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ABSTRACT

The Colorado Student Assessment Program (CSAP) has been underway since 1997. Since then, fourth grade students have been assessed annually on the Colorado Model Content Standards for Reading and Writing. In 1998, testing began on third grade reading. Over the course of time, other grades and subject areas will be phased in. In the spring of 2000, public school eighth grade students will be assessed for the first time on the Colorado Model Content Standards for Mathematics. This guide was developed to help educators prepare for this upcoming mathematics assessment. The purpose of this book is to provide a window into eighth grade mathematics assessment. It is divided into four sections: (1) an overview of CSAP with a question-and-answer section about test development, scoring, and reporting; (2) the "Colorado Model Content Standards for Mathematics Fifth to Eighth Grade Assessment Framework"; (3) referenced tests (sample assessment items) for the "Fifth to Eighth Grade Assessment Framework"; and (4) three sample constructed-response tasks (one 5-minute and two 15-minute tasks) with accompanying scoring guides and student work. (ASK)

Teachers' Guide to the Colorado Student Assessment Program for Eighth Grade Mathematics: An Assessment of Fifth through Eighth Grade Benchmarks

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Teachers' Guide to the Colorado Student Assessment Program for Eighth Grade Mathematics: An Assessment of Fifth Through Eighth Grade Benchmarks

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Special thanks to members of the Colorado Fifth to Eighth Grade Mathematics Assessment Framework Task Force for their expertise and hard work in developing this guide. In particular, Fran Berry's professional knowledge, dedication, and hard work are greatly appreciated.

Also, special thanks to Deborah Lynch for composing the text of Section A, and to Henry Heikkinen, Vonda Kiplinger, Stevi Quate and Don Watson for their careful editing and constructive comments.

Section A

Introduction

The Colorado Student Assessment Program (CSAP) has been underway for three years. Since 1997, fourth graders have been assessed annually on the *Colorado Model Content Standards for Reading and Writing*. In 1998, testing began on third grade reading. Over the next three years, other grades and subject areas will be phased in.

In the spring of 2000, public school eighth graders will be assessed for the first time on the Colorado *Model Content Standards for Mathematics*. The *Teachers' Guide to the Colorado Student Assessment Program for Eighth Grade Mathematics: An Assessment of Fifth through Eighth Grade Benchmarks* was developed to help educators prepare for the upcoming mathematics assessment.

This guide is divided into four sections:

(1) an overview of CSAP with a question-and-answer section about test development, scoring, and reporting;
(2) the *Colorado Model Content Standards for Mathematics Fifth to Eighth Grade Assessment Framework*;
(3) referenced tasks (sample assessment items) for the *Fifth to Eighth Grade Assessment Framework*; and
(4) three sample constructed-response tasks (one, five-minute task and two, fifteen-minute tasks) with accompanying scoring guides and student work.

The sample assessment items are similar to those that may appear in the CSAP for eighth grade mathematics and are provided for illustrative purposes only.

It is natural for teachers and administrators to be anxious about a high profile assessment. The purpose of this booklet is to ease those concerns and provide a window into the eighth grade mathematics assessment.

You are encouraged to share this guide with your colleagues. Feel free to copy any or all parts of the document.

All About CSAP

The Colorado Student Assessment Program (CSAP) has been underway for three years. Since 1997, fourth graders have been assessed annually on the Colorado *Model Content Standards for Reading and Writing*. In 1998, testing began on third grade reading. Over the next three years, other grades and subject areas will be phased in, including the assessment of eighth grade mathematics starting in the spring of 2000.

CSAP Assessment Schedule

1997	4th grade Reading Writing				
1998	4th grade Reading Writing	3rd grade Reading			
1999	4th grade Reading Writing	3rd grade Reading	7th grade Reading Writing 5th grade Mathematics*		
2000	4th grade Reading Writing	3rd grade Reading	7th grade Reading Writing 5th grade Mathematics*	**8th grade** **Mathematics** Science	
2001	4th grade Reading Writing	3rd grade Reading	7th grade Reading Writing 5th grade Mathematics*	**8th grade** **Mathematics** Science	10th grade Reading Writing Mathematics

*All testing will take place in the spring with the exception of the fifth grade mathematics assessment which will take place in the fall.

CSAP measures students' knowledge of the state model content standards and benchmarks. Benchmarks in the state standards are grade-level expectations, specifying what students should know by the end of fourth, eighth, and twelfth grades. The statewide assessments are cumulative exams, based on what students have learned up to the time of the assessment. **Therefore, the spring eighth grade mathematics CSAP will assess students on the fifth through eighth grade benchmarks.**

As educators know, there are many different levels and forms of assessment. The most complete form of assessment takes place in the classroom. Teachers assess students on a daily and weekly basis. Most school districts also regularly assess students in certain grade levels. CSAP is the first large-scale assessment used in Colorado to measure student achievement relative to the state model content standards. Thus, results will yield information about how well schools and students are performing statewide, using a common yardstick — the state content standards.

The following are questions and answers about test development, scoring, and reporting of results. For more information about CSAP, refer to the Colorado Department of Education's web page at www.cde.state.co.us or contact your district assessment director.

Q. What is the purpose of CSAP?

A. The Colorado Department of Education (CDE) is administering CSAP to give parents, the public, and educators a uniform source of information about how well students and schools are achieving. Colorado schools have been engaged in efforts to improve education for all students through standards-based education for the past decade. CSAP is one measure of students' performance relative to the content standards.

Q. What is CSAP based on?

A. CSAP is based on the Colorado Model Content Standards developed by Colorado educators and community members. These standards were reviewed by over 10,000 Coloradans. Though curriculum varies from district to district, expectations of what students are learning are consistent across the state, allowing for a fair and uniform assessment of Colorado students.

Q. How is CSAP developed?

A. CSAP is developed by a team of Colorado educators, curriculum and assessment experts, and the test publisher, CTB/McGraw-Hill. The process includes selecting and developing assessment questions (or items), setting scoring criteria, and establishing performance levels. Two types of assessment items will be used: selected-response and constructed-response. Selected-response items (e.g., multiple-choice, matching, true-false) are items in which the student chooses an answer from among a list of options. Constructed-response items (e.g., short answer, essay) are items in which the student gives a response in his or her own words.

Q. How are assessment items selected?

A. Approximately a year before the administration of a particular CSAP assessment, the test publisher meets with a group of Colorado educators to develop specifications for test development. These specifications include types of items (e.g., multiple-choice, short answer) and a thorough discussion of the meaning and content of the standards and benchmarks.

Approximately seven months before the administration of the assessment, a variety of assessment items, which align with the Colorado standards, are reviewed by Colorado educators to ensure their alignment with standards and corresponding benchmarks. Some items are revised or discarded and new items are either written or obtained from other sources.

Once this initial pool of items is agreed upon, four committees review each assessment item. The content committee, composed of teachers and assessment and curriculum specialists, reviews each item for grade and content appropriateness. Two other committees, composed of educators, parents, community members, and businesspersons, review each question to ensure it is fair, acceptable, and unbiased. The instructional impact committee, another group of educators, reviews all recommendations and then makes a final decision regarding each item. Only if a question passes these screenings is it included in the pool of items from which the assessment will be constructed.

Q. How are assessments scored?

A. Selected-response items are machine-scored. Constructed-response tasks are scored by hand using rubrics (scoring guides) that match the specific task requirements. Hand scoring the constructed-response tasks is a lengthy and intricate process. Generic rubrics outlining general expectations for satisfactory student work will be provided to schools in the demonstration booklets and CSAP updates (available on the CDE web site).

After the assessment is administered, the test publisher selects student papers for each performance item that represent a range of quality from excellent to poor. These "anchor" papers are scored, then given to Colorado educators. The educators evaluate the anchor papers to determine whether the publisher's judgements are in agreement with Colorado's expectations. These papers are then used to train scorers.

Q. Who scores constructed-response items?

A. The scorers are highly-trained individuals employed by CTB/McGraw-Hill. They use the rubrics and the identified anchor papers to score each constructed-response item. A sampling of each scorer's rated student papers is evaluated to ensure constructed-response items are scored reliably and accurately. This evaluation process includes three different types of reviews to ensure that each scorer's ratings are accurate and consistent. Any scorer who produces ratings that are inconsistent with expectations is retrained.

Q. How are performance levels determined?

A. The setting of performance levels begins after the student papers are scored. First, statisticians determine the degree of difficulty for each question and examine the range of scores. The degree of difficulty for each test item is based on students' performance. Questions that a majority of students answered correctly are identified as less difficult, while those questions that only a few students answered correctly are identified as more difficult. Then, items are ordered by degree of difficulty from least to most difficult, creating the Ordered-Item Booklet.

Next, each member of a group of Colorado educators and community representatives takes the student test and reviews the Ordered-Item Booklet. Since the Ordered-Item Booklet arranges test items from least to most difficult, each group member is asked to place "bookmarks" at points he or she considers student performance that is advanced, proficient, and partially proficient. As a group, members then discuss their individual placement of the bookmarks. After two rounds of discussion, individuals may choose to revise their bookmark placements.

The last round of bookmarking finalizes performance levels. The group writes descriptions of these performance levels based on the student skills and knowledge required at each level.

Q. What do performance levels tell us?

A. The four performance levels — Advanced, Proficient, Partially Proficient, and Unsatisfactory — describe what a student knows and is able to do at each level, and provide information to the school and district about the effectiveness of instructional programs. A student at the proficient level has achieved the knowledge and skills defined in the standards.

Q. How are results reported?

A. Assessment results are reported by four performance levels: Advanced, Proficient, Partially Proficient, and Unsatisfactory. Results are reported by performance levels for individual students, schools, districts, and the state. Each student will receive an overall performance level rating on the mathematics assessment, as well as an indication of performance on each mathematics content standard. Individual student scores are released only to the school and school district, which are encouraged to share this information with the student's parents. School, district, and statewide scores are public records. Student results are disaggregated by gender, race/ethnicity, and separate disabling condition. School results are also reported by socioeconomic status as determined by the number of students eligible for free- or reduced-cost lunch and by district size.

Q. How will schools and districts use assessment results?

A. It is up to each district to determine how to use individual student assessment results. By law, only locally-elected school boards may set graduation requirements, establish school curricula, and make policies for grade retention and promotion. Assessment results will most likely be used to aid teachers and administrators in improving curriculum and instruction.

The recently-passed accreditation law, House Bill 98-1267, requires all Colorado school districts to include state assessment results in their accreditation contract. Accreditation contracts must contain plans for improving student achievement on the statewide assessments.

Q. What will the Colorado Department of Education do with assessment results?

A. The Colorado Department of Education is required by law to report CSAP results for the state and for all local school districts. These results are released as public documents. In addition, each year 25% of assessment items are released and made available to educators, the public, and the media.

At this time, there are no state-imposed rewards or sanctions tied to the assessment results. However, CSAP results will be used in the new accreditation process established by the Colorado State Board of Education in accordance with House Bill 98-1267.

Q. Will results be used to compare schools and districts?

A. CSAP results are not intended to show comparisons among schools and districts. In fact, it would be misleading to use only one such measure to make judgments about the overall performance of schools.

Q. How will student anonymity be protected?

A. Only schools and parents will have access to individual student results. To protect student anonymity, smaller school districts (with 15 or fewer students per grade) will not have their results released publicly.

Q. How long do CSAP assessments take?

A. The length of testing differs depending on the content area and grade levels assessed. The mathematics assessment will require three, fifty-minute class periods, spread out over three days.

Q. How will I know how to administer the assessment?

A. Teachers will receive demonstration and test administration booklets in advance of the testing dates. Teachers giving the CSAP mathematics assessment will receive a demonstration booklet about six weeks prior to the assessment, and an administration manual will accompany student test materials. The booklet and manual thoroughly explain how to administer the assessments, contain example assessment items, and answer many questions teachers may have.

District assessment coordinators are encouraged to attend regional assessment training sessions offered by CDE. Participating coordinators are provided with test-administration materials to use in training building-level staff.

Q. Am I being graded or are my students being graded?

A. CSAP is an assessment of students and schools, not of individual teachers. What students have learned up to the time of the assessment and how well students do are the result of many factors, including all previous instruction from earlier grade levels and schools.

Q. Are all students expected to take the CSAP assessments?

A. Every student is expected to take the CSAP assessments, with a few exceptions. Each school district decides, based on guidelines from CDE, those students for whom the assessment is not appropriate. Appropriate accommodations, used in instruction, are allowed to assist students with special needs in taking the assessment. In calculating the percent of students at each performance level, **every** student in the grade being assessed is counted. Students who do not take the assessment are included in the CSAP report as "not assessed."

Section B

COLORADO
MODEL CONTENT STANDARDS
FOR
MATHEMATICS
Fifth to Eighth Grade
Assessment Framework

Colorado Model Mathematics Standards

Fifth to Eighth Grade Assessment Framework Task Force

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Colorado Model Content Standards
MATHEMATICS

1. **Students develop number sense* and use numbers and number relationships in problem-solving situations* and communicate the reasoning used in solving these problems.**

2. **Students use algebraic methods* to explore, model*, and describe patterns* and functions* involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.**

3. **Students use data collection and analysis, statistics*, and probability* in problem-solving situations and communicate the reasoning and processes used in solving these problems.**

4. **Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.**

5. **Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.**

6. **Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic*, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.**

STANDARD 1:

Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems.

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

1.1 demonstrating meanings for integers, rational numbers, percents, exponents, square roots, and pi (π) using physical materials and technology in problem-solving situations;

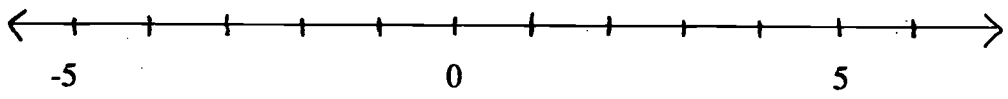
Examples:
(1) Shaded Circles Problem (Section C, page 1)
(2) Equivalent Algebraic Expression Problem (Section C, page 2)
(3) Shade Units on a Grid Problem (Section C, page 3)
(4) The area of a square is 5 square units. Find the length of one side of the square.
(5) Fractions of a Square (Available from Middle Grades Assessment Package 1, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

1.2 reading, writing, and ordering integers, rational numbers, and irrational numbers (namely square roots such as $\sqrt{2}$ and $\sqrt{5}$ and π);

Examples:
(1) Which number is largest?
(a) 2π (b) 2^3 (c) 7.5 (d) $\sqrt{32}$
(2) Number Order Problem (Section C, page 4)
(3) Closest to 0.52 Problem (Section C, page 4)
(4) Write a fraction that is larger than 2/7. Justify how you know your fraction is larger.

(5) Place the following numbers on the number line below, labeling each number:

$-2, \pi, \sqrt{2}, -3.5$



1.3 using multiple representations of numbers with number theory concepts (for example, primes, factors, multiples);

Examples:

(1) Fran is thinking of a number. She says, "When you divide my number by 5, the remainder is 2. My number has two digits, and both digits are odd. The sum of the digits is 12." What is Fran's number? Show how you know.

(2) Cliff has chosen a mystery number. His number is between 30 and 70, and it has exactly three factors. What could his number be? Show how you know.

(3) Secret Number (also, 1.5)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

(4) Disk Sum (also, 6.2)
(Available from Middle Grades Assessment Package 1, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

(5) Tile Signs (also, 4.4)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

1.4 using the relationships among fractions, decimals, and percents, including the concepts of ratios and proportions, in problem-solving situations;

Examples:

(1) More Boys or Girls in Class Problem (Section C, page 5)

(2) Ratio of Red Paint to Total Amount of Paint Problem ((Section C, page 6)

(3) Mary had some pencils. She gave half of her pencils to Steve. Steve then gave one-third of his pencils to Denise. Denise gave one-fourth of her pencils to Josh.
If Josh has five pencils, how many pencils did Mary have at the beginning?
Released mathematics assessment item, Oregon Department of Education

(4) The capacity of an elevator is either 20 children or 15 adults.
If 12 children are currently on the elevator, how many adults can still get on?
Released mathematics assessment item, Oregon Department of Education

(5) Allison's Paper Problem (also, 1.6)
Allison is ordering copy machine paper for the office. She can purchase a 10-pack box with 200 sheets in a pack for $18, or she can get a 5-pack box with 500 sheets in a pack for $20. The third option is to buy a box containing 10 packs of 600 sheets each for $55. Which is the better buy? Explain your thinking at each step and your answer.
Released mathematics assessment item,
Oregon Department of Education

(6) Pizza Price Differences Problem (also, 1.6, 4.4, 6.1)
The Pizza Palace has started to make square pizzas. This sounds good to Myra, who wants her family to get one for dinner. "No way," says her brother Kenji. "It costs too much. A 12-inch round pizza with four toppings costs $11.00 at Mondo's. The Pizza Palace wants $13.00 for their 12-inch square pizza with four toppings."
Show which pizza company charges more per identical size of piece of pizza.
Released mathematics assessment item, Oregon Department of Education

(7) The Better Airline Deal (Section C, page 7) (also, 1.6)

(8) Eating Pizza (Section C, page 8) (also, 1.6)

(9) Similar Triangles (Section C, pages 9-10) (also, 2.2, 4.2, 5.5)

(10) Science Fair (Section C, pages 11-12) (also, 6.1, 6.2)

1.5 developing, testing, and explaining conjectures about properties of integers and rational numbers; and

Examples:

(1) Tracy said, "I can multiply 6 by another number and get an answer that is smaller than 6." Pat said, "No, you can't. Multiplying 6 by another number always makes the answer 6 or larger." Who is correct? Give a reason for your answer.

From *NAEP 1992 Mathematics Report Card*, U.S. Department of Education, © 1993

(2) Is That Really True?
During a stock market report, the announcer was explaining how to figure out a stock market pattern. He multiplied a number by 0.49. The announcer then stated that he could get a very close estimate by dividing the original number by two.
Does dividing a number by two always produce an answer close to the product of the original number and 0.49? Explain your reasoning. Drawings, pictures and calculations may be included in your explanation.

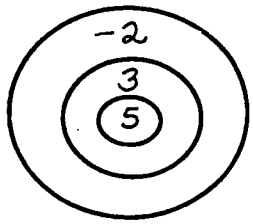
Adapted from original problem by Paul G. Becher, School District of Waukesha, Waukesha, Wisconsin, published by the *Wisconsin Teacher of Mathematics*

(3) Secret Number (also, 1.3)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

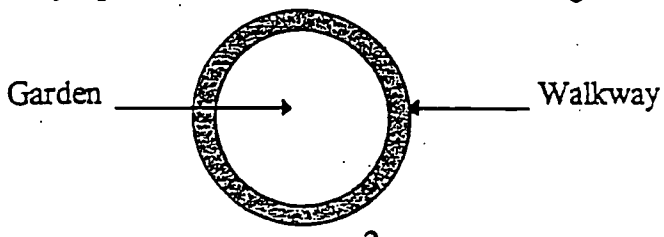
1.6 using number sense to determine and justify the reasonableness of solutions to problems involving integers, rational numbers, and irrational numbers (namely square roots such as $\sqrt{2}$ and $\sqrt{5}$ and π).

Examples:

(1) Allison's Paper Problem (also, 1.4)
(2) Pizza Price Differences Problem (also, 1.4, 4.4, 6.1)
(3) The Better Airline Deal (Section C, page 7) (also, 1.4)
(4) Eating Pizza (Section C, page 8) (also, 1.4)
(5) Refer to the dartboard at the right.
Nick threw 4 darts and got a total score of 2 points. Where did his darts hit?
Explain your answer.



(6) Garden Problem (also, 4.4, 5.1, 5.4)
Sarah planted a circular flower garden that is 15 feet across. She wants to put a gravel walkway around the outside of the flower garden that is 3 feet across.
How many square feet will Sarah have to cover in gravel?



Hint: Area of a circle = π x radius^2

Released mathematics assessment item, Oregon Department of Education

(7) Picnic (also, 6.2, 6.4)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

STANDARD 2:

Students use algebraic methods to explore, model and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

2.1 representing, describing, and analyzing patterns and relationships using hands-on materials, tables, graphs, verbal rules and standard algebraic notation;

Examples:
(1) Expression Representing Number of Hats Problem (Section C, page 13)
(2) Equivalent Algebraic Expressions Problem (Section C, page 14)
(3) Number Missing From Table Problem (Section C, page 15) (also, 2.3, 2.4)
(4) Number Pattern Problem (Section C, page 16) (also, 2.4)

(5) The numbers in the sequence 2, 7, 12, 17, 22, ... increase by fives. The numbers in the sequence 3, 10, 17, 24, 31, ... increase by sevens. The number 17 occurs in both sequences. If the two sequences are continued, what is the next number that will be seen in both sequences? Show how you got your answer.

(6) Missing Values in Proportionality Table (Section C, page 17) (also, 2.3)
(7) Sequence of Triangles Problem (Section C, page 18)
(8) Marcy's Dot Pattern (Section C, page 19) (also, 2.4)
(9) Pat's Pattern (Section C, pages 20-21) (also, 2.2)
(10) Bouncing Ball (Questions 1, 2, and 3 only) (also, 2.4)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

2.2 using variables, expressions, equations, and inequalities to describe patterns in problem-solving situations;

Examples:
(1) Pat's Pattern (Section C, pages 20-21) (also, 2.1)
(2) Similar Triangles (Section C, pages 9-10) (also, 1.4, 4.2, 5.5)

2.3 analyzing functional relationships to explain how a change in one quantity results in a change in another (for example, how the area of a circle changes as the radius increases, or how a person's height changes over time);

Examples:

(1) The width of a rectangle is 3 cm and the length is 4 cm. If the length of the rectangle is doubled, which of the following describes the change in area? (also, 5.5)

(a) The area is doubled.
(b) The area increases by 2 cm^2.
(c) The area increases by 4 cm^2.
(d) The area increases by 8 cm^2.

