DOCUMENT RESUME

ED 431 593 SE 062 483

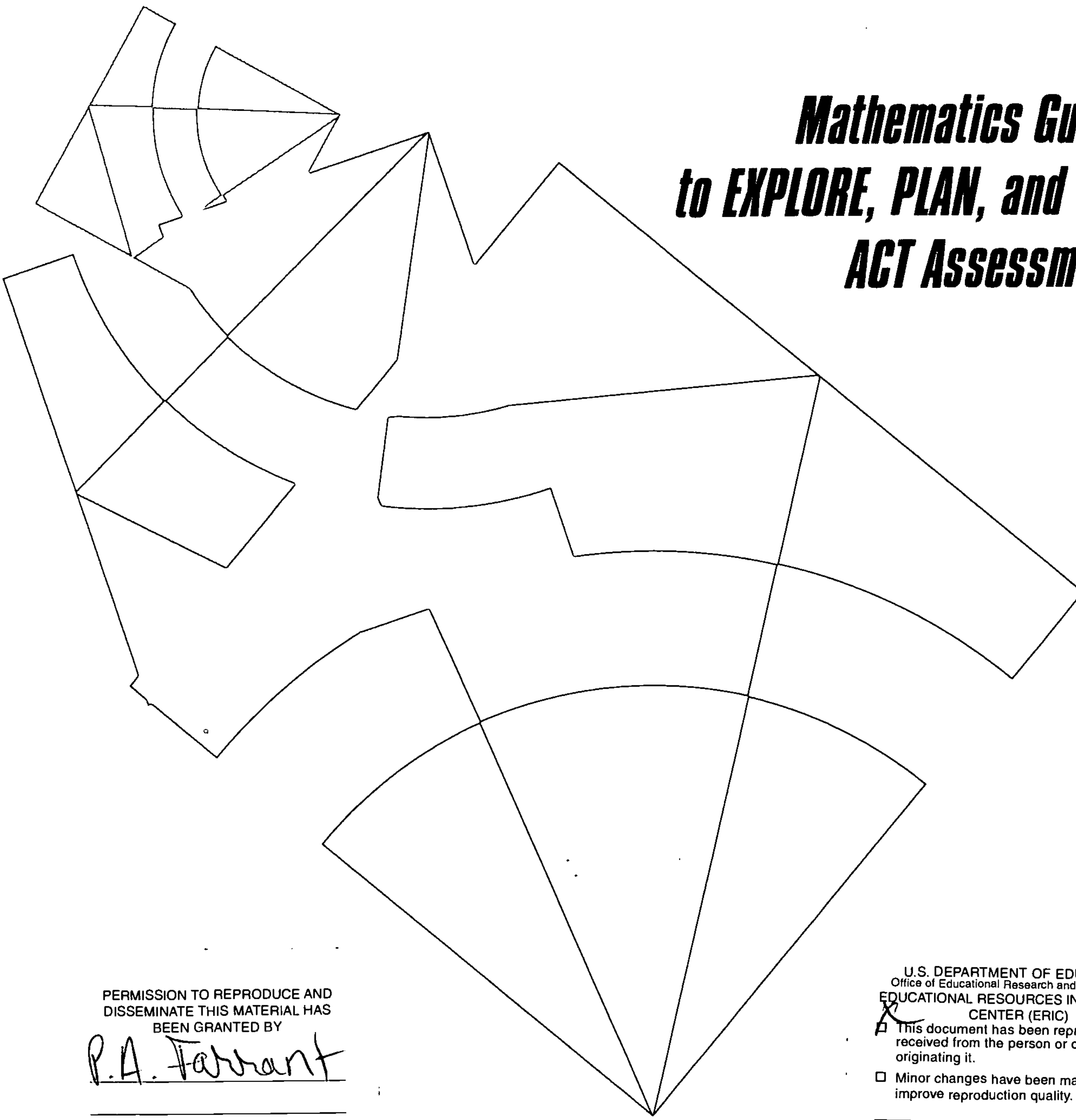
TITLE	Linking Assessment to Instruction in Your Classroom: Mathematics Guide to EXPLORE, PLAN, and the ACT Assessment. Edition 1.
INSTITUTION	ACT, Inc., Iowa City, IA.
REPORT NO	IC-8171
PUB DATE	1996-08-00
NOTE	121p.
AVAILABLE FROM	ACT, Inc., 2201 N. Dodge Street, P.O. Box 168, Iowa City, IA 52243.
PUB TYPE	Guides - Classroom - Teacher (052)
EDRS PRICE	MF01/PC05 Plus Postage.
DESCRIPTORS	*Educational Assessment; Evaluation; Higher Education; *Mathematics Activities; *Mathematics Instruction; Secondary Education; Teaching Methods
IDENTIFIERS	*ACT Assessment

ABSTRACT

In the early 1980s, American College Testing (ACT) saw a need for a system that would respond to the planning and assessment needs of students, parents, teachers, and administrators, and conducted an extensive review of what was being taught in American schools from grade 7 through college-sophomore level in order to identify the important educational goals that should be assessed. The study confirmed that there are skills and understandings which develop over time that are vital to students' success in their post-high school careers, whether they choose to enter the workplace or pursue a postsecondary education. ACT's Educational Planning and Assessment System includes three testing programs: EXPLORE and PLAN tests for eighth- and tenth-graders respectively, and the ACT Assessment taken by high school juniors and seniors. The three instruments are administered at three separate points in a student's secondary school educational experience. By beginning to evaluate students' strengths and weaknesses early in grade 8 and continuing to assess progress through grade 12, educators have the information necessary to monitor and guide students as they prepare for their high school and post-high school endeavors. This guide was prepared for teachers and contains descriptions of the content and skills assessed in EXPLORE, PLAN, and the ACT Assessment. It illustrates the instruction-assessment link by providing a prototype of a mathematics instructional unit and sample classroom assessments. The sample instructional unit and classroom assessments were included to illustrate how classroom-based activities may be linked to assessments and to the skills and understandings measured by the test questions on EXPLORE, PLAN, and the ACT Assessment. These include tasks that engage students in small-group and large-group work, provide opportunities for discussion, require student-led presentations, and include both independent and collaborative work. These activities are intended to illustrate how the skills and knowledge measured by EXPLORE, PLAN, and the ACT Assessment can be taught in the classroom. Contains 27 references. (ASK)

August 1996 - Edition 1

Linking Assessment to Instruction in Your Classroom



Mathematics Guide to EXPLORE, PLAN, and the ACT Assessment

ACT

Contents

LIST OF TABLES

LIST OF FIGURES

Overview of EXPLORE, PLAN, and the ACT Assessment

In the early 1980s, American College Testing (ACT) saw a need for a system that would respond to the planning and assessment needs of students, parents, teachers, and administrators. We conducted an extensive review of what is being taught in American schools, grade 7 through college-sophomore level, in order to identify the important educational goals that should be assessed. The study confirmed that there are skills and understandings, which develop over time, that are vital to students' success in their post-high school careers, whether they choose to enter the workplace or to pursue a postsecondary education. The results of the study laid the foundation for the design and development of our Educational Planning and Assessment System (EPAS). The skills and understandings identified by the study are clearly linked to thinking skills and are described in more detail in Section II.

ACT's Educational Planning and Assessment System includes three testing programs: EXPLORE and PLAN tests for eighth- and tenth-graders, respectively, and the ACT Assessment, taken by high school juniors and seniors. The three instruments are administered at three separate points in a student's secondary-school educational experience.

EXPLORE is intended to be used by all eighth graders. In eighth grade, students need to formulate their plans for high school, and the EXPLORE program helps students investigate and understand a wide range of options. The results from EXPLORE provide educators with the means to structure high school planning and career exploration for students and parents, as well as with a baseline to monitor students' progress. Through EXPLORE, a student's strengths and weaknesses can be identified early in his or her secondary-school educational experience.

The second program in EPAS begins when students enter 10th grade. PLAN is a midpoint review of progress that is being made in high school. For use by all students, PLAN provides direction for educational and career planning that will allow for an adjustment in students' coursework in order to be better prepared to achieve goals after high school.

The ACT Assessment is used by 11th and 12th graders who are considering attending a college or university. Schools, by using the ACT Assessment, not only help those students who are going on to a college or university, but also receive a final measure of the outcomes these students have attained by the time they reach their last two years of high school.

By beginning to evaluate students' strengths and weaknesses early in grade 8 and continuing to assess progress through grade 12, educators have information necessary to monitor and guide students as they prepare for their high school and post-high school goals. EPAS provides schools, parents, and students with:

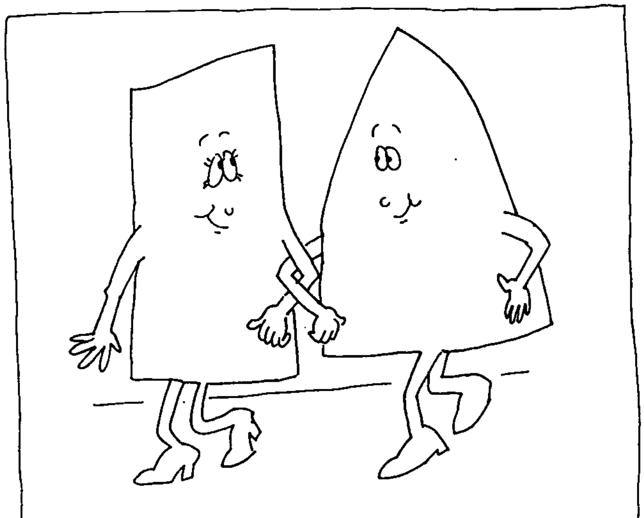
- *a student planning component*, in which students are engaged in a long-term planning process that begins with career exploration and educational planning in grade eight, moves to career and educational planning in grade ten, and concludes with students prepared for life after high school.
- *an assessment component* in each of three programs: EXPLORE, PLAN, and the ACT Assessment. Each program is composed of four tests—English, Mathematics, Reading, and Science Reasoning—that measure what students can *do* with what they *know*. A fourth program, Work Keys, is a program to assess the skills employers are looking for and to help students develop the workplace skills necessary to get the jobs they want after high school.
- *an instructional support component*, offering teachers support in the classroom through ACT's PASSPORT Program, a high school portfolio system in Mathematics, Science, and Language Arts, as well as through instructional support guides like this one.
- *an evaluation component*, providing information that allows schools to research and monitor student performance over time and to assess the strengths and potential weaknesses of school programs.

Taken together, these four components provide students and parents with information they need to help them make the most of their opportunities and to guide them in future educational and career planning.

EXPLORE, PLAN, and the ACT Assessment

Testing Program	Target Grade	Components	Content Areas
EXPLORE	Transition to High School 8th Grade	• Student Planning • Assessment • Instructional Support • Evaluation	• English • Mathematics • Reading • Science Reasoning
PLAN	Midpoint High School Review 10th Grade	• Student Planning • Assessment • Instructional Support • Evaluation	• English • Mathematics • Reading • Science Reasoning
ACT Assessment	Transition to College 11th and/or 12th Grade	• Student Planning • Assessment • Instructional Support • Evaluation	• English • Mathematics • Reading • Science Reasoning

This guide has been prepared for teachers and is, therefore, related to the third element listed above, the instructional support component. Throughout the guide you will find descriptions of the content and skills assessed in EXPLORE, PLAN, and the ACT Assessment, but you will also be introduced to ACT's vision of the instruction-assessment link. We have illustrated the instruction-assessment link by providing a prototype of a mathematics instructional unit and sample classroom assessments.



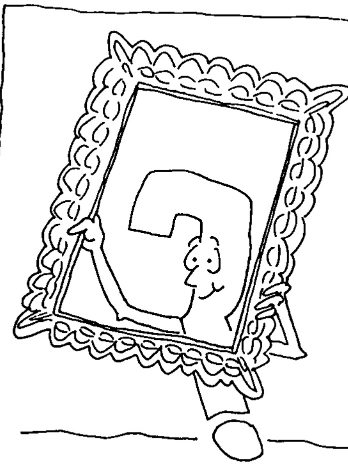
The instruction-assessment link

The sample instructional unit and classroom assessments have been included to illustrate how classroom-based activities may be linked to assessments and to the skills and understandings measured by the test questions in EXPLORE, PLAN, and the ACT Assessment. As you review the instructional activities and the classroom assessments, you will note that they include tasks that engage students in small-group and large-group work, provide opportunities for discussions, require student-led presentations, and include both independent and collaborative work. These activities are intended to illustrate how the skills and knowledge measured by EXPLORE, PLAN, and the ACT Assessment can be taught in the classroom.

The Need for Thinking Skills

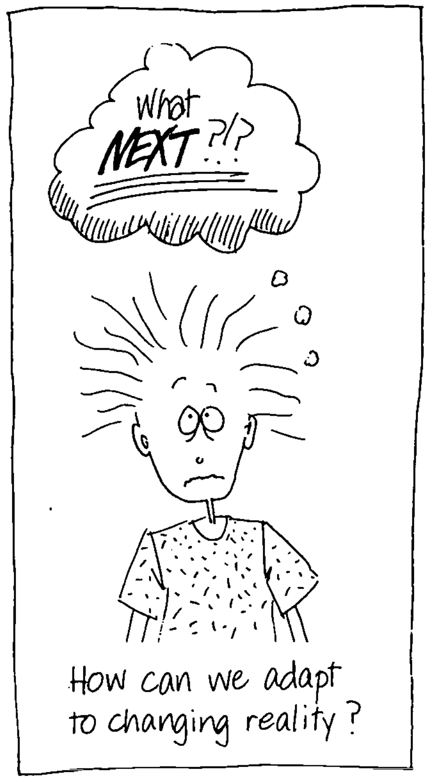
The test questions in EXPLORE, PLAN, and the ACT Assessment focus on a variety of thinking skills; students are asked to select, manipulate, and apply skills, strategies, and processes to solve specific problems in specific contexts. Every student comes to school with the ability to think; it is the ability to analyze, expand, and understand one's thinking that holds the key to future achievement. It is more important now than ever before that students in our schools not only know information, but be able to manage it effectively. Students must be provided with the tools for ongoing learning; understanding, analysis, and generalization skills must be developed so that the learner is able to adapt to a variety of situations. Our belief in the importance of developing thinking skills in learners was a key factor in the development of EXPLORE, PLAN, and the ACT Assessment. The test questions in each of the three programs are designed to determine how skillfully students solve problems, grasp implied meanings, draw inferences, evaluate ideas, and make judgments in subject-matter areas important to success in intellectual work both inside and outside of school.

Thinking skills are especially important at a time when skilled workers are in increasing demand. We live at a time in which many familiar assumptions and values are being questioned. The students in your classrooms will be faced with increasingly complex realities, and their effective adaptation to change will require them to address issues in rational and creative ways, to cope with ambiguities, and to find new means of applying and transferring information.



Getting the "big picture."

Revolutions in communications and other technologies, as well as global environmental and health issues that we all face, require choices and planning for which many of us are not adequately prepared. Currently available information and technology in many fields will soon be obsolete, and there is reason to believe that this trend will continue. Richard Paul, the director of the Foundation for Critical Thinking in Santa Rosa, California, and a prolific writer in the area of critical thinking, has stated that two characteristics of today's world, increasing complexity and an accelerated pace of change, "sound the death knell for traditional methods of learning how to survive. . . . How can we adapt to reality when reality won't give us time to master it before it changes itself, again and again, in ways we cannot anticipate?" (Paul, 1993)



As a classroom teacher, you are integrally involved in preparing today's students for their futures. Such preparation must include the development of thinking skills such as problem solving and decision making. Many agree that only by developing thinking and reasoning skills can our nation's students fulfill their potential. In a recent policy statement, the Council of Chief State School Officers urged that "schools, previously asked to ensure the development of basic skills, now be required to teach all students a new, broad range of skills demanded by the changing contexts in which students live" (quoted in ACT's brochure *Thinking Skills Measured in ACT's Assessment Programs*, 1992, page 1). The Council's statement illustrates the emphasis on thinking skills that is required by the increasingly complex needs of our global economy and by America's need for educated and aware citizens. *Yet what exactly are these thinking skills? How do ACT's assessment programs measure them? How can thinking skills be embedded into the daily instructional activities and classroom assessments in today's classrooms?*

Content and Skills Measured in EXPLORE, PLAN, and the ACT Assessment

Just as it is difficult to agree on the content of a curriculum, it is difficult to determine what the phrase "thinking skills" encompasses. ACT's definition, therefore, is wide ranging, encompassing everything from skills such as performing calculations to more complex skills, such as relating various mathematical concepts. Generally speaking, higher-order skills reflect an ability to select, manipulate, and manage core skills, strategies, and processes in order to solve problems in specific contexts. The acquisition and use of thinking skills, however, is not the product of a linear instructional program; individuals may engage a unique subset of skills each time they encounter a new problem. Similarly, the ease with which individuals use certain skills depends on their experience and the specific learning situations they encounter. Some students, for example, may be able to find the solution to a problem using estimation, while others may prefer to work out the entire problem on paper.

EXPLORE, PLAN, and the ACT Assessment measure thinking skills that are important for success in high school, college, and work. Thus, underlying the three testing programs is the belief that students' preparation for further learning is best assessed by measuring, as directly as possible, the academic skills that students have acquired and that they will need to perform at the next level of learning. These skills are measured through contexts representing the complexity of the tasks students are asked to perform in college and in life. These thinking skills are the focus of the content areas covered by each of the four tests in each program: English, Mathematics, Reading, and Science Reasoning.

English Tests assess skills used in effective writing, including organization, clarity, style, and rhetorical techniques in a given essay. Students analyze a writing situation, selecting answers that best express an idea, achieve a rhetorical purpose, make the statement follow standard conventions of written English or are consistent with the style, tone, and purpose of the passage. Many thinking skills need to be combined and applied in answering the questions.

Students must assess the skills used in effective writing...

Mathematics Tests assess thinking skills required to perform tasks of quantitative reasoning, such as applying and integrating skills in different contexts or analyzing a problem situation to decide which skills to apply.

Analyzing a problem can involve translating English into math...

Reading Tests assess a broad range of thinking skills that include recognizing how details are related to the main idea of a passage, synthesizing information, analyzing the structure of a passage, and drawing conclusions.

Science Reasoning Tests assess thinking skills, processes, and strategies students use while learning and applying science. These skills include comparing experimental designs and methods, comparing assumptions underlying experiments, analyzing and interpreting data, making generalizations, and identifying and evaluating conflicting points of view.


science reasoning involves analyzing experimental designs, such as Archimedes' observations regarding a body immersed in a liquid...

SECTION I

The following sections of this guide will (1) describe the mathematics content areas assessed by EXPLORE, PLAN, and the ACT Assessment and the ways in which these tests measure thinking skills; (2) discuss the importance of the instruction-assessment link; (3) provide a sample instructional unit and classroom assessments for use in a ninth- or tenth-grade mathematics classroom; (4) offer a list of ideas for other instructional units in the mathematics area; and (5) describe the relationship between the mathematics instructional unit and the test questions in EXPLORE, PLAN, and the ACT Assessment. In addition to this Mathematics Guide, there are two other instructional support guides focusing on the other content areas assessed by EXPLORE, PLAN, and the ACT Assessment: Language Arts and Science Reasoning.

MATHEMATICS

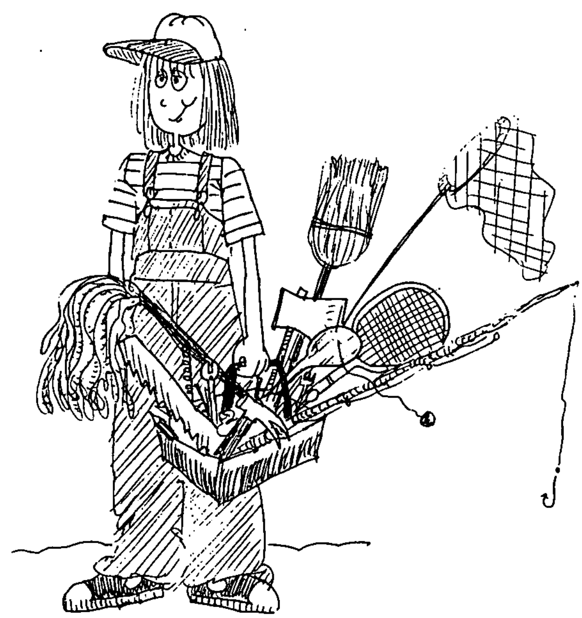
The idea that all students can and must learn mathematics represents a shift from earlier thinking and practices which dictated that learning mathematics requires special ability that many students do not have. The study of mathematics is truly an avenue to success, both in the student's future postsecondary work and in the workplace. Students, as well as parents, need to realize that mathematics achievement is attainable by all, and that success in mathematics will help students solve problems of everyday life, help them participate intelligently in civic affairs, and prepare them for jobs, vocations, or professions.

A Definition of Mathematics

The National Council of Teachers of Mathematics (NCTM) was one of the first organizations to recognize the need for radical change in mathematics education and to develop a set of standards for curriculum and assessment, referred to as the *Standards* (NCTM, 1989, 1991, 1995). In addition to other recommendations, the *Standards* proposed a more "real-life" orientation to mathematics than what was being used in schools at that time. The *Standards* present a broad framework for mathematical processes and concepts rather than a recipe book for understanding formulas and rules. Further, this approach to mathematics integrates technology, including the calculator, into the curriculum and considers assessment to be a necessary tool of contemporary society.

Many theorize that the *Standards* document grew out of the inability of the "back-to-basics" movement of the 1970s to come to terms with key issues of the 1980s and 1990s. The back-to-basics movement emphasized rote memorization of arithmetic facts and the learning of paper-and-pencil algorithms. In doing so, the movement did not address critical components of learning such as higher-order thinking, including problem solving, the changing mathematical skills needed in the work force, findings from cognitive research on teaching and learning, and the existence of inexpensive calculators and computers.

According to the *Standards*, schools are expected to "insure that all students have the opportunity to learn, and to become informed citizens capable of understanding issues in a technological society." Mathematics teachers have been challenged to meet these broad goals, and, accordingly, have begun to shift the emphasis to the thinking skills required to perform tasks of quantitative reasoning. As the *Standards* are incorporated into daily classroom practices, students are challenged to use the full range of their mathematical skills, including such important skills as problem solving, communication, reasoning, and inventiveness. The goal is to teach students how to apply these skills flexibly across a range of problems within a variety of content areas. Describing the use of symmetry in a work of art, devising an algorithm for use in a computer program, or explaining one's reasoning to a peer are some examples of possible applications of mathematical skills important to a student's success in high school, college, and the work force.



The more tools you have in your toolbox, the better equipped you'll be.

ACT's Mathematics Tests

ACT has been aware of and responsive to trends and expectations in mathematics education. In order to define the most important outcomes of mathematics education, ACT has invested considerable time and effort seeking the advice and opinions of teachers, curriculum coordinators, administrators, and course content experts; additionally, state curriculum guides and the most widely used mathematics textbooks have been reviewed. Based on the findings from these discussions, surveys, and reviews, we were able to broadly define the important goals of mathematics education. Briefly stated, the goals are that students be able to understand and apply essential concepts and procedures, be able to reason and to analyze, and be able to transfer problem-solving skills from one context to another.

The content for the Mathematics Tests is based on a detailed analysis of the coursework included in grades seven through fourteen. Content areas for the tests were defined by blending and allocating all of the topics taught across the country, so long as the topics were common to most curriculums. A second criterion used in the identification of mathematics content areas was the degree to which the knowledge and skills underlying the content areas were deemed important to success in later mathematics, in the world of work, or in other facets of one's life. Some of the content areas line up with common course titles; however, it's important to note that not all courses with the same titles are equivalent.

Presently, the Mathematics Tests in EXPLORE, PLAN, and the ACT Assessment are designed to reflect today's mathematics curriculum through a multiple-choice format. The three tests encompass key content areas of the secondary school curriculum, including pre-algebra, algebra, geometry (plane, coordinate, solid, and transformational), statistics/probability, and trigonometry. Proficiency in these content areas is necessary for success as students make transitions from grade-to-grade within secondary school and as they move from high school to college and into the workplace.

The content areas for the three Mathematics Tests are listed in Table 1 on page 12. The table provides the number (and percentage) of test questions in each content area. As you will note, there is a clear progression in the content coverage of the tests from grades eight to twelve. Several points need to be made about the labeling of the content areas, especially at the eighth-grade level. At eighth grade, consistent with the NCTM Standards, statistics/probability does not refer to the content of a statistics course, but to the ability to process data. Similarly, eighth-grade "pre-geometry" deals with using figures and diagrams to solve mathematical problems. At levels higher than eighth grade, content definitions are consistent with standard courses at the high school level. However, many topics, such as statistics and probability, are measured in different content areas but do not constitute a content area in and of themselves. For example, the content area of Basic Statistical/Probability Concepts is listed as a content area in the EXPLORE program but does not appear in either PLAN or the ACT Assessment. This does not mean that PLAN and the ACT Assessment have no statistics or probability test questions. These topics are integrated into the other major content areas of the tests, as appropriate. In many cases, these topics are a part of the Pre-Algebra content area. Also, Plane Geometry is used to distinguish this content area from Coordinate Geometry; Plane Geometry also includes some three-dimensional applications as well as some transformational geometry.

