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ABSTRACT

The Colorado Student Assessment Program (CSAP) has been underway since 1997. Since then, fourth grade students have been assessed annually on the Colorado Model Content Standards for Reading and Writing. In 1998, testing began on third grade reading. Over the course of time, other grades and subject areas will be phased in. In the fall of 1999, public school fifth grade students will be assessed for the first time on the Colorado Model Content Standards for Mathematics. This guide was developed to help educators prepare for the upcoming mathematics assessment. The purpose of this book is to provide a window into fifth grade mathematics assessment. This guide is divided into four sections: (1) an overview of CSAP with a question-and-answer section about test development, scoring, and reporting; (2) the "Colorado Model Content Standards for Mathematics Kindergarten to Fourth Grade Assessment Framework"; (3) referenced tests (sample assessment items) for the "Kindergarten to Fourth Grade Assessment Framework"; and (4) three sample constructed-response tasks (one 5-minute and two 15-minute tasks) with accompanying scoring guides and student work. (ASK)

Teachers' Guide to the Colorado Student Assessment Program for Fifth Grade Mathematics: An Assessment of Kindergarten through Fourth Grade Benchmarks

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An Assessment of Kindergarten Through Fourth Grade Benchmarks

Table of Contents

Special thanks to Deborah Lynch for composing the text of Section I, and to Henry Heikkinen, Vonda Kiplinger, Stevi Quate and Don Watson for their careful editing and constructive comments.

Introduction

The Colorado Student Assessment Program (CSAP) has been underway for two years. Since 1997, fourth graders have been assessed annually on the *Colorado Model Content Standards for Reading and Writing*. In 1998, testing began on third grade reading. Over the next three years, other grades and subject areas will be phased in.

In the fall of 1999, public school fifth graders will be assessed for the first time on the Colorado *Model Content Standards for Mathematics*. The *Teachers' Guide to the Colorado Student Assessment Program for Fifth Grade Mathematics: An Assessment of Kindergarten through Fourth Grade Benchmarks* was developed to help educators prepare for the upcoming mathematics assessment.

This guide is divided into four sections:

(1) an overview of CSAP with a question-and-answer section about test development, scoring, and reporting;
(2) the *Colorado Model Content Standards for Mathematics Kindergarten to Fourth Grade Assessment Framework*;
(3) referenced tasks (sample assessment items) for the *Kindergarten to Fourth Grade Assessment Framework*; and
(4) three sample constructed-response tasks (two, five-minute tasks and one, fifteen-minute task) with accompanying scoring guides and student work.

The sample assessment items are similar to those that may appear in the CSAP for fifth grade mathematics and are provided for illustrative purposes only.

It is natural for teachers and administrators to be anxious about a high profile assessment. The purpose of this booklet is to ease those concerns and provide a window into the fifth grade mathematics assessment.

You are encouraged to share this guide with your colleagues. Feel free to copy any or all parts of the document.

All About CSAP

The Colorado Student Assessment Program (CSAP) has been underway for two years. Since 1997, fourth graders have been assessed annually on the Colorado *Model Content Standards for Reading and Writing*. In 1998, testing began on third grade reading. Over the next three years, other grades and subject areas will be phased in, including the assessment of fifth grade mathematics starting in the fall of 1999.

CSAP Assessment Schedule

1997	4th grade Reading Writing				
1998	4th grade Reading Writing	3rd grade Reading			
1999	4th grade Reading Writing	3rd grade Reading	7th grade Reading Writing **5th grade Mathematics***		
2000	4th grade Reading Writing	3rd grade Reading	7th grade Reading Writing **5th grade Mathematics***	8th grade Mathematics Science	
2001	4th grade Reading Writing	3rd grade Reading	7th grade Reading Writing **5th grade Mathematics***	8th grade Mathematics Science	10th grade Reading Writing Mathematics

*All testing will take place in the spring with the exception of the fifth grade mathematics assessment which will take place in the fall.

CSAP measures students' knowledge of the state model content standards and benchmarks. Benchmarks in the state standards are grade-level expectations, specifying what students should know by the end of fourth, eighth, and twelfth grades. The statewide assessments are cumulative exams, based on what students have learned up to the time of the assessment. **Therefore, the fall fifth grade mathematics CSAP will assess students on the kindergarten through fourth grade benchmarks.**

As educators know, there are many different levels and forms of assessment. The most complete form of assessment takes place in the classroom. Teachers assess students on a daily and weekly basis. Most school districts also regularly assess students in certain grade levels. CSAP is the first large-scale assessment used in Colorado to measure student achievement relative to the state model content standards. Thus, results will yield information about how well schools and students are performing statewide, using a common yardstick — the state content standards.

The following are questions and answers about test development, scoring, and reporting of results. For more information about CSAP, refer to the Colorado Department of Education's web page at www.cde.state.co.us or contact your district assessment director.

Q. What is the purpose of CSAP?

A. The Colorado Department of Education (CDE) is administering CSAP to give parents, the public, and educators a uniform source of information about how well students and schools are achieving. Colorado schools have been engaged in efforts to improve education for all students through standards-based education for the past decade. CSAP is one measure of students' performance relative to the content standards.

Q. What is CSAP based on?

A. CSAP is based on the Colorado Model Content Standards developed by Colorado educators and community members. These standards were reviewed by over 10,000 Coloradans. Though curriculum varies from district to district, expectations of what students are learning are consistent across the state, allowing for a fair and uniform assessment of Colorado students.

Q. How is CSAP developed?

A. CSAP is developed by a team of Colorado educators, curriculum and assessment experts, and the test publisher, CTB/McGraw-Hill. The process includes selecting and developing assessment questions (or items), setting scoring criteria, and establishing performance levels. Two types of assessment items will be used: selected-response and constructed-response. Selected-response items (e.g., multiple-choice, matching, true-false) are items in which the student chooses an answer from among a list of options. Constructed-response items (e.g., short answer, essay) are items in which the student gives a response in his or her own words.

Q. How are assessment items selected?

A. Approximately a year before the administration of a particular CSAP assessment, the test publisher meets with a group of Colorado educators to develop specifications for test development. These specifications include types of items (e.g., multiple-choice, short answer) and a thorough discussion of the meaning and content of the standards and benchmarks.

Approximately seven months before the administration of the assessment, a variety of assessment items, which align with the Colorado standards, are reviewed by Colorado educators to ensure their alignment with standards and corresponding benchmarks. Some items are revised or discarded and new items are either written or obtained from other sources.

Once this initial pool of items is agreed upon, four committees review each assessment item. The content committee, composed of teachers and assessment and curriculum specialists, reviews each item for grade and content appropriateness. Two other committees, composed of educators, parents, community members, and businesspersons, review each question to ensure it is fair, acceptable, and unbiased. The instructional impact committee, another group of educators, reviews all recommendations and then makes a final decision regarding each item. Only if a question passes these screenings is it included in the pool of items from which the assessment will be constructed.

Q. How are assessments scored?

A. Selected-response items are machine-scored. Constructed-response tasks are scored by hand using rubrics (scoring guides) that match the specific task requirements. Hand scoring the constructed-response tasks is a lengthy and intricate process. Generic rubrics outlining general expectations for satisfactory student work will be provided to schools in the demonstration booklets and CSAP updates (available on the CDE web site).

After the assessment is administered, the test publisher selects student papers for each performance item that represent a range of quality from excellent to poor. These "anchor" papers are scored, then given to Colorado educators. The educators evaluate the anchor papers to determine whether the publisher's judgements are in agreement with Colorado's expectations. These papers are then used to train scorers.

Q. Who scores constructed-response items?

A. The scorers are highly-trained individuals employed by CTB/McGraw-Hill. They use the rubrics and the identified anchor papers to score each constructed-response item. A sampling of each scorer's rated student papers is evaluated to ensure constructed-response items are scored reliably and accurately. This evaluation process includes three different types of reviews to ensure that each scorer's ratings are accurate and consistent. Any scorer who produces ratings that are inconsistent with expectations is retrained.

Q. How are performance levels determined?

A. The setting of performance levels begins after the student papers are scored. First, statisticians determine the degree of difficulty for each question and examine the range of scores. The degree of difficulty for each test item is based on students' performance. Questions that a majority of students answered correctly are identified as less difficult, while those questions that only a few students answered correctly are identified as more difficult. Then, items are ordered by degree of difficulty from least to most difficult, creating the Ordered-Item Booklet.

Next, each member of a group of Colorado educators and community representatives takes the student test and reviews the Ordered-Item Booklet. Since the Ordered-Item Booklet arranges test items from least to most difficult, each group member is asked to place "bookmarks" at points he or she considers student performance that is advanced, proficient, and partially proficient. As a group, members then discuss their individual placement of the bookmarks. After two rounds of discussion, individuals may choose to revise their bookmark placements.

The last round of bookmarking finalizes performance levels. The group writes descriptions of these performance levels based on the student skills and knowledge required at each level.

Q. What do performance levels tell us?

A. The four performance levels — Advanced, Proficient, Partially Proficient, and Unsatisfactory — describe what a student knows and is able to do at each level, and provide information to the school and district about the effectiveness of instructional programs. A student at the proficient level has achieved the knowledge and skills defined in the standards.

Q. How are results reported?

A. Assessment results are reported by four performance levels: Advanced, Proficient, Partially Proficient, and Unsatisfactory. Results are reported by performance levels for individual students, schools, districts, and the state. Each student will receive an overall performance level rating on the mathematics assessment, as well as an indication of performance on each mathematics content standard. Individual student scores are released only to the school and school district, which are encouraged to share this information with the student's parents. School, district, and statewide scores are public records. Student results are disaggregated by gender, race/ethnicity, and separate disabling condition. School results are also reported by socioeconomic status as determined by the number of students eligible for free- or reduced-cost lunch and by district size.

Q. How will schools and districts use assessment results?

A. It is up to each district to determine how to use individual student assessment results. By law, only locally-elected school boards may set graduation requirements, establish school curricula, and make policies for grade retention and promotion. Assessment results will most likely be used to aid teachers and administrators in improving curriculum and instruction.

The recently-passed accreditation law, House Bill 98-1267, requires all Colorado school districts to include state assessment results in their accreditation contract. Accreditation contracts must contain plans for improving student achievement on the statewide assessments.

Q. What will the Colorado Department of Education do with assessment results?

A. The Colorado Department of Education is required by law to report CSAP results for the state and for all local school districts. These results are released as public documents. In addition, each year 25% of assessment items are released and made available to educators, the public, and the media.

At this time, there are no state-imposed rewards or sanctions tied to the assessment results. However, CSAP results will be used in the new accreditation process established by the Colorado State Board of Education in accordance with House Bill 98-1267.

Q. Will results be used to compare schools and districts?

A. CSAP results are not intended to show comparisons among schools and districts. In fact, it would be misleading to use only one such measure to make judgments about the overall performance of schools.

Q. How will student anonymity be protected?

A. Only schools and parents will have access to individual student results. To protect student anonymity, smaller school districts (with 15 or fewer students per grade) will not have their results released publicly.

Q. How long do CSAP assessments take?

A. The length of testing differs depending on the content area and grade levels assessed. The mathematics assessment will require three, fifty-minute class periods, spread out over three days.

Q. How will I know how to administer the assessment?

A. Teachers will receive demonstration and test administration booklets in advance of the testing dates. Teachers giving the CSAP mathematics assessment will receive a demonstration booklet about six weeks prior to the assessment, and an administration manual will accompany student test materials. The booklet and manual thoroughly explain how to administer the assessments, contain example assessment items, and answer many questions teachers may have.

District assessment coordinators are encouraged to attend regional assessment training sessions offered by CDE. Participating coordinators are provided with test-administration materials to use in training building-level staff.

Q. Am I being graded or are my students being graded?

A. CSAP is an assessment of students and schools, not of individual teachers. What students have learned up to the time of the assessment and how well students do are the result of many factors, including all previous instruction from earlier grade levels and schools.

Q. Are all students expected to take the CSAP assessments?

A. Every student is expected to take the CSAP assessments, with a few exceptions. Each school district decides, based on guidelines from CDE, those students for whom the assessment is not appropriate. Appropriate accommodations, used in instruction, are allowed to assist students with special needs in taking the assessment. In calculating the percent of students at each performance level, **every** student in the grade being assessed is counted. Students who do not take the assessment are included in the CSAP report as "not assessed."

COLORADO
MODEL CONTENT STANDARDS
FOR
MATHEMATICS
Kindergarten to Fourth Grade
Assessment Framework

Colorado Model Mathematics Standards
Kindergarten to Fourth Grade Assessment Framework Task Force

Fran Berry	Principal Investigator	CONNECT Colorado's Statewide Systemic Initiative
Ken Berry	Elementary Mathematics Teacher-on-Special-Assignment	Jefferson County Schools
Nyla Bristow	Fifth Grade Teacher	Centennial Elementary School Weld County School District 6
Ceri Dean	Senior Associate	Mid-Continent Regional Educational Laboratory (McREL)
Janeane Golliher	Mathematics Coordinator	St. Vrain Valley Schools
Janet Iona	Assistant Principal	Centennial Elementary School Adams Twelve Five Star Schools
Marge Lindskog	Consultant and Retired Elementary Teacher	Pueblo School District 60
Alice Lopez	Teacher-on-Special-Assignment	Mapleton Public Schools
Mattye Pollard-Cole	Mathematics Senior Consultant	School Effectiveness Unit Colorado Department of Education
Sharon Simpson	Teacher/Mentor — STAR Program Teacher-On-Special-Assignment	Cherry Creek Schools
Diane Weaver-Marshall	Sixth Grade Mathematics Teacher	Morey Middle School Denver Public Schools

INTRODUCTION

Colorado Model Content Standards for Mathematics

Responsible and productive members of today's technological society need to have a broad, connected, and useful knowledge of mathematics. The Colorado Model Content Standards for Mathematics are designed to serve as a guide for school districts as they define standards that will enable every Colorado student to develop the mathematical literacy needed for citizenship and employment in the 21st century.

> " Today's students will live and work in the 21st century, in an era dominated by computers, by worldwide communication, and by a global economy. Jobs that contribute to this economy will require workers who are prepared to absorb new ideas, to perceive patterns, and to solve unconventional problems. Mathematics is the key to opportunity for these jobs."[1]

Mathematics is not simply a collection of facts and procedures, and doing mathematics is not simply recalling these facts, nor performing memorized procedures. Mathematics is a coherent and useful discipline that has expanded dramatically in the last 25 years. The mathematics students study in school must reflect these changes, and the ways students study mathematics must capitalize on the growth in our understanding of how students learn.

> " There has been a mentality that you have to be ... special to be successful in mathematics, that you have to be the best and the brightest. Well, we are demystifying mathematics. We can no longer say that ther is any segment of society that doesn't need mathematics."[2]

Three questions have guided the development of the Colorado Model Content Standards for Mathematics: What is mathematics? What does it to mean to know, use, and do mathematics? What mathematics should *every* Colorado student learn?

Responses to these questions have resulted in six goals, adapted from those of the National Council of Teachers of Mathematics[3], that serve as the framework for the Colorado Model Content Standards for Mathematics. The six goals that Colorado students should reach are stated on the following page.

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[1] L. Steen, (1989), "Teaching Mathematics for Tomorrow's World", *Educational Leadership*, 47: 18-22.

[2] Quote by Iris Carl found in A. Wheelock, (1992), *Crossing the Tracks*, (New York: The New Press).

[3] National Council of Teachers of Mathematics, (1989), *Curriculum and Evaluation Standards for School Mathematics*, (Reston, VA: author).

Six Goals for Colorado Students of Mathematics

- ***Become mathematical problem solvers.*** To be problem solvers, students need to know how to find ways to reach a goal when no routine path is apparent. To develop the flexibility, perseverance, and wealth of strategies that are characteristic of good problem solvers, students need to be challenged frequently and regularly with non-routine problems, including those they pose themselves.

