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ABSTRACT

This essay traces the history of two standards--estimation and mental calculation--devised by the National Council of Teachers of Mathematics (NCTM). It is hoped that the history of these standards will illustrate the difficulties encountered by other proposed standards and by the standards movement as a whole. The essay provides an overview of the national standards movement, its rise in the 1980s, and the evolving standards debate. It examines the rationale for the NCTM standards, arising from the recognition that most Americans could not solve simple math problems. It describes the need for proficiency in mental calculation and the command of math facts and basic operations in making such calculations. The document explains that many states quickly followed NCTM's lead in establishing standards, but notes that there were remarkable differences of interpretation among them. It details how California educators tried to implement the NCTM standards only to have their students perform abysmally on a national mathematics test, and how Maryland educators successfully pushed students to learn mathematical reasoning and the ability to solve novel problems. The essay recounts the politics surrounding the NCTM standards, parents' reactions to the changes, ways to measure skills in estimation and mental calculation, and other information. (RJM)

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The Life and Times of a Pair of Standards:

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Estimation and Mental Calculation

by Harriet Tyson

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Standards for Excellence in Education

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by Harriet Tyson

CBE
COUNCIL FOR BASIC EDUCATION
An Independent Voice for Educational Excellence

A national advocate for high standards in K-12 education

The Life and Times of a Pair of Standards: Estimation and Mental Calculation

by Harriet Tyson

Long before there was any talk of national academic standards, the National Council of Teachers of Mathematics (NCTM) initiated a study of what students in American schools needed to know and be able to do in mathematics. In 1989, after a decade of wide-ranging consultation, the council issued a report, *Curriculum and Evaluation Standards for School Mathematics.* Taken as a whole, their proposed mathematics standards were designed to help students grasp the meaning behind numbers and formulas, to teach them how to apply mathematics in the world outside of class, and to equip them with the skills required to prosper in a world saturated with computers and calculators. Two interrelated topics, estimation and mental calculation, were among the "new" topics the mathematics community thought should be taught.

This essay traces the recent history (since 1989) of these two standards as they have traversed the American educational landscape. Their odyssey illustrates the difficulties encountered by other proposed standards, and by the standards movement as a whole, in several ways:

- First, these two topics are perceived as "new" to the school curriculum by this generation of parents. The topics were taught in schools in the nineteenth and early twentieth centuries, but virtually disappeared for most of this century. Although all adults are forced by circumstances to estimate and figure sums in their heads, most do not think that this is "doing math," and many are uncomfortable with the sudden appearance of these topics in their children's schooling.
- Second, there is evidence that suggests estimation and mental calculation are teachable skills—those who study these skills in school become more accurate estimators and mental calculators—but the evidence is skimpy. Similarly, the evidence that the mind is improved by reading good literature, by imagining alternative historical scenarios, or by writing persuasive essays is also skimpy.
- Third, skill in estimation and mental calculation is hard to measure fairly and well. By their very nature, they are mental processes and do not lend themselves to measurement on short-answer, machine-scored tests. The right answer is within a defined range, rather than a single figure. Some estimation test items ask students to reveal their mental strategies by writing a short explanation of how they got their answers. But the process of scoring these constructed answers (like the scoring of essays in English and history) is time-consuming and expensive, and troublesome to testing professionals because the answers are subject to human judgment.

- Fourth, those who are teaching estimation and mental calculation typically had little or no exposure to these challenges during their own schooling and frequently do not see their value. The same can be said of most teachers now being asked to organize scientific experiments for elementary school children or to introduce statistical concepts in social studies. Unless teachers have graduated from strong, academically oriented teacher preparation programs or have studied in high-quality, sustained in-service training programs, they may be either avoiding these topics, teaching them badly, or spending too much time on them.
- Finally, the estimation and mental calculation standards have already acquired a rich political history in the eight years since NCTM brought them back to prominence. Critics have lumped them together with other "new" math standards (such as problem solving), new pedagogies (such as using concrete objects to teach math concepts), new testing approaches (performance tests), and new scoring systems (variable answers and partial credit), dubbing the whole effort as "fuzzy math." All of these approaches have become pawns in a bitter public debate about who should control education, whether there should be national tests, who should develop the tests, and what the consequences of national tests will be for students, teachers, schools, and school districts.

The school topics of estimation and mental calculation, along with the whole suite of NCTM math standards, entered the scene at the moment in history when the national political debate about "national" standards was at its height. To situate these two topics in a larger context, we provide a brief historical sketch of the "national" standards movement.

The National Standards Movement

Many developed countries have national curricula and testing programs. Their citizens have become accustomed to conducting periodic debates about what students should know and what standards they should meet; but in the United States, the idea of a public discussion of curriculum and standards is new. Historically, American educators have not thought much about standards at all, and when they did, it was to consider whether a school program would prepare college-bound youth to score well on college admission tests. Traditionally, the states have provided only broad guidelines, leaving curriculum to the discretion of local school districts.

But in the late 1970s, this pattern began to change. Governors and state education agencies were spurred into more pointed activity by evidence of American slippage in international economic competition, by the unimpressive performance of American students compared to students in other nations, and by disparities in achievement among the races and classes at home. "Accountability" became the watchword in American education, and the states became the focal point of the effort to raise educational standards through testing.

In the late 1970s and early 1980s, hardly anyone questioned the idea that there was a direct cause-and-effect relationship between student achievement and American economic competitiveness. Raising test scores, especially in mathematics and science, became the central preoccupation. States became the senior partners in the venture, and local districts began to accept the challenge to provide a more rigorous academic program in order to prepare students for more rigorous state tests.

But the urge to measure raised a more fundamental question of what was supposed to be measured. Students in Michigan were not studying the same things as students in Alabama. They might be taking quite different tests. So how could states be compared to one another meaningfully? And how could our standing on international tests be determined in a country with such a long and deep tradition of state and local control and with such a patchwork of differing goals, standards, curricula, tests, demographics, and traditions?