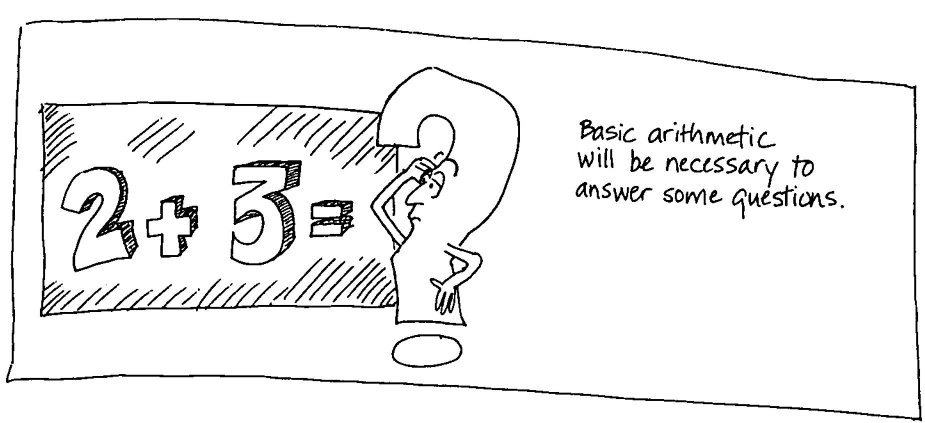
Table 1. Mathematics Test Content Specifications

Content Area	Testing Program		
	EXPLORE	PLAN	ACT Assessment
	Number and Percentage of Items		
Basic Statistical/ Probability Concepts	4 (13%)		
Pre-Algebra	10 (33%)	14 (35%)	14 (23%)
Elementary Algebra	9 (30%)	8 (20%)	10 (17%)
Pre-Geometry	7 (23%)		
Plane Geometry		11 (27%)	14 (23%)
Coordinate Geometry		7 (18%)	9 (15%)
Intermediate Algebra			9 (15%)
Trigonometry			4 (7%)
Total Items	**30**	**40**	**60**

The cognitive levels assessed by the Mathematics Tests are listed in Table 2 on page 14. The numbers (and percentages) of items measuring each cognitive level are reported. A detailed definition of each cognitive level follows.

KNOWLEDGE AND SKILLS (KS)

These items test the direct use of one or more facts, definitions, formulas, or procedures.



DIRECT APPLICATION (DA)

Direct Application test questions are the same as those categorized as Knowledge and Skills, except that DA problems are set in real-world situations. Most familiar types of word problems would fit into this category.

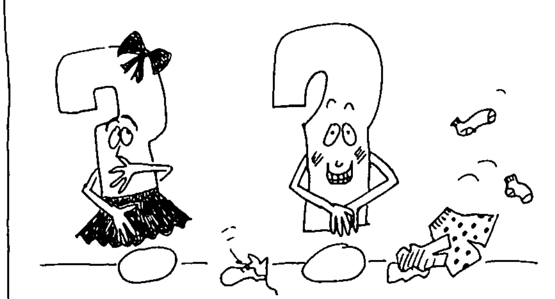
UNDERSTANDING CONCEPTS (UC)

UC items test for depth of understanding of a major concept. These items are often more difficult or complex. UC problems involve reasoning from a particular concept, perhaps drawing a conclusion or making an inference. These items should appear novel to most students, and should elicit a thoughtful, rather than a practiced, response (unlike KS and DA items). A student would answer problems in this category correctly (discounting guessing) only with a significant level of understanding of the concept being tested.

INTEGRATING CONCEPTUAL UNDERSTANDING (IC)

IC questions require an integrated understanding of two or more major concepts. IC problems are not routine. They may require planning, making connections, generalizing, using mathematical insight, making value judgments, synthesizing, or extending conceptual knowledge (applicable also to UC items).

The distinction between KS and DA is not in the degree of complexity, but rather in the setting—purely mathematical as opposed to in-context. On the other hand, setting does not distinguish between the last two categories, UC and IC, but rather the necessity for integrating more than one major concept.



Properly attired math question recoils in horror from stripped-down, bare-bones type.

BEST COPY AVAILABLE

18

Table 2. Mathematics Test Cognitive Specifications

Content Area	Cognitive Level* KS	DA	UC/IC	Total Items
	Number and Percentage of Items			
EXPLORE				
Pre-Algebra	2 (.067)	3 (.100)	5 (.167)	10
Elementary Algebra	3 (.100)	2 (.067)	4 (.133)	9
Geometry	2 (.067)	2 (.067)	3 (.100)	7
Statistics/Probability	1 (.033)	1 (.033)	2 (.067)	4
Total Items	**8**	**8**	**14**	**30**
PLAN				
Pre-Algebra	5 (.125)	6 (.150)	3 (.075)	14
Elementary Algebra	3 (.075)	2 (.050)	3 (.075)	8
Coordinate Geometry	3 (.075)	1 (.025)	3 (.075)	7
Plane Geometry	3 (.075)	3 (.075)	5 (.125)	11
Total Items	**14**	**12**	**14**	**40**
ACT Assessment				
Pre-Algebra	4 (.067)	8 (.133)	2 (.033)	14
Elementary Algebra	5 (.083)	3 (.050)	2 (.033)	10
Intermediate Algebra	5 (.083)	1 (.017)	3 (.050)	9
Coordinate Geometry	6 (.100)	1 (.017)	2 (.033)	9
Plane Geometry	9 (.150)	2 (.033)	3 (.050)	14
Trigonometry	3 (.050)	1 (.017)	0 (.000)	4
Total Items	**32**	**16**	**12**	**60**

*KS = Knowledge and Skills; DA = Direct Application;
UC = Understanding Concepts; IC = Integrating Conceptual Understanding

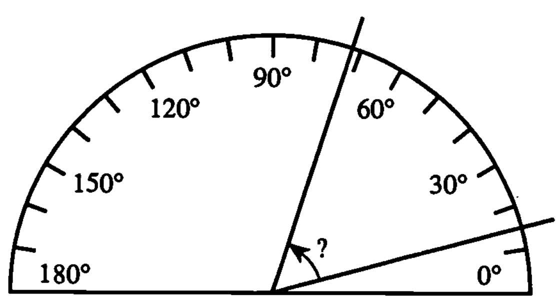
Sample Mathematics Test Questions

The following questions represent the types of questions students may encounter in the Mathematics Tests in EXPLORE, PLAN, and the ACT Assessment. Two of the sample test questions represent types of questions students might encounter on the eighth-grade test, EXPLORE. The types of questions students might encounter on the tenth-grade test, PLAN, and the eleventh-twelfth grade test, the ACT Assessment, are each represented by three test questions.

EXPLORE

Following are two examples of test questions that might appear on the eighth-grade EXPLORE Mathematics Test.

1. Shown below is a protractor in position to measure an angle. Which of the following is closest to the measure of that angle?



A. 15°
B. 45°
C. 57°
D. 72°
E. 87°

Question 1 addresses several basic concepts of measurement, including understanding the concept of degree in angles, knowledge of how to use a protractor and how to read a scale, and the ability to adapt thinking since neither of the rays that form the angle is at the 0° mark.

In order to find the measure of the angle in question, a student might choose one of several strategies. For example, some students might imagine rotating the protractor counterclockwise until the close side of the angle goes through the 0° mark, a rotation of about 15°. That would make the second side go through the (72° - 15°) mark and the angle would measure about 57°—answer C. Other students would more naturally subtract the measure of the angle from 0° to 15° from the measure of the angle from 0° to 72°, rather than imagining rotating the protractor into standard measuring position. While both strategies would yield the correct answer (C), the methods are subtly different.

The most common incorrect answer is 72° (D), which is chosen when students fail to account for the angle not being lined up with the 0° mark. Students who choose answer A could be ignoring the arrow indicating which angle is being measured. Students choosing answer B could simply be misreading the protractor or they may be looking only at the angle and seeing that it resembles a 45° angle. Answer choice E might be eliminated based on the student's knowledge of what a 90° angle looks like. Students who have had experience using protractors should be able to correctly answer this question.

This question is classified KS (Knowledge and Skills) because it is in a purely mathematical context and students may be used to lining up rulers away from the end in order to get a more accurate measurement.

2. A survey taken at a local preschool asked each child for his or her favorite number. The following chart shows the results. How many of the children's favorite numbers were also their age?

Age	Favorite Number				
	2	3	4	5	over 5
3	4	8	7	2	1
4	2	9	9	8	3
5	2	4	1	12	5

A. 14
B. 20
C. 29
D. 34
E. 77

Question 2 involves reading and interpreting data presented in a chart. Students must be able to analyze the chart, understand how the rows and columns are labeled, and draw a conclusion based on the evidence presented in the chart. The correct answer cannot be found by simply locating one number on the chart; rather, students have to take the statement about the children's favorite numbers being their age and connect that statement to the row and column headings of the chart. Students who select the correct answer will determine that there are 8 children who are 3 years old whose favorite number is 3; there are 9 children who are 4 years old whose favorite number is 4; and there are 12 children who are 5 years old whose favorite number is 5. This makes 8 + 9 + 12 = 29 children whose favorite number is their age (answer C). Some students might figure out that the numbers involved (8, 9, and 12) appear on a diagonal line on the chart. Answers A (14) and B (20) might be selected by students who added 4 + 9 + 1 or 7 + 8 + 5 on other diagonals. Students who select answer choice E might do so because of a lack of clear understanding of the problem's question, and/or because they added all the numbers on the chart.

This question is classified DA (Direct Application) because it includes the real-world context of a survey conducted at a preschool.

PLAN

The following are examples of test questions that might appear in the tenth-grade PLAN Mathematics Test.

---

3. Mary went bowling with 4 of her friends. Each girl bowled 2 games. These are the results of the games:

	Game 1	Game 2
Mary	120	130
Fran	124	128
June	118	132
Dee	110	142
Linda	120	140

For those 2 games, who had the highest average?

A. Mary
B. Fran
C. June
D. Dee
E. Linda

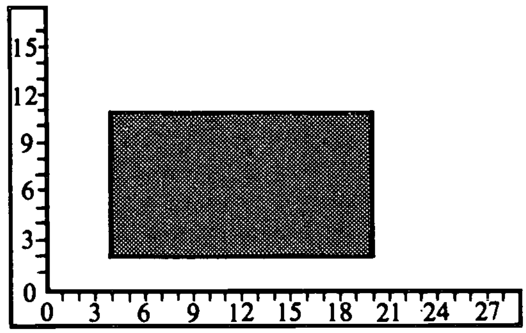
Question 3 is a problem that students may not find difficult, yet it does require some reasoning. A student must decide which numbers to average, based on their reading and understanding of the problem. The normal convention of adding columns of numbers will not lead to the correct answer.

In this problem, there are several opportunities for an insightful student to use shortcuts that will save time and cut down on the possibility of error. One such possibility is to observe that the average is highest when the sum is highest, so it is unnecessary to actually divide all the sums by 2. Another way to simplify the problem is to see that all of the 100's in the girls' scores can be ignored in deciding which average is highest, since everyone is even on that count. Whatever method a student chooses to solve this problem, the result will be that Linda's average is higher than the other averages, and the correct answer is E.

This question is classified DA (Direct Application) because it is a straightforward problem in the real-world context of people bowling and keeping score.

---

4. The carpenter's square, shown below, is marked off in inches. What is the area, in square inches, of the rectangular board in the figure?



A. 16×9

B. 20×11

C. $16 + 9 + 16 + 9$

D. $20 + 11 + 20 + 11$

E. $\sqrt{20^2 + 11^2}$

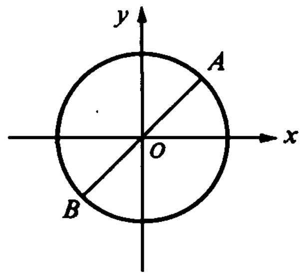
Question 4 addresses the concepts of length and area. Students who reason that the most common way to calculate area for a rectangle is to find its length and width and multiply the two are a good share of the way to the solution to the problem. But they must also understand enough about measurement to see that the length of the board is not 20 inches because the left edge of the board does not start at the 0 inch mark. Similarly, the width of the board is 2 inches less than the 11-inch reading of the top of the board. So, the area is $(20 - 4) \times (11 - 2) = 16 \times 9$, which is answer choice A. Students who do not notice that the bottom-left corner of the board is not at (0,0) could arrive at 20×11 as the area, and that is the most common wrong answer.

There are students who may not have the concepts of area and perimeter clearly separated. Answer choice C is the board's perimeter, and answer choice D is the perimeter calculation that could be made if the student did not notice the 2-inch gap between the bottom of the board and the carpenter's square and the 3-inch gap on the left of the board. Answer choice E represents the distance from the (0,0) point on the carpenter's square to the top right corner of the board. Each of these answer choices has a form that will look familiar to students.

There are alternate methods of finding the area of the board. One is to find the area of the larger rectangle from (0,0) to the upper-right corner of the board and then subtract the areas of the gaps between the board and the carpenter's square. Another method is to imagine moving the board flush against the carpenter's square and find the distances in that new position.

This question is classified DA (Direct Application) because students could count along the ruler to get the length of the sides (a practiced method) and because of the real-world context of carpenters' tools.

5. In the figure below, $\overline{AB}$ is a diameter of the circle graphed in the standard coordinate plane. The circle is determined by the equation $x^2 + y^2 = 25$. The (x,y) coordinates of A are (3,4). What are the (x,y) coordinates of B?



A. (−4,−3)
B. (−4, 3)
C. (−3,−4)
D. (−3, 4)
E. (3,−4)

Problem 5 is an application of symmetry, and there are many good ways for students to approach the problem. Students who have a basic understanding of the concepts of graphing in coordinate geometry will know that answers B, D, and E may be eliminated because none of them are in the same quadrant as point *B*, which would have negative *x*- and *y*-coordinates. Answer A, which also has negative *x*- and *y*-coordinates, is the most common wrong answer.

Students could decide to solve the problem using the concept of formal slope, or a student might reason that point *A* is as much above the *x*-axis as point *B* is below the *x*-axis, and similarly for the distance right/left of the *y*-axis. Other students might choose to draw the figure to scale in order to check whether the *x*- or the *y*-coordinate was less negative and thus rule out answer A, arriving at the correct answer, C.

From a transformational geometry viewpoint, point *B* can be viewed as a double reflection of point *A*, first about the *y*-axis and then about the *x*-axis. The first transformation negates the *x*-coordinate and the second negates the *y*-coordinate, giving answer C. Alternatively, point *B* could be viewed as a reflection of point *A* through the origin or as a 180° rotation of point *A* about the origin. (Each of these resulting transformations also maps the circle and the diameter to themselves.)

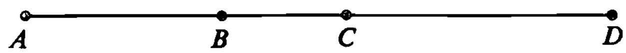
For a formal slope approach, students might use the following procedures. Points *O*, *A*, and *B* are on the same line, so the slope of that line can be calculated based on points *O* and *A* as $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{3 - 0} = \frac{4}{3}$. The slope can also be calculated for points *O* and *B* as $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_B - 0}{x_B - 0} = \frac{y_B}{x_B}$. The only answer for which $\frac{y}{x} = \frac{4}{3}$ is (−3,−4), where $\frac{y}{x} = \frac{-4}{-3}$.

This question is classified UC (Understanding Concepts) because students must decide on a method for choosing between answers A and C.

Following are three examples of test questions that might appear on the ACT Assessment Mathematics Test (for eleventh- and twelfth-grade students).

---

6. In the figure below, points *A*, *B*, *C*, and *D* represent exits on a straight stretch of highway. The distance from *A* to *C* is 24 miles, from *B* to *D* is 32 miles, and the entire distance from *A* to *D* is 50 miles. What is the distance between *B* and *C*?



A. 6
B. 8
C. 18
D. 26
E. 42

To answer question 6, students must understand distance concepts, they must know how to label a diagram accurately in order to determine the missing information, and they need to have an understanding of the relationship between overlapping distances. Students might approach this problem in several ways. One approach is to consider the distances given and note that when distance *AC* is added to distance *BD*, part of the distance (distance *BC*) has been counted twice. The total distance given for *AD* is 50 miles. If distance *AC* (24 miles) is added to distance *BD* (32 miles), the total is 56 miles. From this information, students can correctly reason that 6 miles have been counted twice, and that distance *BC* is 6 miles.

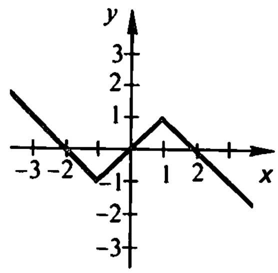
Another excellent strategy would be to assign coordinates, viewing this highway as a number line. For example, if point *A* is at coordinate 0, then point *C* has coordinate 24 and point *D* has coordinate 50. The coordinate for point *B* could then be found by subtracting the distance *BD* (32) from point *D*'s coordinate (50) to get point *B*'s coordinate (18). Knowing that the coordinate of point *B* is 18 and the coordinate of point *C* is 24, it becomes clear that the distance from point *B* to point *C* is $24 - 18 = 6$.

Students might use logical reasoning to eliminate answer choices D (26) and E (42), realizing that distance *BC* has to be less than distance *AC* (24), since *BC* is part of *AC*. Another method, using logical reasoning, is to "try out" the answer choices. For example, if students try answer choice B, making $BC = 8$, then $AB = 24 - 8 = 16$ and $BD = 32 - 8 = 24$; then $AB + BC + CD = 16 + 8 + 24 = 48$, which isn't long enough. Students who not only understand the underlying concepts but who have also had practice in using a variety of strategies increase their chances of correctly finding solutions to such problems.

This question is classified as UC (Understanding Concepts) because, though many textbooks include a few problems similar to this one, students in general do not learn this as a practiced skill but rather have to figure out how to fit the information together.

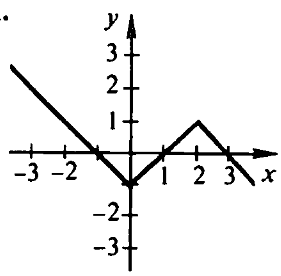
---

7. The graph of $y = f(x)$ is shown below.

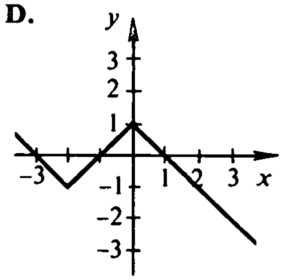


Which of the following is the graph of $y = f(x) + 1$?

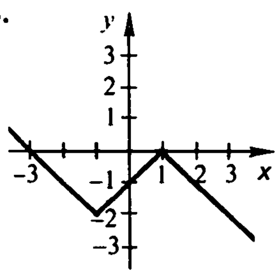
A.



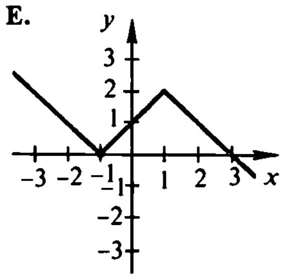
D.



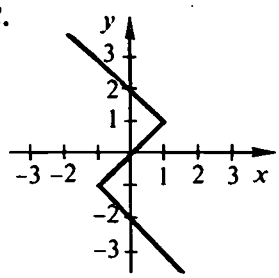
B.



E.



C.

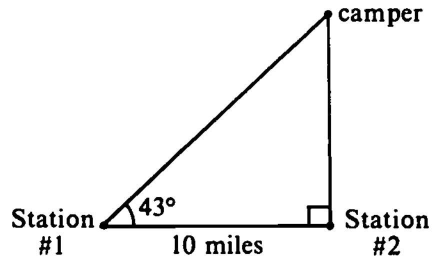


Question 7 requires that students understand underlying concepts of graphical functions. This problem asks students to work with a graphical function where the algebraic form of the function is not a familiar equation. Students have to make graphical connections to the functional form $f(x) + 1$. A student might reason that, for each particular x-value, the new y-value is 1 more than the old y-value, which was $f(x)$, and so the graph would shift up by 1 unit at each x-value. The result of this reasoning would yield the correct graph, answer choice E.

Students who suspect that the new graph is a translation of the old one but are not sure of the direction might be likely to select answer choice A, a popular choice. A student who was unsure of the direction of the translation might want to employ a strategy for checking, such as substituting a more familiar function in place of this $f(x)$, such as x^2. (A linear function may not be a good choice.) Plotting a few values on the curve $y = x^2 + 1$ would make it clear that this is a translation up by 1 unit. Answer choice C is a reflection about $y = x$, which students might choose if they mistakenly incorporate ideas about inverse functions. This question doesn't ask students to solve a problem or come up with a specific solution; it only asks them to think through principles of graphing and functions to arrive at the correct graph.

This question is classified UC (Understanding Concepts) because it looks at translation of equations (a topic covered in most curriculums) at an abstract level.

8. A distress call from a camper is received by 2 ranger stations. Station #1 is 10 miles due west from Station #2. The rangers determine that the camper is located as shown in the diagram below. How many miles is the camper from Station #2 ?



A. $\frac{10}{\sin 43°}$

B. $\frac{10}{\cos 43°}$

C. $10 \sin 43°$

D. $10 \cos 43°$

E. $10 \tan 43°$

Question 8 asks students to apply their knowledge of trigonometry to a real-world situation. One of the practical uses of trigonometry is to find angles and distances that are hard to measure directly. Here, a student who knows basic trigonometric relations in a right triangle can note that the sides of the triangle involved in the problem are the side opposite the 43° angle and the side adjacent to the 43° angle. The tangent function relates the lengths of these sides to the angle measure as follows: $\tan 43° = \frac{d}{10}$ where d is the distance from the camper to Station #2. Solving this for d gives answer choice E. Other answers would be a result of misunderstanding or misapplying trigonometric functions.

While that is a good solution, there are many other ways to approach the problem. One of the more useful observations is that this is almost a 45°-45°-90°-triangle, so the distance from the camper to Station #2 should be about 10 miles, and it should really be a little less than 10 miles because 43° is slightly less than 45°. Approximating sin 43° and cos 43° as about 0.7 puts answer choices A and B at about 14, answer choices C and D at about 7, and answer choice E at about 10, making E the most reasonable choice. Even if students don't feel confident about using sine and cosine, using estimation skills gives them a way to feel confident about the solution.

This question is classified DA (Direct Application) because it is an example of how trigonometry can be used in the real world, and it is similar to many textbook problems.

The sample test questions from the Mathematics Tests in EXPLORE, PLAN, and the ACT Assessment are intended to illustrate some of the ways thinking skills can be utilized when answering test questions. The next section shows how thinking skills help to form the link between instruction and assessment.

THE INSTRUCTION-ASSESSMENT LINK

Instruction and assessment are sometimes viewed as separate activities. When instruction and assessment are viewed as separate activities, a unit of instruction is frequently taught from a textbook, students read and answer questions, and then a final unit test is administered. The results of such unit tests are used to determine if students have learned the material. The assignment of grades occurs next in this nearly linear process with those students "who have learned it" receiving an A or a B and those students "who haven't learned it" receiving a D or an F. When students receive their grades, the process is repeated with the teacher dispensing a new set of information. When instruction and assessment are "delivered" in this way, there is little or no opportunity for students to learn from the test, to use their mistakes as springboards to new knowledge, or to incorporate knowledge from one unit of work into the work of the next unit.

There is now a rich body of evidence that supports the practice of linking assessment with instruction so that assessment is not a single activity or an exercise that stands apart from the instructional process. Rather, the education community now supports the integration of assessment into instructional activities in such a way that the two are viewed as being inseparable. Assessment used in this way can be described as a "feedback loop" linked to instruction. Assessment becomes more than collecting data; its primary use is that of influencing instruction. Linking instruction with assessment places the teacher in the role of a facilitator and places new responsibilities on the students.

As one begins to see assessment as integral to the entire teaching-learning process, then one must also recognize the changes that need to occur in the classroom environment. If you believe that one learns by connecting past experiences and prior knowledge to new concepts and new information, then you have placed the student at the center of the learning experience. Such a view of learning can be visually represented in a map of classroom activities that includes both instruction and assessment (see Figure 1 on page 24).

