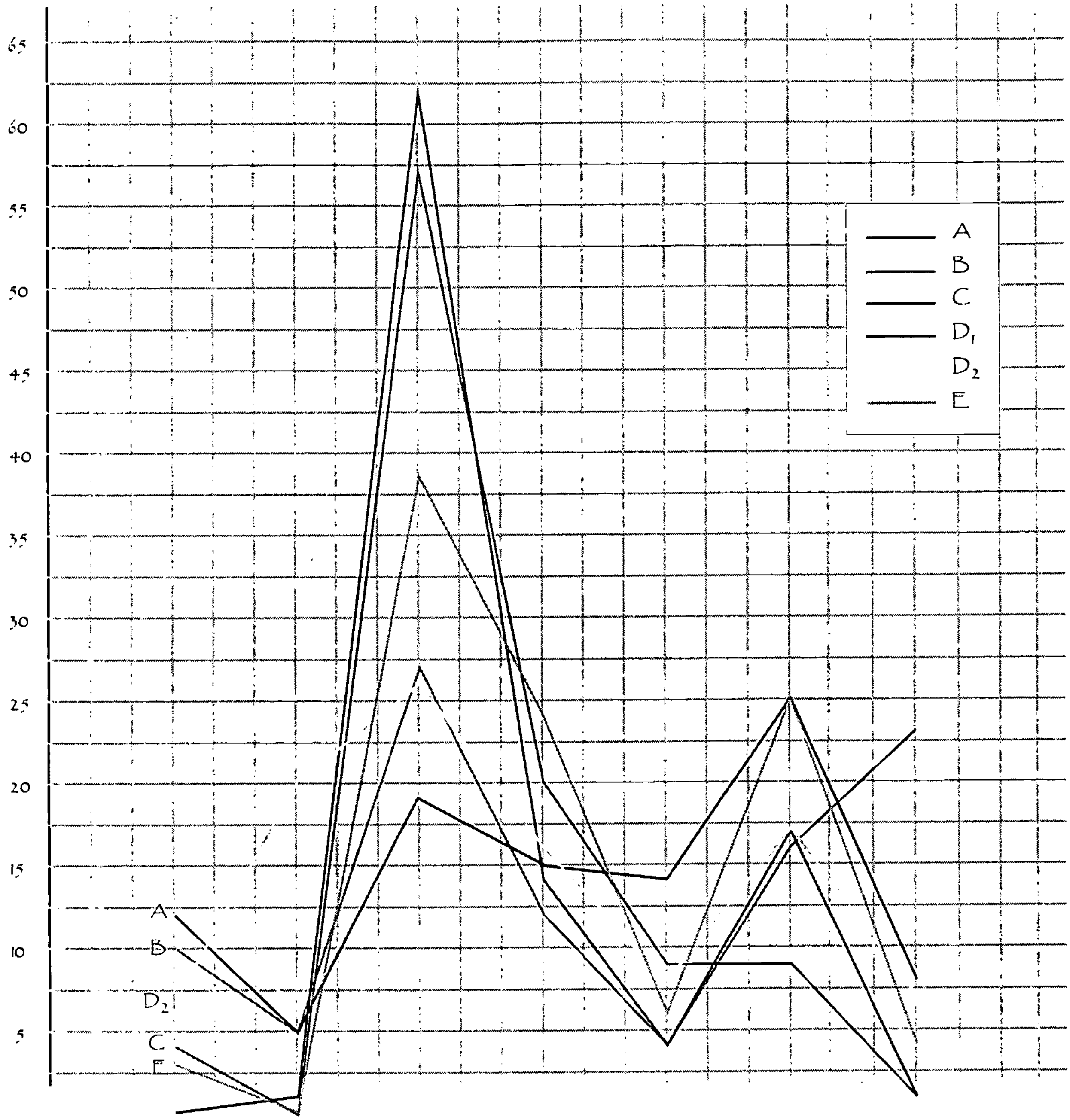
Explaining: included giving a step-by-step explanation (E), giving an answer (GA), pointing out an error (PE), correcting an error (CE), and defending one's reasoning (DR).

