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ABSTRACT

Although geometry and spatial reasoning are important as a way to interpret and reflect on the physical environment and also form the foundation for learning mathematics and other subjects, many early childhood and primary school teachers spend little time instructing their students in these areas. This paper examines how young children learn about space and geometry, discusses how they think about specific concepts in this area, and presents activities and teaching approaches that early childhood educators can use to help them develop. Section 1 of the paper examines how children learn about space and geometry and begins with an examination of Piaget's belief that children have constructed "perceptual space" by infancy but develop ideas about space through action; this is followed by a discussion of children's exploration of shapes by touch, drawing of shapes, and the development of perspective taking. This section also describes levels of geometric thinking--from a holistic, unanalyzed visual beginning through description to an analysis of geometric figures--and discusses the important role of education in this development. Section 2 discusses how children of different ages think about salient mathematical concepts: shape, spatial orientation, and spatial visualization and imagery. Section 3 presents suggestions for instructing young children, including use of manipulatives and pictures, computer manipulatives, and the Agam program to develop the visual language of young children. The paper concludes by noting that it is essential that geometry and spatial sense receive greater attention in instruction and research. (Contains approximately 60 references and 11 figures). (KB)

Geometric and Spatial Thinking in Young Children

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Part II: Mathematics for the Young Child

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Geometric and Spatial Thinking in Young Children

Geometry is the study of space and shape. We study spatial objects such as lines, shapes, and grids; relationships such as "equal in measure" and "parallel"; and transformations such as flips and turns. Spatial reasoning includes building and manipulating mental representations of these objects, relationships, and transformations. For example, we might see in our "mind's eye" what shapes would result from cutting a square from corner to corner.

> Geometry is grasping space...that space in which the child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it (Freudenthal, in National Council of Teachers of Mathematics, 1989, p. 48).

So, geometry and spatial reasoning are important in and of themselves. In addition, they form the foundation of much learning of mathematics and other subjects. Teachers of older students use geometric models for arithmetic when they use grids to illustrate multiplication or circles or bars to illustrate fractions. Unfortunately, however, we too often give geometry short shrift. And this shows up in our children's achievement.

According to extensive evaluations of mathematics learning, elementary students in the United States are failing to learn basic geometric concepts and geometric problem solving. They are under prepared for the study of more sophisticated geometric concepts, especially compared to students from other nations (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980; Fey et al., 1984; Kouba et al., 1988; Stevenson, Lee, & Stigler, 1986; Stigler, Lee, & Stevenson, 1990). For instance, fifth graders from Japan and Taiwan scored more than twice as high as U. S. students on a test of geometry (Stigler et al., 1990). Japanese students in both first and fifth grades also scored much higher (and Taiwanese students only slightly higher) than U. S. students on tests of visualization and paper folding. This may be because Japanese teachers emphasize visual representations for concepts and expect their students to become competent at drawing.

The U.S.'s worst performance on the most recent international comparison was in geometry (TIMSS, National Center for Education Statistics, 1996). Further, geometry showed a smaller-than-average increase, presumably because educators do not emphasize this content (Mullis et al., 1997). Indeed, there are whole districts in which elementary teachers spend virtually no time teaching geometry (Porter, 1989). We could do more, and better, geometry with younger children as well. To help us, we need to understand how children learn about space and geometry, how they think about specific ideas in this area, and what activities and teaching approaches can help them develop.

How Do Children Learn about Space and Geometry?

Piaget's Counter-intuitive View

While Piaget's writings about this topic are long and complex, a few basic findings capture much of what is important for our purposes. Piaget believed that children have constructed "perceptual space" by infancy. However, only much later do they build up *ideas* about space in geometry—what Piaget called "representational space." This is the subject of Piaget and Inhelder's (1967) experiments.

Children's exploration of shapes by touch.. Piaget and Inhelder had children explore hidden shapes by touch and match them to duplicates. Young preschool children initially could discriminate between features such as "closed" or "open." Older children could tell shapes with straight sides from those with curved sides. Only later could they discriminate among shapes such as squares and diamonds. Why would this be so, when children could recognize such figures visually (in perceptual space)? The researchers explain that understanding ideas about shapes (representational space) requires children to coordinate their actions systematically. Younger children, for example, touch one part of a shape only, or perhaps two parts without *relating* the two perceptions. They make decisions based on this limited information. Older children connect one perception to another, building up a complete mental picture of the shape. So, to create *ideas* about shapes, children need to act and connect their actions. Children "can only 'abstract' the idea of such a relation as equality on the basis of an action of equalization, the idea of a straight line from the action of following by hand or eye without changing direction, and the idea of an angle from two intersecting movements" (Piaget & Inhelder, 1967, p. 43).

The main point is that children's ideas about shapes do not come from passive looking. Instead, they come as children's bodies, hands, eyes...and minds...engage in action. In addition, the experiment illustrates that children need to explore shapes extensively to fully understand them. Merely seeing and naming pictures is insufficient. Finally, they have to explore the parts and attributes of shapes.

Children's drawing of shapes.. Making a drawing is an act of representation, not perception, so it also illustrates children's understanding of ideas. Young children's inability to draw or copy even simple shapes again argues that this understanding stems from coordinating their own actions, rather than passive perception. But could this be due simply to motor difficulties? Such difficulties do limit children's drawings. However, Piaget and Inhelder provide many examples that "motor ability" does not explain, such as the child who could draw a pine tree with branches at right angles but could not draw a square with right angles. Also, most children take two years to progress from drawing a (horizontal) square (Fig. 1a) to drawing a rhombus (diamond, Fig. 1b). So, children need far more than a visual "picture."

Again, we see the importance of action and exploration. Children benefit from trying to represent shapes in many ways, from drawing to building specific shapes with sticks or with their bodies.

Children's perspective taking. Piaget also investigated children's understanding of relationships between figures. Instead of only considering shapes in isolation, can they consider a "point of view"? Children perceive straight lines since their earliest years, of course. They can not, however, place objects along a straight path not parallel to the edges of a table. Only at about 7 years of age can they make straight paths by spontaneously 'aiming' or sighting. Similarly, in the "three mountains" task, children constructed a scene from the perspective of a doll. For each new position of the doll around a scene of three mountains, young children re-created the appropriate viewpoint, but it always turned out to be from the same point of view... their own! So, it is not just familiarity or experience, but *connecting* different viewpoints, that develops perspective-taking ability.

This is a critical step along the way to understanding space as adults do. Another step toward developing a full "frame of reference" involves building ideas of horizontal and vertical. For example, children observed jars half-filled with colored water and predicted the spatial orientation of the water level when the jar was tilted. Children first represented the water with a scribble; later, they drew it perpendicular to the sides of the jar, regardless of tilt (Fig. 2). Their satisfaction did not weaken when the researchers placed the water-filled tilted jar next to their drawings! Certainly, this is not merely a perceptual-motor task. Only older children used a larger frame of reference (e.g., tabletop) for drawing the horizontal.

These illustrations are examples of the many types of activities in which children must engage to build a full understanding of space and shape. Children's ideas develop from intuitions grounded in *action*—building, drawing, moving, and perceiving. What types of ideas develop from these intuitions?

