DOCUMENT RESUME

ED 430 038 TM 029 782

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TITLE	Issues in National Assessment of Mathematics--the Transition to Higher Level Study.
PUB DATE	1999-04-00
NOTE	17p.; Paper presented at the Annual Meeting of the American Educational Research Association (Montreal, Quebec, Canada, April 19-23, 1999).
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	College Bound Students; *Educational Assessment; Foreign Countries; *Mathematics Education; National Competency Tests; Secondary Education; *Secondary School Students; *Testing Programs; *Transitional Programs
IDENTIFIERS	*England; *General Certificate of Secondary Education

ABSTRACT

A study was conducted in England in 1997 to consider materials that were available to support students making a transition from compulsory education to the study of mathematics in noncompulsory post-16 education. This report on the study also contains an outline of the basic structures of the English education system and its assessment provisions. Recent changes in the English educational assessment system and to the curriculum with the adoption of the National Curriculum have created considerable interest in the transition students face between the General Certificate of Secondary Education (GCSE) mathematics and the "A level" examinations required for university entrance. Responses by 67 colleges and schools to a questionnaire about the transition were used to select 5 institutions for case studies and interviews about practices. Specific information about difficulties schools and colleges noted about gaps between the two mathematics curricula are identified and grouped into concerns about teaching in the "pre-16" years, teaching related to the GCSE, and teaching for students after the age of 16 who are beginning their A-level courses of study. Five recommendations are made for easing students' transition between the two curricula. Appendixes contain the attainment target for mathematics, a mathematics task, and a list of A-level prerequisites in mathematics. (Contains six references.) (SLD)

Issues in National Assessment of Mathematics- the Transition to Higher Level Study

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This paper is prepared for the:
Annual Meeting of the American Educational Research Association in Montreal Canada
April 1999

Issues in National Assessment of Mathematics - the transition to higher level study

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Introduction and outline of this paper

This paper will report on the results of a study undertaken in England in 1997 (Winter et al, 1997). The initial purpose of the study was to consider the materials which were available to support students making a transition from compulsory education to the study of mathematics in non-compulsory post-16 education.

The basic structures of the English education system and its assessment will be outlined for the non-English reader as will the range of reforms in recent years which have led to concerns about the possible 'gap' which students encounter in this phase of their mathematical education.

The structure and methodology of the study will then be described and the development of the features being examined as the study proceeded will be outlined.

The major part of the paper will then be devoted to the main findings of the study and some of the data which supports them. This will include both questionnaire results and case study data which illuminated and deepened the insights gained from the questionnaire.

Conclusions will then be offered, both in terms of recommendations which were made in the English context and in general terms which have a wider applicability and can offer pointers for effective practice in other education systems.

The English assessment system at ages 16 and 18

This section of the paper will describe the main features of the assessment system, focusing on mathematics, at ages 16 and 18. The system of assessment at age 16 has undergone major changes in the last 10 years and this part will concentrate on outlining the broad structures while the next will look at some of the qualitative changes which have accompanied the structural changes.

The main system of external examinations for students at school leaving age (16 years) has changed in quite fundamental ways twice in short succession recently. First, the system of GCE O levels and CSE examinations (intended for approximately the top 25% and next 35% of the attainment range of students respectively) was superseded by a new unified examination called GCSE, the General Certificate of Secondary Education. This new assessment system incorporated elements from both of its predecessors and was intended to simplify the certification of students of school leaving age as well as introduce for all students of that age a more widely ranging set of assessment methods. This assessment aimed to recognise positive achievement from a more widely ranging set of assessment tasks, including tasks undertaken in school, often both set and marked by the student's teacher. The first examinations of the GCSE were in 1988.

The arrival of the National Curriculum (NC) in 1989 was therefore a major further upheaval at a time when teachers were still developing their understanding of the requirements of the GCSE examination system. This was the first time that England and Wales had had a statutory curriculum for all students and an accompanying statutory assessment scheme applicable to students throughout compulsory education from ages 5 to 16 - divided into Key Stages 1 to 4[1] (DES, 1989; DFE, 1995). It was decided

[1] The age and school year structure of the Key Stages is as follows:
Key Stage 1 Ages 5 to 7 Years R (Reception), 1 and 2

that GCSE examinations at 16 should continue to be the vehicle of assessment for students in Key Stage 4, although a range of tests was introduced to assess students' achievement of the attainment targets of the National Curriculum at the end of each of the other three Key Stages. GCSE assessment was therefore adapted to meet the criteria now required by the NC. An additional feature which assumed significance in relation to A level study (see next paragraph for explanation of A level examinations) was that entries to GCSE mathematics could be made at 3 levels - higher, intermediate and foundation. While the Higher syllabus contains considerably more of the advanced content which might help prepare students for A level study, it is possible to obtain a C grade[2] (widely regarded as the lowest level of 'pass') through study of the Intermediate syllabus.