- ***Learn to communicate mathematically.*** The development of students' power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations where students have an opportunity to read, write, and discuss ideas in the language of mathematics. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking.

- ***Learn to reason mathematically.*** Students who reason mathematically gather data, make conjectures*, assemble evidence, and build an argument to support or refute these conjectures. Such processes are fundamental to doing mathematics.

- ***Make mathematical connections.*** The study of mathematics should provide students with many opportunities to make connections among mathematical ideas (for example, the connection between geometric and algebraic concepts) and among mathematics and other disciplines (for example, art, music, psychology, science, business). The curriculum should portray mathematics as an integrated whole that permeates activities both in and out of school. These connections make mathematics meaningful and useful to each Colorado student.

- ***Become confident of their mathematical abilities.*** As a result of studying mathematics, students need to view themselves as capable of using their growing mathematical power to make sense of new problem situations in the world around them. School mathematics must endow all students with a realization that doing mathematics is a common human activity. Students learn to trust their own mathematical thinking by having numerous and varied experiences.

- ***Learn the value of mathematics.*** In addition to providing the tools to solve problems, mathematics provides a way of thinking about and understanding the world around us. Students should have numerous and varied opportunities to think mathematically about their world. They should also explore the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society.

The following Colorado Model Content Standards for Mathematics provide a new vision of the content students should study in order to achieve these goals. The standards reinforce the need for technical skills, long a goal of school mathematics, and also the need to know when to apply them and why they work. They also broaden considerably the context in which these technical skills might be attained. Students who have a working knowledge of the mathematics in each of these standards will be better able to reason critically, vote responsibly, and work productively in today's complex world.

* A glossary of terms can be found beginning on page 16.

Colorado Model Content Standards

MATHEMATICS

1. Students develop number sense* and use numbers and number relationships in problem-solving situations* and communicate the reasoning used in solving these problems.

2. Students use algebraic methods* to explore, model*, and describe patterns* and functions* involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.

3. Students use data collection and analysis, statistics*, and probability* in problem-solving situations and communicate the reasoning and processes used in solving these problems.

4. Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.

5. Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.

6. Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic*, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.

STANDARD 1:

Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems.

GRADES K-4

In grades K-4, what students know and are able to do includes

1.1 demonstrating meanings for whole numbers, commonly-used fractions and decimals (*for example, 1/3, 3/4, 0.5, 0.75*), and representing equivalent forms of the same number through the use of physical models, drawings, calculators, and computers;

Examples:
(1) Sharing a Cake (Referenced Tasks, pages 1-2)
(2) Draw and label a diagram that shows the fractional value of each piece of a set of tangram pieces, if all seven pieces (i.e., the whole set) are equal to one whole.
(3) An auditorium holds about 7,500 people. About how many auditoriums would it take to hold 100,000 people? Explain how you got your answer. (also, 1.5, 6.4)
(4) Explain what 0.25 means. Describe in as many ways as you can.
(5) Rectangle Problem (Referenced Tasks, page 3) (also, 4.1)
(6) Decimal Problem (Referenced Tasks, page 4)

1.2 reading and writing whole numbers and knowing place-value concepts and numeration through their relationships to counting, ordering, and grouping;

Examples:
(1) Number Sentence Problem (Referenced Tasks, page 5)
(2) In which of the following numbers does the number '5' have the greatest value?
a. 21,543 b. 15,341 c. 69,258 d. 73,645
(3) Marge says that 2 hundreds, 15 tens, 7 ones is another name for the number 357. Is Marge correct? Explain why or why not.
(4) Which of the following represents the greatest number?
a. 2 thousands, 19 tens, 6 ones c. three thousand eight hundred six
b. 3000 + 900 + 6 d. 3,086
(5) Smallest Whole Number Problem (Referenced Tasks, page 6)
(6) Addition Game Problem (Referenced Tasks, page 7)
(7) Laura's Calculator Correction (Referenced Tasks, page 8)

1.3 using numbers to count, to measure, to label, and to indicate location;

Examples:
(1) Game Board Problem (Referenced Tasks, page 9) (also, 4.4)
(2) Grid Problem (Referenced Tasks, page 10) (also, 4.4)
(3) Rule It Out (Referenced Tasks, page 11) (also, 5.1, 5.3)
(4) Name The Graph (Referenced Tasks, page 12) (also, 3.1)

1.4 developing, testing and explaining conjectures* about properties of whole numbers, and commonly-used fractions and decimals (*for example, 1/3, 3/4, 0.5, 0.75*);

Examples:

(1) Given that 3 + Δ = 3, 1/4 + Δ = 1/4, 0.5 + Δ = 0.5, 29 + Δ = ?
Find and explain the value of Δ.

(2) Jana's Number (Referenced Tasks, page 13)

(3) Use examples and drawings to show a third-grader who is having trouble understanding multiplication why 3 X 6 has the same value as 6 X 3. (also, 6.1)

(4) One evening, when baseball player Yogi Berra was hungry after a night game, he phoned a pizza delivery service and ordered a pie to be delivered to his room.
"Do you want your pizza cut into four slices or eight?" Berra was asked.
"Cut it into four," Yogi decided. "I don't think I can eat eight."
Is Yogi Berra making a reasonable statement or not? Explain your reasoning.

1.5 using number sense to estimate and justify the reasonableness of solutions to problems involving whole numbers, and commonly-used fractions and decimals (*for example, 1/3, 3/4, 0.5, 0.75*).

Examples:

(1) 5/6 + 7/8 is closest to which of the following:
a. 1 b. 2 c. 12 d. 12/14

(2) Ken walked 0.4 miles to the grocery store, 0.5 miles to the video store, and 0.75 miles to the park. He said he walked a total distance of 0.84 miles. Is Ken's statement reasonable? Why or why not?

(3) An auditorium holds about 7,500 people. About how many auditoriums would it take to hold 100,000 people? Explain how you got your answer. (also, 1.1 and 6.4)

(4) Pizza Problem (Referenced Tasks, page 14)

STANDARD 2:

Students use algebraic methods to explore, model and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.

GRADES K-4

In grades K-4, what students know and are able to do includes

2.1 reproducing, extending, creating, and describing patterns and sequences using a variety of materials (*for example, beans, toothpicks, pattern blocks, calculators, unifix cubes, colored tiles*);

Examples:
(1) Stick Pattern Problem (Referenced Tasks, page 14)
(2) Given the following pattern:
13, 17, ___, 25, 29, ___, ___
What numbers are missing in this pattern? Explain your answer.
Released mathematics assessment item, Oregon Department of Education
(3) Shape Problem (Referenced Tasks, page 15)

2.2 describing patterns and other relationships using tables, graphs, and open sentences*;

Examples:
(1) Book Problem (Referenced Tasks, page 16) (also, 6.1)
(2) Marvelous Marbles (Referenced Tasks, pages 17-18)
(3) Column Problem (Referenced Tasks, page 19) (also, 2.3)

2.3 recognizing when a pattern exists and using that information to solve a problem;

Examples:
(1) Tile Pattern (Referenced Tasks, page 20)
(2) Column Problem (Referenced Tasks, page 19) (also, 2.2)
(3) Sarah's birthday is on Saturday. She invited one friend on Monday. On Tuesday she invited 2 friends. If she continues to invite one more friend each day than the day before, how many friends in all will be invited by Friday?
Show your work and explain your thinking.
Released mathematics assessment item, Oregon Department of Education

2.4 observing and explaining how a change in one quantity can produce a change in another (*for example, the relationship between the number of bicycles and the numbers of wheels*).

Examples:
(1) Counting Raisins II (Referenced Tasks, page 21) (also, 6.4)
(2) In last year's Fourth of July parade, there were five tricycles (each tricycle has three wheels). This year's parade had seven tricycles. How many more wheels were in the parade this year than last year?

STANDARD 3:

Students use data collection and analysis, statistics, and probability in problem-solving situations and communicate the reasoning used in solving these problems.

GRADES K-4

In grades K-4, what students know and are able to do includes

3.1 constructing, reading, and interpreting displays of data including tables, charts, pictographs, and bar graphs;

Examples:
(1) What Does the Graph Mean? (Referenced Tasks, page 22)
(2) Mystery Graphs (Referenced Tasks, pages 23-24)
(3) Ice Cream Choice (Referenced Tasks, page 25)
(4) Name The Graph (Referenced Tasks, page 12) (also, 1.3)

3.2 interpreting data using the concepts of largest, smallest, most often, and middle;

Example:
(1) The fourth graders in Ms. Simpson's class bought some peanuts that come in small bags. Each student in the class reported how many peanuts were in her or his bag. Here are the numbers each student reported:

Number of Peanuts in a Bag
14 17 15 16 16 18 21 13 15 14 15
15 16 17 17 19 17 16 19 15 19

What is the smallest number of peanuts in a bag?
What is the largest number of peanuts in a bag?
What number of peanuts in a bag occurs most often?
If someone asked you, "About how many peanuts are in a bag?", what would you say? Explain your reasoning.

3.3 generating, analyzing, and making predictions based on data obtained from surveys and chance devices;

Examples:
(1) Favorite Pizza (Referenced Tasks, page 26)
(2) Spinner Problem (Referenced Tasks, page 26)
(3) Is It Fair? (Referenced Tasks, pages 27-28)
(4) Fair Game (Referenced Tasks, pages 29-30)

3.4 solving problems using various strategies for making combinations* (*for example, determining the number of different outfits that can be made using two blouses and three skirts*).

Examples:
(1) The Vending Machine (Referenced Tasks, page 31)
(2) Mr. Jones's Pies (Referenced Tasks, page 32) (also, 6.3)

STANDARD 4:

Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.

<u>GRADES K-4</u>

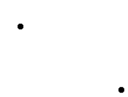
In grades K-4, what students know and are able to do includes

4.1 recognizing shapes and their relationships (*for example, symmetry*, congruence**) using a variety of materials (*for example, pasta, boxes, pattern blocks*);

Examples:
(1) Shape Problem (Referenced Tasks, page 33)
(2) Rectangle Problem (Referenced Tasks, page 3) (also, 1.1)
(3) Line of Symmetry Problem (Referenced Tasks, page 34)
(4) Symmetry in Shapes (Referenced Tasks, page 35)
(5) Thinking About Congruent Triangles (Referenced Tasks, page 36)

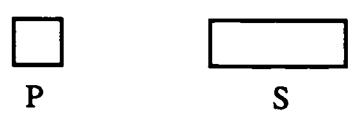
4.2 identifying, describing, drawing, comparing, classifying, and building physical models of geometric figures;

Examples:
(1) In the space below, use your ruler to draw a square with two of its corners at the points shown.



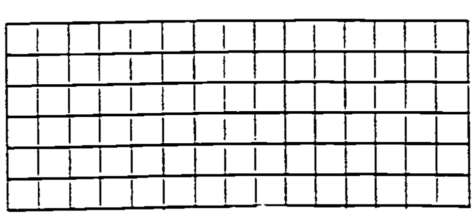
from *Can Students Do Mathematical Problem Solving?*
U. S. Department of Education, 1993.

(2) Shapes: Rectangles and Triangles (Referenced Tasks, pages 37-38)
(3) Building with Squares (Referenced Tasks, pages 39-40) (also, 4.4)
(4) How many shapes like P and how many shapes like S do you need to make a closed box?



4.3 relating geometric ideas to measurement and number sense;

Examples:
(1) Wire Problem (Referenced Tasks, page 41)
(2) Draw a rectangle with length 4 centimeters and width 3 centimeters.
(3) On the grid below, draw a rectangle with perimeter 12 units.



4.3 Examples (continued):

(4) The distance around Keisha's rectangular sandbox is 18 feet. What might be the length of each side. Show your thinking.

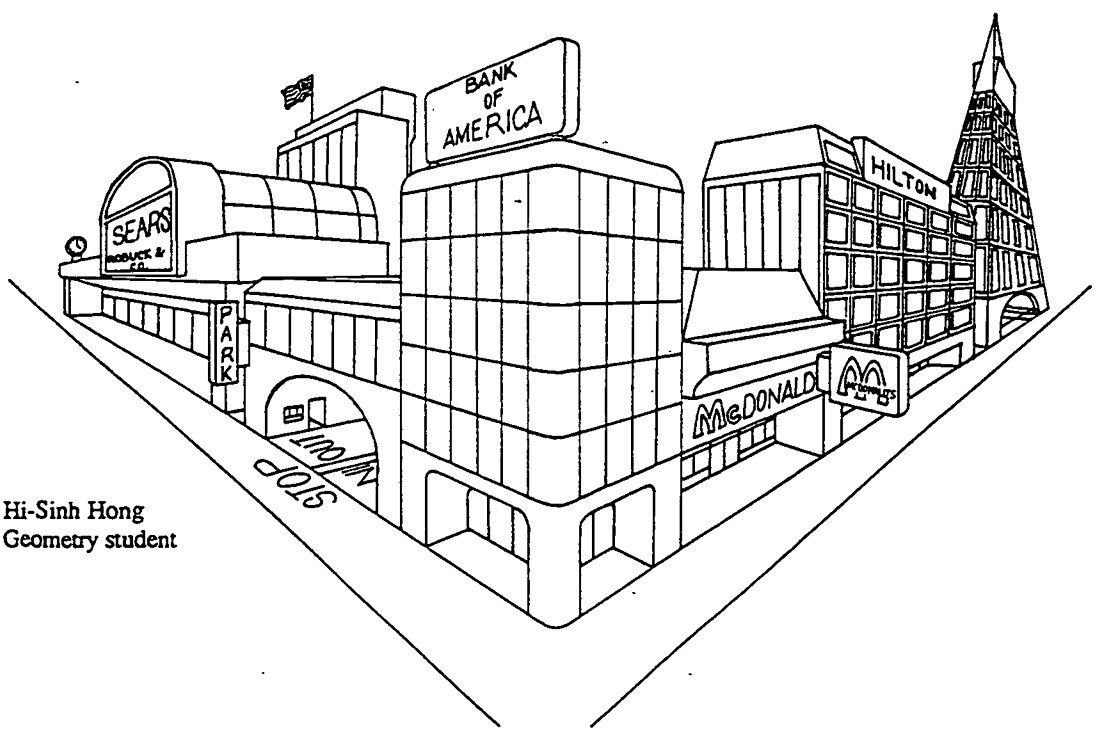
Released mathematics assessment item
Oregon Department of Education

4.4 solving problems using geometric relationships and spatial reasoning* (*for example, using rectangular coordinates* to locate objects, constructing models of three-dimensional objects*);

Examples:
(1) Game Board Problem (Referenced Tasks, page 9) (also, 1.3)
(2) Grid Problem (Referenced Tasks, page 10) (also, 1.3)
(3) Building with Squares (Referenced Tasks, pages 39-40) (also, 4.2)
(4) Cube Problem (Referenced Tasks, page 42)
(5) Rug Problem (Referenced Tasks, page 43)
(6) Triangle Problem (Referenced Tasks, page 44)

4.5 recognizing geometry* in their world (*for example, in art and in nature*).

Examples:
(1) List five objects in your fourth grade classroom that are the shape of a rectangle.
(2) List the shapes you see in the picture below.



Hi-Sinh Hong
Geometry student

STANDARD 5:

Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.

GRADES K-4

In grades K-4, what students know and are able to do includes

5.1 knowing, using, describing, and estimating measures of length, perimeter, capacity, weight, time, and temperature;

Examples:
(1) Pencil Problem (Referenced Tasks, page 45) (also, 5.3)
(2) Paper Clip Problem (Referenced Tasks, page 46) (also, 5.4, 5.5)
(3) Milliliter Problem (Referenced Tasks, page 47)
(4) Scale Problem (Referenced Tasks, page 48)
(5) Rectangle Problem (Referenced Tasks, page 48)
(6) Rule It Out (Referenced Tasks, page 11) (also, 1.3, 5.3)
(7) Time at the Birthday Party (Referenced Tasks, pages 49-50) (also, 5.5)

5.2 comparing and ordering objects according to measurable attributes (*for example, longest to shortest, lightest to heaviest*);

Examples:
(1) Gram Problem (Referenced Tasks, page 51)
(2) Place the following in order from smallest to largest:
1 quart 1 cup 1 gallon 1 pint

5.3 demonstrating the process of measuring and explaining the concepts related to units of measurement;

Examples:
(1) Pencil Problem (Referenced Tasks, page 45) (also, 5.1)
(2) Rule It Out (Referenced Tasks, page 11) (also, 1.3, 5.1)
(3) Bill says that there are four feet in a gallon. Is he correct? Explain why or why not.

