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ABSTRACT

This paper is based on the on-going research work and teaching carried out in the Mathematics Foundations Course (MFC) at Goldsmiths College, University of London. It is argued that teaching adult students mathematical investigation has given insight into the way they learn mathematics better and removes the barriers of mathematics phobia. It should be a tool in its own right which adult students can associate with and clarify, think, predict, survey, and research, and should enable them to solve problems in a variety of ways including other related subjects like science, economics, geography, technology, and art. Furthermore, mathematical investigation encourages adult students to read, understand, and respond to questions and explanations better. It makes them confident in mathematics learning by making them familiar with mathematical words and their meanings. Moreover, adult students become more confident when talking about mathematics. It involves using processes which will lead to the understanding of mathematical concepts and rules and generates mathematical discussions. This paper invites discussion of mathematical investigation as well as other issues concerning research on all aspects of adult mathematics learning and teaching considered to be vital to increase our understanding and enhance the status of adult mathematics education. This paper concludes that mathematical investigation provides a basis for improving the quality of teaching and learning mathematics, which will lead to an achievement in mathematics education, therefore, leading to an improvement in standards. A critical mathematics curriculum is also discussed. (ASK)

TEACHING ADULT STUDENTS MATHEMATICAL INVESTIGATIONS .2

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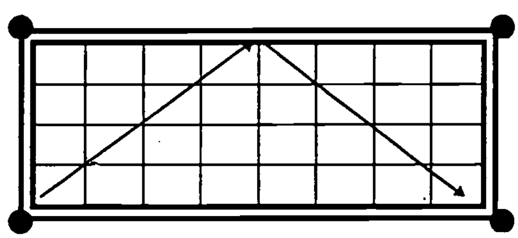

Start

PAPER PRESENTED AT THE ADULTS LEARNING MATHEMATICS - 4 CONFERENCE AT THE UNIVERSITY OF LIMERICK, IRELAND 4th JULY - 6th JULY 1997

TEACHING ADULT STUDENTS MATHEMATICAL INVESTIGATION - 2

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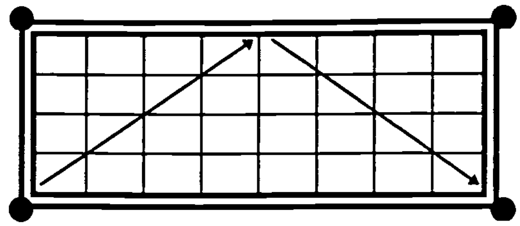
Abstract:

The theme of this paper is 'Teaching Adult Students Mathematical Investigation (TASMI - 2)' It is based on the on-going research work and teaching carried out for the last 7 years of the Mathematics Foundations Course (MFC), at Goldsmiths College University of London (Angiama, RO, 1992, 1994, 1995, 1996).

I have argued that 'Teaching Adult Students Mathematical Investigation', is given an insight into the way they learnt mathematics better and removes the barriers of mathematics phobia. It should be a tool in its own right which adult students can associate with and clarify, think, predict, survey, research and should enable them to solve problems in a variety of ways including other related subjects like science, economics, geography, technology and art.

Furthermore, mathematical investigation encourages adult students to read, understand and respond to questions better and explanations. It makes them to become confident in mathematics learning by making them familiar with mathematical words and their meanings. Moreover, adult students become more confident when talking about mathematics. It involves using processes which will lead to the understanding of mathematical concepts, rules and generates mathematical discussions. This paper invites the participants attending the fourth Adults Learning Mathematics - A Research Forum (ALM-4), conference their views of what they consider 'Mathematical Investigation' to be, as well as other issues concerning research on all aspects of adult mathematics learning and teaching - vital to increase our understanding and enhance the status of adult mathematics education.

The Cockcroft Report (1982), stated, "the idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in very many field". The paper concludes, 'Mathematical investigation' provides a basis for improving the quality of teaching and learning mathematics which will lead to an achievement in mathematics education and, therefore, leads to an improvement on standards. It calls for a critical mathematics curriculum.



INTRODUCTION

The Billiard table shown above is said to be a little odd because it has only four pockets and the base is divided into squares. Equally, the rules of the game are a little odd too, considering that only one Billiard ball is used and it is always struck from the same corner of 45 degrees to the side. In other words, the ball hits at an angle of 45 degrees from the corner and then bounces off of the sides at 45 degrees.

The theme of the investigation, is to investigate what happens for tables of different sizes, for instance, 2 x 6, 5 x 10 and 4 x 8 etc, etc.

Rosanne Benn (1997), has argued that ' the process of teaching and learning mathematics as a language involving reading, talking and writing is a practical and useful one in the practice of helping adults learning mathematics'. This is particularly true with mathematical investigation work with adults which requires that the student should do the work independently.

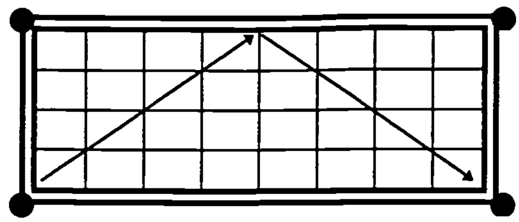
AIMS

- Reading mathematics requires that the learner must read, understand and respond to questions as well as explanations and extract the relevant information from the text

  The intention will be for adult students and young people to become confident by making them familiar with mathematical words and their meaning, and to make mathematics accessible to students, no matter what levels of English.

- Talking mathematics aims at teacher-student relationship which is often seen as fundamentally narrative with the narrating done by the teacher and passive listening by the student. Alternatively teaching and learning mathematics aims can be based on the concept of dialogue. Knowledge of mathematics can be systematically developed by dialogue not only between teacher and students but also between students through group work. By working with others while learning mathematics adult students can, through discussion groups and dialogue, break patterns of dependency, social relationships and isolated learning. They should be able to speak audibly and with confidence and understand what is heard including use of a mathematical dictionary.

- Writing Mathematics requires that the learner is to use mathematics effectively in everyday situations. They will need to be able to express mathematical ideas in a communicable written form. In addition, students should be able to solve problems and set out solutions fully and logically written form. They should be encouraged to discuss their investigation work with each other and use constructive criticism to develop ideas.



Start

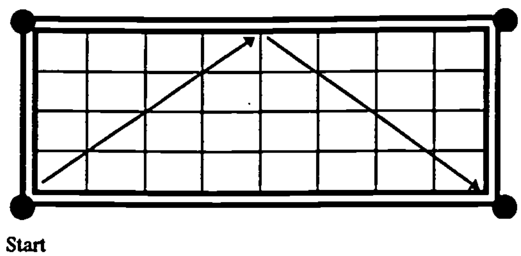
METHOD OF ANALYSIS AND TEACHING STYLE (1)

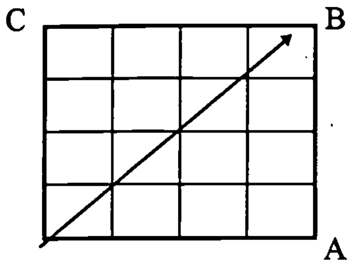
I now want to consider, how the students investigate what happens for table of different sizes. In order to do this, while researching as a practitioner, (Kathy Stafford, 1997), I have adopted the following investigational teaching styles, learning strategies and questioning. This has profound effect on the teaching and to the practice of helping adults learn mathematical investigation better.

- Before starting any mathematics investigation, students are encouraged to introduce the problem in their own words and show how they are going to try and solve the problem. When working alone, in this type of work, the pressure is on the student to produce all the ideas and write down what s/he thinks.
- Adult students are also being encouraged to show all their working and to collect their results that the work has produced. Always tabulate results if they can - thus, that is to say, put the results in a logical order, and see if they can find any rules or patterns.
- If they think they have spotted a rule or a pattern, say what it is then check, by trying out their rule on some new data and seeing if the pattern continues.
- If the method a student has chosen does not work, then they are advised to try and find another method and start again, because mathematicians often have to do this.
- Furthermore, we encourage our students to try and add something of their own, that is to say, something original to the problem, do a little research about the topic, their investigation, which they are expected to read about the work any mathematicians have done about the topic.
- Finally, students are expected to state any conclusion drawn from the purpose of the investigation, information of evidence sought and method which they think their work entitles them to make.