Other: The other category includes Reading: meaning reading or paraphrasing the problem to oneself with no response from others (RS), sometimes to announce "I'm going on," or reading the problem as a means of focusing or re-focusing the group (RA). The reading category was initially separate, because Group A had used it fairly often, but it turned out not to be used very much by the other groups. Other also includes such things as irrelevant connectors (IC), such as "Ooopy, doopy, dooo," a noise used in Group B to fill the silence, and smart comments (SC), such as Josh's remarks on his own and other's intelligence.

I started with Group A partly because the group was so focused and businesslike and therefore represented so much of what, as teacher, I thought I wanted to see in students' behavior when they were working together. When I attempted to code the talk in Group B I had to make several changes. The first change was the addition of the Work Display category under Talking while Working, because when Josh announced his results and his group members did not respond, I could not really code that as Working Alone, because that wasn't his intent, nor could I code it as Working Together. Once I had established the Work Display code I returned to Group A and realized that many of the statements I had coded as Working Alone were better described as Work Display. Another category established for Group B and used mostly in Group B was Defending one's Reasoning (DR).

As soon as I started coding Group C, I realized there was a major difference in their working together talk. Whereas much of the Working Together talk for Groups A and B had been socially connecting talk such as, "Wait. You have to solve the equation, Jason." or "Let's do (f) now. 'Cause I want to get this problem done," because their mathematical talk had already been coded as Working Alone or Work Display, for Group C and subsequently D_1, the Working Together talk included much more of their mathematical talk because they did very little, if any, Working Alone or Work Display talk.

The graph shows a profile for each group based on the percentage of on-task turns in each of the categories: Working Alone (WA), Work Display (WD), Working Together (WT), Questioning (Q), Repeating (R), Explaining (E), and Other (O). The profile shows clearly the emphasis on working together in Groups C and D_1. There is also less working alone and work display. It is also interesting to note that the more heterogeneous Group E's profile points often fall between those of the Cooperative groups and those of the Businesslike and Competitive groups.



Advantages and Disadvantages; Explanations and Insights

There are advantages and disadvantages for each of the four styles. In terms of efficiency the Businesslike and Teacher/Student Groups, A and D_2 had the advantage. Both were focused on getting the problems done with the lead student setting the pace. The disadvantage in these groups showed up in the procedural orientation of their explanations. The explanations in Group A were strictly about how to do the problem, for example, Kao Pou's explanation for Ngoan of how to find the sides of the larger similar triangle and Ngoan's explanation for Kou Sio that, "Ten is a one with a point there." Paul's explanations did generalize beyond the particular problem, for example when he explained subtracting a negative number, and Mike had the opportunity to rephrase them, re-apply them and try to make them his own, but they still focused on procedures. Working with Mike in a Teacher/Student relationship was probably as good for Paul as it was for Mike, because Paul had the tendency to rush through the problems and miss some of the details and nuances, as he did with the Broken Telephone Pole and at least one other problem in this assignment. So Mike had the effect of slowing him down and making him think and explain his work. It is true that the problems Groups A and D_2 were doing on these transcripts were fairly routine applications and exercises that required the use of procedures, and that could have affected the totally procedural nature of their explanations.

There were some advantages to the competitive approach of Group B. Apparently the social relationship of off-task banter and rude insults helped to build a relationship in which group members had to defend their mathematical reasoning. Because members of Group B were much more likely to point out other's errors (9 times as compared to at most 2 in other groups), they were also much more likely to have to defend or at least explain themselves (8 times compared to at most 2 in other groups), something that did not happen to any great extent in any of the other groups. In addition to having to make more mathematical arguments Group B, had several group insights, which occurred just as they were about to go off-task. While individuals in the other groups had new insights, there were no other group inspired insights. Could it be that off-task talk is good for something after all? For example, the group gains some insight into the relation of horizontal and vertical scaling to the steepness of the graph, a concept that will be discussed formally later in the year.