5.4 using the approximate measures of familiar objects (*for example, the width of your finger, the temperature of a room, the weight of a gallon of milk*) to develop a sense of measurement;

Examples:
(1) Pace Problem (Referenced Tasks, page 52)
(2) Paper Clip Problem (Referenced Tasks, page 46) (also, 5.1, 5.5)

5.5 selecting and using appropriate standard and non-standard units of measurement in problem-solving situations.

Examples:
(1) Paper Clip Problem (Referenced Tasks, page 46) (also, 5.1, 5.4)
(2) Clothespin Problem (Referenced Tasks, page 53)
(3) Mr. Brown Problem (Referenced Tasks, page 54)
(4) Time at the Birthday Party (Referenced Tasks, pages 49-50) (also, 5.1)

STANDARD 6:

Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.

<u>GRADES K-4</u>

In grades K-4, what students know and are able to do includes

6.1 demonstrating conceptual meanings for the four basic arithmetic operations of addition, subtraction, multiplication, and division;

Examples:
(1) Multiplication Problem (Referenced Tasks, page 55)
(2) Use examples and drawings to show a third-grader who is having trouble understanding multiplication why 3 X 6 has the same value as 6 X 3. (also, 1.4)
(3) Sharing 25...
In each situation below, four friends want to "share 25" as equally as possible. Show or explain how to "share 25" in each situation.
1. Four friends shared 25 balloons as equally as possible.
2. Four friends shared $25 as equally as possible.
3. Four friends shared 25 cookies as equally as possible.
from *Writing in the Math Class*, © 1995, Math Solutions Publications
(4) Write a story that goes with each of the following problems.
You do not need to compute the answer to the problems.
1. 395 + 1,006
2. 700 - 268
3. 27 x 5
4. 49 ÷ 6
(5) Book Problem (Referenced Tasks, page 16) (also, 2.2)

6.2 adding and subtracting commonly-used fractions and decimals using physical models (*for example, 1/3, 3/4, 0.5, 0.75*);

Examples:
(1)

What is the total shaded in the picture shown above?
(2) Show 1/2 + 5/8 with a picture or diagram so that a younger student could understand the sum.
(3) Show 0.9 - 0.5 with a picture or diagram so that a younger student could understand the difference.
(4) Amy and Eric's dad made some chocolate chip cookies for the family. The cookies were so big, he decided to cut each one in half. Amy ate seven halves and Eric ate five halves. How many whole cookies did they eat altogether? Explain your thinking.

6.3 demonstrating understanding of and proficiency with basic addition, subtraction, multiplication, and division facts* without the use of a calculator;

Examples:
(1) List three multiplication problems which have an answer of 24.
(2) List three division problems which give an answer of 4.
(3) Write an addition problem and a subtraction problem which both have an answer of 9.

6.4 constructing, using, and explaining procedures to compute and estimate with whole numbers;

Examples:
(1) Mr. Jones's Pies (Referenced Tasks, page 32) (also, 3.4)
(2) Jessie's Marbles (Referenced Tasks, page 56) (also, 6.5)
(3) An auditorium holds about 7,500 people. About how many auditoriums would it take to hold 100,000 people? Explain how you got your answer. (also, 1.1 and 1.5)
(4) Elena Problem (Referenced Tasks, page 57)

6.5 selecting and using appropriate methods for computing with whole numbers in problem-solving situations from among mental arithmetic, estimation, paper-and-pencil, calculator, and computer methods.

Examples:
(1) Jessie's Marbles (Referenced Tasks, page 56) (also, 6.4)
(2) Sam can purchase his lunch at school. Each day he wants to have juice that costs 50¢, a sandwich that costs 90¢, and fruit that costs 35¢. His mother has only $1.00 bills. What is the least number of $1.00 bills that his mother should give him so he will have enough money to buy lunch for five days?

from *NAEP 1996 Mathematics Report Card*
U.S. Department of Education, 1997

(3) Subraction Problem (Referenced Tasks, page 58)

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GLOSSARY

Colorado Model Content Standards for Mathematics

Kindergarten to Fourth Grade Assessment Framework

Algebraic methods — The use of symbols to represent numbers and signs to represent their relationships.

Basic facts — Addition facts through 10 (0 + 0, 1 + 0, ..., 10+ 10), subtraction facts which are the inverses of the addition facts (20 - 10, ..., 1 - 0, 0 - 0), multiplication facts (1 x 1, 1 x 2, ..., 10 x 10), and division facts which are the inverses of the multiplication facts (1 ÷ 1, 2 ÷ 1, ... , 100 ÷ 10).

Capacity — The volume of a container given in units of liquid measure. The standard units of capacity are the liter and the gallon.

Combinations — Subsets chosen from a larger set of objects in which the order of the items doesn't matter (for example, the number of different committees of three that can be chosen from a group of twelve members).

Congruent or the concept of congruence — Two figures are said to be congruent if they are the same size and shape.

Conjecture — A statement that is to be shown true or false. A conjecture is usually developed by examining several specific situations.

Coordinate system (also called rectangular coordinate system) — A method of locating points in the plane or in space by means of numbers. A point in a plane can be located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect had the coordinates (0, 0) and is called the origin.

Function — A relationship between two sets of numbers (or other mathematical objects). Functions can be used to understand how one quantity varies in relation to another, for example, the relationship between the number of cars and the number of tires.

Geometry — A branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, and two- and three-dimensional figures.

Mental arithmetic — Performing computations in one's head without writing anything down. Mental arithmetic strategies include finding pairs that add up to 10 or 100, doubling, and halving.

Model — To make or construct a physical or mathematical representation.

Number sense — An understanding of number. This would include number meanings, number relationships, number size, and the relative effect of operations on numbers.

Open sentence — A statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown. For example, $3 + x = 5$.

Patterns — Regularities in situations such as those in nature, events, shapes, designs, and sets of numbers (for example, spirals on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12, ...).

Probability — The likeliness or chance of an event occurring.

Problem-solving situations — Contexts in which problems are presented that apply mathematics to practical situations in the real world, or problems that arise from the investigation of mathematical ideas.

Spatial visualization (also called spatial reasoning) — A type of reasoning in which a person can draw upon one's understanding of relationships in space, the three-dimensional world. For example, spatial reasoning is demonstrated by one's ability to build a three-dimensional model of a building shown in a picture. A person who uses spatial visualization is said to have spatial sense.

Statistics — The branch of mathematics which is the study of the methods of collecting and analyzing data. The data are collected on samples from various populations of people, animals, or products. Statistics are used in many fields, such as biology, education, physics, psychology, and sociology.

Symmetry — The correspondence in size, form, and arrangement of parts on opposite sides of a plane, line, or point. For example, a figure that has line symmetry has two halves which coincide if folded along its line of symmetry.

COLORADO
MODEL CONTENT STANDARDS
FOR
MATHEMATICS

REFERENCED TASKS
Kindergarten to Fourth Grade
Assessment Framework

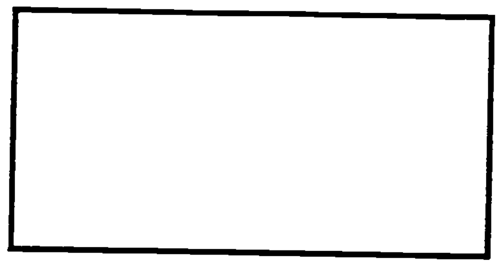
NOTE: The examples included in this framework are meant to illustrate the grade-level expectations in the *Kindergarten to Fourth Grade Mathematics Assessment Framework*. These are **not** actual CSAP test items, although the actual items will be similar to these in content and form. In addition, several of the examples illustrate more than one standard and/or expectation.

Each example is coded with a box in the lower right-hand corner which indicates which expectation(s) is illustrated by the example. For example, [2.3] means the example illustrates grade-level expectation 2.3 under Mathematics Standard 2.

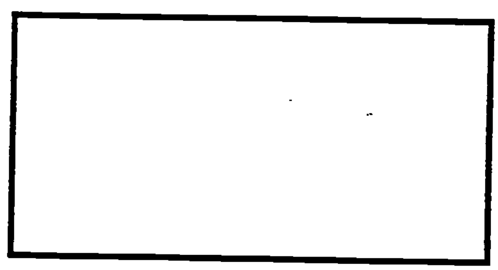
Sharing a Cake

Rafael and his sister Luisa bake a small cake to share with their four cousins. Below you see the shape of the cake.

Show one way to cut the whole cake into equal pieces for Rafael, Luisa, and the cousins.



Now show a different way to cut the whole cake into equal pieces for everyone.

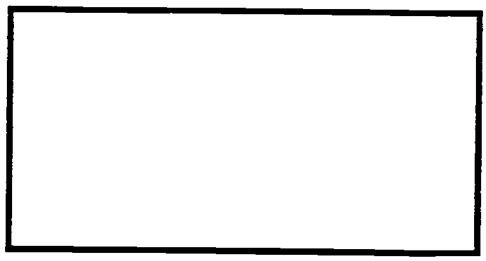


If the cake is cut into equal pieces, what fraction of the cake is Rafael's piece?

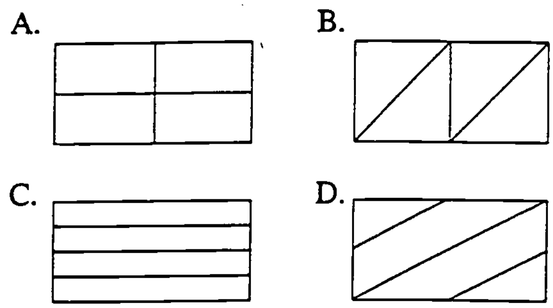
Just before they cut the cake, two friends walk in the door!

1) On the drawing below, show how to cut the cake so that Rafael, Luisa, the four cousins, and the two friends can have the same amount.

2) On the drawing, shade in the cake that is for their two friends and tell what fraction of the cake you have shaded.



K8. Which rectangle is NOT divided into 4 equal parts?



1.1; 4.1

I2. 0.4 is the same as

A. four

B. four tenths

C. four hundredths

D. one-fourth

S-2

S2. Here is a number sentence.

$2000 + \square + 30 + 9 = 2739$

What number goes where the $\square$ is to make this sentence true?

Answer: ______________________

1.2

T-2

T2. What is the smallest whole number that you can make using the digits 4, 3, 9 and 1 ? Use each digit only once.

Answer: ______________________________

V4. In a game, Mysong and Naoki are making addition problems. They each have four cards like these.

1	2	3	4

The winner of the game is the person who can make the problem with the largest answer.

Mysong placed the cards like this.

$$\begin{array}{r} 43 \\ +\ 21 \\ \hline \end{array}$$

Naoki placed the cards like this.

$$\begin{array}{r} 31 \\ +\ 24 \\ \hline \end{array}$$

Who won this game? ____________________

How do you know? ____

____

Write numbers in the squares below to show how you would place the cards to beat both Mysong and Naoki.

$$\begin{array}{r} \square\square \\ +\ \square\square \\ \hline \end{array}$$

V-4

Grade 4 Question: Laura's Calculator Correction

The Task

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work.

Laura wanted to enter the number 8375 into her calculator. By mistake, she entered the number 8275. Without clearing the calculator, how could she correct her mistake?

Without clearing the calculator, how could she correct her mistake another way?

Did you use the calculator on this question?

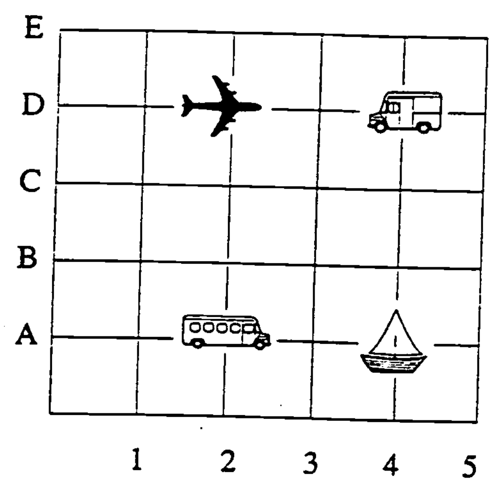
Yes No

BEST COPY AVAILABLE

From *Can Students Do Mathematical Problem Solving?* National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

L-3

L3. This is a game board.



Which object is located at (2, D)?

A. The plane 

B. The truck 

C. The bus 

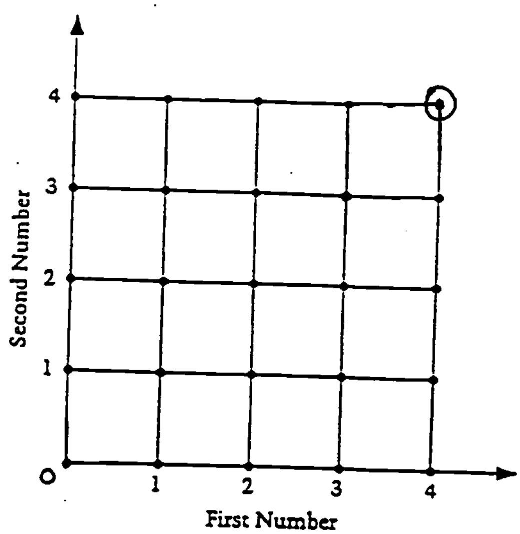
D. The boat 

9

1.3; 4.4

Grid Problem

On the grid below, the dot at (4, 4) is circled. Circle two other dots where the first number is equal to the second number.



Rule It Out

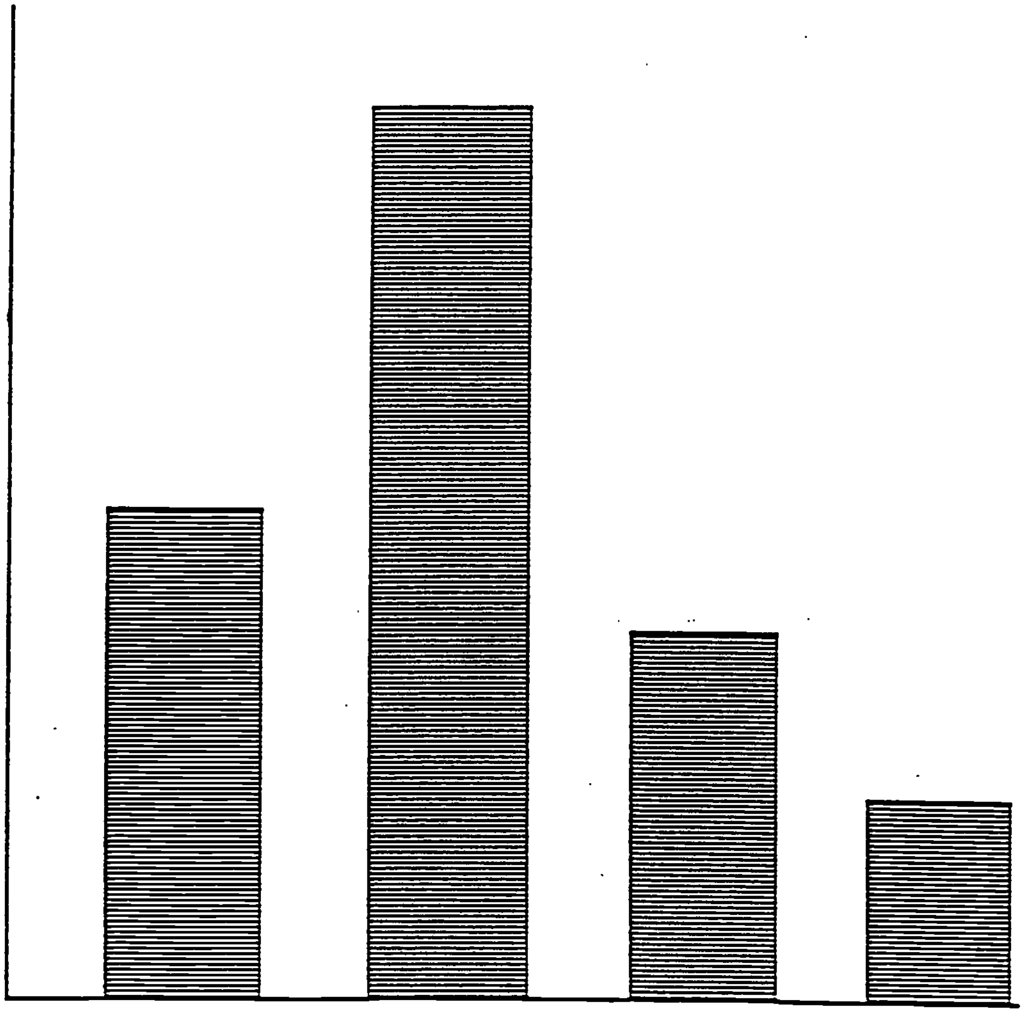
1. Draw a line segment 10 cm long in the space below.