The van Hieles' Levels of Geometric Thinking

Pierre and Dina van Hiele say that students' ideas about geometry progress through levels (van Hiele, 1986; van Hiele-Geldof, 1984). From a holistic, unanalyzed visual beginning, they learn to describe, then analyze geometric figures. At the visual level they can only recognize shapes as wholes and can not form mental images of them. A given figure is a rectangle, for example, because "it looks like a door." They do not think about the attributes, or properties, of shapes. At the next, descriptive/analytic level they do recognize and characterize shapes by their properties. For instance, a student might think of a square as a figure that has four equal sides and four right angles. Many students do not reach this level until middle or even high school.

Why do some students proceed so slowly? The van Hieles believe that education is required for progress through these levels. Many mathematics curricula do not help. The little geometry they include is often *all at the earliest level* and does not extend children's thinking beyond that level. Teachers can do much to improve this situation.

So, we should enrich the geometry learning of our students by going beyond typical curriculum materials. Much beyond naming shapes builds students' visual level thinking. They might make shadows and identify shapes in different contexts, all the while describing their experiences. Especially at the early levels, children should manipulate concrete geometric shapes and materials so that they can "work out geometric shapes on their own." They might combine, fold, and create shapes, or copy shapes on geoboards, by drawing, or by tracing.

Children who are ready to explore the next level can investigate the parts and attributes, or properties, of shapes. They might measure, color, fold, or cut to identify properties of figures. For example, children could fold a square to figure out equality of sides or angles or to find symmetry (mirror) lines. They might sort shapes by their attributes (all those with a square corner here) or play "guess my shape" from attribute clues.

For all children, we should understand that the ideas that underlie children's use of simple verbal labels (e.g,. for words like "square" or "triangle") may be vastly different from what we assume.

Shape

Let's examine more of what we know of the origins of shape concepts. What are our most basic images, or visual prototypes, of shapes?

People in a Stone Age culture with no geometric concepts were asked to choose a "best example" of a group of shapes, such as a group of quadrilaterals and near-quadrilaterals (Rosch, 1975). People chose a square and circle more often, even when close variants were in the group. For example, the group with squares included square-like shapes that were not closed, had curved sides, and had non-right angles. It seems that we might have "built-in" preferences for closed, symmetric shapes.

In addition, culture shapes certain preferences. We conducted an extensive examination of materials that teach children about shapes from books, toy stores, teacher supply stores, and catalogs. With few exceptions, these materials introduce children to triangles, rectangles, and squares in rigid ways. Triangles are usually equilateral or isosceles and have horizontal bases. Most rectangles are horizontal, elongated shapes about twice as long as they are wide. No wonder so many children, even throughout elementary school, say that a square turned is "not a square anymore, it's a diamond."

These visual prototypes have strong influence. One kindergartner impressed his teacher saying he knew that a shape (Fig. 3a) was a triangle because it had "three straight lines and three angles." Later, however, she said Fig. 3b was not a triangle.

Teacher: Doesn't it have three straight sides?

Child: Yes.

Teacher: And what else did you say triangles have to have?

Child: Three angles. It has three angles.

Teacher: Good! So...

Child: It's still not a triangle. It's upside down!

So, visual prototypes can rule children's thinking. What should we do? We should ensure that our children experience many different examples of a type of shape. For example, Figure 4a shows a rich variety of triangles that would be sure to generate discussion. We should also show nonexamples that, when compared to similar examples, help focus attention on the critical attributes. For example, the nonexamples in Figure 4b are close to the examples to their left, differing in one attribute.

Specifically what visual prototypes and ideas do preschool children form about common shapes? We recently conducted several studies with hundreds of children, ages 3 to 6 years. In the first study (Clements, Swaminathan, Hannibal, & Sarama, in press), we used the same line drawings we previously used with elementary students for comparison purposes. Children identified circles quite accurately, only a few of the youngest children chose the ellipse and curved shape (Fig. 5). Children identified squares fairly well. Younger children tended to mistakenly choose nonsquare rhombi ("diamonds" such as No. 3 in Fig. 6). They were less accurate recognizing triangles and rectangles, though their averages (e.g., 60% for triangles, Fig. 7) are not remarkably smaller than those of elementary students (64-81% for K-6). Children's visual prototype seems to be of an isosceles triangle. Young children tended to accept "long" parallelograms or right trapezoids (shapes 3, 6, 10, and 14 in Fig. 8) as rectangles. So, children's prototypical image of a rectangle seems to be a four-sided figure with two long parallel sides and "close to" square corners.

In the second study (Hannibal & Clements, 1998), we asked children ages 3 to 6 to sort a variety of *manipulative* forms. We found that certain mathematically irrelevant characteristics affected children's categorizations: skewness, aspect ratio, and, for certain situations, orientation. With these manipulatives, orientation had the least effect. Most children accepted triangles even if their base was not horizontal, although a few protested. Skewness, or lack of symmetry, was more important. Many rejected triangles because "the point on top is not in the middle." For rectangles, on the other hand, many children accepted non-right parallelograms and right trapezoids. Also important was aspect ratio, the ratio of height to base. Children preferred an aspect ratio near one for triangles; that is, about the same height as width. Other forms were "too pointy" or "too flat." Children rejected both triangles and rectangles that were "too skinny" or "not wide enough."

What implications do these findings have? First, a level of geometric thinking exists *before* the visual level. Children who cannot reliably identify circles, triangles, and squares might be considered at a *pre-recognition* level. Their prototypes are just forming. So shapes that are closed and "rounded" are circles; shapes with four near-equal sides with approximately right angles are squares, and four-sided shapes with approximate parallelism of opposite "long" sides are rectangles. As children develop, these prototypes develop. This is educational, not merely maturational, growth. If the examples and nonexamples children experience are rigid, so will be their prototypes. Many children learn to accept only isosceles triangles. Others learn richer concepts, even at a young age. One of the youngest 3-year-olds in our research scored higher than every 6-year-old. Such children have good experiences with shapes, including rich, varied examples and nonexamples and discussions about shapes and their characteristics. Of course, it is always important to get our language "straight." Many of our 4-year-olds stated that they distinguished triangles by "three points and three sides." Half of these children, however, were not sure what a "point" or "side" was! So, early talk can clarify the meanings of such terms.

Then, children can learn to explain why a shape belongs to a certain category -- "It has three straight sides." Eventually, they can internalize such arguments; for example, thinking, "It is a weird, long, triangle, but it has three straight sides!"

This leads to another implication. The "visual" level is not just visual. Appearances usually dominate children's decisions, but they are also learning and sometimes using verbal knowledge. Using such verbal knowledge accurately takes time and can initially appear as a "setback." Children may initially say a square has "four sides the same and four points." Because they have yet to learn about perpendicularity, some accept any rhombus as a square. Their own description convinces them even though they feel conflicted about the "look" of this "new square." Eventually, however, this conflict can be beneficial, as they come to understand more properties of squares.

The findings also imply changes for educational practice. Too often, teachers and curriculum writers assume that students in early childhood classrooms have little or no such knowledge, even of simple shape identification (Thomas, 1982). Obviously, this belief is incorrect; preschool children exhibit working knowledge of shapes. Instruction should build on this knowledge and move beyond it.

Indeed, education should begin early. Shape concepts begin forming in the preschool years and stabilize as early as age 6. So, an ideal period to learn about shapes is between 3 and 6 years of age. We should provide varied examples and nonexamples and help children understand attributes of shapes that are mathematically relevant as well as those (orientation, size) that are not. So, examples of triangles and rectangles should include a wider variety of shapes, including "long," "skinny" and "fat" examples.

