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ABSTRACT

This report researches the use of a standard deck of playing cards in entry-level college mathematics classrooms. The study looks at published research on the use of cards, and reviews pedagogic concerns directly related to the implementation of playing cards in the classroom--including the appropriateness of manipulatives, the link to cooperative learning, and societal perceptions of card playing. The article discusses natural connections and relationships between cards and topics in current college mathematics curricula, and suggests some diverse activities and demonstrations that directly address topics and techniques in contemporary mathematics. The study also demonstrates that a deck of cards can be useful in modeling, motivating, and teaching the techniques of: elementary probability, elementary statistics, set theory, abstract and linear algebra and their notations, business mathematics, Game Theory, and technical mathematics. Content-focused interactive activities present an alternative approach for math-anxious adults and non-traditional students, who may be disenchanted with a traditional presentation of the mathematics curriculum. Figures 1-3 illustrate major steps in the evolution of the deck as a model and manipulative; generalizations and notions required to access the methods of algebra through playing cards; and kinds of cards and symmetries prior to 19th century advances. (AS)

Cards in the Classroom:

Mathematics and Methods

Robert N. Baker
Assistant Professor of Mathematics
University of Alaska Southeast-Ketchikan Campus

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University of Alaska Southeast-Ketchikan Campus
2600 Seventh Avenue
Ketchikan, Alaska 99901

Campus phone: 907-225-6177
Office phone: 907-228-4502
FAX: 907-225-3624
email: robert.baker@uas.alaska.edu

Abstract

This paper is a report from work, research and reflections on the use of a standard deck of playing cards in entry level college mathematics classrooms. It begins with a look at published research on the use of cards, and follows with brief overviews of pedagogic concerns directly related to implementation of playing cards in the classroom: manipulatives, cooperative learning, and social influences. The article then articulates natural connections and relationships between cards and topics in current college mathematics curricula. Along the way it suggests some of the many and diverse activities and demonstrations which directly address--through illustration or motivation--topics and techniques of contemporary mathematics.

Introduction

It was chance that I introduced the set of playing cards in the classroom--while an adjunct at The University of Montana. Since then I have been more deliberate. The standard deck of playing cards is an inexpensive manipulative that is rich in mathematics and is found commonly around the world. It also has the potential to be a very powerful tool for teachers and students of mathematics.

The deck motivates concerns at the foundation-level of several branches of mathematics. It can illustrate simple concepts for developmental students. As complexity increases in the curriculum, the deck's structure is rich enough that prior activities can be extended to illustrate the new complications. It is useful in modeling, motivating and teaching the techniques of: elementary probability; elementary statistics; set theory; abstract and linear algebra and their notations; business mathematics; Game Theory; and technical mathematics.

Content-focused interactive activities present an alternative approach for students disenchanted with a traditional presentation of the mathematics curriculum. The cards enable a concrete and user-friendly introduction to, and motivator of, the more formal and abstract concerns of several branches of academic mathematics. They can be used in the cooperative setting for skills practice, concept and technique illustration and interaction, and in open explorations. Once a notation has been developed, the deck is also useful for concept development through traditional demonstrations in the lecture format.

The standard deck of playing cards is a natural tool for mathematics teachers in two-year and community colleges. It is simple to use, but not so simplistic as to be insulting to adults, and it is not mathematically trivial. Decks are inexpensive, and commonly available. The cards and games for them can illustrate concepts spanning the first two to three years of "non-math-major" college mathematics. The use of cards as a focus of mathematical content enables techniques for the classroom that are particularly helpful among the math-anxious, adults and non-traditional students.