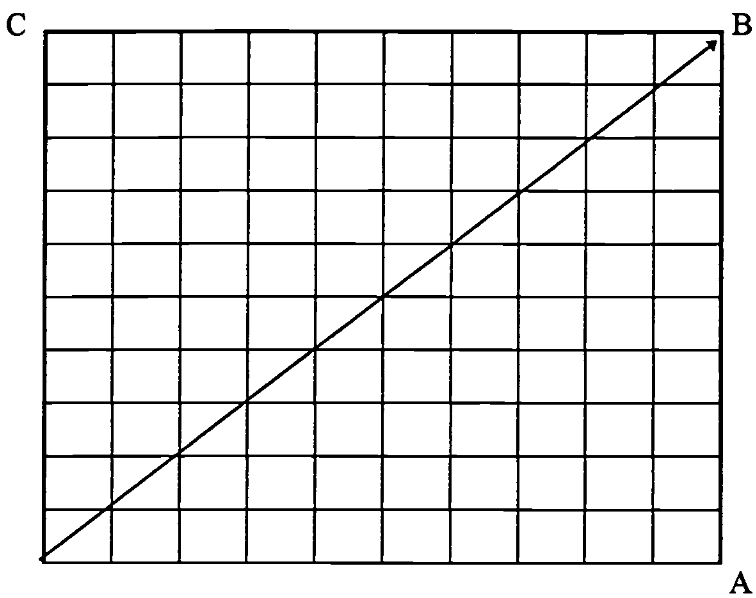
METHOD OF ANALYSIS AND TEACHING STYLE (2)

To investigate this, they are instructed first to draw Billiard tables of different sizes and follow the routes which the ball has made, including making note of hole number the ball ends up in, and the number of sides the ball rebounds off. Second, they will try to find patterns and rules which will be discussed among themselves and the write up of the investigation, plus results and conclusions to be drawn. In addition, students will try to investigate the number of pockets the ball goes in, lines created and the number of rebounds off the side, for different size tables.

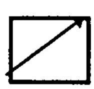




4 up 4 across
1 line
pocket B
no bounces

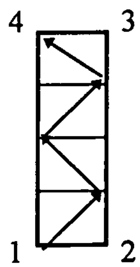


10 square up & 10 across
1 line
Pocket B
No bounces

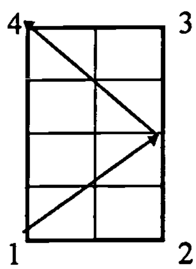


1 by 1
1 line
Pocket B
No bounces

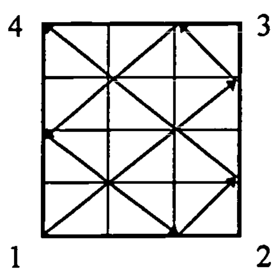
As you can see the billiard ball always ended up in pocket B. The reason for this is that a ball struck from any corner of a square 45 degrees will cut the square up in half diagonally, hence reaching the opposite corner pocket (furthest away from where it was struck). There is a rule here. The rule obviously can apply to any other size table, provided that A = B, where A => square up B => squares across.



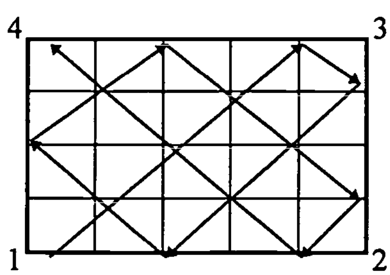
4 up 1 across
4 lines
Pocket 4
3 bounces



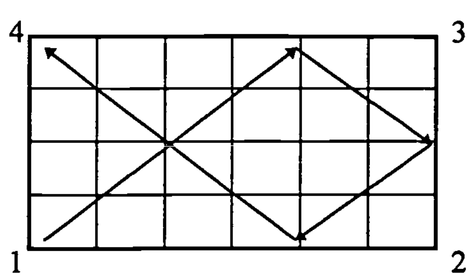
4 up 2 across
2 lines
Pocket 4
1 bounce



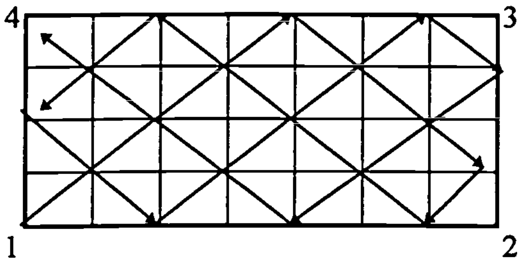
4 up 3 across
6 lines
Pocket 4
5 bounces



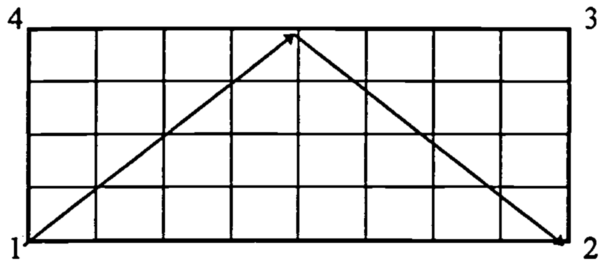
4 up 5 across
8 lines
Pocket 4
7 bounces



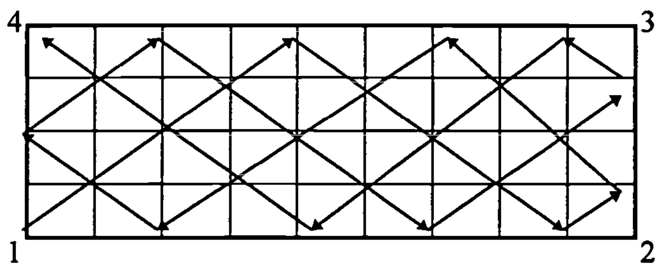
4 up 6 across
4 lines
Pocket 4
3 bounces



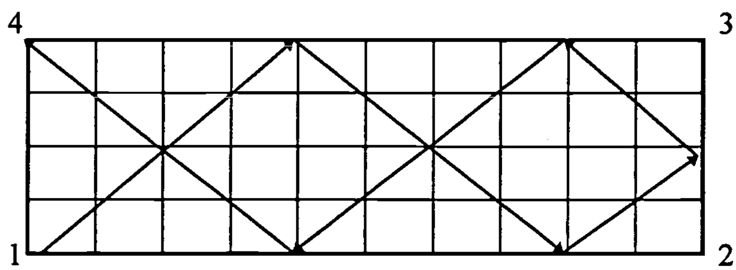
4 up 7 across
10 lines
Pocket 4
9 bounces



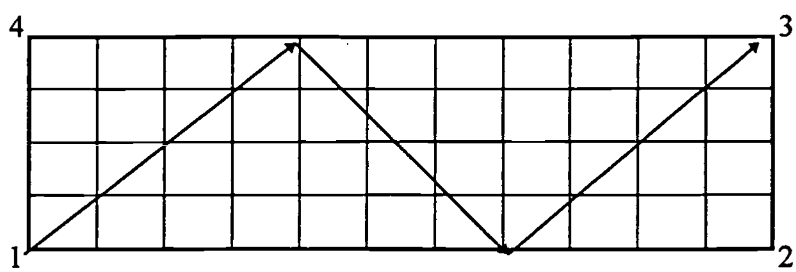
4 up 8 across
2 lines
Pocket 2
1 bounce



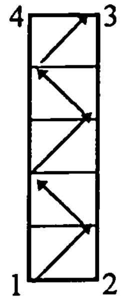
4 up 9 across
12 lines
Pocket 4
11 bounces



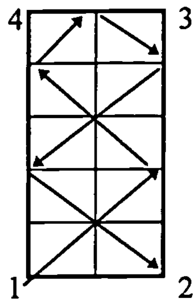
4 up 10 across
6 lines
Pocket 4
5 bounces



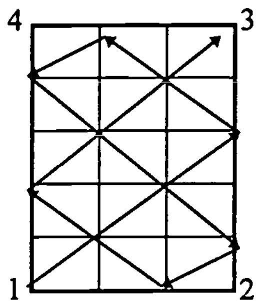
4 up 12 across
3 lines
Pocket 3
2 bounces



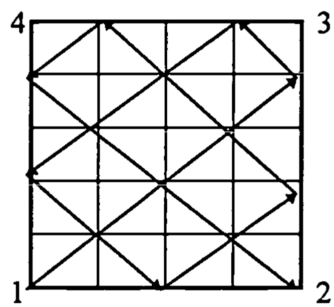
5 up 1 across
5 lines
Pocket 3
4 bounces



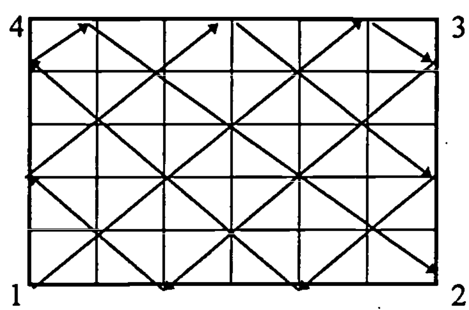
5 up 2 across
6 lines
Pocket 2
5 bounces



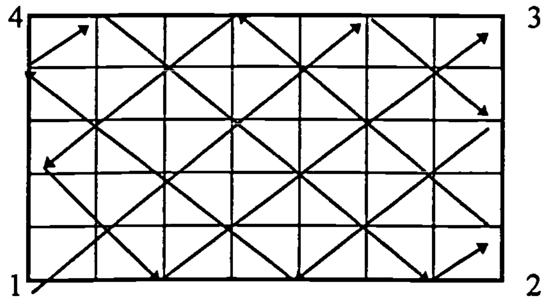
5 up 3 across
7 lines
Pocket 3
6 bounces