(2) The length of each side of a square is doubled. How will the area change? (also, 5.5)
(3) Point on a Line Problem (Section C, page 22) (also, 4.3)
(4) Number Missing From Table Problem (Section C, page 15) (also, 2.1, 2.4)
(5) Missing Values in Proportionality Table (Section C, page 17) (also, 2.1)

2.4 distinguishing between linear and nonlinear functions through informal investigations;

Examples:

(1) Number Missing From Table Problem (Section C, page 15) (also, 2.1, 2.3)
(2) Number Pattern Problem (Section C, page 16) (also, 2.1)
(3) Marcy's Dot Pattern (Section C, page 19) (also, 2.1)
(4) Bouncing Ball (Questions 1, 2, and 3 only) (also, 2.1)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

2.5 solving simple linear equations in problem-solving situations using a variety of methods (informal, formal and graphical) and a variety of tools (physical materials, calculators and computers).

Examples:

(1) I'm thinking of a number. If you multiply it by 4 and add 11, you get 35. What is my number?
(2) I'm thinking of a number. Twice the number added to three times the **same** number gives a result of 85. What is my number?

STANDARD 3:

Students use data collection and analysis, statistics, and probability in problem-solving situations and communicate the reasoning used in solving these problems.

GRADES 5-8

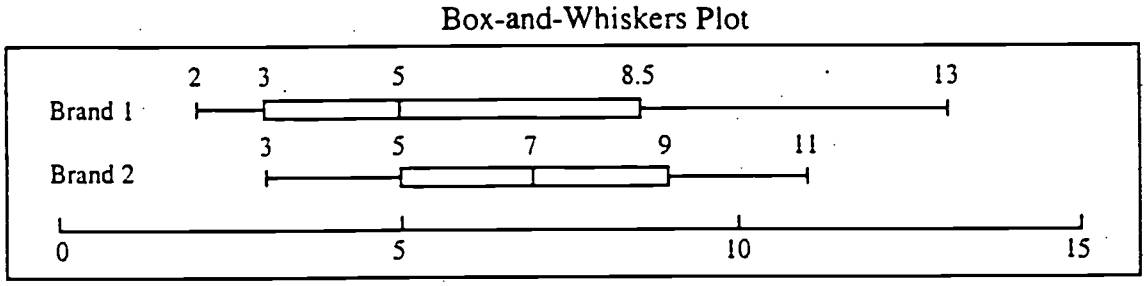
As students in grades 5-8 extend their knowledge, what they know and are able to do includes

3.1 reading and constructing displays of data using appropriate techniques (for example, line graphs, circle graphs, scatter plots, box plots, stem-and-leaf plots) and appropriate technology;

Examples:

(1) Speed of Car from Graph Problem (Section C, page 23)
(2) Temperature on Table and Thermometers (Section C, page 24) (also, 5.3)
(3) Sit-ups Problem (Section C, page 25)
(4) How Much Salad Is Eaten? (Section C, page 26) (also, 3.4)
(5) Life Expectancy (Section C, page 27-28) (also, 3.4)
(6) The Vet Club (Section C, pages 29-30) (also, 3.2, 3.4)

(7) Pepperoni Pizza Problem
A group of students is trying to determine which of two brands of frozen "supreme" pizza has the most pepperoni. The students decide to collect data on the number of pepperoni slices on a sample of each brand of pizza by visiting several supermarkets and examining samples in the frozen food section. The students made the following plot based on their data:



Draw as many conclusions as you can based on the box-and-whiskers plot shown above. Use statistical terms such as median and range in your writing. (also, 3.2)

(8) Free Throw Shots Problem
Eighteen basketball players practiced free throw shots. The number of shots made out of 50 free throws by each player is shown below: (also, 3.2)

12	32	42	50	39	38
27	34	42	35	48	36
13	15	23	16	25	43

Make a stem-and-leaf plot for the data. Find the mean, median, and range for the data.

3.2 displaying and using measures of central tendency, such as mean, median, and mode, and measures of variability, such as range and quartiles;

Examples:

(1) The mean of Eddy's ten test scores is 86. His teacher throws out the top and bottom scores, which are 40 and 100. What is the mean of the remaining set of scores? Show how you got your answer.
Released mathematics assessment item, Oregon Department of Education

(2) Hazardous Chemicals Problem
Ten hazardous chemicals have been rated with positive and negative safety values. They are about to be mixed together. If their mean value is -1, there is no danger of explosion. The chemist knows eight values: 7, 4, -1, -3, 5, -3, -7, and -4. What will the safety values of the other two chemicals have to be so there is no danger of explosion? Show how you got your answer. (also, 6.2)
Released mathematics assessment item, Oregon Department of Education

(3) Samantha made the following scores on unit tests for the term: 92, 98, 15, 92, 87, 92. Samantha's teacher said her grade would be based on the mean of her grades. Samantha argued that her grade should be based on the median score of her grades. Find the mean and median of Samantha's grades. Which do you think **best** reflects Samantha's work for the term? Explain your reasoning.
Released mathematics assessment item, Kentucky Department of Education

(4) The Vet Club (Section C, pages 29-30) (also, 3.1, 3.4)

(5) Pepperoni Pizza Problem (also, 3.1)

(6) Free Throw Shots Problem (also, 3.1)

(7) Best Guess? (also, 3.4)
(Available from Middle Grades Assessment Package 1, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

(8) Library Books (also 3.4)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

3.3 evaluating arguments that are based on data analysis;

Examples:

(1) Trash Can Problem (Section C, page 31)

(2) The Allowance Mystery (Section C, page 31)

3.4 formulating hypotheses, drawing conclusions, and making convincing arguments that are based on data analysis;

Examples:

(1) How Much Salad Is Eaten? (Section C, page 26) (also, 3.1)

(2) Life Expectancy (Section C, page 27-28) (also, 3.1)

(3) The Vet Club (Section C, pages 29-30) (also, 3.1, 3.2)

(4) Best Guess? (also, 3.2)
(Available from Middle Grades Assessment Package 1, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

(5) Library Books (also, 3.2)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

3.5 conducting experiments or simulations to determine probabilities;

Examples:
(1) Chance of Picking Red Marble Problem (Section C, page 32)
(2) Probability of Even-Numbered Chip Problem (Section C, page 33)
(3) Dice Game Problem (also, 6.2)
Travis and Nathan were trying to be clever and invent a new dice game to fool a friend. They made two die that each had the following numbers on it: -3, -2, -1, 0, 1, 2. Before they could invent a game that insured that they would win, they did a little problem solving to determine the odds using these new dice. If you were to roll the two dice 100 times, what sum would be rolled most often?
Explain your thinking at each step and your answer.
Released mathematics assessment item, Oregon Department of Education
(4) Three dimes are tossed at the same time. What is the probability that **exactly** two of the coins will be heads **or** two will be tails?
Explain your thinking at each step and your answer.
Released mathematics assessment item, Oregon Department of Education

3.6 making predictions and comparing results using both experimental and theoretical probability drawn from real-world problems; and

Examples:
(1) Number of Blue Pens in Drawer Problem (Section C, page 34)
(2) Odd or Even? (Section C, page 35) (also, 3.7)

3.7 using counting strategies to determine all the possible outcomes from an experiment (for example, the number of ways students can line up to have their picture taken).

Examples:
(1) There are 8 players in a darts tournament. Each player plays one game against each of the other players. How many darts games will be played in the tournament?
Explain your thinking at each step and your answer.
Released mathematics assessment item, Oregon Department of Education
(2) Picture Frames
Imagine that you have an unlimited number of pieces of wood with which you will make picture frames. The pieces are 3 inches, 5 inches, and 10 inches long.
How many different rectangular frames (including square) can you make when you have three different lengths to work with? Now extend this problem to 4, 5, or more different lengths. How many different rectangular frames (including square) can you make when you have 4 different lengths or 5 different lengths or 6 different lengths and so on?
From problem written by Connie Laughlin, Marquette University, Milwaukee, Wisconsin, and Joe Georgeson, Glen Hills Middles School, Glendale, Wisconsin, published by the *Wisconsin Teacher of Mathematics*
(3) Odd or Even? (Section C, page 35) (also, 3.6)

STANDARD 4:

Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

4.1 constructing two- and three-dimensional models using a variety of materials and tools;

Examples:
(1) Spatial Visualization Problem (Section C, page 36) (also, 4.2)
(2) Birdhouse Problem (Section C, page 37)
(3) Double the Dotted Line Segment Problem (Section C, page 38) (also, 4.2)

4.2 describing, analyzing, and reasoning informally about the properties (for example, parallelism, perpendicularity, congruence, similarity) of two- and three-dimensional figures;

Examples:
(1) Properties of Parallelograms Problem (Section C, page 39)
(2) Lines of Symmetry Problem (Section C, page 40)
(3) Which Two Triangles Are Similar? Problem (Section C, page 41)
(4) Spatial Visualization Problem (Section C, page 36) (also, 4.1)
(5) A rectangle with a perimeter of 24 inches is cut into two squares. What are the dimensions of the rectangle? Use drawings and/or words to explain your answer.
Released mathematics assessment item, Oregon Department of Education
(6) Properties of Quadrilaterals Problem (Section C, page 42)
(7) Similar Triangles (Section C, pages 9-10) (also, 1.4, 2.2, 5.5)
(8) Double the Dotted Line Segment Problem (Section C, page 38) (also, 4.1)
(9) Building Problem (Section C, page 43) (also, 5.2, 6.1)

4.3 using coordinate geometry to explore concepts and solve problems;

Examples:
(1) Point on a Line Problem (Section C, page 22) (also, 2.3)
(2) Likely Coordinates Problem (Section C, page 44)
(3) The vertices of an isosceles triangle are (2,0), (10,0), and (6,6). Find the area of the triangle. (also, 5.4)
(4) The vertices of a square are (-4, -5) and (-4, 1). Find two other points that could be vertices of the square.

4.4 solving problems involving perimeter and area in two dimensions, and involving surface area and volume in three dimensions; and

Examples:
(1) Area of Paper Not Covered Problem (Section C, page 45) (also, 5.4)
(2) Triangle Problem (Section C, page 46) (also, 4.5)
(3) Area of a Rectangle Problem (Section C, page 46) (also, 5.1)
(4) Square Problem (Section C, page 47) (also, 5.4)
(5) Area of Parallelogram Problem (Section C, page 48)
(6) Place Card Problem (also, 5.1)
Julia is making place cards for the homecoming dance.
How many 1 1/2 x 3-inch cards can she cut from a 9-inch square of paper?
Explain your thinking at each step and your answer.
Released mathematics assessment item, Oregon Department of Education
(7) Henry is in charge of making name tags. Each name tag should be 4 inches by 2 inches. Henry has two sizes of paper he can use to make the name tags. One size measures 12" x 18" and the other measures 24" x 36". If Henry needs to make 28 name tags, which size paper should he use to create the least amount of waste. Explain your thinking at each step and your answer(s).
Released mathematics assessment item, Oregon Department of Education
(8) Pizza Price Differences Problem (also, 1.4, 1.6, 6.1)
(9) Trapezoid Problem (Section C, page 49)
(10) Garden Problem (also, 1.6, 5.1, 5.4)
(11) Cubes Problem (Section C, pages 50-51) (also, 5.5)
(12) Compare the Lakes Problem (Section C, page 52) (also, 5.1)
(13) Greendale Garbage (Section C, page 53) (also, 5.1, 5.4)
(14) Grass for Goats (also, 5.1, 5.4)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)
(15) Tile Signs (also, 1.3)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

4.5 transforming geometric figures using reflections, translations, and rotations to explore congruence.

Examples:
(1) Rotated Three-Dimensional Figure Problem (Section C, page 54)
(2) Half-Turn of Shaded Figure Problem (Section C, page 55)
(3) Rotated Triangle Problem (Section C, page 56)
(4) Translations and Reflections Problem (Section C, page 57)
(5) Reflection Problem (Section C, page 57)

STANDARD 5:

Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

5.1 estimating, using, and describing measures of distance, perimeter, area, volume, capacity, weight, mass, and angle comparison;

Examples:
(1) Estimate Size of Angle Problem (Section C, page 58)
(2) Area of Paper Not Covered Problem (Section C, page 45) (also, 4.4, 5.4)
(3) Area of a Rectangle Problem (Section C, page 46) (also, 4.4)
(4) Amount of Water Used Problem (Section C, page 59) (also, 6.3)
(5) What is the **most** appropriate unit for measuring the weight of a pencil? (also, 5.6)
(a) liter (b) gram (c) kilogram (d) kiloliter
(6) Which of these is the **most** appropriate unit to measure the amount of medicine to give a child? (also, 5.6)
(a) milliliter (b) liter (c) kiloliter (d) kilogram
(7) Consider each of the following sentences. Which statement is **unlikely** to be true?
(a) A basketball player is 2 meters tall.
(b) Chip bought a liter of milk at the grocery store.
(c) A professional football player weighs 120 kilograms.
(d) The temperature dropped to 25° Celsius and it began to snow.
(8) Measure Rectangle Problem (Section C, page 60)
(9) Draw a rectangle 2 inches wide and 3 1/2 inches long.
From *Can Students Do Mathematical Problem Solving?*
U. S. Department of Education, 1993
(10) Place Card Problem (also, 4.4)
(11) Garden Problem (also, 1.6, 4.4, 5.4)
(12) Buying Strapping Tape Problem (Section C, page 61) (also, 6.2)
(13) Compare the Lakes (Section C, page 52) (also, 4.4)
(14) Greendale Garbage (Section C, page 53) (also, 4.4, 5.4)
(15) Grass for Goats (also, 4.4, 5.4)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

5.2 estimating, making, and using direct and indirect measurements to describe and make comparisons;

Examples:
(1) Height of Tree Problem (Section C, page 62)
(2) Longest Pace Problem (Section C, page 63)
(3) Approximate Length of Pencil Problem (Section C, page 64)
(4) Comparing Triangles (Section C, pages 65-66) (also, 5.5)
(5) Building Problem (Section C, page 43) (also, 4.2, 6.1)

5.3 reading and interpreting various scales including those based on number lines, graphs, and maps;

Examples:
(1) Distance on Map Problem (Section C, page 67)
(2) Scale Problem (Section C, page 68)
(3) Temperature on Table and Thermometers (Section C, page 24) (also, 3.1)
(4) Number Line Problem (Section C, page 68)

5.4 developing and using formulas and procedures for determining measures to solve problems;

Examples:
(1) Area of Paper Not Covered Problem (Section C, page 45) (also, 4.4, 5.1)
(2) Triangles in Trapezoid Problem (Section C, page 69)
(3) Square Problem (Section C, page 47) (also, 4.4)
(4) Garden Problem (also, 1.6, 4.4, 5.1)
(5) The vertices of an isosceles triangle are (2,0), (10,0), and (6,6). Find the area of the triangle. (also, 4.3)
(6) Greendale Garbage (Section C, page 53) (also, 4.4, 5.1)
(7) Stopping Sneaky Sally (Section C, page 70)
(8) Lotsafun Amusement Park (Section C, page 71)
(9) Errol's Evergreen Farm (Section C, page 71)
(10) Grass for Goats (also, 4.4, 5.1)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

5.5 describing how a change in an object's linear dimensions affects its perimeter, area, and volume;

Examples:
(1) New Rectangle Problem (Section C, page 72)
(2) Kyle works at Greenhorn Ranch during vacations. They are building a rectangular corral using an existing fence for one side and 48 feet of new fencing for the remaining sides. How wide and how long should they make the corral to get the largest possible area? Explain your thinking at each step and your answer.
Released mathematics assessment item, Oregon Department of Education
(3) Comparing Triangles (Section C, pages 65-66) (also, 5.2)
(4) Similar Triangles (Section C, pages 9-10) (also, 1.4, 2.2, 2.3, 4.2)
(5) The width of a rectangle is 3 cm and the length is 4 cm. If the length of the rectangle is doubled, which of the following describes the change in area? (also, 2.3)
(a) The area is doubled. (b) The area increases by 2 cm^2.
(c) The area increases by 4 cm^2. (d) The area increases by 8 cm^2.
(6) The length of each side of a square is doubled. How will the area change? (also, 2.3)
(7) Cubes Problem (Section C, pages 50-51) (also, 4.4)

5.6 selecting appropriate units and tools to measure to the degree of accuracy required in a particular problem-solving situation.

Examples:
(1) Recognizing Precision Problem (Section C, page 73)
(2) What is the **most** appropriate unit for measuring the weight of a pencil? (also, 5.1)
(a) liter (b) gram (c) kilogram (d) kiloliter
(3) Which of these is the **most** appropriate unit to measure the amount of medicine to give a child? (also, 5.1)
(a) milliliter (b) liter (c) kiloliter (d) kilogram

STANDARD 6:

Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

6.1 using models to explain how ratios, proportions, and percents can be used to solve real-world problems;

Examples:
(1) Pizza Price Differences (also, 1.4, 1.6, 4.4)
(2) Building Problem (Section C, page 43) (also, 4.2, 5.2)
(3) Science Fair (Section C, pages 11-12) (also, 1.4, 6.2)

6.2 constructing, using, and explaining procedures to compute and estimate with whole numbers, fractions, decimals, and integers;

Examples:
(1) People Boarding Bus Problem (Section C, page 74)
(2) Hazardous Chemicals Problem (also, 3.2)
(3) Brian's Bicycle Problem
Brian's bicycle shop had 48 water bottles for sale. On Monday, Brian marked the water bottles down to $5 each and sold 1/2 of them. On Tuesday, he marked the remaining bottles down to $4 each and sold 1/2 of what was left. On Wednesday, he marked the bottles down to $3 and sold 1/3 of the remaining bottles. On Thursday, he marked the water bottles he had left down to $2 and sold them all. If Brian purchased the bottles for $3 each, how much did he gain or lose? Explain your thinking at each step and your answer. (also, 6.4)
Released mathematics assessment item, Oregon Department of Education
(4) Double Discount (Section C, page 75) (also, 6.4)
(5) Buying Strapping Tape (Section C, page 61) (also, 5.1)
(6) Science Fair (Section C, pages 11-12) (also, 1.4, 6.1)
(7) Dice Game Problem (also, 3.5)
(8) Disk Sum (also, 1.3)
(Available from Middle Grades Assessment Package 1, published by Dale Seymour Publications — refer to Section B, page 17 for more information)
(9) Picnic (also, 1.6, 6.4)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

6.3 developing, applying and explaining a variety of different estimation strategies in problem-solving situations, and explaining why an estimate is acceptable in place of an exact answer;

Examples:
(1) Use of Estimates Problem (Section C, page 76)
(2) Heart Beats Problem (Section C, page 77)
(3) Amount of Water Used Problem (Section C, page 59) (also, 5.1)
(4) Estimated Time for Songs to Play Problem (Section C, page 78)

6.4 selecting and using appropriate methods for computing with commonly-used fractions and decimals, percents, and integers in problem-solving situations from among mental arithmetic, estimation, paper-and-pencil, calculator and computer methods, and determining whether the results are reasonable.

Examples:
(1) Number of Students at Beaton High School Problem (Section C, page 79)
(2) Total Distance Traveled by Ball Problem (Section C, page 80)
(3) Number of Marbles in Bag Problem (Section C, page 81)
(4) Rate of Fuel Consumption Problem (Section C, page 82)
(5) Percent Increase in Price Problem (Section C, page 83)
(6) Running Problem (Section C, page 84)
(7) Double Discount (Section C, page 75) (also, 6.2)
(8) Brian's Bicycle Problem (also, 6.2)
(9) Sheets of Paper Needed Problem (Section C, page 85)
(10) Treena's Budget Problem (Section C, page 85)
(11) Picnic (also, 1.6, 6.2)
(Available from Middle Grades Assessment Package 2, published by Dale Seymour Publications — refer to Section B, page 17 for more information)

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Colorado Model Content Standards for Mathematics

Fifth to Eighth Grade Assessment Framework

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Special Note:
Several tasks referenced in this Guide are from the Middle Grades Assessment Packages 1 and 2, developed by the project Balanced Assessment for the Mathematics Curriculum.
Balanced Assessment packages, comprising addtional tasks and instructional support, are published by Dale Seymour Publications. Further information can be obtained from the publisher, or the project website: http://www.educ.msu/MARS

Section C

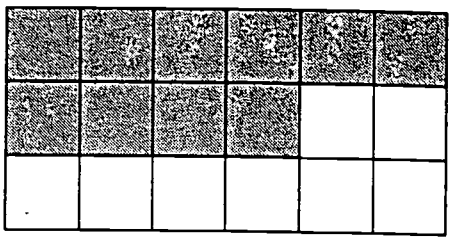
COLORADO
MODEL CONTENT STANDARDS
FOR
MATHEMATICS

REFERENCED TASKS
Fifth to Eighth Grade
Assessment Framework

NOTE: The examples included in this framework are meant to illustrate the grade-level expectations in the *Fifth to Eighth Grade Mathematics Assessment Framework.* These are **not** actual CSAP test items, although the actual items will be similar to these in content and form. In addition, several of the examples illustrate more than one standard and/or expectation.

Each example is coded with a box in the lower right-hand corner which indicates which expectation(s) is illustrated by the example. For example, 2.3 means the example illustrates grade-level expectation 2.3 under Mathematics Standard 2.

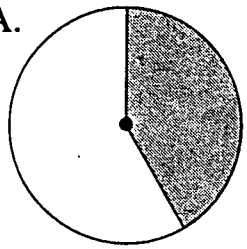
K1.



K-1

Which circle has approximately the same fraction shaded as that of the rectangle above?

A.



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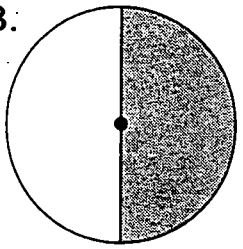
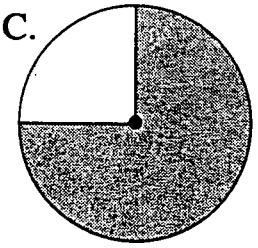
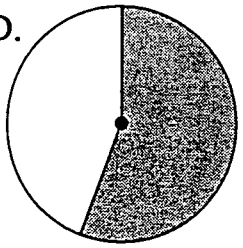


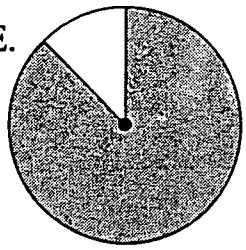
C.