Also, children can and should discuss the parts and attributes of shapes. Activities that promote such reflection and discussion include building shapes from components. For example, children might build squares and other polygons with toothpicks and marshmallows. They might also form shapes with their bodies, either singly or with their friends.

We should encourage children to describe why a figure belongs or does not belong to a shape category. Visual (prototype-based) descriptions should, of course, be expected and accepted, but property responses should also be encouraged. They may initially appear spontaneously for shapes with stronger and fewer prototypes (e.g., circle, square). They should be especially encouraged for those shape categories with more possible prototypes, such as triangles. In all cases, the traditional, single-prototype approach must be extended. Books can be found that feature many examples of each shape category. Also, take children on a shape hunt or shape walk, giving special attention to nonprototypical shapes.

Early childhood curricula traditionally introduce shapes in four basic level categories: circle, square, triangle, and rectangle. The idea that a square is not a rectangle is rooted by age five (Clements et al., in press; Hannibal & Clements, 1998). Is it time to re-think our presentation of squares as an isolated set? If we try to teach young children that "squares are rectangles"—especially through direct telling—will we confuse them? If, on the other hand, we continue to teach "squares" and "rectangles" as two separate groups, won't we be blocking children's transition to more flexible categorical thinking? Probably the best approach is to present many examples of squares and rectangles, varying orientation, size, and so forth, including squares as examples of rectangles. If children say "that's a square," you might respond that it is a square that is a special type of rectangle, and you might try double-naming ("it's a square-rectangle"). Older children can discuss "general" categories, such as quadrilaterals and triangles, counting the sides of various figures to choose their category. Also, encourage them to describe why a figure belongs or does not belong to a shape category. Then, you can say that because a triangle has all equal sides, it is a special type of triangle, called an equilateral triangle. They can also "test" right angles on rectangles with a "right angle checker."

We should also teach children about composing and decomposing shapes from other shapes. In our Building Blocks™ software and curriculum development project[1], we give children felt squares, triangles, hexagons, trapezoids and diamond shapes and ask them to form

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[1] National Science Foundation, grant number ESI-9730804, "Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-based Materials Development." Contact the author for additional information about these computer, manipulative, and print curriculum materials.

other shapes from these shapes. We also challenge them to make a larger figure that is the same shape as the original shape. Finally, we challenge them to make squares out of triangles. Children can build shapes with many different shape sets. (We discuss similar transformation activities in the following section.)

Spatial Thinking

Why "Spatial Sense?"

Why do we need to develop children's "spatial sense," especially in mathematics classes? The main reason is that:

> Spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world (National Council of Teachers of Mathematics, 1989, p. 48)

Further, spatial ability and mathematics achievement are related. While we do not fully understand why and how, children who have strong spatial sense do better at mathematics. This relationship, however, is not straightforward. Sometimes, "visual thinking" is "good" but sometimes it is not. For example, many studies have shown that children with specific spatial abilities are more mathematically competent. However, other research indicates that students who process mathematical information by verbal-logical means outperform students who process information visually (for a review, see Clements & Battista, 1992).

Similarly, some imagery in mathematical thinking can cause difficulties. An idea can be *too* closely tied to a single image. For example, connecting the idea of "triangles" to a single image such as an equilateral triangle with a horizontal base restricts young children's thinking.

Spatial ability is important in learning many topics of mathematics. The role it plays, however, is elusive and, even in geometry, complex. Let us sort out what we mean by spatial abilities and spatial sense and then return to the role of spatial sense in mathematical thinking. To have spatial sense you need spatial abilities. Two major abilities are spatial orientation and spatial visualization.

Spatial Orientation: Maps and Navigation

Spatial orientation is knowing where you are and how to get around in the world; that is, understanding and operating on relationships between different positions in space, especially with respect to your own position. Young children learn practical navigation early—as all adults responsible for their care will attest. What, however, can they understand and represent about spatial relationships and navigation? For example, at what age can they use and create maps? When can they build "mental maps" of their surroundings?

While at first, talk of maps may seem developmentally premature, research has shown that even preschoolers are not without abilities. For example, 3-year-olds can build a simple, but meaningful map with landscape toys such as houses, cars, and trees (Blaut & Stea, 1974). Not as certain is what *specific abilities* and *strategies* they are using. For example, kindergarten children making models of their classroom cluster furniture correctly (e.g., they put the furniture for a dramatic play center together), but may not relate the clusters to each other (Siegel & Schadler, 1977). Also, it is unclear what kind of "mental maps" young children possess. Some researchers believe that people first learn to navigate only by noticing landmarks, then by routes, or connected series of landmarks, then by scaled routes, and finally by putting many routes and locations into a kind of "mental map." Only older preschoolers learn scaled routes for familiar paths; that is, they know about the relative distances between landmarks (Anooshian, Pascal, & McCreath, 1984). Even young children, however, can put different locations along a route into some relationship, at least in certain situations. For example, they can point to one location from another even though they never walked a path that connected the two (Uttal & Wellman, 1989).

So, while we know young children have some competencies in navigating and making mental maps, it is less certain what these are. We do know that learning spatial orientation, and eventually understanding maps, is a long-term process. Even the youngest children, however, possess capabilities on which to build. Children slowly develop many different ways to represent the locations of objects in space. Infants associate objects as being near a person such as a parent (Presson & Somerville, 1985), but cannot associate objects to distance landmarks. Toddlers and

3-year-olds can place objects in pre-specified locations near distant landmarks, but "lose" locations that are not specified ahead of time once they move. So, they may be able to form simple frameworks, such as the shape of the arrangement of several objects, that has to include their own location. With no landmarks, even 4-year-olds make mistakes (Huttenlocher & Newcombe, 1984). Kindergartners build local frameworks that are less dependent on their own position. They still rely, however, on relational cues such as being close to a boundary. By third grade, children can use larger, encompassing frameworks that include the observer of the situation.

Finally, neither children nor adults actually have "maps in their heads"—that is, their "mental maps" are not like a mental picture of a paper map. Instead, they are filled with private knowledge and idiosyncrasies and actually consist of many kinds of ideas and processes. These may be organized into several frames of reference. The younger the child, the more loosely linked these representations are. These representations are spatial more than visual. Blind children are aware of spatial relationships by age 2, and by 3 begin to learn about spatial properties of certain visual language (Landau, 1988).

What about physical maps? We have seen that 3-year-olds have some capabilities building simple "maps." There are many individual differences in such abilities. In one study, most preschoolers rebuilt a room better using real furniture than toy models. For some children, however, the difference was slight. Others placed real furniture correctly, but grouped the toy models only around the perimeter. Some children placed the models and real furniture randomly, showing few capabilities (Liben, 1988). Even children with similar mental representations may produce quite different maps due to differences in drawing and map-building skills (Uttal & Wellman, 1989).

Most children can learn *from* maps. For example, 4 to 7 year-olds had to learn a route through a playhouse with six rooms. Children who examined a map beforehand learned a route more quickly than those who did not. As with adults, then, children learn layouts better from maps than from navigation alone. Even preschoolers know that a map represents space. More than 6 or 7-year-olds, however, they have trouble knowing where they are in the space. Therefore, they have difficulty using information available from the map relevant to their own position (Uttal & Wellman, 1989). By the primary grades, most children are able to draw simple sketch-maps of the area around their home from memory. They also can recognize features on aerial photographs and large-scale plans of the same area (Boardman, 1990).