Josh: You did it wrong [Josh thinks Jason's graph is wrong because it does not rise as fast as his]

Jason: Leave the mic alone. I did not do it wrong.

Josh: Yes you did.

Jason: No I just, I just, I just did them in different things

David: You do it like this Josh

Jason: So I did five here

David: There. And it won't fall off any more.

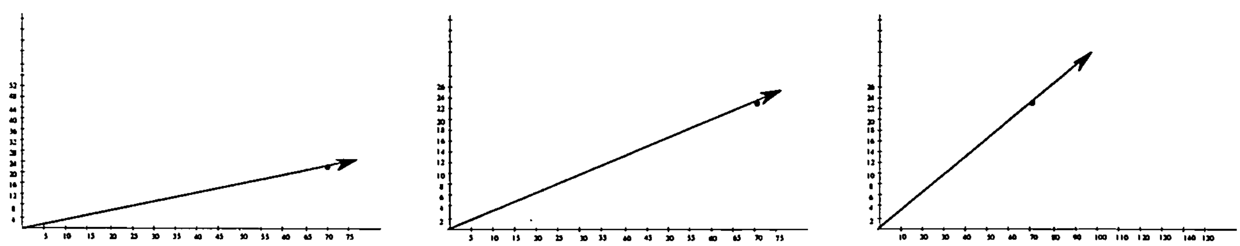
Jason: And I did four for the weight of the wire . .

Josh: No your graph is supposed to go up

David: No, no, no .

Jason: It is goin up. It's goin at a slower pace though . see . yours is by twos. Mine is mine is by four

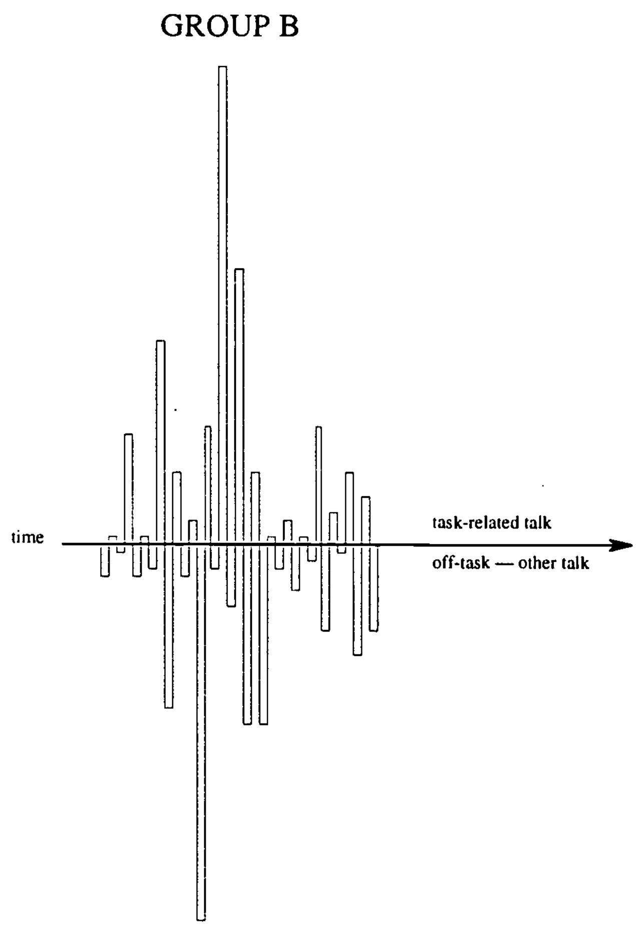
David: Mine's by ten's it goes pweechoa. [because he used tens on the horizontal axis] Imagine if it was by a hundreds. Then it'd go, it'd go phweee