D.



E.



C-1

1.1

P15. Which of these expressions is equivalent to y^3 ?

A. $y + y + y$

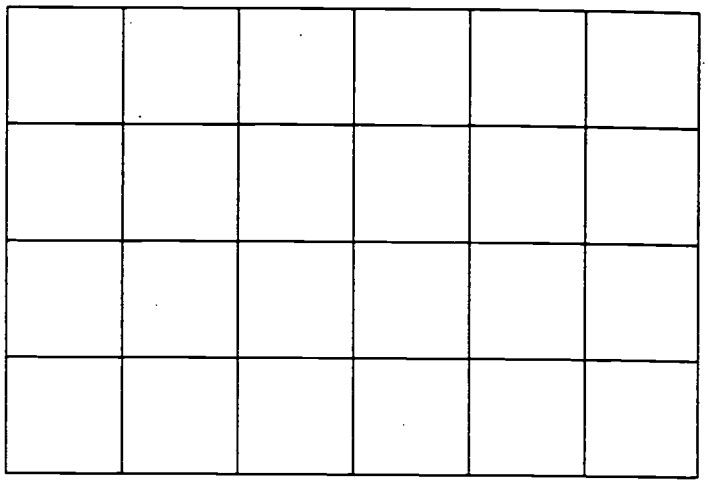
B. $y \times y \times y$

C. $3y$

D. $y^2 + y$

P-15

N19. Shade in $\frac{5}{8}$ of the unit squares in the grid.



N-19

Alternate Problems:

(a) Shade in 75% of the unit squares in the grid.

(b) Shade in 0.375 of the unit squares in the grid.

Q8. Which list shows the numbers from smallest to largest?

A. 0.345, 0.19, 80%, $\frac{1}{5}$

B. 0.19, $\frac{1}{5}$, 0.345, 80%

C. 80%, 0.19, $\frac{1}{5}$, 0.345

D. $\frac{1}{5}$, 80%, 0.345, 0.19

Q-8

1.2

Modified

Of the following, which is closest in value to 0.52 ?

A $\frac{1}{50}$

B $\frac{1}{5}$

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

NAEP 1992 Mathematics Report Card for the Nation and the States.
National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 1993.

1.2

Q5. Three-fifths of the students in a class are girls. If 5 girls and 5 boys are added to the class, which statement is true of the class?

A. There are more girls than boys.

B. There are the same number of girls as there are boys.

C. There are more boys than girls.

D. You cannot tell whether there are more girls or boys from the information given.

Q-5

V3. To mix a certain color of paint, Alana combines 5 liters of red paint, 2 liters of blue paint, and 2 liters of yellow paint. What is the ratio of red paint to the total amount of paint?

A. $\frac{5}{2}$

B. $\frac{9}{4}$

C. $\frac{5}{4}$

D. $\frac{5}{9}$

V-3

The Better Airline Deal

Marvin and Gloria want to fly from San Francisco to Los Angeles together.

Two airlines have special deals which Marvin and Gloria are considering.

El Paso Air	Air Garcia
Buy One Ticket and Take a Friend Free	40% Off All Sales of Two or More Tickets

Both airlines charge the same for a single ticket.

Decide which offer Marvin and Gloria should choose.

Explain and show your work to support your choice.

1.4
1.6

C-7

Eating Pizza

You are invited to go out for pizza with several friends. When you get there, you find your friends sitting at two different tables.

You can join either group. If you join the first one, there will be a total of 4 people in the group and all of you will be sharing 6 small pizzas.

If you join the other group, there will be 6 people in the group and all of you will be sharing 8 small pizzas.

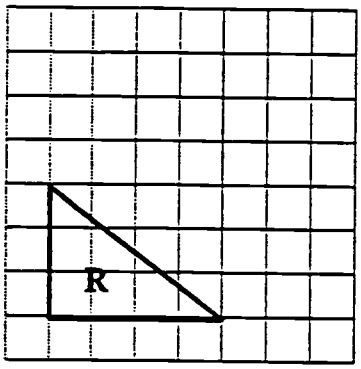
If your goal was to get as much pizza to eat as possible, which group would you join?

Explain your thinking and show your math.

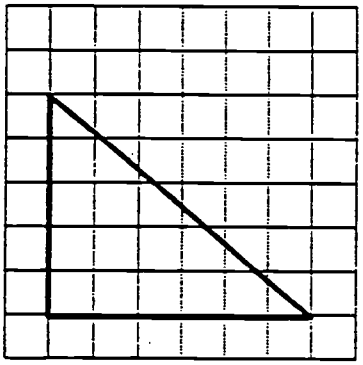
1.4
1.6

Similar Triangles

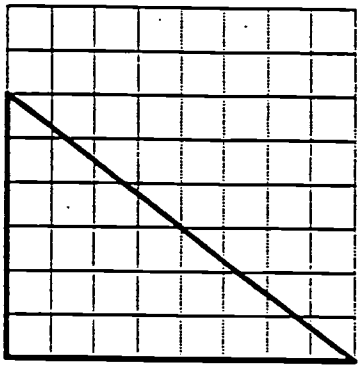
1. Which of the triangles below is similar to Triangle R?



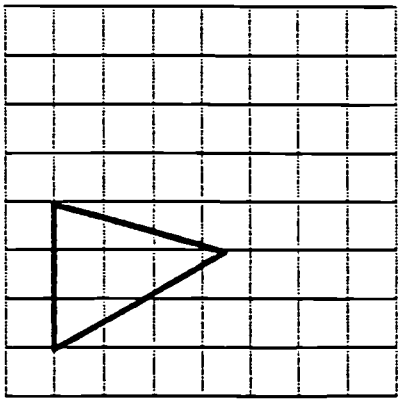
Triangle 1



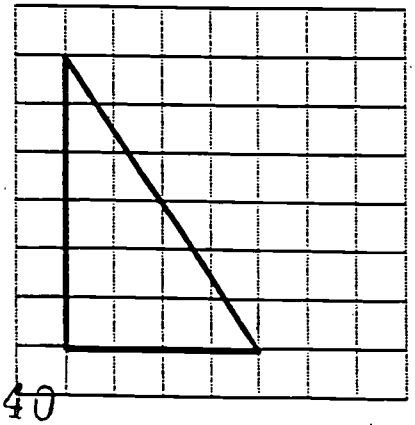
Triangle 2



Triangle 3



Triangle 4



40

1.4
2.2
4.2
5.5

2. Describe how you decided which triangle was similar to R.

3. If Triangle R is enlarged, what will height and base lengths be for the new triangle for the following scale factors:

	Height	Base
– scale factor of 2.5	_____	_____
– scale factor of 10	_____	_____
– scale factor of n	_____	_____

4. If Triangle R is enlarged, what will the area be for the new triangle for the following scale factors:

– scale factor of 2.5 _____

– scale factor of 10 _____

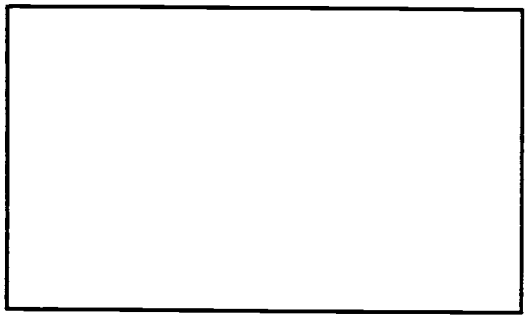
– scale factor of n _____

1.4
2.2
4.2
5.5

Science Fair

Three middle schools are going to have a science fair. The science fair will be in an auditorium. The amount of space given to each school is based on number of students. Bret Harte Middle school has about 1,000 students, Malcolm X Middle School has about 600 students, and Kennedy Middle School has about 400 students.

1. The rectangle below represents the auditorium. **Divide the rectangle** to show the amount of space each school should get based on number of students. Label each section **BH** for Bret Harte, **MX** for Malcolm X, or **K** for Kennedy.



2. What *fraction* of the space should each school get based on number of students? Show your mathematical reasoning.

School BH

School MX

School K

1.4
6.1
6.2

3. If the schools share the cost of the science fair based on number of students, what *per cent* of the cost should each school pay?

School BH

School MX

School K

4. If the cost of the science fair is $300.00, how much should each school pay based on number of students? Justify your answers.

School BH

School MX

School K

1.4
6.1
6.2

Q1. Juan has 5 fewer hats than Maria, and Clarissa has 3 times as many hats as Juan. If Maria has n hats, which of these represents the number of hats that Clarissa has?

A. $5 - 3n$

B. $3n$

C. $n - 5$

D. $3n - 5$

E. $3(n - 5)$

Q-1

P10. If m represents a positive number, which of these is equivalent to $m + m + m + m$?

A. $m + 4$

B. $4m$

C. m^4

D. $4(m + 1)$

P-10

2.1

J18. The table represents a relation between x and y.

J-18

What is the missing number in the table?

A. 2

B. 3

C. 4

D. 5

E. 6

x	y
1	1
2	?
4	7
7	13

Extension for Benchmark 2.3:

How does a change in the value of x affect the value of y?

Extension for Benchmark 2.4:

Does this represent a linear or nonlinear relationship? Explain how you know.

2.1
2.3
2.4

C-15

Modified

A	B
2	5
4	9
6	13
8	17
14	?

If the pattern shown in the table were continued, what number would appear in the box at the bottom of column *B* next to 14 ?

A 19

B 21

C 23

D 25

E 29

Extension for Benchmark 2.4:

Does this represent a linear or nonlinear relationship? Explain how you know.

NAEP 1992 Mathematics Report Card for the Nation and the States.
National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 1993.

2.1
2.4

L14. The table shows the values of x and y, where x is proportional to y.

x	3	6	P
y	7	Q	35

L-14

Find the values of P and Q.

Extension for Benchmark 2.3:

How does the change in the value of x affect the value of y?

C-17

2.1
2.3

Modified

S1. Here is a sequence of three similar triangles. All of the small triangles are congruent.

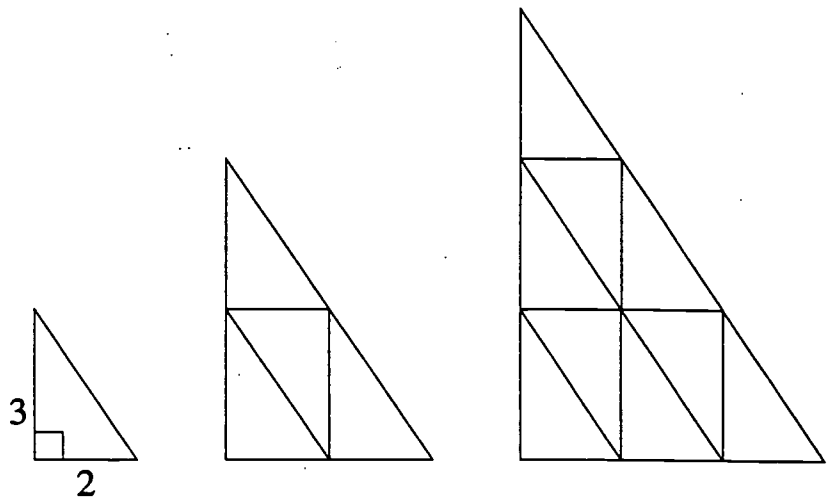


Figure 1 Figure 2 Figure 3

a. Complete the chart by finding how many small triangles make up each figure.

Figure	Number of small triangles
1	1
2	
3	

S-1a

b. The sequence of similar triangles is extended to the 8th Figure. How many small triangles would be needed for Figure 8?

C-18

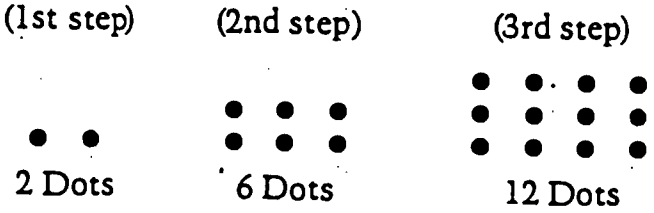
2.1

Grade 8 Question: Marcy's Dot Pattern

The Task

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.



Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots.

Explain or show how she could do this and give the answer that Marcy should get for the number of dots.

Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

C-1950

2.1

Pat's Pattern

For homework Pat's teacher asked him to look at the pattern below and draw the figure that would come next.

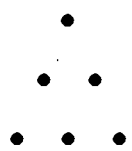


Figure 1

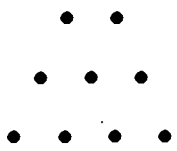


Figure 2

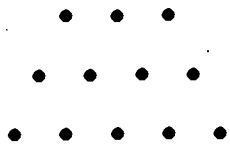


Figure 3

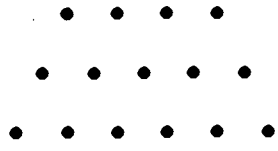


Figure 4

Pat does not know how to find the next figure.

1. Draw the next figure for Pat.

2.1
2.2

2. Write a description for Pat telling him how you knew which figure comes next. Make sure that Pat can understand how to continue the pattern himself.

3. Write a rule that describes how to draw any Figure n.

2.1
2.2

I8. A straight line on a graph passes through the points (3,2) and (4,4). Which of these points also lies on the line?

A. (1,1)

B. (2,4)

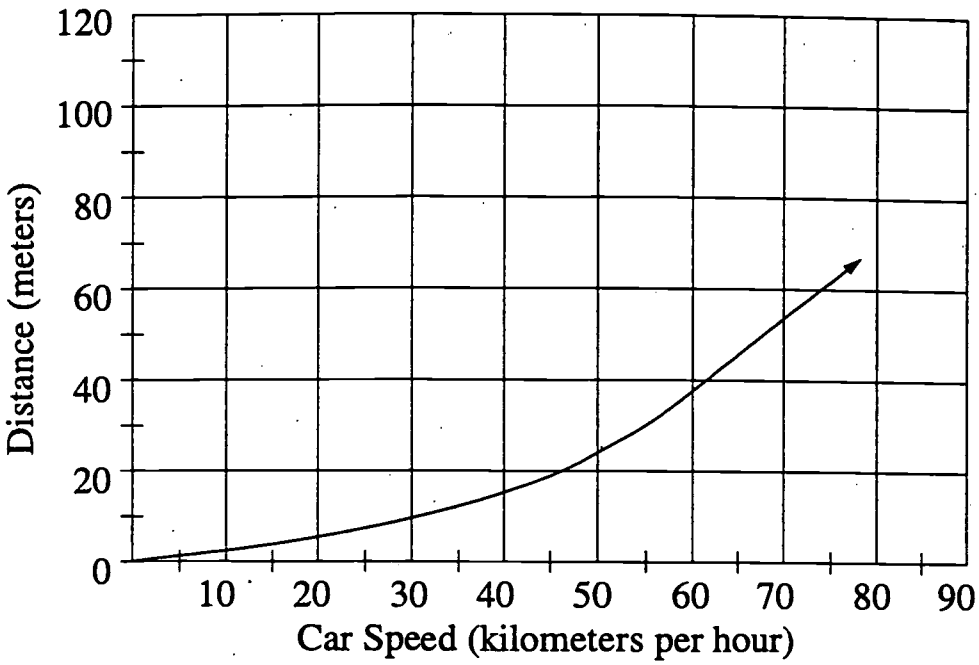
C. (5,6)

D. (6,3)

E. (6,5)

2.3
4.3

O1. The graph shows the distance traveled before coming to a stop after the brakes are applied for a typical car traveling at different speeds.



A car traveling on a highway stopped 30 m after the brakes were applied. About how fast was the car traveling?

A. 48 km per hour

B. 55 km per hour

C. 70 km per hour

D. 160 km per hour

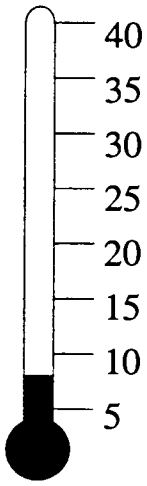
P17. This table shows temperatures at various times during the week.

TEMPERATURES					
	6 a.m.	9 a.m.	Noon	3 p.m.	8 p.m.
Monday	15°	17°	20°	21°	19°
Tuesday	15°	15°	15°	10°	9°
Wednesday	8°	10°	14°	13°	15°
Thursday	8°	11°	14°	17°	20°

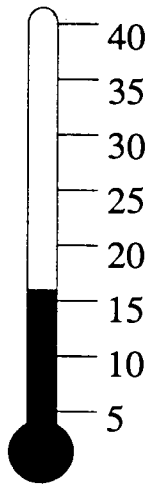
Which thermometer shows the temperature at 8 p.m. on Monday?

P-17

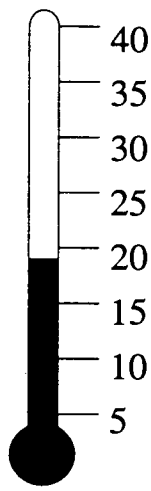
A.



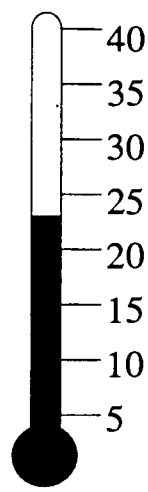
B.



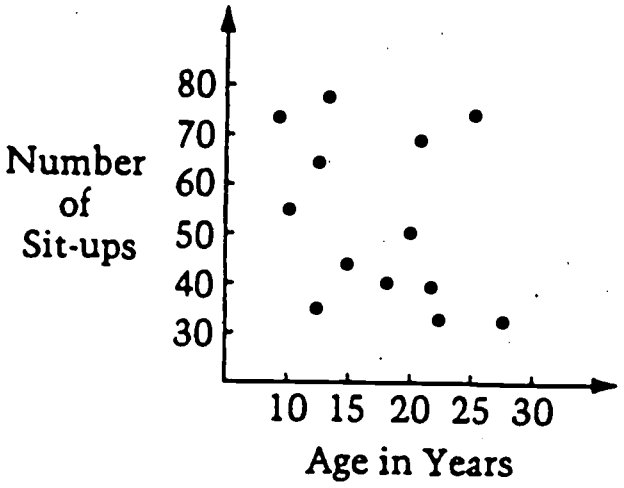
C.



D.



3.1
5.3



In the graph above, each dot shows the number of sit-ups and the corresponding age for one of 13 people. According to this graph, what is the median number of sit-ups for these 13 people?

A 15

B 20

C 45

D 50

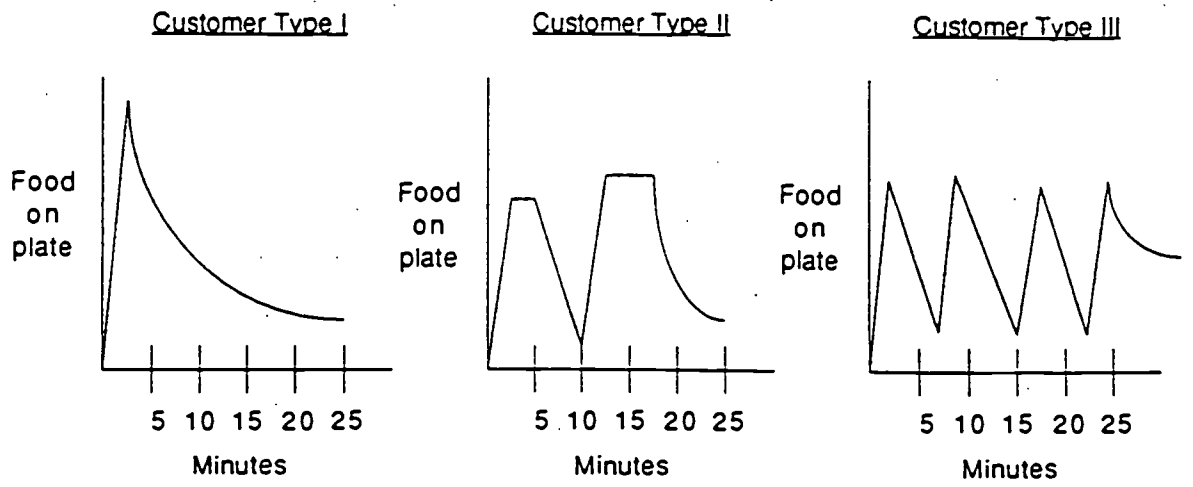
E 55

NAEP 1992 Mathematics Report Card for the Nation and the States.
National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 1993.

How Much Salad Is Eaten?

You have been asked by the owner of a salad bar restaurant to observe customers and to report on how they help themselves at the salad bar. Customers are allowed to return to the salad bar as often as they wish.

Here are graphs you have made for the three types of customers.



Based upon the graphs, describe each customer type.