Pedagogy

On research

An exhaustive review of the literature, both electronic and text-based, led in 1995 to the sure conclusion that there are no papers or reports published in the past couple decades that relate the standard deck of playing cards to the standard American mathematics curriculum. A few articles had been written about using the deck to teach elementary pattern recognition. A few articles and conference-presenters have illustrated card tricks as motivators to learn mathematics. Many probability texts give problems with cards to demonstrate counting techniques; these assume student familiarity with the deck and its structure. (Baker, 1995) In recent years I have noticed an increase in the number of conference presentations related to the use of games and manipulatives in the secondary and post-secondary mathematics classroom, but still I have seen nothing published.

This paper presents some of my findings and uses for the deck. My results lead me to encourage the mathematically-valid use of cards and card-games in the mathematics classrooms of America. My chance introduction of the deck to the classroom in 1990 has led me to continually refine and enhance the content-based usefulness of the deck in 0-, 100- and 200-level college mathematics classes. (I must acknowledge the support and encouragement given me in these efforts by the mathematics faculty, and particularly Dr. Johnny Lott, at The University of

Montana, and the faculty and administration at the University of Alaska Southeast-Ketchikan Campus). Student response has been universally positive in both their content-achievement and attitude-toward-mathematics. These results have been presented variously in several professional venues and have received enthusiastic response from peers (including conferences by the Mathematical Association of America and Montana Academy of Sciences in 1996, National Council of Teachers of Mathematics in 1997 and 1999, and American Mathematical Association of Two-Year Colleges in 1998).

The mathematical power of the deck comes not only from what it is (a discrete set with several equivalence classes built-in), but also from how it is used. The educational use of the deck has natural ties to two concerns of educational theory and practice: manipulatives and cooperative learning. Research in these realms agrees with my own results--reasoned and content-centered use of manipulatives and cooperative activities yields positive results.

I have found that content-centered cooperative activities with the deck are especially successful with adults and "non-traditional" students of mathematics. Many such students are already familiar with the deck, and some games with it--to many of these students it is a rare and valuable opportunity to be able to enhance the class with their knowledge and experience. The deck itself has a diverse, multi-cultural history of use and development--crossing race, gender, religion, economic class, and culture in its 1000-year history.

Cards are popularly used for intellectual diversion by people for diverse reasons. Card games are a low-sweat activity during hot summers in the south; in the north, they provide an indoor activity to help folks stave off cabin fever during long, cold winters. In my classrooms, they are used as a carrier of mathematics and civility.

On manipulatives

A popular question of late: Why, all of a sudden, are all college students required to take math classes? Part of the answer relates directly to job training. Many employers of well-paying jobs require such employees to be good at manipulating things--whether it be information or inventory, it is constantly being counted, evaluated, organized, packaged, distributed and documented. The mathematics of manipulation is inherent to mass distribution of goods, knowledge of it is required for "good jobs" in our free-market economy. The college degree is assurance to employers that the holder of such degree is qualified for the intellectual demands of the job. The mathematics classroom is a natural place to introduce students to these quantitative needs of society and the workplace.

In a different realm, psychologists, educational researchers in mathematics and American writers share a similar conclusion about abstractions as encountered in mathematics and the human being. "We must have a concrete idea of anything, even if it be an imaginary idea, before we can comprehend it." (O. Henry, 1935) To many students, the abstractions of mathematics are more accessible if they are learned as generalizations of a concrete, physical reality. Throughout our history, humans have started with "things", then developed and abstracted representations of them, then played with the representations, and then generalized on their properties. This natural progression has recently been reintroduced to mathematics education in America: it has proved to be a very powerful teaching tool as schools move toward "math for all".

Manipulatives became a fad, and then a copyright word. Many teachers now equate--and restrict--the mathematical concerns of manipulation to those items found exclusively in Educational Materials catalogs. Some are good tools, but they cannot be considered a panacea for mathematics education. Many flooded the math-ed marketplace just because, through clever design, they can be used to physicalize a single topic in mathematics. Too many of them take longer to learn than the mathematical concept they were designed to illustrate, too many illustrate the topic but not the connection to either mathematics or the material world. Manipulatives are only toys and tools, and should not be used as crutches, nor as ends in themselves. (Burrill, 1997)

Educational manipulatives and activities do need to be efficient regarding class time, content focus and teacher prep-time. Meanwhile, students do need to know how to manipulate and document the flow of things. They need to know the subtleties that arise when doing so, including special case scenarios. Experience with such activities is a good thing. The deck is a tool which satisfies the needs of students, teachers and coursework.