What accounts for differences and age-related changes? Maturation and development are significant. Children need mental processing capacity to update directions and location. The older they get, the more spatial memories they can store and transformations they can perform. Such increase in processing capacity, along with general experience, determines how a space is represented more than the amount of experience with the particular space (Anooshian et al., 1984). Both general development and learning are important.

Though young children possess impressive initial abilities, they have much to learn about maps. For example, preschoolers recognized roads on a map, but suggested that the tennis courts were doors (Liben & Downs, 1989)! In addition, older students are not competent users of maps. School experiences fail to connect map skills with other curriculum areas, such as mathematics (Muir & Cheek, 1986).

Fundamental is the connection of primary to secondary uses of maps (Presson, 1987). Even young children form primary, direct relations to spaces on maps. They must grow in their ability to treat the spatial relations as separate from their immediate environment. These secondary meanings require people to take the perspective of an abstract frame of reference ("as if you were there") that conflicts with the primary meaning. You no longer imagine yourself "inside," but rather must see yourself at a distance, or "outside," the information. Such meanings of maps challenge people into adulthood, especially when the map is not aligned with the part of the world it represents (Uttal & Wellman, 1989). Adults need to connect the abstract and concrete meanings of map symbols. Similarly, many of young children's difficulties do not reflect misunderstanding about space, but the conflict between such concrete and abstract frames of reference. In summary, children (a) develop abilities to build relationships among objects in space, (b) extend the size of that space, and (c) link primary and secondary meanings and uses of spatial information.

These findings re-emphasize that we must be careful how we interpret the phrase "*mental map*." Spatial information may be different when it is garnered from primary and secondary sources...such as maps.

What about the mathematics of maps? Developing children's ability to make and use mental maps is important, and so is developing geometric ideas from experiences with maps. We should go beyond teaching isolated "map skills" and geography to engage in actual mapping, surveying, drawing, and measuring in local environments (Bishop, 1983). Such activities can begin in the early years.

Our goal is for children to both read and make maps meaningfully. In both of these endeavors, four basic questions arise: Direction—which way?, distance—how far?, location—where?, and identification—what objects? To answer these questions, students need to develop a variety of skills

Children must learn to deal with mapping processes of *abstraction*, *generalization*, and *symbolization*. Some map symbols are icons, such as an airplane for an airport, but others are more abstract, such as circles for cities. Children might first build with objects such as model buildings, then draw pictures of the objects' arrangements, then use maps that are "miniaturizations" and those that use abstract symbols. Some symbols may be beneficial even to young children. Over reliance on literal pictures and icons may hinder understanding of maps, leading children to believe, for example, that certain actual roads are red. A teacher might have each child pick some object in the room, and denote its location with an "X" on their maps. Children could exchange maps, trying to identify the mystery object and thereby test the usefulness of the map (Downs, Liben, & Daggs, 1988).

Related are the ideas of boundaries to create two regions, one inside and one outside a curve. A U.S. map uses closed curves as boundaries for states. Ask children if they ever "marked off" a region for their play, as in a sandbox.

As children work with model buildings or blocks, give them experience with *perspective*. For example, they might identify block structures from various viewpoints, matching views of the same structure that are portrayed from different perspectives, or try to find the viewpoint from which a photograph was taken. Such experiences address such confusions of perspective as preschoolers "seeing" windows and doors of buildings in vertical aerial photographs (Downs & Liben, 1988).

Similarly, children need to develop more sophisticated ideas about *direction*.. Young children should master environmental directions, such as above, over, and behind. They should develop navigation ideas, such as left, right, and front, and global directions such as north, east, west, and south, from these beginnings. Such ideas, along with *distance* and *measurement* ideas, might be developed as children build and read maps of their own environments. For example, children might mark a path from a table to the wastebasket with masking tape, emphasizing its continuity. With the teacher, children could draw a map of this path (some teachers take photographs of the wastebasket and door and glue these to a large sheet of paper). Items appearing alongside the path, such as a table or easel, can be added to the map.

Perspective and direction are particularly important regarding the alignment of the map with the world. Some children of any age will find it difficult to use a map that is not so aligned. Teachers should introduce such situations gradually and perhaps only when necessary.

A final mathematical idea is that of *location*.. Children might use cutout shapes of a tree, swing set, and sandbox in the playground and lay them out on a felt board as a simple map. They can discuss how moving an item in the schoolyard, such as a table, would change the map of the yard. On the map, locate children shown sitting in or near the tree, swing set, and sandbox. Plan scavenger hunts on the playground, in which students give and follow directions or clues.

As we have seen, young children learn to relate various reference frames. Can they also use traditional mathematical ideas such as *coordinates*? Again, we see there is a long developmental process, but definite early competencies on which to build. Regarding the former, intermediate grade students still struggle with organizing 2D space (Clements et al., 1998). They need further experiences structuring and working with two-dimensional grids to develop precise working concepts of grids, grid lines, points, and the overall structure of order and distance relationships in a coordinate grid. On the other hand, 3-year-olds can extrapolate, or extend, lines from each axis of a simple grid. Between 4 and 6 years most children learn to extrapolate lines from positions on both axes and determine where they intersect (Somerville, Bryant, Mazzocco, & Johnson, 1987). Some 4-year-olds can use a coordinate reference system, whereas most 6-year-olds can (Blades & Spencer, 1989). Therefore, even young children can use coordinates that adults provide for them. However, when facing traditional tasks, they and their older peers may not yet be able or predisposed to spontaneously make and use coordinates for themselves.

Computer activities can facilitate learning of navigational and map skills. Young children can abstract and generalize directions and measurement working with the Logo turtle (Clements, Battista, Sarama, Swaminathan, & McMillen, 1997; Clements & Meredith, 1994; Goodrow, Clements, Battista, Sarama, & Akers, 1997). Giving the turtle directions such as forward 10 steps, right turn, forward 5 steps, they learn orientation, direction, perspective, and measurement concepts. Walking paths and then recreating those paths on the computer help them abstract, generalize, and symbolize their experiences navigating. For example, one kindergartner abstracted the geometric notion of "path" saying, "A path is like the trail a bug leaves after it walks through purple paint." A first grader explained how he turned the turtle 45°: "I went 5, 10, 15, 20...45! [rotating her hand as she counted]. It's like a car speedometer. You go up by fives!" (Clements & Battista, 1991). This child is mathematizing turning. She is applying a unit to an act of turning and using her counting abilities to determine a measurement.

Coordinate-based games on computers can help older children learning location ideas (Clements et al., 1998). When children enter a coordinate to move an object but it goes to a different location, the feedback is natural, meaningful, nonevaluative, and particularly helpful.

Spatial Visualization and Imagery

Spatial visualization is understanding and performing imagined movements of two- and three-dimensional objects. To do this, you need to be able to create a mental image and manipulate it. An image is not a "picture in the head." It is more abstract, more malleable, and less crisp than a picture. It is often segmented into parts. As we saw, some images can cause difficulties, especially if they are too inflexible, vague, or filled with irrelevant details.