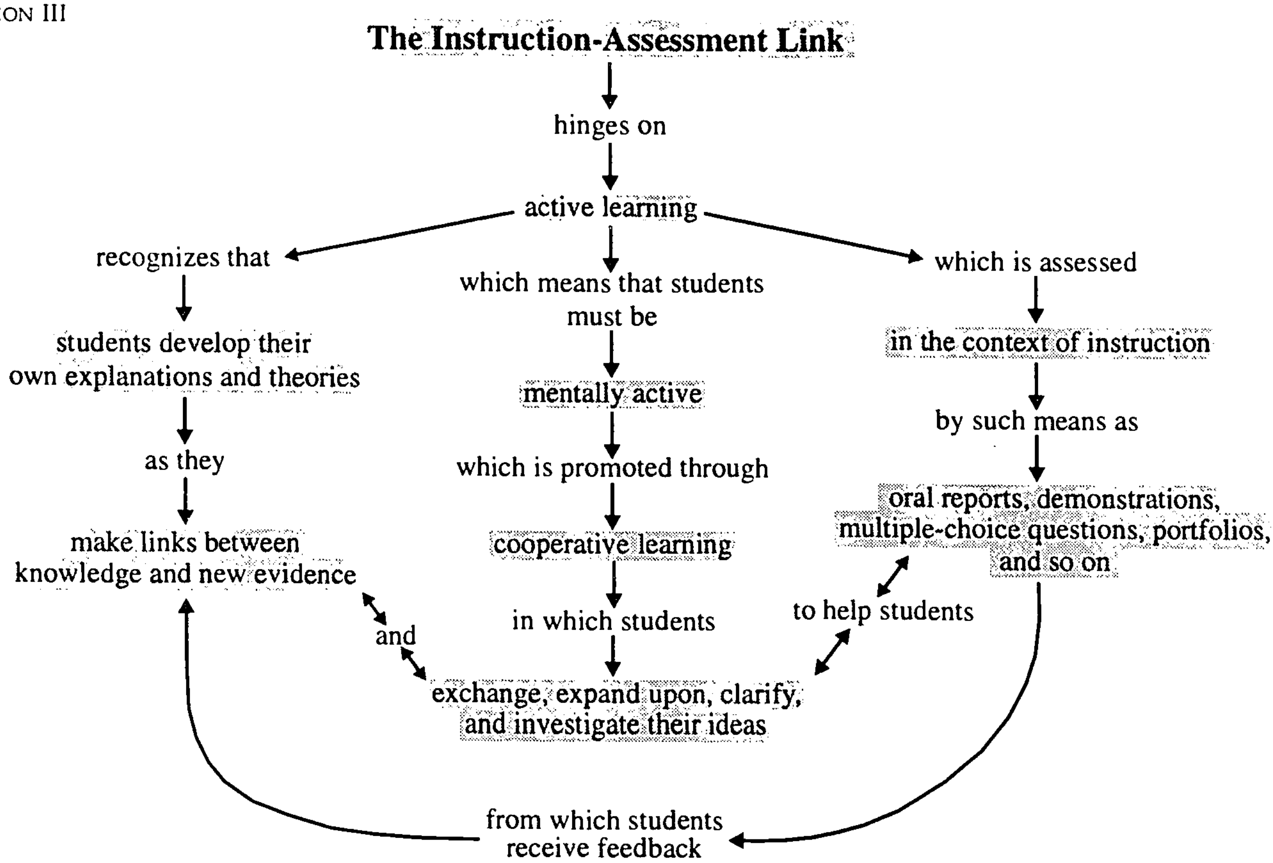


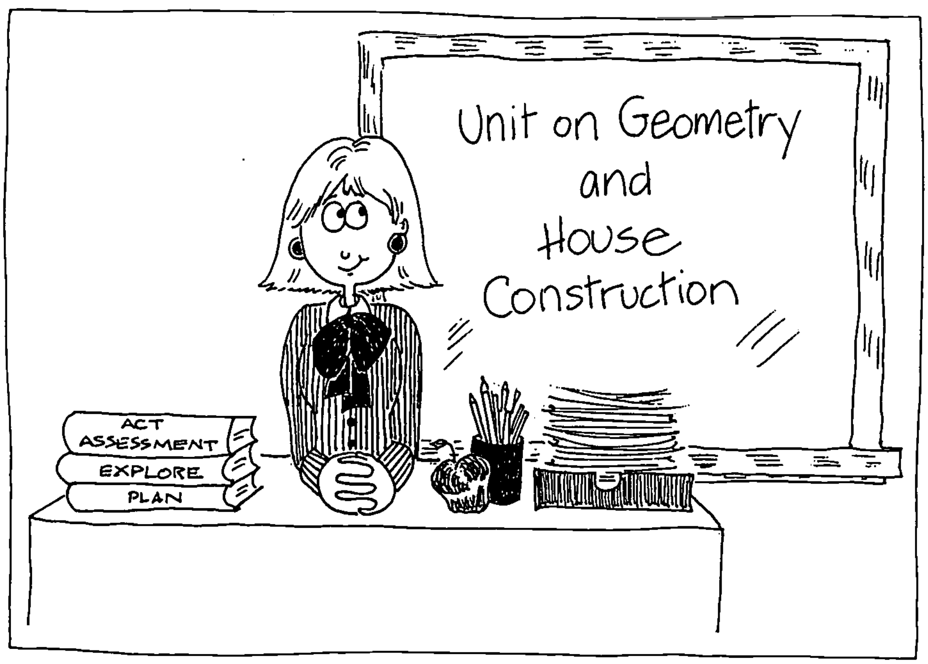
Figure 1. Activities Comprising the Instruction-Assessment Link

The classroom activities mapped in Figure 1 provide a visual representation of the instruction-assessment link (adapted from Greenwood, 1996). Assessment is embedded in the day-to-day functioning of the classroom. We are using the term "embedded assessments" to describe those assessments that are primarily teacher developed and that are integral to the instructional process. Such assessments are designed to improve student learning and to help teachers determine students' instructional needs. Embedded assessments:

- help to focus teachers and learners on the important elements of the curriculum
- are indistinguishable from instruction
- provide information about student progress and the instructional process
- use multiple methods of assessment
- occur daily and are cumulative
- allow students to show what they know and can do
- provide opportunities for student self-assessment

As teachers expand the types of assessment tools used in the classroom, they are providing an opportunity to increase their understanding of students' learning. The instruction and assessment activities that are illustrated in Figure 1 place the student at the center of the learning experience. When students are placed at the center of learning, they begin to learn to question, and, in doing so, they begin to trust their intuition. Having a strong sense of self can have unlimited benefits for students when they are called upon to demonstrate their proficiency in more formal testing situations such as EXPLORE, PLAN, and the ACT Assessment.

An example of the instruction-assessment link is provided in the next section of this guide. We have developed an instructional unit and sample classroom assessments for use in a ninth- or tenth-grade pre-geometry or geometry class. Many of the skills and understandings embedded in the unit are those that are assessed at varying degrees of sophistication in the Mathematics Tests from EXPLORE, PLAN, and the ACT Assessment. Although we chose ninth and tenth grades as the focal points for the instructional unit, it could be used with older or younger students who are studying pre-geometry or geometry.



SECTION IV

MATHEMATICS INSTRUCTIONAL UNIT AND SAMPLE CLASSROOM ASSESSMENTS

Unit on Geometry and House Construction

The mathematics unit that follows relates the study of geometry with house construction. The unit is provided as an example of how classroom instruction and assessment, linked with an emphasis on thinking skills, can help students practice skills and understandings they will need in the classroom and in their lives beyond the classroom. It is these very skills and understandings that are represented in the Mathematics Tests in EXPLORE, PLAN, and the ACT Assessment. It is our belief that as students engage in such classroom activities and assessments, they will be using the types of skills and strategies that they will also need to use in more formal testing situations.

The unit asks students to gather information from various sources that describe the techniques, tools, or mathematical concepts involved in the construction of a house. Students would also learn about the detailed planning involved in constructing a new home, analyzing many aspects of the building process. They would be involved in applying geometry to the world beyond the classroom, using mathematical reasoning to explain conclusions and generalizations, and communicating about mathematics orally and in writing. The unit attempts to engage students in a variety of thought-provoking activities, including small- and large-group discussions, reflective writing, presentations of logical reasoning, and independent and collaborative activities. By helping your students, through daily classroom instruction and assessment, to engage in strategies for thinking, you will be helping them to succeed in many types of situations.

The unit that follows was developed to illustrate various types of activities and assessment tasks that encourage the development of thinking skills. Included in this section, you will find a description of the unit and its development; a school vision statement; course goals and benchmarks for a geometry class; seventeen classroom activities, seven of which have been expanded to include detailed classroom assessments; and a list of selected resources.

RELATIONSHIP OF UNIT TO ACT'S ASSESSMENTS

The primary purpose of including the unit in this guide is to illustrate the link between classroom-based activities and assessments and the skills and understandings measured by EXPLORE, PLAN, and the ACT Assessment. The skills and understandings forming the basis for the instructional unit are provided in a list of goals and benchmarks that can be found on pages 32–35 in this section. As you review the list, you will note that some of the benchmarks are highlighted; those that are highlighted represent the benchmarks that are assessed in full or in part by the Mathematics Tests in EXPLORE, PLAN, and the ACT Assessment.

A Vision for Student Learning

Based on conversations with faculty and staff at schools engaged in the process of curriculum renewal, we have found that faculties of successful schools have a clear vision of what they believe students will need to know and be able to do, both now and in the future. At best, a school's vision for student learning is the result of a staff's careful consideration of the many ways in which students learn and teachers teach. Successful schools have often involved parents, students, and the community in educational planning. A clearly articulated vision of a school's educational goals provides a common language or framework for everyone to work from. It can also prompt teachers to reflect on how their course goals fit into and work toward the common vision. With this in mind, we wrote a vision statement as the first step in developing the unit; next, course goals and corresponding benchmarks for a geometry course were developed. The vision statement, goals, and benchmarks have served as the foundation for the activities and assessments in the instructional unit. A sample vision statement for a school appears with the goals and corresponding descriptive benchmarks for a geometry course on pages 32–35 of this guide.

Purpose of the Unit

We chose house construction as the topic for this unit for several reasons. First, many students may be involved at some point in their lives in the remodeling or construction of either a home, a business, or a building in their community. Second, since mathematics is both practical and theoretical in nature, this unit attempts to make the theoretical practical by involving students in the uses of geometry in the real world.

The unit engages students in an in-depth look into the field of home construction, addressing both exterior and interior house construction issues. The intent of the unit is to encourage students to develop an understanding of geometry as they learn how its properties and concepts can be directly applied in various contexts. Some of the unit activities prompt students to develop and build upon their understanding of geometric figures and to encourage a greater understanding of measuring and measurement systems. Other activities help students to understand the connections and relationships among mathematical topics and to develop mathematical strategies in order to solve problems and make decisions in the classroom and beyond. The instructional unit also attempts to foster in students positive attitudes, perceptions, and dispositions about learning. Finally, the unit provides students with opportunities to transfer their skills and understandings beyond the classroom—to other disciplines and to other "real-world" situations.

Some activities encourage a greater understanding of measuring and measurement systems.

As you will see, this is not intended to be a ready-to-use unit. Its main purpose is to illustrate how an instructional unit can incorporate assessment. To actually use this unit in the classroom would require a considerable amount of preparation of materials for students. (For those teachers who wish to explore these directions, a list of resources is provided on pages 103–104). For the purposes of this document, we have tried to paint a reasonably clear picture of how the unit could work in the classroom, leaving room for you, the reader, to use your imagination and envision how it might work with your students.

The unit, as written, would need to be supplemented by a variety of textbook materials and perhaps by other resources such as computer software and the like. Some of the instructional activities may not be seen as directly developing geometry skills, but they are included because they provide a context for the unit, connecting geometry to the real world. Basic geometry skills not addressed in this unit would, of course, be taught through other units.

CONTENT OF THE UNIT

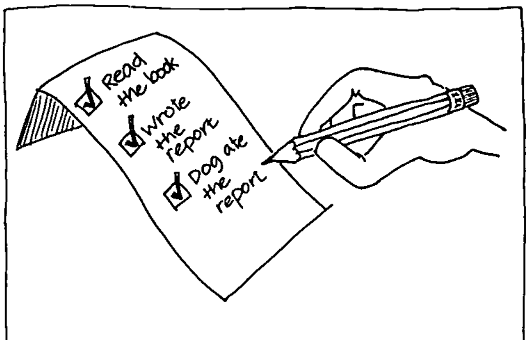
The content of the unit is based primarily on skills and understandings in the areas of geometry and measurement. However, the unit also focuses on using number theory concepts, exploring mathematical relationships, communicating mathematical ideas and concepts, developing thinking skills, solving mathematical problems, and developing skills for lifelong learning. These skills and understandings, which were drawn from an extensive review of standards documents, state curriculum guides, and other resources, such as the *Systematic Identification and Articulation of Content Standards and Benchmarks* and *The ASCD Curriculum Handbook*, were written as end-of-course benchmarks for a geometry or pre-geometry course. A large portion of these classroom goals and benchmarks reflect the same skills and understandings as those measured by EXPLORE, PLAN, and the ACT Assessment. As stated previously, the goals and benchmarks can be found on pages 32–35; they appear again in chart form on pages 38–41 to show the correspondence between goals and benchmarks of the course and activities and assessments of the unit.

Based on such factors as your interests or those of your students, the available resources, and the length of the class period, a unit such as this could last anywhere from several intensive weeks to a semester. While this unit is intended to be taught as part of a geometry course, it could also be related to various art or tech-prep classes, although the selection of benchmarks would change according to the context in which the unit would be taught. Also, we hoped this topic might inspire students to investigate other mathematical ideas or concepts and to make connections with various other disciplines.

Since families have an important role in education, this unit affords opportunities for family members and friends to become involved in various activities or to serve as consultants to the class.

EMBEDDED ASSESSMENTS

Many of the activities created for the unit contain ideas for possible embedded assessments. As stated previously, we are using the term "embedded assessment" to describe those assessments that are primarily teacher developed and that are an integral part of the curriculum. Such assessments are designed to improve student learning and to help teachers determine students' instructional needs. The unit suggests various ways in which you might document or record students' learning during an activity. Forms for recording or collecting information include such options as classroom observations, checklists, and questionnaires. Some assessments also include a rating scale, a scoring rubric, or an answer key for evaluating student learning.



Checklists are one way to document students' learning.

Please note that the activities in this unit are not necessarily tied to any one form of assessment; therefore, any of the embedded assessments suggested in this unit could be modified or altered to meet the needs of your students. Since this unit was designed as a prototype, only seven of the unit's seventeen activities have been developed in greater detail, providing actual samples of embedded assessment tasks and scoring criteria. The seventeen activities, including the seven which have related embedded assessments, can be found on pages 42-101. As you review the activities and embedded assessments, you will find that the goals and benchmarks covered by each activity/assessment are excerpted from the longer list, which appears on pages 32-35.

The criteria for an embedded assessment (that is, the specific traits, attitudes, and knowledge that define the skills and understandings expressed in the broad goals and benchmarks for the course) can be generated by the teacher and/or the students. More and more, teachers seem to be making sure that students are aware of the purposes of the activities and of the skills and understandings that will be assessed; the embedded assessments included here are based on the assumption that students will know exactly what they are being evaluated on.

The embedded assessments measure most, though not necessarily all, of the benchmarks listed for an activity because an activity may encourage the development of more skills and understandings than can be assessed at one time. The sample embedded assessments presented here were designed to be used at different points during the unit, thus providing multiple assessments and sources of evidence. For example, one of the assessments is a checklist for a math log/journal that could be used with many of the unit's activities. Another assessment, linked to the activity of discussing the connections between house construction and geometry, is a chart that could be used at the beginning of a unit and throughout the unit. Still another sample assessment is linked to the activity of creating a three-dimensional scale model of a house, a project that would occur near the end of the unit and that would illustrate students' understanding of various mathematical ideas and concepts. The students' projects could be assessed using a scoring rubric.

VISION STATEMENT

To encourage every student to develop independence as a lifelong learner, to acquire and integrate essential knowledge and skills, to assume productive roles in the community and the interdependent world, to make effective decisions based on understandings of themselves and the world around them, to communicate effectively in diverse situations, to think creatively, logically, and critically in the classroom and beyond, and to affirm the worth and dignity of self and others.

GOALS AND BENCHMARKS

Course: Geometry

(Note: Highlighting denotes those benchmarks that are assessed in full or in part by EXPLORE, PLAN, and the ACT Assessment.)

Goal: **To develop an understanding of two- and three-dimensional geometric figures**

a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures
b. Investigates, deduces, and explains properties of and relationships between geometric figures
c. Classifies figures in terms of properties, including congruence and similarity, and applies these relationships in solving problems
d. Understands transformations and uses them as a tool to solve problems
e. Develops and applies formulas for perimeters, circumferences, areas, and volumes of geometric figures
f. Understands and uses properties of lines, angles, triangles, quadrilaterals, and circles in solving problems
g. Understands perspectives and cross-sections of three-dimensional figures

Goal: **To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments**

a. Understands why measurements are used, ways in which measurements arose, and ways in which measurements are standardized
b. Understands measurement, using appropriate instruments to accurately determine angle, length, weight, mass, time, etc.
c. Uses units of measure when expressing quantities and chooses appropriate units of measure for the situation
d. Understands limitations and precision of measurement instruments
e. Understands differences and relations between length, area, and volume

f. Converts from one unit of measurement to another within the same system or between systems
g. Makes and reads scale drawings

Goal: To examine numbers in the context of broader mathematical systems

a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers
b. Understands how to use calculators and computer software, recognizing their limitations
c. Selects appropriate computation methods, for example using mental arithmetic, paper and pencil, calculators, and/or computers
d. Represents numbers and operations in a variety of equivalent forms, using models, diagrams, and symbols
e. Understands and applies percents, ratios, and proportions in a wide variety of situations
f. Applies number theory concepts (primes, divisibility, number patterns, etc.) in order to solve problems
g. Understands the purposes for formulas and how formulas work

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

a. Explores relationships among various mathematical concepts or topics
b. Models real-world objects with geometric figures, and models complex geometric figures with simpler geometric figures
c. Recognizes that some geometric figures have concise algebraic representations and vice versa, and uses these alternate representations to solve problems
d. Understands that any mathematical model, graphic or algebraic, is limited in how well it can represent how the world works
e. Uses mathematical patterns, including numeric and geometric patterns, to solve problems
f. Explores relationships between real-world problems and accepted mathematical models
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields
h. Describes how seemingly different mathematical systems may be essentially the same
i. Understands the nature of axiomatic systems, the reasons for developing such systems, and the role of proof; realizes that results proven for one system do not necessarily hold in another system

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking
d. Understands the role of definitions and notation in mathematical thinking and communication
e. Recognizes the importance of providing and communicating both complete and accurate mathematical information

Goal: To apply thinking skills to develop greater understanding across the curriculum

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations
b. Recognizes correct logical arguments as well as fallacies in and assumptions underlying an argument
c. Makes inferences and predictions based on logic and probability
d. Recognizes when mathematical information is incomplete, redundant, or extraneous

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

a. Identifies and analyzes problems
b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem
e. Selects and applies appropriate tools and technology to solve problems
f. Determines if the solution of a mathematical or real-world problem is reasonable
g. Looks back for patterns that can be useful for solving other problems
h. Knows that if one strategy does not work another strategy might

Goal: To develop and utilize the skills and attitudes of a lifelong learner

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
b. Understands the value of continual mathematics learning throughout life

c. Gathers and uses mathematical information from various resources
d. Understands that mathematics can be (and is) built on mathematical investigation or inquiry done by others
e. Becomes aware of the importance of mathematical discoveries and mathematicians across cultures and throughout history
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning
g. Reads about mathematics with understanding
h. Understands the role of justification in the mathematical learning process
i. Accepts new knowledge critically, through analysis, reflection, and inquiry

Activities and Assessments for the Unit

On pages 42–101 are descriptions of seventeen activities that could be included in a unit on the uses of geometry in the construction of a house; seven of the activities have been expanded to include embedded classroom assessments. The activities were developed to involve students in reading building plans, constructing scale drawings and scale models, and observing the ways that geometry is important to house construction. A related goal was to engage students in measurement activities. Students will also be involved in learning to work in small and large groups as well as independently to investigate, discuss, and plan for "building" a house. In addition, they will make plans for finishing the interior of the house.

Involve students in reading blueprints.

Please note that the activities and assessments developed for this unit are only suggestions. You are the best judge of what is necessary and relevant for your students. Therefore, you are encouraged to examine the list of activities, using those that apply, modifying others as needed, and deleting those that are not appropriate for your students. Although presented in a particular order, the sequence of activities could change. It is assumed that the unit would be used near the beginning of a geometry course and that during the time students are engaged in the activities described here, they are likely to also be engaged in math activities from their geometry textbooks.

Following is a list of activities developed for the unit. An asterisk (*) denotes that embedded assessments have been developed to accompany the activity. Related goals and benchmarks on which each activity is based can be found on the chart on pages 38–41.

Activity A:	Discussing House Construction and Geometry*
Activity B:	Keeping a Math Log/Journal*
Activity C:	Visualizing the Construction Process
Activity D:	Reading About and Discussing Building Plans
Activity E:	Selecting a House Plan
Activity F:	Investigating Geometric Constructions with Drafting Tools
Activity G:	Creating a Scale Drawing of a Room*
Activity H:	Scheduling Work on a House
Activity I:	Exploring Building Methods and Tools
Activity J:	Selecting Materials for a House*
Activity K:	Uses of Perimeter, Area, and Volume in House Construction*
Activity L:	Determining Costs of Materials
Activity M:	Creating Illusions in a House
Activity N:	Exploring Patterns and Transformations
Activity O:	Exploring Symmetry in Wall Coverings*
Activity P:	Constructing a Three-Dimensional Scale Model*
Activity Q:	Solving Various House Construction Problems

Table 3. Correspondence Among the Goals/Benchmarks and the Activities/Assessments

✓ indicates the goals and benchmarks addressed by the activity

* indicates the goals and benchmarks measured by the assessment

Course Goals and Corresponding Benchmarks	A Discussing Construction	B Keeping a Log/Journal	C Visualizing Construction	D Investigating Bldg Plans	E Selecting a House Plan	F Geometric Constructions	G Creating Scale Drawings	H Scheduling Work	I Exploring Bldg Methods	J Selecting Materials	K Perimeter, Area, and Volume	L Determining Costs	M Creating Illusions	N Exploring Patterns	O Exploring Symmetry	P Constructing Scale Models	Q Solving Problems
Goal: To develop an understanding of two- and three-dimensional geometric figures																	
a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures						✓	✓*		✓				✓	✓	✓*	✓*	
b. Investigates, deduces, and explains properties of and relationships between geometric figures	✓			✓	✓	✓			✓				✓	✓	✓		
c. Classifies figures in terms of properties, including congruence and similarity, and applies these relationships in solving problems				✓										✓	✓*	✓	
d. Understands transformations and uses them as a tool to solve problems														✓	✓*		
e. Develops and applies formulas for perimeters, circumferences, areas, and volumes of geometric figures				✓							✓					✓	
f. Understands and uses properties of lines, angles, triangles, quadrilaterals, and circles in solving problems				✓		✓			✓					✓		✓	
g. Understands perspectives and cross-sections of three-dimensional figures					✓		✓										
Goal: To understand the structure and uses of measurements systems and to measure accurately using appropriate instruments																	
a. Understands why measurements are used, ways in which measurements arose, and ways in which measurements are standardized						✓					✓						
b. Understands measurement, using appropriate instruments to accurately determine angle, length, weight, mass, time, etc.						✓		✓	✓						✓	✓	
c. Uses units of measure when expressing quantities and chooses appropriate units of measure for the situation						✓	✓*	✓	✓						✓*	✓*	
d. Understands limitations and precision of measurement instruments						✓	✓		✓					✓	✓*		
e. Understands differences and relations between length, area, and volume						✓					✓						
f. Converts from one unit of measurement to another within the same system or between systems						✓	✓	✓			✓						

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
g. Makes and reads scale drawings				✓	✓	✓	✓*	✓			✓					✓*	
Goal: To examine numbers in the context of broader mathematical systems																	
a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers					✓		✓*	✓	✓		✓	✓				✓	
b. Understands how to use calculators and computer software, recognizing their limitations												✓					
c. Selects appropriate computation methods, for example using mental arithmetic, paper and pencil, calculators, and/or computers					✓				✓		✓	✓					
d. Represents numbers and operations in a variety of equivalent forms, using models, diagrams, and symbols							✓*					✓					
e. Understands and applies percents, ratios, and proportions in a wide variety of situations				✓		✓	✓*		✓		✓	✓				✓*	
f. Applies number theory concepts (primes, divisibility, number patterns, etc.) in order to solve problems																	
g. Understands the purposes for formulas and how formulas work						✓	✓				✓					✓	
Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world																	
a. Explores relationships among various mathematical concepts or topics	✓*	✓							✓								
b. Models real-world objects with geometric figures, and models complex geometric figures with simpler geometric figures									✓							✓*	
c. Recognizes that some geometric figures have concise algebraic representations and vice versa, and uses these alternate representations to solve problems																	
d. Understands that any mathematical model, graphic or algebraic, is limited in how well it can represent how the world works									✓								
e. Uses mathematical patterns, including numeric and geometric patterns, to solve problems														✓	✓		
f. Explores relationships between real-world problems and accepted mathematical models	✓*			✓	✓	✓	✓										✓
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields	✓*	✓*		✓	✓				✓	✓	✓	✓	✓	✓	✓	✓	✓
h. Describes how seemingly different mathematical systems may be essentially the same																	
i. Understands the nature of axiomatic systems, the reasons for developing such systems, and the role of proof; realizes that results proven for one system do not necessarily hold in another system															✓		

✓ indicates the goals and benchmarks addressed by the activity

* indicates the goals and benchmarks measured by the assessment

Course Goals and Corresponding Benchmarks	A Discussing Construction	B Keeping a Log/Journal	C Visualizing Construction	D Investigating Bldg Plans	E Selecting a House Plan	F Geometric Constructions	G Creating Scale Drawings	H Scheduling Work	I Exploring Bldg Methods	J Selecting Materials	K Perimeter, Area, and Volume	L Determining Costs	M Creating Illusions	N Exploring Patterns	O Exploring Symmetry	P Constructing Scale Models	Q Solving Problems
Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation																	
a. Participates in group discussions involving mathematics, synthesizing the ideas presented	✓*		✓	✓	✓			✓	✓	✓*		✓	✓			✓*	✓
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing	✓*	✓*	✓	✓	✓		✓*	✓	✓	✓*		✓			✓*	✓*	
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking		✓*		✓			✓*		✓	✓					✓	✓*	✓
d. Understands the role of definitions and notation in mathematical thinking and communication		✓*					✓										
e. Recognizes the importance of providing and communicating both complete and accurate mathematical information	✓	✓*	✓				✓										
Goal: To apply thinking skills to develop greater understanding across the curriculum																	
a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations	✓*	✓*		✓	✓			✓	✓	✓		✓			✓*	✓*	✓
b. Recognizes correct logical arguments as well as fallacies in and assumptions underlying an argument		✓*			✓				✓	✓							
c. Makes inferences and predictions based on logic and probability	✓	✓*	✓		✓			✓		✓		✓					
d. Recognizes when mathematical information is incomplete, redundant, or extraneous			✓					✓									
Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond																	
a. Identifies and analyzes problems		✓*	✓		✓			✓	✓	✓*							✓
b. Generates and accepts divergent or alternative approaches and solutions to a problem		✓	✓		✓			✓	✓	✓*		✓				✓	✓
c. Chooses from among various solutions strategies the one that seems best		✓*	✓		✓			✓	✓	✓*	✓	✓				✓*	✓

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
d. Formulates a plan to solve a problem		✓	✓					✓	✓	✓*	✓	✓				✓*	✓
e. Selects and applies appropriate tools and technology to solve problems		✓*				✓				✓*				✓			✓
f. Determines if the solution of a mathematical or real-world problem is reasonable		✓			✓			✓		✓	✓	✓					✓
g. Looks back for patterns that can be useful for solving other problems		✓								✓*							✓
h. Knows that if one strategy does not work another strategy might		✓							✓	✓*		✓					✓
Goal: To develop and utilize the skills and attitudes of a lifelong learner																	
a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings	✓	✓*	✓	✓	✓			✓		✓*		✓			✓	✓	✓
b. Understands the value of continual mathematics learning throughout life	✓	✓		✓				✓			✓	✓		✓	✓		
c. Gathers and uses mathematical information from various resources		✓	✓		✓				✓	✓							✓
d. Understands that mathematics can be (and is) built on mathematical investigation or inquiry done by others																	
e. Becomes aware of the importance of mathematical discoveries and mathematicians across cultures and throughout history																	
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning	✓*		✓		✓			✓	✓		✓	✓					✓
g. Reads about mathematics with understanding		✓*		✓						✓							
h. Understands the role of justification in the mathematical learning process																✓*	
i. Accepts new knowledge critically, through analysis, reflection, and inquiry		✓*	✓									✓					

50

49

ACTIVITY A: DISCUSSING HOUSE CONSTRUCTION AND GEOMETRY*

Purpose of Activity/Assessment

- To engage students in conversations about constructing a building (house, apartment building, etc.)
- To encourage students to reflect on their knowledge by discovering connections between topics and by formulating questions that will facilitate further exploration
- To develop students' awareness of mathematical concepts or terminology
- To encourage students to develop appropriate group participation skills

Description of Activity

The teacher might begin the unit by asking the students to share their thoughts and ideas about house construction and geometry, establishing a starting place for both students and teacher. The teacher could begin the discussion by saying the following: "What do you know about house construction? What do you know about geometry? Explain in more detail the idea(s) you have mentioned. How do you think geometry is used in building? How could we find out if we are correct in our assumptions about geometry and house construction?" The teacher could guide and encourage the students to reflect on their knowledge, developing their awareness of the connections between the two topics. For instance, students might make such comments as "You use geometry in getting measurements," or "You need a math education to do construction." In addition, the teacher or several student volunteers could record the ideas suggested, creating a diagram to graphically illustrate relationships or connections between house construction and geometry.