During this time there was much less radical change happening to the major academic programmes for students in the period of post compulsory education from ages 16 to 18. This period of education in England and Wales is when students who wish to continue beyond school education into University or other Higher Education take courses of study leading to qualifications required for entry into those institutions. For students taking 'academic' courses of study this largely means GCE A (Advanced) level qualifications. These are specialised courses of study which students follow for two years and the grades they achieve in the final examinations are a major factor in determining which University or College they can gain admission to. Students normally follow 3 A level subjects (or occasionally 2 or 4) thus leading to a much more specialised programme of study than they had experienced in their compulsory education before age 16 when they would have studied the ten subjects of the National Curriculum. Thus, for those students studying mathematics, the subject changes from being a minor part of their experience to a major one, often accompanied by other related subjects such as Physics, leading to a strong orientation of their work towards mathematics. This is heightened for some students who choose to study 'double mathematics' i.e. 2 A levels in Mathematics and Further Mathematics. As an essential preparation for Higher Education study of mathematics there were few dramatic changes to the content of mathematics syllabuses and assessment at this level although there has been a gradual move towards modularisation of A level courses.

Because the A level courses were not changing in line with examinations pre-16, the differences, which will be examined in the next section, between the requirements of the old O levels (for higher achieving students) and the new GCSE courses in mathematics led to a perceived 'gap' developing for students making this transition. This 'gap' has led to concerns from those involved in mathematics education at many levels - in schools, universities and in government. (See, for example, Tackling the Mathematics Problem, 1995; Dearing, 1996)

Qualitative changes in mathematics education

The changes described above - of moving from one examination system to another at age 16 with no change in the succeeding exam at age 18 - need not have led to any particular difficulties if it were not for some of the particular features of the changes to the system pre-16. These changes centred on changes in emphasis on particular parts of the mathematics curriculum and changes to the procedures used in the summative assessments.

Key Stage 2	Ages 8 to 11	Years 3, 4, 5 and 6

Key Stages 1 and 2 are known as Infant and Junior and are spent in Primary School.

Key Stage 3	Ages 12 to 14	Years 7, 8 and 9
Key Stage 4	Ages 15 and 16	Years 10 and 11

Key Stages 3 and 4 are spent in Secondary School.

Post 16 students may stay in school or attend college for further non-compulsory study. In school these years are known as Years 12 and 13 or 'Sixth form'.

[2] GCSE qualifications are graded A*, A, B, C, D, E, F, G and U (ungraded) but grades A* to C are considered the equivalent to a pass at the old O level and therefore more highly valued. Different grades are available to candidates depending on which tier of examination they enter. Currently grades A*-C are available to students entering the Higher tier, grades B-E at the Intermediate tier and grades E-G at the Foundation tier.

The changes to the mathematics curriculum which are of particular concern here have two components: firstly, the algebra content of the curriculum and secondly, the new emphasis on the development of mathematical processes. Both of these changes have had 'knock on' effects on other parts of students' mathematical learning.

The changes to the algebra content of the curriculum are dealt with fully in Rosamund Sutherland's paper, also presented in this symposium, and this paper will therefore only touch on them. The previous emphasis, particularly for higher attaining students and therefore those most likely to continue to higher studies in mathematics, was on the development of fluency in a wide range of manipulative techniques. This was achieved by a great deal of practice in the use of techniques which were often taught in quite a formal way and by students' use of learnt, often memorised, rules and formulae. This skill was valued by those teaching mathematics at higher levels, i.e. A level and higher. The new syllabuses had a broader view of the use of algebra, placing more emphasis on aspects such as generational activities, pattern formation and observation and trial-and-improvement methods of solution. While these new skills are also considered to be important, the loss of a high level of technical skill is perceived by some to have been a factor in the widening of the 'gap' students face when beginning A level courses (see Teaching and Learning Algebra pre-19, 1997)

The other major change to the curriculum pre-16 is the introduction of an emphasis on 'Using and Applying Mathematics'. This is the title of one of the Attainment Targets[3] in the National Curriculum. These Attainment Targets describe the main areas of the mathematics curriculum which are to be taught and assessed during compulsory education. Using and Applying has become known as the 'process' attainment target, i.e. that which describes the way in which students are expected to work mathematically (see appendix 1). The programme of study for this aspect of mathematics is divided into three main areas:

Making and monitoring decisions to solve problems;
Communicating mathematically; and
Developing skills of mathematical reasoning.

These 'strands' all develop throughout the Key Stages as students become more sophisticated in their use of mathematical reasoning and show more independence in their use of the mathematics they have learned in the other attainment targets. While the effects of the changes previously described in algebra are often considered detrimental to students' performance in higher mathematics, the introduction of these process skills has generally been approved by those teaching students at higher levels as their independence and ability to tackle unfamiliar tasks is often improved.