People's first images are static. They can be mentally re-created, and even examined, but not transformed. For example, you might attempt to think of a group of people around a table. In contrast, dynamic images can be transformed. For example, you might mentally "move" the image of one shape (such as a book) to another place (such as a bookcase, to see if it will fit). In mathematics, you might mentally move (slide) and rotate an image of one shape to compare that shape to another one. Piaget argued that most children cannot perform full dynamic motions of images until the primary grades. However, preschool children show initial transformational abilities.

How can we build imagery for young children? Manipulative work with shapes, such as tangrams, pattern blocks, and other shape sets, provides a valuable foundation. After such explorations, engage children in puzzles in which they see only the outline of several pieces (Fig. 9a). Have them find ways to fill in that outline with their own set of tangrams (Fig. 9b). Even more challenging to spatial visualization and imagery are "quick image" activities (Clements, in press; Yackel & Wheatley, 1990). Children briefly see a simple configuration on the overhead, then try to reproduce it. The configuration is shown again for a couple of seconds as many times as necessary. Older children can be shown a line drawing and try to draw it themselves (Yackel & Wheatley, 1990). This often creates interesting discussions revolving around "what I saw." In Fig. 10, different children see three triangles, "a sailboat sinking," a square with two lines through it, a "y in a box," and even 7 different lines. Having children use many different media to represent their memories and ideas with the "hundred languages of children" (Edwards, Gandini, & Forman, 1993) will help them build spatial visualization and imagery.

Spatial Sense Revisited

Spatial sense includes two main spatial abilities: spatial orientation and spatial visualization and imagery. Other important knowledge includes how to represent ideas in drawing and how and when to use such abilities.

This view clears up some confusion regarding the role of spatial sense in mathematics thinking. "Visual thinking" and "visual strategies" are not the same as spatial sense. Spatial sense as we describe it—all the abilities we use in "making our way" in the spatial sphere—is related to mathematical competencies (Brown & Wheatley, 1989; Clements & Battista, 1992; Fennema & Carpenter, 1981; Wheatley, Brown, & Solano, 1994).

Visual thinking, as in the first van Hiele level of geometric thinking, is thinking that is tied down to limited, surface-level, visual ideas. Children move beyond that kind of visual thinking as they learn to manipulate dynamic images, as they enrich their store of images for shapes, and as they connect their spatial knowledge to verbal, analytic knowledge. Teachers might encourage children to describe why a shape does or does not belong to a shape category.

Instructional Materials

Manipulatives and Pictures

Research supports the use of manipulatives in developing geometric and spatial thinking in young children (Clements & McMillen, 1996). Using a greater variety of manipulatives is beneficial (Greabell, 1978). Such tactile-kinesthetic experiences as body movement and manipulating geometric solids help young children learn geometric concepts (Gerhardt, 1973; Prigge, 1978). Children also fare better with solid cutouts than printed forms, the former encouraging the use of more senses (Stevenson & McBee, 1958).

If manipulatives are accepted as important, what of pictures? Pictures can be important; even children as young as 5 or 6 years (but not younger) can use information in pictures to build a pyramid, for example (Murphy & Wood, 1981). Thus, pictures can give students an immediate, intuitive grasp of certain geometric ideas. However, pictures need to be sufficiently varied so that students do not form limited ideas. However, research indicates that it is rare for pictures to be superior to manipulatives. In fact, in some cases, pictures may not differ in effectiveness from instruction with symbols (Sowell, 1989). But the reason may not lie in the "nonconcrete" nature of the pictures as much as it lies in their "nonmanipulability"—that is, that children cannot act on them as flexibly and extensively. Research shows that manipulatives on computers can have real benefit.

Computer Manipulatives

Instructional aids help because they are manipulatable and meaningful. Therefore, computers can provide representations that are just as real and helpful to young children as physical manipulatives. In fact, they may have specific advantages (Clements & McMillen, 1996). For example, some computer manipulatives offer more flexibility than their noncomputer counterparts. Elastic Lines (Harvey, McHugh, & McGlathery, 1989) allows the student to instantly change both the size (i.e., number of pegs per row) and the shape of a computer generated geoboard. Children and teachers can save and later retrieve any arrangement of computer manipulatives. Similarly, computers allow us to store more than static configurations. They can record and replay sequences of our actions on manipulatives. This helps young children form dynamic images.

You can do things on computers that you can not do with physical manipulatives. For example, you could have the computer automatically draw shapes symmetrical to anything you draw. Or, you could use a computer manipulative that allows you to perform new actions with shapes. In one activity in our Building Blocks™ curriculum for preschool to grade 2, children fill in puzzle outlines using an extended set of pattern blocks. Here, a child made a combination of 2 green triangles by gluing, then duplicated this unit to fill the outline (see Fig. 11). That is psychologically different from covering it with 20 separate triangles. For a challenge, they find a way to use the fewest blocks to fill the outline. (Note that you can also choose to glue two triangles and create a blue rhombus.)

Computers can help children become aware of, and mathematize, their actions. For example, very young children can move puzzle pieces into place, but they do not think about their actions. Using the computer, however, helps children become aware of and describe these motions (Clements & Battista, 1991; Johnson-Gentile, Clements, & Battista, 1994). In another Building Blocks™ activity, children are challenged to build a picture with physical shapes and copy it onto the computer, requiring the use of specific tools for geometric motions.

The Agam Program

An artist and educational researcher created the Agam program to develop the "visual language" of children ages 3 to 7 years (Eylon & Rosenfeld, 1990). The activities begin by building a visual alphabet. For example, the activities introduce horizontal lines in isolation. Then, they teach relations, such as parallel lines. In the same way, teachers introduce circles, then concentric circles, and then a horizontal line intersecting a circle. The curriculum also develops verbal language, but always following a visual introduction. Combination rules involving the visual alphabet and ideas such as large, medium, and small, generate complex figures. As words combine to make sentences, the elements of the visual alphabet combine to form complex patterns and symmetric forms.

The approach is structured. Instruction proceeds from passive identification to memory to active discovery, first in simple form (e.g., looking for plastic circles hidden by the teacher), then in tasks that require visual analysis (e.g., finding circles in picture books). Only then does the teacher present tasks requiring reproduction of combinations from memory. The curriculum repeats these ideas in a large number of activities featuring multiple modes of representation, such as bodily activity, group activity, and auditory perception.

The results of using the program, especially for several consecutive years, are positive. Children gain in geometric and spatial skills and show pronounced benefits in the areas of arithmetic and writing readiness (Razel & Eylon, 1990). These results support systematic long-term instruction in the domain of geometry and spatial thinking in early childhood (Razel & Eylon, 1990). Children are better prepared for all school tasks when they gain the thinking tools and representational competence of geometric and spatial sense.

Summary and Conclusions

Without doubt, geometry is important for several reasons. It offers us a way to interpret and reflect on our physical environment. It can serve as a tool for the study of other topics in mathematics and science. As important is spatial thinking, which supports geometry and creative thought in all mathematics. Given their importance, it is essential that geometry and spatial sense receive greater attention in instruction and in research. Unfortunately, U.S. children's performance in geometry and spatial reasoning is woefully lacking. Educators need to understand more about learning and teaching geometry and spatial sense.

Educational and psychological research has shown us that children build ideas about shapes from action, rather than merely passive viewing. Children need to explore shapes fully, including their parts, attributes, and transformations. They need to represent them in drawings, buildings, dramatizations, and verbal language. The shapes they experience should include rich, varied examples and nonexamples of shape categories. Activities based on this research are appropriate and beneficial for preschoolers.