House Construction →	Relationships	← Geometry
• build things • use certain nails • use tools (hammers, wrenches) • takes a long time • get cement, build the foundation • put up walls • put on sides of the house and the roof • put on drywall • put on floors • put in counters and other stuff like that • do the wiring • put in carpet • paint walls and put in doors	• you use geometry in getting measurements • you make scale drawings for the architecture • you use geometry to place windows, doors • you need a math education to do construction	• it has to do with shapes • it is hard • you make things out of shapes • has to do with angles, lines • you do graphs

(Note: The examples given are based on the assumption that students are beginning the instructional unit in the fall semester.)

How Student Learning Could Be Assessed

Teacher Notes — Throughout the unit, students will be using a graphic organizer called a K-W-L chart, which can be found on page 45. After the class discussion, each student could receive a K-W-L chart and be asked to fill in the first column of the chart—"What I *know* (K) about house construction and/or geometry." After completing this task, pairs of students might brainstorm about what they hope to discover or learn during the unit. Each student would then fill in the second column of the chart—"What I *want* (W) to find out about house construction and geometry"—either during or after the brainstorming session. The teacher should look over the charts at this point to see what students know and want to learn. The charts will then be returned to the students so that they can add to the chart on an ongoing basis. The teacher will collect the charts at various intervals throughout the unit in order to see and make note of the skills or understandings students are developing. The students can fill in the final column—"What I have *learned* (L) about house construction and geometry"—whenever they believe they have found a satisfactory answer to a question. The chart has the potential to illustrate what students have learned from various sources as well as whether they have found answers to all of their questions. Students would return to the chart at the end of the unit to discuss how their learning has evolved. They might also use their completed charts as the basis for a final entry in their math log/journal (see Activity B, page 46).

Resources or Materials Needed

- Paper/chalkboard to list ideas
- One or more K-W-L charts for each student (see Ogle, 1986)

Optional Activity

A surveyor could be invited to talk to the class about the role of surveyors in the field of construction and the mathematical principles involved in surveying a lot that a house or other building might be built on. The surveyor could describe the techniques and tools he/she uses to determine the area of a piece of land, including the lengths and directions of the area's boundaries and the contours of the land's surface. In addition, the surveyor could describe how he/she records the information gathered on a piece of land.

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

b. Investigates, deduces, and explains properties of and relationships between geometric figures

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

a. Explores relationships among various mathematical concepts or topics
f. Explores relationships between real-world problems and accepted mathematical models
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
e. Recognizes the importance of providing and communicating both complete and accurate mathematical information

Goal: To apply thinking skills to develop greater understanding across the curriculum

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations
c. Makes inferences and predictions based on logic and probability

SECTION IV Goal: To develop and utilize the skills and attitudes of a lifelong learner

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
b. Understands the value of continual mathematics learning throughout life
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning

Graphic Organizer

Name: ____

K-W-L Chart		
What I *Know* **about** ____________	**What I *Want* To Find Out** **about** ____________	**What I Have *Learned*** **about** ____________

ACTIVITY B: KEEPING A MATH LOG/JOURNAL*

Purpose of Activity/Assessment

- To develop students' understanding of mathematical concepts, principles, and terminology
- To develop students' awareness of the connections between mathematical topics, various disciplines, and other applied fields
- To encourage students to communicate their thoughts, interpretations, and/or questions through writing

Description of Activity

Students will be engaged in a variety of activities throughout the house construction unit, such as reading information on house construction, discussing mathematical concepts and ideas, and applying their mathematical knowledge through various hands-on activities. The students could keep a math log/journal in which they might respond to and reflect upon what they have read and are learning. They could be given time each day to record their ideas, which may include noting how what they are learning in their geometry class connects with their other experiences; reflecting on their understanding of and growth in geometry; relating their thoughts about mathematics in general; responding to written prompts or open-ended questions posed by the teacher; formulating logical arguments by using sound mathematical reasoning; etc. The students might also indicate their successes and/or need for help, and make comments or pose questions they have. The logs/journals may also include problem solving ideas or solutions, or graphic representations of information. At the conclusion of the unit, the students could create a final entry in their log/journal that reflects what they have learned about themselves, geometry, and house construction. The teacher will review the students' logs/journals periodically to note their thinking processes and to provide feedback to the students about their successes, questions, or concerns.

A math log.

How Student Learning Could Be Assessed

Math Log Checklist — A checklist could be used to evaluate students' entries in the math log/journal. The criteria for the checklist could be generated by the teacher and/or students. The checklist may be developed in various ways: it may cover a wide range of skills and understandings, it may cover a specific set of skills and understandings, or it may change as the unit unfolds to reflect new skills and understandings being learned. The checklist on page 49, for example, assesses such broad skills as mathematical understandings, the use of mathematical language, and mathematical reasoning or argumentation. It also assesses the students' abilities to make inferences and generalizations, to ask thoughtful questions, and to reflect on their own thinking processes. The math log/journal could continue throughout the entire unit.

Teachers could use the checklist in several ways. For example, teachers could be sure students understand that the checklist is to be used during the entire unit and that all the criteria on the checklist should be illustrated *somewhere* in the journal but certainly all would not be used in each entry. Another approach might be for a teacher to specify certain checklist criteria to be used in certain entries, or for a teacher to ask students to choose one or two criteria from the checklist to focus on as they write each journal entry. The checklist could be used either by the teacher or students to evaluate entries in the math logs/journals.

Resources or Materials Needed

- Either a spiral-bound notebook, or looseleaf paper and a small 3-ring binder, for each student

Related Goals and Benchmarks

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

a. Explores relationships among various mathematical concepts or topics
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking
d. Understands the role of definitions and notation in mathematical thinking and communication
e. Recognizes the importance of providing and communicating both complete and accurate mathematical information

Goal: To apply thinking skills to develop greater understanding across the curriculum

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations
b. Recognizes correct logical arguments as well as fallacies in and assumptions underlying an argument
c. Makes inferences and predictions based on logic and probability

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

a. Identifies and analyzes problems
b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem
e. Selects and applies appropriate tools and technology to solve problems
f. Determines if the solution of a mathematical or real-world problem is reasonable
g. Looks back for patterns that can be useful for solving other problems
h. Knows that if one strategy does not work another strategy might

SECTION IV **Goal: To develop and utilize the skills and attitudes of a lifelong learner**

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
b. Understands the value of continual mathematics learning throughout life
c. Gathers and uses mathematical information from various resources
g. Reads about mathematics with understanding
i. Accepts new knowledge critically, through analysis, reflection, and inquiry

Math Log Checklist

(The blanks at the end are for additional criteria generated by the teacher and/or students.)

Name: ______________________________

Criteria	**Yes/No** (include dates that log/journal is reviewed)	**Comments**
Makes connections between mathematics and other disciplines or applied fields		
Generally uses precise mathematical language and notation		
Explains own thinking; may use mathematical models, drawings, facts, etc.		
Provides both complete and accurate mathematical information		
Uses logical reasoning to make conjectures, to construct arguments, and/or to validate and prove conclusions		
Comments on arguments, recognizing fallacies or underlying assumptions		
Makes inferences and/or generalizations based on logic and probability		
Analyzes problems, selecting various solution strategies and tools		
Poses thoughtful questions		
Reflects on and assesses own thinking and learning		
Writes about mathematical materials read with understanding		
Analyzes new information gained		

ACTIVITY C: Visualizing the Construction Process

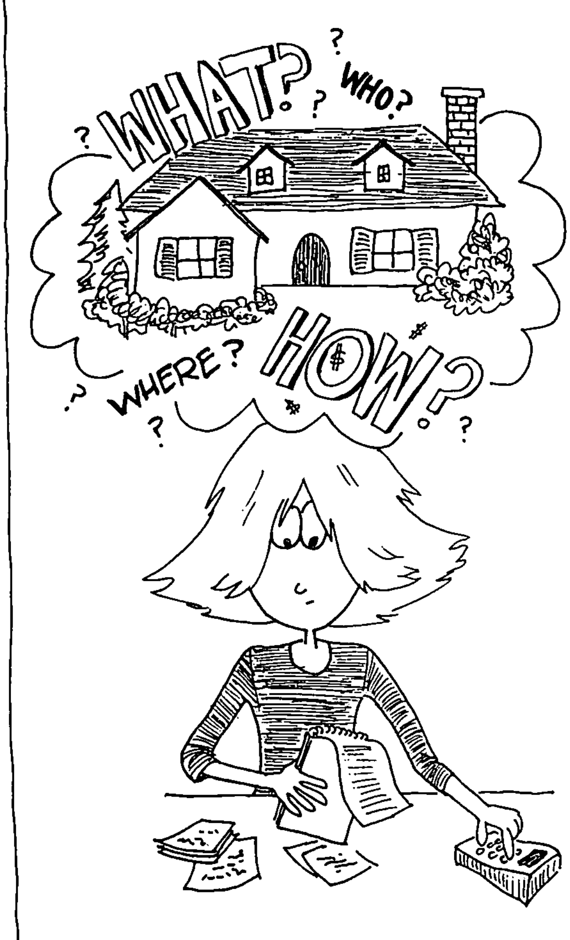
Purpose of Activity/Assessment

- To help students to think carefully about and sequence the various steps in the construction process
- To help students obtain information from other resources and incorporate this information into their predictions
- To help students negotiate a class list

Description of Activity

Students could volunteer to provide an oral synopsis of the discussion held previously about house construction and geometry, with the teacher providing additional information or clarification as needed. Afterward, students could be encouraged to think about and try to visualize all the steps necessary to the building process. In small groups, the students could generate a list and predict the order in which each step would be completed, using logical reasoning to support their decisions and assumptions. The students might generate a list such as the following:

1. Find a lot
2. Select a house plan
3. Dig a hole
4. Pour concrete
5. Build the walls and floors
6. Put on the roof
7. Do the inside stuff (plumbing/wiring)
8. Finish outside
9. Finish inside
10. Paint/decorate
11. Landscape



Students could try to visualize the steps in the building process.

After the students generate their small-group lists, information about the construction process could be given to students, perhaps through videos, slides, or photographs. In addition, the teacher could invite a contractor to visit the class. The class could prepare questions for the contractor, such as 1) What are your responsibilities on the job? 2) What tools do you use? 3) What mathematical knowledge do you utilize when building a home, apartment, or other building? 4) What kinds of costs does a contractor bear? 5) In what ways could people save time or money by doing some of the work themselves? 6) In your opinion, what are the pros and cons of being one's own contractor? 7) How much time do you spend building a house? 8) What happens if you don't finish the house on time? 9) What is the most difficult step in building a house?

As a class, students could create a class list from their original lists and from the additional information they have received, using a chalkboard, an overhead, or a projected computer screen. Students are almost certain to discover that their small-group lists are organized in different ways and have different amounts of detail, and that the extra information the class received conflicts with their original lists. For example, one source might suggest choosing the house plan first, then finding a lot for it. But, it is equally valid for students to reason that they should choose the lot first and then find a house plan to fit the lot. Students may even discover that some of the extra information is in conflict. The class will need to negotiate how the steps should be described, and perhaps it would even be necessary to negotiate an acceptable process for trying to agree on the steps. This is an important place for the teacher to facilitate the process, knowing when to let students work out their own solutions and when to provide suggestions, support, and feedback.

How Student Learning Could Be Assessed

Anecdotal Notes — The teacher might want to informally note students who are participating in the small-group discussions, communicating their ideas, applying thinking skills, etc.

Resources or Materials Needed

- Chalkboard, overhead projector, or projected computer
- Videos, slides, or photographs about home construction
- Possibly a timetable for house construction for the teacher to use as a reference or for students to compare with their list at the end of the activity

Related Goals and Benchmarks

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
e. Recognizes the importance of providing and communicating both complete and accurate mathematical information

Goal: To apply thinking skills to develop greater understanding across the curriculum

c. Makes inferences and predictions based on logic and probability
d. Recognizes when mathematical information is incomplete, redundant, or extraneous

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

a. Identifies and analyzes problems
b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem

Goal: To develop and utilize the skills and attitudes of a lifelong learner

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
c. Gathers and uses mathematical information from various resources
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning
i. Accepts new knowledge critically, through analysis, reflection, and inquiry

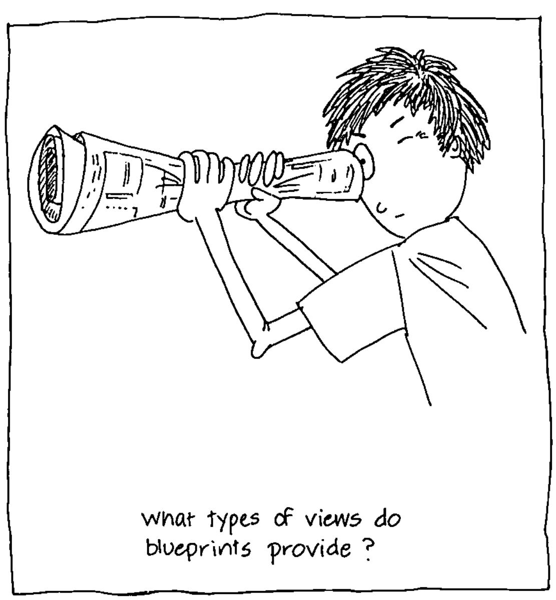
ACTIVITY D: READING ABOUT AND DISCUSSING BUILDING PLANS

Purpose of Activity/Assessment

- To encourage students to explore geometric figures and their properties
- To encourage students to integrate new information with their prior knowledge
- To develop students' understanding of mathematical terminology
- To enhance students' awareness of various measuring tools
- To facilitate students' exploration of the techniques of perspective

Description of Activity

Throughout the unit, the teacher will be providing the students with written information about the building process. First, students will be reading information about the initial step in the construction process—the planning stage. The teacher could introduce or review mathematical terminology pertinent to the building process like congruence, similarity, proportion, scale, etc., as well as providing some concrete examples of each. In order to help guide the students' reading, the teacher might ask them to think about the following questions while reading: 1) Who do you think uses building plans?, 2) What types of information do building plans provide?, 3) What types of views do building plans provide?, 4) Why are these different views needed?, 5) In what ways are building plans useful?, 6) How might the building plans reflect congruence?, similarity?, proportion?, scale?, and 7) What tools or technology can be used to create building plans? Time could be given during the class to read the articles, followed by a discussion of the information in either large or small groups.



How Student Learning Could Be Assessed

Math Log Checklist — Students could use their math log/journals to comment on what they have learned through their reading and from thinking about the questions posed by the teacher. The teacher could assess each student's log/journal entry using the checklist developed for this unit, which can be found on page 49.

Resources or Materials Needed

- Information about the building process
- Sample building plans

Optional Activity

The teacher could invite an architect or draftsperson to join the class to describe his/her responsibilities on the job, the tools used, and the mathematical knowledge utilized when designing a home, apartment, or other building.

Related Goals and Benchmarks

Goal: **To develop an understanding of two- and three-dimensional geometric figures**

b. Investigates, deduces, and explains properties of and relationships between geometric figures
c. Classifies figures in terms of properties (including congruence and similarity), and applies these relationships in solving problems
e. Develops and applies formulas for perimeters, circumferences, areas, and volumes of geometric figures
f. Understands and uses properties of lines, angles, triangles, quadrilaterals, and circles in solving problems

Goal: **To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments**

g. Makes and reads scale drawings

Goal: **To examine numbers in the context of broader mathematical systems**

e. Understands and applies percents, ratios, and proportions in a wide variety of situations

Goal: **To understand the connections and relationships among various mathematical topics and their applications in the real world**

f. Explores relationships between real-world problems and accepted mathematical models
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: **To effectively communicate mathematical ideas through writing, speaking, and visual representation**

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking

Goal: **To apply thinking skills to develop greater understanding across the curriculum**

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations

Goal: **To develop and utilize the skills and attitudes of a lifelong learner**

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
b. Understands the value of continual mathematics learning throughout life
g. Reads about mathematics with understanding

ACTIVITY E: SELECTING A HOUSE PLAN

Purpose of Activity/Assessment

- To guide students' exploration of geometric figures and their properties
- To develop students' understanding of how to make conjectures
- To encourage students' use of sound mathematical reasoning to justify a choice
- To encourage students to participate in small-group discussions

Description of Activity

In order to encourage students' understanding of the first step of the building process—the planning stage—students could be given several house designs and descriptions of several building sites to choose from. Each house design should include the following information: the estimated construction cost, dimensions of the house, etc. The building site descriptions give students a context for their decision, providing, for example, dimensions and location, contours, vegetation, and aesthetics of views in different directions. At the same time, the imaginary owner's personal and financial situation (physical and personal needs of the family, income, etc.) would be described to the students. Not all of the house designs would be appropriate for all building sites, and some of the sites might not be suitable for house construction.

Depending on the needs of the students, the teacher might facilitate a discussion of geometric terms and relations in the context of the house designs and site descriptions. The discussion might include such things as students finding examples to illustrate the concept of point, line, and plane as well as parallel lines, obtuse and acute angles, and right triangles. Building sites may come in different geometric shapes, and the teacher could ask students which geometric shapes would make good sites for a house and why. Students could be led to visualize that the intersection of two planes is a line, as evidenced from the roof peaks and valleys. Also, students might be asked, among other things, why various right angles on the house do not look like right angles on the picture. The teacher and students may discover many other connections between geometry and house construction while examining the building plans.

Students could then move into small groups to study the information and generate a list of all of the reasonable combinations for a house design and building site. During the group discussions, the teacher would guide students in making conjectures using the information provided. For example, students might conjecture that "If the lot has a steep slope, house design X would not be appropriate." Groups would also be asked to discuss the pros and cons of each of the listed combinations.

The groups' lists would then be merged into a class list illustrating the reasonable combinations. Any disagreements among the groups could be resolved by letting the groups involved explain why a combination was or was not on their list. If the class agreed that a combination was not reasonable, it should be removed from the class list. If there was disagreement, the combination should stay on the class list.

After the class list is completed, all students should be able to nominate from among the combinations. When nominating a combination, students should give one or more reasons why the combination is best for the imaginary buyer. Other students may second the nomination with their reasons.

Then, students could cast their vote for the combination they think best. Results will be tabulated and the winning combination announced.

How Student Learning Could Be Assessed

Anecdotal Notes — The teacher could note students using logical reasoning based on the information provided about the imaginary buyer, the house designs, and the building site descriptions.

Math Log Checklist — Students could be asked to record some of their conjectures in their math log, which would be evaluated along with the other entries according to the checklist on page 49.

Resources or Materials Needed

- Descriptions of the building sites and of the potential homeowner
- Multiple copies of several different house plans. (Note: The house plans might be furnished by an architect or contractor who is working with the class, or they might be found in various books in the library or for sale at hardware stores or lumber companies. See resources on page 104.)

Optional Activity

Arrangements might be made in order to give the geometry students first-hand experience with a house that is currently under construction. A local contractor, perhaps a parent or friend, might consult with the students. Some students might engage in a building project themselves through a tech-prep class.

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

- b. Investigates, deduces, and explains properties of and relationships between geometric figures
- g. Understands perspectives and cross-sections of three-dimensional figures

Goal: To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments

- g. Makes and reads scale drawings

Goal: To examine numbers in the context of broader mathematical systems

- a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers
- c. Selects appropriate computation methods, for example using mental arithmetic, paper and pencil, calculators, and/or computers

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

- f. Explores relationships between real-world problems and accepted mathematical models
- g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

- a. Participates in group discussions involving mathematics, synthesizing the ideas presented
- b. Uses mathematical language and diagrams to effectively express ideas orally and in writing

SECTION IV

Goal: To apply thinking skills to develop greater understanding across the curriculum

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations
b. Recognizes correct logical arguments as well as fallacies in and assumptions underlying an argument
c. Makes inferences and predictions based on logic and probability

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

a. Identifies and analyzes problems
b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
f. Determines if the solution of a mathematical or real-world problem is reasonable

Goal: To develop and utilize the skills and attitudes of a lifelong learner

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
c. Gathers and uses mathematical information from various resources
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning

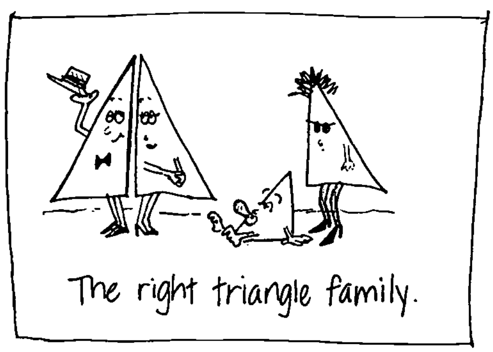
ACTIVITY F: INVESTIGATING GEOMETRIC CONSTRUCTIONS WITH DRAFTING TOOLS

Purpose of Activity/Assessment

- To show students how to use drafting tools in order to construct drawings
- To allow students to experiment with geometric constructions

Description of Activity

The teacher could begin this activity by introducing students to various drafting tools, such as T-squares, triangles, circle guides, protractors, compasses, and rulers. Then, students could be asked to construct various figures such as a square, a triangle, a cube, etc. Students could then move on to construct, for example, a line parallel to a given line through a given point, a line perpendicular to a given line through a given point, a circle that has two given points as the endpoints of a diameter, an angle bisector, and an enlargement or reduction of a given figure. The students could also draw diagrams to represent geometry theorems they have learned.



How Student Learning Could Be Assessed

Scoring Rubric — The teacher could select one or two of the constructions the students worked with previously, asking them to list the steps they used to make the construction. The rubric could assess the correctness of the students' construction and the clarity of the explanation, which would include a diagram illustrating the sequence of steps.