Accompanying the changes in curriculum content have been changes to assessment procedures. Clearly, the introduction of Using and Applying Mathematics, as a compulsory part of the curriculum to be reported on at the end of each Key Stage, required appropriate forms of assessment to monitor students' progress in it. The usual written tests were widely felt to be inappropriate to this sort of activity and unable to adequately represent students' engagement with the statements. It is difficult to design an examination question to test students' ability to: 'select, trial and evaluate a variety of possible approaches', for example. GCSE examinations, introduced shortly before the National Curriculum, already incorporated a compulsory element of assessed coursework[4] and the NC clearly reinforced the need for this wider range of assessment methods. Students' assessment in GCSE mathematics therefore includes a compulsory assessment of Using and Applying Mathematics, normally through the assessment of a small number of tasks each undertaken over a short period of time, often about two weeks, during lessons and homework, under the supervision of the teacher. These tasks provide students with the opportunity to make decisions about the mathematics they use, to communicate it effectively and to apply their skills in reasoning. (See appendix 2 for some examples). Since these tasks need to address the requirement that students 'use and apply mathematics in practical tasks, in real-life problems and within mathematics itself' they give the

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[3] The National Curriculum now (following changes to early more complicated models) has 4 Attainment Targets: 1: Using and Applying Mathematics, 2: Number and Algebra, 3: Shape, Space and Measures and 4: Handling Data. The mathematical content is arranged for assessment purposes in levels - now 8 levels plus 'exceptional performance', but in earlier versions (referred to later in this paper) 10 levels.

[4] Coursework can account for up to 20% of the marks in a GCSE examination.

opportunity for assessment to address mathematics in contexts beyond the abstract environment of mathematics and to broaden students' understanding of the contexts in which mathematics is useful. The skills that students gain by working on such tasks are also found to be an advantage to them in their further study of mathematics.

Another important change in assessment of mathematics at GCSE level is the increased use of questions divided into smaller sub-tasks. The need to assess against the tight criteria in the attainment targets has led to this change, so that the content of papers can be itemised against the attainment targets it is intended to assess. This is certainly perceived by those teaching mathematics at higher levels to be a problem, leading, they believe, to students being unable to tackle 'multi-step' problems of the type they will face during A level study.

All of these factors go some way to explaining why there has been considerable interest in the transition students now face between GCSE mathematics and A level mathematics. In the next section of this paper I will describe the study undertaken at the University of Bristol to look into some aspects of this transition.

Study of the transition of students from GCSE to A level mathematics

This study was carried out on behalf of SCAA (School Curriculum and Assessment Authority)[5] in order to address a recommendation of a previous report reviewing the qualifications for 16-19 year olds in England and Wales (Dearing, 1996). The recommendation was that the range of curriculum materials available to support courses to bridge the gap between GCSE and A level should be reviewed. This formed the brief for the research study.

The areas which were examined included GCSE and A level syllabuses, publishers' catalogues of commercially produced materials and teachers' views of the extent and nature of any 'gap' and the steps they took to support students through it.

A questionnaire was distributed to a range of 200 schools and colleges and, following responses, numbering 67, to this questionnaire a small number (5) were selected for further study through interviews and the development of small scale case studies of practice.

The questionnaire was designed to identify views on the issues faced by different institutions in relation to the transition of mathematics students. Respondents were asked what they considered the three most important **mathematical** pre-requisites (bold in original questionnaire) for success on an A level course and asked to say why each was important. The results are given in Appendix 3. What was most interesting about the responses to this question was the wide range of pre-requisites identified. Despite the emphasis on mathematical pre-requisites, many respondents chose to highlight other requirements they felt important such as 'coverage of the Higher[6] syllabus', 'commitment', 'logic and reasoning', 'enjoyment of mathematics' and 'problem solving'. The most popular response was concerned with algebraic skills, as could have been expected, but this range of responses showed teachers' breadth of perceptions of what made a successful mathematician at A level.

Another question which yielded important results was one which asked respondents whether they perceived a 'gap' between GCSE and A level mathematics:

34 perceived a gap
22 did not perceive a gap
5 said 'yes and no'
6 had no view

These comments need to be considered in the light of responses to the next question, concerning strategies employed to deal with a perceived gap or factors preventing such a gap arising. Schools would give similar responses having given opposite responses to the previous question. For example, a

[5] SCAA, the government body responsible for the curriculum and its assessment in schools, has now been merged with the body responsible for the administration of vocational qualifications and renamed QCA (Qualifications and Curriculum Authority).
[6] See footnote 2.

school saying they did perceive a gap might say they therefore used a modular[7] A level syllabus to help overcome it while a school saying they did not perceive a gap would say this was because they used a modular A level. It is therefore difficult to isolate cause from effect and any perceived problem from the strategies used to deal with it. More detail will be given on the responses to this question in a later consideration of the report's findings.

The case studies will also be dealt with in more detail later but it is important to state that they were seen by the research team as a vital part of the project since they enabled a discussion of effective practice in a meaningful and contextualised way. We found that the factors which seemed to contribute to successful teaching at A level were closely linked to what mathematics was to the teachers concerned. This was an outcome of the research which was followed up in the small scale study reported by Laurinda Brown in her paper in this symposium.