2. Draw a line segment twice the length of your first line. Write down its length next to it.

3. Now draw a line segment that is half the length of your <u>first</u> <u>line</u>. Write down its length next to it.

1.3 ; 5.1 ; 5.3

NAME THE GRAPH



1. What do you think this might be the graph of? Put names and numbers on the graph to show what you mean.

2. Write down everything you know from your graph.

1.3; 3.1

Jana's Number

Jana asks some students to guess her number.

Katie:	Is it an even number?
Jana:	Yes.
Ervin:	Is your number more than 50?
Jana:	Yes.
Chua:	Is your number less than 100?
Jana:	Yes.
David:	Does it divide evenly by 5?
Jana:	Yes.
Adam:	I know it! There's only one number it can be!

Adam is wrong.

There is more than one number that Jana could be thinking of.

List all the numbers that could be Jana's number.

Grade 4 Question: Pizza

The Task

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work.

José ate $\frac{1}{2}$ of a pizza.

Ella ate $\frac{1}{2}$ of another pizza.

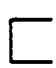
José said that he ate more pizza than Ella, but Ella said they both ate the same amount. Use words and pictures to show that José could be right.

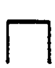
From *Can Students Do Mathematical Problem Solving?* National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

1.5

STICK PATTERN PROBLEM

In the pattern above, which figure would be next?

A  **B** 

C  **D** 

Released item from *NAEP 1992 Mathematics Report Card for the Nation and the States*,

L-4

L4. These shapes are arranged in a pattern.

○△○○△△○○○△△△

Which set of shapes is arranged in the same pattern?

A. ★□★□★★□□★★□□

B. □★□□★□□□★□□□□

C. ★□★★ □□★★★□□□

D. □□★★□★□□★★□★

2.1

I7. Tanya has read the first 78 pages in a book that is 130 pages long. Which number sentence could Tanya use to find the number of pages she must read to finish the book?

A. $130 + 78 = \square$

B. $\square - 78 = 130$

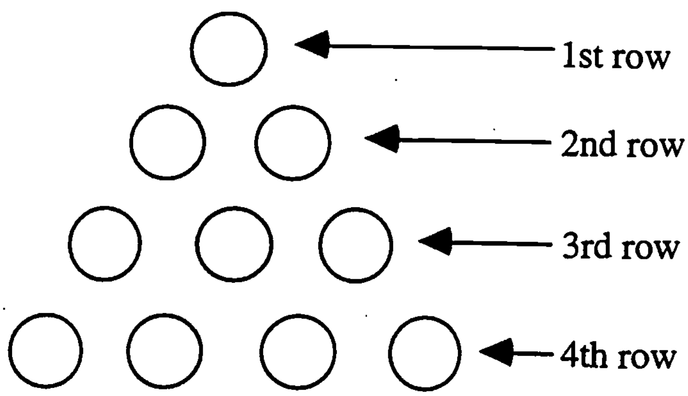
C. $130 \div 78 = \square$

D. $130 - 78 = \square$

Marvelous Marbles

Dorian sets up ten marbles in a triangle like the one you see below.

1) Make the triangle bigger by drawing three more rows of marbles.



2) How many marbles are in the triangle now?

2.2

3) If the triangle keeps growing, how many marbles will be in the twelfth row? Explain how you know.

J-5

J5. What do you have to do to each number in Column A to get the number next to it in Column B?

Column A	Column B
10	2
15	3
25	5
50	10

A. Add 8 to the number in Column A.

B. Subtract 8 from the number in Column A.

C. Multiply the number in Column A by 5.

D. Divide the number in Column A by 5.

K-6

K6. Here is the beginning of a pattern of tiles.

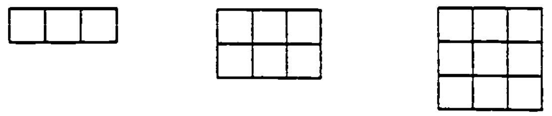


Figure 1 Figure 2 Figure 3

If the pattern continues, how many tiles will be in Figure 6 ?

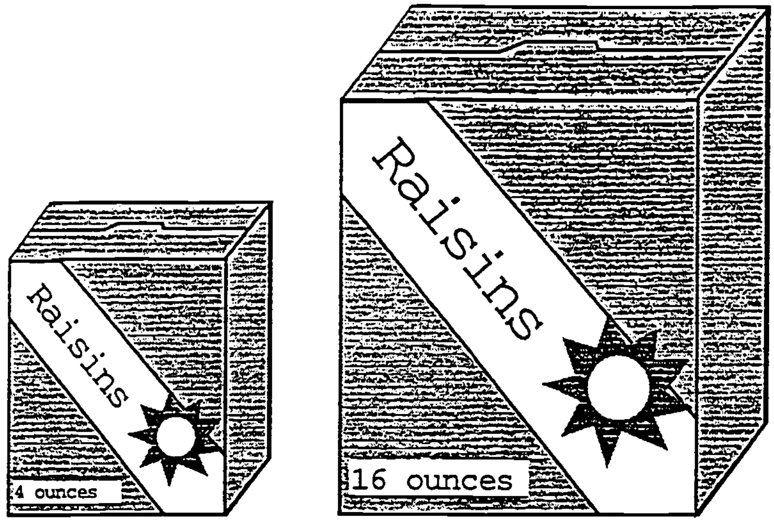
A. 12

B. 15

C. 18

D. 21

Counting Raisins II



Marcus has two different size boxes of raisins. He counts 62 raisins in the 4 ounce box.

Estimate how many raisins are in the 16 ounce box.

Explain your estimate and show your work so Marcus can see how you decided.

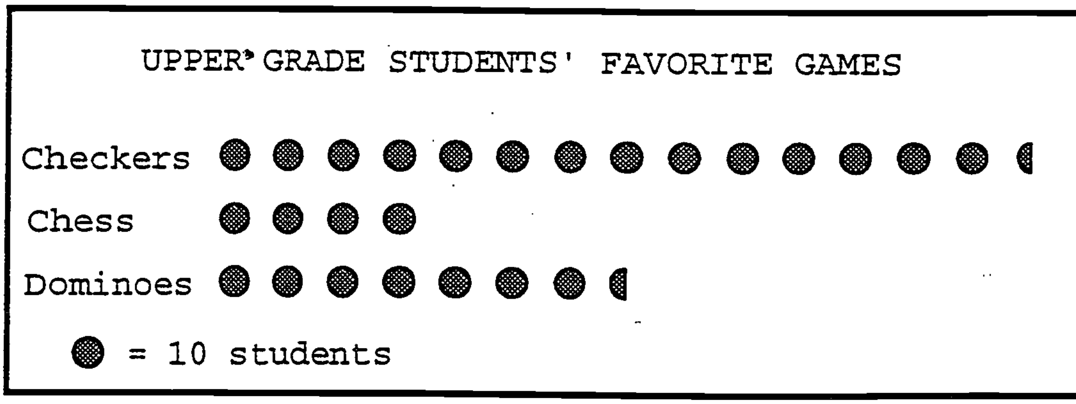
2.4; 6.4

What Does the Graph Mean?

Mrs. Cortines, the school principal, plans to buy new games for her students. She has enough money to buy 30 game sets. Here is her shopping list:

Checkers	buy 10 sets
Chess	buy 10 sets
Dominoes	buy 10 sets
TOTAL	30 sets

Then she sees this fourth grader's graph:



Mrs. Cortines looks at the graph and says, "I want to buy what the students like...I guess I shouldn't buy exactly the same number of sets of each game."

Write a new shopping list for Mrs. Cortines. Use the information in the graph to help you. Explain how you know that your list matches what the students like.

Mystery Graphs

Look at the five graphs on the next pages. Each graph shows something about a classroom of fourth graders.

1. Which of the five graphs do you think shows:

   a. The number of cavities that the fourth graders have? ______

   b. The ages of the fourth graders' mothers? ______

   c. The heights of the fourth graders, in inches? ______

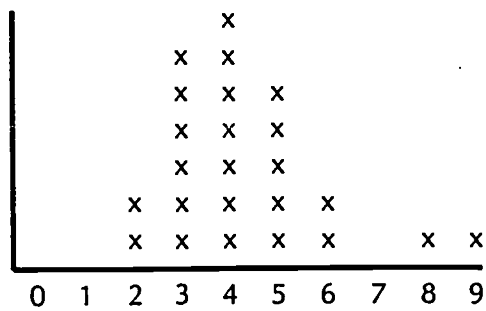
   d. The number of people in the fourth graders' families? ______

2. Explain why you think the graph you picked for **c** is the one that shows the heights of fourth graders.

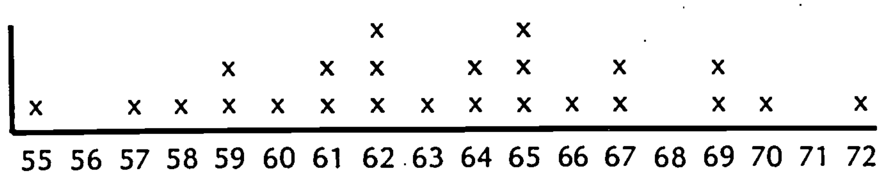
3. Why do you think the other graphs <u>don't</u> show the fourth graders' heights?

from *Measuring Up: Prototypes for Mathematics Assessment*, Mathematical Science Education Board, National Research Council, National Academy Press, Washington, DC, 1993.

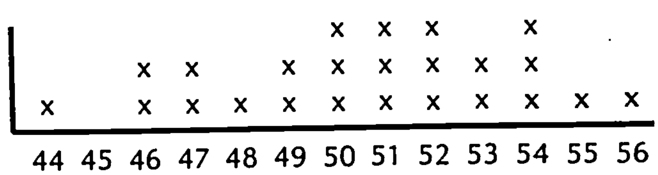
Graph 1



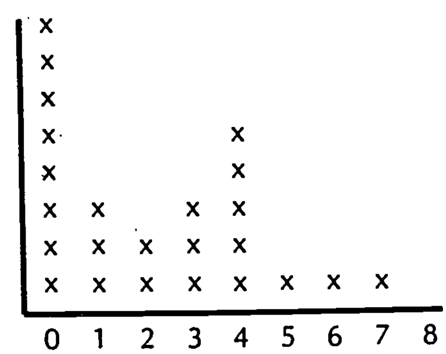
Graph 2



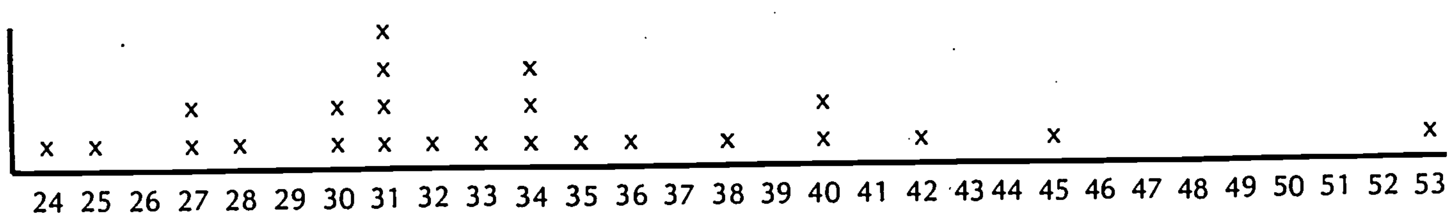
Graph 3



Graph 4



Graph 5



from *Measuring Up: Prototypes for Mathematics Assessment*, Mathematical Science Education Board, National Research Council, National Academy Press, Washington, DC, 1993.

Ice Cream Choice

Be sure to complete both parts of this problem.

On a field trip, Mrs. Haysom's students chose free ice cream samples. One student, Ana, kept track of the choices on a page in her journal.

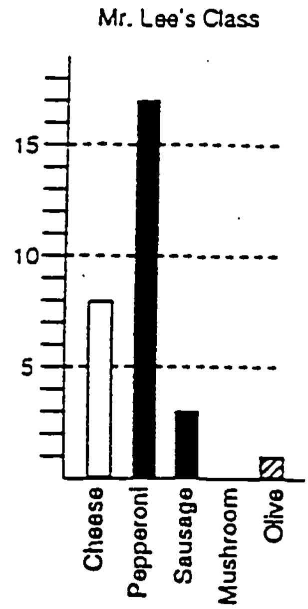
> On our ice cream field trip, 13 students chose chocolate, 10 students chose vanilla, and 7 students chose chocolate chip.

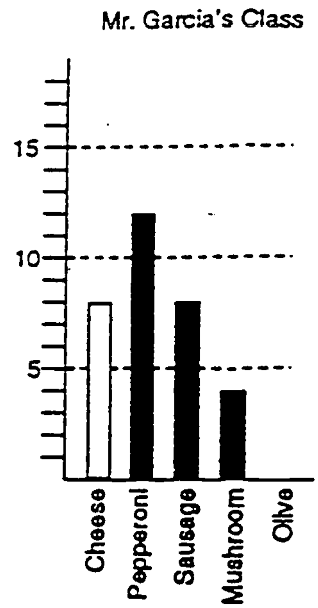
1) Make a graph, chart, drawing, or table that shows this information.

2) By looking at the information, what can you say about the ice cream flavors these students like?

FAVORITE PIZZA

The cafeteria manager asked you to find out which kind of pizza the fourth grade liked best. The fourth grade classes made graphs of their survey data to help you:





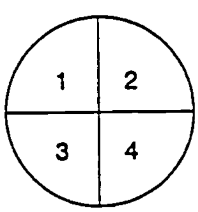
Write to the cafeteria manager advising her about the kinds and amounts of pizza to serve at a fourth grade pizza party.

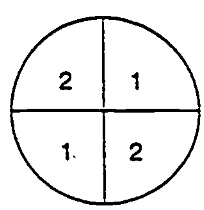
Be sure to give reasons for your choices.

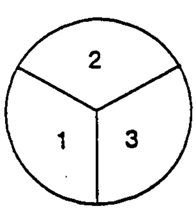
3.3

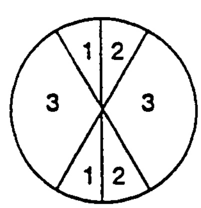
SPINNER PROBLEM

You want to spin a 2. Which spinner would give you the best chance of spinning a 2 on the first spin?

A.


B.


C.


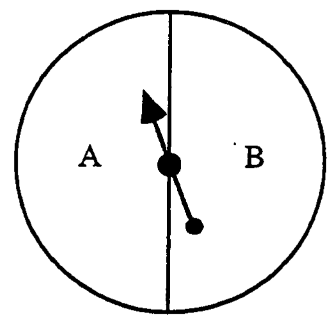
D.