Resources or Materials Needed

- Paper and pencil
- Drafting tools

66

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures
b. Investigates, deduces, and explains properties of and relationships between geometric figures
f. Understands and uses properties of lines, angles, triangles, quadrilaterals, and circles in solving problems

Goal: To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments

a. Understands why measurements are used, ways in which measurements arose, and ways in which measurements are standardized
b. Understands measurement, using appropriate measurement instruments to accurately determine angles, length, weight, mass, time, etc.
c. Uses units of measure when expressing quantities and chooses appropriate units of measure for the situation
d. Understands limitations and precision of the tools of measurement
e. Understands differences and relations between length, area, and volume
f. Converts from one unit of measurement to another within the same system or between systems
g. Makes and reads scale drawings

Goal: To examine numbers in the context of broader mathematical systems

e. Understands and applies percents, ratios, and proportions in a wide variety of situations
g. Understands the purposes for formulas and how formulas work

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

f. Explores relationships between real-world problems and accepted mathematical models

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

e. Selects and applies appropriate tools and technology to solve problems

ACTIVITY G: CREATING A SCALE DRAWING OF A ROOM*

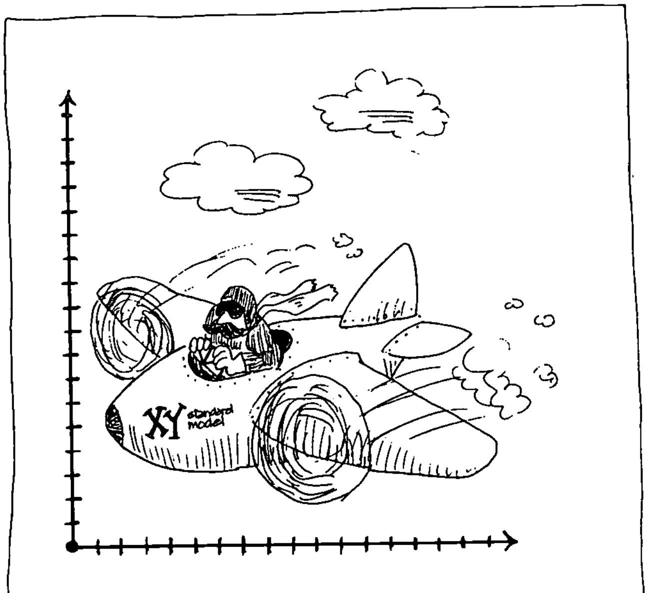
Purpose of Activity/Assessment

- To develop students' understanding of proportion
- To increase students' skill in reading, interpreting, and making scale drawings

Description of Activity

After a house plan is chosen, students could move into small groups; each small group could select a slip of paper that would designate which parts of the house plan they will be working with. For example, Group 1—living room, bathroom, and driveway; Group 2—kitchen/dining area, master bedroom, and basement; and Group 3—bedroom, bathroom, and walkways outside. All the students will learn about the external structure of the house, comparing notes periodically on decisions made regarding the interior parts of the house.

The teacher will demonstrate how to make a scale drawing of a deck or patio, on both graph paper and unlined drawing paper; during the demonstration, the teacher will discuss the concepts of proportion and scale and explain the thinking used in creating the scale drawing. The teacher will also review the concept of ratios, demonstrating how special features, like chairs and tables, should be in proportion to the scale used for the drawing. After the demonstration, the students will select a room from their respective parts of the house and create a scale drawing of it, using the dimensions given on the building plans. The scale drawing should provide a detailed floor plan of a specific room, including select furnishings and a descriptive key. Either the teacher could provide the scale for all students to use, or individual students could determine the scale for their particular rooms. Similarly, a key (symbols for the door openings, windows, closet areas, stairways, outlets, etc.) could be provided, the class could develop a key, or individual students could develop their own keys. (The teacher might provide various examples of architectural drawings in order to focus students' attention on the symbols and notations used.) The students could also submit written explanations of how they completed their drawings, including any mathematical calculations they completed.



Graphing in the (x,y) plane.

How Student Learning Could Be Assessed

Rating Scale — A qualitative rating scale, such as the one on page 63, could be used to assess the students' mathematical skills and understandings, how well they communicated their ideas, and the quality and accuracy of the scale drawing.

Resources or Materials Needed

- Graph paper and unlined drawing paper
- Copies of architectural drawings, including keys

Optional Assessment

Paper and Pencil Tasks — The students could be assigned various problems that would give them practice in converting from one scale to another and one measurement system to another; the problems could give real-world dimensions to be converted to a small scale, or scale dimensions to be converted to real-world measurements. The problems might require the use of either paper and pencil or calculators. Problems such as these are found in some geometry textbooks.

Optional Activity

Students might make a scale drawing of a room in their home, or of a room in some other building.

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

- a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures
- g. Understands perspectives and cross-sections of three-dimensional figures

Goal: To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments

c. Uses units of measure when expressing quantities and chooses appropriate units of measure for the situation
d. Understands limitations and precision of the tools of measurement
f. Converts from one unit of measurement to another within the same system or between systems
g. Makes and reads scale drawings

Goal: To examine numbers in the context of broader mathematical systems

a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers
d. Represents numbers and operations in a variety of equivalent forms, using models, diagrams, and symbols
e. Understands and applies percents, ratios, and proportions in a wide variety of situations
g. Understands the purposes for formulas and how formulas work

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

f. Explores relationships between real-world problems and accepted mathematical models

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking
d. Understands the role of definitions and notation in mathematical thinking and communication
e. Recognizes the importance of providing and communicating both complete and accurate mathematical information

Rating Scale

CRITERIA AND SCORING
Quality of Work Has worked diligently to construct a clear, neat drawing Has illustrated functionality through the arrangement of the furnished room |------------------------|------------------------| little evidence — some evidence — much evidence
Accuracy of Work Has accurate computation skills Has carefully drawn furnishings in proportion to the stated scale |------------------------|------------------------| little evidence — some evidence — much evidence
Mathematical Skills and Understandings Has thoughtfully selected an appropriate scale Shows understanding of ratios and proportions Has carefully chosen appropriate tools and units of measurement |------------------------|------------------------| little evidence — some evidence — much evidence
Organization of Information Has included necessary information in the drawing (scale, mathematical notations, descriptive key, etc.) |------------------------|------------------------| little evidence — some evidence — much evidence
Communication of Ideas Provides a well-reasoned explanation of processes used Uses appropriate mathematical language to express ideas Refers to various aspects of the scale drawing to explain own thinking |------------------------|------------------------| little evidence — some evidence — much evidence

Comments:

Names/Group: ______________________ **Date:** __________

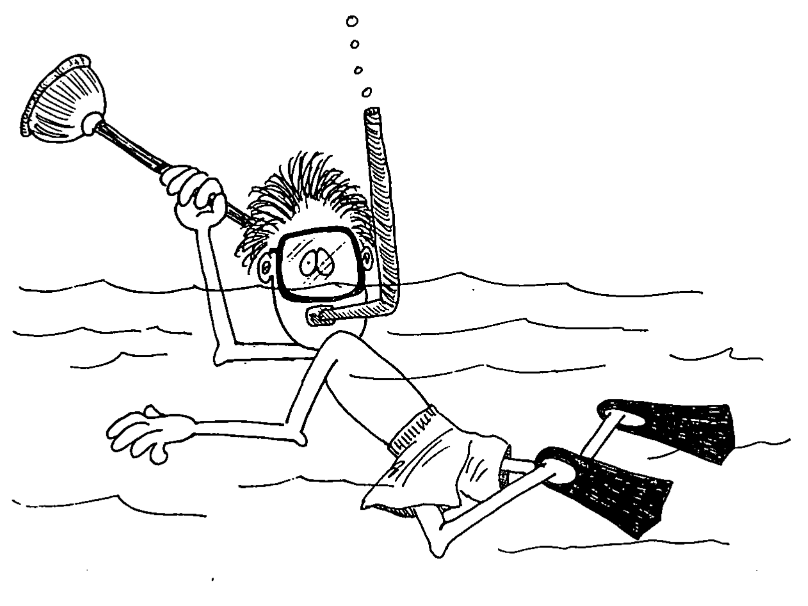
ACTIVITY H: SCHEDULING WORK ON A HOUSE

Purpose of Activity/Assessment

- To develop students' problem-solving skills and decision-making skills
- To give students practice in the estimation of time
- To help students construct a graph, chart, or timeline

Description of Activity

This activity relates the building of a house to other quantitative topics such as estimating and scheduling time. Using the class list of steps in building a house (negotiated in Activity C), students could tentatively order the steps, listing them on a large sheet of paper. Afterward, the students could add the people who would complete each of the construction steps (carpenters, electricians, plumbers, roofers, concrete layers, etc.). The list could then be represented as a chart, graph, or timeline with estimates of how long each group of workers would need to complete their work, assuming good weather and no major problems. A contractor could be brought in to describe the process he/she goes through in developing a construction schedule, as well as discussing the class schedule and workers' labor costs.



If the plumber does not come when scheduled, what will you do?

How Student Learning Could Be Assessed

Paper and Pencil Tasks — The students could, in small groups, determine how to represent, using a graph, chart, or timeline, the work to be completed on the house. The students need to include such information as who would work, when they would work, what tasks they would complete, how long it would take to complete a task, and which groups of workers would need to work at the same time.

Math Log Checklist — Students could write an entry in their math logs about the scheduling and graphing processes. The entry could be assessed using the checklist on page 49.

Optional Activity

It would be possible to continue this activity by introducing students to project management techniques, such as PERT (Program Evaluation and Review Technique) Analysis, which is used to schedule complex projects. There are also many project management computer-software packages on the market.

Resources or Materials Needed

- Copies of calendars to use for estimating time
- Possibly a sample work schedule to be used as a reference for the teacher or the students

Related Goals and Benchmarks

Goal: **To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments**

b. Understands measurement, using appropriate measurement instruments to accurately determine angles, length, weight, mass, time, etc.
c. Uses units of measure when expressing quantities and chooses appropriate units of measure for the situation
f. Converts from one unit of measurement to another within the same system or between systems
g. Makes and reads scale drawings

Goal: **To examine numbers in the context of broader mathematical systems**

a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers

Goal: **To effectively communicate mathematical ideas through writing, speaking, and visual representation**

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing

Goal: **To apply thinking skills to develop greater understanding across the curriculum**

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations
c. Makes inferences and predictions based on logic and probability
d. Recognizes when mathematical information is incomplete, redundant, or extraneous

Goal: **To select and apply mathematical strategies to solve problems in the classroom and beyond**

a. Identifies and analyzes problems
b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem
f. Determines if the solution of a mathematical or real-world problem is reasonable

Goal: **To develop and utilize the skills and attitudes of a lifelong learner**

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
b. Understands the value of continual mathematics learning throughout life
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning

ACTIVITY I: EXPLORING BUILDING METHODS AND TOOLS

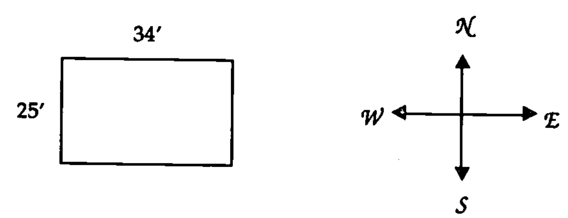
Purpose of Activity/Assessment

- To help students see that many construction methods are based on geometry
- To develop students' ability to construct proofs
- To help students deal with the inherent imprecision in real-world applications

Description of Activity

The first activity is to stake the outlines for the foundation of the house design chosen by the class. As a preparation exercise, the class could brainstorm different ways for constructing this shape, making sure angles and side lengths are all accurate. For example, students might suggest using a protractor to measure angles and a yardstick to measure side lengths; another method might be to measure side lengths with a tape measure and measure the lengths of diagonals to be sure the angles are correct. After brainstorming, students could divide into small groups and each group take a different one of the brainstormed methods to justify as producing the correct shape. Justifications can be based on theorems and properties that the students have already covered or that can be found in geometry textbooks that might be available to the students. For example, students might base their justifications on the Pythagorean Theorem, on the theorem stating that diagonals of a triangle are congruent, or on the theorem stating that two coplanar lines perpendicular to the same line are parallel to each other.

A group's justification could be something like the following:



First, put down a stake at the NW corner. Measure 25′ south using a tape measure and directional compass. Place a stake there. Then go straight east 34′ and place a stake. By going straight east, the angle at the SW corner is a right angle (or close enough as long as you're not building near the north pole). Continue north 25′, placing a stake. Since N, S, E, and W are perpendicular (or close enough), and since the opposite sides are the same length, the angles are all 90°, and the shape is a rectangle of the correct size.

If some of the methods do not produce the correct shape, it is up to the group to show that the method does not work, perhaps with a counterexample. Different groups will have tasks of different difficulties. If there are more than enough methods to go around, some groups could do more than one. Some of the methods may never be proven as correct or incorrect, which is appropriate since unsolved problems are a part of mathematics. Later in the course some of the students may be motivated to work further on the proofs.

To actually stake the foundation, the class could be divided into one group for each of the methods that have been proven correct. Then, on the school grounds or at a nearby park, each group of students could use their assigned method to stake the foundation. After groups have finished staking their foundations, groups could switch to check each other's work. One observer from the original group might stay to observe the second group at work and certify measurements. Measurements of each side and each diagonal should be made, in addition to checking angles as desired. These measurements should be recorded, perhaps on a diagram. (If it is not feasible to stake full-sized versions of the foundation, then the teacher could ask students to stake scaled-down versions, perhaps on the school grounds or at a park, or perhaps in tubs of dirt or on paper.)

The next part of the activity involves analyzing each of the methods to determine how accurate they were for creating the foundation shape. Each of the staking groups could decide on the accuracy of the method they used for creating the foundation shape and on the accuracy of the method they checked for another group. Groups should get together with the group that checked their measurements, in order to agree on the accuracy of the method. As the results are agreed upon, they should be posted on the chalkboard or in another spot where the class can view the report. Accuracy may not necessarily be based on a number; the result could be a description.

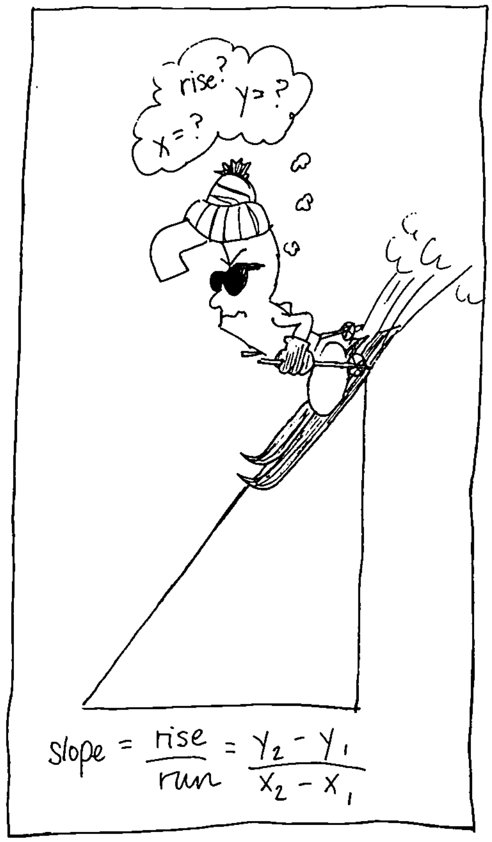
After the reports are posted, the class could review the results. The class may request further analysis from some of the groups in order to make the results more comparable across methods.

To help show how applications of mathematics must deal with imprecision in the real world, groups could analyze the various methods for creating the foundation outline. Students would consider the precision of the measurement instruments used, and then they would do a geometric construction of the foundation outline with measurements off by the maximum imprecision of the measurement tools. Groups should see how far off the shapes of the foundation could possibly be from the desired shape when they are still following a construction method that works perfectly in theory. The teacher may wish to discuss the idea of errors compounding or errors cancelling.

Results of this experiment could be posted on the board. After that, the class could select the method they think would be the most practical at the construction site.

The next activity could be to research methods used by actual construction workers to lay out a foundation. One interesting and relatively easy way to obtain this information would be to invite a contractor to come and explain how it is normally done and to answer other questions. (If it is not feasible for a contractor to visit the classroom, the groups might be assigned library research.) If the method chosen by the class differed from the method used on the job, the class could work to justify which method is really better.

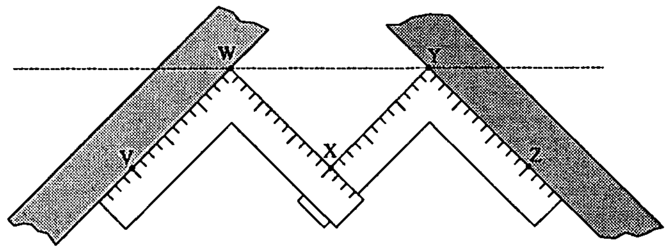
A carpenter could also be asked to visit the classroom. Students could ask about "tricks of the trade" for accomplishing certain tasks, such as cutting boards to the correct angles, making sure walls are straight, adjusting things to correct for imperfections in materials or in parts of the house already nailed in place, putting stairways at the correct angle, and the like. Students could ask how builders use various tools, such as a mitre box or a carpenter's square, and use various techniques to ensure horizontal, vertical, and parallel lines and right angles. (The teacher might consult with the carpenter in advance to determine which methods might have underlying geometry.) It is then the task of the class to justify the methods used in the field as producing the correct results, using geometry theorems as appropriate. Students might work on this task in small groups and then report their progress to the class. As before, the class might not solve all of the problems right away. (If a carpenter is unable to visit the classroom, these methods may be researched in local libraries and the other parts of the activity could then be based on that research.)



How Student Learning Could Be Assessed

Paper and Pencil Tasks — The class could be given a new problem that is related to justifying a particular technique used by a carpenter. For example, a carpenter might use two carpenter's squares to make sure that the lumber he cuts will form congruent angles with a wall. Students could write a justification to show that the pieces of lumber cut will form congruent angles. Students can also be asked to explain their problem-solving processes by reference to various theorems and axioms. Problems such as these appear in some geometry textbooks.

BEST COPY AVAILABLE



Resources or Materials Needed

- A place to lay out a full-sized foundation
- Stakes and string; carpenter's tape measures; perhaps a carpenter's framing square; other tools appropriate for student's methods of laying out foundation
- Photographs, diagrams, and/or reading materials about constructing the structural framework of a house
- Videos illustrating house construction techniques

Optional Activity

Teachers might encourage students to experiment with a number of geometric figures in order to discover which ones are rigid and which are non-rigid. (The vertex angles of figures are allowed to change, but side lengths must not change.) After some experimentation, the class could study trusses used to support roofs, perhaps examining photographs, diagrams, or detailed drawings of various kinds of trusses. From these connections, the class can make conjectures about rigidity and tie these conjectures back to congruence theorems such as side-side-side, and why there is no side-side-side-side theorem for quadrilaterals.

A carpenter could be consulted about how builders measure the pitch or slope of a roof, and the teacher could help the class connect the coordinate geometry concept of slope to the slope of a roof. Students could use the concept of slope to calculate the length of the rafters in the attic of their house, given a specific slope.

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures
b. Investigates, deduces, and explains properties of and relationships between geometric figures
f. Understands and uses properties of lines, angles, triangles, quadrilaterals, and circles in solving problems

Goal: To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments

b. Understands measurement, using appropriate measurement instruments to accurately determine angles, length, weight, mass, time, etc.
c. Uses units of measure when expressing quantities and chooses appropriate units of measure for the situation
d. Understands limitations and precision of measurement instruments

SECTION IV

Goal: To examine numbers in the context of broader mathematical systems

a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers
c. Selects appropriate computation methods, for example using mental arithmetic, paper and pencil, calculators, and/or computers
e. Understands and applies percents, ratios, and proportions in a wide variety of situations

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

a. Explores relationships among various mathematical concepts or topics
b. Models real-world objects with geometric figures, and models complex geometric figures with simpler geometric figures
d. Understands that any mathematical model, graphic or algebraic, is limited in how well it can represent how the world works
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking

Goal: To apply thinking skills to develop greater understanding across the curriculum

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations
b. Recognizes correct logical arguments as well as fallacies in and assumptions underlying an argument

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

a. Identifies and analyzes problems
b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem
h. Knows that if one strategy does not work another strategy might

Goal: To develop and utilize the skills and attitudes of a lifelong learner

c. Gathers and uses mathematical information from various resources
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning

ACTIVITY J: SELECTING MATERIALS FOR A HOUSE*

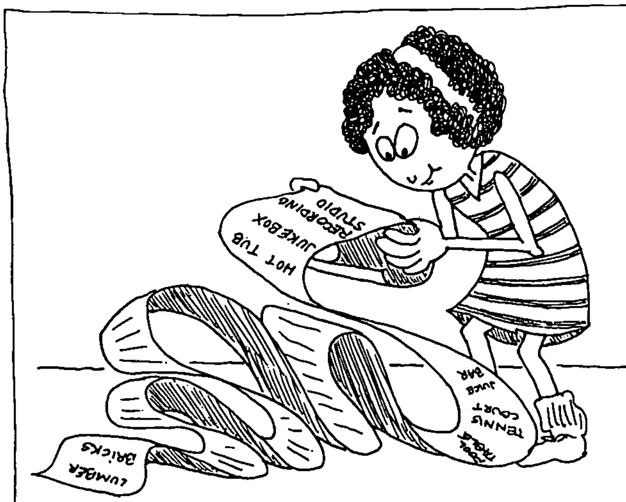
Purpose of Activity/Assessment

- To encourage students to gather information from a variety of sources
- To develop students' problem-solving strategies
- To develop students' cooperative skills
- To encourage students to effectively communicate their ideas and understandings to their peers.
- To give students practice in recording information on a chart

Description of Activity

This activity provides the context for Activities K (Uses of Perimeter, Area, and Volume in House Construction) and L (Determining Costs of Materials).

The students, as a class, could talk in more detail about the next step in the building process—the list of materials needed to build a home. Because materials are essential to the building process, the students could generate a preliminary list of materials needed for the construction of house and could pool their knowledge about various types of building materials.



Students could generate a list of materials needed to build a home.

After the discussion, the students could move into their designated groups as chosen in Activity G (Creating a Scale Drawing of a Room). Each group could assign various tasks to individual group members, including responsibilities and timelines for collecting information about building materials. Students will be given a chart (page 76) on which to record the information they gather about selecting materials such as lumber, flooring, wall coverings, fixtures, etc. needed for their assigned rooms. As they collect information, the students may discover such things as how long the materials will last, the cost and installation requirements for each, etc. Students might bring in samples or pictures of materials in various price ranges. They should be given time to collect the information and to discuss and share their findings with one another so that informed decisions could be made about their respective rooms in the house. Students will need to ensure that all the materials they select fit together aesthetically. In addition to the chart, students will receive a questionnaire (page 75) about the problem-solving processes of the group to complete with the members of their group. (Option: Each group of students could focus on selecting materials for only one room of the house.)

After a tentative plan has been formed, each student would complete the chart, outlining the features of a specific type of material he or she was responsible for. The teacher would provide assistance and support during the information-gathering process and would review the charts when completed. (Note: This chart will be used again in Activity L: Determining Costs of Materials.)

How Student Learning Could Be Assessed

Questionnaire — As a group, the students would complete a questionnaire designed to encourage cooperative working relationships and to help them develop an effective plan of action. The questionnaire could ask such things as: 1) Can you explain the task and its key elements?, 2) Have you thought about and discussed possible plans for completing the task?, 3) Can you list the steps to be taken to complete the task?, 4) Did your group use an appropriate strategy for each step or sub-step?, 5) Explain how this task is similar to other tasks you have encountered, and 6) Did the strategies your group chose or the decisions your group made work?

Resources or Materials Needed

- Books, magazines, or advertisements to help facilitate the students' investigations. (Note: Students could gather other information or materials by visiting local businesses, placing phone calls, etc.)

Optional Activity

The students could talk with retail sales people concerning sale prices, discounts for buying in bulk, seasonal savings, etc.

Related Goals and Benchmarks

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking

SECTION IV

Goal: To apply thinking skills to develop greater understanding across the curriculum

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations
b. Recognizes correct logical arguments as well as fallacies in and assumptions underlying an argument
c. Makes inferences and predictions based on logic and probability

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

a. Identifies and analyzes problems
b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem
e. Selects and applies appropriate tools and technology to solve problems
f. Determines if the solution of a mathematical or real-world problem is reasonable
g. Looks back for patterns that can be useful for solving other problems
h. Knows that if one strategy does not work another strategy might

Goal: To develop and utilize the skills and attitudes of a lifelong learner

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
c. Gathers and uses mathematical information from various resources
g. Reads about mathematics with understanding

Group Questionnaire

This questionnaire is designed to be used by all the members of a group to guide their planning and development of an effective plan of action and to reflect on their problem-solving strategies and decision-making skills.

Directions:

1) As a group, carefully read each question.
2) Place a "✓" and a date in the scoring box that applies.
3) Discuss each question with the members of your group and write a group response.

Scoring		Questions	Responses
Yes	Not quite	Can you explain the task(s) and its key elements?	Task(s):
Yes	Not quite	Have you thought about and discussed possible plans for completing the task(s)?	Best solution:
Yes	Not quite	Can you list steps for completing the task(s)?	Steps in process:
Yes	Not quite	Did you use an appropriate strategy for each step and sub-step?	Strategies used:
Yes	Not quite	Explain how this task is similar to other tasks you have encountered.	Explanation:
Yes	Not quite	Did the strategies your group chose and the decisions your group made work?	Reasons:

Names/Group: ______________________________

Building Materials Comparison Chart

(The blanks at the end are for student-generated criteria.)