On cooperation

Self-governance is the foundation of our democratic government. It has become popular also for administering public services and even departments within corporations. Committee work is voluminous in business, industry and government. These requirements of our society and workplace often involve solving problems with quantitative ramifications. Success often depends on interpersonal skills and the ability to communicate technical and quantitative information. Establishing the attitude that "problem solving is a cooperative human endeavor" has inherent value in our culture and our schools (even in math class!).

The research agrees, and my experience agrees with it. When well-reasoned and structured, and implemented with mathematics in mind, the techniques of cooperative learning provide very useful classroom tools. Even when poorly implemented, establishment of a cooperative approach has yielded worst-case results of non-negative impact on student progress through mathematics curriculum. Yet cooperation is not a panacea for teachers--it is only a tool which augments, without replacing, traditional demonstrations, lectures and direct instruction. (Baker, 1996)

One challenge to the implementation of new classroom techniques is to maintain content focus with all activities. I use the deck to help randomly assign groups for in-class activities and assignments--whether card-based or not. It can be quickly preset to accommodate up to thirteen groups sized four or less. I shuffle one card per student, while introducing an activity. I let the class decide when the cards have been sufficiently randomized, then pass the deck to a student to take the top card and iterate the process. Those with the same 'kind' seek each other and then stake-out a place to work.

Such random assignment reduces the feelings and anxieties some students have when being manipulated into small group work. After the first few attempts students know the process, and can form their work groups and be on-task in little time. They sift, sort and establish themselves in the room--which provides real-life experience with manipulation-of-resources. To further integrate group-work into the curriculum, I've had students document who they worked with and when, and what card brought them together. This provides raw data for practice with data collection and statistical analyses, experience with documenting technical but non-numeric information, and direct exposure to the workings of chance--in a safe setting.

Establishment of a cooperative attitude and approach can yield many psychologic and social benefits for your students and your classroom, as well as provide useful curriculum support. Cooperation and interaction are particularly useful tools with liberal arts and non-traditional students. (Tobias, 1990) Teaching and practicing card games, taking statistics on the games, and developing mathematics-based strategies are all activities which are well-suited to effective small-group work and to steady progress through the curriculum. Try it, but ease into it at your own comfort level--only when the teacher is confident will it be comfortable for the students also. (A comfortable work environment is desirable for most humans, and this includes students and teachers!)

Cards and Society

Ming Dynasty books praised cards as superior to all other amusements: convenient to carry, they stimulate thinking, and could be played by a group of four without annoying conversation, and without the "difficulties which accompanied playing chess or meditation" (James & Thorpe, 1994). In the 1990's, these properties of card playing still hold, and closely match properties beneficial for study in the modern classroom--relatively quiet, thought provoking, small-group activities which engage students and focus on mathematics content (provided by text and teacher). Playing card activities can be constructed amenable to humans' short attention spans and schools' scheduling diversities.

The fact that cards can be played in almost any circumstance without restrictions of time, place, weather, or qualification of partners, has made them a popular social recreation for a thousand years in many diverse societies (see Figure 1). Surveys by the American Game Industry in the 1950's showed that playing cards was the most popular casual recreation in the nation (but acknowledged the rate was declining due to radio and television) (Encyclopedia Americana, 1959). Cards provide an informal and non-intimidating focus for interaction among families, communities, and even with strangers in town. Cards have been, and

FIGURE 1:

A Socio/economic Model: Four "Suits"