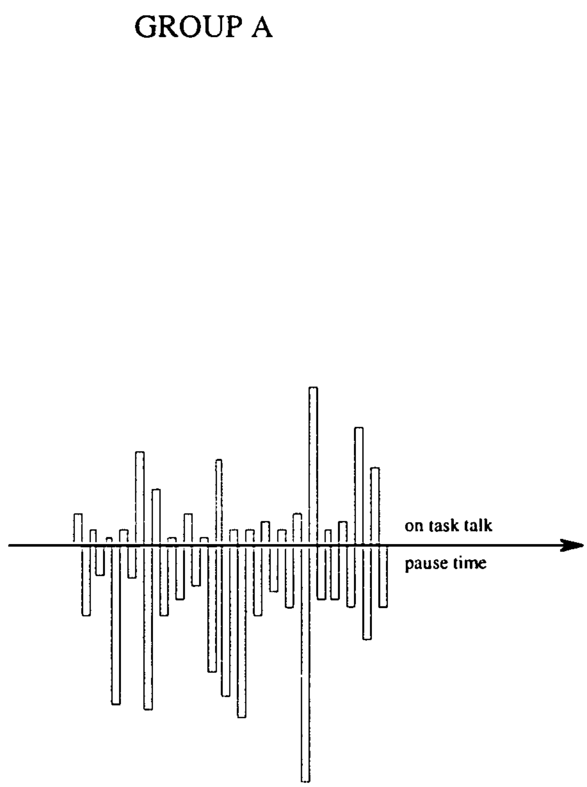


Jason Josh David

Jason is the catalyst, but David takes it a step further, and while he doesn't articulate it well (that or I'm confused by his use of the referent "it"), he's got the general idea. So the whole group, while on the verge of playing around, and still trying to one-up each other (except for Jason who is defending his graph) with clever silliness, develops a notion of the effect of different scalings of the horizontal and vertical axes on the steepness of the graph.

Group B was off-task more than any other group, but they were off-task in a different way from the Cooperative groups. Except for one line when David mentions that his family might be moving so he would be changing schools again, the off-task talk is totally trivia. It's all chatter, nothing that any of the participants would have any investment in. That must make it easy to ignore and non-invasive to the thought process, because time after time, group members announced solutions to problems they had been working on while the conversation continued. It's almost as if the trivia talk serves as a cover for their work, as pauses serve other groups. The diagrams in figure illustrate Group B's on-task/off-task talk and Group A's on-task/pause alternation. There are no pauses of more than two seconds on the Group B tape. The graphs are different sizes because the first is in terms of word count and the second is based on time. The point is the alternation between on and off-task or talking and pausing. The heights are not comparable across graphs.





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On-task percentages and times are in the following chart. In spite of all their off-task talk Group B did problems at about the same rate as Group A[3] and managed to fit as much, actually more mathematics related talk into the same amount of time as the other groups.

Six Groups	Description of Problems	Number and % Turns on Task	% On-task Turns with Teacher	Ratio Time to Turns on Task	Total Time
A. Kao Pou, Ngoan, Kou Sio Jan 5	Routine applications of ratios, writing ratios and equations	150 98%	3%	9.52 secs.	23 min. 48 sec.
B. Josh, Jason, David Jan 3	Develop connections- linear graph, similar triangles, equal ratios	205 54%	4%	6.36 secs.	21 min. 43 sec.
C. Jessica, Veronica, Ngan Apr. 4	Stating reasons for steps in solving equations involving rational expressions	103 56%	14%	9.47 secs.	16 min. 15 sec.
D_1 Hugo, Hector Apr 6	Applications involving the Pythagorean theorem, solving equations.	156 79%	22%	10.25 secs.	26 min. 39 secs.
D_2 Paul, Mike Apr. 6	Applications of the Pythagorean theorem and practice simplifying algebraic expressions	163 96%	10%	9.81 secs.	26 min. 39 secs.
E. Martha, Carmen, Brian, Richard Jan 7	Group test: Using graphs and writing and solving equations involving ratios.	223 94%	16%	10.29 secs.	38 min. 15 secs.