As can be seen from the description given above, the initial brief of looking at the provision of resources to support the transition to A level was, in the final report, a less important factor than others relating to teaching issues. The concerns of teachers in providing an effective transition for students lay much less in the materials available than in the range of experiences they provided for students before, during and after the transition to further study. This led the research team to the conclusion that effective teaching was the key to a successful experience for students and that the development of teachers' understanding was a more important goal than the development of curriculum materials. The next sections will consider the three main areas of findings of the report and offer some of the data which support them.

Findings of the research

These issues fall into three main areas which will be dealt with separately here. They are as follows:

- **Teaching pre-16**. This area relates to factors in the organisation and content of the teaching of mathematics to students pre-16 and the effects these factors have on the ease of students' transition to A level study.
- **GCSE factors**. This area relates to factors linked to the assessment procedures and the skills learned in GCSE mathematics.
- **Teaching post-16**. This area relates to factors concerned with teaching practices employed with students beginning their A level courses of study and how these can serve to reduce any transition difficulties students may face.

In each of these sections quotes from teachers interviewed as part of the development of case studies will be used to illustrate their views and attitudes to the relevant issues.

Teaching mathematics pre-16

Five factors were identified in this area. The issues are concerned with the extent to which schools see the goal of GCSE grades as being the primary target at which they and their students are aiming during their mathematics work in Key Stage 4. This attitude is encouraged by some of the regulations applied to schools - in particular the fact that GCSE results are published and receive major prominence in both local and national media. These results are used to compare explicitly the 'performance' of schools and, since funding directly follows student numbers, any effect of attracting students to a school through achieving good results is strongly desired by schools. This leads to the dangerous possibility that a student's needs for preparation for further study will be neglected in the push for the shorter term goals of high grades at GCSE. A level grades are also published but receive less attention than the fairly easily understood statistic of '% of students gaining 5 or more A* to C grades at GCSE'. Since one of the aims of the English education system at the moment is to increase the proportion of young people who stay in full time education post-16, this unnatural discontinuity seems to be counter productive.

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[7] Modular syllabuses are those which divide content into smaller units which are assessed separately at the end of the teaching of the unit rather than at the end of the whole course.

The five factors are listed below, along with a sample of statements by teachers, from both the questionnaires and interviews, which relate to each. In addition, where applicable, statistics from the questionnaire data are quoted.

1. *Institutions which planned for their students' learning needs post-16 saw less of a gap than those which saw Key Stage 4 and post-16 teaching as quite separate.*

Several respondents said that they began teaching topics during the GCSE course which would be needed early in A level: 'Students who take the Extension Module at GCSE are already experiencing some of the topics in the first Pure Maths module', 'We stress which topic is A level and will be seen again', 'Many of the topics that students initially cover in A level courses are covered by top sets in GCSE', 'We still do geometrical proofs occasionally and set coursework requiring algebraic proof', 'You can do a module or two early, in year 11. Also, if you've got a syllabus that is suitable to do it and you've got teaching time, you can integrate elements of the A level course into the GCSE course', 'There's some stuff about uncertainty in Pure which seems to follow on from work on Data Handling in the National Curriculum so that seems fine.'

One school took this strategy to great lengths: 'Problems are alleviated if students who are capable of achieving an advanced level qualification are identified early. At the end of year 8, sixty+ candidates are chosen to complete the GCSE syllabus by the end of year 10 and may go on to study 2 A level modules, without too much pressure, in year 11.'

Some kept high expectations of all students to ensure no-one was disadvantaged: 'Until year 11 mocks everyone works towards Higher level', 'Not sticking to the level of questions that GCSE requires but keeping A level in mind.'

'We try to make it clear what A level involves!' - from a school which tried to emphasise the continuous nature of students' mathematical learning. Similarly, 'Almost all GCSE teachers also teach A level, hence they understand the demands and requirements of A level.'

Schools with large numbers entering A level courses are able to exploit this: 'An awareness that the majority of yr. 11 students will want to study A level influences our teaching - i.e. we will be looking ahead rather than seeing level 9/10 as an end point.'

2. *Previous experience of some Higher tier GCSE content is important for students starting A level.*

Several teachers felt that if students experienced sufficient Higher level material they would not have problems: 'If the GCSE course has substantial numbers doing level 10 work, I see no problem', 'At A* level they do sine and cosine rule - things fall into place more quickly at students spend more time 'playing' with sine and cosine.' 11 respondents stated that some coverage of Higher level content was an essential pre-requisite.

3. *Competence in using algebraic techniques is particularly important.*

'Greater emphasis on algebra than NC requires' was a strategy used by several teachers. 'Good quantity of algebra every year from year 7', ' Algebra is taught quite formally and the basic techniques are reinforced so an understanding is built up', 'Covered in depth and frequently', 'Extend the most able algebraically' - these comments all relate to an increased emphasis on algebra over and above what is considered necessary at GCSE level. 44 out of 67 respondents included algebraic skills as one of their three chosen essential pre-requisites for success at A level.