Voices began to be raised for more coherence, consistency, and stability, not only for the purposes of gauging our international standing and rescuing our economy, but also for the benefit of a highly mobile student population. Loudest among those voices was the federal government, speaking through the publication of *A Nation at Risk* (1983). The booklet did not call for national "standards" because that would have been seen as an intrusion on states' prerogatives, but it did call for national "goals," which included higher expectations for all students and, implicitly, a more national (but not federal) approach to curriculum reform.

Meanwhile, NCTM had been working independently on mathematics reform for a decade. Without government encouragement or assistance, the nation's mathematics teachers and their counterparts in higher education had been looking at student achievement data, at research on teaching and learning, at the effects of computers on the mathematical sciences, and at the uses of mathematics in modern life. The council's final document, *Curriculum and Evaluation Standards for School Mathematics* (1989), could easily have been titled *Advice to States About Mathematics Education*, for it was never in the minds of the authors to launch a "national" standards movement. NCTM, like nearly everyone else, accepted the idea that states control standards and assessment and school districts control and implement curriculum. NCTM's pioneering effort was praised by leaders across the political spectrum. Its debut in 1989 coincided with widespread frustration over the low educational expectations of many states, over the crazy-quilt of curriculum and standards across the nation, and over the fact that in the United States there was no captain at the helm of education.

The touting of "national standards" and the call for states to fall in line with the recommendations of the national subject matter organization crested in 1989 at the Williamsburg Summit on education. Governors and leaders in business and education, liberal and conservative alike, were still focused on the link between poor performance by U.S. students and sagging American economic productivity. Given the tendency of states to keep standards low and the Constitutional reality of state control, many reformers thought that only national, discipline-based, nongovernmental entities were in a position to promulgate "high and rigorous" or "world-class" standards for education in the United States. The eager acceptance of NCTM's standards in 1989 may have reflected a widespread wish that *somebody* would step up to the plate and perform the role that the Constitution does not permit the federal government to play.

The newly released math standards stimulated a call for standards in other disciplines. But most disciplined-based teacher organizations operate on a shoestring and could not have mounted the effort that NCTM had organized. Gingerly, the federal government (prompted by the highly respected Diane Ravitch, who served as Assistant Secretary of Education under President Bush) began to let contracts to groups of educators, who were asked to create a brace of standards documents in the core curricular areas.

The federal contract for science education standards went to the National Research Council, an arm of the National Academy of Science. Its *National Science Education Standards* was published in 1996. These standards asserted the importance of integrating mathematics and science content and called for the development of skills in estimation; graphic representation; statistical methods to describe, analyze, evaluate, and make decisions; the use of geometry in solving problems; and the use of statistics and probability in science.

Years earlier, in 1990, Project 2061, an arm of the American Association for the Advancement of Science, published *Science for All Americans*, which was a graceful exposition of standards in science education. That document was followed by *Benchmarks for Science Literacy* in 1993, which set forth reasonable checkpoints for measuring student attainment in science. Among other things, it called for the teaching of estimation and number sense (mental math) in the early grades, which merged into graphic representation, probability, and statistics at higher grade levels.

Key to both the mathematics and science standards was the idea that far too many Americans had failed to prosper in traditional programs, which were largely rote and insufficiently focused on conceptual understanding and application. Rather than allow the majority to slide into mathematical and scientific dysfunction in later life after the rules and formulas had faded from memory, math and science leaders hoped to equip people with a depth of understanding that would help them to retain and use their knowledge in adult life. This approach was in sharp contrast to the nation's traditional indifference to the great masses of innumerate and scientifically illiterate graduates of public schools.

But slowly, the fervor for "national" standards began to subside. The first draft of the national history standards set off a political firestorm that resulted in its condemnation by 99 members of the United States Senate. Although the history standards were revised to the satisfaction of most critics, the episode had soured many people, especially conservatives, on the idea that the federal government could intelligently sponsor such an effort, even if it kept its hands off of substance. The first draft of the standards for English language arts was so fiercely criticized by review panels that President Clinton canceled the contract with the National Council of Teachers of English.

Critics began to charge that the emerging standards were tainted by federal money and apt to lead to federal attempts to control education. The distinction between "federal" and "national" became blurred, and any idea attached to either "federal" or "national" began to lose its political appeal.

Also, the national economic crisis ebbed, and American industrial productivity surged ahead of Japan, Germany, and other rivals. Although the schools had been blamed for the economic slump of the 1980s and early 1990s, nobody congratulated them for bringing about our impressive economic revival—a point well made by Larry Cuban, a professor of education at Stanford University.[1] Although business and industry leaders would continue to support national standards and national tests, their voices would be drowned out by politicians with a case to make against anything the national government did. The argument that we needed national education standards to compete in the world economy lost its legs.

[1] Cuban, Larry, "The Great School Scam." *Education Week*, June 16, 1994, p. 44.

Nevertheless, the idea that we should have definite standards, that they should be high, and that virtually all students should meet them has survived a batch of low-quality efforts, the shift in the national zeitgeist toward local control, and a lessening fascination with U.S. standing on international assessments.

The Evolving Standards Debate

Discussion of standards in the late 1990s has shifted toward the inherent value of a rigorous education in the academic disciplines—for the sake of the individual rather than for the sake of international comparisons. The current standards movement, in which the Council for Basic Education (CBE) has taken a leading part, projects the idea that clear and specific standards are necessary to halt the downward drift of American education. There is a growing consensus that standards are of great benefit to teachers, who need to know toward what goals they and their students should aim. With the present system of norm-referenced tests, parents know only how their children stand in relation to other children in the test-taking group, and group norms can be low. The only national test now given is the National Assessment of Educational Progress (NAEP), but it measures the performance of school districts and states, not individual students. But voluntary national assessment of individual students' performance on standards will tell teachers and parents whether their students meet an absolute standard.