The off-task talk in the cooperative groups was more often about people or issues that were of some importance to the participants, so the participants were generally not doing much mathematics while they were talking. The cooperative groups did not get as many problems done during a given time as the Competitive, Businesslike, or Teacher/Student groups. For example, Hugo and Hector did not complete as many problems as Paul and Mike[4], but they spent more time and talk on each problem. So each member had a chance to talk about the problem several times and work through it from more than his own perspective. The Cooperative groups stayed with problems and checked them until they were convinced they were right. The earlier excerpt with Hugo and Hector is an extreme example.

Talk with the teacher

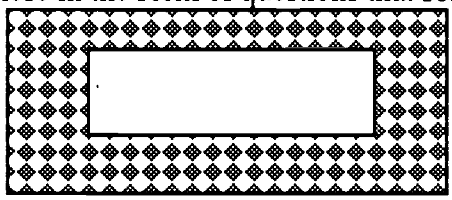
In addition to examining students' use of language in their groups I considered the dialogue between teacher and student as I circulated among the groups responding to questions and requests for help. In addition to talking out-loud to themselves, repeating explanations, explaining to others, repeating solutions steps and calculations, and reading aloud, students used talk with the teacher in some different ways. The examples that follow illustrate some of these additional uses.

[3]The two groups sat next to each other, and I actually checked this by listening to the Group A tape. Group B was noisy enough that I could overhear them working on the same problems as Group A. They kept up the same pace until they got to the very last one which was about VW Bugs, and they started talking about Beetles.

[4]But nobody kept up with Paul when he was on a roll.

Kenya often did not say what she meant in her initial response to a direct question. She'd give fast list of unrelated or incorrect options until she got to the one she wanted. Generally the last one would be the one that worked. In some ways this is just a form of working out loud, but Kenya had a consistent habit of doing this, and it took me a while to realize that in most cases she was not confused, nor was she waiting for me to pick the right one. She was usually quite certain of her choice, but had difficulty explaining why. In the following excerpt she had listed several incorrect possible ways to solve problem GG-37

GG-37. a) The large rectangle below is 20 cm by 50 cm and the small rectangle is 9 cm by 30 cm. What two subproblems do you need to solve to find the area of the shaded region? Write these in the form of questions that someone else will have to answer.



Teacher: .. .because you said a lot of things. I want you to think about what it . should be..

Kenya You subtract one-thousand, . um, two-seventy from one thousand?

Teacher Okay, but **why** would that, why would that work?. . .

Kenya 'Cause that's what it **equals** [a little irritated]. That in the middle

Kou Oh, co, 'cause the whole thing is one thousand and this is, this is um two-thirty, two-thirty and you just take it out.

On another problem about football tickets, adult and student: 105 tickets were sold, some adult and some student.

Teacher Okay, but if you, wait you're guessing fo•rty tickets for adults, right?

Kenya Umhmm.

Teacher So how many student tickets, if she sold a hundred and five?

Kenya Forty times, minus one hundred and five .[high pitch but firm, as in "of course"]

Teacher Okay

Kenya Sixty-five [firmly]

Then a little later:

Teacher Like if this were, like if this were x

Kenya Be less than x, has to be what

Teacher You know it has to be less, but

Kenya But one-oh-five

Teacher One-oh-five what. . . .

Kenya Times x, less, **minus** x [with certainty, and I didn't

Teacher Okay, write that down.

Kenya One-oh-five minus x

<u>Richard</u> found it helpful to verbalize for himself. So he anticipated or talked along with, as well as repeated what the teacher is saying. Often he would insist that I not move on to another group until he had generalized and repeated a sequence of steps he would take to solve a problem in his own words. In the following example he first interjected repetitions of what the teacher was saying as he wrote out the solution of the system of equations, and then later in the problem started to anticipate what I would say. Some where in the middle of the following exchange we switch roles. Carmen also used this approach.