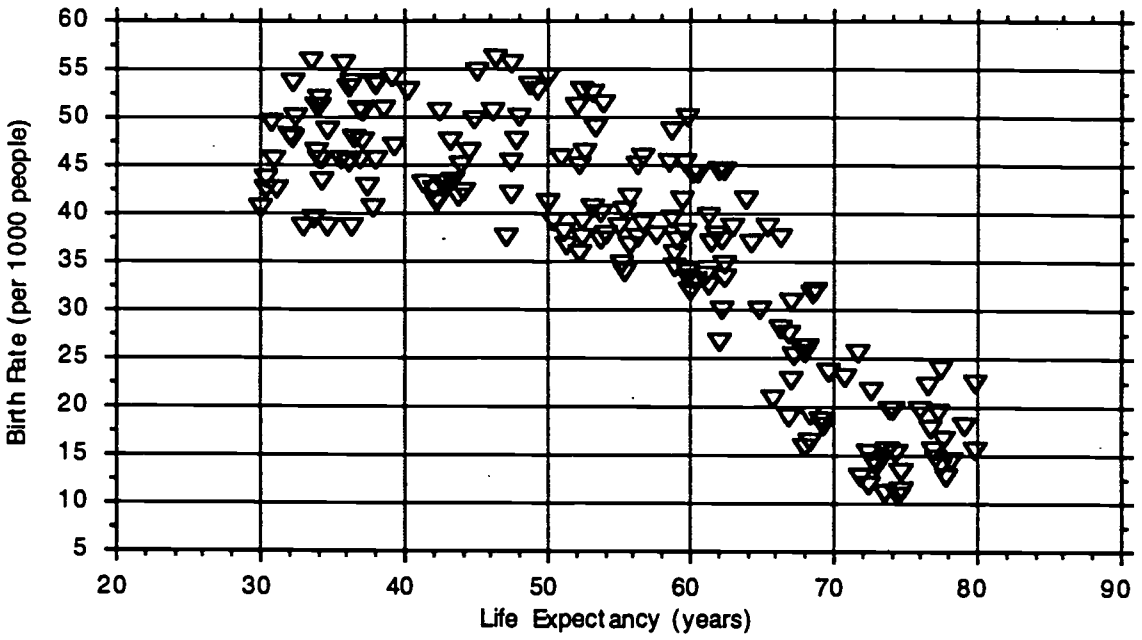
3.1
3.4

C-26

Life Expectancy

The birth rate of a country is a measurement of the number of babies born alive each year for every 1000 people. For example, if the birth rate of a country is 30, and a village has 1000 people, there will be—on the average—30 births each year. If a town has 10,000 people, we might expect 300 babies, and so forth.

Below is a graph of birth rate against life expectancy for 184 countries.



1) In one country, the life expectancy of people is 75 years. Use the graph to answer this question:
In what range would you expect the birth rate to be?

3.1
3.4

2) In another country, the birth rate is 50. Based on the graph, what can you say about the life expectancy there?

3) Based on the information in the graph, describe the relationship between birth rate and life expectancy.

3.1
3.4

Jenny is writing a newsletter article about the members of the Future Veterinarians Club. She asked each of the members: **"What pets live in your house?"** Her notes look like this:

B. J.	*"none"*	*J. M.*	*"1 dog"*
A. P.	*"1 dog, 1 cat"*	*J. Z.*	*"none"*
K. K.	*"2 dogs"*	*S. F.*	*"3 cats"*
R. B.	*"1 bird, 1 dog, 4 goldfish"*	*E. P.*	*"none"*
G. L.	*"1 dog"*	*K. B.*	*"1 bird"*
W. F.	*"none"*	*S. L.*	*"2 dogs, 1 bird, 2 fish"*
J. A.	*"1 rabbit"*	*D. L.*	*"6 fish"*

3.1
3.2
3.4

1. Your job is to prepare a graph to go with Jenny's article.

   Organize the information into a graph that will show how many of her friends have no pets, one pet, two pets, etc.

Jenny plans to title the article:

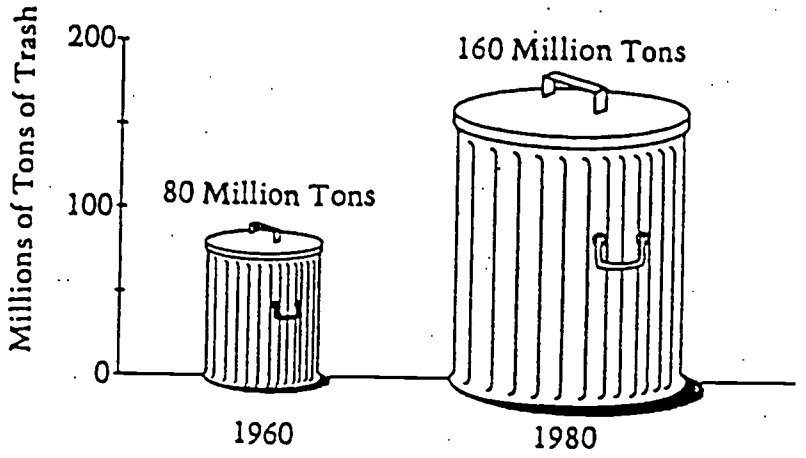
TYPICAL FUTURE VETERINARIANS CLUB MEMBER HAS _?_ HOUSE PETS

2. What number should Jenny put in the blank? _____

3. Explain why the number you chose is the best number to complete the headline.

3.1
3.2
3.4

THE UNITED STATES
IS PRODUCING MORE TRASH



The pictograph shown above is misleading. Explain why.

3.3

NAEP 1992 Mathematics Report Card for the Nation and the States.
National Center for Education Statistics, Office of Educational Research and Improvement,
U.S. Department of Education, 1993.

The Allowance Mystery:

Year	Total Yearly Allowance	Total Family Income
1992	$250	$25,000
1993	$260	$26,000

The table above shows Terry's total yearly allowance and his family's total income for the year. **Between 1992 and 1993 the cost of living increased by 8%.**

Terry's family was discussing the change in his allowance between 1992 and 1993:

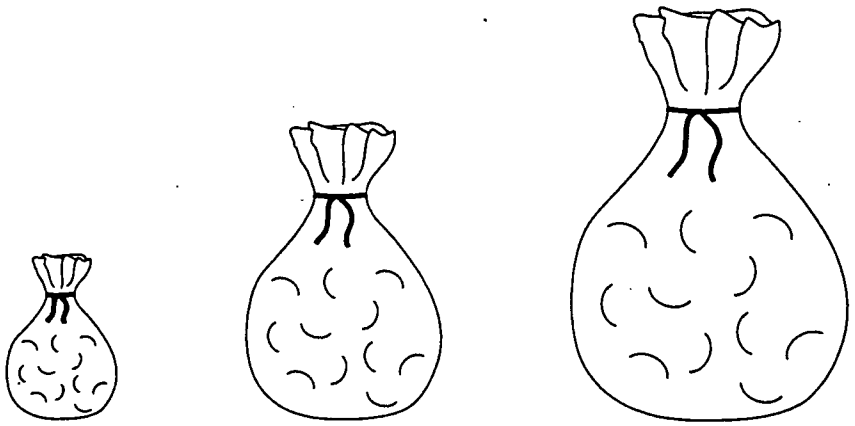
- Terry's older sister, home from college, insisted that Terry's allowance had increased.
- Terry complained that the allowance had gone down.
- Terry's mother maintained that, in fact, there had been no change in the allowance system.

Describe how each person's opinion could be valid.
Use mathematics to justify each point of view.

Wisconsin Student Assessment System Performance Assessment Sampler. Wisconsin Department of Public Instruction, Madison, WI, 1995.

3.3

M3. There is only one red marble in each of these bags.



10 marbles 100 marbles 1000 marbles

Without looking in the bags, you are to pick a marble out of one of the bags. Which bag would give you the greatest chance of picking the red marble?

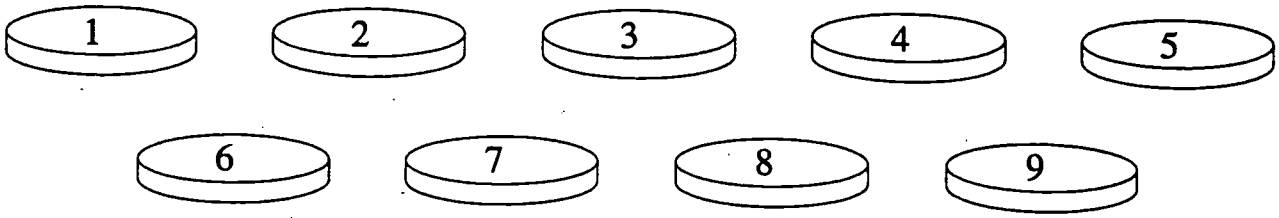
A. The bag with 10 marbles

B. The bag with 100 marbles

C. The bag with 1000 marbles

D. All bags would give the same chance.

N18. The nine chips shown are placed in a jar and mixed.



Madeleine draws one chip from the jar. What is the probability that Madeleine draws a chip with an even number?

A. $\frac{1}{9}$

B. $\frac{2}{9}$

C. $\frac{4}{9}$

D. $\frac{1}{2}$

K7. A drawer contains 28 pens; some white, some blue, some red, and some gray. If the probability of selecting a blue pen is $\frac{2}{7}$, how many blue pens are in the drawer?

A. 4

B. 6

C. 8

D. 10

E. 20

Odd or Even?

Michiko and Patricia are going to play a spinner game. These are the rules:

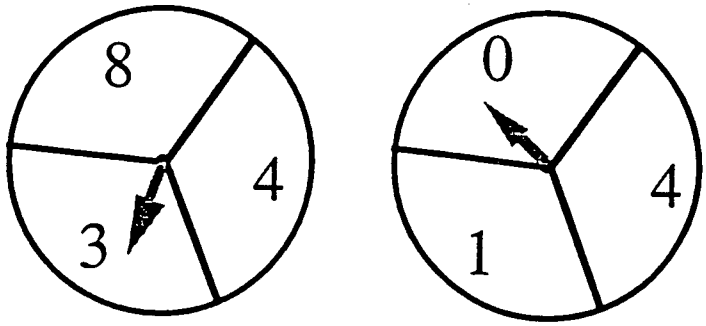
When it is a player's turn, she spins both spinners.

Then she adds the two numbers that the arrows point to.

If the sum is odd (1, 3, 5, 7, 9...), Michiko wins, even if it was not her turn.

If the sum is even (0, 2, 4, 6, 8...), Patricia wins, even if it was not her turn.

Patricia tries a test spin, first. Here is what she spins:



The sum from the first spin is 3, because 3 + 0 = 3. Michiko wins.

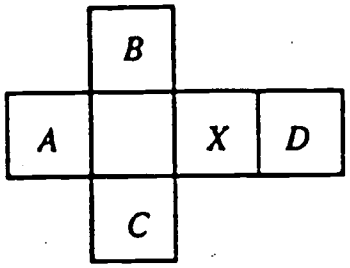
Michiko says, "I like this game. I have a better chance to win it than you do."

Patricia says, "No, I have a better chance to win it than you do."

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

3.6
3.7



The squares in the figure above represent the faces of a cube which has been cut along some edges and flattened. When the original cube was resting on face *X*, which face was on top?

A *A*

B *B*

C *C*

D *D*

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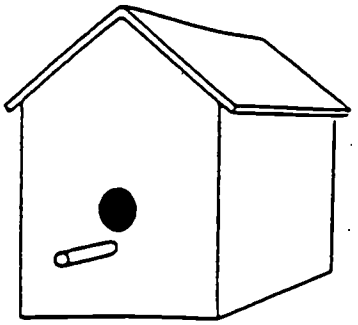
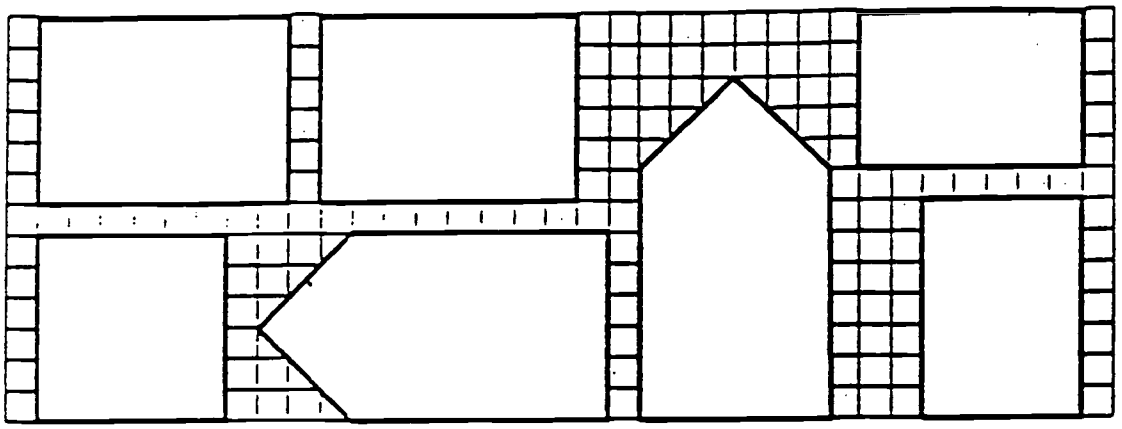
National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 1993.

4.1
4.2



Keisha is absent on the day that you cut the pieces for the birdhouses. Mr. Ramírez asks you to cut the pieces for her birdhouse. After you cut them, you decide to write her a note explaining what each piece should be used for.



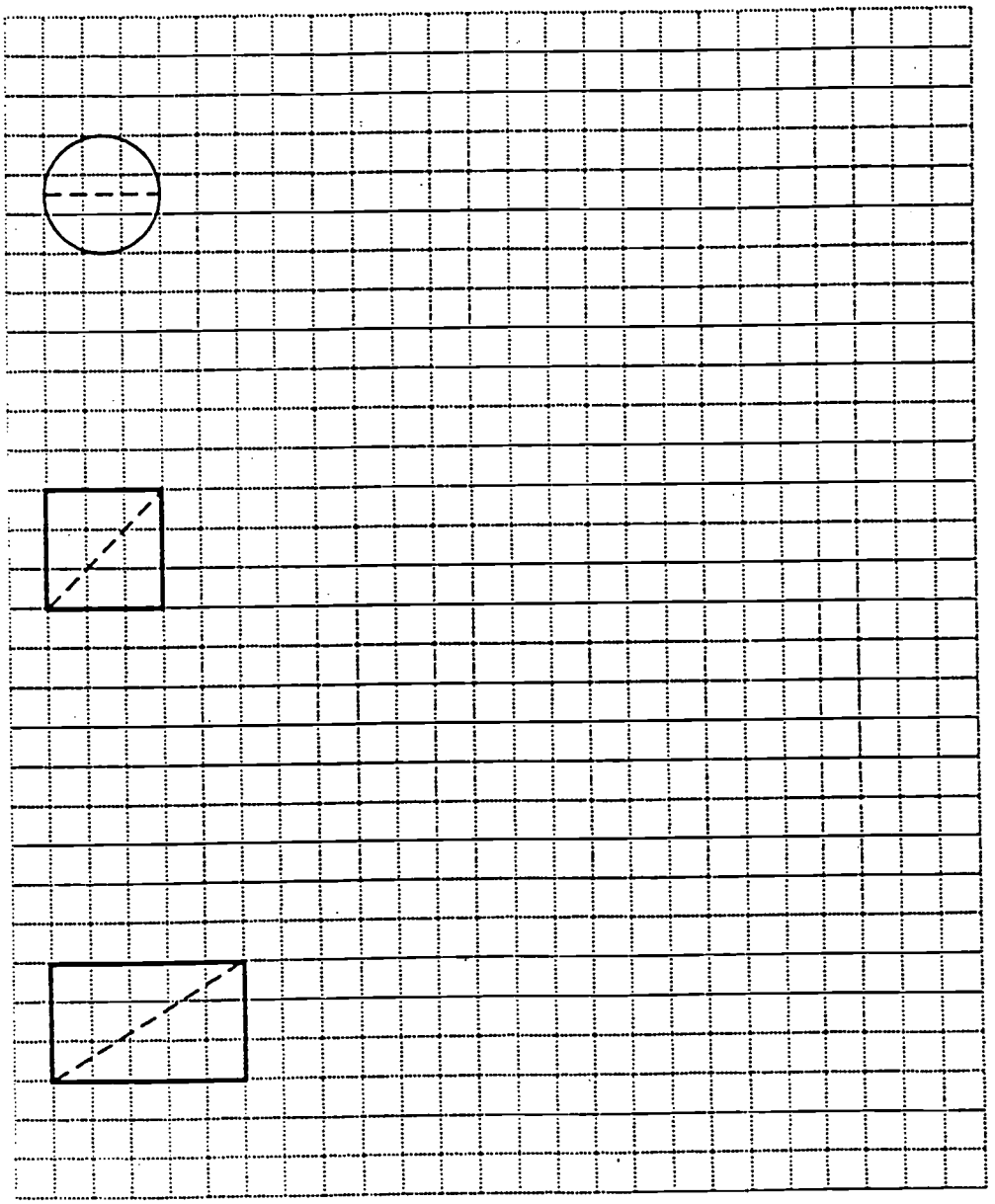
Write a note to Keisha explaining what each piece you cut should be used for. Use geometric terms to refer to the pieces.

Be sure to complete both parts of this problem.

Double the Dotted Line Segment

Part 1

For each of the figures shown below, draw a new enlarged figure that has the same shape. The dotted line segment in each new figure should be double the length of the dotted line segment in the original figure.



Part 2

Describe mathematically how the pairs of old and new figures are the same and how they are different.

4.1
4.2

J11. A quadrilateral MUST be a parallelogram if it has

J-11

A. one pair of adjacent sides equal

B. one pair of parallel sides

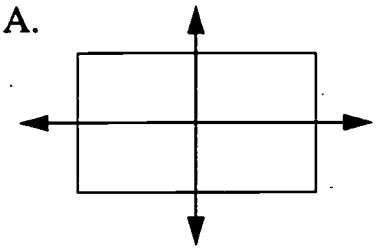
C. a diagonal as axis of symmetry

D. two adjacent angles equal

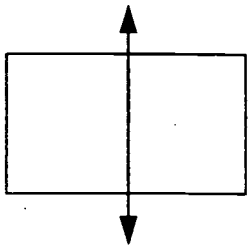
E. two pairs of parallel sides

M2. Which shows all of the lines of symmetry for a rectangle?

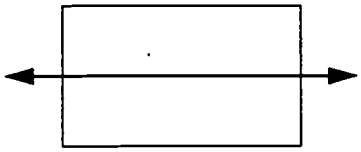
A.



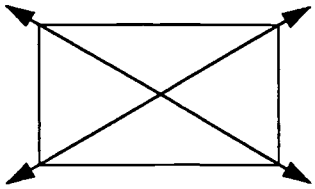
B.



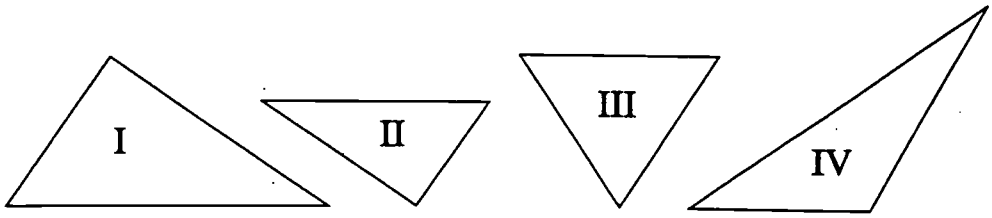
C.



D.



J15.



Which two triangles are similar?

A. I and II

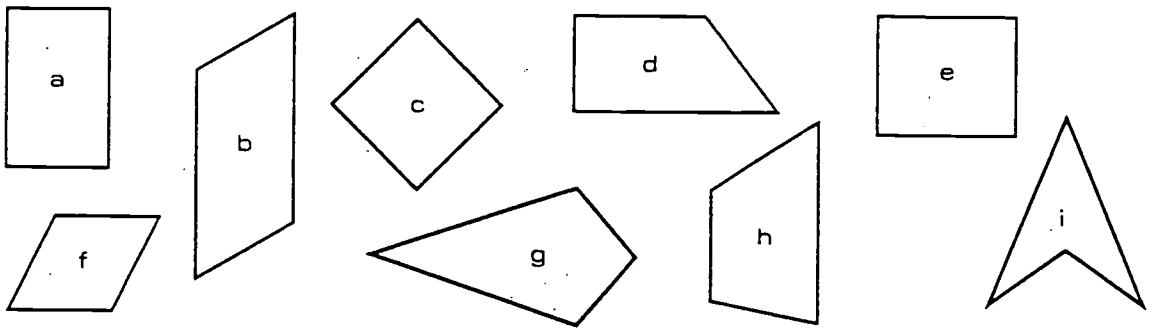
B. I and IV

C. II and III

D. II and IV

E. III and IV

Select the figures that satisfy each of the sets of properties below and state their names.



1. Has 4 sides.
   Opposite sides are congruent.
   Opposite sides are parallel.
   Opposite angles are congruent.
   Figures:_____ Name:_____

2. Has 4 sides.
   All sides are congruent.
   Opposite sides are parallel.
   All angles are right angles.
   Figures:_____ Name:_____

3. Has 4 sides.
   Opposite sides are congruent.
   Opposite sides are parallel.
   All angles are right angles.
   Figures:_____ Name:_____

4. Has 4 sides.
   Only two sides are parallel.
   Figures:_____ Name:_____

5. Has 4 sides.
   Has two pairs of congruent adjacent sides.
   Figures:_____ Name:_____

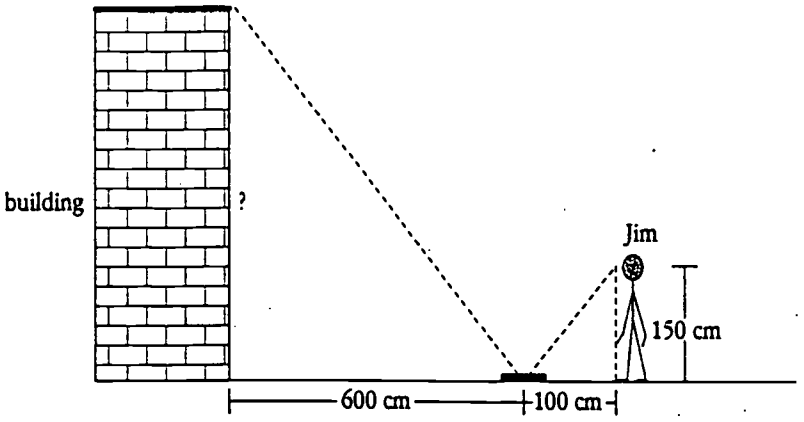
6. Has 4 sides.
   All sides are congruent.
   Opposite sides are parallel.
   Opposite angles are congruent.
   Figures:_____ Name:_____

C-42

4.2

Jim and Qin-Zhong, students in Mr. Anwar's class, are using the mirror method to estimate the height of their school building. They have made the following measurements and sketch:

Height from the ground to Jim's eyes = 150 cm
Distance from the middle of the mirror to Jim = 100 cm
Distance from the middle of the mirror to the building = 600 cm



A. Use what you know about similar triangles to find the building's height from the given measurements. Explain your work.