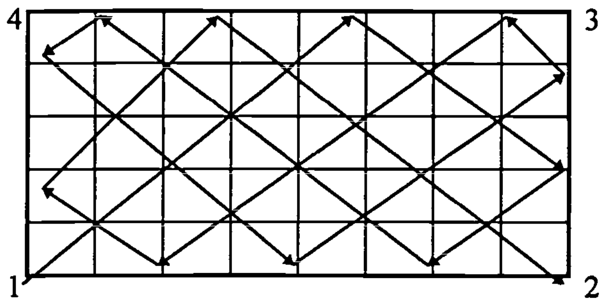
5 up 4 across
8 lines
Pocket 2
7 bounces



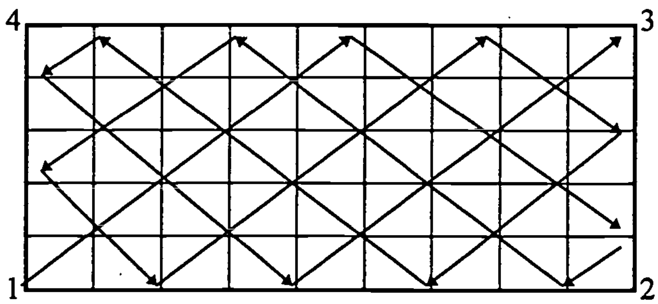
5 up 6 across
10 lines
Pocket 2
9 bounces



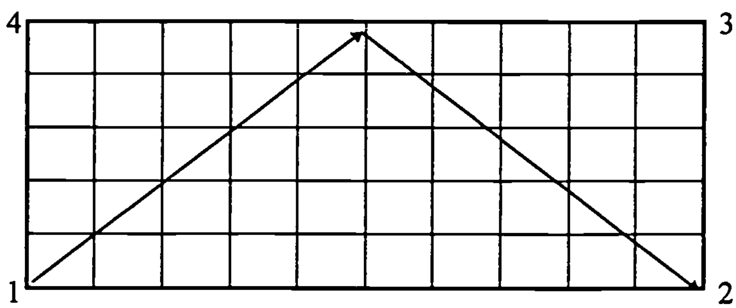
5 up 7 across
11 lines
Pocket 3
10 bounces



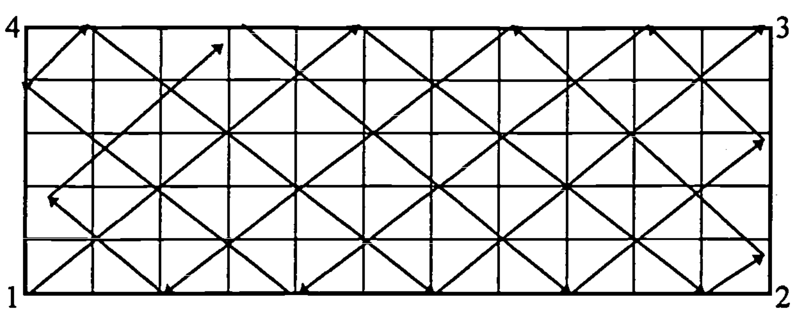
5 up 8 across
12 lines
Pocket 2
11 bounces



5 up 9 across
13 lines
Pocket 3
12 bounces



5 up 10 across
2 lines
Pocket 2
1 bounce



5 up 11 across
15 lines
Pocket 3
14 bounces

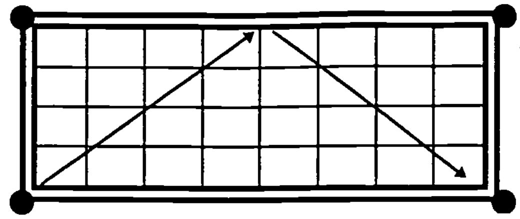


TABLE R0A/1

SIZE OF TABLE	LINES CREATED	POCKET HOLE ENDED UP IN
4.1	4	4
4.2	2	4
4.3	6	4
4.4	1	3
4.5	4	4
4.6	4	4
4.7	10	4
4.8	2	2
4.9	12	4
4.10	6	4
4.11	14	4
4.12	3	3

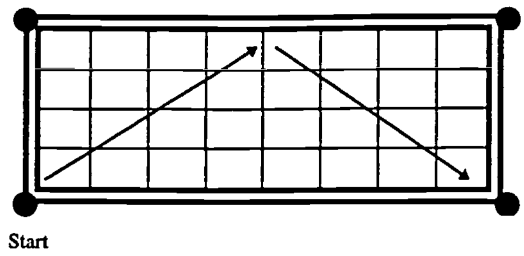
RULE

On observation, all squares across which are odd, ie 4, 3*, the billiard ball will end up in the pocket hole 4. All squares across which are divisible by 4, will end up in either the pocket 2 or 3. The pocket hole is (or can be), determined by the previous hole that the ball ended up in.

If it was pocket 3, then the next number (right hand digit, which is divisible by 4, the ball ends up in pocket 2.) The sequence of events goes like this, 3, 2, 3, 2, 3, 2 .... An explanation as to why the ball ends up in pocket 2 or 3 say, 4, 8, 12, 16 is as follows:

In Table ROA/1, shown above, say, 4, 16 the ball struck from 45 degrees would cover 4 squares across. 16 is 4 x 4, so at the fourth line it would end up in the pocket 2, because after the last rebound off the side the ball is travelling downwards and the previous hole was 3 on size 4, 12.

Rosanne Benn (1997, p.25), has said that 'the use of group work encourages problem generation and solving approaches to learning mathematics. Group work encourages the exploring and discovering of mathematics in a concrete and human process. Through discussion, adults can learn to articulate their point of view, listen to others, ask appropriate questions, how to recognise and respond to mathematically relevant challenges and in these ways to develop their mathematical conceptions and their applications'.



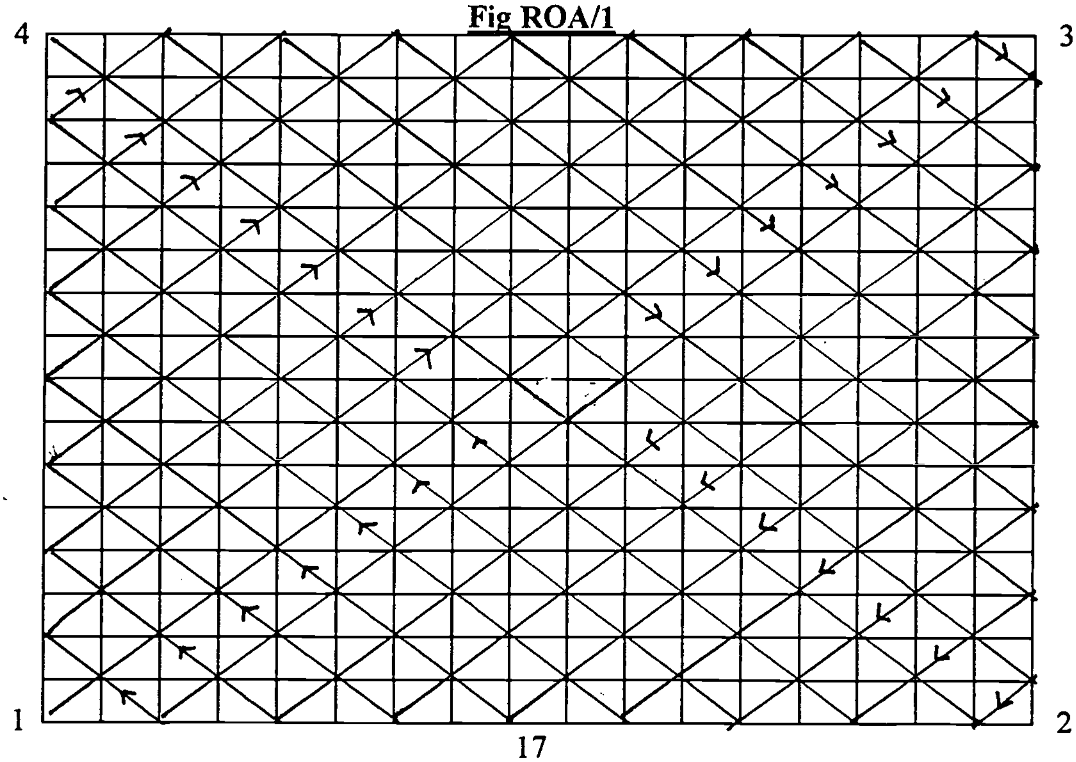
One such group, having completed size of tables from a constant of 4 squares up and 'X' squares across, they are still struck with a problem. For example, what happens on a table size of 8 squares across and 7 squares? The group have argued that knowing the rule for the table size of A, X does not help to find out about table size 8, 7, so instead, they have tried to find out by, writing down all the results from tables and then try to find a rule which fits in for all size tables. They discussed among themselves that when a billiard table whose base is divided into squares, this can be very helpful, the task says that the ball is always struck at 45 degrees and rebounds at the same angle.

This then is easy to do without a protractor because the ball always cuts a square in half diagonally. They also found out that drawing each different size table is quite tiring and therefore, sought for a general solution of the problem by searching for a formula.