Name: ____

Points of Comparison	**Information about** ______________________ (flooring, wall coverings, etc.)				
Available varieties of this material					
This variety is made of					
Cost per unit					
Installation method					
Durability rating					
Available colors					
Available textures, designs, or styles					
Available sizes					
Maintenance considerations					

ACTIVITY K: USES OF PERIMETER, AREA, AND VOLUME IN HOUSE CONSTRUCTION*

Purpose of Activity/Assessment

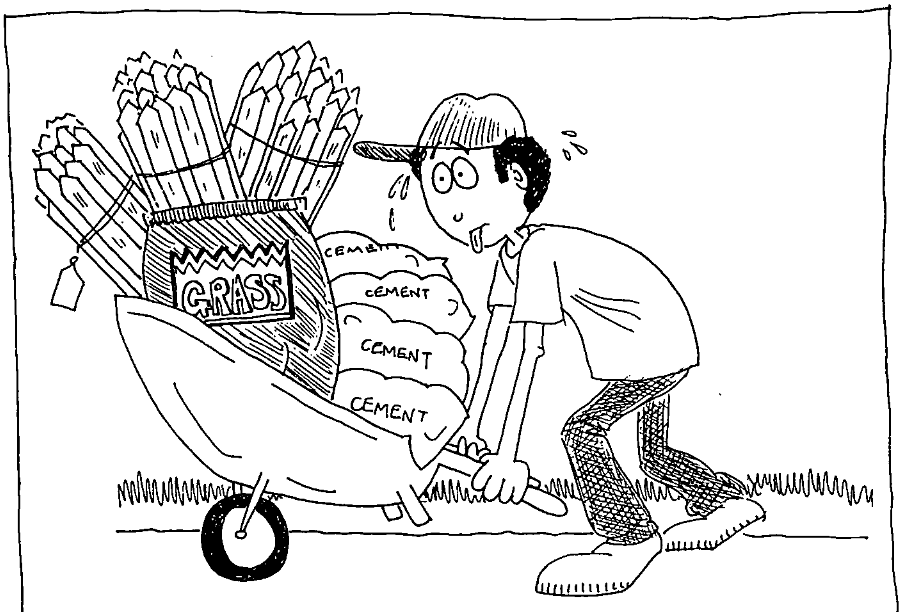
- To enable students to apply area, perimeter, and volume
- To develop students' understanding and use of mathematical formulas for determining the quantities needed for various materials for the house, such as lumber, sheetrock, baseboard trim, concrete, etc.

Description of Activity

As a class, the students could be encouraged to rediscover or recall formulas or methods for determining the perimeter, area, and volume of various geometric figures. The teacher could model how to use the scale drawing to calculate the area and perimeter of the entire house, first, by finding the actual perimeter and area of the drawing, and second, by determining the perimeter and area of the house represented by the drawing. The students could then be given time to calculate the areas and perimeters of various rooms in the house based on the scale drawing, using paper and pencil, and calculators.

The students could then move into their designated groups in order to create a list of the materials needed for each of their respective rooms. The students could calculate the amounts of materials for the floors, walls, ceilings, etc. (using area) and for the baseboards, doors, window trim, etc. (using perimeter). The students could also determine amounts such as the perimeter of the yard (used when installing fencing), the area of the yard (used when laying sod or sowing grass seed), etc. In addition, the students could figure out the volume of concrete needed for the sidewalks and the basement, etc. and other measures such as the attic volume (used in determining the size of an attic fan), the total area of the windows (used in determining heating and cooling needs), and the total volume of the house (used in determining heating and cooling needs). (Note: The list of quantities of materials should be saved for use in Activity L: Determining Costs of Materials.)

The teacher might also provide the students with several problems that reflect non-standard areas, such as finding wall areas in rooms with half-round windows. The students could provide each other with additional practice in working with the concepts of perimeter, area, and volume by developing word problems for their classmates to solve.



Students can calculate the perimeter of a yard for a fence, the area of the yard for grass seed, the volume of concrete needed for the basement...

How Student Learning Could Be Assessed

Multiple-Choice Questions and Answer Key — The multiple-choice questions on pages 81–85 could be given to groups of students to evaluate the depth of their understanding of perimeter, area, and volume. The students might use paper and pencil or calculators. This is an opportunity for students to discuss problem-solving strategies and processes. The answers would be scored using the answer key on page 85.

Resources or Materials Needed

- Calculators and paper and pencil for making calculations and for recording data and organizing work
- Rulers for measuring scale drawings to convert into actual house measurements

Optional Activity

Using the concept of slope (perhaps studied in relation to the Optional Activity in Activity I), students could calculate for their house plan: 1) the volume of the attic space in the house that would need to be vented, depending on various slopes of the roof, 2) the quantity of shingles needed to cover a roof of varying pitches, and/or 3) the length of the rafters needed for the varying roof pitches.

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

e. Develops and applies formulas for perimeters, circumferences, areas, and volumes of geometric figures

Goal: To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments

a. Understands why measurements are used, ways in which measurements arose, and ways in which measurements are standardized
e. Understands differences and relations between length, area, and volume
f. Converts from one unit of measurement to another within the same system or between systems
g. Makes and reads scale drawings

Goal: To examine numbers in the context of broader mathematical systems

a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers
c. Selects appropriate computation methods, for example using mental arithmetic, paper and pencil, calculators, and/or computers
e. Understands and applies percents, ratios, and proportions in a wide variety of situations
g. Understands the purposes for formulas and how formulas work

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

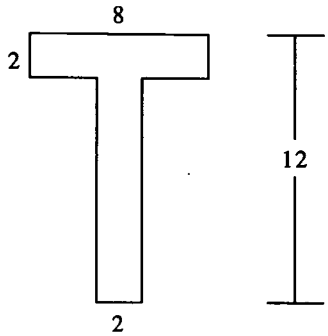
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem
f. Determines if the solution of a mathematical or real-world problem is reasonable

Goal: To develop and utilize the skills and attitudes of a lifelong learner

b. Understands the value of continual mathematics learning throughout life
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning

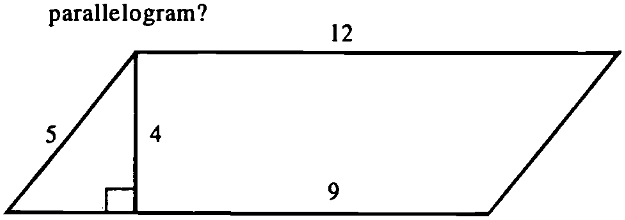
Multiple-Choice Questions

1. The T-shaped region shown below has all square corners and has dimensions, in feet, as indicated. What is the area, in square feet, of the region?



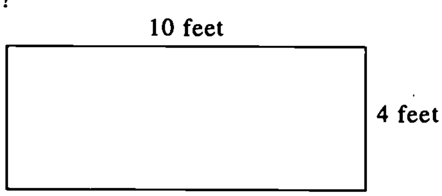
A. 24
B. 36
C. 40
D. 96
E. 100

2. In the figure below, all distances are measured in inches. What is the area, in square inches, of this parallelogram?



A. 36
B. 45
C. 48
D. 54
E. 60

3. What is the perimeter, in feet, of the rectangle shown below?



A. $\sqrt{4^2 + 10^2}$
B. 14
C. 20
D. 28
E. 40

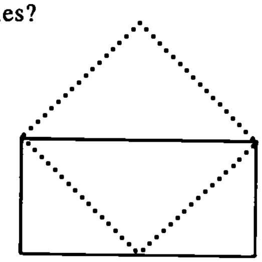
4. If the circumference of a circle is 16π units, then how many units long is its radius?

A. 2
B. 4
C. 8
D. 16
E. 32

5. How many inches in circumference is a circle whose area is 16π square inches?

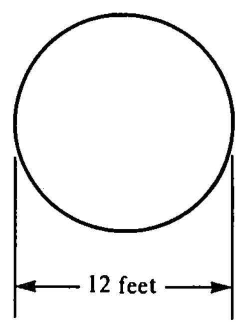
A. 4π
B. 8π
C. 16π
D. 32π
E. 64π

6. The area of the dotted square shown below is 4 square inches. What is the area of the solid rectangle, in square inches?



A. $\sqrt{2}$
B. $2\sqrt{2}$
C. $4\sqrt{2}$
D. 2
E. 4

7. In square feet, what is the area of the circle below?



A. 6π
B. 12π
C. 24π
D. 36π
E. 144π

8. Starting at point *A*, Bob walks 10 feet due east, then turns 15° to the left and walks another 10 feet. He then turns another 15° to the left, walks another 10 feet, turns left another 15°, and so on until he returns to his starting point. How far does he walk, to the nearest foot?

A. 120
B. 140
C. 150
D. 240
E. 260

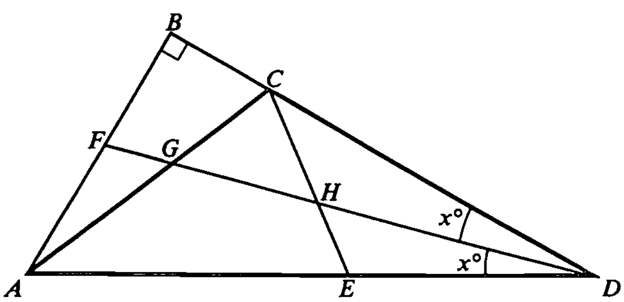
9. The perimeter of a regular polygon (all sides and all angles congruent) is 24 inches, and each side is 4 inches long. If a circle is drawn (circumscribed) in such a way that it passes through all vertices of the polygon, what is the area of that circle, in square inches?

A. 4π
B. 8π
C. 12π
D. 16π
E. $24\sqrt{3}$

10. Mary Anne's bicycle has tires with a diameter of 24 inches. She runs over a small piece of bubble gum that gets stuck to her front tire. Rolling in a straight line, the front tire goes around 100 times (without slipping) before the bubble gum sticks to the ground and comes off. Which of the following best approximates the number of inches the bubble gum is from where it was picked up on her front tire?

A. 1,200
B. 2,400
C. 4,800
D. $1,200\pi$
E. $2,400\pi$

11. In the figure below, which line segment represents an altitude of $\triangle ACD$?



A. $\overline{AB}$
B. $\overline{AC}$
C. $\overline{CE}$
D. $\overline{DF}$
E. $\overline{DG}$

12. What is the area, in square centimeters, of a semicircle with a radius of 2.4 centimeters?

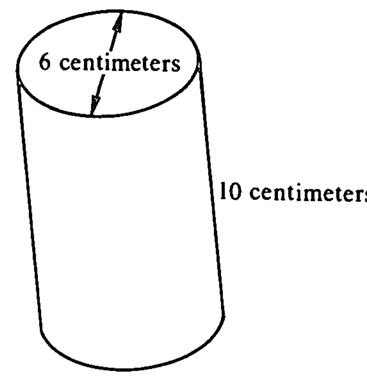
A. 1.44π
B. 2.4π
C. 2.88π
D. 4.8π
E. 5.76π

13. The perimeter of a rectangular flower bed is 42 feet. If the length is shortened by 3 feet, with the width staying constant, what will be the new perimeter of the bed, in feet?

A. 9
B. 12
C. 30
D. 36
E. 39

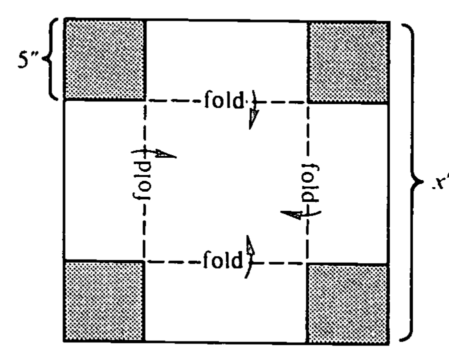
14. What is the outside surface area, in square centimeters, of the curved surface (that is, not including the ends) of a right cylinder with height 10 centimeters and diameter 6 centimeters, illustrated below?

(Note: The formula for the area of this curved surface is $A = 2\pi rh$, where h = height and r = radius.)



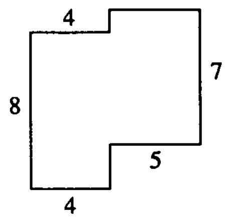
A. 12π
B. 20π
C. 30π
D. 60π
E. 120π

15. A square, shown below, has sides that are x inches long. If smaller squares with sides 5 inches long are cut from the 4 corners of the square, as shown, and the sides are bent up along the dotted lines, it forms an open box. In terms of x, what is the volume of this box, in cubic inches?



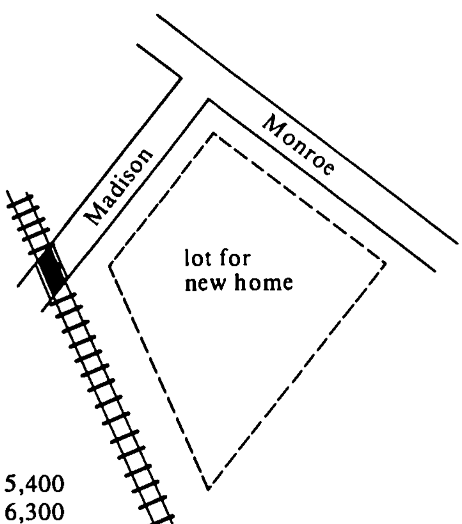
A. $5(x - 10)^2$
B. $(x - 5)^3$
C. $x^2 - 100$
D. $25x^2$
E. $25(x - 5)^2$

16. Angles in the figure below are right angles; lengths of sides are given in centimeters. What is the area, in square centimeters, of the figure?



A. 67
B. 68
C. 72
D. 76
E. 135

17. A lot for a new home is located on the corner of the perpendicular streets Madison and Monroe. The front of the lot, on Madison, is 60 feet long; the side of the lot, on Monroe, is 90 feet long; the back lot line, parallel to Madison, is 120 feet long; and the remaining lot line is as shown below. What is the area, in square feet, of the lot?



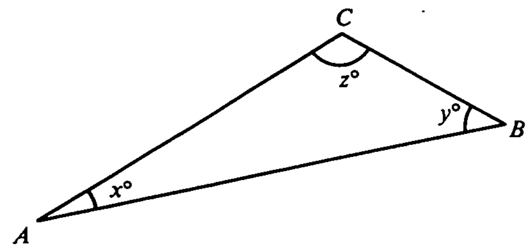
A. 5,400
B. 6,300
C. 8,100
D. 9,000
E. 10,800

18. For an unknown value of x, a rectangle has area $(6x^2 + x - 35)$ square feet and width $(2x + 5)$ feet. Which of the following must be the length of the rectangle, in feet?

A. $6x^2 - 3x - 45$
B. $6x^2 - x - 4$
C. $4x - 7$
D. $3x + 7$
E. $3x - 7$

19. Which of the following 3 combinations of values, when known, will determine the area of $\triangle ABC$, shown below?

(Note: The notation AB denotes the length of $\overline{AB}$.)



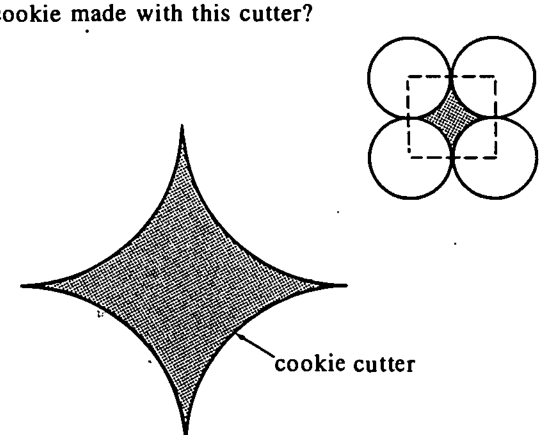
I. $x°$, $y°$, and z
II. AC, $x°$, and $y°$
III. AB, AC, and BC

A. I only
B. II only
C. III only
D. I and III only
E. II and III only

20. What is the perimeter, in feet, of a square that has a diagonal of length 6 feet?

A. $3\sqrt{2}$
B. $6\sqrt{2}$
C. $12\sqrt{2}$
D. 18
E. 24

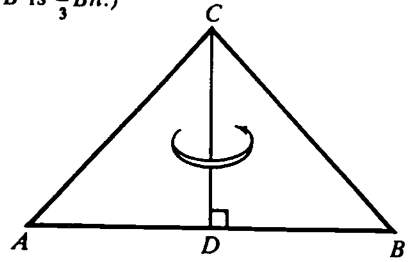
21. In order to estimate the amount of cookie dough to make, Sharon wants to know the area of a cookie made with the cookie cutter shown below. Notice that such a cookie would exactly fill the space between 4 tangent circles of radius 3 inches whose centers form a square. What is the area, in square inches, of 1 cookie made with this cutter?



A. $36 - 4\pi$
B. $36 - 9\pi$
C. 36
D. 9π
E. 36π

22. Sides $\overline{CA}$ and $\overline{CB}$ are each 6 inches long and $\overline{AB}$ is 8 inches long in isosceles triangle $\triangle ABC$ below. If the triangle is rotated about altitude $\overline{CD}$ to form a cone, what is the volume, in cubic inches, of that cone?

(Note: The volume of a cone with height h and base of area B is $\frac{1}{3}Bh$.)



A. 16π

B. 64π

C. $\frac{32\pi}{3}\sqrt{5}$

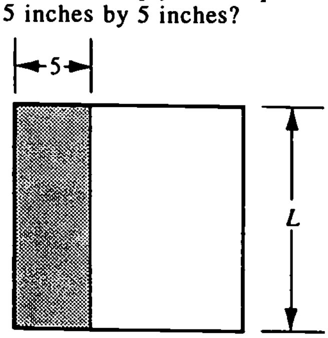
D. $\frac{32\pi}{3}\sqrt{13}$

E. $\frac{64\pi}{3}$

23. If the area of a square 2 inches by 2 inches is increased by 12 square inches, what are the dimensions of the new square, in inches?

A. 4 by 4
B. 5 by 5
C. 8 by 8
D. 14 by 14
E. 16 by 16

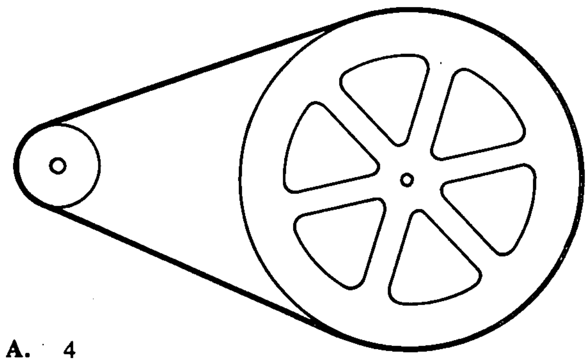
24. Let L represent the length, in inches, of the side of a plywood square. A carpenter saws off a 5-inch-wide strip from one side, as shown in the diagram below. Which of the following expressions would give the area of the remaining (unshaded) rectangle, in square inches, for an original plywood square of any size bigger than 5 inches by 5 inches?



A. $L^2 - 5L$
B. $L^2 - 25$
C. $L^2 - 5$
D. $L^2 + 25$
E. $L^2 + 5L$

25. A motor drives a water pump using the belt and pulley system shown in the figure below. If one pulley has a radius of 12 inches and the other has a radius of 3 inches, how many revolutions will the small pulley make for each revolution of the large pulley?

(Note: Assume the belt will not slip along the pulleys.)

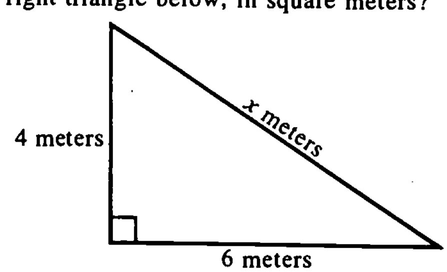


A. 4
B. 8
C. 15
D. 16
E. 15π

26. The surface area of a sphere is given by the formula $A = 4\pi r^2$, where r is the radius of the sphere. What is the surface area, in square centimeters, of a sphere whose *diameter* is 20 centimeters?

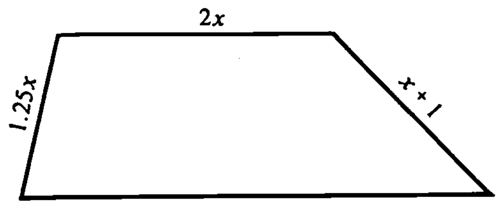
A. 80π
B. 160π
C. 400π
D. $1{,}600\pi$
E. $6{,}400\pi$

27. Which of the following expressions gives the area of the right triangle below, in square meters?



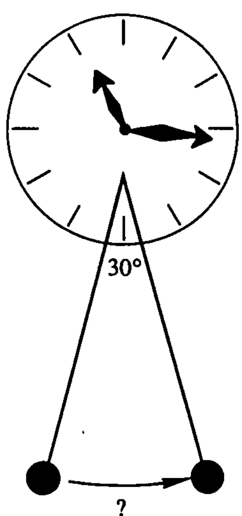
A. 12
B. $\sqrt{40}$
C. $10 + x$
D. $12x$
E. $24x$

28. The length of all 4 sides of the quadrilateral below are expressed in meters. Which of the following expresses its perimeter, in meters?



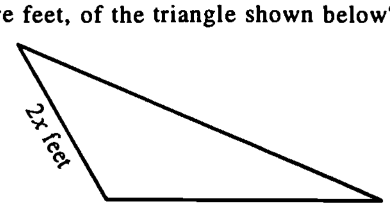
A. $5.25x + 3$
B. $6.25x + 3$
C. $130x + 3$
D. $2.25x^2 + 2.125x + 5$
E. $5x^4 + 10x^3 + 5x^2$

29. The pendulum of a grandfather clock is 18 inches long and swings through an angle of 30° in 1.5 seconds. How many inches does the bob at the bottom of the pendulum travel in those 1.5 seconds?



A. 3π
B. 4.5π
C. 6π
D. 27π
E. 40.5π

30. Which of the following is an expression for the area, in square feet, of the triangle shown below?

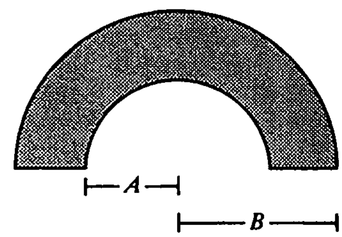


A. $3x$
B. $3x^2$
C. $6x$
D. $6x^2$
E. Cannot be determined from the given information

31. How many coordinate units in length is the perimeter of a quadrilateral whose vertices in the standard (x,y) coordinate plane are, in order, (7,0), (3,0), (3,8), and (7,11) ?

A. 24
B. 27
C. 28
D. 30
E. 38

32. In square inches, what is the area of the half-annulus below (the shaded area between the two half-circles) with inner radius A inches and outer radius B inches?



A. $\frac{\pi}{2}(B^2 - A^2)$
B. $\pi(B^2 - A^2)$
C. $\frac{\pi}{2}(B - A)^2$
D. $\pi(B - A)^2$
E. $2\pi(B - A)$

Answer Key

Item	Key	Item	Key
1.	B	17.	C
2.	C	18.	E
3.	D	19.	E
4.	C	20.	C
5.	B	21.	B
6.	E	22.	C
7.	D	23.	A
8.	D	24.	A
9.	D	25.	A
10.	E	26.	C
11.	A	27.	A
12.	C	28.	B
13.	D	29.	A
14.	D	30.	E
15.	A	31.	C
16.	A	32.	A

ACTIVITY L: DETERMINING COSTS OF MATERIALS

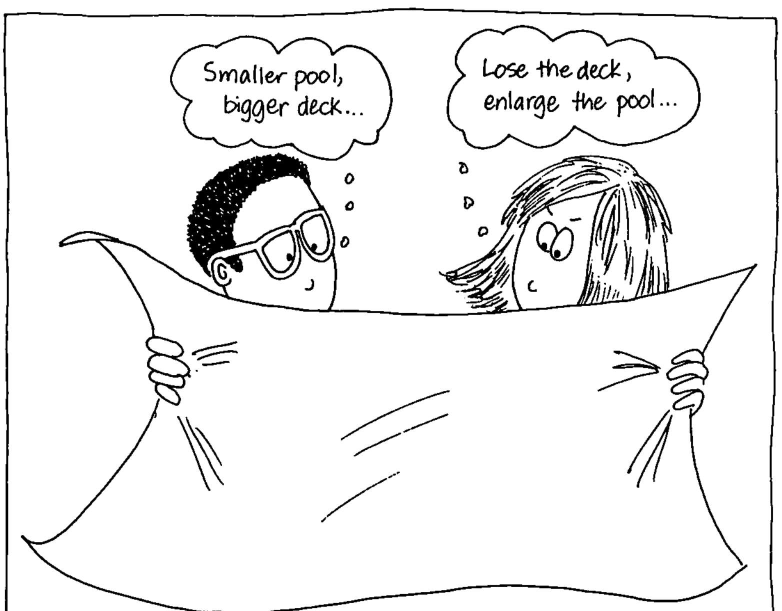
Purpose of Activity/Assessment

- To build upon and enhance students' knowledge of perimeter, area, and volume
- To develop students' computation and estimation skills
- To encourage students to justify their mathematical conclusions

Description of Activity

For this activity, the students will use the information gathered and the decisions made earlier in Activity J (Selecting Materials for a House). The groups could work on determining the quantities needed and associated costs for their respective parts of the house, using the calculations for area, perimeter, and volume completed previously in Activity K. For example, using the information about the price of the wall covering they selected in Activity J and the amounts of wall coverings determined in Activity K, students would need to determine the actual quantity needed and the cost of the wall coverings. To determine quantities, they would need to subtract the areas of doors, windows, and other openings from the total area of the walls.

Once the quantities are calculated, the estimated costs for each room can be calculated and then presented to the class. (Note: The students could be given information about costs for the exterior of the house.) Each student could then figure the total cost of materials, including labor costs if available, in order to arrive at an estimated price for the entire house. The students could be asked to evaluate how well the class kept within the home buyer's set price range and to provide options for cutting costs, if necessary. For example, students may be asked to cut 10% from the costs of their respective rooms. As a group or in small groups, the students could examine how much wasted material there might be and ways in which they could reduce the amount of waste, justifying their conclusions through mathematical computations.