The absence of any uncontaminated source of information about how individual students are doing on tasks that really matter is one of the great motivating forces behind both voluntary national standards and voluntary national tests. Another impetus is the increasing mobility of American families. Children suffer gaps in their education that will never be filled and are exposed to wasteful repetition and widely varying standards of performance. To the extent that national standards, freely chosen by educational authorities at all levels, become a reality, these and many other well-known problems in American education will begin to be addressed.

The dream of a more coherent national system of education has faded, but states are working hard to lift student achievement by developing their own standards, closely or loosely based on standards proposed by professional organizations. Standards proposed by national subject matter organizations are now having to earn their own living because these groups have no power, other than the power of persuasion, to change what states, localities, and teachers do. In truth, that has always been the reality. It is just that many people hoped, for a while, that the whole effort would lead to much greater consistency across the country than has turned out to be the case.

The Rationale for the NCTM Standards

NCTM's recommendations were influenced by overpowering evidence that most Americans, even well-educated ones, cannot solve simple, practical math problems. Many adults who passed through the traditional school mathematics curriculum cannot easily balance a checkbook, calculate the tip on a restaurant check in their heads, figure how much paint or lumber to buy for a home project, or convert decimals to percents. Fractions paralyze most adults because they never learned the concept of fractions and have forgotten the mechanics for manipulating them. Their minds go blank when they encounter very large numbers—the population of China, the distance to a star, or any other number with many zeros. They often draw fallacious conclusions from the statistics they read in the newspaper.

While there is strong evidence that nearly all students learn to add, subtract, multiply, and divide during their twelve years of schooling, an astonishingly large percentage of the population cannot apply those operations to anything real. Widely reported studies of American ineptness in mathematics and popular books published in the 1980s (*Innumeracy* by John Allen Paulos, *Math Anxiety* by Sheila Tobias) made it clear that the traditional way of teaching mathematics in school was not working, at least not for the great majority. Moreover, the societal stakes were getting higher; many more people needed to know more about mathematics than ever before. NCTM's proposed changes were designed to augment a system that filtered out the talented few and replace it with a system that would enable nearly everyone to use mathematics in life.

NCTM was convinced that the two standards examined in this essay are important to meet. But the public still has questions. Why do people need to know how to estimate and do mental math? Will time spent on these skills be stolen from mastering the basics? How are these skills being taught and measured? Are they "fuzzy math," as some critics contend? Or are they serious and useful components of learning mathematics?

The Need

The need to estimate and calculate on one's feet is as old as civilization. When farm land was divided or sold, precise measurement was neither feasible nor affordable; measurement involved "stepping off" distances. Farmers the world over practiced the art of adjusting their stride so that reasonably exact distances could be determined. The skill of stepping off was taught at home or at work and sometimes in rural schools in the United States. When armies traveled cross-country, they needed to count off miles as they marched; soldiers perfected a measured pace that would add up to an even mile or kilometer.[2]

As late as 1920, schoolteachers in the United States conducted "math bees," in which students had to stand (and thus lose recourse to paper and pencil) and listen to a teacher read off a string of numbers to add or subtract. The winner was the one who came closest to the exact number. "Horseback calculation" was a valued skill when many transactions were made on horseback.

Despite all the practical reasons for being adept at estimation and mental math, and despite the fact that most people continued to use these operations daily, the formal study of estimation and mental calculation gradually disappeared from the schools as the twentieth century wore on. Measurement technology improved, and most commercial transactions moved indoors where adding machines or paper and pencils replaced mental calculation. Local schools became absorbed into larger and larger "systems" with required curricula and textbooks tailored more toward urban needs (i.e., arithmetic for the many, business math for a few, and college preparatory math for the very few).

For the past three or four generations, elementary students have been taught math facts and the mechanics of manipulating whole numbers, fractions, decimals, and percents. Secondary students have studied geometry, but most have come away empty-handed. Many studied algebra, and then never used it again in their lives. Some students gradually absorbed the meaning behind algorithms; most did not. Some students were happy to be working pages and pages of abstract numerical problems; a larger number didn't see the point. "Word" problems, which stumped a great many students, often posed wildly implausible situations, and many students did not think they were worth solving. A small percentage of students with a natural aptitude for mathematics, and perhaps a penchant for brainteasers, went on to study algebra, which was the gatekeeping course to trigonometry and calculus in high school. Those who cleared those hurdles went on to selective colleges and profitable careers.

[2] Duncan, David R., and Litwiller, Bonnie H., "Two Folk Methods for Estimating Distance." In *NCTM Yearbook: Estimation and Mental Calculation*, Harold L. Schoen and Marilyn J. Sweng, eds., 1986.

The society at large saw these students as naturally good at math. The rest, who graduated from high school with just enough arithmetic to get by in the world, were seen as naturally poor at math. The belief that there were natural winners and losers in math was so deeply imbedded in the national consciousness that hardly anyone considered the possibility that there was something wrong with the way we were teaching it or that more people could be "good" at math if they, and their teachers, were better taught.

As a rule, children and adults in this generation have little tolerance for estimations, approximations, or probabilities and seem unaware that their lives are dominated by them. But they are. The teaching guide to an educational film series, *The Challenge of the Unknown*,[3] describes the truth of the matter:

> Our world runs on estimates—on projections, tentative conclusions, approximations. They shape our thinking and guide our decisions.... Because many problems crop up quickly or must be answered almost immediately, we use estimates. The world often places a higher premium on a readily available, workable answer than on slow-moving exactitude. We rarely have the luxury of collecting and processing all the necessary information, performing in-depth analysis or calculating alternative solutions. Wages are negotiated based on projected rates of inflation. Research studies are budgeted and staffed based on careful estimates of the nature and duration of the work. Faculty are hired and tuition set after estimating costs and numbers of incoming students.