As a result, the group have drawn a 23, 33 square size table. The whole table is covered with squares which have been cut through once. If we wanted to find the pocket hole ended up in and the number of lines created, then we proceed as follows:

For 16, 17 (16 squares up, 17 squares across). We counted 16 squares up, make a mark (>) and do the same with 17 across (see Figure ROA/1 below). The method the group adopted was that they go 2 pieces of paper and covered the rest of the table leaving only the size of table needed and you can use your pen or pencil and follow through the lines, always keeping the number of lines you counted.

Fig ROA/1



The result so, for a table of 16, 17 as in Fig ROA/1 is 32 lines and pocket 4.

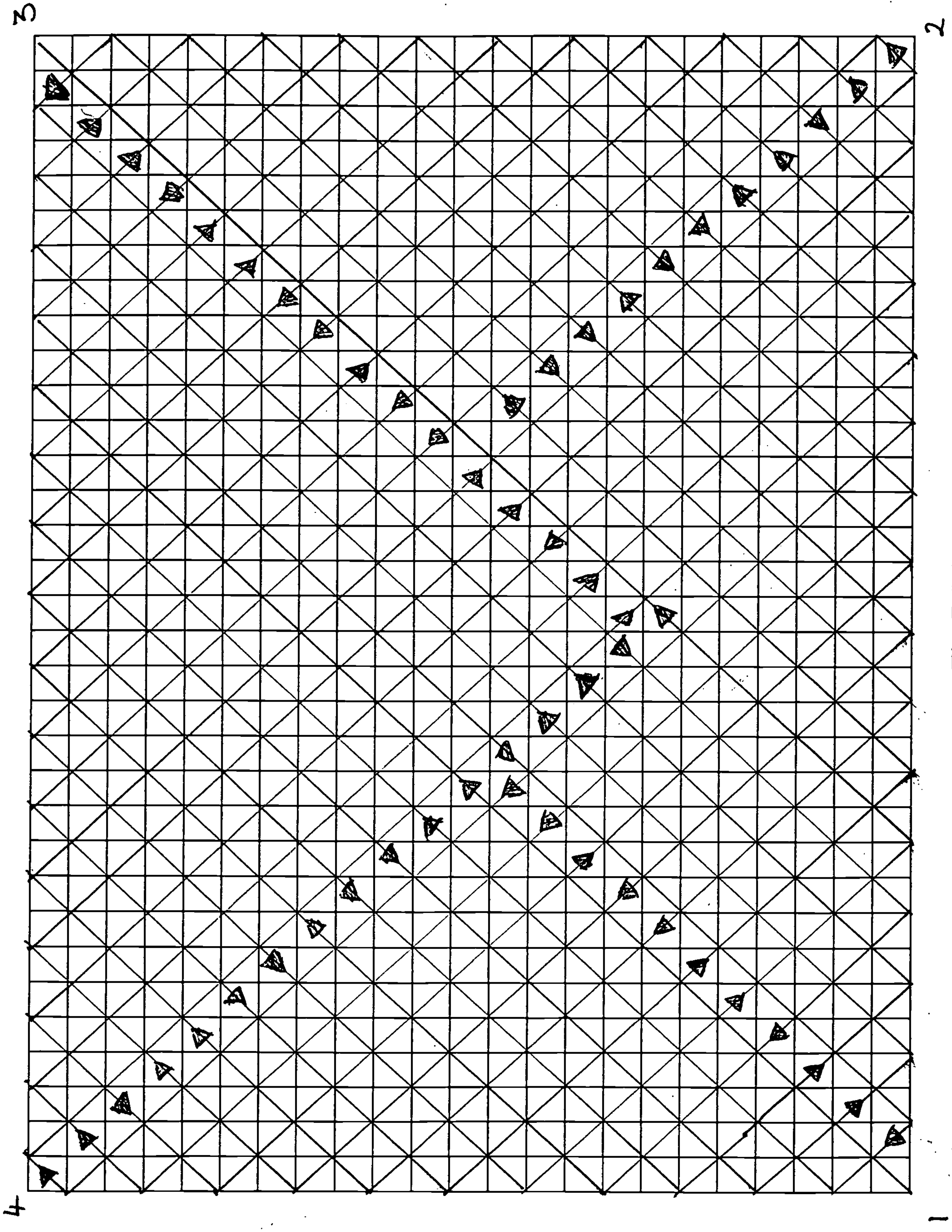


Fig ROA/2

TABLE ROA/2 RESULTS

WIDTH	LENGTH	PERIMETER	BOUNCES	LINES	HOLE
1 sq	1 sq	4cm	0	1	3
1	2	6cm	1	2	2
1	3	8cm	2	3	3
1	4	10cm	3	4	2
1	5	12cm	4	5	3
1	6	14cm	5	6	2
1	7	16cm	6	7	3
1	8	18cm	7	8	2
1	9	20cm	8	9	3
1	10	22cm	9	10	2

TABLE ROA/3 RESULTS

WIDTH	LENGTH	PERIMETER	BOUNCES	LINES	HOLE
2 sq	1 sq	6cm	1	2	4
2	2	8cm	0	1	3
2	3	10cm	3	4	4
2	4	12cm	1	2	2
2	5	14cm	5	4	4
2	6	16cm	2	3	3
2	7	18cm	7	4	4
2	8	20cm	3	2	2
2	9	22cm	9	4	4
2	10	24cm	4	3	3

TABLE ROA/4 RESULTS

WIDTH	LENGTH	PERIMETER	BOUNCES	LINES	HOLE
3 sq	1 sq	8cm	2	3	3
3	2	10cm	3	4	2
3	3	12cm	0	1	3
3	4	14cm	5	6	2
3	5	16cm	6	7	3
3	6	18cm	1	2	2
3	7	20cm	8	9	3
3	8	22cm	9	10	2
3	9	24cm	2	3	3
3	10	26	11	12	2

TABLE ROA/5 RESULTS

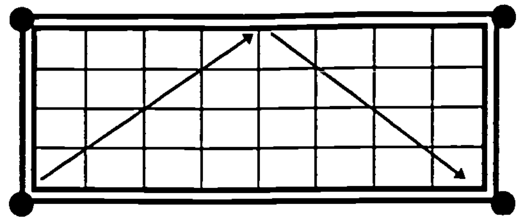
WIDTH	LENGTH	PERIMETER	BOUNCES	LINES	HOLE
4 sq	1 sq	10cm	3	4	4
4	2	12cm	1	2	4
4	3	14cm	5	6	4
4	4	16cm	0	1	3
4	5	18cm	7	8	4
4	6	20cm	3	4	4
4	7	22cm	9	10	4
4	8	24cm	1	2	2
4	9	26cm	11	12	4
4	10	28cm	5	6	4

TABLE ROA/6 RESULTS

WIDTH	LENGTH	PERIMETER	BOUNCES	LINES	HOLE
5 sq	1 sq	12cm	4	5	3
5	2	14cm	5	6	2
5	3	16cm	6	7	3
5	4	18cm	7	8	2
5	5	20cm	0	1	3
5	6	22cm	9	10	2
5	7	24cm	10	11	3
5	8	26cm	11	12	2
5	9	28cm	12	13	3
5	10	30cm	1	2	2

TABLE ROA/5 RESULTS

WIDTH	LENGTH	PERIMETER	BOUNCES	LINES	HOLE
6 sq	1 sq	14cm	5	6	4
6	2	16cm	2	3	3
6	3	18cm	1	2	4
6	4	20cm	3	4	2
6	5	22cm	9	10	4
6	6	24cm	0	1	3
6	7	26cm	11	12	4
6	8	28cm	5	6	2
6	9	30cm	3	4	4
6	10	32cm	6	7	3



In general, after observing and looking at the results of the tables it is important to look for a relationship between the number of times the ball rebounds is related to the length of the tables sides, the perimeter of the table and the area of the table. For example, take the letter K to represent the number 4 and K to represent the number 16, then we can formulate some kind of relationship with holes and pockets.

(X-n x 2) equals P. h equals [X - P]		(X - n (x 2) equals P n equals [X - P/n]
A/even	B/even	equals pocket 2, 3 and 4
A Odd	B Odd	 equals pocket 3
A Odd Even	B Even Odd	 equals pocket 2 equals pocket 4
Do	B ÷ A, if divisible then, hole equals 2 or 3.	

If B ÷ A equals an odd number, then this implies 3. Similarly if B ÷ A equals an even number, then this implies that the number is 2.

From this we can infer that if B equals 16, then the hole is 2, provided that A is an even number. Equally if A equals 16 then the hole is 4, provided that the number is even. Hence, the results from tables on the next pages are arranged differently. But this time round we have cancelled ‘bounces’ and ‘perimeter’, the reason being that we want to fit in more results to find a rule to which hole the ball goes in.