B. With your group, use the mirror method to estimate the height of the same object or building you worked with in Problem 5.1. In your answer, include all the measurements your group made, and explain in words and drawings how you used the measurements to find the object's height.

C. How does the height estimate you made using the shadow method compare with the height estimate you made using the mirror method? Do you think your estimates for the object's height are reasonable? Why or why not?

4.2
5.2
6.1

Connected Mathematics, Michigan State University, Dale Seymour Pub., Palo Alto, CA 1996.

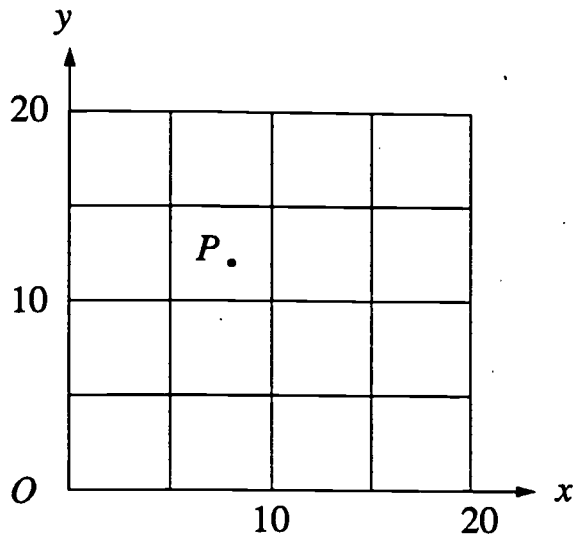
J16. Which of the following are most likely to be the coordinates of point P ?

A. (8, 12)

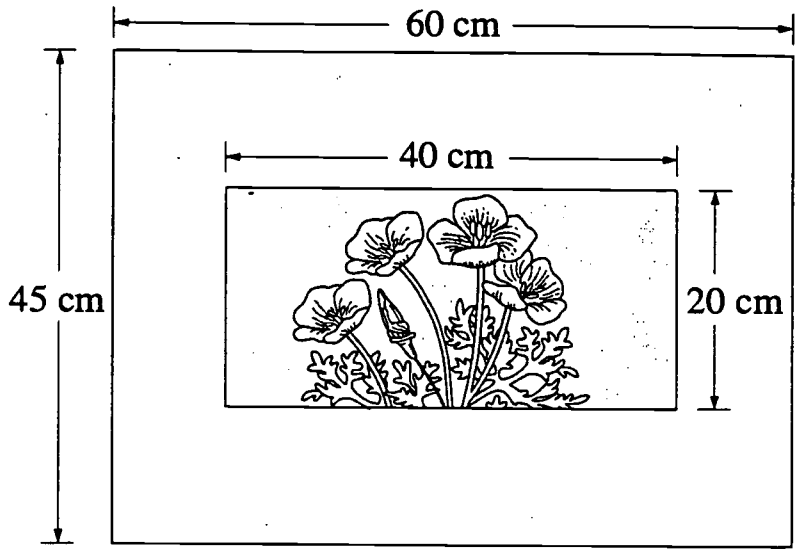
B. (8, 8)

C. (12, 8)

D. (12, 12)



J10. A rectangular picture is pasted to a sheet of white paper as shown.



What is the area of the white paper not covered by the picture?

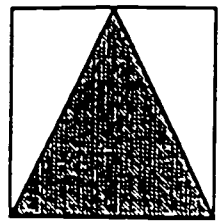
A. 165 cm^2

B. 500 cm^2

C. 1900 cm^2

D. 2700 cm^2

4.4
5.4



If the area of the shaded triangle shown above is 4 square inches, what is the area of the entire square?

A 4 square inches

B 8 square inches

C 12 square inches

D 16 square inches

E Not enough information given

4.4
4.5

NAEP 1992 Mathematics Report Card for the Nation and the States.
National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 1993.

K5. The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is the area of the rectangle in square centimeters?

Answer:_____

4.4
5.1

K-5

S2. The figure consists of 5 squares of equal size. The area of the whole figure is 405 cm^2.

Find the area of one square.

Answer _____ square centimeters

Find the length of the side of one square.

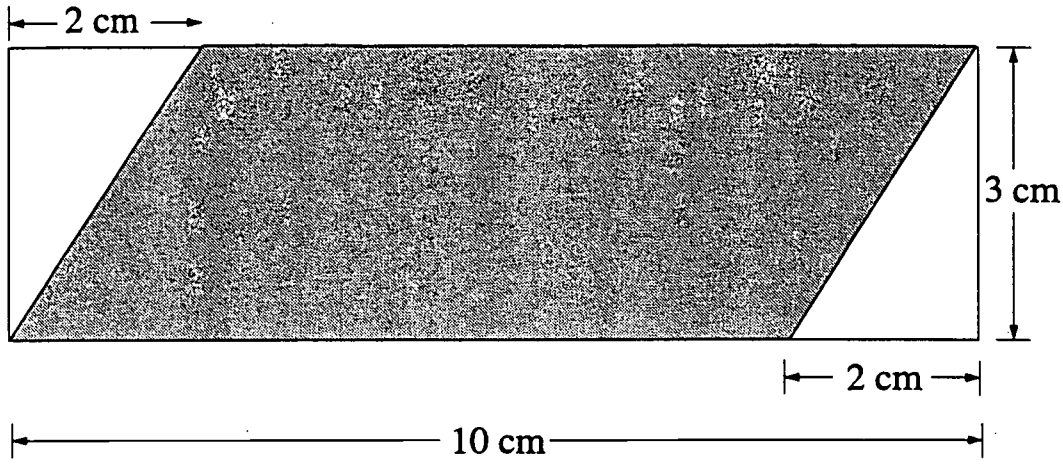
Answer _____ centimeters

Find the perimeter of the whole figure in centimeters

Answer _____ centimeters

S-2a

V4. The figure shows a shaded parallelogram inside a rectangle.

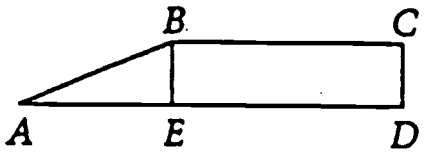


What is the area of the parallelogram?

Answer:_____

V-4

4.4



The area of rectangle *BCDE* shown above is 60 square inches. If the length of *AE* is 10 inches and the length of *ED* is 15 inches, what is the area of trapezoid *ABCD*, in square inches ?

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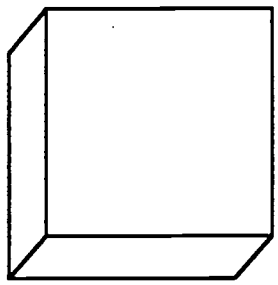
National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 1993.

C-49 80

4.4

Cubes



1) What is the volume of a cube whose edges each measure 3 centimeters?

2) What is the surface area of a cube whose edges each measure 3 centimeters?

4.4
5.5

3) A student named Eddie says,

"No matter what size the cube is, the number you get when you calculate its surface area is always twice as big as the number you get when you calculate its volume."

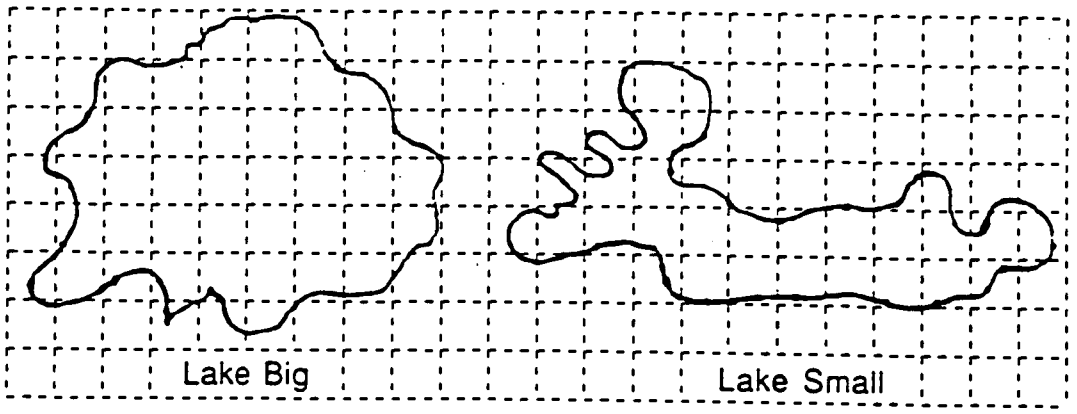
Is Eddie right? Show how you know.

4.4
5.5

Sample Open-Ended Problem

Compare the Lakes

□ = 1 sq. mi



a. Estimate the area of the lakes in the picture above. Describe how you could estimate the area of the lakes.

b. Compare the perimeters of the lakes. Describe how you could estimate the perimeter of each lake. Find the perimeter of the lakes.

c. Do you think the lakes were properly named. Explain.

4.4
5.1

The city of Greendale has set aside a piece of land on which to bury its garbage. The city plans to dig a rectangular hole with a base measuring 500 feet by 200 feet and a depth of 75 feet.

The population of Greendale is 100,000. It has been estimated that, on average, a family of four throws away 0.4 cubic foot of compacted garbage a day. How could this information help Greendale evaluate the plan for a waste site?

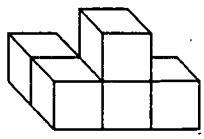
A. How much garbage will this site hold?

B. How long will it take before the hole is filled?

4.4
5.1
5.4

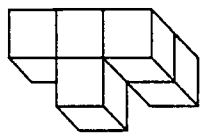
Connected Mathematics, Michigan State University, Dale Seymour Pub., Palo Alto, CA 1996.

K3. This figure will be turned to a different position.

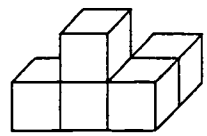


Which of these could be the figure after it is turned?

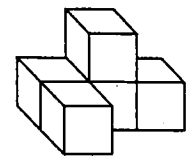
A.



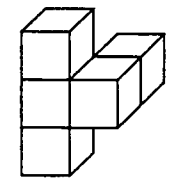
B.



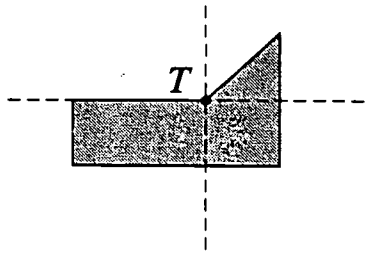
C.



D.



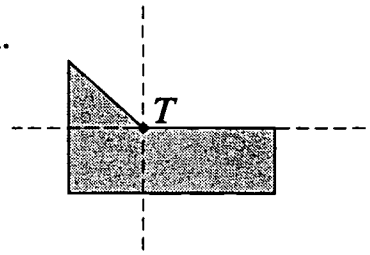
M5. A half-turn about point T in the plane is applied to the shaded figure.



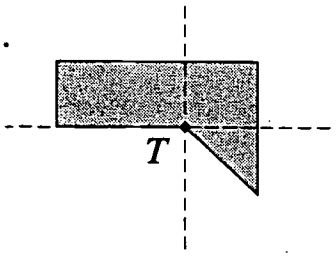
M-5

Which of these shows the result of the half-turn?

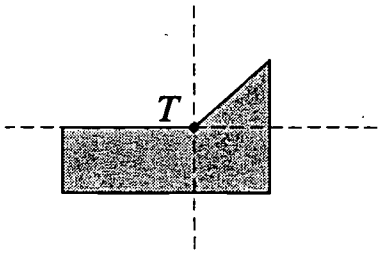
A.



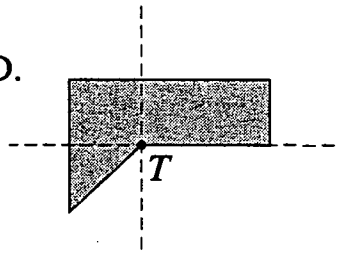
B.



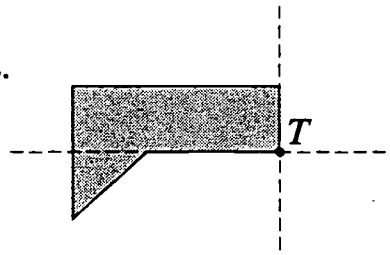
C.



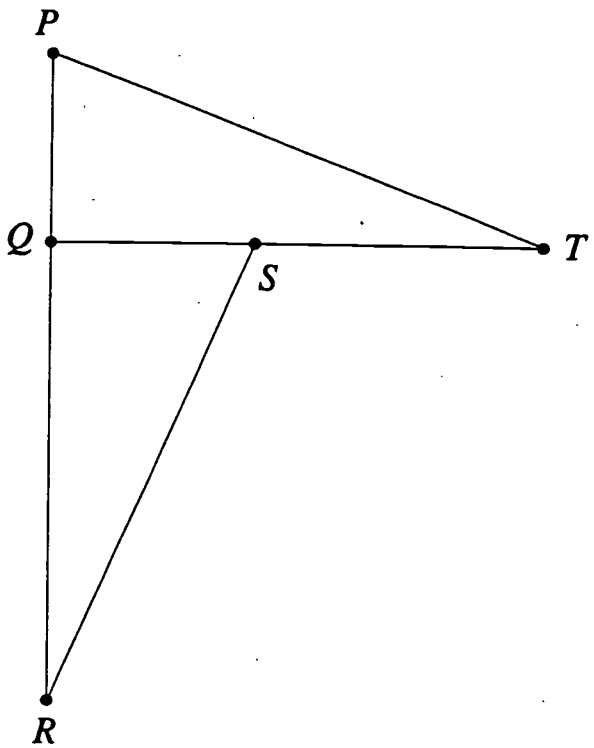
D.



E.



O8. Triangle PQT can be rotated (turned) onto triangle SQR.



What point is the center of rotation?

A. P

B. Q

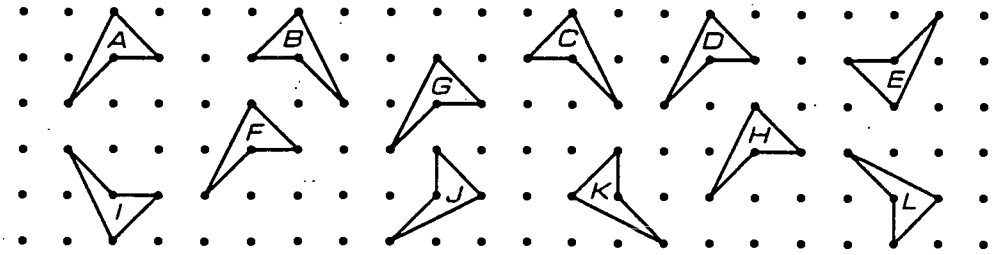
C. R

D. S

E. T

Translations and Reflections Problem

1. a. Which figures are *translation* (slide) images of figure A? Indicate the slide arrow.



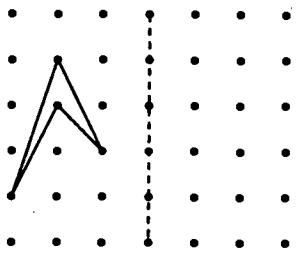
b. Which figures are *reflection* (flip) images of figure A? Find the flip lines.

4.5

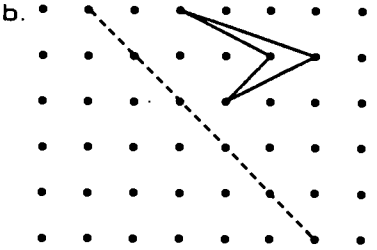
Reflection Problem

2. Draw each *flip* image using the dotted line as the flip line.

a.



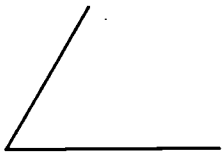
b.

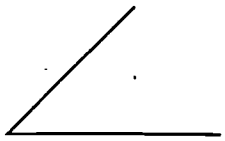


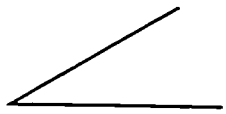
c. Explain how you know that the figure you drew is congruent to the original figure.

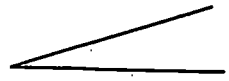
4.5

N15. Which of these angles has a measure closest to 30° ?

A. 

B. 

C. 

D. 

N-15

C-58

5.1

Q6. The Smith family uses about 6000 L of water per week. Approximately how many liters of water do they use per year?

A. 30 000

B. 240 000

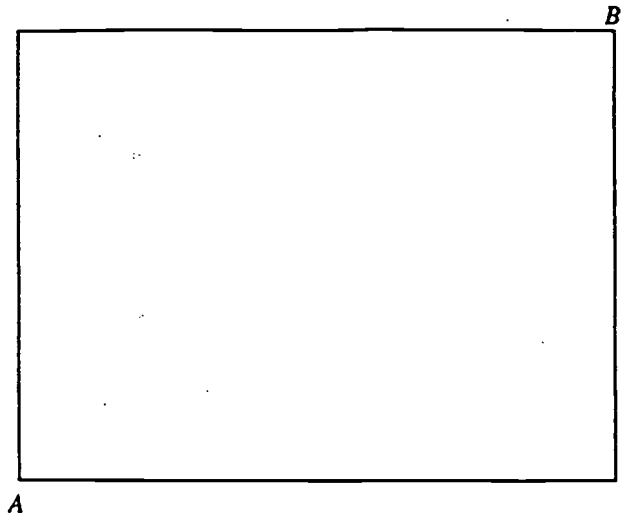
C. 300 000

D. 2 400 000

E. 3 000 000

Q-6

5.1
6.3



Use your centimeter ruler to make the following measurements to the nearest centimeter.

What is the length in centimeters of one of the longer sides of the rectangle?

What is the length in centimeters of the diagonal from A to B ?

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National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 1993.

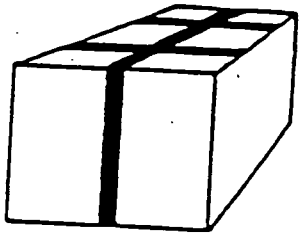
C-60 91

5.1

BUYING STRAPPING TAPE

The public library is moving into a new building. Your class volunteered to pack three hundred cartons of books for the move. You are in charge of buying strapping tape to secure the boxes. When you ask how much tape to buy, your teacher says, "The cartons are 18 in long, 16 in wide, and 12 in high. We will tape them once around the long way, and use two strips across the top."

Estimate how much tape the class will need for the job.



At the store, you find the following packages of strapping tape:

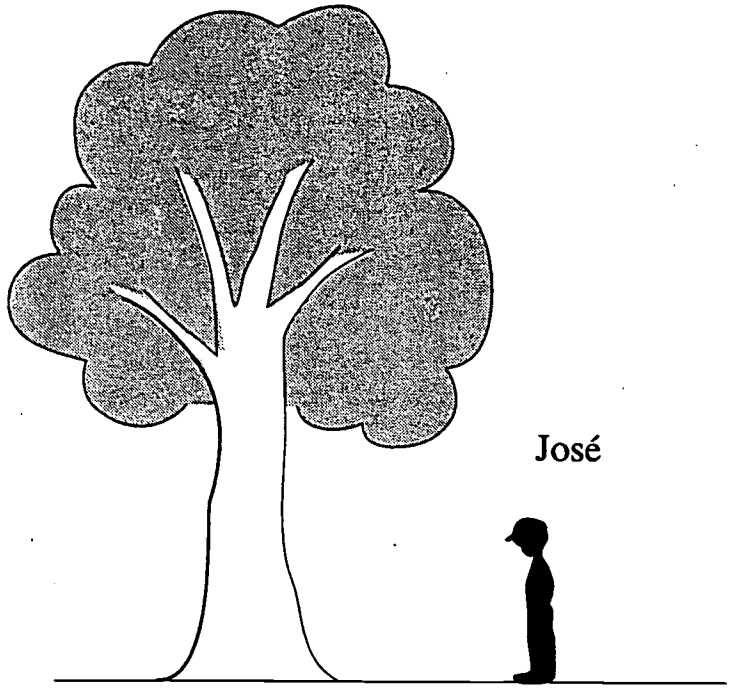
50	yards	@	$.99 per roll
100	yards	@	1.79 per roll
200	yards	@	2.99 per roll

How many rolls of each size will you buy? Explain how you arrived at your decision. How much money will you need to purchase the tape?

Released by the Connecticut State Department of Education.

5.1
6.2

L8.



José is 1.5 m tall. About how tall is the tree?

A. 4 m

B. 6 m

C. 8 m

D. 10 m

L12. Four children measured the width of a room by counting how many paces it took them to cross it. The chart shows their measurements.

Name	Number of Paces
Stephen	10
Erlane	8
Ana	9
Carlos	7

Who had the longest pace?

A. Stephen

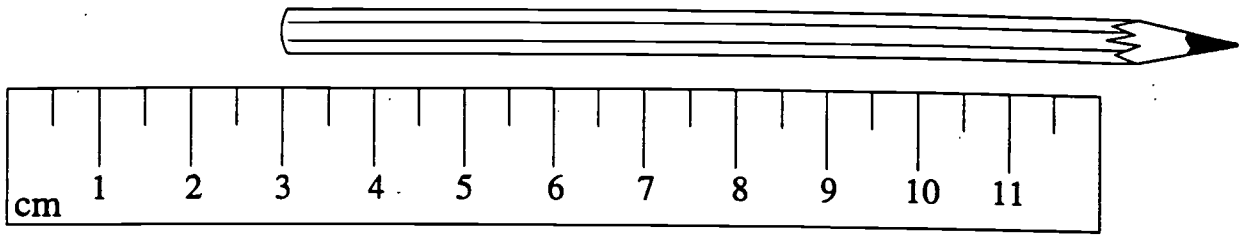
B. Erlane

C. Ana

D. Carlos

L-12

P11.