CASTE: China ≈a.d. 1000	*"peasant"*	*"military"*	*"professional"*	*"religious"*

Representations ("pips") used in the physical model (deck) of the theoretic model (of society):

Chinese Pip-shape: a.d.1000	coins	strings of coins	tens of strings of coins	myriads of tens of strings of coins
Egyptian Pip-shape: a.d. 1200	sticks, or wands	daggers, or swords	precious stones, or pentacles	chalices
Northern European Pip-shape: a.d.1400	♣ **"clubs"**	♠ **"spades"**	♦ **"diamonds"**	♥ **"hearts"**
Modern Surveys: "Circle one U.S., 1995	≤ $15,000/yr.	< $45,000/yr.	≤ $100,000/yr.	> $100,000/yr."

Further distinction relative to the model: the most common social associations between castes/classes are indicated by the color of the pips.

	black	**black**	**red**	**red**

Caption for Figure 1.

Here are major steps in the evolution of the deck as a model and manipulative. Changes were mainly due to traceable perturbations in the different interpretations of language and symbols among various cultures, as the deck spread around the world. In its 1000-year history, the look of the deck has been influenced by social theory, culture and language, growth of technology, consumer relations, isomorphisms and other symmetries.

The deck itself was the first physical model of an ancient classification scheme, which has remained popular in modern mathematical modeling procedures. Students interested in any of these influences can be lured through them, via an examination of the history of the deck, into a mathematical setting.

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continue to be a sociable influence as well as recreational activity--both at home and while traveling.

On resistance to cards

The introduction of playing cards to western society during the Renaissance came via the same source as came the reintroduction of formal mathematics--the Arabic world. Both playing cards and mathematical analyses were opposed by popes, princes and potentates of Western civilization. Nevertheless, their use spread rapidly among the clergy, royalty and the masses--throughout Europe and later to the Americas. They were so immediately popular in Europe that Gutenberg was known to print decks with fine art on the backs, sold cheaply to generate income during lean times with his fledgling printing press business (James & Thorpe, 1994).

Their use is still opposed by a vocal few Americans, ostensibly because they can be the focus of gambling. These objections over-estimate the influence of the cards; among those prone to gamble, pebbles and a curb provide sufficient focus. Meanwhile, gaming machines have been legalized in several states, and are employed "for amusement only" in states where gambling is illegal. These machines are programmed to win in the long run, always; this can be demonstrated mathematically. Teaching the mathematics of card-games is the most convincing argument against engaging in such forms of gambling-with-a-hope-to-gain.

Some peers have suggested that many college students don't know the deck, so it shouldn't be used in the classroom. I have found through trial, however, that teaching the deck is more efficient than trying to generate a comprehendible background in the other sciences commonly used in traditional word-problems. Meanwhile, many students, and especially non-traditional students, do know the deck, and are often glad at the opportunity to contribute their knowledge and experience.

The assumptions needed to examine the deck mathematically are few; they need not be over-simplified to be accessible to novice students. This cannot be said of probabilistic problems from genetics, business and industry.

Curriculum

Cards and probability

Science and society are probabilistic ventures. The laws of probability overshadow and influence measurement in advanced physics, design in industry, inventory in retail, and rates on your insurance. Free enterprise in conjunction with the variability of people makes probabilistic gambles out of all free-market business ventures. While the formal study of probability began with a king's desire to win when gambling, the mathematical field of probability has expanded its domain continually since--in part because of the human propensity to gamble, in part because of the world's propensity to change.

The human concern for "close enough" is no longer restricted to only "horse-shoes and hand-grenades". Measurement and manufacture are never exact, but only within given specifications. Mass production and engineering only require 'good enough to work, usually'. Insurance and pharmacology industries have mastered the art of determining 'when is soon enough, usually'. We live in a rapidly changing and volatile world; 'usually' and 'probably' are the best we can say, usually. The Newtonian formula to predict all the future has yet to surface, and few yet know the mathematical interpretation of a "60% chance of rain tomorrow".