4. *Teaching Higher tier content to students who will take the Intermediate tier examination.*

'Intermediate candidates learn some Higher work in extra classes in year 11', 'Teach level 9 and 10 algebra to those taking Intermediate level' - both these teachers are preparing students for possible A level well in advance of GCSE entry.

5. *Teaching in Key Stages 3 and 4 is seen as a step towards further learning rather than an end in itself.*

Some teachers' responses in this area indicated that no particular examination was their aim, rather they were aiming to enable students to continue to learn mathematics: 'Teaching '<u>through</u>' the GCSE exam, e.g. summing series, first ideas of calculus, etc. Include some 'self-teaching' of concepts at KS4. Considerable emphasis on problem solving techniques at KS4.'

'The whole testing, setting, homework, etc. ethos takes improvement very seriously and throughout all the sets pupils are encouraged to resolve problems as they occur' - students' own learning is stated to be the priority in this school. A similar aim is expressed rather differently by another teacher: 'One of the aims of the faculty is to develop mathematical thinking for themselves. We avoid giving tricks/short cuts/routines. Students have to struggle for themselves on a problem. Allow students to share ideas, work in small groups to encourage participation. Students are exposed to problem solving environment from year 7.'

A teacher in a post-16 college wanted to see a more flexible approach to problems from students on entry: 'It would be useful if students came with the idea that problems can be reduced to equations which can then be solved analytically. They often expect to be able to work out answers directly or by trial and error.'

GCSE factors

In this part of the outcomes factors are considered relating to the structures of the GCSE examination itself and aspects of the particular mathematics being learnt by students. Both positive and negative factors were identified here, with some of the new 'process' aspects of the National Curriculum being seen as a positive advantage for students beginning A level study. However, an important negative factor has been introduced by the importance to schools of their GCSE results. Since these are published and affect perceptions of the standing of schools, the highest achievement in GCSE examinations can assume undue importance over the natural progression of students' learning.

1. *GCSE examinations and content were seen as intruding into the natural progression of mathematics learning for some higher attaining students.*

One clearly angry and resentful teacher stated: 'Main reason is stupid emphasis on statistics in National Curriculum. It accounts for 25% of available written paper marks - and should be returned to the roughly 10% found before the National Curriculum was foisted on schools.' Although this comment is not directly related to GCSE content it reflects similar comments relating to the way in which National Curriculum content has changed and diverted time and attention away from algebraic activities. Referring to a change in the National Curriculum which was happening at the time of the questionnaire, one teacher said, 'It helps that 1998 sees a change in GCSE so that more algebra can be tested at the highest levels.'

One teacher said, ' In the abstract - there's GCSE, there's A level - gap - let's add some content that will fill that gap. But the content that was put in did not make a linear progression from GCSE. It went tangentially sideways. So you add in matrices which led to nothing. It appears in A level, but it's not even in the common core. It appears in Further Maths at A level in many syllabuses' and 'The amount of time you've got - you'll be spending about two lessons on the Normal Distribution if you're lucky, which is nonsense. You may be able to teach people on a memory basis to pass the hoop of a GCSE question but in terms of learning mathematics it's absolutely pointless.'

2. *GCSE coursework has been used as a way of tackling some of the advanced topics which it is useful for students to meet. It also develops their problem solving skills.*

One teacher cited GCSE coursework as an important factor which prevented problems arising for students in the transition to A level. 4 questionnaire respondents said that they do work on developing thinking during the GCSE course.

One teacher said '...there's no reason why you can't double up a piece of coursework for GCSE and A level. We're not wanting GCSE to falsely limit a student and the coursework is an area in which they can go as far as they please, if they're showing above and beyond level 10 then this is okay. The coursework has to be well chosen to bridge that sort of gap.'

'The sort of lines I'll be looking at are opportunities for them to start thinking about things A level-ish. That seems to be the most clear way to get new mathematics.'

'Another one has done some work on arithmetic progressions and has ended up deriving all the things for that.... It's more the coursework task - laying a fertile sowing ground for when they're in the sixth form.'

3. *Schools which see their aims for students solely in terms of achieving the highest possible GCSE grade can narrow the mathematical syllabus which they meet.*

'There's a tension between covering all of the syllabus to be fair and knowing that they can get an A with 50% of the marks, isn't there? Wasn't the idea of GCSE to test them on what they do know and yet at the extension tier - getting 50% of the marks to get the grade now doesn't seem right.'

'We very much teach for the GCSE exam. Supporting pupils to do well in this exam is considered very important. This could detract from A level but the school wants pupils to do as well as possible in GCSE.'
'I think they probably spend far too long in their final year revising... so that they get out of the habit of learning new things...but it doesn't seem to enable them to see the course as a whole.'