Which of these is closest to the length of the pencil in the figure?

A. 9 cm

B. 10.5 cm

C. 12 cm

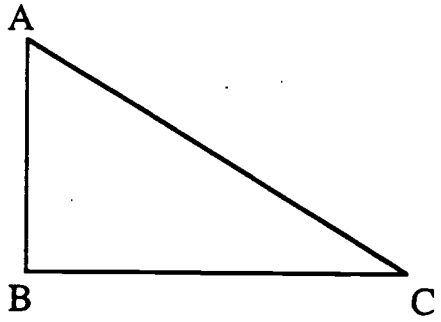
D. 13.5 cm

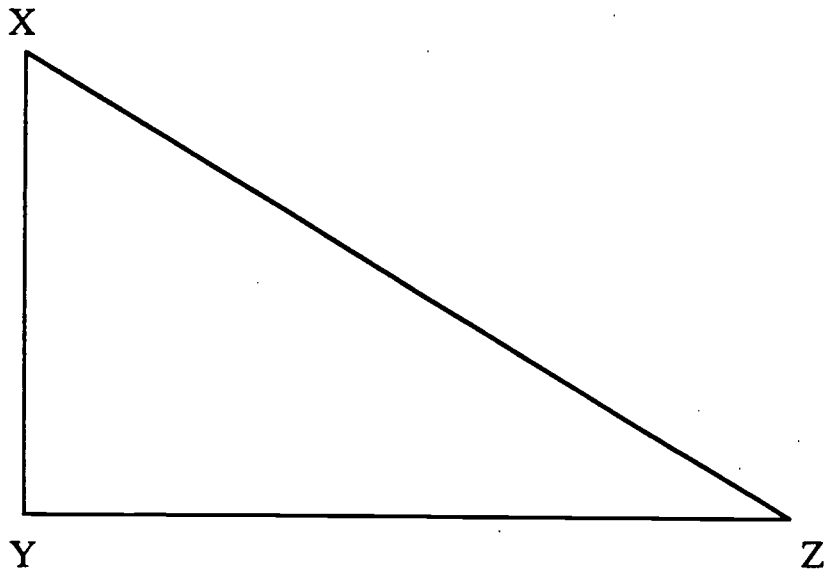
P-11

Comparing Triangles

Triangle ABC is similar to triangle XYZ (that is, the two triangles are exactly the same shape, but are different sizes). Make the measurements you need that will help you compare each feature of the two triangles:

perimeter
size of angles
area





5.2
5.5

Here is an accurate comparison of the bases of the two triangles (check your measurements to see that you agree).

Base YZ is twice as long as base BC.

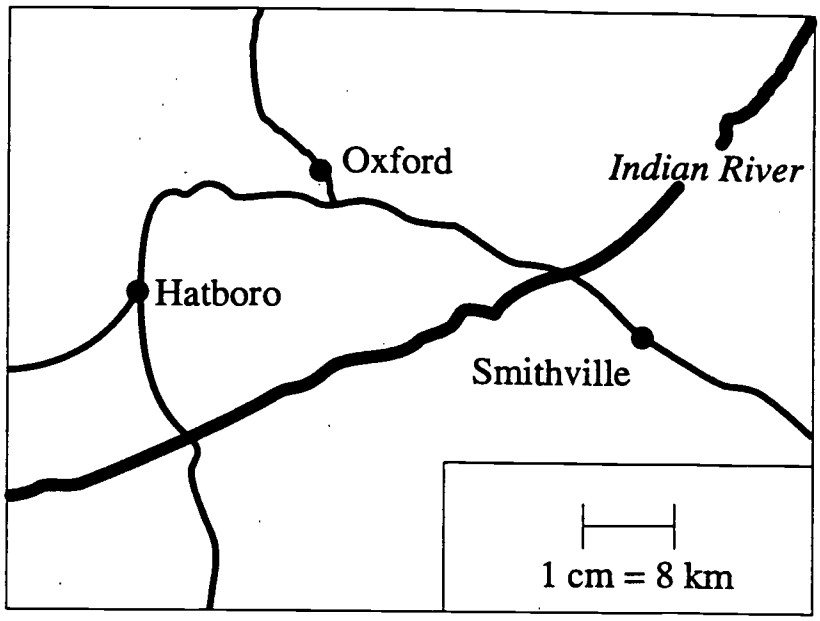
Compare the perimeters of the two triangles. Support your comparison by giving your measurements.

Compare the sizes of the angles in one triangle to the other triangle. Support your comparison by giving your measurements.

Compare the areas of the two triangles. Support your comparison by giving your measurements.

5.2
5.5

J17. One centimeter on the map represents 8 kilometers on the land.



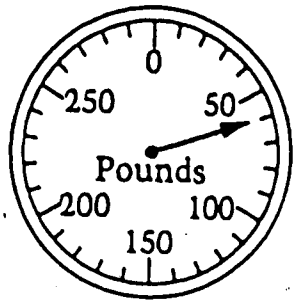
About how far apart are Oxford and Smithville on the land?

A. 4 km

B. 16 km

C. 35 km

D. 50 km



What is the weight shown on the scale?

A 6 pounds

B 7 pounds

C 51 pounds

D 60 pounds

5.3

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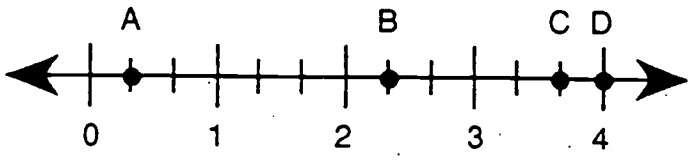
National Center for Education Statistics, Office of Educational Research and Improvement,
U.S. Department of Education, 1993.

What is the value of point B on the number line?



F 4

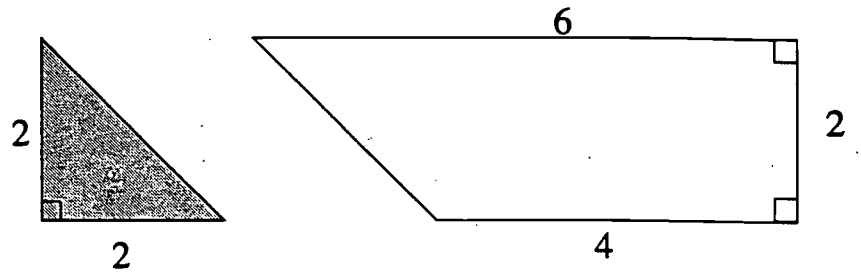
G $3\frac{2}{3}$

H $2\frac{1}{3}$

J $\frac{1}{3}$

5.3

R10.



How many triangles of the shape and size of the shaded triangle can the trapezoid above be divided into?

A. Three

B. Four

C. Five

D. Six

R-10

Stopping Sneaky Sally

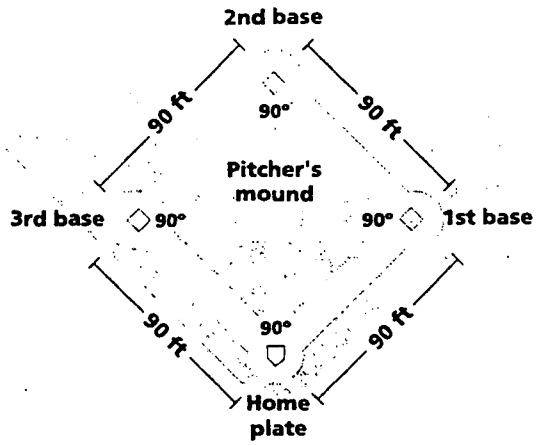
The Pythagorean Theorem can be used in situations like the following, in which you need to find a missing length in a right triangle.

Horace Hanson is the catcher for the Humbolt Bees baseball team. Sneaky Sally Smith, the star of the Canfield Cats, is on first base. Sally is known for stealing bases, so Horace is keeping a sharp eye on her.

The pitcher throws a fastball, and the batter swings and misses. Horace catches the pitch. Out of the corner of his eye, he sees Sally take off for second base.

Problem 4.1

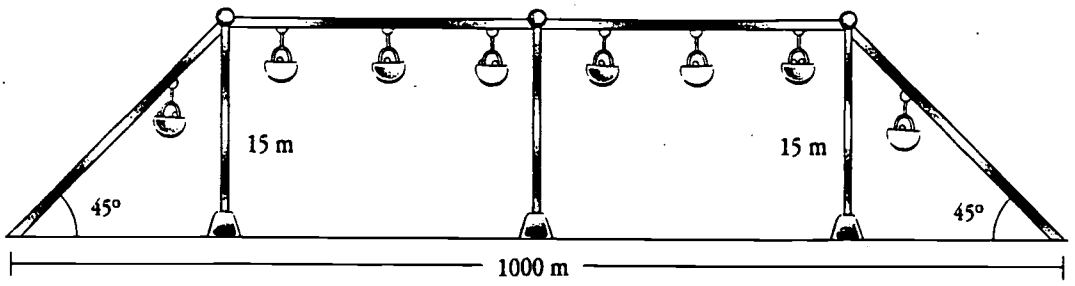
How far must Horace throw the baseball to get Sally out at second base? Explain how you found your answer.



Problem 4.1 Follow-Up

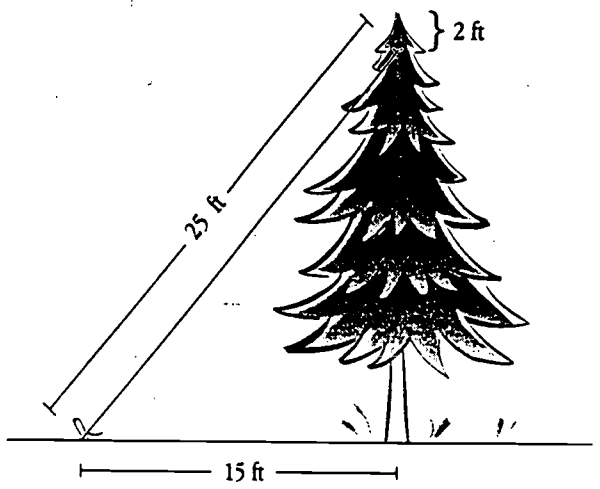
The shortstop is standing on the baseline, halfway between second base and third base. How far is the shortstop from Horace?

Lotsafun Amusement Park has a new ride, the Sky Breaker. The starting and ending points of the ride are separated by 1000 meters. The tram cars glide along a cable that rises at a 45° angle from the ground until it reaches a height of 15 meters. The cable runs parallel to the ground for most of the ride, eventually sloping down again at a 45° angle with the ground. How long, to the nearest tenth of a meter, is the cable for the Sky Breaker ride?



5.4

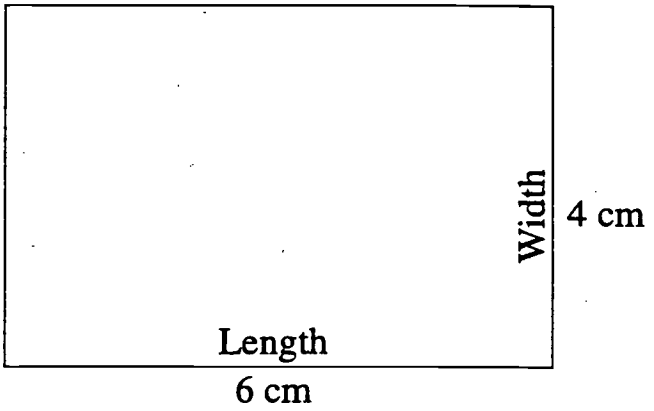
At Errol's Evergreen Farm, the taller trees are braced by a wire extending from 2 feet below the top of the tree to a stake in the ground. What is the tallest tree that can be braced with a 25-foot wire staked 15 feet from the base of the tree?



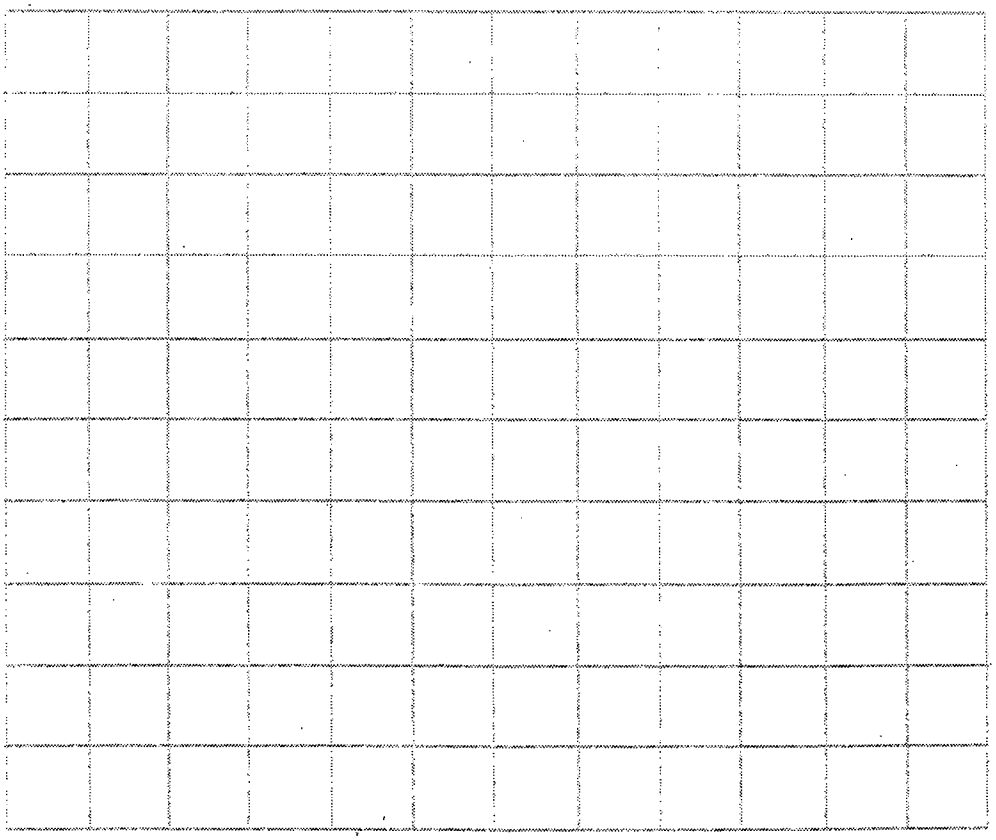
5.4

Connected Mathematics, Michigan State University, Dale Seymour Pub., Palo Alto, CA 1996.

U2.



a. In the space below, draw a new rectangle whose length is one and one half times the length of the rectangle above, and whose width is half the width of the rectangle above. Show the length and width of the new rectangle in centimeters on the figure.



U-2a

b. What is the ratio of the area of the new rectangle to the area of the first one?

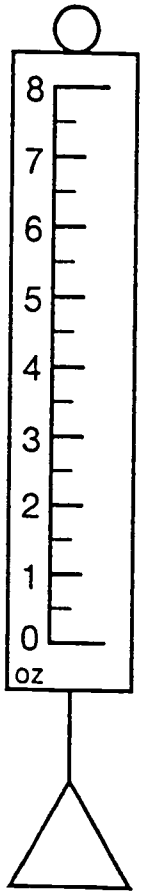
Show your work.

C-72

5.5

Recognize precision

What is the greatest precision that can be achieved with this scale?



A Nearest $\frac{1}{4}$ ounce

B* Nearest $\frac{1}{2}$ ounce

C Nearest 1 ounce

D Nearest 8 ounces

C-73

5.6

I2. Two groups of tourists each have 60 people. If $\frac{3}{4}$ of the first group and $\frac{2}{3}$ of the second group board buses to travel to a museum, how many more people in the first group board buses than in the second group?

A. 2

B. 4

C. 5

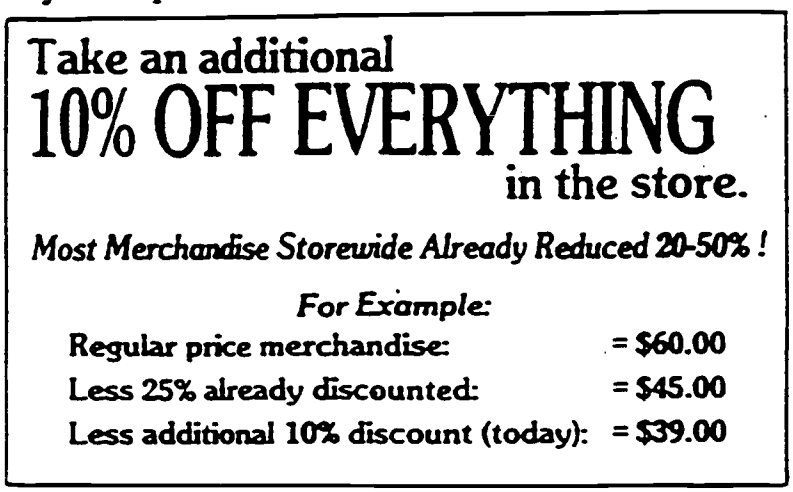
D. 40

E. 45

DOUBLE DISCOUNT

At a department store sale, you are buying a $50 sweater that you selected from a table that says "25% OFF." You also have a coupon for an additional 10% off on any purchase.

The cashier takes 25% off the original price and then takes an additional 10% off. She asks you for $33.75. Write what you would explain to the cashier to justify why this price is not as good as the bargain claimed in the coupon.



Released by the Connecticut State Department of Education.

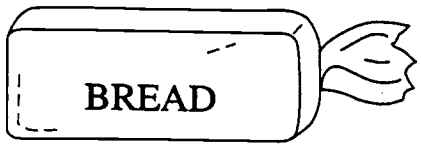
C-75

6.2
6.4

I7. Prabhu had $5 to buy milk, bread, and eggs. When he got to the shop he found that the prices were those shown below:

$1.50

$1.29



$1.44

At which of these times would it make sense to use estimates rather than exact numbers?

A. When Prabhu tried to decide whether $5 was enough money

B. When the clerk entered each amount into the cash register

C. When Prabhu was told how much he owed

D. When the clerk counted Prabhu's change

P13. A person's heart is beating 72 times a minute. At this rate, about how many times does it beat in one hour?

A. 420 000

B. 42 000

C. 4 200

D. 420

P-13

U1. Teresa wants to record 5 songs on tape. The length of time each song plays for is shown in the table.

Song	Amount of Time
1	2 minutes 41 seconds
2	3 minutes 10 seconds
3	2 minutes 51 seconds
4	3 minutes
5	3 minutes 32 seconds

ESTIMATE to the nearest minute the total time taken for all five songs to play and explain how this estimate was made.

Estimate: _____

Explain:

U-1a

K6. Last year there were 1172 students at Beaton High School. This year there are 15 percent more students than last year. Approximately how many students are at Beaton High School this year?

A. 1800

B. 1600

C. 1500

D. 1400

E. 1200

L11. A rubber ball rebounds to half the height it drops. If the ball is dropped from a rooftop 18 m above the ground, what is the total distance traveled by the time it hits the ground the third time?

A. 31.5 m

B. 40.5 m

C. 45 m

D. 63 m

N16. Jan had a bag of marbles. She gave half of them to James and then a third of the marbles still in the bag to Pat. She then had 6 marbles left. How many marbles were in the bag to start with?

A. 18

B. 24

C. 30

D. 36

N-16

N17. A car has a fuel tank that holds 35 L of fuel. The car consumes 7.5 L of fuel for each 100 km driven. A trip of 250 km was started with a full tank of fuel. How much fuel remained in the tank at the end of the trip?

A. 16.25 L

B. 17.65 L

C. 18.75 L

D. 23.75 L

N-17

O2. If the price of a can of beans is raised from 60 cents to 75 cents, what is the percent increase in the price?

A. 15%

B. 20%

C. 25%

D. 30%

O-2

O9. Luis exercises by running 5 km each day. The course he runs is $\frac{1}{4}$ km long. How many times through the course does he run each day?

Answer:_____

Raymond must buy enough paper to print 28 copies of a report that contains 64 sheets of paper. Paper is only available in packages of 500 sheets. How many whole packages of paper will he need to buy to do the printing?

6.4

NAEP 1992 Mathematics Report Card for the Nation and the States.
National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 1993.

Grade 8 Question: Treena's Budget

The Task

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

Treena won a 7-day scholarship worth $1,000 to the Pro Shot Basketball Camp. Round-trip travel expenses to the camp are $335 by air or $125 by train. At the camp she must choose between a week of individual instruction at $60 per day or a week of group instruction at $40 per day. Treena's food and other expenses are fixed at $45 per day. If she does not plan to spend any money other than the scholarship, what are all choices of travel and instruction plans that she could afford to make? Explain your reasoning.

Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

6.4

Section D

COLORADO
MODEL CONTENT STANDARDS
FOR
MATHEMATICS
Fifth to Eighth Grade
Assessment Framework

Sample Constructed-Response Tasks
with Accompanying Scoring Guides
and Student Work

Constructed Response Task #1
Marcy's Dot Pattern

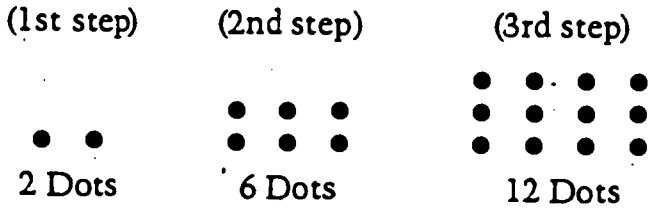
Source: *Can Students Do Mathematical Problem Solving?*
National Center for Education Statistics,
Office of Educational Research and Improvement,
U. S. Department of Education, 1993.

Grade 8 Question: Marcy's Dot Pattern

The Task

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.



Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots.

Explain or show how she could do this and give the answer that Marcy should get for the number of dots.

Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Possible Solution

The explanation should include one of the following ideas with no false statements:

a) For each successive step, the number of rows and the number of columns is increasing by 1, forming a pattern. For example, the first step shows a pattern of dots that consists of one row by two columns, the second step shows a pattern of dots that consists of two rows by three columns, the third step three rows by four columns, and so on. Continuing in this pattern, the twentieth step would have 20 x 21, or 420, dots.

b) Look at successive differences between consecutive steps. The differences 4, 6, 8, 10, ... form a pattern. There are 19 differences forming the pattern 4, 6, 8, 10, ... 38, 40, and this sum is equal to (9 x 44) + 22, or 418. However, 2 must be added for the first step, yielding a response of 420.

The solution to this task requires students to analyze several steps in a pattern of dots in order to conjecture about a general rule for determining the number of dots for any particular step in the pattern. Additionally, students are required to use their rule to find the number of dots at a particular step in an extension of the pattern where it no longer is convenient to draw all of the intermediate dot figures. One approach is to think of the steps in the pattern as consisting of dots in rows and columns and to realize that the number of dots in the nth step can be expressed as
$x_n = n(n+1)$ for n= 1,2,3 ... and thus
$x_{20} = 20(20+1) = (20)21 = 420$.

Other approaches are possible and students could use arithmetic or algebraic concepts to explain their reasoning. Although a few students did write an algebraic equation to express a rule for the general term in a recursive relationship, it was neither expected nor necessary for students to do so.

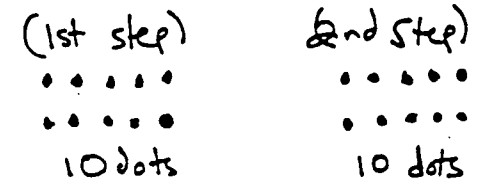
Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Rating and Performance Category

0 **No Response**

1 **Incorrect -- The work is completely incorrect or irrelevant, or the response states, "I don't know."**

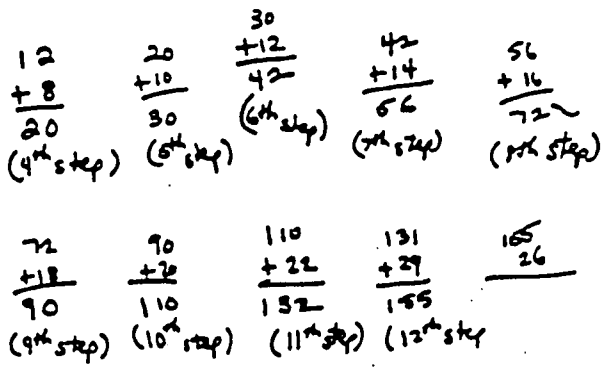
It is difficult to discern an explanation for this INCORRECT response. One possibility is that the student apportioned the total of 20 dots in the three steps shown into two 2 x 5 sets.



Rating and Performance Category

2 **Minimal -- An attempt to generalize the pattern on a superficial level or to draw all 20 pictures in the pattern (with a clear understanding of the pattern).**

This MINIMAL response illustrates a student's attempt to display the first 12 steps in the pattern. There is some understanding of the number of total dots in each entry but no attempt is made to explain the pattern in terms of rows and columns.



Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Rating and Performance Category

3 **Partial -- The response has communicated a partially correct generalization of the pattern.**

This PARTIAL response does begin to formulate an explanation of the total number of dots for an entry. However, the last sentence incorrectly uses the term "multiply" in an attempt to discuss the 20th step. At this point, the explanation falters.

When the pattern starts with 2 dots the next step is to add 4 dots to it and the 3rd step is to add 6 dots to it so every time there is a new step you add 2 dots to the last amount you added on to the last step. You would multiply two dots on and on until you reached the 20th step

Rating and Performance Category

4 **Satisfactory -- The response contains a completely correct generalization of the pattern but does not include -- or incorrectly states -- the number of dots (420) in the 20th step.**

This SATISFACTORY response provides sufficient evidence of how to generate the various steps in the pattern by multiplying the number of rows times the number of columns. However, the student does not determine the number of dots in the 20th step.

Multiply each step by 1# higher such as

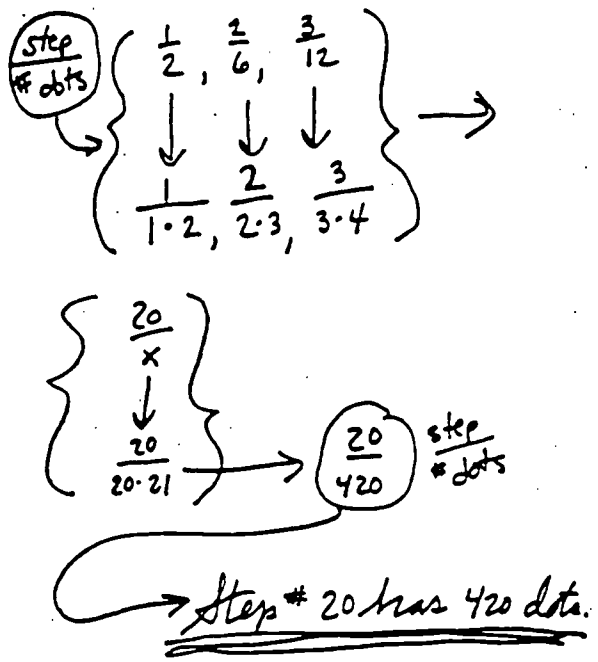
$1 = 1 \times 2$
$2 = 2 \times 3$
$3 = 3 \times 4$
$4 = 4 \times 5$

Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Rating and Performance Category

5 **Extended -- This response contains a completely correct generalization of the pattern and specifies that there are 420 dots in the 20th step.**

This is a strong EXTENDED response. The student clearly related the number of dots in a step to an appropriate multiplication rule. This student then moves directly from step three to step 20 and determines the correct number of dots for that step.



Can Students Do Mathematical Problem Solving? National Center for Education Statistics, Office of Educational Research and Improvement, U. S. Department of Education, 1993.

Constructed Response Task #2
Eating Pizza

Source: *New Standards Released Tasks,*
National Center for Education and the Economy,
Washington, DC, 1995.

Eating Pizza

In this task the student decides which group of friends to join if his or her goal is to eat as much pizza as possible.

This task is designed primarily to assess conceptual understanding in the Number and Operation Standard. It also assesses skills and mathematical communication. Students must reason proportionally to decide which group they should join, make comparisons between amounts of pizza, draw logical conclusions, and communicate information effectively.

Circumstances of Performance

When used as an assessment, this task is intended to take about 5 minutes. It may be used as part of a set of short tasks (for instance, in combination with other released New Standards tasks). Sets of short tasks should aim for a total estimated time of not more than 45 minutes. All New Standards short tasks are designed with the assumption that each student has a calculator, a ruler, and a pencil.

When used as a formal assessment, New Standards provided the following directions to teachers:

Before Students Begin

Make sure the students understand that today is an assessment of their individual work, so they may not work with or talk to each other. This is an "open book" assessment , which means that students may use their mathematics books or other resources they usually use in class during today's assessment. The suggested amount of time for the assessment itself is 45 minutes, but use your judgment: give students ample time so they do not have to rush.

During the assessment

You may read any or all tasks out loud to students who need help reading them. If you normally translate directions for some students, you may do so with this assessment. (If you normally have one student translate for another, you may allow this, but limit it to the first few minutes.)

While students work, do not give any hints or encourage any particular approach. Remind students that they may use calculators, as long as they show which operations they did in their booklet. Encourage them to write what makes sense to them and to persist until it makes sense.

Mathematical Analysis

Product Student Creates	Links to the Standards
A comparison of the ratio of the amount of pizza to the number of people, resulting in the amount of pizza each person gets.	Number and Operation Concepts • reasons proportionally to solve problems involving equivalent fractions or equal ratios • interchanges fractions, decimals, and percents • consistently and accurately adds, subtracts, multiplies, and divides
Explanation of thinking	Mathematical Communication • shows an understanding of the concept by explaining the idea to another person

Eating Pizza

You are invited to go out for pizza with several friends. When you get there, you find your friends sitting at two different tables.

You can join either group. If you join the first one, there will be a total of 4 people in the group and all of you will be sharing 6 small pizzas.

If you join the other group, there will be 6 people in the group and all of you will be sharing 8 small pizzas.

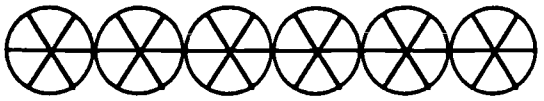
If your goal was to get as much pizza to eat as possible, which group would you join?

Explain your thinking and show your math.

Scoring "Eating Pizza"

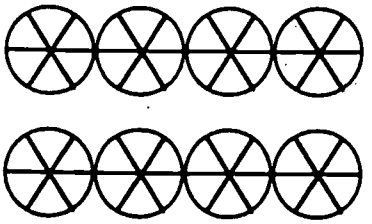
Possible Responses

First Group



Each person gets one whole pizza and three pieces each from the remaining two pizzas.

Second Group



Each person gets one whole pizza and two pieces each from the remaining two pizzas.

I would pick group 1 because I would get $\frac{9}{6}$ or $1\frac{1}{2}$ pizzas instead of $\frac{8}{6}$ or $1\frac{1}{3}$ pizzas.

Another response

I would pick the first one because you would get 9 slices of pizza for each person. If you sit at the second table you would only get 8 slices per person.

1) $6 \times 6 = 36$

$$4\overline{)36}\ 9$$

Village Pizza has 6 slices in a pizza.

2) $6 \times 8 = 48$

$$6\overline{)48}\ 8$$

Task Rubric 5 minute task/two point rubric

To be "Ready for Revision (R)," the response essentially accomplishes the task by the student's choice of the correct group (the first one) and by providing supporting evidence for that choice. There are several possible methods that could be used to arrive at the correct answer. Three of these are shown in the anchor papers for an R. Readers should expect to see additional methods used successfully in the papers they score.

A paper with a minor error such as a calculation error could still be an R if the student demonstrates understanding in the work.

If the reader is not confident that a brief note to the student would suffice to elicit a revision that accomplishes the task, the response is an "M": more instruction needed.

[response #082627]: score R

The student has selected the correct group and provided supporting evidence for that choice. The student's thinking is explained and the calculation is shown.

[response #052440]: score R

The student has selected the correct group and provided supporting evidence for that choice through the use of a labeled picture.

[response #081066]: score R

The student has selected the correct group and provided supporting evidence for that choice. The student's thinking is explained and the math is shown. The student's rounding the answer does not detract from the solution of the problem.

[response #083292]: score R

The student has chosen the second group, which is wrong; however, that choice is consistent with a calculation error in dividing the number of pieces in the first group's pizza by 6 instead of by 4. Since the student divides, accurately, by the number of people in the case of the second group, a brief note to the student should suffice to correct this.

[response #042493]: score M

The student has chosen incorrectly ("it would be the same") and has based the choice on the fact that, in each case, there are two more pizzas than people. More than a brief note would be necessary.

[response #048644]: score M

The student has made the correct choice of group but has not explained thinking or mathematically justified choice. More than a brief note would be necessary.

Anchor Set List for Eating Pizza

Grade level: Middle Length of task: 5 minutes

These anchors are listed in the order in which they are referenced in the task rubric.

Student Responses	Score
082627	R
052440	R
081066	R
083292	R
042493	M
048644	M

Eating Pizza

You are invited to go out for pizza with several friends. When you get there, you find your friends sitting at two different tables.

You can join either group. If you join the first one, there will be a total of 4 people in the group and all of you will be sharing 6 small pizzas.

If you join the other group, there will be 6 people in the group and all of you will be sharing 8 small pizzas.

> If your goal was to get as much pizza to eat as possible, which group would you join?
>
> Explain your thinking and show your math.

I would pick the first one because you would get 9 slices of pizza for each person. If you sit at the second table you would only get 8 slices per person.

Village Pizza has 6 slices in a small pizza

① $6 \times 6 = 36$

$4\overline{)36}$ → 9

② $6 \times 8 = 48$

$6\overline{)48}$ → 8

Eating Pizza

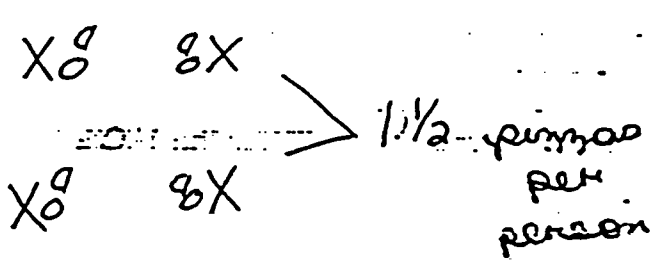
You are invited to go out for pizza with several friends. When you get there, you find your friends sitting at two different tables.

You can join either group. If you join the first one, there will be a total of 4 people in the group and all of you will be sharing 6 small pizzas.

If you join the other group, there will be 6 people in the group and all of you will be sharing 8 small pizzas.

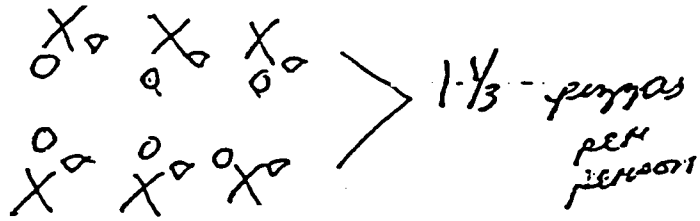
> If your goal was to get as much pizza to eat as possible, which group would you join?
>
> Explain your thinking and show your math.

1st table



Key
X - people
O - pizza
Q - half a pizza
∇ - third of a pizza

2nd table



I would join the 1st group. You get 1 1/2 pizza which is more than 1 1/3.

Eating Pizza

You are invited to go out for pizza with several friends. When you get there, you find your friends sitting at two different tables.

You can join either group. If you join the first one, there will be a total of 4 people in the group and all of you will be sharing 6 small pizzas.

If you join the other group, there will be 6 people in the group and all of you will be sharing 8 small pizzas.

> If your goal was to get as much pizza to eat as possible, which group would you join?
>
> Explain your thinking and show your math.

$6 \div 4 = 1.5 = 2$

$8 \div 6 = 1.\overline{3} = 1$

I would join the first group because I would be getting more pizza. To figure this out I first divided the number of pizzas in the first group (6) by the number of people there would be if I joined them (4). I did the same thing to the second group. Then I rounded my answer to the first group (1.5) and got 2. I rounded my answer to the second group ($1.\overline{3}$) and got 1. This is why I would join the first group.

Eating Pizza

You are invited to go out for pizza with several friends. When you get there, you find your friends sitting at two different tables.

You can join either group. If you join the first one, there will be a total of 4 people in the group and all of you will be sharing 6 small pizzas.

If you join the other group, there will be 6 people in the group and all of you will be sharing 8 small pizzas.

If your goal was to get as much pizza to eat as possible, which group would you join?

Explain your thinking and show your math.

If my goal was to eat as much pizza as I could I would join the 2nd. group because if the pizza has 4 pieces in them per pizza because if I joined the first group I would only get four because 6 pizzas x 4 pieces equals 24 and 24 divided by 6 equals 4. If I joined the 2nd one I would get 5 or more because 8x4=32 and 4/32=5 or more because it has a remainder of 2

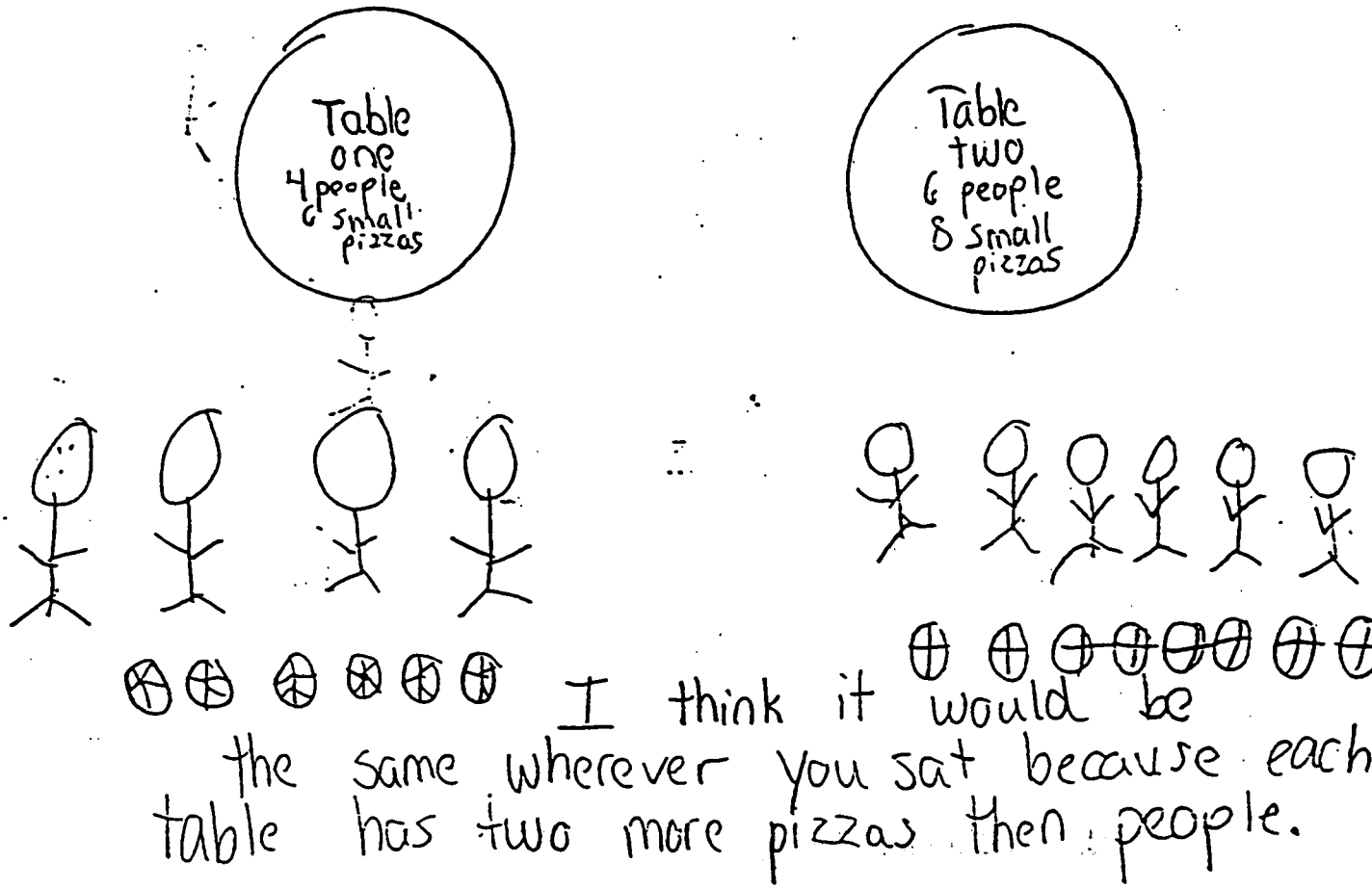
Eating Pizza

You are invited to go out for pizza with several friends. When you get there, you find your friends sitting at two different tables.

You can join either group. If you join the first one, there will be a total of 4 people in the group and all of you will be sharing 6 small pizzas.

If you join the other group, there will be 6 people in the group and all of you will be sharing 8 small pizzas.

> If your goal was to get as much pizza to eat as possible, which group would you join?
>
> Explain your thinking and show your math.



I think it would be the same wherever you sat because each table has two more pizzas then people.

Eating Pizza

You are invited to go out for pizza with several friends. When you get there, you find your friends sitting at two different tables.

You can join either group. If you join the first one, there will be a total of 4 people in the group and all of you will be sharing 6 small pizzas.

If you join the other group, there will be 6 people in the group and all of you will be sharing 8 small pizzas.

If your goal was to get as much pizza to eat as possible, which group would you join?

Explain your thinking and show your math.

6/4 = 3/2

8/6 = 4/3

The group with 4 people and 6 pizzas

Constructed Response Task #3
Odd or Even?

Source: *New Standards Released Tasks,*
National Center for Education and the Economy,
Washington, DC, 1995.

Odd or Even?

The student is asked to predict the winner of the game involving two spinners. Each spinner is divided into thirds. The numbers 3, 4, and 8 are on one spinner. The numbers 0, 1, and 4 are on the other spinner. The first player spins both spinners. She adds the two numbers that the arrows point to. If the sum is even, player #1 wins. If the sum is odd, player #2 wins.

This task is designed primarily to assess conceptual understanding in the Statistics and Probability Standard. It also assesses skills and mathematical communication. Students must determine the theoretical probability for each game, make comparisons between probabilities, draw logical conclusions, and communicate information effectively.

Circumstances of Performance

When used as an assessment, this task is intended to take about 15 minutes. It may be used as part of a set of short tasks (for instance, in combination with other released New Standards tasks). Sets of short tasks should aim for a total estimated time of not more than 45 minutes. All New Standards short tasks are designed with the assumption that each student has a calculator, a ruler, and a pencil.

When used as a formal assessment, New Standards provided the following directions to teachers:

Before Students Begin

Make sure the students understand that today is an assessment of their individual work, so they may not work with or talk to each other. This is an "open book" assessment , which means that students may use their mathematics books or other resources they usually use in class during today's assessment. The suggested amount of time for the assessment itself is 45 minutes, but use your judgment: give students ample time so they do not have to rush.

During the assessment

You may read any or all tasks out loud to students who need help reading them. If you normally translate directions for some students, you may do so with this assessment. (If you normally have one student translate for another, you may allow this, but limit it to the first few minutes.)

While students work, do not give any hints or encourage any particular approach. Remind students that they may use calculators, as long as they show which operations they did in their booklet. Encourage them to write what makes sense to them and to persist until it makes sense.

Number and Operation Concepts	Geometry and Measurement Concepts	Functions and Algebra Concepts	Statistics and Probability Concepts	Problem Solving and Reasoning	Mathematical Skills and Tools	Mathematical Communication	Putting Mathematics to Work

Mathematical Analysis

Product Student Creates	Links to the Standards
A sample space for a set of spinner outcomes	Statistics, Probability • makes predictions based on experimental or theoretical probabilities • determines probabilities of events • constructs sample spaces Problem Solving • extracts pertinent information from situations
A note explaining who has a better chance of winning	Mathematical Communication • uses mathematical language and representations • considers purpose and audience

Odd or Even?