3.3

Is It Fair?

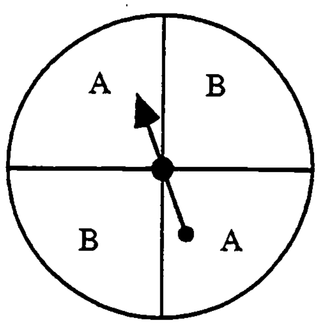
In Ms. Bigby's fourth-grade class, she spins a spinner each day to see whether team A or team B will go to lunch first.

1) Is there an equal chance of team A or team B going to lunch first, using Spinner 1? Explain your thinking.



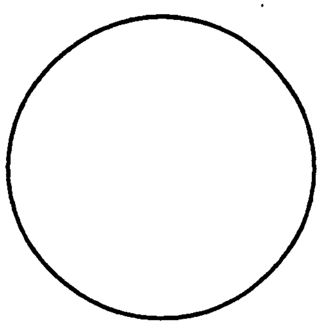
Spinner 1

2) Bobby, a new boy in team A, thinks he will have a greater chance to go to lunch first if Ms. Bigby uses Spinner 1, rather than Spinner 2. Do you agree or disagree? Explain.



Spinner 2

3) Design a different spinner that will give team A a greater chance of going to lunch first. Include both team A and team B on your spinner.

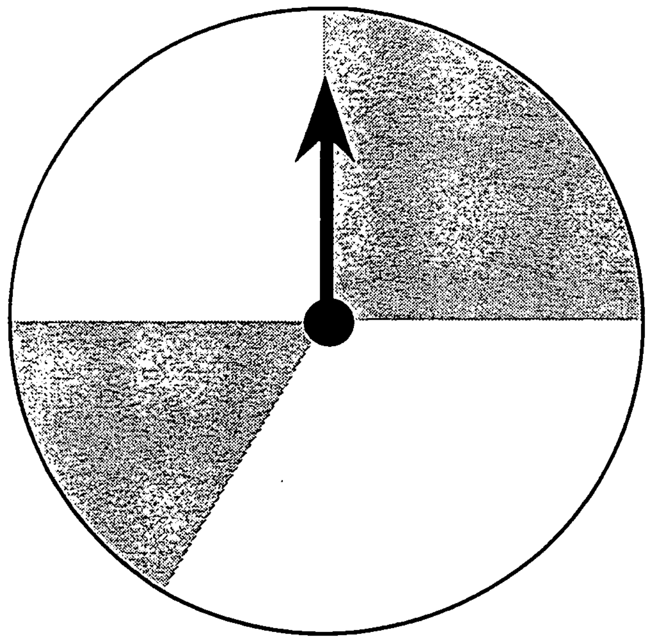


Your spinner

Explain why your spinner will give team A a greater chance of going to lunch first.

Fair Game?

Imagine that you and a friend are playing a game with this spinner:



Here are the rules:

- The two of you take turns spinning.
- For every spin, you get a point if the spinner lands on a shaded part.
- Your friend gets a point if the spinner lands on an unshaded part.
- The first person to get to 25 points wins.

Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

Be sure to complete both parts.

The Vending Machine

Maria wants to buy a 75-cent snack from a vending machine. The machine takes only nickels, dimes, and quarters. Maria has 7 nickels, 5 dimes, and 2 quarters.

Part 1
Show all of the different ways she could pay for the snack. You may use words, diagrams, or charts.

Part 2
Which of your ways uses the fewest number of coins? Explain why this is true.

From "A Sampler of Mathematics Assessment - 1994", developed by the California Department of Education

Mr. Jones's Pies

Mr. Jones is a baker. He has decided to sell 4 different flavors of pie. Each flavor will be sold in 3 sizes: large, medium and small. For example, one **kind** of pie will be *large apple,* and a different kind will be *medium apple.*

1) How many different kinds of pies will he sell? Make a picture or a diagram or a table that shows all the different kinds.

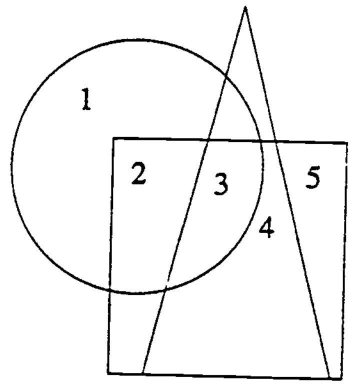
One day Mr. Jones made **a total of 60 pies.**

2) He made the same number of each flavor. How many of each flavor did he make?

3) He made the same number of each size. How many of each size did he make?

K-1

K1. Here is a figure.



Which number is in the square and the circle but is NOT in the triangle?

A. 2

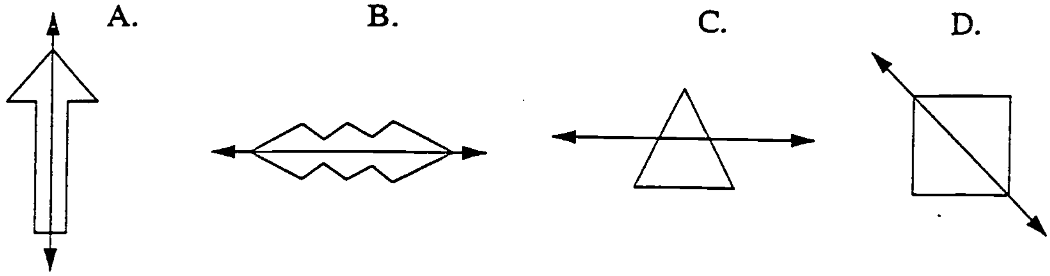
B. 3

C. 4

D. 5

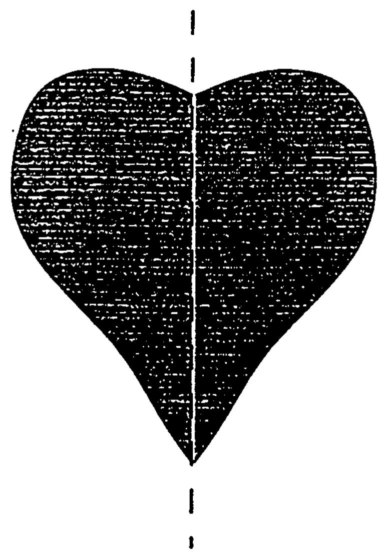
J2. Which of these does NOT show a line of symmetry?

A. B. C. D.



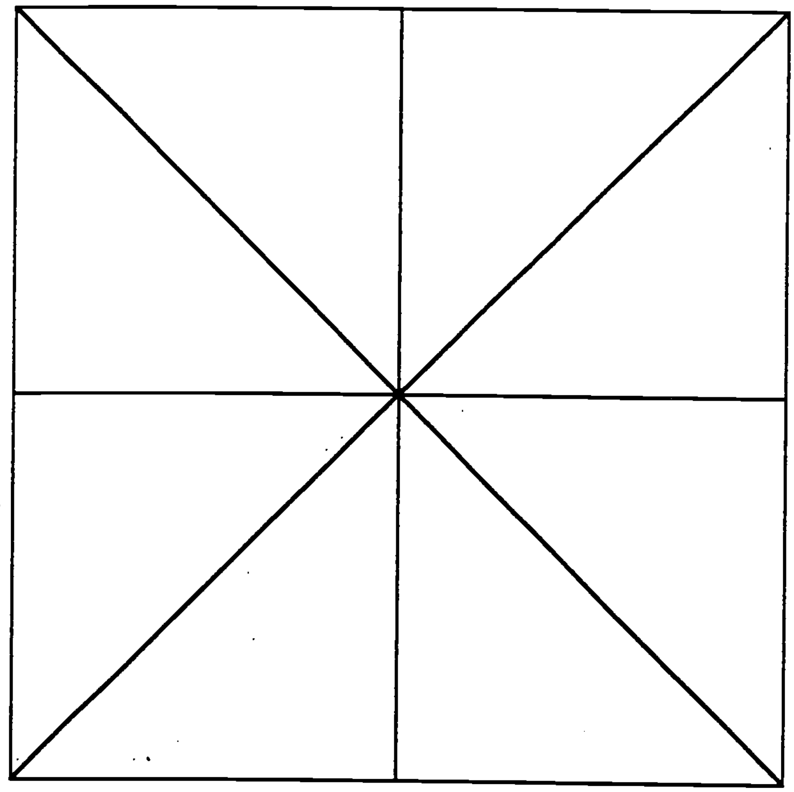
Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.

Thinking About Congruent Triangles



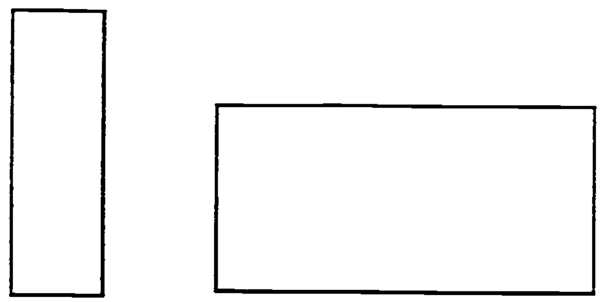
Your teacher told you in class today that the square above is divided into 8 congruent triangles. After you got home your best friend called and said he did not know what that meant. What would you say to your friend to help him understand?

From *Competency-Based Education Assessment Series, Fourth Grade Mathematics*, Ohio Department of Education, 1997.

Shapes: Rectangles and Triangles

Dear Student,

We are interested in what you know about rectangles and triangles.
Please look at these two rectangles:



1) Tell us the ways you think these rectangles are different from each other:

2) Tell us the ways you think the two rectangles are alike:

3) What is the same about all *triangles*?

4) Please draw two triangles that are as different from each other as you can make them. Use the space below.

5) Tell us the ways your triangles are different from each other.

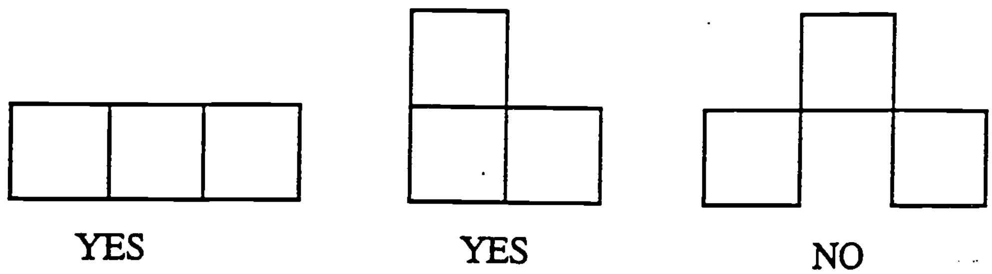
Thank you,

The Triangle/Rectangle Coordinating Committee

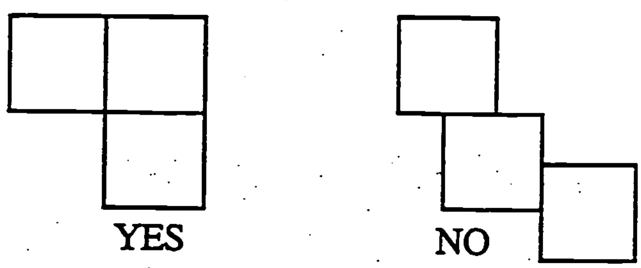
Building with Squares

On the next page, you will draw shapes made from four squares. There are two rules for your shapes:

1) All squares have to touch at least one other square with at least one side.



2) The sides must join evenly with each other and not overlap.



4.2; 4.4

On the grid below, draw all the shapes you can make using four squares. If you see that one shape is the same as another but just turned a different way, erase it or cross it out.

K-7

K7. A thin wire 20 centimeters long is formed into a rectangle. If the width of this rectangle is 4 centimeters, what is its length?

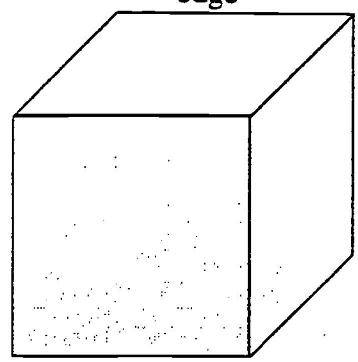
A. 5 centimeters

B. 6 centimeters

C. 12 centimeters

D. 16 centimeters

L5. This picture shows a cube with one edge marked. How many edges does the cube have altogether?



A. 6

B. 8

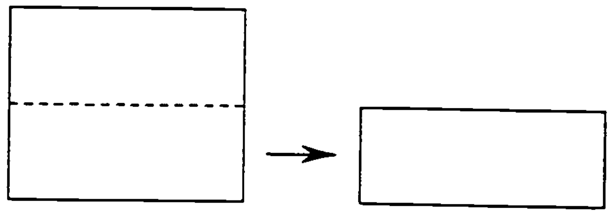
C. 12

D. 24

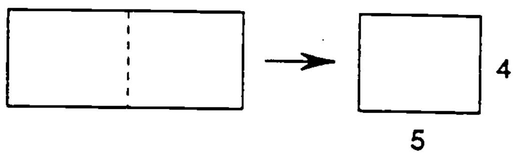
L-5

Rug Problem

Kim folded the rug in half.



Then she folded it again.

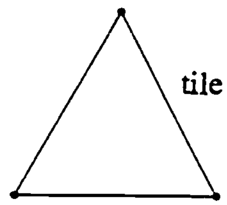


How big was the rug when it was opened?

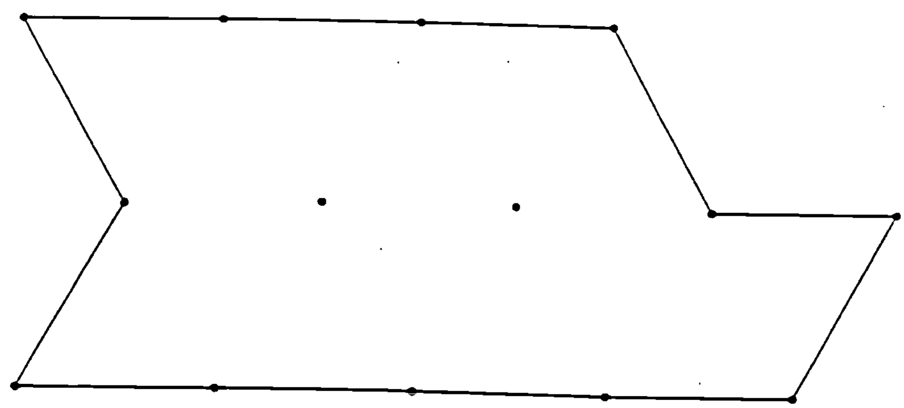
A. 4 × 5
B. 5 × 8
C. 4 × 10
D. 8 × 10

From *A Sampler of Mathematics Assessment*, California Department of Education, 1991.

U1. The triangle represents one tile in the shape of a triangle.



How many tiles will it take to cover the figure below?



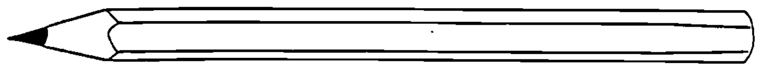
Number of tiles: ____________________

Use the figure above to show how you worked out your answer.

U-1

K-5

K5. About how long is this picture of a pencil?



A. 5 cm

B. 10 cm

C. 20 cm

D. 30 cm

5.1; 5.3

S-5

S5. Here is a paper clip.

← Length →

About how many lengths of the paper clip is the same as the length of this line?

Answer: ______________________________

46

5.1; 5.4; 5.5

M-7

M7. Which of these would most likely be measured in milliliters?

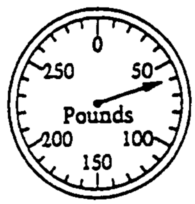
A. The amount of liquid in a teaspoon

B. The weight (mass) of a pin

C. The amount of gasoline in a tank

D. The thickness of 10 sheets of paper

Scale Problem



What is the weight shown on the scale?