RT-87. Solve each of the following systems for x and y.

d) $x + 2y = 1$
$3x - 2y = -5$

Richard: I did this

Teacher: Oh, I see, I see. This, this equals one, just the one, here, okay. N•o•w on the n•ext step, you were gonna subtract two y from both sides–

Richard: The front?

Teacher: On both sides, minus two y and a minus two y Okay.

Richard: Two y Two y

Teacher: And that leaves **just** x. equals one minus two y. Okay.

Richard: just x equals Okay.

Teacher: N•o•w that's useful because now you can repl•a•ce, **this x** with one minus two y . [(1 - 2y)] . . Okay? . .

Richard: This x, right here, right?

Teacher: Yeah, in the second equation.

three parens, yeah

Richard: three one minus two y

Teacher: And minus two y again, because that's already there. Umkay, equals

Richard: Equals.

Teacher: Right. Now you can solve that.

Richard: This 'd be three, this'd be negaive six ___ an' then minus two y

Teacher: minus

Richard: You put the y's all together an' then you subtract this.

Teacher: Yeah so this way it's thre•e ___ minus ___

Richard: three minus eight times y

Teacher: Okay, and now you just go ahead as usual. Subtract three, right there..........

Breeana often used her own ways of solving and describing her solutions and often would put forth her method as a challenge to what was being suggested by the teacher, the materials, or another student. In this case we were focusing on representing problems with equations involving ratios. Using this approach she could show that she knew how to solve the problem but still get an explanation or direction to another approach. Sometimes she would argue stubbornly for an approach that contained a misconception. She was always interested in short-cuts and tried to do the work in her head rather than write it out. Her confrontative approach worked well for creating and sustaining the discussion and usually, but not always, resolving the issue to everybody's advantage.

Breeana: Could you instead of doing the over-and-equal-thing, could you just go if 100 vitamins cost 1.89 you times that by three and then, then you put it aside and then you split that in half by 50?

Teacher: You could, but you can write an algebraic/

Breeana: Equals $6.62?

Teacher: You can write an algebraic equation which is what you want that helps, so you can write like what, 100 is related to its price.

Breeana: So 100. [Writing 100] is related to [she writes... $\frac{100}{1.89}$]

Teacher: And then 350, equals 350 over its price which we don't know. [She writes x...]
Okay now if you don't like it this way you can turn them both upside down ....
[Breeana does this]
over 350 [following along as Breeana is writing $\frac{1.89}{100} = \frac{x}{350}$.]
And this gives you a way to organize it and solve it as an equation.

Breeana: So you just ... multiply this by that and then put it over 100.

And in conclusion, she succeeds in reducing the recommended approach to a procedure.

David who followed Josh when working in Group B, and who participated in the discussion of the problems with such statements as "Hey, wait how'd you get this?" was much more articulate when talking with the teacher. He was one of the few students with whom I could discuss a problem without having to see his paper. He used the opportunity to talk to the teacher to practice using mathematical language, something that rarely showed up in his group tapes.

David: *Ms. Kysh.* Do we have ta, um, write a algebraic equation down here . like, um, we did those?

Teacher: Right. umhmm.

David: All right, um, what does this . . just use algebra to solve the equation . . is that, is that what/

Teacher: Yeah, then um, then show how to use the algebra to solve the equation you g·o·t.

David: O·hhh like what we were do ing __

Teacher: What we were doing right here. Yuh.

David: All right . . **Okie** doke.

And later the same day.

David: Ms. Kysh, when they said the perimeter of a rectangle, the perimeter is all the sides added up, right?

Teacher: Right.

And a little later.

David: Ms. Kysh, the length is three times the width. This'd be x and this'd be $x \div 3$, right?

Teacher: Yeah, it might be easier if,um, you guess the width first.