Both precise calculation and mental figuring require an automatic command of math facts and basic operations, as well as a firm grip on place value. But mental calculations require some additional skills: quickly establishing high and low boundaries for the probable answer, rounding to manageable or compatible numbers, restructuring the problem to a more mentally manageable form, and adjusting the answer to compensate for the mental liberties you have taken with the numbers. Good estimates of distance, length, height, and volume involve techniques such as subdividing, sampling, and iteration of units. These are all teachable skills. Because they have not been taught in school for several generations, these distinct skills have atrophied.

The reemergence of estimation and mental calculation as school topics and as "standards" that students should be required to meet owes a lot to the educational deficiencies revealed by the near-universal use of calculators and computers in nearly every walk of life. One wrong key stroke and the bill of sale can be wildly erroneous; a wrong answer to a complex set of engineering calculations can result in a threat to public health and safety. People need to

[3] *The Challenge of the Unknown: Teaching Guide.* New York and London: W.W. Norton & Company, 1986.

stay conscious while using these machines, continually estimating probable answers as a check against the reasonableness of the machine's answers. So just as advancing technology once made estimation and mental mathematics seem obsolete, it now makes them necessary once again.

The NCTM Standards' Impetus to Mathematics Reform in the States

Nearly all of the states were quick to follow NCTM's lead, partly because their standards made sense to a lot of people and partly because many state-level specialists and teacher leaders had been deeply involved in developing the standards document and were already familiar with and committed to the proposed changes.

Although nearly all states claim to have based their standards on NCTM's, there are remarkable differences of interpretation among them. As to estimation and mental calculation, some states wrote fully developed sequences that progressed through the K-12 program with increasing complexity; Maryland, Georgia, Ohio, Utah, Connecticut, and Pennsylvania are examples. Other states subsumed the topic under measurement or number sense, and still others issued such brief documents that it is hard to tell whether estimation and mental calculation are part of the program or not.

Some states have developed their own testing programs, which measure progress on these two standards. Others have purchased tests that include items to assess these topics. Still others use tests that avoid the issue.

Most state education agencies organized training opportunities to help teachers meet new mathematics standards, and continue to operate those programs. But even in states that have not mounted training efforts, many teachers nationwide have been able to study the math they missed in school and college, as well as teaching strategies, in regional workshops sponsored by NCTM. Although many of the teachers who have taken advantage of these opportunities found delight in this fresh way of looking at mathematics, others seem to have gotten the words but not the music.

The Strange Case of California

Despite clear instances of progress attributable to the NCTM standards, the document harbored deficiencies which in some states, notably California, led to confusion and failure. Harold Stevenson, a professor of mathematics at the

University of Michigan who has been studying mathematics education in the United States and in Asia for fifteen years, recently commented on the NCTM standards:

> Of course, we want children to value mathematics, to be mathematical problem solvers, to be confident of their ability, to be able to reason and communicate mathematically. Certainly, students must develop a number sense, have concepts of whole number operations, and the other kinds of skill and knowledge indicated under NCTM's curriculum standards. But the published standards do not integrate these two important components, the general attitudes and mathematical skills. Another important aspect is how realistic it is for teachers to implement these vague ideas in their everyday teaching. In many cases, schools elected to emphasize students' general attitudes and fail to provide them with opportunities to learn fundamental mathematical skills.[4]

Stevenson's assessment can be seen in the working out of mathematics reform in California. The state's 1991-92 Mathematics Framework was a radical interpretation (or misinterpretation) of NCTM mathematics. The California approach was based on the philosophy of "constructivism." Although it is a potent and widely respected theory of learning, it is neither a curriculum nor a pedagogy. While constructivism was lightly implied in the NCTM standards, it became the keystone (or millstone) of the California mathematics framework.

Although one can find enthusiasts who call themselves "constructivist" teachers, what they are doing is a stab in the dark at best. Based on the well-grounded constructivist theory that people "construct their own knowledge," many math educators in California leapt to the conclusion that children should invent their own algorithms, rather than being taught them directly. In another world, having an eight-year-old reinvent multiplication might have been a worthwhile pursuit, but in a world of scarce resources and limited time, it was impossibly inefficient. On the theory that "knowledge is socially constructed," children invented arithmetic operations in small groups, and many teachers were reluctant to intervene when the groups went off track. In many places in California, students had no textbooks—only manipulative materials. Although neither the NCTM nor the California Mathematics Framework of 1991-92 said, "Don't teach math facts and times tables," many California teachers were told by state math leaders to shun facts and algorithms, and many did.

Opinion is divided on whether masses of California teachers actually neglected the teaching of math facts and basic operations, and if so, why they strayed from time-honored teacher wisdom about the importance of nailing down the basics in the first few grades. Some believe that American teachers have become so

[4] Stevenson, Harold, Facsimile communication to Diane Ravitch, September 11, 1997.

passive and obedient that they do what they are told, and only what they are told. Others believe that those teachers who failed to teach the basics did so because the new approaches—"hands-on" math, learning in groups, and a conceptual approach—were more fun. Those with a more conspiratorial mind-set say that the tight-knit cadre of NCTM zealots in California imperiously foisted off their half-baked theories on a teacher cadre prone to faddism in the first place.

Even if constructivist theory turns out to have some useful applications in the classrooms of the future, its premature use in the California Mathematics Framework and criteria for textbook adoption overshot the capacity of most California teachers by miles. Anticipated funds for a statewide in-service training program evaporated, and in consequence, most elementary teachers lacked the conceptual understanding required to teach their students using constructivism, let alone help their students discover mathematical understandings on their own. On the 1996 mathematics test conducted by the highly regarded National Assessment of Educational Progress (NAEP), California students did very badly.