Probability has grown to hold significant importance in a person's mathematical literacy. Introductory probability courses have become an entry-level key to college mathematics. It was just such a course in which I first used a deck of cards. After having taught from three different texts, I concluded that there is no topic at that level which can't be motivated by an investigation of the deck. I further noticed that students benefited from having a consistent thread through the many seemingly disjointed topics of the course.

When teaching basic set theory, Venn diagrams and subscript notation, the deck gives a hands-on and visual representation of what the symbolic manipulations are doing. It is there 'in spades' for demonstration of counting techniques with combinations and permutations. Games provide a household use for motivation and application of random variables. Good strategies for games are guided by expected values and conditional probabilities. (Note that the two modes prevalent for application of conditional probabilities are both commonly encountered in card games. One concerns "how my hand can be made better", the other concerns "how does my hand compares to the competitor's".)

Keeping score in games gives a natural source of data. The score-pad can be used to teach methods of compiling data, and to practice basic arithmetic skills. Results can then be analyzed to generate statistical information, and this in turn can be compared to theoretic probabilities. Such activities motivate the techniques of statistics, and clearly illustrate the interplay between probability and statistics.

<u>Cards and computers</u>

With the advent of computers has come an escalating use, throughout society, of finite discrete systems. Meanwhile, world population continues to grow exponentially. Social dependence on technology means that everyone needs to become facile with it, and especially with computer structures. This requires exposure to the mathematics behind the models; it requires that schools make math accessible to all--not just the few that need the calculus in their job training.

S. Brent Morris of the National Security Agency has used the deck to model advanced concerns for dynamic computer memories (discs, bubbles, etc.)--via the 'perfect shuffle'. The perfect shuffle--used to keep track of where certain cards end-up even though the deck is ostensibly being randomized--was once a favorite among dealers of the commercial card game Faro. (Here is a prime example of the human desire to shuffle without randomizing--thus maintaining both control of outcomes and the impression of a random event!) While difficult to perform a perfect shuffle on a physical deck, a bit of simple modular arithmetic nicely describes the operation. Manipulation of the physical deck gives an accessible introduction to the algebraic model used in the large discrete sets known as dynamic computer systems--to describe popular manipulations of memory and networks.

My graduate work at The University of Montana included an investigation into static computer memories (random access, read only). The hardware concerns of computer memories lead to an addressing hierarchy in the software which is very similar to the structure inherent in the standard deck of cards. I used the smaller set (deck) to introduce the algebraic concerns of static computer memory structures--including set construction, representation, manipulation of arrays, and scripted notation. (See Fig. 2 for two algebraic representations of the deck).

Meanwhile, the fully-defined nature of games enabled programmers and manufacturers to use them for initial efforts to construct dedicated computer applications. Game-playing computers had been developed by the 1950's, and were hailed as a major step toward a "smart" machine. (Asimov, 1989) Hand-held computer games were on the market a decade before desk-top computers. The card

FIGURE 2

Set Notation and Representation

"The diamond ace, the spade ten"

To construct D via its 'suits', define the four sets:

d = {all cards with a diamond for pip} = $\{d_a, d_2, d_3, \ldots, d_j, d_q, d_k\}$

c = {all cards with a club for pip} = $\{c_a, c_2, \ldots c_q, c_k\}$

s = {all cards with a spade for pip} = $\{s_a, s_2, s_3, \ldots s_k\}$

h = {all cards with a heart for pip} = $\{h_a, h_2, \ldots h_q, h_k\}$.

Whence it makes sense to define the set of suits, S = {d, c, s, h}

with four elements (each of which is itself a set with thirteen elements) and define

$$D = \bigcup_{x \in S} x = d \cup c \cup s \cup h = \{d_a, \ldots, d_k, c_a, \ldots, c_k, \ldots, s_7, \ldots, h_q, h_k\}$$

"The ace of diamonds, the ten of spades."