Perimeter equals (W + L) x 2 Bounces equals lines - 1
When W equals width and L equals length

TABLE ROA/12

Size	Created	Pocket
1, 1	1	3
1, 2	2	2
1, 3	3	3
1, 4	4	2
1, 5	5	3
1, 6	6	2
1, 7	7	3
1, 8	8	2
1, 9	9	3
1,10	10	2

Size	Created	Pocket
2, 1	2	4
2, 2	1	3
2, 3	4	4
2, 4	2	2
2, 5	6	4
2, 6	3	3
2, 7	8	4
2, 8	4	2
2, 9	10	4
2,10	5	3

Size	Created	Pocket
3, 1	3	3
3, 2	4	2
3, 3	1	3
3, 4	6	2
3, 5	7	3
3, 6	2	2
3, 7	9	3
3, 8	10	2
3, 9	2	3
3,10	12	2

TABLE ROA/13

Size	Created	Pocket
4, 1	4	4
4, 2	2	4
4, 3	6	4
4, 4	1	3
4, 5	8	4
4, 6	4	4
4, 7	10	4
4, 8	2	2
4, 9	12	4
4,10	6	4

Size	Created	Pocket
5, 1	5	3
5, 2	6	2
5, 3	7	3
5, 4	8	2
5, 5	1	3
5, 6	10	2
5, 7	11	3
5, 8	12	2
5, 9	13	3
5,10	2	2
5,11	15	3

Size	Created	Pocket
6, 1	6	4
6, 2	3	3
6, 3	2	4
6, 4	4	2
6, 5	10	4
6, 6	1	2
6, 7	12	4
6, 8	6	2
6, 9	4	4
6,10	7	3
6,11	16	4

TABLE ROA/14

Size	Created	Pocket
7, 1	7	3
7, 2	8	2
7, 3	9	3
7, 4	10	2
7, 5	11	3
7, 6	12	2
7, 7	1	3
7, 8	14	2
7, 9	15	3
7,10	16	2
7,11	17	3

Size	Created	Pocket
8, 1	8	4
8, 2	4	4
8, 3	10	4
8, 4	2	4
8, 5	12	4
8, 6	6	4
8, 7	14	4
8, 8	1	3
8, 9	16	4
8,10	8	4

TABLE ROA/15

9, 1	9	3
9, 2	10	2
9, 3	3	3
9, 4	2	2
9, 5	12	3
9, 6	13	2
9, 7	4	3
9, 8	15	2
9, 9	16	3
9, 10	18	2
9, 11	19	3
9, 12	6	2
9, 13	21	3
9, 14	22	2
9, 15	7	3

10, 1	10	4
10, 2	5	3
10, 3	12	4
10, 4	6	2
10, 5	2	4
10, 6	7	3
10, 7	16	4
10, 8	8	2
10, 9	18	4
10, 10	1	3
10, 11	20	4
10, 12	10	2
10, 13	22	4
10, 14	11	3

11, 1	11	3
11, 2	12	2
11, 3	13	3
11, 4	14	2
11, 5	15	3
11, 6	16	2
11, 7	17	3
11, 8	18	2
11, 9	19	3
11, 10	20	2
11, 11	1	3
11, 12	22	2
11, 13	23	3
11, 14	24	2

12, 1	12	4
12, 2	6	4
12, 3	4	4
12, 4	3	3
12, 5	16	4
12, 6	2	4
12, 7	12	4
12, 8	4	2
12, 9	6	4
12, 10	10	4
12, 11	12	4
12, 12	1	3
12, 13	24	4

TABLE ROA/16

13, 1	13	3
13, 2	14	2
13, 3	15	3
13, 4	16	2
13, 5	17	3
13, 6	18	2
13, 7	19	3
13, 8	20	2
13, 9	21	3
13, 10	22	2
13, 11	23	3
13, 12	24	2
13, 13	1	3

14, 1	14	4
14, 2	7	3
14, 3	16	4
14, 4	8	2
14, 5	18	4
14, 6	9	3
14, 7	2	4
14, 8	10	2
14, 9	22	4
14, 10	11	3
14, 11	24	4
14, 12	12	2
14, 13	26	4

15, 1	15	3
15, 2	16	2
15, 3	5	3
15, 4	18	2
15, 5	3	3
15, 6	6	2
15, 7	21	3
15, 8	22	2
15, 9	7	3
15, 10	4	2
15, 11	25	3
15, 12	8	2
15, 13	27	3
15, 14	28	2

16, 1	16	4
16, 2	8	4
16, 3	18	4
16, 4	4	4
16, 5	20	4
16, 6	10	4
16, 7	22	4
16, 8	2	4
16, 9	24	4
16, 10	12	4
16, 11	26	4
16, 12	6	4
16, 13	28	4
16, 14	14	4

RESULTS OF SIZES OF TABLES RANGING FROM 1,1 TO 16,16

1,1 = 1 square along & 1 square up

Table ROA/17

Line 1/ Hole 3	2/2	3/3	4/2	5/3	6/2	7/3	8/2	9/3	10/2	11/3	12/2	13/3	14/2	15/3	16/2	17/3	18/2	19/3	20/2	21/3	22/2	23/3	25/2	25/3	26/2	27/
1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	1,10	1,11	1,12	1,13	1,14	1,15	1,16	1,17	1,18	1,19	1,20	1,21	1,22	1,23	1,24	1,25	1,26	1,27
2,2	2,4	2,6	2,8	2,10	2,12	2,14	2,16	2,18	2,20	2,22	2,24	2,26	2,28	2,30	2,32	2,34	2,36	2,38	2,40	2,42	2,44	2,46	2,48	2,50	2,52	2,54
3,3	3,6	3,9	3,12	3,15	3,18	3,21	3,25	3,27	3,30	3,33	3,36	3,39	3,42	3,45	3,48	3,51	3,54	3,57	3,60	3,62	3,66	3,69	3,72	3,75	3,78	3,81
4,4	4,8	4,12	4,16	4,20	4,24	4,28	4,32	4,36	4,40	4,44	4,48	4,52	4,56	4,60	4,64	4,68	4,72	4,76	4,80	4,84	4,88	4,92	4,96	4,100	4,104	4,10
5,5	5,10	5,15	5,20	5,25	5,30	5,35	5,40	5,45	5,50	5,55	5,60	5,65	5,70	5,75	5,80	5,85	5,90	5,95	5,100	5,105	5,110	5,115	5,120	5,125	5,130	5,13
6,6	6,12	6,18	6,23	6,30	6,36	6,42	6,48	6,54	6,60	6,66	6,72	6,78	6,84	6,90	6,96	6,102	6,108	6,114	6,120	6,126	6,132	6,138	6,144	6,150	6,156	6,16
7,7	7,14	7,21	7,28	7,35	7,42	7,49	7,56	7,63	7,70	7,77	7,84	7,91	7,98	7,105	7,112	7,119	7,126	7,133	7,140	7,147	7,154	7,161	7,168	7,175	7,182	7,18
8,8	8,16	8,24	8,32	8,40	8,48	8,56	8,64	8,72	8,80	8,88	8,96	8,104	8,112	8,120	8,128	8,136	8,144	8,152	8,160	8,168	8,168	8,176	8,194	8,192	8,200	8,21
9,9	9,18	9,27	9,36	9,45	9,54	9,63	9,72	9,81	9,90	9,99	9,108	9,117	9,126	9,135	9,144	9,153	9,162	9,271	9,180	9,189	9,198	9,207	9,216	9,225	9,234	9,24
10,10	10,20	10,30	10,40	10,50	10,60	10,70	10,80	10,90	10,100	10,110	10,120	10,130	10,140	10,150	10,160	10,170	10,180	10,190	10,200	10,210	10,210	10,220	10,230	10,240	10,260	10,2
11,11	11,22	11,33	11,44	11,55	11,66	11,77	11,88	11,99	11,110	11,111	11,122	11,133	11,144	11,155	11,166	11,177	11,188	11,199	11,210	11,221	11,232	11,243	11,254	11,265	11,276	11,2
12,12	12,24	12,36	12,48	12,60	12,72	12,84	12,96	12,108	12,120	12,132	12,144	12,156	12,168	12,180	12,192	12,204	12,216	12,228	12,240	12,252	12,264	12,276	12,288	12,300	12,312	12,3
13,13	13,26	13,39	13,52	13,65	13,78	13,91	13,104	13,117	13,130	13,143	13,156	13,169	13,182	13,195	13,208	13,221	13,234	13,247	13,260	13,273	13,286	13,299	13,312	13,325	13,338	13,3
14,14	14,28	14,42	14,56	14,70	14,84	14,98	14,112	14,126	14,140	14,154	14,168	14,182	14,196	14,210	14,224	14,238	14,252	14,266	14,280	14,294	14,308	14,322	14,336	14,350	14,364	14,3
15,15	15,30	15,45	15,60	15,75	15,90	15,105	15,120	15,135	15,150	15,165	15,180	15,195	15,210	15,225	15,240	15,255	15,270	15,285	15,300	15,315	15,330	15,345	15,360	15,375	15,390	15,4
16,16	16,32	16,48	16,64	16,80	16,98	16,114	16,130	16,146	16,160	16,178	16,194	16,210	16,228	16,242	16,258	16,274	16,290	16,306	16,322	16,336	16,354	16,370	16,386	16,402	16,418	16,4

Table ROA/18

Line 2/ Hole 4	4/4	5/4	8/4	10/4	12/4	14/4	16/4	18/4	20/4	22/4	24/4	26/4	28/4	30/3
2,1	2,3	2,5	2,7	2,9	2,11	2,13	2, 15	2,17	2,19	2,21	2,23	2,25	2,27	2,29
4,2	4,1	4,3	4,5	4,7	4,9	4,11	4,13	4,15	4,17	4,19	4,21	4,23	4,25	4,27
6,3	6,9	6,1	6,21	6.5	6,7	8,7	6,11	6,13	8,13	6,17	6,19	8,19	6,23	6,25
8,4	8,2	8,6	8,10	8,3	8,5	8,7	8,9	8,11	8,13	8,15	8,17	8,19	8,21	8,23
10,5	10,15	8,6	8,10	10,1	10,3	8,7	10,7	10,9	10,11	10,13	8,17	10,17	10,19	10,21
12,6	12,3	12,9	8,10	12,10	12,1	8,7	12,5	12,7	12,9	12,11	12,13	12,15	12,17	12,19
14,7	14,21	14,25	8,10	12,10	12,1	14,1	14,3	14,5	14,7	14,9	14,11	14,13	14,15	14,17
1,8	16,4	16,12		16,6	16,10	16,14	16,1	16,3	16,5	16,7	16,9	16,11	16,13	16,15

Rules

Having made all the necessary results tables, we could see many patterns and as a result come up with a set of rules and formulas, starting with the simplest rule.