Though children learn about maps over many years, even preschoolers possess initial abilities in spatial orientation, including navigation, map reading, and map making. Working from physical materials and models to two-dimensional maps, including computer representations, can help children with the mapping processes of abstraction, generalization, and symbolization, as well as with ideas such as perspective, direction, measurement, and location. Likewise, young children can develop spatial visualization abilities as they work to develop images of two- and three-dimensional objects and learn to transform these images. Spatial orientation and visualization are critical components of spatial sense.

Children effectively learn about space and shape through active engagement with manipulatives, drawings, and computers. A growing number of educational programs are available as teaching resources.

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Figure 1

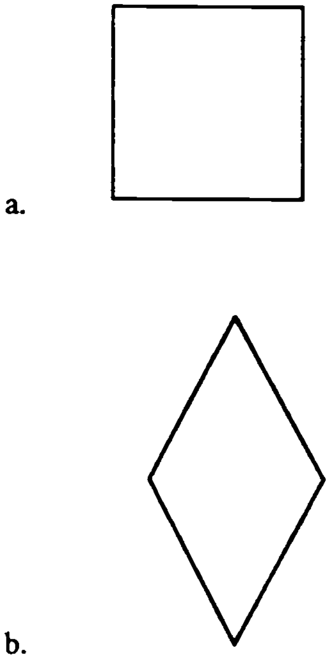
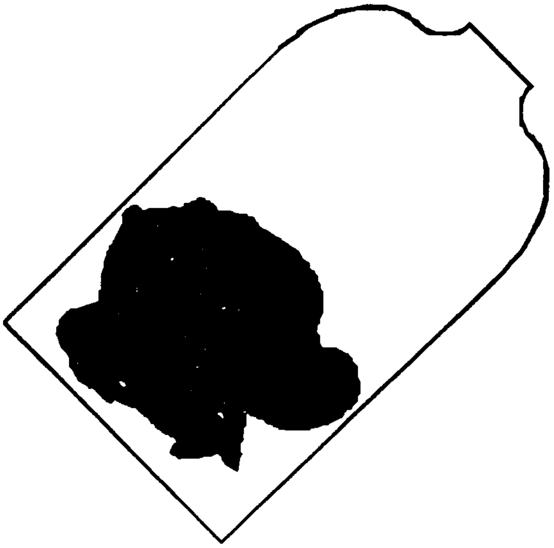


Figure 2 (a) Up to 4 or 5 years of age, children represent water in a jar with scribbles. (b) from 4 to 6 years, they represent the water as a surface perpendicular to the sides of the jar.

a.



b.

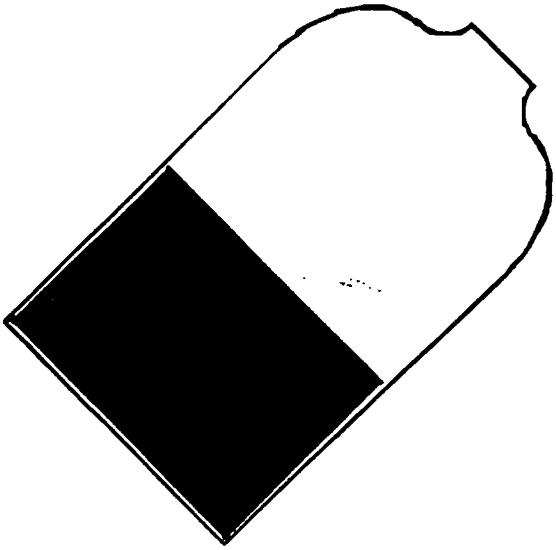


Figure 3

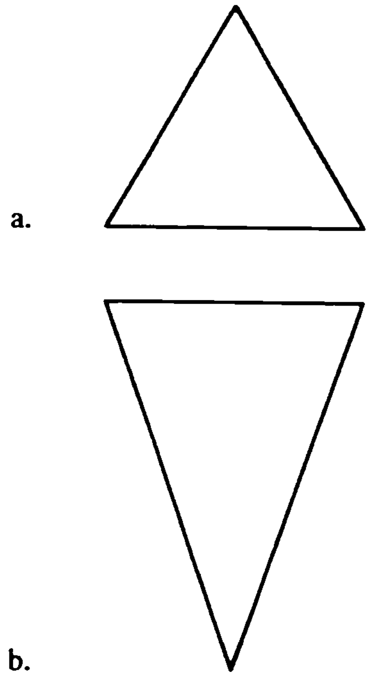
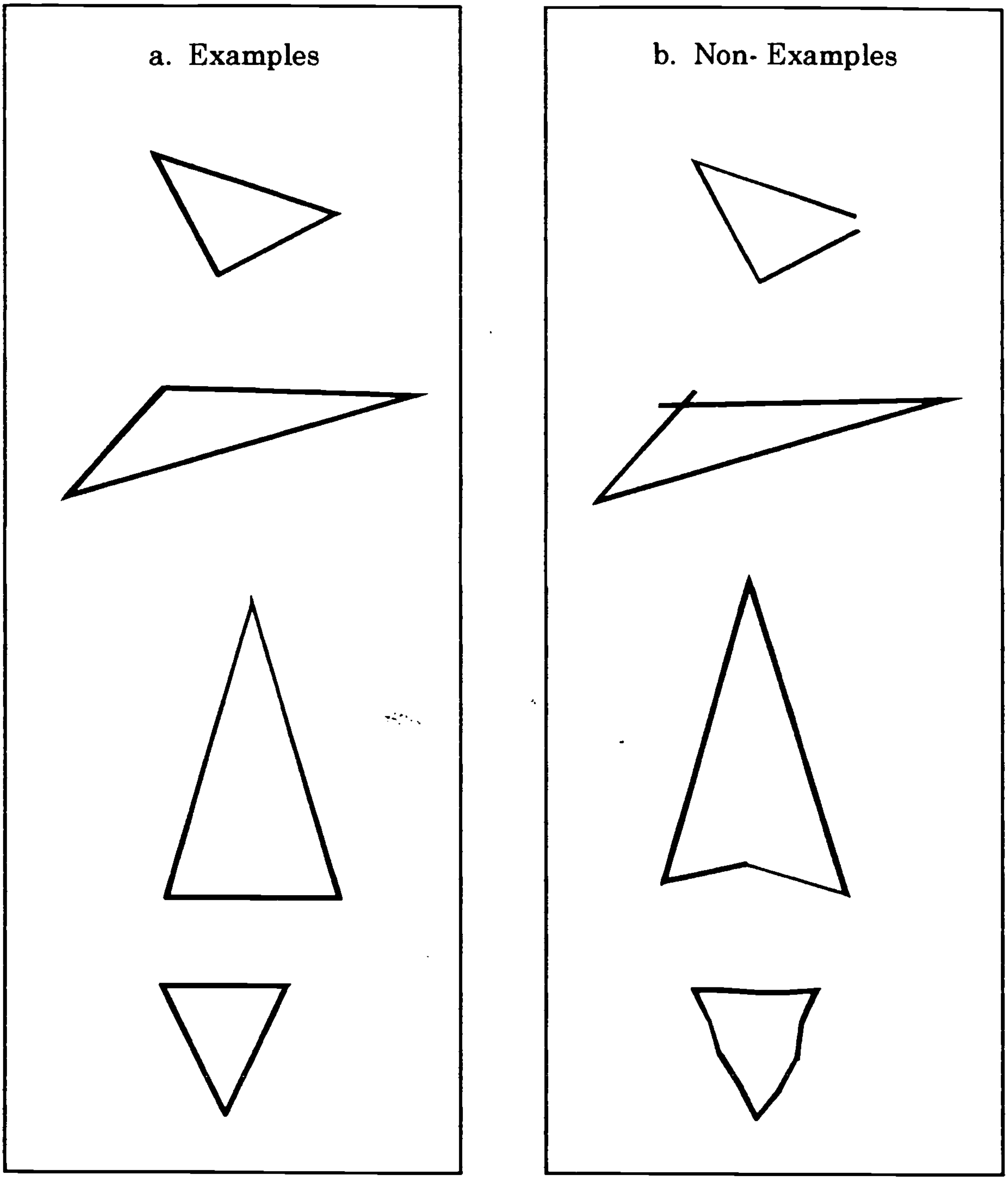