The poor test results in California ignited a backlash, and an ugly battle ensued over who got seats on the committee that would revise the mathematics framework. That revision is still in contention, and it is therefore too early to see its character. But observers say it will, at a minimum, emphasize computation in first grade and delay the use of calculators.

Successful Implementation in Maryland

Maryland, like California, was quick to embrace the NCTM approach, but its interpretation was more sensible and balanced than California's. The Maryland mathematics educators thought that students needed to be pushed to deeper levels of understanding in math and should be required to demonstrate that understanding on performance tests. That deeper knowledge included mathematical reasoning, conceptual understanding, the ability to solve novel problems, the ability to communicate about mathematics, and skill in estimation, mental calculation, statistics, and probability.

It is impossible to measure deep understanding on short-answer, machine-scored tests, and commercial testing companies were initially reluctant to expand their tests to include the open-ended items that Maryland wanted. So Maryland developed its own tests. It enlisted teachers in the development and scoring of the tests, not only as a way of getting the job done but also as a way of educating large numbers of teachers about the mathematics reform program and its nuances. That Maryland embarked on this comparatively expensive but more authentic approach, and that it has not retreated from it, owes much to the leadership of Governor William Donald Shaeffer, who presided over the educational reform effort in the early 1990s.

Professor Stevenson noted that the NCTM's 1989 document's vagueness lent itself to many interpretations. California and Maryland illustrate Professor Stevenson's point. The case can be made that the NCTM standards sent a small signal that fact mastery and basic computation should be de-emphasized, and therefore that California's excesses can be laid at the doorstep of the NCTM. But the case can also be made that the states, which aggressively claim jurisdiction over public education, are ultimately responsible for their mistakes. Under that assumption, it is California, not NCTM, that bears responsibility for the radical and disastrous approach it took, and it is Maryland, not NCTM, that can take credit for a more balanced approach and a steadier implementation plan.

Teachers

Chris Rosen started teaching in a Prince George's County (Maryland) elementary school in 1985 while in her fifties, having switched to teaching after a career as a psychologist. "I had never done very well in math in school and had a lot of trouble with statistics in my Ph.D. program, so when I got into the classroom, I thought I needed a lot of help with math. I subscribed to *The Arithmetic Teacher*, which carried articles describing the NCTM standards and how you might teach to them." She attended an NCTM-sponsored regional conference and took advantage of a month-long "Governor's Academy" in the summer. There, she fell in love with mathematics for the first time.

When Rosen taught first grade, she taught her children basic addition and subtraction, and also introduced estimation by asking students to answer simple questions: Which of two objects is heavier? What is the approximate height and width of the blackboard? Students then practiced actual measurement by weighing, measuring, and counting, checking their estimations against actual measurements. "I found that kids got better at all kinds of estimation tasks as they had more practice. I also noticed that they became better observers."

Rosen also taught math in the upper elementary grades, where she taught kids "tricks to figure things out when you don't have a calculator. It doesn't do any good to have a calculator if you don't know which operation to use," said Rosen. By 1990, the state invited Rosen and many other teachers to develop problems for the state assessment, which began in 1991.

Rosen had one encounter with a dissatisfied parent. As school ended one day, a mother drove up on a motorcycle. "She was really tough and on the offensive," Rosen reported. "What is this junk about guessing the number of jelly beans in a jar!" demanded the mother. "I'll bet when you go to the grocery store, you tote up the cost of the things you drop in the basket," said Rosen. After a few more homely examples, the mother said, "Oh, is that what this is about! I guess it's okay, then," and left.

Chris Rosen retired from teaching in 1994, still aglow with the love of mathematics and teaching, still strongly in support of NCTM mathematics. "So much calculation is rote; students spend years doing 50 long division problems a week. It is a case of overkill." Like many other reform-minded teachers of mathematics, Rosen thinks that kids are easily capable of going far beyond what most people consider the basics.

Amy Brittan has been teaching elementary school in neighboring Montgomery County, Maryland, for six years. This year, she is teaching second grade, and estimation and mental math are part of the county's curriculum for students at that level. She is teaching students how to round numbers to the nearest ten, how to make measurement estimates of nearby objects, and how to follow up the estimates with precise measurement. When Brittan teaches computation, she starts with manipulative materials to help students understand the concepts and then moves them to more abstract paper-and-pencil computation—a practice universally used by Japanese teachers.

How does Brittan know that her students are on track with basic computation? "I require my students to keep math journals, and I can tell whether they know what they're doing by reading their journals." She says that her students quickly get better at writing about math. (Quick improvement in writing about math has also been noted on NAEP.) Also, Brittan gets objective feedback about how her students are doing because the county has an instructional management system that assesses children throughout the year—each marking period in the case of elementary students. Brittan has seen some poorly written questions on those assessments, but the county has a mechanism for feedback from teachers. "I've seen problems I complained about rewritten the following year."

The county also imbeds estimation activities into the science and social studies curriculum. In science, students learn how to count animal populations—figures that can never be precise, but which can be closer to the truth through disciplined estimation techniques. In social studies, third graders must rely on mathematical estimation to learn about conditions that support populations.

Chris Rosen and Amy Brittan seem at ease with the new topics and with the underlying purposes of the reform goals and purposes. Both have taught in Maryland, which implemented mathematics reform soberly and invested the time and money to do it as well as it could be done. But they are probably not very representative of American teachers as a whole.