A	B	
13	26	13 denotes to the width or squares up 26 denotes to the length or squares across

For this size table, to find the hole pocket, lines and bounces follow instructions below:

if A>B do $\frac{A}{B}$ If A<B do $\frac{B}{A}$

Here it is $\frac{B}{A}$ which equals 2, and 2 denotes the number of lines created

If $\frac{A}{B}$ or $\frac{B}{A}$ = even number → pocket 2
= odd number → pocket 3

Bounce number = lines - 1

So the pocket hole is 2, number of lines is 2 and bounces = 2 - 1
= 1 bounce

This rule only works if one number ie width or length is divisible by the other.

To sum up the rules in shorter terms:

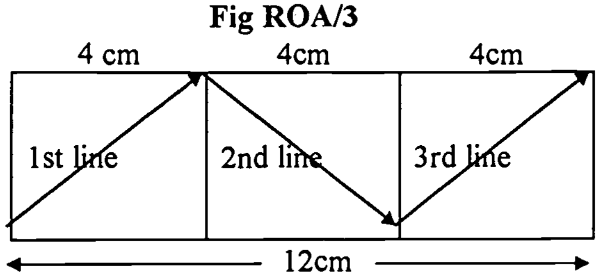
Find out if any one number is divisible by the other
Then do $\frac{B}{A}$ or $\frac{A}{B}$ = line number

No of bounces = line - 1

The reason for why this rule works that if it is a divisible number ie the width will fit in the length for χ times

Fig ROA/3

Fig 1 4cm



In Fig ROA/3 the ball always travelled from left → right and never the opposite. When the ball makes its first bounce, it would have travelled the same length as its width which is 4. As the total length is 12cm 4cm fits into 12, 3 times, so on the 3rd line it will end up in a pocket.

If neither numbers are divisible by the other, then these rules apply to them.

The width (square up) = A
The length (squares across) = B

1. If A is odd
   B is odd then the resulting pocket = 3

2. If A is odd
   B is even = pocket 2

3. If A is even
   B is odd = pocket 4

An example for rule (1) from results tables

1,9	→	3	9,5	→	3
3,3	→	3	11,13	→	3
5,7	→	3	13,11	→	3
7,1	→	3	15,17	→	3

An example for rule (2)

1,8	→	2	11,14	→	2
3,6	→	2	11,16	→	2
5,19	→	2	13,2	→	2
9,12	→	2	15,4	→	2

An example for rule (3)

2,7	→	4	6,5	→	4
4,5	→	4	8,11	→	4
4,3	→	4	10,11	→	4
6,7	→	4	12,9	→	4

If both length and width are even numbers then a more complicated method is used to find the pocket hole. This is because pocket 2, 3 and 4 are all open.

1. If B is divisible by A, vice versa (see Rule 1)

2. If not Do $\frac{A}{2}$. If even go to (3), if odd go to (4)

3. Do $\frac{B}{2}$ = even → pocket 2
   = odd → pocket 4

4. Do $\frac{B}{2}$ = odd → pocket 3
   = even → pocket 2

Examples

6,8 → $\frac{6}{2} = 3$ (odd)

$\frac{8}{2} = 4$ (even)

even → 2 checked in results from tables.

4,10 → $\frac{4}{2} = 2$ (even)

$\frac{10}{2} = 5$ (odd)

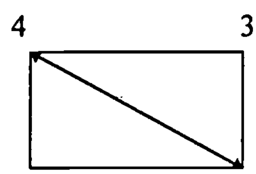
odd → 4 checked in results table

6,14 → $\frac{6}{2} = 3$ (odd)

$\frac{14}{2} = 7$ (odd

odd → 3 checked in results table

I've dealt only with size tables in which the width was shorter than the length. To find a size table for A > B first you must find A< B (width shorter than the length B)



1 **Fig 2** 2

Fig 2

As shown in Fig 2, pocket 4 is opposite pocket 2. Pocket 3 is not opposite pocket 1 as a sense that the ball never goes into pocket 1.

Fig ROA/4

A > B	A < B
2	4
3	3
2	4
4	2
4	2

→

6,4	4,6	8,6	6,8	10,6	6,10
2	4	4	2	3	3

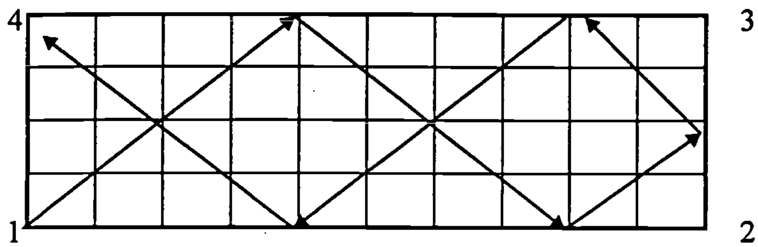
As shown the reverse size table results in the opposite pocket hole.

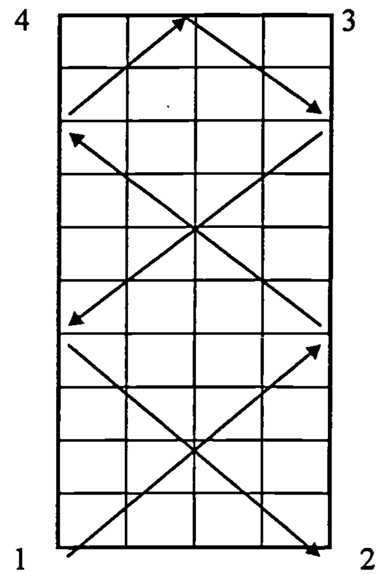
Example 1

Find the pocket hole for a billiard table size 10cm by 4cm (1cm = 1 square). First find the pocket hole for 4 cm by 10

= 4

The opposite hole to 4 is 2. Therefore 10 by 4 → pocket 2



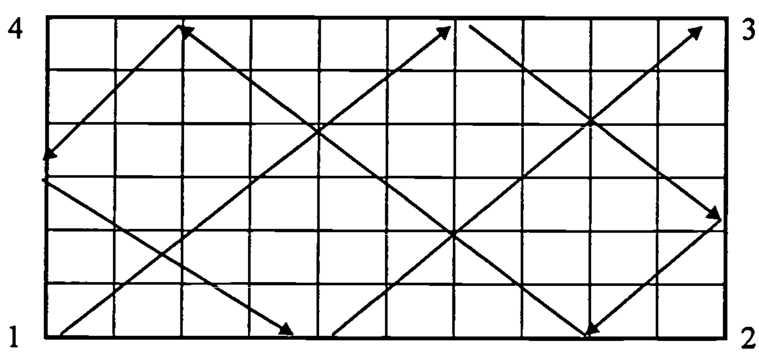


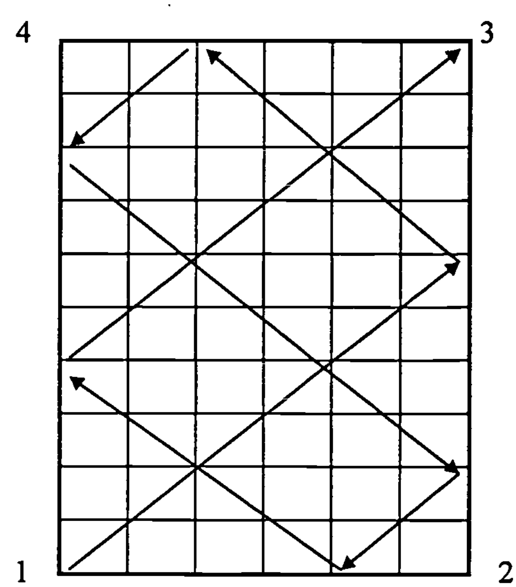
Example 2

Find the pocket hole for a table size 10 by 6.