Figure 4

Triangles



a. Examples

b. Non- Examples

Fig. 5. Student marks circles. Adapted from (Razel & Eylon, 1991)

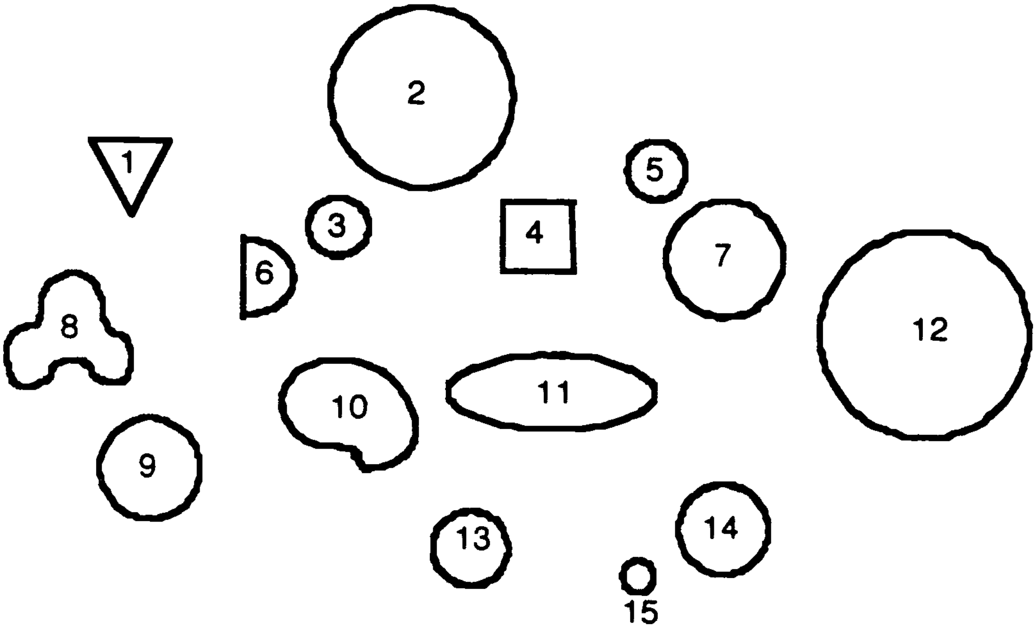


Fig. 6. Student marks squares. Adapted from (Razel & Eylon, 1991)

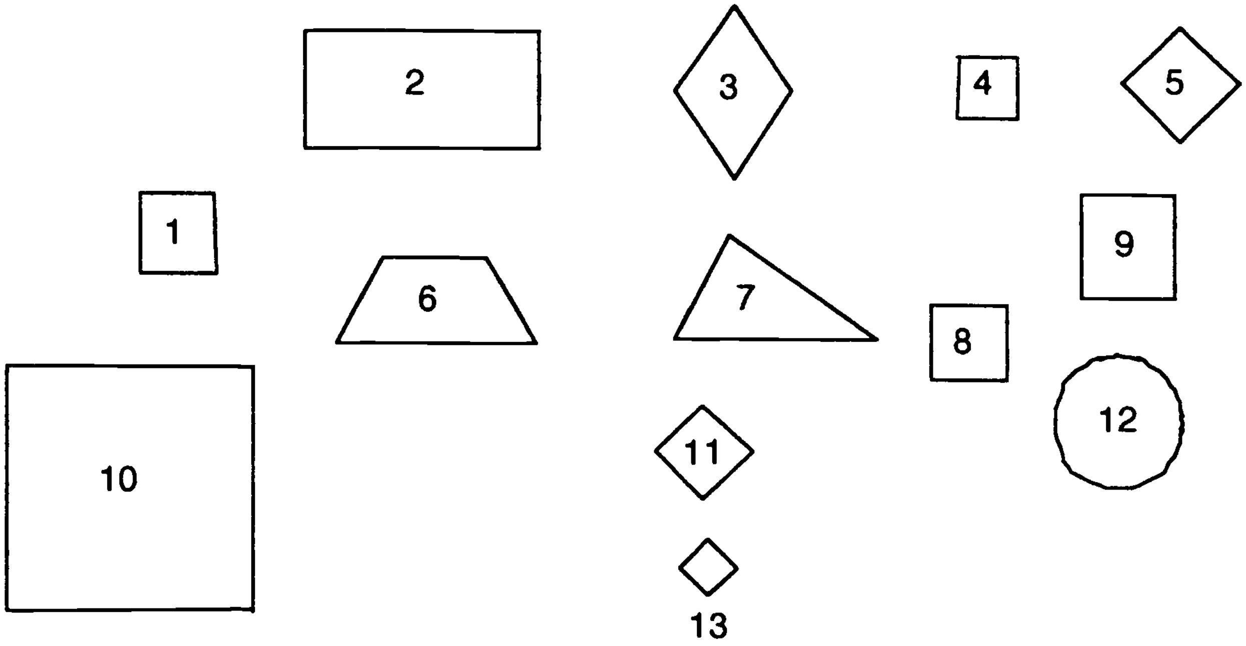


Fig. 7. Student marks triangles. Adapted from (Burger & Shaughnessy, 1986) and (Clements & Battista, 1991)