A 6 pounds

B 7 pounds

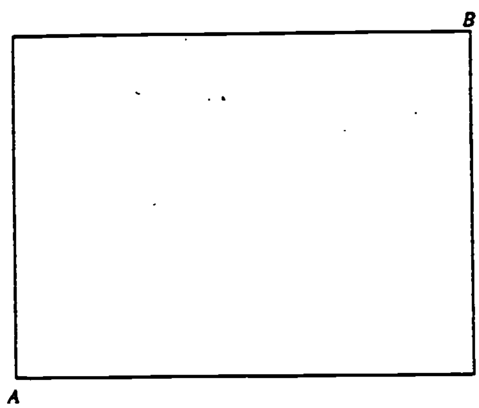
C 51 pounds

D 60 pounds

5.1

Rectangle Problem

(Size reduced from original)



Use your centimeter ruler to make the following measurements to the <u>nearest centimeter.</u>

What is the length in centimeters of one of the longer sides of the rectangle?

Answer: ______________

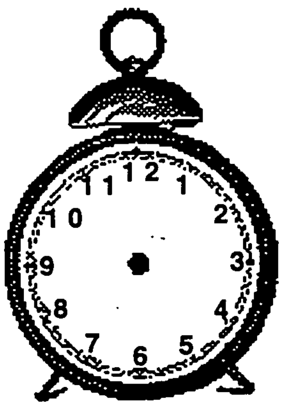
What is the length in centimeters of the diagonal from *A* to *B* ?

Answer: ______________

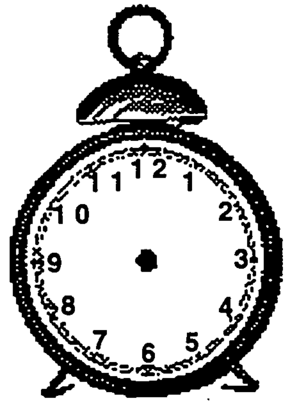
5.1

Time at the Birthday Party

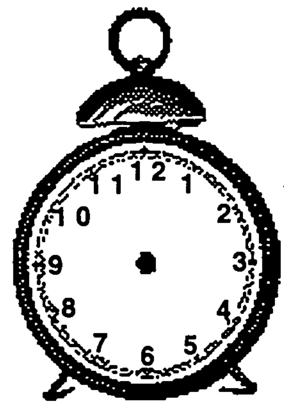
Draw hands on each clock to show the time given. You may use the drawings to help answer the questions on the next page.



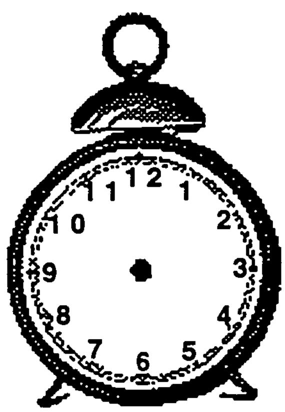
The party started at 1:00 o'clock.



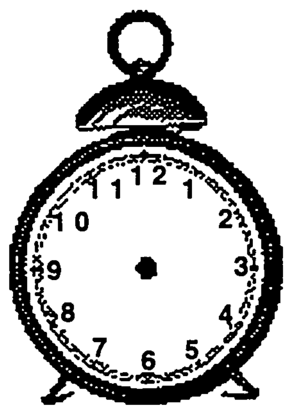
The party was over at 3:30 o'clock.



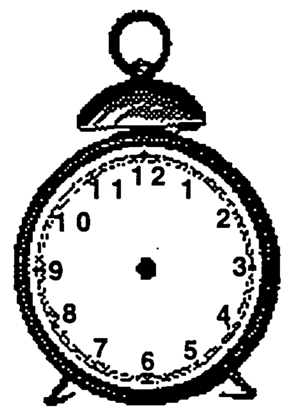
Mary and Tom opened their presents at 2:15 o'clock.



Mary and Tom 's mother cut the cake at 2:45 o'clock.



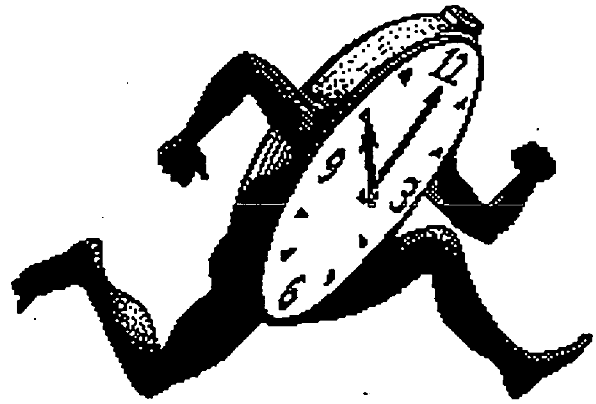
Susan's mother picked her up 10 minutes before the party was over.



It took Mary and Tom 1 hour and 40 minutes to clean up after the party was over.

From *Competency-Based Education Assessment Series, Fourth Grade Mathematics*, Ohio Department of Education, 1997.

5.1 ; 5.5



How long did the birthday party last?____________

If Susan got to the party when it started, how long was she at the party?____________________

What time did Mary and Tom finish cleaning up?________________

If the cake was cut as soon as Tom and Mary finished opening their presents, how long did it take them to open their presents?___________

After the party started, how long did Tom and Mary wait to open their presents?_______________

In the box below write another problem about time at Tom and Mary's Birthday Party.

From *Competency-Based Education Assessment Series, Fourth Grade Mathematics*, Ohio Department of Education, 1997.

J-6

J6. Which of these is largest?

A. 1 kilogram

B. 1 centigram

C. 1 milligram

D. 1 gram

L-8

L8. Four children measured the width of a room by counting how many paces it took them to cross it. The chart shows their measurements.

Name	Number of Paces
Stephen	10
Erlane	8
Ana	9
Carlos	7

Who had the longest pace?

A. Stephen

B. Erlane

C. Ana

D. Carlos

L6. The weight (mass) of a clothespin is 9.2 g. Which of these is the best estimate of the total weight (mass) of 1000 clothespins?

A. 900 g

B. 9 000 g

C. 90 000 g

D. 900 000 g

L-6

T3. Mr. Brown goes for a walk and returns to where he started at 07:00. If his walk took 1 hour and 30 minutes, at what time did he start his walk?

Answer:______________________________

T-3

J-4

J4. 25×18 is more than 24×18. How much more?

A. 1

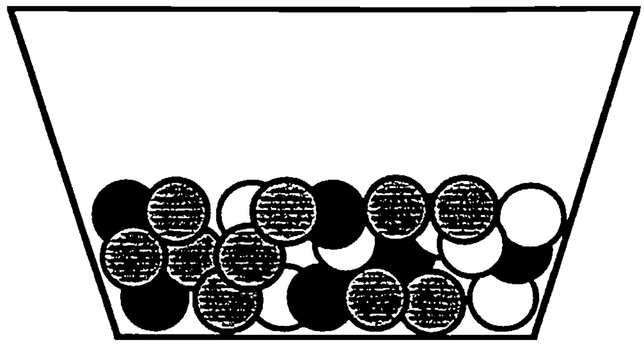
B. 18

C. 24

D. 25

Jessie's Marbles

Jessie keeps her marbles in this bowl. Right now there are 37 marbles in the bowl.



About how many marbles will the whole bowl hold?

Explain why you think your guess is reasonable.

6.4; 6.5

J-8

J8. Elena worked 57 hours in March, 62 hours in April, and 59 hours in May. Which of these is the BEST estimate of the total number of hours she worked for the three months?

A. 50 + 50 + 50

B. 55 + 55 + 55

C. 60 + 60 + 60

D. 65 + 65 + 65

I3. When you subtract one of the numbers below from 900, the answer is greater than 300. Which number is it?

A. 823

B. 712

C. 667

D. 579

COLORADO
MODEL CONTENT STANDARDS
FOR
MATHEMATICS
Kindergarten to Fourth Grade
Assessment Framework

Sample Constructed-Response Tasks
with Accompanying Scoring Guides
and Student Work

Constructed-Response Task #1
Pizza Problem

Source: *Can Students Do Mathematical Problem Solving?*
National Center for Education Statistics,
Office of Educational Research and Improvement,
U. S. Department of Education, 1993.

Grade 4 Question: Pizza

The Task

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work.

José ate $\frac{1}{2}$ of a pizza.

Ella ate $\frac{1}{2}$ of another pizza.

José said that he ate more pizza than Ella, but Ella said they both ate the same amount. Use words and pictures to show that José could be right.

Possible Solution

Jose would be right if the size of his pizza was larger than the size of Ella's pizza. More generally, students are expected to communicate by pictures and/or words that half of a larger quantity is more than half of a smaller quantity.

Students with only a naive understanding of the meaning of "1/2" in the context of the given task are likely to indicate "1/2 = 1/2" because they do not realize the potential for the two quantities being compared, the pizzas, to be different in size. Students with a higher level of comprehension can show some evidence that size is an important factor but are unable to convey how the comparison of the two pizzas is related to their relative sizes. Students with the highest level of understanding of the meaning of "1/2" in the context of the given problem can demonstrate responses that, at least informally, demonstrate what the fraction 1/2 means in terms of relative sizes of pizzas.

Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Scoring Guide and Sample Responses

<u>Rating and Performance Category</u>

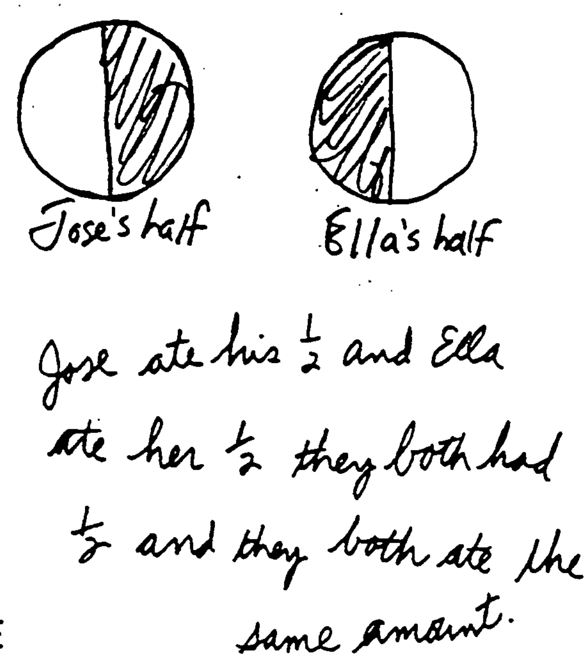
0 No Response

1 Incorrect -- The work is completely incorrect or irrelevant, or the response states, "I don't know."

This INCORRECT response does not involve the concept of one-half of a whole pizza.

2 Minimal -- Student responds that "1/2 is always 1/2" indicating an awareness of fractional parts. Other responses may include only references to number of pizzas or to toppings.

This MINIMAL response indicates an understanding of the concept of 1/2 as a fractional part of a whole, but states 1/2 is always equal to 1/2.

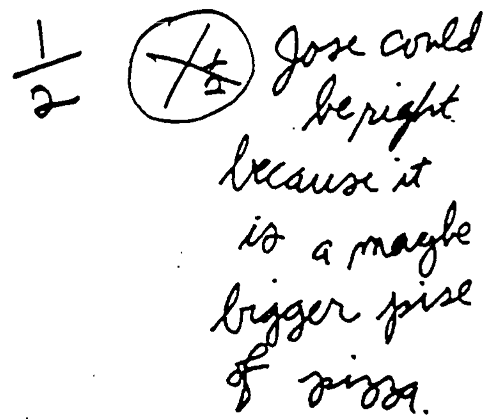


Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Rating and Performance Category

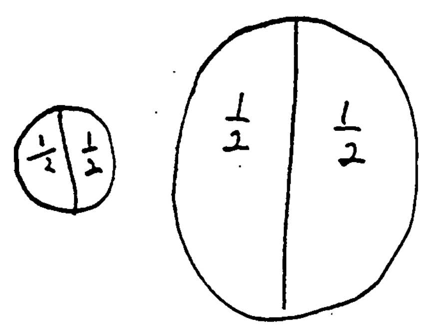
3 **Partial** -- Student makes statements such as "Jose's pizza has bigger pieces" that begin to demonstrate an awareness of the idea of relative size.

This PARTIAL response does give an indication that Jose's pizza may be larger.



4 **Satisfactory** -- Student displays responses that connect figuraly the relationship between the difference in the relative size of Jose's and Ella's pizzas but are not clear in explaining that relationship.

This SATISFACTORY response uses diagrams to clearly show two different-sized pizzas and to illustrate that the respective halves of those pizzas are not the same size.



BEST COPY AVAILABLE

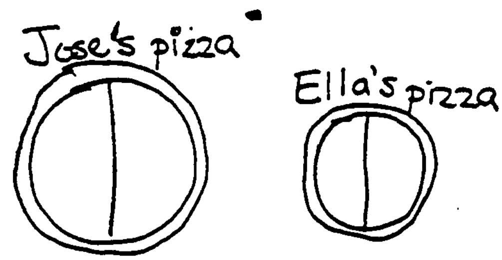
Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Rating and Performance Category

5 **Extended -- Student explains and/or demonstrates a clear understanding of fractional part and relative size.**

This strong EXTENDED response provides drawings of two different-sized pizzas, each divided into halves and labelled appropriately. The student also has written a clear and accurate description of the situation.

José could be right because his pizza could be bigger than Ella's.



Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Constructed-Response Task #2
Symmetry in Shapes

Source: *New Standards Released Tasks,*
National Center for Education and the Economy,
Washington, DC, 1995.

Symmetry in Shapes

In this task students are asked to draw two shapes that have line symmetry and to mark each line of symmetry with a dotted line. The prompt explains line symmetry and gives an example of a heart with the line of symmetry shown by a dotted line.

This task is designed to assess conceptual understanding of line symmetry. It fits into the Geometry and Measurement Standard.

Circumstances of Performance

When used as an assessment, this task is intended to take about 5 minutes. It may be used as part of a set of short tasks (for instance, in combination with other released New Standards tasks). Sets of short tasks should aim for a total estimated time of not more than 45 minutes. All New Standards short tasks are designed with the assumption that each student has a calculator, a ruler, and a pencil.

When used as a formal assessment, New Standards provided the following directions to teachers:

Before Students Begin

Make sure the students understand that today is an assessment of their individual work, so they may not work with or talk to each other. This is an "open book" assessment , which means that students may use their mathematics books or other resources they usually use in class during today's assessment. The suggested amount of time for the assessment itself is 45 minutes, but use your judgment: give students ample time so they do not have to rush.

During the assessment

You may read any or all tasks out loud to students who need help reading them. If you normally translate directions for some students, you may do so with this assessment. (If you normally have one student translate for another, you may allow this, but limit it to the first few minutes.)

While students work, do not give any hints or encourage any particular approach. Remind students that they may use calculators, as long as they show which operations they did in their booklet. Encourage them to write what makes sense to them and to persist until it makes sense.

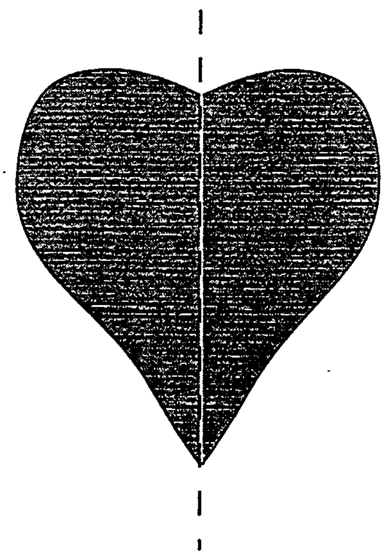
Arithmetic and Number Concept	Geometry and Measurement Concepts	Functions and Algebra Concepts	Statistics and Probability Concepts	Problem Solving and Reasoning	Mathematical Skills and Tools	Mathematical Communication	Putting Mathematics to Work

Mathematical Analysis

Product Student Creates	Links to the Standards
Two shapes with line symmetry	Conceptual Understanding • identifies, classifies, and names geometric figures by specific attributes and properties e.g., symmetry

Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.