Teaching mathematics post-16

This area was the one in which most respondents to the questionnaire had comments to make. This indicated part of the problem - that solutions are employed rather late by tackling difficulties at the start of the A level course, rather than developing students' skills earlier in order to avoid problems arising.
One factor not listed below was the effect of the particular course of study being followed. Several different A level syllabuses are available to schools and colleges and many felt that the choice of the right one for their context was vital. In particular, many institutions choose modular[8] syllabuses which they feel give students a 'gentle' start to the work and a manageable workload from the start. A level is seen as being a series of small hurdles rather than one major and perhaps daunting one. Those who felt that choice of A level syllabus was a factor in easing transition all used a modular syllabus.

1. ***Uncertainty about which students will be taking the course until late summer prevents some transition measures being taken earlier.***
This is a particular problem for schools and colleges taking new students who have come from other institutions. 'We were going to do an induction course in the summer but it just wasn't possible.' 'I also have a 'problem' in the sense that we receive students from our own institution and several others.' 'Many of the 'possible solutions' are difficult to implement when an F.E. college has no say in their prior learning.'
The other side of this problem is experienced by schools who send their students to another institution for A level courses and feel they cannot prepare them properly: 'We have little contact/liaison with the sixth form providers in our area and so do not know how to help our students prepare for A level. A significant number drop out because A level was 'too hard'.'

2. ***Some schools and colleges offer an induction period or extra materials for students to work on before the course starts, or in its first few weeks.***
This solution was one used by a very large number of respondents to deal with a perceived 'gap' or to prevent one from arising. Some typical descriptions are: 'Everyone wanting to do A level must complete an in-house algebra booklet before the start of year 12.' 'Bridging pack - school produced resource given out at Induction in July for work during summer holiday - on algebra and trig.' 'We do a six week bridging course for all first year A level students.' 'Pre-A level course - improving algebraic and logical reasoning.' 'Bridging the gap course after GCSE for those not confident with level 8+ materials.'
Some schools use this as an activity to help students understand the expectations of the course they are about to begin: 'We set a 'vacation pre-A level' holiday 'homework' booklet which we collect in in September. We get students to do a self analysis of this section by section as well as marking it.' 'After GCSEs have finished, the students are expected to return to school in June for a bridging course of material.... This is in the hope of showing Intermediate candidates the level/quantity they are about to face.'

3. ***Commercially produced materials are not widely used, although colleges make more use of materials to support students' independent study.***
Several schools, as noted above, produce their own in-house materials focusing on the areas of mathematics they feel are most of concern. Colleges, with more emphasis in their teaching methods on independent study, sometimes offer commercial materials to students to work on independently: 'We direct students to reference materials.' 'Students who have not done Higher GCSE are directed, when the need is identified, to certain elements of the GCSE course and work on self-study material to help close any gap.' 'A range of self-study materials are located in the Maths Workshop to support the

[8] See footnote 7.

work of the taught classes.' 'Any students can 'drop-in' to the lunch time Workshop when they feel the need.'

An alternative approach is to use a formal course at a lower level than A level to introduce students to higher level work: 'We are hoping to offer the MEI Foundations of Advanced Mathematics one year course in order to prepare the 'weaker' candidates and provide a sound foundation for progression to an A level course.' 'Offer BTEC National Level III, Maths for Higher Education as an alternative, not necessarily for weaker students, but for students who are able to work independently but yet do not tend to do well in exams.'

4. *Early parts of the A level course are often seen as a transition period easing students into new ways of working.*

The early part of an A level course is often seen as a transition period, with time being taken from the course to teach or revise some lower level material, partly to boost students' confidence and partly to establish 'foundations': 'A brief revision and assessment programme at the beginning of the A level course.' ' Only study P1 module at the start of year 12. (Simple grounding)' 'At the start of the course, revision and bridging work on algebra and trig.' 'Several weeks of work on algebra at start of A level course.' 'Give a course on basics in algebra particularly for first month or so of A level course.'

Starting from 'where the students are' is recognised as important by many: 'Track to what the students currently know and understand and take it from there.' 'What is needed is for A level teaching to start where the students are, not where we might like them to be.' 'Constantly I hark back to GCSE but pushing things forward.'

5. *Teachers value process skills and personal qualities as highly as some mathematical knowledge.*

This statement is illustrated by the statistics from the questionnaire (in Appendix 3). Comments include: 'Maturity and motivation play a great part in students' succeeding' 'Students are encouraged to work together.' 'Making students aware of the importance of the personal skills and encouragement to analyse their approach.'

One school's rather hard line approach is to 'present a heavy workload initially and then the less determined 'drop out'!'

6. *Teachers' and students' views of mathematics are important in the way the transition is handled and a strong shared sense of the purpose of mathematics can reduce students' difficulties.*

This factor emerged most strongly in the more detailed interviews which formed the case studies, but questionnaire responses also reflected the role of the teachers' views and beliefs about mathematics: 'Teachers show enthusiasm for their subject.' 'Problems arise when teachers have not changed their style or moved to make allowance for the NC.' 'Major factor is approach of teachers in early months of SMP course.'