The most recent evidence on what teachers are actually doing in the classroom comes from the Third International Mathematics and Science Study (TIMSS) report on eighth-grade mathematics and science teaching in the United States, which concluded

> These findings suggest that the instructional habits and attitudes of U.S. mathematics teachers are only beginning to change in the direction of implementation of mathematics reform recommendations [note: "reform recommendations" here refers to the entire suite of standards proposed by NCTM]. Teachers' implementation of the reform still concentrates on isolated techniques, rather than the central message, which is to focus lessons on high-level mathematical thought.[5]

The analysis of the American data for TIMSS resulted in another report, *Splintered Vision: An Investigation of U.S. Science and Mathematics Education.*[6] The data were collected only three years after the NCTM standards were published, and the authors therefore did not offer any conclusions about these reforms. At that time, U.S. teachers were found to be generally familiar with reform goals and topics, but many merely paid lip service to them. The report observed that many techniques associated with the reform were in use, but also noted that "these techniques can be used with or without engaging students in real mathematical thinking." Accompanying videotapes of American teachers at work revealed many examples of these techniques being conducted in the absence of high-quality mathematical content.

Although much has changed since those data were collected, it is fair to say that the implementation of "thinking math" is still spotty and sometimes clumsily done. It is also true that American students' performance on NAEP mathematics assessments has been inching upward. But given the high probability that implementation is neither widespread nor smooth, champions of standards math cannot yet claim credit for the small improvement, nor can opponents claim that the standards are the cause of the mediocre performance of American students in mathematics.

[5] *Pursuing Excellence: A Study of U.S. Eighth-Grade Mathematics and Science Teaching, Learning, Curriculum, and Achievement in International Context.* Washington, DC: Office of Educational Research and Improvement, U.S. Department of Education, 1996.

[6] Schmidt, William H., McKnight, Curtis C., and Raizen, Senta A., *Splintered Vision: An Investigation of U.S. Science and Mathematics Education: Executive Summary.* Lansing, MI: U.S. National Research Center for the Third International Mathematics and Science Study, Michigan State University, 1996.

Politics

The NCTM standards, including those for estimation and mental calculation, have been drawn into an acrimonious political war over the proposed national tests in reading and mathematics. In their effort to stop President Clinton's proposed tests, opponents have visited the sins of the English/language arts standards (too much "whole language," not enough explicit phonics) on the mathematics standards by calling them "whole math." They have invoked bad memories of the failed "new math" effort of the 1960s by calling NCTM mathematics "new new math"—a dubious parallel at best. (While new math was highly abstract, NCTM math is very concrete. New math was designed to quickly generate candidates for science and engineering degrees; NCTM math is designed to produce mathematical literacy for all.)

Some leading polemicists have dubbed NCTM math as "fuzzy math" and drawn examples of estimation activities and test items with a range of right answers to prove their case for fuzziness. National testing opponents have used these examples to discredit NCTM, the tests, and the President. Through no fault of their own, estimation and mental calculation have become hostages in a war that seems more connected to partisan politics and the proposed national tests than to the practical merits of teaching these topics or to the goals of assessment.

Whatever the merits or demerits of national testing, it is clear that much of the political battle now being waged is being fueled by "spin." In the case of estimation, for example, opponents deride what they call "guess and check," which sounds frivolous. "Front-end estimation" (what teachers call it) requires students to use mental strategies to estimate the probable answer to a problem and then work the problem to get the exact answer. That sounds—and is—rigorous and useful.

In reading education, an apparent synthesis has come after a half-century of hysterical political battles between phonics and whole language partisans. Dogmatism, refusal to accept solid research findings that contradict one's position, vilification of opponents and their motives, and the invocation of unrelated ideological symbols (patriotism, tradition, discipline, and rigor versus innovation, meaning, freedom, and creativity) have characterized the reading debate throughout its chaotic history.

Similar postures and claims to higher purpose are beginning to characterize the math war and are getting in the way of honorable debate and empirical resolution. While the battle over the proposed national eight-grade mathematics test rages on, thoughtful public consideration of what should come first in the teaching of math, how much time to spend on what, how to balance skills and concepts, how to write and score open-ended test items, and what teachers need to know in order to succeed is being obscured by a veritable wall of polemics. Worthy debates are occurring about when students should be allowed to use calculators, but these debates are not in public.

Although the political critics of standards-based math may have inflamed the natural skepticism of the public about something new, they have not addressed an overpowering fact: What passes for "traditional" math in most schools does not work for most people, and the consequences of its failure are evident all around us. But a well-entrenched failure is hard to dislodge, and even the best reform proposals go through a long, ungainly phase and are slow to take root.

Parents

A front-page article in the November 2, 1997, *Washington Post* reported that "some parents lament that their youngsters can't do the simplest calculations without using beans or their fingers." Michele Zuckerman of Loudoun County, Virginia, "said that her fifth-grade son has been identified as gifted and gets straight A's in math, but that he can't make change in his head."

> I gave him $5 to go to McDonalds, and the item he wants will cost $2.38, and I ask him how much change he's supposed to bring back," she said. "He can do it if he sits down and figures it out, but a fifth-grader should be able to figure that in his head, and he can't. Until last year, I saw him using his fingers for basic subtraction.

This parent's complaint seems justified, but it is not clear whether her son's incapacity can be blamed on NCTM mathematics, as the article implies, or on the absence of it. We do not know whether he has failed to internalize the rules of subtraction (drill and practice) or whether he internalized them too soon and lost confidence in his ability to figure in his head (which can happen). And we don't know whether or not his teachers taught him mental calculation skills, or if they did, whether they taught it badly.

Whatever the truth of this particular case, parents in many places suspect that their children's teachers are lingering too long on beans, blocks, sticks, and interesting projects, waiting for the miracle of "understanding" to occur and avoiding the drill and practice on math facts that most parents (as well as the NCTM) think is necessary. A Westport, Connecticut, parent who is generally sympathetic to the objectives of the NCTM standards and does not want his child spending years and years doing long division problems is nevertheless dismayed at the assignments his elementary school child brings home.

> They are 90% art work and 10% math," he said. "There's a limited amount of time allowed for learning math, and that way doesn't seem very efficient. The kids still have to master the basics.