Students could provide options for cutting costs.

How Student Learning Could Be Assessed

Scoring Rubric — The students could complete a costing sheet based on the information obtained earlier by each group, composing a written explanation of the final estimated cost of the house and ways in which to possibly cut expenses. The scoring rubric could be written to assess the completeness and accuracy of students' calculations, the problem-solving skills demonstrated, and the mathematical reasoning involved.

Resources or Materials Needed

- Costing sheets for each group, one for each item to be costed (carpet, wallpaper, paint, fixtures, etc.)

Optional Activities

Students could develop various word problems for their classmates to solve based on the information generated by the class. They could also formulate conditional statements based on the costs of various building materials. For example, if the price of lumber rose 10%, the cost of the house would increase 3%.

As a group or individually, students could interview realtors, loan officers, etc. about their job responsibilities and their connections to the building process.

Related Goals and Benchmarks

Goal: **To examine numbers in the context of broader mathematical systems**

a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers
b. Understands how to use calculators and computer software, recognizing their limitations
c. Selects appropriate computation methods, for example using mental arithmetic, paper and pencil, calculators, and/or computers
d. Represents numbers and operations in a variety of equivalent forms, using models, diagrams, and symbols
e. Understands and applies percents, ratios, and proportions in a wide variety of situations

Goal: **To understand the connections and relationships among various mathematical topics and their applications in the real world**

g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: **To effectively communicate mathematical ideas through writing, speaking, and visual representation**

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing

Goal: **To apply thinking skills to develop greater understanding across the curriculum**

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations
c. Makes inferences and predictions based on logic and probability

Goal: **To select and apply mathematical strategies to solve problems in the classroom and beyond**

b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem
f. Determines if the solution of a mathematical or real-world problem is reasonable
h. Knows that if one strategy does not work another strategy might

Goal: **To develop and utilize the skills and attitudes of a lifelong learner**

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
b. Understands the value of continual mathematics learning throughout life
f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning
i. Accepts new knowledge critically, through analysis, reflection, and inquiry

ACTIVITY M: CREATING ILLUSIONS IN A HOUSE

Purpose of Activity/Assessment

- To encourage students to become aware of optical illusions
- To develop students' understanding of how various materials, colors, techniques, etc. can create optical illusions in a house

Description of Activity

Students could examine a variety of optical illusions, considering how the illusions trick the eye into seeing things that aren't there as well as seeing objects of the same size as being very different in size. After examining a number of illusions, students will discuss the ways illusion is used and then apply this concept to the interior of a house.

The teacher could invite an interior decorator or a home economics teacher to the class to describe how various materials, colors, shapes, lines, objects, etc. can create illusions in a room, like making a room seem smaller or bigger than it actually is. The teacher might bring pictures to share with the class of various kinds of illusions in interior decoration or fashion. The students could experiment with various materials in order to draw a picture or make a model of an illusion in one room of the house. The students could show how the illusion changes a room by making both a "before" and "after" picture or model. (Option: Students could work with a partner, in their designated groups, or on their own to create the illusion. Students might consider using a computer graphics program to create their designs.)

Tricking the eye into seeing something that isn't there.

How Student Learning Could Be Assessed

Math Log Checklist — Students could write an entry in their math log, describing the process they used to create the illusion in the room. The entry could be evaluated using the checklist on page 49.

Resources or Materials Needed

- Examples of optical illusions (Escher prints, for example)
- Photographs of illusions in interior decorating
- Paper
- Colored pencils, crayons, markers, and tools for geometric construction
- Materials for building room models

Optional Activity

Students could look for optical illusions, either in books or in the world around them.

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures
b. Investigates, deduces, and explains properties of and relationships between geometric figures

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

a. Participates in group discussions involving mathematics, synthesizing the ideas presented

ACTIVITY N: EXPLORING PATTERNS AND TRANSFORMATIONS

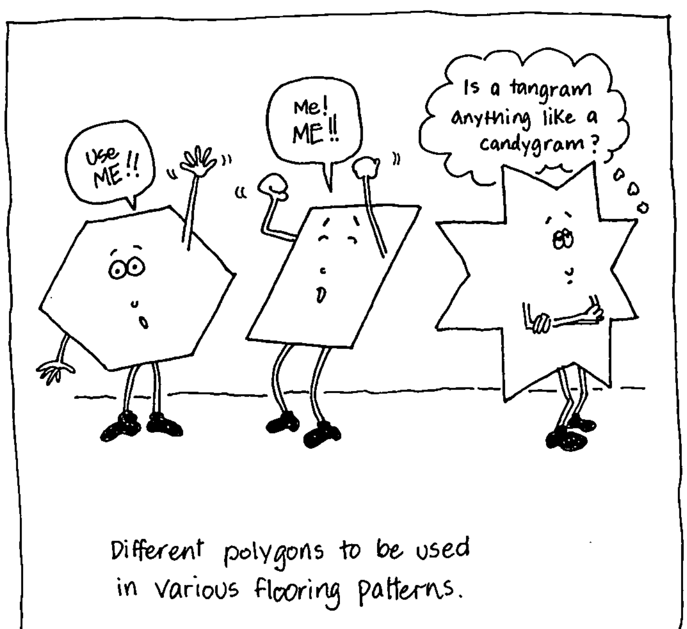
Purpose of Activity/Assessment

- To develop students' understanding of patterns, transformations, and tessellations
- To encourage students' use of inductive reasoning to identify patterns

Description of Activity

In their exploration of building materials, the students would have investigated different types of flooring, including vinyl, carpet, tiles, etc. The teacher could ask those students who investigated flooring to share what information they gathered about different types of floor patterns. The teacher may also show advertisements that provide examples of different types of floor patterns. The students could be given tangrams, stencils, and other materials to explore patterns.

As the students conducted their explorations, the teacher could provide them with information and guidance about various types of transformations, including reflections, rotations, and translations, and ways these transformations can be used to create patterns. In addition, students could explore the uses of tessellations in patterns. Afterward, the students could create their own pattern for a tiled area or room in the house (either a wall or floor), using various materials and geometric figures.



Different polygons to be used in various flooring patterns.

How Student Learning Could Be Assessed

Rating Scale — Students' products could be assessed using a rating scale that evaluates their understanding of transformations. (Note: An alternative to a rating scale would be to develop a checklist.)

Resources or Materials Needed

- Manipulatives such as tangrams for practicing transformations and creating patterns
- Stencils
- Paper
- Colored pencils, crayons, markers, etc.
- Straightedge instruments and compasses for constructing flooring patterns

Optional Assessments

Performance Task — Students could solve problems with tangrams, using them to construct various figures and to determine perimeters or areas of figures.

Paper and Pencil Tasks — The students could also solve various word problems related to amounts of flooring to be used, different polygons to be used in various flooring patterns, or examinations of how transformations relate to various flooring patterns. Problems such as these can be found in many geometry textbooks.

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures
b. Investigates, deduces, and explains properties of and relationships between geometric figures
c. Classifies figures in terms of properties (including congruence and similarity), and applies these relationships in solving problems
d. Understands transformations and uses them as a tool to solve problems
f. Understands and uses properties of lines, angles, triangles, quadrilaterals, and circles in solving problems

Goal: To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments

d. Understands limitations and precision of the tools of measurement

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

e. Uses mathematical patterns, including numeric and geometric patterns, to solve problems
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

e. Selects and applies appropriate tools and technology to solve problems

Goal: To develop and utilize the skills and attitudes of a lifelong learner

b. Understands the value of continual mathematics learning throughout life

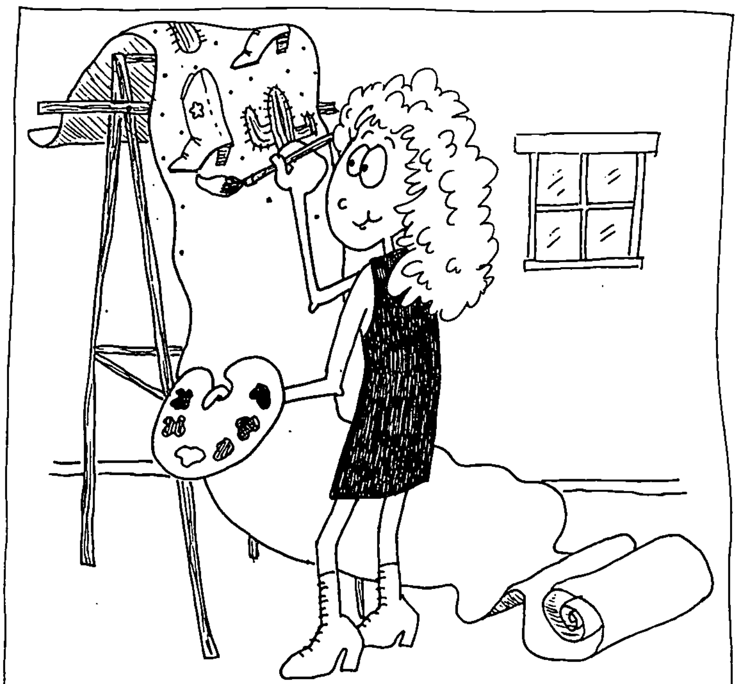
ACTIVITY O: EXPLORING SYMMETRY IN WALL COVERINGS*

Purpose of Activity/Assessment

- To develop students' understanding of symmetry
- To enhance students' knowledge of and skill with transformations

Description of Activity

The teacher might begin this activity by asking the students who investigated different types of wall coverings to share with the class what they discovered, especially information in regard to symmetry. Then, using either discarded wallpaper books or small wallpaper samples, students could explore the concept of symmetry, investigating line and rotational symmetry, learning how reflections and symmetry are related, and examining geometric designs for various types of symmetry. Following this investigation, students could be asked to create their own wallpaper design for one room in the house. The wallpaper design should be the full width of a piece of actual wallpaper and a section long enough to adequately display the symmetry of the design. Students will need to consider the effects of color on symmetry and how the pattern continues from one strip of wallpaper to the next.



Students could design their own wallpaper for one room in the house.

How Student Learning Could Be Assessed

Rating Scale — The final product could be assessed using the Rating Scale on page 94; students will be aware of all the criteria before creating their design. A scale of 1-4 is used to rate both the design and the student's ability to explain, either orally or in writing, the mathematical principles and processes involved in the construction of the design, as well as the symmetrical aspects of the design.

Resources or Materials Needed

- Discarded wallpaper books or wallpaper samples
- Large wallpaper-sized pieces of paper
- Rulers, compasses, and protractors; stencils, tangrams, and other templates
- Colored pens, crayons, or markers

Optional Assessment

Paper and Pencil Tasks — The students could solve various word problems involving reflections, rotations, etc.

Optional Activity

Students could use their knowledge of transformations to create a wallpaper or stencil border for a room in the house.

Related Goals and Benchmarks

Goal: To develop an understanding of two- and three-dimensional geometric figures

a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures
b. Investigates, deduces, and explains properties of and relationships between geometric figures
c. Classifies figures in terms of properties (including congruence and similarity), and applies these relationships in solving problems
d. Understands transformations and uses them as a tool to solve problems

Goal: To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments

b. Understands measurement, using appropriate measurement instruments to accurately determine angles, length, weight, mass, time, etc.
c. Uses units of measure when expressing quantities and chooses appropriate units of measure for the situation
d. Understands limitations and precision of the tools of measurement

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

e. Uses mathematical patterns, including numeric and geometric patterns, to solve problems
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields
i. Understands the nature of axiomatic systems, the reasons for developing such systems, and the role of proof; realizes that results proven for one system do not necessarily hold in another system

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking

Goal: To apply thinking skills to develop greater understanding across the curriculum

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations

Goal: To develop and utilize the skills and attitudes of a lifelong learner

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
b. Understands the value of continual mathematics learning throughout life

Rating Scale

4 = Complete Evidence   3 = Partial Evidence   2 = Minimal Evidence   1 = No Evidence

This Wallpaper Design Project in Geometry:

Shows Students' Understanding of Symmetry and Transformations				
• Design accurately illustrates either reflectional or rotational symmetry or both	4	3	2	1
• Student can accurately identify the symmetry (reflectional or rotational) in the design	4	3	2	1
• Student can describe transformations (translations, reflections, rotations, glide reflections, dilations) or tessellations used in the design	4	3	2	1
Makes Effective Use of Color and Pattern				
• Pattern shows functionality and/or originality	4	3	2	1
• Design shows interesting use of color	4	3	2	1
• Design indicates how two strips of the design would fit together (the "match" on the wallpaper)	4	3	2	1
Demonstrates Effective Communication Using Written or Spoken Language				
• Student explains (either in writing or orally) how the design was made	4	3	2	1
• Student uses mathematical language/terminology when describing the design and the processes used to create it	4	3	2	1
Shows Evidence of Careful Construction				
• Design shows evidence of careful planning	4	3	2	1
• Design shows evidence of accurate measurement	4	3	2	1
• Design shows evidence of accurate and appropriate geometric construction techniques	4	3	2	1
• Design is neatly executed	4	3	2	1
• Student utilized appropriate measurement tools to construct design	4	3	2	1

Comments:

Name: ______________________________ **Date:** __________

ACTIVITY P: CONSTRUCTING A THREE-DIMENSIONAL SCALE MODEL*

Purpose of Activity/Assessment

- To enhance students' understanding and application of mathematical concepts
- To assess students' group participation skills.

Description of Activity

Using building plans of the house that the class has been working with, groups of students will each construct a three-dimensional scale model of the entire house, including at least a few basic furnishings made to scale. Students could use class time to construct their models, using whatever materials the group selects. They will need to work cooperatively to determine a usable scale, the materials to be used, and the construction methods, as well as to actually construct the model. They also need to develop a plan for deciding which students will be responsible for which aspects of construction.

After the model is completed, each group will make an oral presentation to the class, in which every student will participate, discussing mathematics and its connections to home construction. They will also explain why they chose the materials and methods they used, what problems they encountered during construction, and what they learned from making the model. Students will need to use various math skills to determine ratios and proportions when creating the scale, to construct geometric figures, and to measure accurately.

Following the presentation, students might arrange to display their models in a case in a school corridor or entryway or in a building such as a public library or a senior citizen's center.

(Option: Each group could work together to construct a three-dimensional scale model of only one room of the house selected from those rooms that their group had been responsible for.)

How Student Learning Could Be Assessed

Scoring Rubric — Each scale model, along with each group's presentation to the class, will be scored by the teacher on the basis of the rubric on page 98. It is assumed that students would have access to the rubric criteria prior to constructing their model and giving their presentation.

At the bottom of the rubric or on the back of the sheet, space is provided for teachers to make notes about the contributions of individual students to the project and/or the presentation. Students might also want to write about the construction process in their math logs/journals.

Resources or Materials Needed

- Materials for constructing models (Note: Students should have been thinking since nearly the beginning of the unit about materials they might use and have begun saving things such as empty cereal boxes, scraps of balsa wood, or whatever materials they decide to use.)

Related Goals and Benchmarks

Goal: **To develop an understanding of two- and three-dimensional geometric figures**

a. Describes, visualizes, draws, and constructs two- and three-dimensional geometric figures
c. Classifies figures in terms of properties (including congruence and similarity), and applies these relationships in solving problems
e. Develops and applies formulas for perimeters, circumferences, areas, and volumes of geometric figures
f. Understands and uses properties of lines, angles, triangles, quadrilaterals, and circles in solving problems

Goal: **To understand the structure and uses of measurement systems and to measure accurately using appropriate instruments**

b. Understands measurement, using appropriate measurement instruments to accurately determine angles, length, weight, mass, time, etc.
c. Uses units of measure when expressing quantities and chooses appropriate units of measure for the situation
g. Makes and reads scale drawings

Goal: **To examine numbers in the context of broader mathematical systems**

a. Understands and uses the processes of computation and estimation with fractions, decimals, and integers
e. Understands and applies percents, ratios, and proportions in a wide variety of situations
g. Understands the purposes for formulas and how formulas work

Goal: **To understand the connections and relationships among various mathematical topics and their applications in the real world**

b. Models real-world objects with geometric figures, and models complex geometric figures with simpler geometric figures
g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: **To effectively communicate mathematical ideas through writing, speaking, and visual representation**

a. Participates in group discussions involving mathematics, synthesizing the ideas presented
b. Uses mathematical language and diagrams to effectively express ideas orally and in writing
c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking

Goal: **To apply thinking skills to develop greater understanding across the curriculum**

a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations

Goal: **To select and apply mathematical strategies to solve problems in the classroom and beyond**

b. Generates and accepts divergent or alternative approaches and solutions to a problem
c. Chooses from among various solution strategies the one that seems best
d. Formulates a plan to solve a problem

Goal: **To develop and utilize the skills and attitudes of a lifelong learner**

a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
h. Understands the role of justification in the mathematical learning process

Scoring Rubric for Constructing a Three-Dimensional Scale Model

Scoring	4	3	2	1
Mathematical Accuracy	The model accurately matches the building plan, showing precision in changing from one scale to another. The group has selected an appropriate scale.	The model nearly matches the building plan. The group has selected an appropriate scale.	The model partially matches the building plan, though there may be major inaccuracies in the measurement and/or the application of the scale. The group has selected an awkward scale.	The model has only a slight resemblance to the building plan and shows evidence of careless and imprecise measurement and/or application of the scale. The group has selected an awkward or inappropriate scale.
Appearance/ Materials	The model was carefully and neatly constructed. Materials were selected with extreme care. Many details were added to the interior of the model, as well as to the exterior.	The model was constructed with reasonable care and neatness. Materials were chosen with considerable care, and some details were added to the interior of the model, as well as to the exterior.	The model was constructed in a somewhat careless manner and/or the materials were not chosen wisely for the project. Only a few details were added to the interior or the exterior.	The model was not carefully or neatly constructed, and materials were selected with little care. Few or no details were added to the interior or to the exterior.
Communication and Organization	The presentation provided a well-developed explanation of the mathematical skills used, what materials were selected and why, the process they used, and the problems they encountered during construction. Students used accurate and appropriate mathematical language in their presentation.	The presentation provided an adequately developed explanation of the mathematical skills used, what materials were selected and why, the process they used, and the problems they encountered during construction. Students used appropriate mathematical language in their presentation.	The presentation provided a somewhat sketchy explanation of the mathematical skills and materials used, the process they used, and the problems they encountered during construction. Students used some appropriate mathematical language in their presentation.	The presentation provided a limited explanation of the mathematical skills and materials used, the process they used, and the problems they encountered during construction. Students used little or no mathematical language in their presentation.
Participation	All students helped with the construction of the model, contributed to the oral presentation, and participated in the planning and decision-making processes.	Most students helped with the construction of the model, contributed to the oral presentation, and participated in the planning and decision-making processes.	Some students helped with the construction of the model, contributed to the oral presentation, and participated in the planning and decision-making processes.	Work on the project and/or the presentation appears to have been done by one student, or not done at all.

ACTIVITY Q: SOLVING VARIOUS HOUSE CONSTRUCTION PROBLEMS

Purpose of Activity/Assessment

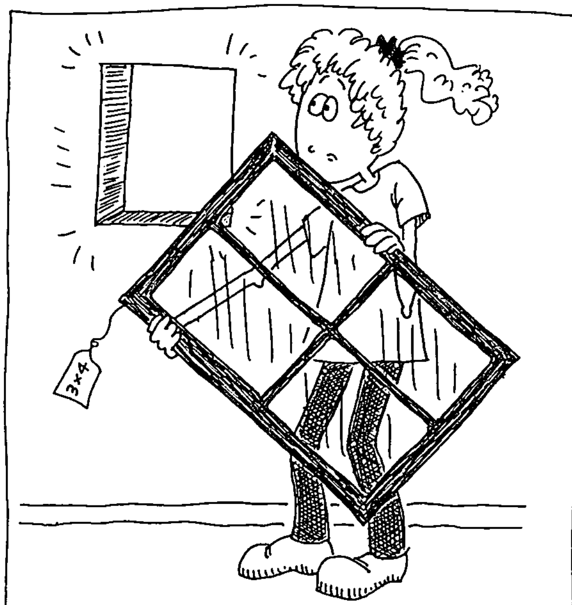
- To stimulate students' problem-solving and decision-making skills

Description of Activity

Students could be given "situation cards" at various intervals throughout the house construction unit. Each card could describe a problem or situation that might actually occur during the building of a house. Students could work individually or in their groups to solve the problems. Students could also generate problem situations for their classmates to solve.

How Student Learning Could Be Assessed

Scoring Rubric — The teacher could provide students with possible scenarios that would affect a construction schedule. For example, the plumber is not able to come when he/she is scheduled, the lighting fixtures that are to be installed are stolen from the garage, the wrong tile is laid, the door is cut in the wrong wall, etc. What will you do? How many other workers would be affected by this change? How would it affect the scheduled completion date for the house? Students could work individually or with a peer to explain what strategies they would use to solve the problem. A rubric could be written to assess students' problem-solving strategies, as well as the accuracy of their mathematical thinking. (Option: Students, individually or in groups, could write scenarios of problems to be solved by their classmates.)



Each card could describe a problem that might actually occur during the building of a house.

BEST COPY AVAILABLE

Resources or Materials Needed

- Situation cards about problems encountered during the house construction unit.

Optional Assessment

Math Log Checklist — The situations might be used as a prompt for an entry in the math log/journal. The entry in the math log/journal could be evaluated using some of criteria on the checklist on page 49.

Related Goals and Benchmarks

Goal: To understand the connections and relationships among various mathematical topics and their applications in the real world

- f. Explores relationships between real-world problems and accepted mathematical models
- g. Investigates connections and interrelationships between mathematics and other disciplines or applied fields

Goal: To effectively communicate mathematical ideas through writing, speaking, and visual representation

- a. Participates in group discussions involving mathematics, synthesizing the ideas presented
- c. Uses mathematical models, drawings, facts, properties, and relationships to explain own thinking

Goal: To apply thinking skills to develop greater understanding across the curriculum

- a. Uses inductive and deductive reasoning to make conjectures, to construct arguments, and to validate and prove conclusions and generalizations

Goal: To select and apply mathematical strategies to solve problems in the classroom and beyond

- a. Identifies and analyzes problems
- b. Generates and accepts divergent or alternative approaches and solutions to a problem
- c. Chooses from among various solution strategies the one that seems best
- d. Formulates a plan to solve a problem
- e. Selects and applies appropriate tools and technology to solve problems
- f. Determines if the solution of a mathematical or real-world problem is reasonable
- g. Looks back for patterns that can be useful for solving other problems
- h. Knows that if one strategy does not work another strategy might

Goal: To develop and utilize the skills and attitudes of a lifelong learner

- a. Monitors own thinking and performance; asks clarifying questions to clear up own misunderstandings
- c. Gathers and uses mathematical information from various resources
- f. Exhibits dispositions (cooperation, independence, persistence, etc.) conducive to effective mathematical learning

Extending the Unit

Even though the creation of a three-dimensional scale model seems to be the culminating event for the unit, it would not have to be the end of the unit. The students' experiences throughout the unit might lead to a number of follow-up activities or experiences, such as students volunteering their time to local organizations involved with housing or investigating other mathematical issues, topics, or questions. Some students might have an interest in exploring the applied fields of carpentry or interior design. Since the math logs/journals are an ongoing activity, students might want to write reactions to the activities they have engaged in and to reflect on all they have learned throughout the unit. If the unit ended shortly after the creation of the three-dimensional scale model, many opportunities for additional growth could be lost.

TEACHER RESOURCES

Ballard, Scott T. (1987). *How to Be Your Own Architect: A Residential Design Handbook*. White Hall, Virginia: Betterway Publications.

Blachford, Laurie, ed. (1995). "Get in Gear." Better Homes and Gardens® *Kitchen Planning Guide*. Des Moines, Iowa: Special Interest Publications, Magazine Group of Meredith Corporation, p. 100-115.

Blackwell, William. (1984). *Geometry in Architecture*. New York: Wiley

Butler, Gayle, ed. (Winter 1995/96). "How to Choose a Contractor." Better Homes and Gardens® *Remodeling Ideas*, p. 17. Des Moines, Iowa: Special Interest Publications, Publishing Group of Meredith Corporation.

Cook, Desmond. (1966). *Program Evaluation and Review Technique: Applications in Education*. Washington, D.C.: U.S. Department of Health, Education, and Welfare.

Dixon, John Morris. (1991). "Geometry Meets Tradition." *Progressive Architecture* 72: 100-103 (April).

Evans, Robin. (1995). *The Projective Cast: Architecture and its Three Geometries*. Cambridge Massachusetts: MIT Press.

Herzog, Jacques. (1989). "The Hidden Geometry of Nature: Six Projects." *Assemblage* 9: 80-107.

Hochswender, Cynthia, ed. (1993). "Kitchen Shopper's Home Companion." *Decorating Remodeling's Kitchen & Bath Custom Planner®*. New York: NYT Women's Magazines, p. 81-103.

Hochswender, Cynthia, ed. (Fall 1993). "Bath Shopper's Home Companion." *Decorating Remodeling's Kitchen & Bath Custom Planner®*. New York: NYT Women's Magazines, p. 129-138.

Hoffer, Alan R., Roberta Koss, et al. (1996). *Secondary Math: An Integrated Approach, Focus on Geometry*. Menlo Park, California: Addison-Wesley Publishing Company, Inc.

Hopster, Gene. (1981). *How to Design and Build Your Dome Home*. New York: Berkley Publishing Group.