David: Oh, all right.

Teacher: Call the width x, and then do the length.

David: All right.

These exchanges were all at a distance. In other words I was working with nearby groups and was not in a position to see David's work. Most students relied on my being able to see their papers when they asked their questions, and I would have had to be standing beside their group.

CONCLUSIONS IMPLICATIONS:

What did I learn?

This section cannot really be called "conclusions" because this is still very much a work in progress.
In relation to the question:

- What can we learn about students' understanding of mathematics based on their talk?

there is a lot more work to be done to show the relationship of their talk as exemplified in the samples in this paper to their understanding of mathematics as demonstrated in their written work and by their oral problem solving a year later.

In relation to the question:

- How do students use talk to build understanding of mathematics?

the group work transcripts were an eye-opener that challenged many of my teacherly expectations. What appeared to be the worst group, often loud and off-task, actually accomplished as much or more work than any other group. Not only did they accomplish as much work, their discussion went farther, because their social behavior involved challenging each other. To add to that, some of their best work was done right at the transition between on and off-task talk. So does off-task talk help? Should it be encouraged? While I would hesitate to recommend encouraging off task talk, I think that it is possible that teachers (and others) may be spending too much time worrying about squelching it. Certainly it should not interfere with the work of others, but a little off-task talk could actually be helpful. The next question then is should we encourage competition? I'm sure there are some who would say yes, but there must be other ways in which we can develop students' abilities to question each other and explain their reasoning. These students had very little, if any, prior school experience working in groups. We needed to spend more time in this class developing communication skills.

While the *Businesslike* group was very efficient in completing problems, they tended to reduce the problems to procedures. In fact, the procedural orientation of all the groups was disappointing, but in hindsight to be expected. While the curriculum is designed to go beyond practice exercises, it had to be designed for use in an Algebra 1 course, so there are still many procedural problems. But the drive to 'get the work done' and to 'cover the expected curriculum' is alive and well in both teacher and students.

The cooperative groups did discuss the mathematics in the problems, but they took a lot longer to get the problems done, checking and double checking and then checking with the teacher. But all of this checking meant doing a lot of mathematical talk, with each group member repeating the solution him or herself. My teacherly inclination wanted them to speed up. That this was not necessarily an appropriate reaction was borne out by two facts. The members of the cooperative groups generally got the problems done, as homework if not in class. And on the group part of the final, which included ten open response word problems, some of which students had had a chance to work on before, Hugo, Hector, Martha, and Carmen correctly completed all ten, four more than any other group.

The teacher/student groups seemed to work well except that the leaders were sometimes too efficient in reducing problems to procedures. Notice that this comment is made by the very person who wanted to speed up the cooperative groups.

Reading the group transcripts gave me some insights into individuals that I would not have expected from my work with them as I moved group to group. For example Josh had much more to offer the group than I would have expected and David much less based on his well articulated questions to the teacher. I was reassured to learn the Carmen, Kou Sio, and Mike, all followers, were doing the work themselves, stopping the people they were working with to ask them to explain what they did not understand. There were very few instances when anyone asked for an answer, and that was usually just to check. In fact Mike, Kou Sio, and Carmen often insisted on a better explanation. Would they have just copied if the tape recorder weren't there? Based on the tapes I don't think so. There was on attempt to copy that was recorded. During the group test Carmen asked someone in the next group for his answer. But the group didn't use the answer they got because Brian pointed out that it was wrong. The group that gave the answer missed the problem. In my work with high school teachers I find that some hesitate to have their students work in groups because they are worried about students copying. At least in these groups copying was less of a problem than some teachers fear.

There is much more to be learned from these transcripts and others like them. While my focus has been on students and their talk, listening the to tapes and reading the transcripts is a very humbling experience, especially for a teacher who finds her experience in a traditional classroom with the institutional demands to cover the curriculum so at odds with what she has learned and is learning about how students learn mathematics.

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