Also, many parents have been irked by the way in which many reform-minded math teachers have interpreted "open-ended questions." Parents usually know that problems can often be solved in more than one way (one definition of "open-ended") but balk at the idea that there can be more than one right answer (another definition). Those more-than-one-right-answer situations usually turn on ambiguities in the phrasing of word problems, in which differing interpretations can lead to different "right answers." Opinions differ on whether children should get used to the ambiguity of the real world or be protected from it until they are older.

But what if the teacher defines an "open-ended" question as "How many Froot Loops will it take to cover the floor?" A Georgia teacher who served on the state's mathematics textbook adoption committee gave that example in answer to a frustrated publisher's question: "What do *you* mean by an open-ended question?" The teacher probably thought the question was a fun exercise in estimating volume, but it probably would not qualify as a real-world application of the topic except in the household of Dennis the Menace. The interchange caused a media furor and a parent uprising, which in turn led to a change in Georgia's textbook adoption law permitting school districts to buy books not approved by the state adoption committees.

Measuring Skills in Estimation and Mental Calculation

Assessment is a chronic problem with respect to estimation and mental math. Although Maryland is satisfied with its homegrown, teacher-based solution to scoring students' written justifications on estimation questions, the measurement of estimation skills remains a problem in most other states and in the testing community.

The Iowa Test of Basic Skills has a component on estimation, and the scoring guides provide a range of right answers. Since the late 1970s, the Iowa Test has probed estimation in measurement and geometry as well as numerical estimation. For numerical estimation problems, a small element of speed is built into the test in order to force students to estimate rather than calculate on paper.

Steve Dunbar, co-author of the Iowa Test, notes that good estimators are usually good calculators. As a statistician, he believes that skill in estimation is important. "Statisticians are professional estimators," he says. But he notes that the whole area of mental math is hard for teachers to grasp, especially elementary teachers.

The National Assessment of Educational Progress (NAEP) began to include estimation items on its assessment in 1972-73. According to researcher Robert Reys of the University of Missouri, students performed poorly on those items in the first test, and the next two administrations of the NAEP math test brought the same results. Performance has always been higher on exact computation than on computational estimates.[7] Over the years, NAEP has increased the scope of estimation questions beyond whole numbers; fractions, decimals, and percents are now included, according to Reys. The answers must be within established acceptable ranges. Also, NAEP controls the time the student is given to answer the question in order to increase reliability. But timing is tricky, and determining optimal time requires prior experimentation with students.

NAEP and the Iowa Test have come up with solutions to the problem of assessing estimation. But Reys observes that "'Getting the right answers for the wrong reasons' has discouraged the authors of many standardized tests from including questions related to computational estimation."

However hard the test makers struggle with the writing of good problems, the time allowed for answering, and the determination of scoring rubrics, the fact remains that estimation and mental calculation are as alien to the paper-and-pencil format as they are to calculators. If one really wants to know whether a student is using solid estimation and mental calculation strategies, whether he is evading those strategies by scribbling furiously, or whether he is simply guessing, one must get him to think aloud or else write about how he came up with the answer. That helps the teacher a lot, but it does not answer the drums in the distance that are beating for accountability. Nor does it help researchers, who need large-scale data to discover better and best ways to teach people how to estimate and do math in their heads.

In addition, creating good test items is getting harder. Where once test writers could invent arcane or improbable situations to test students' ability to apply various operations (remember the two speeding trains problem?), today's questions are supposed to be seen by students as problems they might actually encounter in life. But according to most item writers, there are a finite number of situations that a thirteen-year-old, for example, would see as lifelike, so the same questions keep being remodeled.

The difficulty of coming up with fresh and appropriate questions is intensified by public concerns about test quality and test security. Increasingly, test makers are required to publish tests after they have been administered, and the used items must be retired and replaced by new ones in order to preserve test security.

[7] Reys, Robert E., "Evaluating Computational Estimation." In *National Council of Teachers of Mathematics Yearbook: Estimation and Mental Calculation.* Reston, VA: National Council of Teachers of Mathematics, 1986, p. 225.

The assessment dilemma will not disappear. The policy makers and test developers will have to settle for superficial measures of student progress on estimation questions, or spend the money, as Maryland does, to get at something deeper (which may suffer from technical problems), or hope that teachers will bring students along to higher levels by using their own one-on-one, informal methods of assessment.

Gaps in Teacher Education

In college mathematics departments, students who are planning to become teachers are very unlikely to encounter estimation and mental calculation or to practice it if they are not already naturally proficient at it. Such practical skills are not the province of college mathematics departments, which aim to prepare students for advanced study in pure mathematics.

Mathematics departments presume that their students already grasp underlying concepts in mathematics, although recent research suggests otherwise. A 1985 study of undergraduate math students at the University of California at Berkeley, most of whom had done well in high school calculus and geometry, revealed that most students had great difficulty explaining simple geometric problems (Shoenfeld). At Michigan State University, math majors had difficulty making sense of division with fractions, connecting mathematics to the real world, and coming up with explanations that went beyond a restatement of the rules (Ball, 1988). Borko and colleagues (1990) asked teacher candidates to generate a story problem based on this mathematical sentence: 1 3/4 divided by 1/2. Sixty-nine percent of the elementary education majors were unable to do so, but a surprising 55 percent of the mathematics majors and minors, who were planning to teach math in secondary schools, were also unable to imagine a situation that called for that division problem. All could "work the problem" according to the mechanical rules they had been taught, but in trying to devise a real-life problem, many created one that involved dividing by 2, not by 1/2. Clearly, even these math majors did not understand the concept of dividing by 1/2. Not only does their deficiency in conceptual understanding lead to error, but it predicts inadequate teaching.

The task of providing future teachers with the conceptual base of mathematics and an understanding of its applications, including the ability to estimate and calculate in their heads, has fallen to the schools of education by default. How well they do their jobs is important because there is overwhelming evidence that the more teachers know about mathematics, the better they are at teaching all aspects of the mathematics curriculum.