Scoring "Symmetry in Shapes"

Possible Responses

Any shapes, other than the heart shape, with at least one line of symmetry are acceptable. Some responses may show shapes with more than one line of symmetry.

R/M Rubric for Symmetry in Shapes

To be "Ready for Revision (R)," the response essentially accomplishes the task or provides convincing evidence that the student has the mathematical power to do so. The key issue is whether the response shows "line of symmetry" in graphic representations. Ideally the response will contain two shapes, each drawn to include a single line of symmetry represented by a dotted line. Variations on this are acceptable provided the response still clearly indicates student understanding.

For instance: Multiple lines of symmetry may be presented. Solid lines instead of dotted may be drawn. Several shapes may be presented. They may have inner, unsymmetrical details drawn but it is the <u>outline</u> shape to be considered. Wording or measurements may accompany shapes as further explanation. If only one shape is drawn and an accurate line of symmetry marked, the shape must be of sufficient complexity to assure the scorer that line symmetry is understood. Lack of drawing skills should not penalize the student unless it is severe enough to interfere with clarity of communication as to whether or not the student has demonstrated sufficient understanding of the question

"More Instruction Needed (M)": If the reader is not confident that a note to the student would suffice to elicit a revision that accomplishes the task, the response is an "M".

These papers may not show a clear understanding of the task. There may be several shapes drawn, which may include one or more correct items, but the overall level of understanding is confusing. One or mores shapes may be drawn, but with an incorrect line of symmetry. Multiple lines on a shape may be drawn, one of which may be correct. But the addition of other, incorrect lines added lends doubt to the clarity of the response. The original heart-shaped example may be copied and presented as the only response, which is not considered sufficient. A fragmented shape, or individual lines, are also considered insufficient although those parts may form mirror images.

[response #014386] : score R

This response contains two shapes with multiple, correctly-drawn lines of symmetry plus clearly documented statements supporting these choices. A regular octagon has additional lines of symmetry not shown in this response.

[response #084381] : score R

This is an example of a "typical" response showing two shapes with single dotted lines marking a correct line of symmetry on each.

[response #020537] : score R

This student has drawn more than the requested two shapes with solid lines correctly denoting their symmetrical aspect.

[response #027841] : score R

In this response, a single shape is drawn and its line of symmetry marked. It is of sufficient complexity to assure the scorer that there is an understanding of line symmetry.

[response #026042] : score M

This response contains two shapes, both showing two dotted lines. While one shape (the square) is entirely correct, this is not true for the triangle as the horizontal line depicted is not a line of symmetry thus showing evidence of a lack of conceptual understanding.

[response #084249] : score M

On this paper two shapes are drawn, but neither have any line of symmetry indicated. It is unclear why this student chose to make a dot in one corner of each shape.

[response #083478] : score M

This student has drawn a shape and marked a line through it, but clearly lacks the required understanding of a symmetrical figure.

Anchor Set List for Symmetry in Shapes

Grade level: Elementary Length of task: 5 minutes

These anchors are listed in the order in which they are referenced in the task rubric.

Student Responses	Score
014386	R
084381	R
020537	R
027841	R
026042	M
084249	M
083478	M

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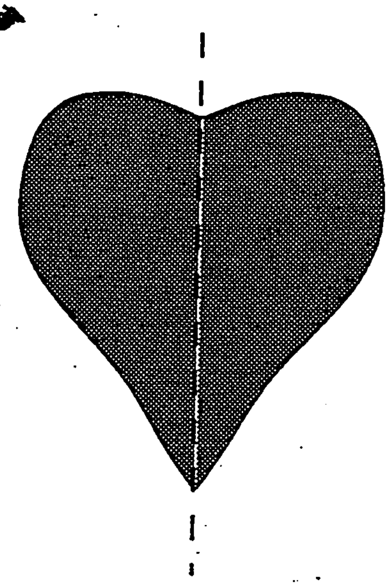
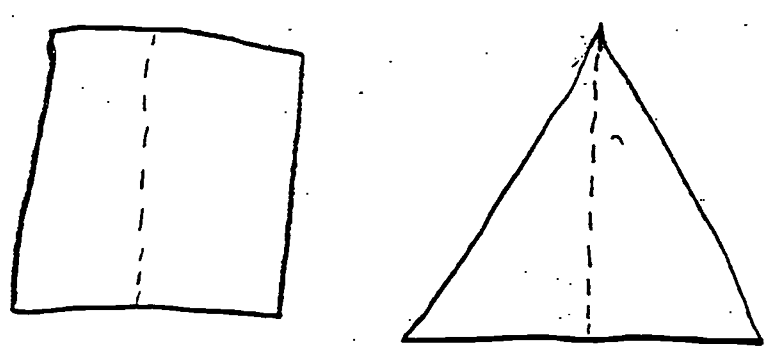
Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



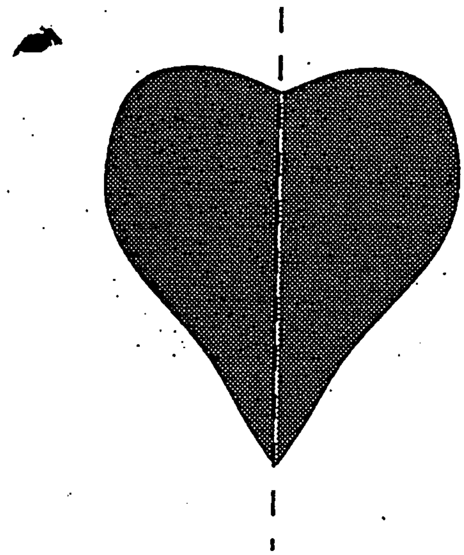
In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.



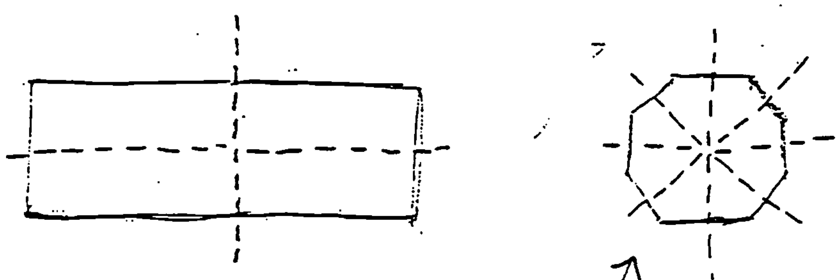
084381
181

Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.



I drew these shapes, and dotted it through the middle because if I had a paper like these shapes I could fold it in half and it will be the same.

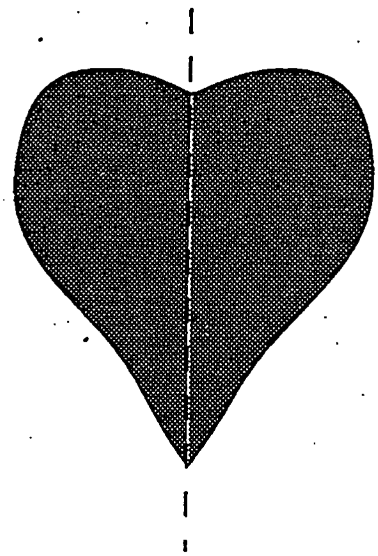
I could see lots of lines of semetry in this shape because the shape has many parts that you can fold to make symetry.

1971

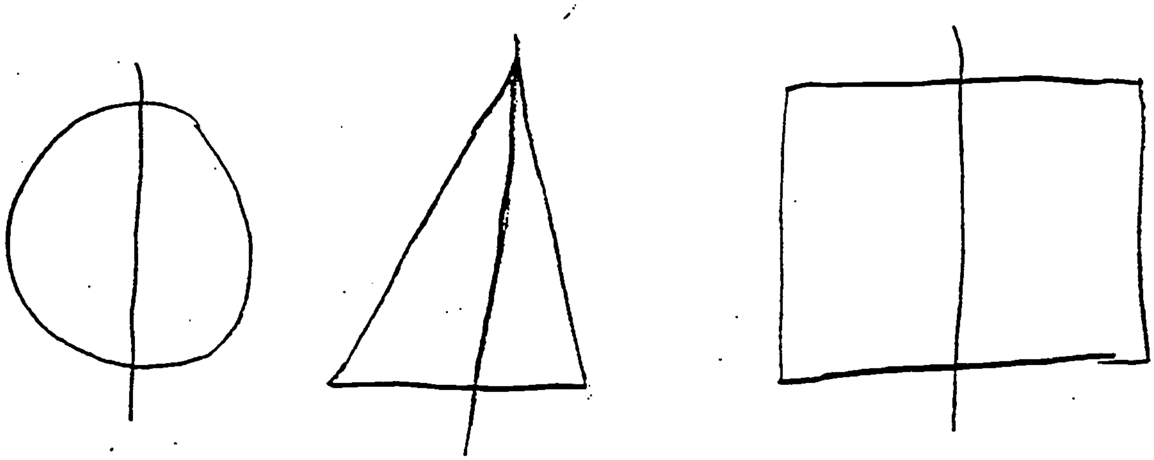
014386

Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.

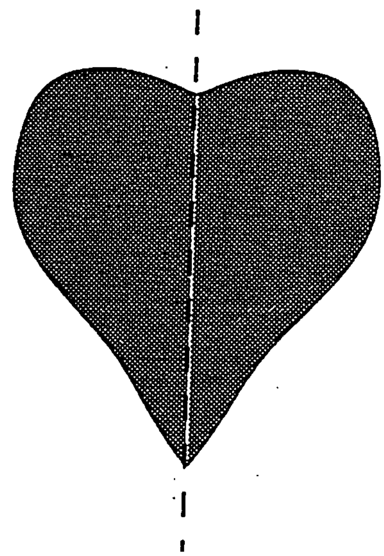


1521

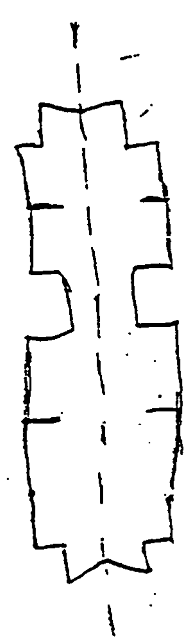
020537

Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



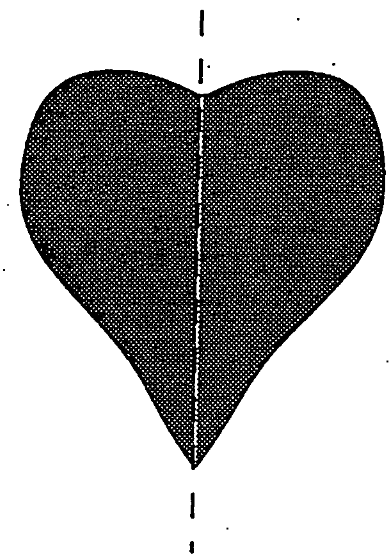
In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.



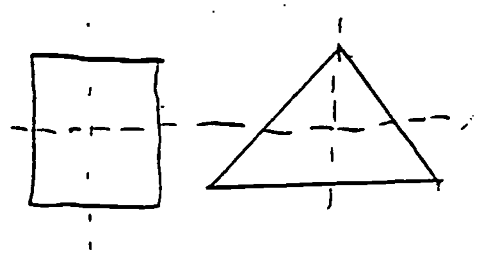
027841
181

Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.

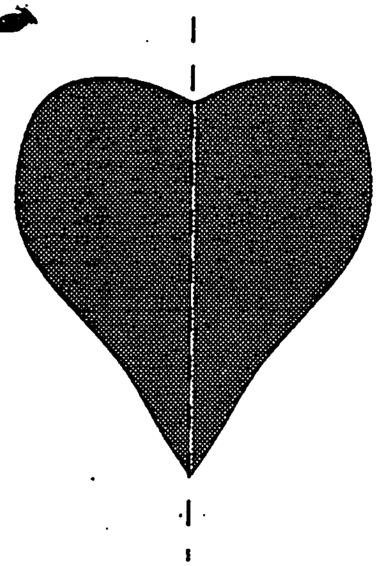


1 of 1

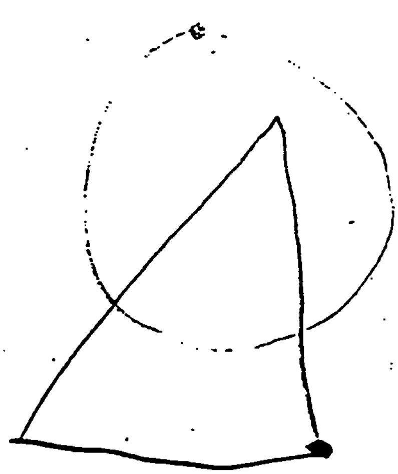
026042

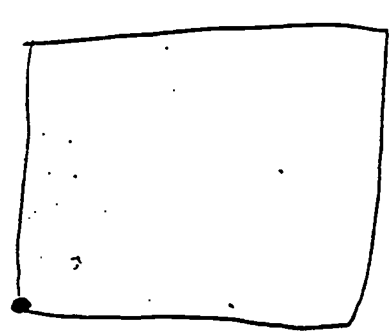
Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.

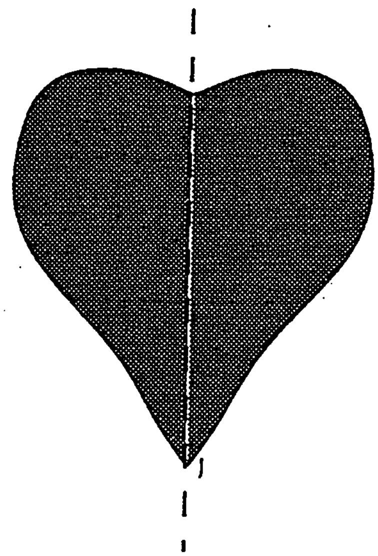




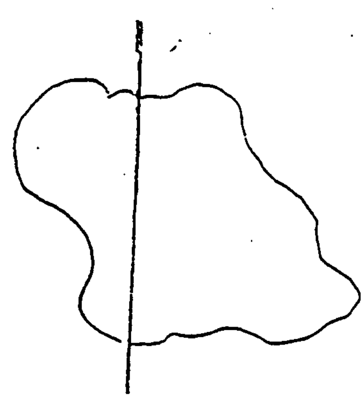
084249
181

Symmetry in Shapes

Some shapes have line symmetry. This means that you can you fold the shape so that it folds back onto itself perfectly. The heart shape below has one line of symmetry.



In the space below, draw two different shapes that have line symmetry. Mark the line of symmetry on each shape with a dotted line.



1 of 1
083478

Constructed-Response Task #3
Fair Game?

Source: *New Standards Released Tasks,*
National Center for Education and the Economy,
Washington, DC, 1995.

Fair Game?

Task Description

In this task students are asked to decide whether or not a spinner game for two children is fair. The spinner shows four sections, two shaded and two unshaded. One child gets points when the spinner lands on a shaded section, and the other child gets points when the spinner lands on an unshaded section. Students need to compare the shaded sections to the unshaded sections to determine whether each child has the same chance of winning and whether or not the game is fair.

This task is designed to assess conceptual understanding of probability as described in the Statistics and Probability Concepts performance standard. To a lesser extent it assesses beginning proportional reasoning as described in the Arithmetic and Number Concepts performance standard.

Circumstances of Performance

When used as an assessment, this task is intended to take about 15 minutes. It may be used as part of a set of short tasks (for instance, in combination with other released New Standards tasks). All New Standards short tasks are designed with the assumption that each student has a calculator, a ruler, and a pencil.