The case study interviews allowed us to probe the views of the teachers involved and to develop a picture of what it was which motivated their teaching. The further work which developed from this aspect of the study is considered in more detail in Laurinda Brown's paper. What is important here is the central role teachers have in supporting the students they work with at this level. Students' views of mathematics are developing through their school careers and at this point, when they first work with deeper ideas and stronger requirements of rigour, the strength and coherence of their teacher's view of what mathematics is for helps them to develop a firm foundation of their own.

Recommendations and conclusions

The five recommendations of the study will now be stated and then the wider issues concerning the applicability of these findings to other contexts will be discussed.

Recommendations:

1. That the influence of GCSE examinations on prospective A level students' progression in mathematics be recognised. In particular, the effects of tiering on the appropriateness of students' experience of algebraic techniques and the effects on continuity and progression of the long period between the examinations and the start of A level courses should be acknowledged.
2. That advice be given to schools and colleges concerning the period between GCSE and A level so that this time can usefully contribute to progress in students' mathematical learning.

3. That, in any evaluations of higher levels of National Curriculum content, consideration be given to the appropriateness of topics covered so that progression to A level is facilitated.
4. That successful ways of working during the early parts of A level courses, such as those identified in this study, be more widely disseminated.
5. That development work should be focused on the areas mentioned above rather than on the production of additional curriculum materials.

The points which can be drawn out of this study for a wider audience concern the appropriateness with which courses which we offer students relate to one another. Are there clear links which make transitions smooth? Does the content of one course prepare students effectively for the mathematics they will next meet? Do the requirements for accountability and reporting of performance hinder effective learning through emphasising unhelpful points in a student's learning? How do the teacher's actions and underlying beliefs affect the students' experiences?

Our evidence is that in the transition from GCSE to A level mathematics many of the features of the English system of reporting of results are not supportive, although creative solutions are found which enable students to succeed. Many teachers' views of mathematics as a continually developing experience for students leads them to work with their students in ways which minimise transition difficulties - by developing skills which will be needed later and by working to develop students' independent learning for example. This is an issue of far wider applicability than just this transition point in the English system. What support do students in a range of contexts need in order to ease any transition between courses? How can courses be designed to encourage this support while also recognising the important effects that individual teachers have and maximising the positive benefits of these effects? How can teacher development nurture and develop this 'story' of mathematics which is so important to effective teaching?

The issues of mismatch in content are also dealt with creatively by many teachers, using opportunities to introduce advanced ideas as an explicit introduction to the idea of A level mathematics at an early stage. This factor also seems to offer opportunities in other contexts. Taking a view of the learner's overall experience rather than simply focusing on the immediate mathematics being learned is a strategy which can be effective in many teaching situations.

Students' skills in Using and Applying mathematics are, for many teachers, an important part of their preparation for further study. This is surely true in a wide range of contexts, given the need for independence, decision making, communication and reasoning skills in all areas of life and further study. In this respect many of the changes which have been made to the English curriculum in the last ten or so years can be recognised for the positive effects they have had on students' longer term learning potential. The shorter term issues of 'content' are being addressed and this needs to be done without losing sight of the advantages inherent in these 'process' parts of the curriculum.

The teaching and learning issues which arose out of this small piece of research were wide ranging and, we believe, applicable to a wide variety of different contexts. The teacher is a vital factor in the process and the way in which the teacher chooses to work mathematically with his/her students because of the teacher's beliefs and experiences has a major effect on the learning of his/her students. This is explored further in Laurinda Brown's paper.

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■ Level 1

Pupils use mathematics as an integral part of classroom activities. They represent their work with objects or pictures and discuss it. They recognise and use a simple pattern or relationship, usually based on their experience.

■ Level 2

Pupils select the mathematics for some classroom activities. They discuss their work using familiar mathematical language and are beginning to represent it using symbols and simple diagrams. They ask and respond appropriately to questions including 'What would happen if...?'.

■ Level 3

Pupils try different approaches and find ways of overcoming difficulties that arise when they are solving problems. They are beginning to organise their work and check results. Pupils discuss their mathematical work and are beginning to explain their thinking. They use and interpret mathematical symbols and diagrams. Pupils show that they understand a general statement by finding particular examples that match it.

■ Level 4

Pupils are developing their own strategies for solving problems and are using these strategies both in working within mathematics and in applying mathematics to practical contexts. They present information and results in a clear and organised way, explaining the reasons for their presentation. They search for a pattern by trying out ideas of their own.

■ Level 5

In order to carry through tasks and solve mathematical problems, pupils identify and obtain necessary information; they check their results, considering whether these are sensible. Pupils show understanding of situations by describing them mathematically using symbols, words and diagrams. They make general statements of their own, based on evidence they have produced, and give an explanation of their reasoning.

■ Level 6

Pupils carry through substantial tasks and solve quite complex problems by breaking them down into smaller, more manageable tasks. They interpret, discuss and synthesise information presented in a variety of mathematical forms. Pupils' writing explains and informs their use of diagrams. Pupils are beginning to give a mathematical justification for their generalisations; they test them by checking particular cases.