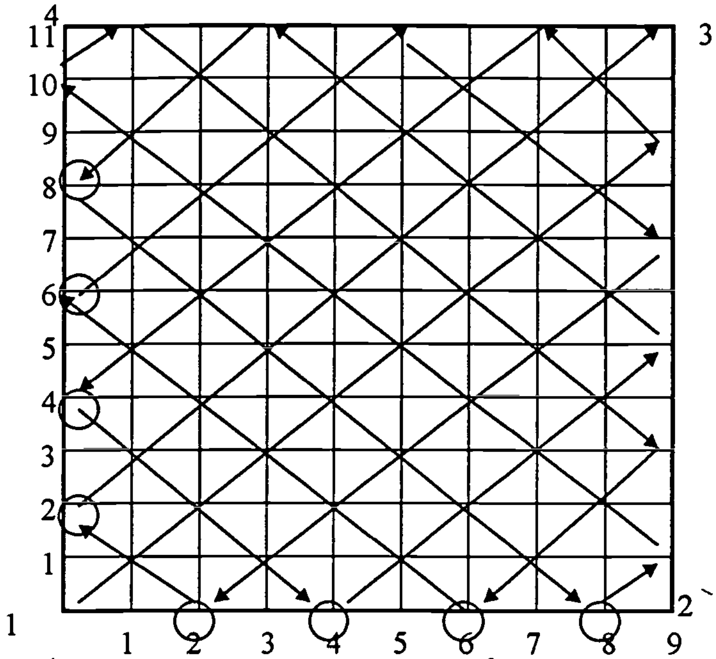
6 by 10 → pocket 3

10 by 6 therefore is still pocket 3



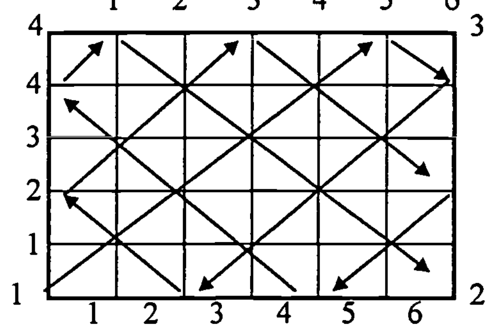


An explanation for why the following rules work
can be seen in 4 diagrams



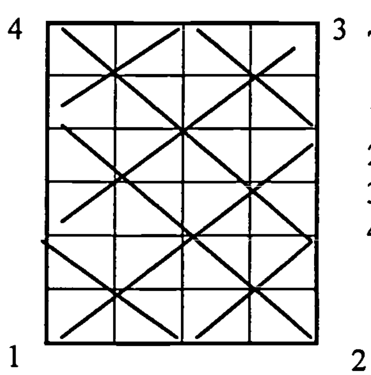
This table is 11 x 9 (odd and odd)
From our rule A is odd
B is odd
Resulting pocket hole = 3
which is correct

For the ball to get into pocket 4 or 2 the width or length must be an even number. This is because the ball bounces to the pocket at every even number shown by the rings O.



This table is 5 by 6
A is odd
B is even = pocket 2
The ball will never go into pocket 3 or 4 because they are closed
For the ball to go to pocket 4 A must be even. Plus for the ball to go in pocket 3 A & B must be odd

If the width and length are even then the possibility of the ball ending up in pocket 2, 3 or 4 is true. As shown below



There are 3 different sets of sequences

1. 4 3 4 2	4 3 4 2	4 3 4 2
2. 3 2 3 2	3 2 3 2	3 2 3 2
3. 4 4 4 3	4 4 4 3	4 4 4 3
4. 4 4 4 4	4 4 4 4	4 4 4 4

The first set of sequences work for when
$\frac{A}{2}$ is even → then $\frac{B}{2}$ = even → pocket 2
= odd → pocket 3
However the 3rd sequence mainly works on factors of 8, ie 8, 16, 32, 40. If the length is a factor of the width then the ball ends up in pocket 3. If not it ends up in pocket 4

Rule 3

So far we can find the pocket hole for different size table using the rules already described to find the lines created and the number of bounces there are basically 2 formulas

1. $\left(\frac{A}{2} + \frac{B}{2}\right) - 1$ This formula is used to calculate the number of lines created for a size table whose width = even length = even number

   A = Width
   B = length

2. (l + W) - 1 This formula is used for size table of:
   L = Length
   W - Width

Width	Length
odd	even
even	odd
odd	odd

The number of bounces = lines - 1. To test this formula, find out the number of lines created and bounces for the following size tables.

1.	4 by 10	4.	6 by 10	7.	9 by 4	10.	5 by
2.		5.	10 by 14	8.	8 by 5	11.	3 by
3.	6 by 8	6.	7 by 8	9.	6 by 5		

Solutions

1. 4 by 10 = even and even
   $\left(\frac{A}{2} + \frac{B}{2}\right) - 1 = \left(\frac{4}{2} + \frac{10}{2}\right) - 1 = (2 + 5) - 1 = 7 - 1 = 6$ lines
   number of bounces = 6 -1 = 5 bounces → checked in results from table.

2. 4 by 6 even and even
   $\left(\frac{A}{2} + \frac{B}{2}\right) - 2 = \left(\frac{4}{2} + \frac{6}{2}\right) - 1 = (2 + 3) - 1 = 5 - 1 = 4$ Lines
   4 - 1 = 3 bounces checked in results from table

3. 6 by 8
   $\left(\frac{A}{2} + \frac{B}{2}\right) - 1 = \left(\frac{6}{2} + \frac{8}{2}\right) - 1 = (3 + 4) - 1 = 7 - 1 = 6$ Lines
   6 - 1 = 5 bounces checked in results from table

4. 6 by 10 = even and even
   $\left(\frac{A}{2} + \frac{B}{2}\right) - 1 = \left(\frac{6}{2} + \frac{10}{2}\right) - 1 = (3 + 5) - 1 = 8 - 1 = 7$ lines
   7 -1 = 6 bounces checked in results from table.

5. 10 by 14 = even and even
   $\left(\frac{A}{2} + \frac{B}{2}\right) - 1 = \left(\frac{10}{2} + \frac{14}{2}\right) - 1 = (5 + 7) - 1 = 12 - 1 = 11$ lines
   11 -1 = 10 bounces checked in results from table.

6. 7 by 8 = odd & even
   (L + W) - 1 = (7 + 8) - 1 = 15 - 1 = 14 Lines
   14 - 1 = 13 bounces checked in results from table

7. 9 by 8 = odd & even
(L + W) - 1 = (9 + 4) - 1 = 12 - 1 = 12 Lines
12 - 1 = 11 bounces checked in results from table

8. 8 by 5 = odd & even
(L + W) - 1 = (8 + 5) - 1 = 13 - 1 = 12 Lines
12 - 1 = 11 bounces checked in results from table

9. 6 by 5 = odd & odd
(L + W) - 1 = (6 + 5) - 1 = 11 - 1 = 10 Lines
10 - 1 = 9 bounces checked in results from table

10. 6 by 7 = odd & odd
(L + W) - 1 = (5 + 7) - 1 = 12 - 1 = 11 Lines
11 - 1 = 10 bounces checked in results from table

11. 3 by 7 = odd & odd
(L + W) - 1 = (3 + 7) - 1 = 10 - 1 = 9 Lines
9 - 1 = 8 bounces checked in results from table

SUMMARY OF ALL RULES

To find the pocket hole

A	B		
odd	odd	=	pocket 3

A	B		
odd	even	=	pocket 2

A	B		
even	odd	=	pocket 4

A	B
even	even

1. If either W or L is divisible by the other:
$\frac{B}{A}$ if $A < B$ $\frac{A}{B}$ if $A > B$
→ even → pocket 2
→ odd → pocket 3

2. If not do $\frac{A}{2}$ = even → do $\frac{B}{2}$ if even → pocket 2; odd → pocket 4
If $A > B$, first find B. If the result is →
3 then $A > B = 3$
2 then $A > B = 4$
4 then $A > B = 2$

To find numbers of lines and bounces

If either W or L is divisible by the other:

Do $\frac{B}{A}$ if $A < B$ or do $\frac{A}{B}$ if $A > B$

= numbers of lines

Bounces = lines - 1

If neither are divisible

$$F1 = \left(\frac{A}{B} + \frac{B}{2}\right) - 1$$

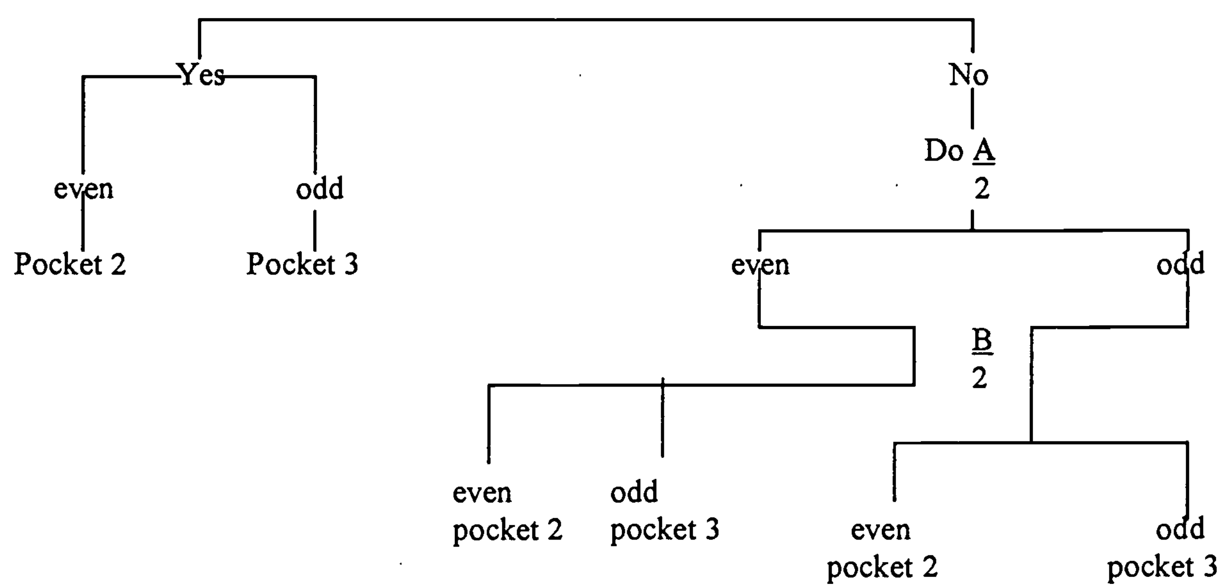
$$F2 = (L + W) - 1$$

Width	Length	
even	even	use F1
odd	even	use F2
even	odd	use F2
odd	odd	use F2

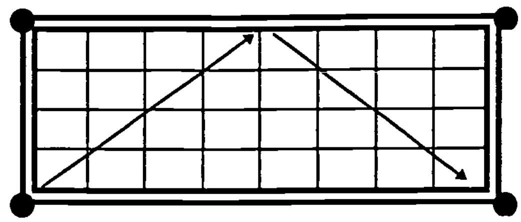
FIG ROA/5

$\frac{B}{A}$ or $\frac{A}{B}$

Divisible



Width	Length			
Even	Even		→	$\left(\frac{A}{2} = \frac{B}{2}\right) - 1$
Odd	Odd	)		
Even	Odd	)	→	(L + W) - 1
Odd	Even	)		



Start

HOW ONE GROUP REACTED TO THE INVESTIGATION

Certainly, the evidence in this investigation suggests that adult students gain more when they know what work they are personally accountable for and what to do when they have finished. This is how they reacted to the investigation:

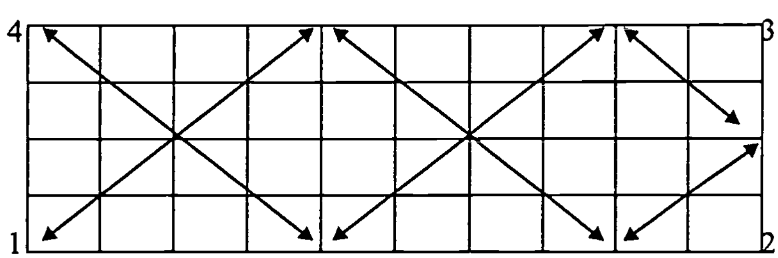
"Our first intention was to do the hidden faces of cubes investigation, [Angiama, RO (1995). We realised that we were not really going to get stuck into it, so we changed our minds but instead we choose to investigate the strange billiard table. We came up with three things that we should be able to find out at the end of the investigation.

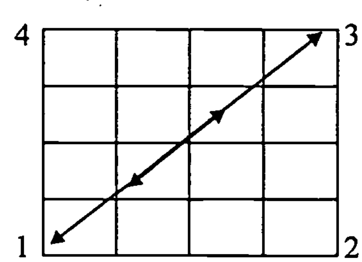
These are the eventual pocket number, number with line created and the number of bounces. We set about by testing our tables after tables with the width as always being the same and the length being varied. When we collected the results from the different tables, we immediately saw a sequence of patterns for the pocket numbers.

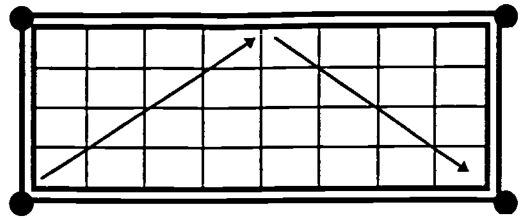
Hence, we did the same for the billiard tables, each time collecting results and having done enough on results, the really hard and long part came up, that is trying to find a rule or formula and we are expected to find 3 rules or formulas, one for the pocket numbers, number of lines created and one for the number of bounces created. In order to do this, we tried concentrating on one type of billiard table, assuming that A equals a constant and B equals a X. Here, we seemed to have found a rule but when we applied it to another size table, it did not work. However, we found out which tables had the same rules, sometimes doing most of the working in our heads.

In the end there were 6 rules for finding the pocket hole, one for the number of bounces and 2 formulas for the number of lines created. A vital part of this investigation which helped us arrive to our rules and formulas was experimenting. We suddenly noticed that there were more than rule or formulas, because we grouped all the size of tables together, according to which rule or formula works for them but after all this, things became clearer, easier and 'Alas', we found out the solutions for what needed to be investigated.

Doing an investigation or practical task near the end of the course is not suitable because our examinations are coming up and we need to revise. This limits the time allowed to the task. The investigation could be extended or improved if we had more time. By this we mean the pocket holes, or one pocket hole. Yet another extension could be that the ball is hit from pocket 3 or 4 and even 2 balls could be used, both struck at different ends of the table as shown below."







In conclusion, I would argue that group work rewards for everyone achieving their individual target often produce high motivation and one study found 22% (percent) greater involvement of adult students in their work using these strategies. however, critics have argued that group work deemed poor when students simply share the same work and the same answers but with investigative mathematics there was evidence of sharing good practice with whole group.

'Through discussion, adults can learn to articulate their point of view, listen to others, ask appropriate questions, how to recognise and respond to mathematically relevant challenges and in these ways to develop their mathematical conceptions and their applications'. The students take an active part in the lessons and demonstrate their developing independence and sense of responsibility.

Equally important is that the teacher as a researcher demonstrates personal attributes, technical competence, and subject knowledge that will promote the students' learning in an atmosphere of respect and confidence and the students enjoy each other's company, are seen to be mutually supportive and treat each other with courtesy and respect.

During the investigation, the teacher as a researcher constantly raises the intellectual level of the verbal exchanges which took place in the learning and teaching environment when we found out that a billiard table with 4 holes is a good investigation. An ordinary table would have 6 holes. The ball is struck at 45 degrees, so it is easy to follow the ball with the help of squares. The reason why it was suggested was because the strange billiard table should have only 4 holes and because too, if you don't, then the ball will always go into the top centre pocket.

We found out that there were only 4 possible sets sequences for the results as obtained in the tables thus:

- 23 23 23 23 23 23 23 23 23 ....
- 43 42 43 42 43 42 43 42 43 42 ....
- 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 ....
- 4 4 4 3 4 4 4 2 4 4 4 3 4 4 ....

From here it was possible to establish the rules and so if the width is even in the number of squares, no matter what the length is, it would always give the set of results : 23 23 23 .. and 4 4 4 34 4 4 2 ... If the width is odd and the length is varying, then the set of results would be 43 42 43 42 43 42 ... and if the width is a multiple of 8, that is to say, 26, 31 it's obvious that the result is going to be 4 4 4 4 4 4 4 4 4 4 4.

Therefore, in practice if we had a strange billiard table which was not divided into squares, then its width and length can be seen to be measured. For example, a billiard table measures 0.5m in width and 0.8 in length. The solution would be that the equivalent of a square is 1cm squared and its side would be 1cm thus:

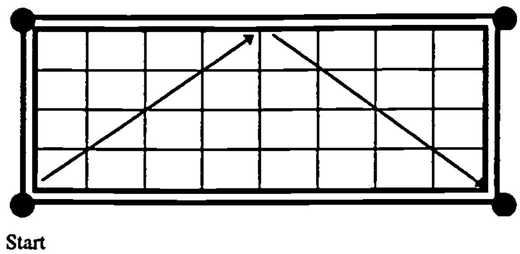
0.5 x 100	.	50 cm
0.8 x 100	.	80 cm
50 by 80	.	implies (>) odd and even

Here we have a rule > > > > > > > > > > > pocket 2 as illustrated in tables.

So, (L + W) - 1 = (50 + 80) - 1
1320 - 1 >>>>>>>>>>>>>129 lines
rebounds = 129 - 1 = 128

One closes this paper with feelings of challenges and excitement that the argument has come to an end and to leave the last word with The Cockcroft Report (1982), which stated, 'the idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in very many field.'

Mathematics investigation (MI), provides a basis for improving the quality of teaching and learning mathematics and, therefore, leads to an improvement on standards. The paper calls for a critical mathematics curriculum for adult students re-learning mathematics.



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