Larson, Roland E., Laurie Boswell, and Lee Stiff. (1995). *Geometry: An Integrated Approach*. Lexington, Massachusetts: D.C. Heath and Company.

Reardon-Tagore, Kelly, ed. (Spring 1995). *House Beautiful Kitchens/Baths®*. New York: The Hearst Corporation, Volume 16, No. 1.

Weidhaas, Ernest R. (1974). *Architectural Drafting and Construction*. Boston: Allyn and Bacon.

Wetzel, Gerhard. (1994). "A Report on Data Analysis of Building Geometry by Computer." *Journal of the Society of Architectural Historians*, 53, no.4: 448-460 (December).

Williams, T. Jeff. (1981). *Basic Carpentry Techniques*. San Ramon, California: Ortho Books, Chevron Chemical Company.

Wills, Margaret Sabo, ed. (1994). "How to Finance Your Remodeling." In *House Beautiful Kitchens/Baths®*, Volume 15, No. 3, p. 27, 28, and 30. New York: The Hearst Corporation.

"Working with Geometry." (1986). *Architectural Review*, 180, No. 1074: 61 (August).

House Plans

Alberta Home and Community Design Branch. (1983). *New House Planning & Idea Book: Make Your New Home Affordable, Energy-Efficient, and Comfortable*. Andover, Massachusetts: Brick House Publishing Company.

Anderson, L. O. (1972). *Build Your Own Low-Cost Home: Complete Working Drawings and Specifications for Eleven Homes Suitable for Year-Round and Vaction Use, With Step-by-Step Construction Information*. New York: Dover Publications.

Favorite American Home Plans. (1993). New York: Archway Press.

Gingras, Net. (1986). *Multi-level Homes: Split-Level, Bi-Level, Hillside Designs*. Farmington Hills, Michigan: Home Planners.

Multi-Level Home Plans. (1989). Topeka, Kansas: Garlinghouse.

New Home Plans for 1995. (1994). Topeka, Kansas: Garlinghouse.

128 Home Plans for Varying Building Budgets. (1986). Farmington Hills, Michigan: Home Planners.

Paul, Samuel. (1967). *Apartments: Their Design and Development*. New York: Reinhold Publishers.

Payne, Lauren, ed. (1992). *Women's Day Favorite Home Plans*. Newport Beach, California: Hachette Magazines, Inc., Volume III, No. 1.

Single-Level Home Plans. (1994). Middleton, Connecticut: Garlinghouse.

300 Best Selling Home Plans. (1989). Middletown, Connecticut: L. E. Garlinghouse.

200 Narrow-Lot Home Plans: Stylish Plans for Houses Less than 60' Wide. (1993). Tucson, Arizona: Home Planners.

STUDENT RESOURCES

Nonfiction

Agostini, Franco. (1986). *Math and Logic Games*. New York: Harper/Collins.

Anno, Mitsumasa. (1987). *Anno's Math Games*. New York: Putnam.

Anno, Mitsumasa. (1989). *Anno's Math Games II*. New York: Putnam.

Anno, Mitsumasa. (1991). *Anno's Math Games III*. New York: Putnam.

Anno, Mitsumasa. (1970). *Topsy-Turvies: Pictures to Stretch the Imagination*. New York: Weatherhill.

Anno, Mitsumasa. (1989). *Topsy-Turvies: More Pictures to Stretch the Imagination*. New York: Philomel/Putnam Publishers Group.

Ardley, Neil. (1984). *Making Metric Measurements*. Danbury, Connecticut: Franklin Watts.

Beckmann, Petr. (1971). *A History of Pi*. Boulder, Colorado: Golem Publishers.

Bendick, Jeanne. (1989). *How Much and How Many: The Story of Weights and Measures*. Danbury, Connecticut: Franklin Watts.

Blocksma, Mary. (1989). *Reading the Numbers*. New York: Viking Penguin.

Branley, Franklyn. (1972). *Think Metric!* New York: Harper.

Buxton, Laurie. (1986). *Mathematics for Everyone*. New York: Schocken.

Gardner, Martin. (1956). *Mathematics, Magic, and Mystery*. New York: Dover.

Hoffman, Paul. (1989). *Archimedes' Revenge*. New York: Fawcett.

Hofstadter, Douglas. (1989). *Godel, Escher, Bach: An Eternal Golden Braid*. New York: Random House.

Huff, Darrell. (1954). *How to Lie with Statistics*. New York: Norton.

Huntley, H. E. (1970). *Divine Proportion: A Study in Mathematical Beauty*. New York: Dover.

Kidder, Tracy. (1985). *House*. New York: Avon Books.

Lieber, Lillian. (1944). *The Education of T.C. Mits*. New York: Norton.

Luckiesh, M. (1965). *Visual Illusions: Their Causes, Characteristics, and Applications*. New York: Dover.

Paulos, John Allen. (1982). *Mathematics and Humor*. Chicago: University of Chicago Press.

Pedoe, Dan. (1983). *Geometry and the Visual Arts*. New York: Dover.

Peterson, Ivars. (1989). *The Mathematical Tourist: Snapshots of Modern Mathematics*. New York: Freeman.

Shannon, George. (1985). *Stories to Solve: Folktales from Around the World.* New York: Greenwillow.

Fiction:

Abbott, Edwin. (1987). *Flatland: A Romance of Many Dimensions.* New York: Viking.

Base, Graeme. (1990). *The Eleventh Hour: A Curious Mystery.* New York: Harry N. Abrams, Inc.

Burger, Dionys. (1983). *Sphereland.* New York: Harper.

Carroll, Lewis. (1985). *Alice in Wonderland.* New York: Scholastic.

Doyle, Arthur Conan. (1985). *The Adventures of Sherlock Holmes.* New York: Berkley.

Grifalconi, Ann. (1986). *The Village of Round and Square Houses.* New York: Little, Brown.

Juster, Norton. (1977). *The Dot and the Line: A Romance in Lower Mathematics.* New York: Random House.

Juster, Norton. (1988). *The Phantom Tollbooth.* New York: Knopf.

Mori, Tuyosi. (1986). *Socrates and the Three Little Pigs.* New York: Philomel/ Putnam Publishers.

Nozaki, Akihiro. (1985). *Anno's Hat Tricks.* New York: Philomel.

Sachar, Louis. (1989). *Sideways Arithmetic from Wayside School.* New York: Scholastic.

Swift, Jonathan. (1990). *Gulliver's Travels.* New York: Viking Penguin.

Tompert, Ann. (1990). *Grandfather Tang's Story.* New York: Crown Books for Young Readers.

Biographies

Albers, Donald J. and G. L. Alexanderson. (1986). *Mathematical People: Profiles and Interviews.* Chicago: Contemporary Books.

Ferris, Jeri. (1988). *What Are You Figuring Now? A Story About Benjamin Banneker.* Minneapolis: The Lerner Group.

Ipsen, D.C. (1989). *Archimedes: Greatest Scientist of the Ancient World.* Springfield, New Jersey: Enslow.

Marks, Robert W. (1983). *The Dymaxion World of Buckminster Fuller.* New York: Doubleday.

Ritchie, David. (1986). *The Computer Pioneers: The Making of the Modern Computer.* New York: Simon and Schuster.

Poetry

Pappas, Theoni. (1991). *Math Talk: Mathematical Ideas in Poems for Two Voices*. San Carlos, California: Wide World Tetra.

Computer Software

Autosketch. National Educational Software Distributors: Autodesk, Inc. (Simplified computer-assisted drafting program.)

Escher Sketch. Santa Barbara, California: Intellimation. (Students can design their own tessellations using transformation templates.)

Geometer's Sketchpad. (1991). Berkeley, California: Key Curriculum Press. (A tool for dynamically constructing, manipulating, and investigating geometric figures.)

Geometry Supersupposer. Pleasantville, New York: Sunburst/Wings for Learning. (Students perform constructions on shapes, view them in the graph history, and use the spreadsheet to analyze data.)

Geometric Supposer: Circles. Pleasantville, New York: Sunburst/Wings for Learning. (Students investigate properties of circles by making constructions and changing scales.)

Geometric Supposer: Quadrilaterals. Pleasantville, New York: Sunburst/Wings for Learning. (Students experiment with and hypothesize about quadrilaterals.)

Geometric Supposer: Triangles. Pleasantville, New York: Sunburst/Wings for Learning. (Students experiment with and hypothesize about triangles.)

Geometry Inventor. Pleasantville, New York: Sunburst/Wings for Learning. (Students manipulate geometric constructions; software includes measurement tools.)

Kidcad. Torrance, California: Davidson and Associates. (A 3-D design tool that lets students design and decorate houses, forts, and other structures with hundreds of different building tools and materials.)

Macdraft. Concord, California: Innovative Data Design. (Introductory computer-assisted drafting program.)

Mathlab Series III. (1990). CAE Software. (Uses on-screen tools—ruler, compass, grids—to teach specific concepts focusing on area, ratio, proportion, and circles.)

SECTION V

Suggested Ideas for Other Instructional Units

This section provides a listing of possible classroom activities that could be used in a secondary mathematics class. The following list of ideas is intended as a starting place for teachers interested in encouraging or enhancing thinking skills in their classrooms. The ideas/activities presented below are suggestions for teachers to review, think about, and/or build upon to best meet the needs of their students. Any idea or activity created for a unit or course should be reflective of the skills and understandings that are believed to be important for students to know and be able to do. In addition, the activities should actively engage students in the learning process by encouraging the construction of meaning, self-reflection, and thoughtful inquiry. The following list of questions could be used as a guide or as a cross-check as you are developing additional units and instructional activities.

Does the activity or unit . . .

- encourage students to *construct meaning* through connecting what is being learned with what is already known and experienced?
- allow students to engage in *thoughtful processes* of inquiry (research, experimentation, and background reading), and to find answers to their own and others' questions?
- encourage *modes of thinking conducive to discovery and learning,* such as keeping an open mind and looking for biases and stereotypes?
- engage students in *active use of thinking skills* such as making inferences and generalizations and drawing conclusions?
- engage students in *applying the knowledge and understandings* they have acquired both in the classroom and beyond the classroom, including problem-solving and decision-making skills?
- encourage both *independent and collaborative learning*?

Does each assessment . . .

- connect to the instructional goals and benchmarks?
- reflect the skills and understandings that are most important for students to learn?
- enhance students' learning?
- engage students in answering questions or performing tasks that are meaningful and purposeful?
- include criteria that are clearly stated and understood by the students?
- provide evidence that yields information teachers can use to make valid inferences about each student's learning?

Unit Ideas and Corresponding Instructional Activities

A unit on travel could engage students in considering the concepts of time and distance, financial calculations, and estimation.

1. Gathering information and brochures on various types of vacations or vacation spots
2. Calculating how long it would take to save money for the trip
3. Working with time zones in order to plan travel time and activities
4. Making decisions on where to go based on time available, money saved, personal tastes, etc.
5. Calculating distances
6. Choosing the means of transportation
   - reading airline, bus, and train schedules
   - determining mileage for one's own car or a rental car and estimating the gasoline needed
   - estimating time involved for the various types of transportation
7. Reading advertisements for sales on clothes or gear needed for the trip
8. Using spatial skills to determine the best way to pack a suitcase or the trunk of a car

A unit on starting one's own business could give students opportunities to investigate the complexities of business mathematics.

1. Conceiving an original idea for a product or service to sell
2. Gathering information on potential competitors
3. Researching costs for starting a business
4. Calculating costs for developing a product
5. Predicting/estimating profits and losses
6. Contacting banks to learn about business loans
   - requirements for receiving a loan
   - interest rates
   - payment schedules/total costs
7. Writing proposals for financial backing
8. Analyzing tax codes for busines implications

A unit on personal money management could help students discover how money affects their lives.

1. Making budgets (personal and family)
2. Investigating checking and savings accounts
3. Balancing checkbooks
4. Learning about investments (stocks and bonds)
5. Calculating taxes
6. Learning about sales and discounts

7. Planning for the future (college funds, retirement income, etc.)

A unit on mathematics in the news could help students learn about the importance of mathematics throughout their lives.

1. Collecting news stories from newspapers or TV/radio news shows that involve mathematics
2. Trying to determine or verify the validity and accuracy of the numbers used
3. Looking at statistics and how they are used
   - how the statistics are determined
   - how the statistics are interpreted
   - whether the statistics really mean what they are interpreted to mean
4. The variety of statistics
   - medical
   - sports
   - educational
   - sociological

A unit on the uses of calculus in the world of engineering could make the purposes of calculus clear to students.

A unit on how the various branches of mathematics differ and overlap could help students make conscious connections among the various areas.

SECTION VI

Relationship Between the Mathematics Unit and the Test Questions in EXPLORE, PLAN, and the ACT Assessment

The purpose of this section is to highlight linkages between what an instructional unit teaches and what ACT's Mathematics Tests measure. The geometry course goals and benchmarks listed on pages 32–35 illustrate skills and understandings that are believed to be most important for students to know and be able to do. This list of benchmarks is based on a review of standards documents and many state curriculum guides, so students who prepare themselves to meet course benchmarks such as these should be well-prepared for future success. Those benchmarks assessed in full or in part by EXPLORE, PLAN, and the ACT Assessment are highlighted on the list. Approximately 80% of the geometry course benchmarks are highlighted, which illustrates the breadth of the EPAS programs. In addition to the geometry course goals and benchmarks, there are skills and understandings that students should be developing in other areas of mathematics. Though the content area chosen for the instructional unit was geometry, the Mathematics Tests in EXPLORE, PLAN, and the ACT Assessment cover other areas of mathematics equally well, focusing on mathematical reasoning. (See pages 11–12 for a discussion of test content areas.)

The first course goal, to develop an understanding of two- and three-dimensional geometric figures, covers most of the explicit content of a geometry course. You will note that every one of the benchmarks under this first goal is assessed by one or more of EXPLORE, PLAN, or the ACT Assessment. Nearly every activity in the instructional unit works directly toward this goal, but there is a great deal of variety in the activities. Some of the activities that occur early in the unit, such as C and E, allow time for reviewing concepts and finding models of geometric constructs in the real world. Activities F and I bring in other geometric relations and theorems. The all-important concepts of length, area, and volume play a role in several activities, but are particularly stressed by Activity K.

The first goal covers important knowledge for students to develop, but it is also important for students to connect these topics with what they already know and to develop their reasoning ability, drawing from their knowledge base as appropriate in order to solve problems, to make conclusions, and to focus investigation and learning. Each activity in the instructional unit works to connect geometry with house construction, but the instructional unit as a whole provides a deeper context than the collection of individual activities, allowing students to see connections and develop their knowledge base in a setting that is more like the real world. The real world does not allow one to focus solely on geometry-related tasks, so there are activities with little explicit "geometry," such as when students must research materials in Activity J.

A set of benchmarks that are assessed by the Mathematics Tests in EXPLORE, PLAN, and the ACT Assessment are those that can be found under the fourth goal, to understand the connections and relationships among various mathematical topics and their applications in the real world. Preparing students to be able to make and draw upon connections among mathematics topics and to the real world will be good preparation for ACT's Mathematics Tests, but it will also be valuable as students enroll in various math courses and as they make plans for their future.

Communication has always been an important part of mathematics and mathematics education, but the standards movement made it an explicit piece of the standards framework, which has elevated the importance attached to mathematical communication. The instructional unit included in this guide encourages students to practice and develop a variety of communication skills. Oral communication skills are a part of nearly every activity, with, for example, Activity A (Discussing House Construction and Geometry) involving descriptive whole-class communication, E (Selecting a House Plan) involving persuasive class discussion, and K (Uses of Perimeter, Area, and Volume in House Construction) centering on small-group discussions. Written communications come in several varieties as well, with Activity F (Investigating Geometric Construction Using Drafting Tools) involving written justifications, O (Exploring Symmetry) producing graphical designs, and the ongoing math log (introduced in Activity B) allowing students to share and organize their thinking about mathematical topics, mathematical thinking, and mathematics itself. Students develop reading and listening/questioning skills in activities C and E, where they must get information from various sources. ACT's Mathematics Tests assess communication skills, mainly through students' reading of the quantitative problem descriptions, including the graphics.

Proof is an important part of mathematics, and, in the past, a geometry course was often used to introduce and refine proof skills. Activity F contains some interesting opportunities for students to investigate practical methods used by construction workers, with the goal being to see whether the methods can be justified by geometric argument. Many other activities, such as C and I, provide students with an opportunity to justify their statements and conclusions. Depending on the rules the teacher establishes, these justifications can be more or less informal. Activities N and O also provide opportunities for students to construct geometric justifications and for the teacher to coach students in proof skills. Students who are skilled in making and recognizing logical deductions and proofs will be better prepared for further mathematics.

Problem solving is often seen as the heart of mathematics. Activity Q (where students must solve problems in the construction process) may be the activity that most directly addresses this course goal, but problem solving is a part of every activity. For example, Activity E challenges students not only to consider the various combinations of building sites and house plans to see which would be best, but also to find a method for the class to decide which combination they will use for the remainder of the unit. Even Activity K (focusing on perimeter, area, and volume), which covers topics students have been exposed to for years, is used to enhance student's problem-solving skills, partly through small-group discussions of students' attempts at and successful solutions to problems. The embedded assessment in that activity contains challenging problems, and ACT's Mathematics Tests also challenge students to demonstrate what they know about problem solving.

Other activities, such as Activities E, F, G, I and P, tap measurement skills. Activities N and O allow students to use transformational geometry. Many of the activities, such as A, H, L, M, and Q, help students develop their problem-solving skills as they work, individually and in groups, to make decisions about geometry and house construction.

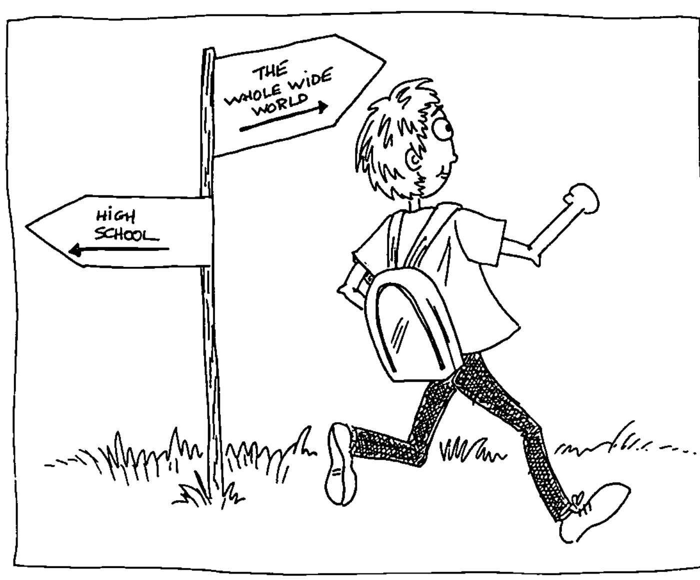
Thinking skills is a separate goal, but the conscious use of thinking skills underlies every activity in the unit, as does the goal of developing the skills and attitudes of a lifelong learner of mathematics. The ultimate goal of education is, as the vision statement on page 32 states, for students to develop independence as a lifelong learner. The purpose of the unit is to pave the way for students to make connections, to solve problems, to gain knowledge, and to reason with this knowledge, the same important knowledge and thinking processes that the Mathematics Tests on EXPLORE, PLAN, and the ACT Assessment measure.

This instructional support guide, and the two others in the content areas of Language Arts and Science Reasoning, were developed to provide teachers with a resource for learning about the skills and understandings underlying the test questions in EXPLORE, PLAN, and the ACT Assessment. The guides include instructional units and classroom assessments as a means of illustrating the different approaches that can be taken to help students develop and use thinking skills. The classroom assessments include performance assessments, multiple-choice questions, demonstrations, projects, and the like. Evaluation tools such as rating scales, observational checklists, and scoring rubrics are also included. The assessments can be used to gauge student progress in developing the skills and understandings necessary for future success. As stated previously, the instructional units were not developed as "ready-to-use" units but rather are provided as materials to be modified to suit the needs of individual teachers and classrooms. It is hoped that the instructional materials and assessments will serve as resources for teachers as they endeavor to integrate assessment with instructional processes.

ACT recognizes that teachers are the essential link between instruction and assessment, and we are committed to providing teachers with the support they need to assist them in their continued efforts to provide quality instruction. We are interested in knowing your thoughts about the utility of this guide. If you would like to submit a response about the guide's usefulness, please feel free to send your comments to:

EPAS Instructional Support Guides
Elementary and Secondary School Programs
Development Division
ACT
2201 N. Dodge Street
P.O. Box 168
Iowa City, IA 52243

Instruction and assessment—assessment and instruction—a necessary and desirable partnership.



REFERENCES USED IN DEVELOPING THE GUIDE

American Association for the Advancement of Science. (1993). "The Mathematical World." *Benchmarks for Science Literacy*, p. 209-234. New York: Oxford University Press.

American College Testing. (1992). *EPAS: Thinking Skills Measured in ACT's Assessment Programs*. Iowa City, Iowa: ACT.

Association for Supervision and Curriculum Development. (1994). *ASCD Curriculum Handbook*. Alexandria, Virginia: The Education and Technology Resources Center, ASCD.

Barnes, Cynthia A., ed. (1992). *Critical Thinking: Educational Imperative*. ERIC Publication Number 77, New Directions for Community Colleges. San Francisco: Jossey-Bass.

Baron, Mark and Floyd Boschee. (1995). *Authentic Assessment: The Key to Unlocking Student Success*. Lancaster, Pennsylvania: Technomic Publishing Company, Inc.

Garaway, G. B. (1994). Language, culture, and attitude in mathematics and science learning: A review of literature. *The Journal of Research and Development in Education*, 27, No. 2: 102-111.

Gardner, Howard. (1983). *Frames of Mind: The Theory of Multiple Intelligences*. New York: Basic Books, Inc., Publishers.

Gardner, Howard. (1993). *Multiple Intelligences: The Theory in Practice*. New York: Basic Books, A Division of Harper Collins, Publishers.

Herman, Joan L., Pamela R. Aschbacher, and Lynn Winters. (1992). *A Practical Guide to Alternative Assessment*. Alexandria, Virginia: Association for Supervision and Curriculum Development.

Hill, Bonnie Campbell and Cynthia Ruptic. (1994). *Practical Aspects of Authentic Assessment: Putting the Pieces Together*. Norwood, MA: Christopher-Gordon Publishers, Inc.

Kendall, John and Robert J. Marzano. (1994). *The Systematic Identification and Articulation of Content Standards and Benchmarks*. Aurora, Colorado: Mid-continent Regional Educational Laboratory.

Marzano, Robert J. (1992) *A Different Kind of Classroom: Teaching with the Dimensions of Learning*. Alexandria, Virginia: Association for Supervision and Curriculum Development.

Marzano, Robert J., Ronald S. Brandt, Carolyn Sue Hughes, Beau Fly Jones, Barbara Z. Presseisen, Stuart C. Rankin, and Charles Suhor. (1988). *Dimensions of Thinking: A Framework for Curriculum and Instruction*. Alexandria, Virginia: Association for Supervision and Curriculum Development.

Marzano, Robert J., Debra Pickering and Jay McTighe. (1993). *Assessing Student Outcomes: Performance Assessment Using the Dimensions of Learning Model*. Alexandria, Virginia: Association for Supervision and Curriculum Development.

National Assessment of Educational Progress. (1992). *Mathematics Assessment: Framework*. The College Board.

REFERENCES

National Council of Teachers of Mathematics. (1993). *Assessment Standards for School Mathematics*. Reston, Virginia: NCTM. Draft.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards*. Reston, Virginia: NCTM.

National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics*. Reston, Virginia: NCTM.

National Council of Teachers of Mathematics. (1995). *Assessment Standards for School Mathematics*. Reston, Virginia: NCTM.

Newmann, Fred M. and Gary G. Wehlage. (1995). *Successful School Restructuring*. A Report to the Public and Educators by the Center on Organization and Restructuring of Schools. Madison, Wisconsin: Wisconsin Center for Education Research.

Paul, Richard. (1993). *Critical Thinking: How to Prepare Students for a Rapidly Changing World*. Santa Rosa, California: Jossey-Bass.

Perrone, Vito, ed. (1991). *Expanding Student Assessment*. Alexandria, Virginia: Association for Supervision and Curriculum Development.

Stenmark, Jean K., ed. (1991). *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*. Reston, Virginia: NCTM.

Stiggins, Richard J. (1994). *Student-Centered Classroom Assessment*. New York: Macmillan College Publishing Company.

Stiggins, Richard J., Evelyn Rubel, and Edys Quellmalz. (1986). *Measuring Thinking Skills in the Classroom*. Washington, D.C.: National Education Association.

Rhodes, Lynn K. (1993). *Literacy Assessment: A Handbook of Instruments*. Portsmouth, New Hampshire: Heinemann.

Stiggins, Richard J. and Nancy Faires Conklin. (1992). *In Teachers' Hands: Investigating the Practices of Classroom Assessment*. Albany, New York: State University of New York.

State Curriculum Guides consulted: Alabama, Arkansas, California, Colorado, Florida, Hawaii, Idaho, Indiana, Iowa, Kansas, Kentucky, Maine, Maryland, Michigan, Minnesota, Mississippi, Missouri, Nevada, New Jersey, New Mexico, North Carolina, North Dakota, Ohio, Oklahoma, South Carolina, South Dakota, Tennessee, Texas, Vermont, Virginia, West Virginia, Wisconsin, and Wyoming.