Michiko and Patricia are going to play a spinner game. These are the rules:

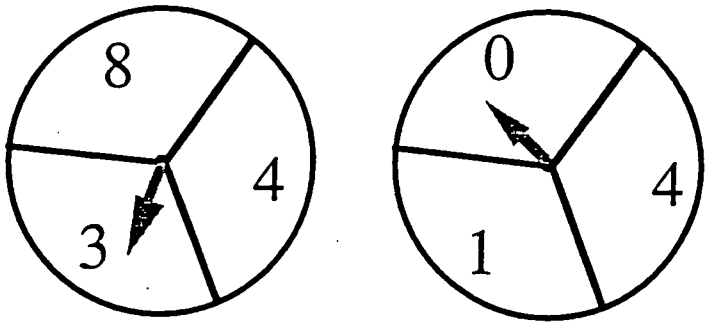
When it is a player's turn, she spins both spinners.

Then she adds the two numbers that the arrows point to.

If the sum is odd (1, 3, 5, 7, 9...), Michiko wins, even if it was not her turn.

If the sum is even (0, 2, 4, 6, 8...), Patricia wins, even if it was not her turn.

Patricia tries a test spin, first. Here is what she spins:



The sum from the first spin is 3, because 3 + 0 = 3. Michiko wins.

Michiko says, "I like this game. I have a better chance to win it than you do."

Patricia says, "No, I have a better chance to win it than you do."

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

Scoring "Odd or Even?"

Possible Responses

Response 1

+	3	4	8
0	[3]	(4)	(8)
1	(4)	[5]	[9]
4	[7]	(8)	(12)

There are 5 out of 9 ways to get an even sum. There are 4 out of 9 ways to get an odd sum.

Dear Girls
Patricia is correct. She has 5 chances out of 9 chances to win. Michiko only has 4 chances out of 9 chances to win.

Response 2

8 + 0 = 8 Even
8 + 1 = 9 Odd
8 + 4 = 12 Even
3 + 0 = 3 Odd
3 + 1 = 4 Even
3 + 4 = 7 Odd
4 + 0 = 4 Even
4 + 1 = 5 Odd
4 + 4 = 8 Even

There are nine ways to win. Therefore we know the chances can't be equal. Five of those chances are even. Only four of the chances are odd.

$$\frac{5}{9} > \frac{4}{9}$$

Dear Girls,
As you can see Patricia has a $\frac{1}{9}$ better chance to win.

Full Rubric for "Odd or Even?"

In this task, students are asked to predict the winner of a game involving two spinners. To be Ready for Revision (R), the response must show that the student understands probability and uses a sample space to make correct decisions based on that understanding. A successful response lists possible combinations (sample space), discusses the likelihood of winning (5/9 vs. 4/9 or 5 to 4), selects a winner based on the data presented, and justifies conclusions reached.

If the reader is not confident that a note to the student would suffice to elicit a revision that accomplishes the task, the response is an "M." Any paper not meeting the above minimum requirements is an M.

> [Notes from NCTM Standards pages 109–110: Students in the middle grades must ...develop an understanding of the relationship between the numerical expression of a probability and the events that give rise to these numbers (e.g., 2/5 as it relates to the probability of choosing a red marble from a hat). Students must not only understand the relationship between the numerical expression and the probability of the event but realize that the measure of certainty or uncertainty varies as more data are collected.]

4: Accomplishes the Task

The response achieves the purposes prompted, by:

- showing an understanding of probability;
- including the sample space of all given possibilities (all possible sums when spinning both spinners);
- determining each player's probability of winning the game;
- selecting the correct winner based on the data; and
- justifying decisions and/or supporting conclusions.

The following responses contain evidence of clear understanding of the task. They have all of the characteristics of a "4" response.

[074863]: score 4

In the last sentence of this response, the student appears to be exploring alternative solution strategies. This neither adds to, nor detracts from, the score of 4.

[068790]: score 4

Although the communication in this response is not polished, the requirements for a 4 are clearly met.

[053140]: score 4

The strengths of this response include a well-organized sample space and effective use of ratios.

3: Ready for Revision

For a 3, a response must:

- show an understanding of probability;
- include enough combinations to show understanding of correct sample space;
- make a prediction as to who will win the game based on the data presented; and
- justify conclusions made.

Responses earning a 3 may not state mathematically each player's probability of winning the game. Conclusions reached as a result of minor computational errors are acceptable as long as those conclusions follow logically from the data displayed. The justification/support may not be as convincing as in a 4 response. Any weaknesses in a three response would not indicate the need for further instruction.

The following responses contain evidence of understanding of the task. They have all of the characteristics of a "3" response. It should be kept in mind that "3" papers do not indicate a need for further instruction.

[065128]: score 3

This response contains an example of a calculation error (0 + 4 = 1), which leads to an incorrect conclusion. The decisions made, however, follow logically from the data displayed.

[080518]: score 3

Even though the student indicates the likelihood of winning the game, the accompanying description lead us to believe that s/he might not have a clear understanding of the problem. The conclusion does not support the data.

[043291]: score 3

Although the student lists all possible combinations, determines the likelihood of winning (tally marks), and predicts a winner, the justification is weak.

[079845]: score 3

There is no discussion of the likelihood of winning.

2: Partial Success

Part of the task is attempted. The response shows some understanding of probability, the chances of winning, and some of the combinations are displayed (sample space attempted). The student makes the choice of a winner and attempts to defend that choice. The response indicates that the student understands the concept of the game.

The following responses contain partial evidence of understanding of the task. They have all of the characteristics of a "2" response. It should be kept in mind that "2" papers indicate that there is a need for further instruction.

[043316]: score 2

Numbers from the same spinner have been added to obtain extraneous combinations.

[074564]: score 2

Numbers from the same spinner have been added to obtain extraneous combinations. In addition, some combinations have been repeated. The justification on page one is not connected to the use of the spinners. The reference of 2 to 1 odds of winning on the second page also indicates a lack of understanding.

1: Engaged with Little Success

The student's response does not demonstrate an understanding of the problem posed. No progress is made toward a solution. The student's response might be based on a hunch/guess or intuition about probability; or might show no mathematical work even though a correct decision might be made.

The following responses do not contain evidence of understanding of the task. They have all of the characteristics of a "1" response.

[063229]: score 1

The student attempts only a guess.

[043733]: score 1

The student demonstrates an understanding of odd and even numbers, but fails to make progress toward a solution to the problem.

[056343]: score 1

Even though the student predicts the correct winner, there is no evidence that the conclusion was based on mathematics.

[053038]: score 1

No mathematical combinations are provided. Therefore, the paper does not meet the requirements of a 2 paper. No justification is made for the solution to the problem.

Anchor Set List for Odd or Even?

Grade level: Middle Length of task: 15 minutes

These anchors are listed in the order in which they are referenced in the task rubric.

Student Responses	Score
074863	4
068790	4
053140	4
065128	3
080518	3
043291	3
079845	3
043316	2
074564	2
063229	1
043733	1
056343	1
053038	1

Odd or Even

Michiko and Patricia are going to play a spinner game. These are the rules:

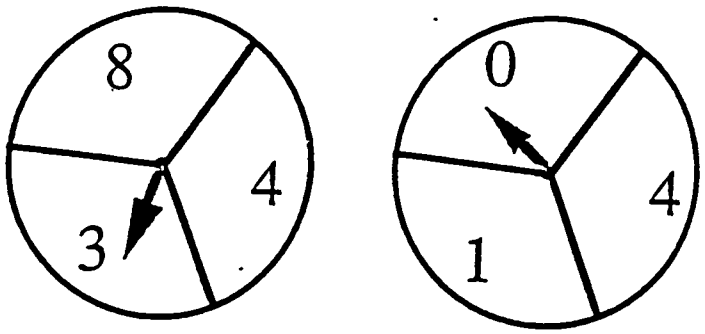
When it is a player's turn, she spins both spinners.

Then she adds the two numbers that the arrows point to.

If the sum is odd (1, 3, 5, 7, 9...), Michiko wins, even if it was not her turn.

If the sum is even (0, 2, 4, 6, 8...), Patricia wins, even if it was not her turn.

Patricia tries a test spin, first. Here is what she spins:



The sum from the first spin is 3, because 3 + 0 = 3. Michiko wins.

Michiko says, "I like this game. I have a better chance to win it than you do."

Patricia says, "No, I have a better chance to win it than than you do."

To figure out this problem all you have to do is add all of the numbers together, if patricia was even and michiko was odd than this is what you do...

michiko (odd)	patricia (even)
8+1=9	8+0=8
4+1=5	8+4=12
3+4=7	4+0=4
3+0=3	4+4=8
4	3+1=4
	5

8+0=8 4+0=4 3+0=3
8+4=12 4+4=8 3+1=4
8+1=9 4+1=5 3+4=7

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So the conclusion is that Patricia has a better chance because most of the numbers come out even when you are adding even numbers and there are 4 even numbers 2 odd ones so most likely the even numbers would win

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074863 D-27

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

even	odd
8+0	8+1
8+4	3+0
3+1	3+4
4+0	4+1
4+4	

Patricia has a better chance of winn
because she has five combonations
win, while Michiko only has four.

Odd or Even

Michiko and Patricia are going to play a spinner game. These are the rules:

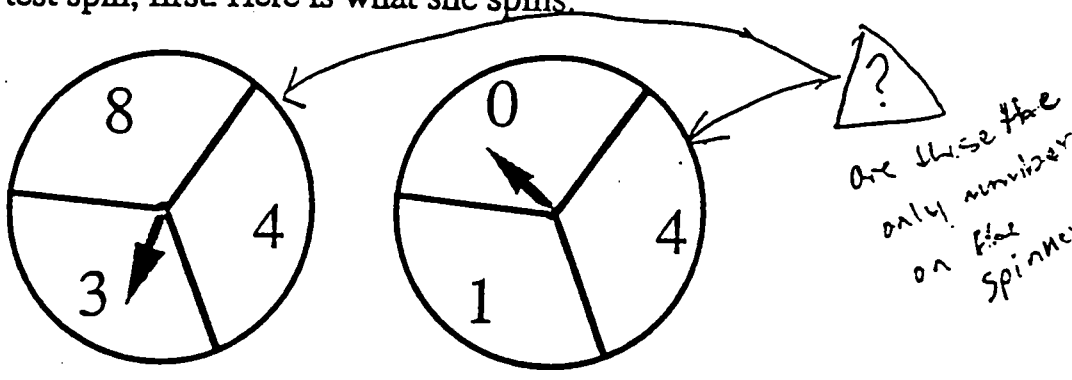
When it is a player's turn, she spins both spinners.

Then she adds the two numbers that the arrows point to.

If the sum is odd (1, 3, 5, 7, 9...), Michiko wins, even if it was not her turn.

If the sum is even (0, 2, 4, 6, 8...), Patricia wins, even if it was not her turn.

Patricia tries a test spin, first. Here is what she spins:



The sum from the first spin is 3, because 3 + 0 = 3. Michiko wins.

Michiko says, "I like this game. I have a better chance to win it than you do."

Patricia says, "No, I have a better chance to win it than than you do."

$8+0=8-P$
$8+4=12-P$
$8+1=9-M$
$3+0=3-M$
$3+4=7-M$
$3+1=4-P$
$4+0=4-P$
$4+4=8-P$
$4+1=5-M$

Pat could win about 5 time or there is 5 ways she could win, but there is only 4 ways for Michiko to win so Patricka actually has a better chance.

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

Sums

8 + 0 = 8
4 + 0 = 4
3 + 0 = 3

$\frac{5}{9}$ chances of getting evens

8 + 1 = 9
4 + 1 = 5
3 + 1 = 4

$\frac{4}{9}$ chances of getting odd's

8 + 4 = 12
4 + 4 = 8
3 + 4 = 7

Patricia has a better chance in winning because she has $\frac{5}{9}$ chances to get an even # and Michelle has $\frac{4}{9}$ chances of getting odd #'s

Odd or Even

Michiko and Patricia are going to play a spinner game. These are the rules:

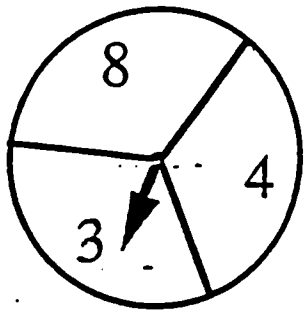
When it is a player's turn, she spins both spinners.

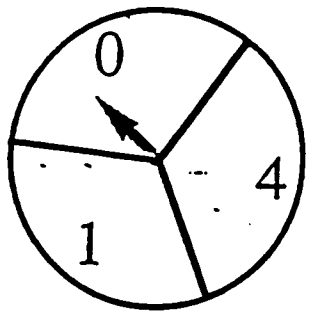
Then she adds the two numbers that the arrows point to.

If the sum is odd (1, 3, 5, 7, 9...), Michiko wins, even if it was not her turn.

If the sum is even (0, 2, 4, 6, 8...), Patricia wins, even if it was not her turn.

Patricia tries a test spin, first. Here is what she spins:





The sum from the first spin is 3, because 3 + 0 = 3. Michiko wins.

Michiko says, "I like this game. I have a better chance to win it than you do."

Patricia says, "No, I have a better chance to win it than than you do."

Michiko has a better chance to win because 5 out of 9 she would win.

4+3=4-m1
4+9=8-p1
4+8=12-p2
1+4=5-m2
1+3=4-p3
1+8=9-m3
0+3=3-m4
0+8=8-p4
0+4=1-m5
n=5
p<4

D-31 065128 (1/2) p1

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

Dear Michiko,
you have a better chance winning because 5 out of 9 spins would be odd.

Dear Patricia,
Your friend Michiko has a better chance of winning because you would only have 4 out of 9 spins that would be even.

Odd or Even

Michiko and Patricia are going to play a spinner game. These are the rules:

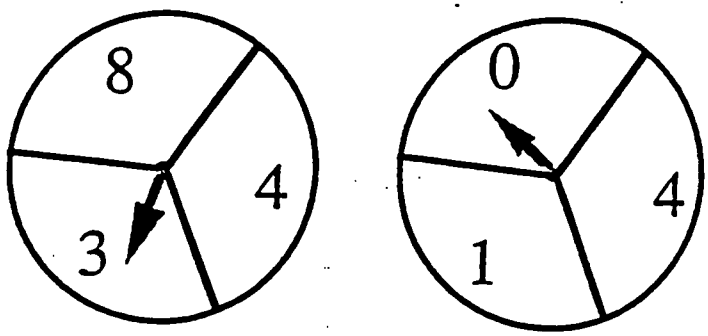
When it is a player's turn, she spins both spinners.

Then she adds the two numbers that the arrows point to.

If the sum is odd (1, 3, 5, 7, 9...), Michiko wins, even if it was not her turn.

If the sum is even (0, 2, 4, 6, 8...), Patricia wins, even if it was not her turn.

Patricia tries a test spin, first. Here is what she spins:



The sum from the first spin is 3, because 3 + 0 = 3. Michiko wins.

Michiko says, "I like this game. I have a better chance to win it than you do."

Patricia says, "No, I have a better chance to win it than than you do."

3+1=4(even)
3+0=3(odd)
3+4=7(odd)
4+0=4(even)
4+1=5(odd)
4+4=8(even)
8+0=8(even)
8+1=9(odd)
8+4=12(even)

even - 5

odd - 4

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Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

Patricia is right because There is a better chance of her getting even numbers than there is Monika getting an odd number.

If you add each number with it possibility that the other might land on there is more evens than there is odds. There also is 2 even numbers on each spinner and one odd.

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

Michiko's (odd)	Patricia's (even)
1111	1111

8+0=8
8+4=12
8+1=9
4+0=4
4+4=8
4+1=5
3+0=3
3+4=7
3+1=4

Patricia has a little better chance, but since there aren't that many numbers it is hard to find out who really has the advantage.

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

8+0 = 8 ✓
8+1 = 9 -
8+4 = 12 ✓
3+0 = 3 -
3+4 = 7 -
3+1 = 4 ✓
4+0 = 4 ✓
4+1 = 5 -
4+4 = 8 ✓

Patricia has a better chance of winning for when you look at all the possibilit for the sums of all the numbers, the are more even answers.

Dear Michiko,

I'm sorry to say, but Patricia is the one wit the atvantage. When all the possibilities are looked at for the addition, there comes out to be more even answers than odd. To make sure of my answer, you can try it too. All you have to do is add all the numbers to each other. Make sure you don't forget any pairs!

Sicerely,

Dear Patricia,

Congratulations! You were right! You do have a better chance to win. when you add each possibl pairs of numbers together, there comes out to be more even sums than odd. Therefore the atvantage is yours!

Sincerely,

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

All possible combinations

8 + 0 = 8
8 + 3 = 11
8 + 4 = 12
8 + 1 = 9
8 + 4 = 12

3 + 4 = 7
3 + 0 = 3
3 + 1 = 4
3 + 4 = 7

4 + 0 = 4
4 + 1 = 5
4 + 4 = 8

1 + 0 = 1
1 + 4 = 5
4 + 0 = 4

# of odd	# of Evens
8	7

Michiko is right she has a better chance of winning.

Odd or Even

Michiko and Patricia are going to play a spinner game. These are the rules:

When it is a player's turn, she spins both spinners.

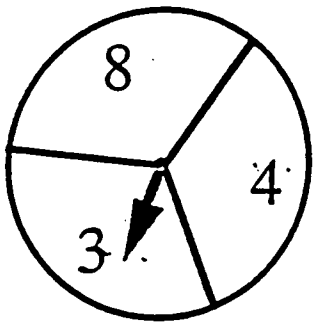
Then she adds the two numbers that the arrows point to.

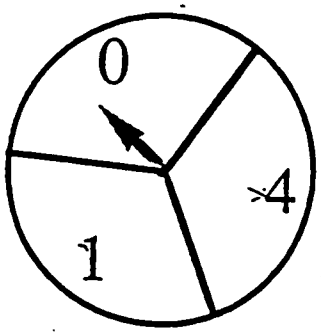
If the sum is odd (1, 3, 5, 7, 9...), Michiko wins, even if it was not her turn.

If the sum is even (0, 2, 4, 6, 8...), Patricia wins, even if it was not her turn.

Patricia tries a test spin, first. Here is what she spins:

3+0=3 odd
8+6=8 even
4+0=4 odd
1+0=1 odd
0+0=0 even





The sum from the first spin is 3, because 3 + 0 = 3. Michiko wins.

Michiko says, "I like this game. I have a better chance to win it than you do."

Patricia says, ~~"No, I have~~ a better chance to win it than than you do."

4+4=8 P wins
4+4=8 P wins
8+1=9 M wins
1+8=9 M wins
8+3=11 M wins
3+8=11 M wins
8+4=12 P wins
4+8=12 P wins
8+0=8 P wins
9+0=9 P wins

There are 2 odds and 4 evens
odd + odd = even, even + even = even
odd + even = odd
odd + 0 = odd
even + 0 = even
The evens outnumber the odds therefore Pat has a better chance

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Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

Pat,
You have a better chance because if one arrow lands to odd and the other to even the outcome will be odd, if both arrows are even the answer will be even, and if both arrows point to odd it will be even. So the evens out number the odds 2 to 1.

Midika
You are wrong because if an odd and an even are spun it is a odd, but if two evens are spun it is always a even, and two evens spun is always a even, ~~that's~~ 2 evens to 1 odd better luck next time!

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

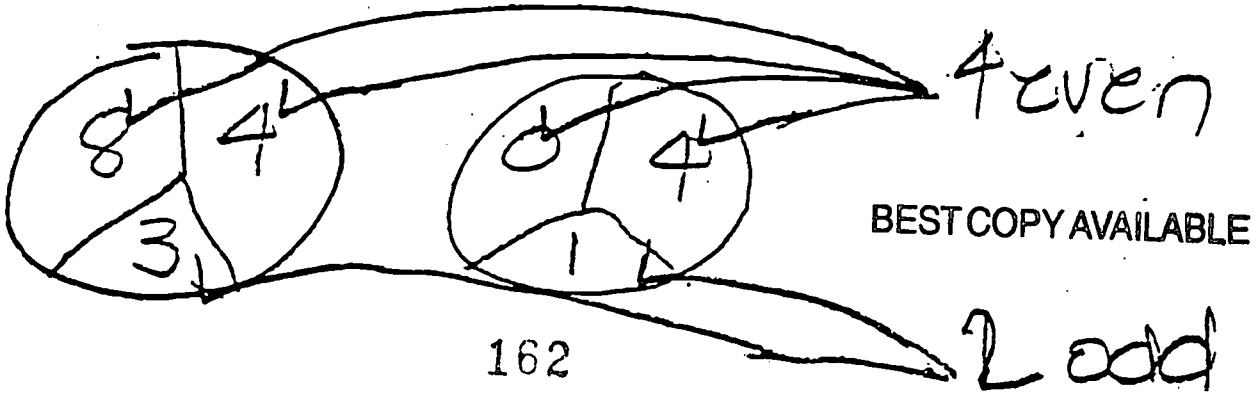
Grils yo Both have a fifty fifty chanch of winning

Because there are the same number of evens and ods

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

I have not a clue how to do - but best ein taking a guess the Patricia has a better chance to win than michiko because I like evens better than odds; and because there are more even numbers than odd numbers on the spinner game.



Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

Michiko,
I'm sorry to say but Patricia has a better chance of winning because whenever the sum is even she wins. There are more evens in the game than odds so the probability of her winning is greater because an even plus an even equals an even.

Patricia,
You do have the better chance of winning because there are more evens in the game. So because an even + and even = an even, You will win more.

Use mathematics to decide which girl is right.

Write a note to both girls explaining how you know who has the better chance of winning.

Patricia, you have a ~~better~~ chance of winning because there are more combinations that add up to an even number, than for an odd

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