■ Level 7

Starting from problems or contexts that have been presented to them, pupils introduce questions of their own, which generate fuller solutions. They examine critically and justify their choice of mathematical presentation, considering alternative approaches and explaining improvements they have made. Pupils justify their generalisations or solutions, showing some insight into the mathematical structure of the situation being investigated. They appreciate the difference between mathematical explanation and experimental evidence.

■ Level 8

Pupils develop and follow alternative approaches. They reflect on their own lines of enquiry when exploring mathematical tasks; in doing so they introduce and use a range of mathematical techniques. Pupils convey mathematical meaning through consistent use of symbols. They examine generalisations or solutions reached in an activity, commenting constructively on the reasoning and logic employed, and make further progress in the activity as a result.

■ Exceptional performance

Pupils give reasons for the choices they make when investigating within mathematics itself or when using mathematics to analyse tasks; these reasons explain why particular lines of enquiry are followed and others rejected. Pupils apply the mathematics they know in familiar and unfamiliar contexts. Pupils use mathematical language and symbols effectively in presenting a convincing reasoned argument. Their reports include mathematical justifications, explaining their solutions to problems involving a number of features or variables.

Appendix 2

TASK A: Boots

A boot size is the sum of the numbers within a boot.

Boot size $\mathbf{B}_5$ is shown in this square grid.

$\mathbf{B}_5 = 5 + 9 + 13 + 14 = 41$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Square grids come in different sizes.

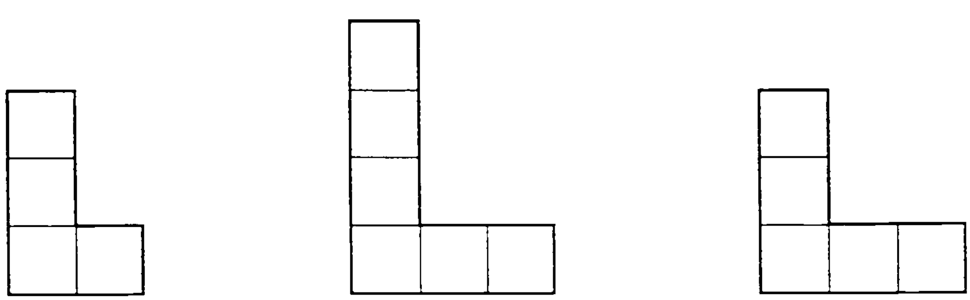
What is the boot size, $\mathbf{B}_5$, in this square grid?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Investigate boot sizes in different square grids.

Boots can come in many shapes.

Here are 3 examples.



Investigate the sizes of different shaped boots in different grids.

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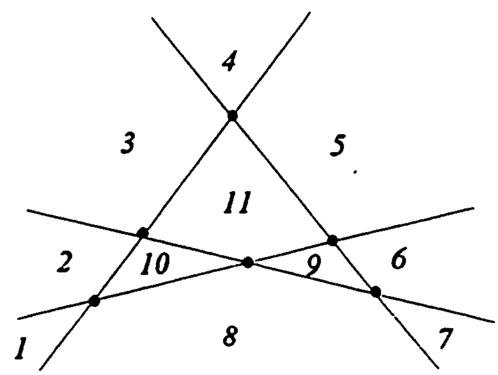
The diagram shows 4 lines which cross over.



They make 6 cross-over points which are marked with dots. They make 11 regions which are marked 1 to 11.

The regions marked 9, 10 and 11 are **closed**. The other regions are open.

1. Draw any number of lines which cross over.

   Count and record,

   (i) the number of cross-over points,
   (ii) the number of regions,

   and

   (iii) say which regions are closed and which regions are open.

2. **Investigate** the relationships between the number of lines, the maximum number of cross-over points and the maximum number of regions

Appendix 3

Essential pre-requisites for A level

The following is a summary of responses to question 5 in the questionnaire, in which institutions were asked to identify the three most important **mathematical** pre-requisites for success on an A level course. Responses were grouped into broad areas, e.g. algebraic, problem solving, determination, etc., and then these were grouped under the four headings below. The number given is the number of times an area was mentioned.

Mathematical content areas:

Algebraic	44
Trigonometry	7
Numerical	5
Quadratics	2
'Basic' concepts	2
Geometric	1
Gradients	1

Mathematical process areas:

Logic and reasoning	14
Problem solving	9
Proof	5
Generalising	4
Multi-step problems	3
Precision and rigour	2
Writing maths	2
Modelling	1
Estimation/checking	1
Reading maths	1

Prior mathematical attainment or experience:

Coverage of higher syllabus	11
Minimum A/B GCSE	6
C or above at GCSE	5
Minimum grade A GCSE	2
Teacher recommendation	1
Good GCSE coursework	1

Personal qualities:

Commitment	14
Enjoyment of maths	9
Determination	9
Willingness to have a go	4
Understanding	3
Confidence	2
Study skills	2
Maturity/initiative	1
Aptitude	1
'Feel' for maths	1

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