To construct D via its 'kinds', we need to define thirteen sets.

$\bar{a}$ = {all cards with exactly one pip} = $\{a_d, a_c, a_s, a_h\}$

$\bar{2}$ = {all cards with exactly two pips} = $\{2_d, 2_c, 2_s, 2_h\}$

up through to the set of kings, $\bar{k} = \{k_d, k_c, k_s, k_h\}$.

With these we can define the set of kinds, K = $\{\bar{a}, \bar{2}, \bar{3}, \ldots, \bar{q}, \bar{k}\}$

with thirteen elements, and now give D as:

$$D = \bigcup_{x \in K} x = \bar{a} \cup \bar{2} \cup \ldots \cup \bar{k} = \{a_d, a_c, \ldots, 2_c, \ldots, 5_h, \ldots, q_s, \ldots, k_c, k_s, k_h\}$$

Caption for Figure 2.

The generalizations and notation required to access the methods of algebra, can take the familiar and make it look bizarre. Set constructions involve defining appropriate elements, and from them appropriate collections of sets (resulting in sets of sets). Here the common deck is described two ways using set notation--where the important property is given priority in words and notation.

Subsets of the standard deck, set D, can be related to piles of cards. There are at least four characteristics (equivalence classes) which are visually apparent on each card (suit, color, kind, type). With the deck, union of two subsets simply means to combine piles, while intersection requires sifting through the elements of piles and gathering only those cards which satisfy two (or more) characteristics.

A generalization through notation can lead these to be seen as isomorphic to the co-product of one element, a card x, dually indexed, and written as

$$D = \{x_j^i \mid i \in S, j \in K\}.$$

game Solitaire was among the first diversions to make the transition to the world of personal computers. It remains popular as a computer-based entertainment even among people who don't regularly play cards.

Cards and Game Theory

Early in this century, John von Neumann elucidated a mathematical generalization of the concerns encountered in playing card games. His rigorous integration of the algebra of discrete systems with the concerns of probability and methods of optimization, is now known as mathematical Game Theory. This theory has become a leading model for the quantitative concerns of modern business and industry in our free-market economy. It addresses, mathematically, questions such as: How do humans make decisions in the face of conflict, complicated further by having only incomplete information? You know what you've got, but you don't know all that your competitors have, and you don't know what the future holds ... what should you do?

This generalization was possible because card games are structured much as mathematical investigations. Game descriptions typically define the patterns which are considered desirable for the game-at-hand; the descriptions go on to postulate the basic operations which will be allowed. Such descriptions often give theorems as rules, and offer strategies for attack--much like many modern mathematics texts. We can use card game descriptions to motivate and illustrate a useful, hands-on method for reading technical text, and for examining the axiomatic structure itself. Once known, a game can then be played to illuminate and practice diverse mathematical methods, techniques and skills.

In playing card games, operations and evaluations are motivated simply because "that's how this game is played, and this is how it works." The analogous mathematical explanation would be, "that's how this axiomatic system is played, and this is how it works." Definitions and rules--aka postulates, theorems and known equalities--are given to justify every move.

Also like most math texts, descriptions of "the game" in written form often require intense scrutiny in order to gain a working understanding. Text is extraordinarily focused on the manipulation of the given set of things according to a given set of "guidelines"; the tight focus of the text requires similar focus by the reader. Rules are given, irregularities to the rules are explained, special-case applications of the rules are illustrated, and examples are worked out for the reader. (Sounds like the traditional mathematics text!) For serious exercise with well-defined operations on a variety of sets, read game descriptions in the quintessential book of game rules According to Hoyle (Frey, 1991) . (Relations defined in a game are more arbitrary than algebraic relations defined on the real numbers, though, by the nature of all axiomatic systems, they are just as sure.)