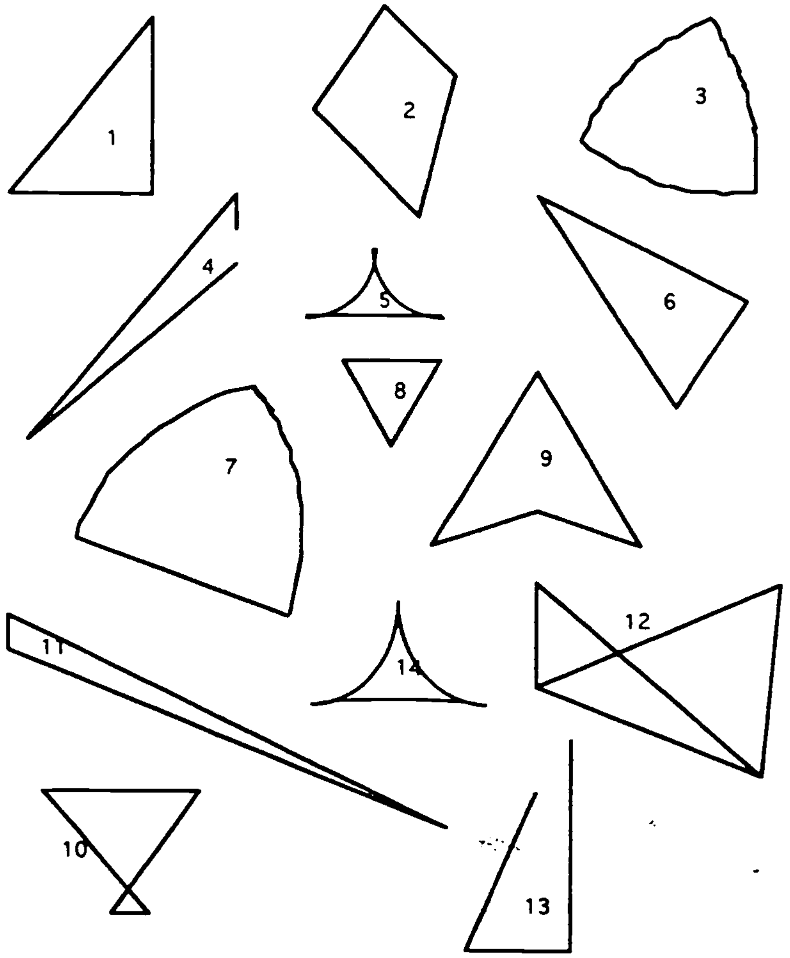


Fig. 8. Student marks rectangles. Adapted from (Burger & Shaughnessy, 1986) and (Clements & Battista, 1991)

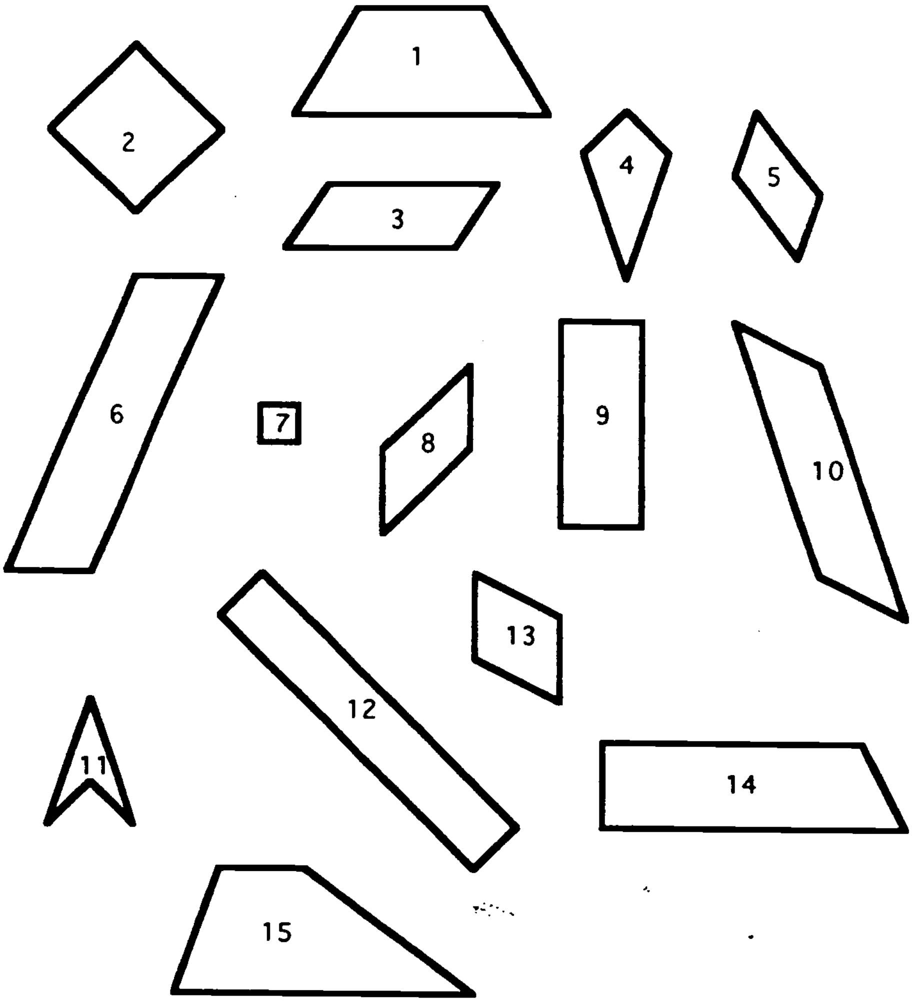


Fig. 9

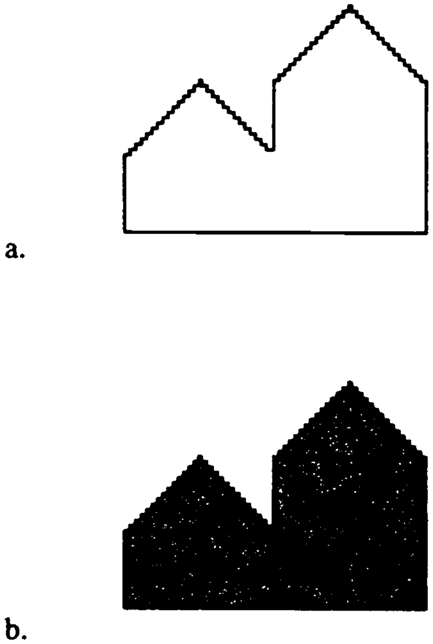


Fig. 10

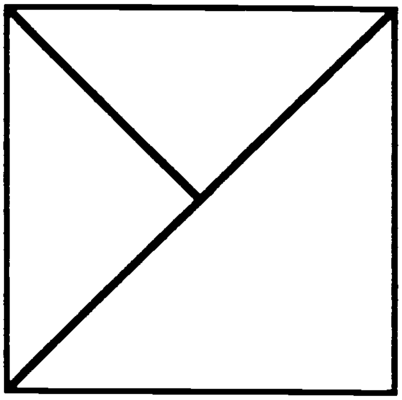
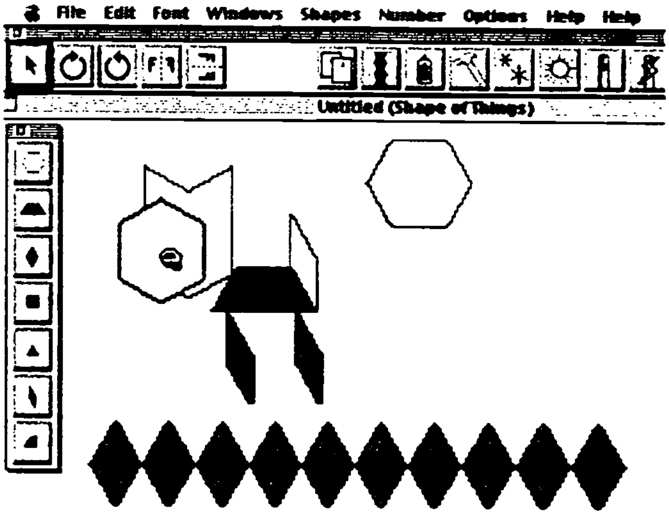


Fig. 11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