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Arithmetic and Number Concepts	Geometry and Measurement Concepts	Function and Algebra Concepts	Statistics and Probability Concepts	Problem Solving and Mathematical Reasoning	Mathematical Skills and Tools	Mathematical Communication	Putting Mathematics to Work

Mathematical Analysis of This Task

Links to the Performance Standards

Standard #4: Statistics and Probability Concepts

- predicts and finds out why some outcomes are more likely, less likely, or equally likely

Standard #1: Arithmetic and Number Concepts

- describes and compares quantities by using simple fractions, decimals, and whole numbers up to 1,000,000

Holistic Mathematical Analysis of This Task

This problem represents an early excursion into theoretical probability. Elementary school students are not expected to figure out the exact quantitative probabilities. However, in qualitative terms, students should see that the pointer is more likely to land on the unshaded part of the spinner.

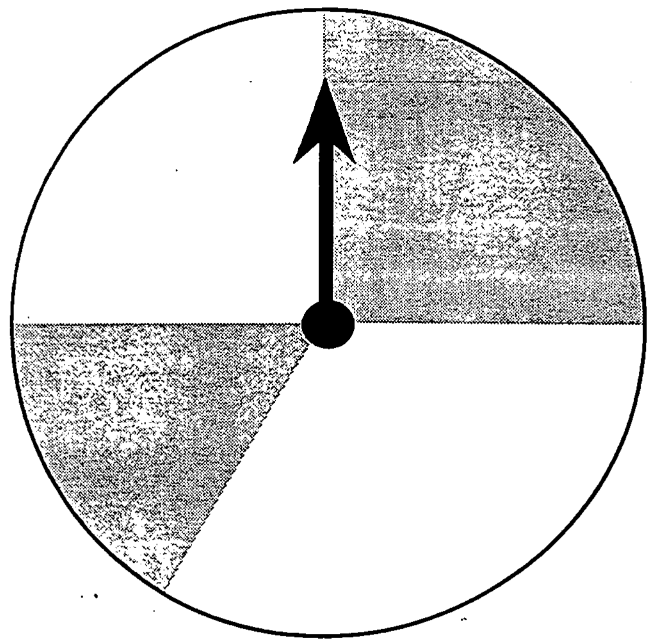
This may seem obvious to adults, but it's worth examining how we know that. It's tempting to say, "it's just like fractions," but there's a difference: in the circular, "pie" representation of fractions, the *area* of the piece represents the quantity; in a spinner, it is the *angle* of the needle that determines the outcome of the spin. Since the lines radiate from the center, these two are equivalent in this case—but even if the shading were only along the perimeter, the meaning of the spinner would be clear, though the fraction analogy (unless you mean a fraction of the circumference) would be lost.

The main stumbling block in this problem for students is the confusion over equally likely outcomes. Each side has two pieces, after all. If you had put two blue and two red cubes in a bag and drew them out (replacing each cube after drawing it out) to figure out the game, it would be fair. But here, we assume that, if it is a fair spinner, each *angle* is equally likely, not each piece. This is a continuous rather than a discrete means of generating randomness: in this way a spinner is unlike dice, coins, or blocks pulled from a bag.

A final note on fairness. Notice that the prompt makes no distinction between the terms "fair" and "an equal chance of winning." To many elementary-school students, these concepts are equivalent; nevertheless we adults need to keep in mind that they are not always the same. Equal chance is mathematical. Fairness is value-laden and embodies many subtleties. If both parties think it's fair, is it fair? If your sister is older, is it fair that she get the bigger room? If you like chocolate better than I do, is it fair to give you a bigger piece? If I have just won a lot of games, is it fair to let you have the unshaded part? Though it would make things a lot simpler, equality is not equity.

Fair Game?

Imagine that you and a friend are playing a game with this spinner:



Here are the rules:

- The two of you take turns spinning.
- For every spin, you get a point if the spinner lands on a shaded part.
- Your friend gets a point if the spinner lands on an unshaded part.
- The first person to get to 25 points wins.

Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

110

Scoring "Fair Game?"

Possible Responses

My friend and I do not have the same chance of winning. If you look carefully at the spinner you will see that the top left unshaded section of the spinner and the top right shaded section of the spinner are both the same size. But the bottom shaded section is much smaller than the bottom unshaded section of the spinner. Since the shaded part of the spinner is smaller than the unshaded part, the spinner is more likely to land on unshaded. That means my friend has a better chance of winning than I do, so the game is not fair.

Full Rubric for "Fair Game"

Separating R from M

To be "Ready for Revision (R)," the response must either accomplish the task or provide convincing evidence of the mathematical power to do so.

The key issue which separates the R's from the M's focuses on identifying the relationship of the size of the parts to the probability of winning the game.

For a score of "R", the response must show recognition of the unfairness of the game and provide some evidence to support that conclusion. The response notes that the parts are unequal and explains how that affects the chances of winning.

Ideally the evidence comes from drawings, charts and commentary, but this is not a requirement of work scored "Ready for Revision"; the response need only convince us that enough has been learned about the concepts for an unassisted revision to accomplish the task and solve the problem.

If the reader is not confident that a note to the student would suffice to elicit a revision that accomplishes the task, the response is scored "M": more instruction needed.

An "M" response may fail to identify the relationship between the probability of winning and the size of the parts. An "M" response may state that the game is fair, or it may correctly state that the game is unfair, but give no explanation or use faulty reasoning.

4: Accomplishes the Task

The response achieves the prompted purpose, if it clearly indicates that the student understands the relationship between the size of the parts and the probability of winning. The response must either explicitly state that the game is not fair or that the friend has a better chance of winning and relate this to the relative sizes of the shaded and unshaded portions of the spinner in the explanation. Fractional parts may be specified but are not required by the prompt to achieve a score of "4".

Response #011667: score 4

The response clearly addresses issues of fairness and chance of winning. The student states that more of the spinner is unshaded than shaded.

Response #031480: score 4

Although it does not directly respond to the fairness question, the response does explicitly state that the friend has a better chance of winning the game. The response contains written commentary and drawings which show that the student recognizes that the unequal sections of the spinner make the game unfair.

3: Ready for Revision

The response gives evidence that the student can revise the work to a "4" with the help of a note or a focusing question. The student does not need a dialogue or additional teaching. Any overlooked issues, misleading assumptions, or errors in execution—to be addressed in the revision—do not subvert the scorer's confidence that the student has learned the relevant mathematics to accomplish the task. The response must show that the student recognizes that the game is unfair and provide some explanation. The explanation may be somewhat incomplete or unclear. For example, the response may state that the size of the shaded section is smaller than that of the unshaded section without explaining why that makes the game unfair. Minor errors such as stating both shaded sections are smaller than the unshaded sections may be evident.

Response #028915: score 3

The explanation, though very brief, answers the question and *implies* unfairness by stating "the parts are not even." Since it does not explicitly state either that the game is unfair, or that the friend has a better chance of winning, or how the student came to this conclusion, it is scored "3".

Response #011653: score 3

The response shows the student recognizes the unfairness of the game. The explanation contains a minor flaw in stating that both of the shaded parts are smaller than both of the unshaded parts.

2: Partial Success

Part of the response is successful, but there is a lack of evidence—or evidence of lack—in some areas needed to accomplish the whole task. Part of the task is accomplished, but the student is not ready to revise the work without a conversation or more teaching. The student has attempted to address the prompt. The response indicates that the game is not fair. Either there is no explanation as to why the game is unfair or the explanation clearly does not support the conclusion or errors are evident.

Response #030981: score 2

The response states that the game is not fair. The explanation makes no mention of unequal sections and indicates a lack of understanding of the rules of the game.

Response #084239: score 2

The response states that the game is not fair. The explanation is incoherent and does not support the fairness decision.

1: Engaged with Little Success

The response shows an attempt to use the information given in the problem. It may contain actual fragments of appropriate material from the situation in the prompt, and may show effort to accomplish the task, but with little or no success. The response indicates that the student misinterprets or does not respond to the given prompt. The response states that the game is fair or may correctly state that the game is unfair, but uses reasoning that contradicts the conclusion.

Response #020760: score 1

The response says that the game is fair and describes both the shaded and non-shaded sections as halves.

Response #020828: score 1

The response fails to show recognition that the game is not fair. Drawings illustrate taking turns, but do not show the unequal sections, indicating a lack of understanding of the prompt.

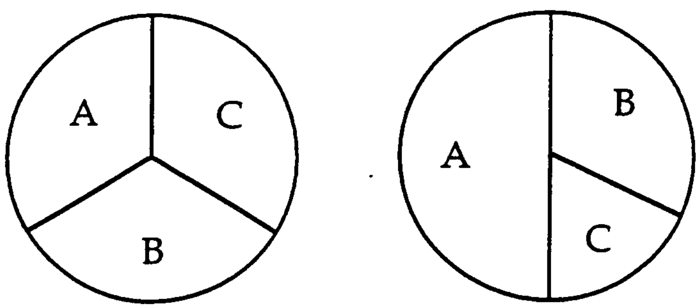
0: No Response or Off Task.

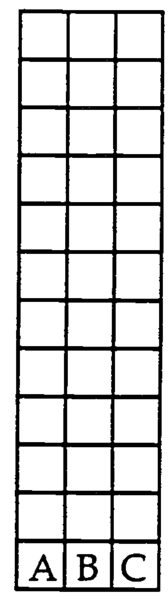
There is no evidence that the task was engaged. When the response is blank, it is scored a NR. When there are marks, words or drawings unrelated to the task, it is scored OT.

Issues for Instruction and Learning

When More Instruction is Needed

If students have not had much experience with probability, they will need many opportunities to engage in a wide variety of probability experiments. Consider making spinners, some with equal sections and some with unequal sections, and recording sheets, like this:





One day students spin the spinners with equal sections and keep track of which letters "win." ("Winning" means the letter that gets to the top of the chart first.) At the end of the day ask them to post all the recording sheets on which A "won" in one area of your chalkboard, the ones on which B "won" in another area and the ones on which C "won" in a third area. Students can work in pairs to spin the spinners and record which letter "wins". The next day have them conduct the same experiment, but with the spinners that have sections of unequal size. Again post the entire class's results. Discuss which letters "won" most often on each day and why the results differed on the two days.

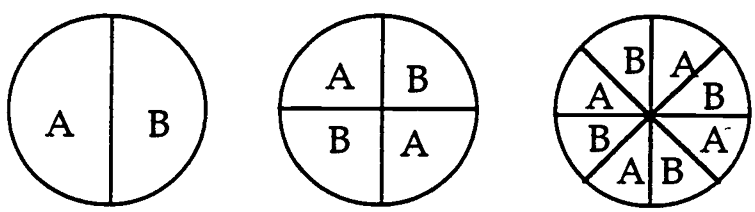
When Students Make Revisions

A score of "3" on these tasks indicates that they represent strong work in the context of a first draft. However, consider asking your students to revisit these tasks to revise their work so that it fully accomplishes the task. If this is the first time your students will be revising a piece of mathematics, it might be a good idea to discuss the differences between "a good first draft" and "fully accomplishing the task". Consider making transparencies of a "3" response (perhaps one of the anchor papers) and asking your class to figure out what features of this response need to be revised.

At first, a note from you to each student can focus them on the particular aspect of their response which needs more thought. As students gain experience in the revision process and become more conscious of the benefits of revising and more proficient at revising their work, they should become increasingly capable of finding errors or omissions in their work themselves.

Extensions

Consider asking students to design several fair spinners for this game. Some examples are:



You can take this extension further by asking students to share the different designs they came up with. The class could then investigate if there is or isn't a difference in how fair these spinners actually are. Does it make a difference, for example, if the first spinner or the third spinner above gets used? This could lead to a class discussion about the difference between theoretical probability and the actual results in a small sample. The class would have to decide how to compare the spinners. How many times does each spinner need to be tested? Is one game on each spinner enough?

The following book is excellent resource for teaching probability:

Math By All Means
Probability: Grades 3-4
by Marilyn Burns

Anchor Set List for Fair Game?

Grade level: Elementary   Length of task: 15 minutes

These anchors are listed in the order in which they are referenced in the task rubric.

Student Responses	Score
011667	4
031480	4
028915	3
011653	3
030981	2
084239	2
020760	1
020828	1

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Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

The game is not fair because There is more unshaded than Shaded parts. So that means you both don't have the same chance of winning because there is more unshaded. So your friend would win.

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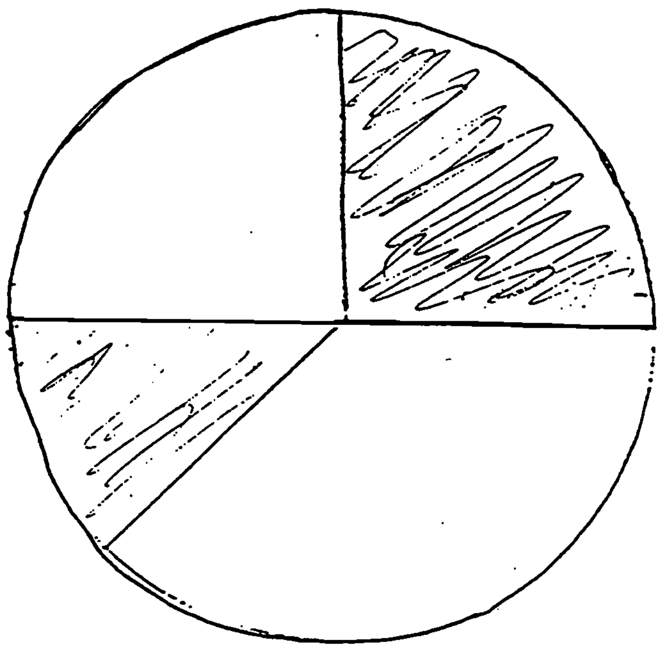
Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

No. My friend has a better chance ob winning because her parts are bigger than mine. My friend wauld probably land on hers more often than on mine.

Here the parts are equal

1. →



← 2. Down here the parts are not equal.

Key
▩ my part
☐ my friend part

Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

no, because The parts are not even

Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

I do not think this game is fair because I do not have a good chance of wining. because both of my shaded parts are smaller than my friends white part. My friend has a bett chance of wining. Her part of the spinner are bigger.

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Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

No because the shaded part is unequal so your friend could spin it and land on the un shaded then you spin you might land on th shaded ~~so~~ it will get your friend a head.

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Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

it no fair becans both of the half unshade and shade
25 if get unshade.

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Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

We have the same chance of winning because its on half shaded and half not and that equals $\frac{1}{2}$

Is fair because you wouldn't be able to call that apoint because it half on the white but it wouldn't not be apoint

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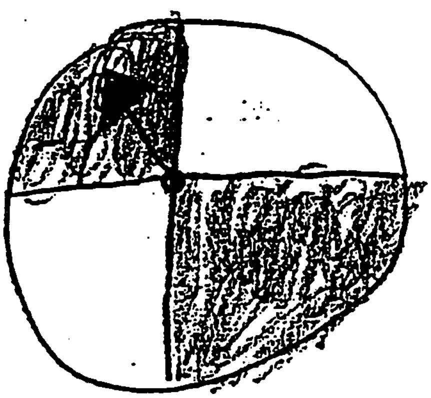
Do you and your friend each have the same chance of winning?

Decide whether the game is fair and explain how you decided.

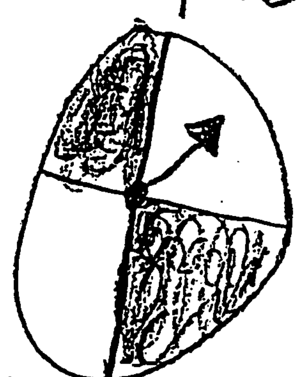
That When I spin and land on black I get a ponte. When he spins and lands on wight he gets a ponte.

I Spines



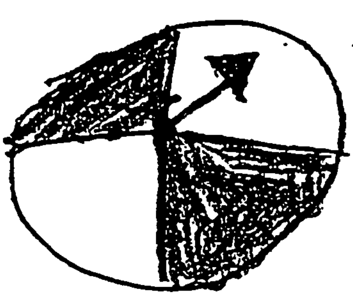
I got a pont!!!

I spines



I have to try agen

he spines



he gets a pont.

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