Dr. Kathe Rasch, Director of Graduate and Pre-Service Programs at Maryville University in St. Louis, believes that well-designed mathematics courses for future teachers can provide most students with an understanding of the connection between mental calculation and formal mathematics. Students can learn and execute strategies for mental calculation, but she says it is very hard to get them to use those strategies in other contexts—for example, using mathematics to investigate the significance of loss of life in the Civil War. "It takes people a while to get it. They balk. And the people who were good at computation balk the most," says Rasch. Asked whether it is humanly possible to educate teachers so that all the elements of "thinking math" can be well executed, she said, "Not entirely. Teachers need lifetime career development, and beginning teachers need a lot of structure." Public support for in-service training in school districts is therefore crucial, but first parents will need to be convinced that the effort will really help their children prosper in mathematics.

The Future

Most thoughtful people support goals such as the teaching of estimation and mental math, problem solving, statistics, and probability. These topics appear not only in NCTM-based textbooks but also in traditional programs such as Saxon Math. Most people agree that it is better to understand the underlying concepts than not to understand them. The remaining disputes are about what kind of pedagogy, timing, and assessment fosters the learning of those topics and the gathering of understanding.

Describing the problem as "concepts" versus "algorithms" or as "rote math" versus "understanding" may obscure its inherent complexity. Instead of thinking in those polarities, we need to ask how all three aspects of math learning—automatic facility in manipulating numbers, conceptual understanding, and application—become linked in an effective way. How much teaching time, how much practice, and what kind of recycling of topics at higher levels of sophistication do students need in order to consolidate their knowledge of particular topics?

There are, in addition, issues that go beyond standards, curriculum, pedagogy, textbooks, and tests. Professor Uri Treisman at the Charles Dana Center for Science and Mathematics Education at the University of Texas has studied high-achieving, high-poverty school districts in the state. He has seen both traditional (Saxon Math, for example) and NCTM-based programs produce unexpectedly good results, but he notes that in the successful districts, teachers in both kinds of programs supplement their teaching with activities that compensate for the weaknesses of both approaches. Treisman concludes that the school culture and the quality of the teachers are more important variables than the philosophy of math teaching.

Some elements of the reform agenda are being tarnished, fairly or unfairly, in the larger partisan struggle at the national level. Some ideas and techniques will be discredited by poor implementation or by the failure of school systems to insist that new teachers be well prepared in mathematics. There may, or may not, be another "back to the basics" movement. But even if there is, most observers believe that much of the new mathematics program will survive because it is driven by profoundly altered circumstances in the world and by a societal demand to educate a much wider swath of the American population than we are accustomed to educating.

A widely accepted, fine-tuned set of standards in mathematics, as well as in all the academic subjects, has the potential to strengthen teachers and teaching. Standards are urgently needed to guide the work of teachers toward specific goals and results. Also, widely accepted standards bring stability and depth to the work of teaching. In Japan, for example, teachers have the luxury of refining their lesson presentations over time because the national curriculum is fairly stable. There, teachers can exchange knowledge and experience not only with teachers in the same school, but also with other teachers of the same subject anywhere in the country. Also, nationally accepted standards have the potential for bringing a much-needed academic focus to teacher education. Finally, and most importantly, well-defined standards can awaken both students and parents to what all students can achieve and to what the world expects and values.

NCTM, as planned in 1989, is reviewing and integrating its three standards documents in light of the last eight years of experience. A first draft of the new version of the new standards is due in the fall of 1998. According to Glenda Lappan, professor of mathematics at Michigan State University and one of the leaders of the original effort, the new version of the standards will clarify and exemplify the standards to aid in interpretation and use:

> Having learned from attempts of teachers, administrators, and policy makers to use the original documents, we have a better idea of how the mathematical strands in the standards play out over time. The next iteration of standards will be grounded in what has been learned both from the use of the standards and from research. For example, the next version of standards will be more explicit about the need for students to develop deep understanding of algorithms and proficiency and comfort in using them. The standards are an attempt to provide help in designing programs that have focus, coherence, and balance in mathematics curriculum, teaching, and assessment, and that set higher expectations

> for students' learning. As new strategies for improving mathematics education are developed, they have to be devised so that teachers can manage the demands of moving from where they are to where they need to be to improve their own practice and their students' learning. This means teachers will need to learn new mathematics as well as new pedagogical strategies. Professional development for teachers needs to be focused, coherent, long-term, and specifically related to improving teachers' knowledge and their classroom practice.

The synthesis that eluded the field of reading for more than half the century seems to be occurring already in mathematics. A merging of the best expressions of NCTM math (which appears to bond a good many students to the study of mathematics and help them persist through difficulties) and the best expressions of traditional programs (which appear to nail down basic skills without killing student curiosity and enthusiasm for the subject) seems to be under way.

We are only beginning to take standards (national or not) seriously in the United States. The exciting and uncertain life and times of estimation and mental calculation, chronicled in this essay, are emblematic of the journey of many other standards in other disciplines as they work their way through the American educational and political system. There will be problems and misunderstandings. Some ideas, techniques, and sequences will be abandoned or adjusted based on experience. Some will be too hard to implement. Some standards will endure long enough for a critical mass of parents and teachers to really understand and value them and come to see them as both "basic" and "traditional."

Whatever the fate of particular standards, the very idea of them has refocused everybody's attention on academic outcomes for all students. The national focus of the 1960s and 1970s on civil rights, student rights, gender equity, the inclusion of handicapped children, and school finance equalization has not been diminished by the standards movement. Rather, it has brought more discipline to all of the arrangements designed to address those concerns because they are now harnessed to clear purposes and the achievement of actual results. The national discussion of standards is bringing more intelligence and coherence to curriculum, testing, teaching, and teacher education. It is a vital conversation.

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