Cribbage is a very good example of this phenomena: descriptions of the game generally include several pages of rules and examples of scoring and play, including rules for irregularities in play (what to do when someone turns over the wrong card, etc). Reading them with deep thought, fortitude and a deck in your hand, the game can be explicated. With only little live demonstration, however, the trial and error of playing this game actively enables most students (aged ten and up) to become quickly competent at playing and scoring. Playing actively puts novice players into positions where they seek guidance from the rules, and must refer often to them. (If only this habit would generalize to mathematics learning!)

In our goal-focused age, winning is a popular focus. The classroom is designed to be a safe atmosphere with a content goal common to all--even when used to teach competition. Losing a game in math class carries no serious consequences, and can motivate educationally beneficial coping strategies. The student's own desire to win is a natural motivation to improve at pattern recognition, and to practice the skill of focusing-on-a-given-task. Mistakes can be elucidated mathematically to develop and practice alternative problem solving strategies. Meanwhile, a content-focused classroom is a safe place to guide interaction skills and character development for winners and losers alike.

Cards and basic skills

Playing a card game is, like doing mathematics, a participation sport; players in both settings must learn to operate accurately and to recognize valued patterns. Practice can be as valuable as understanding, and often sheds light on the rules and structure of the given game. Only by playing often do special-case conditions arise to justify the special-case rules given in the text. Only through repeated observations do patterns and regularities become apparent, and thus useful in guiding student strategy and skills development.

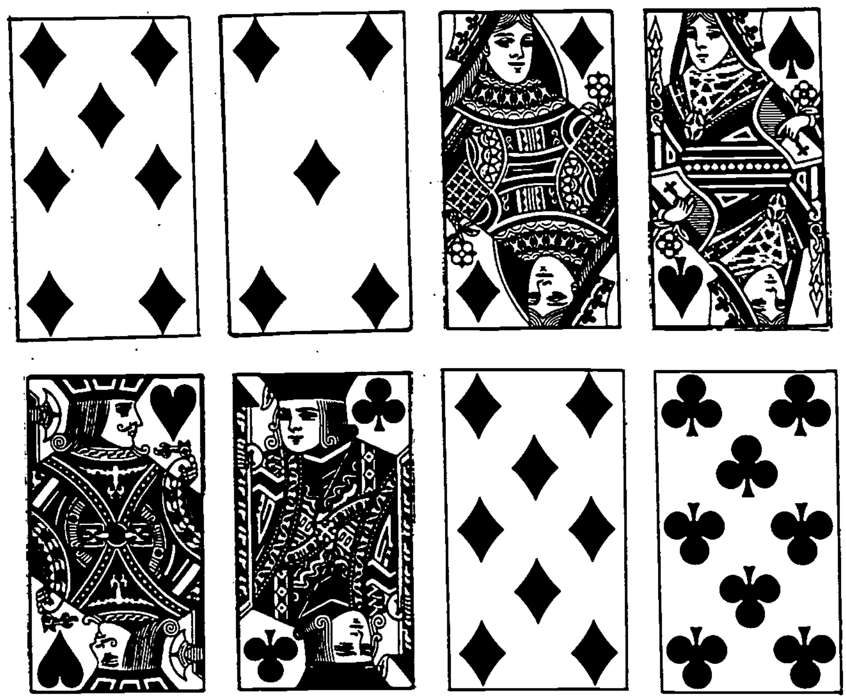
The world's most popular card games have always involved recognizing and manipulating patterns of cards, typically numeric in nature. Games are often accompanied by the need to keep a score, thus motivating an immediate concern for array organization and practice with basic arithmetic skills. Games that require scoring also give drill and practice in application and appreciation of random variables, work with ordered sets, and evaluation of real-valued functions.

User-friendly mathematics

Cards provide a manipulative that is inexpensive, easily stored, and kinesthetically accessible to most humans. The standard deck is imbued with a rich mix of patterns and symmetries (see Figure 3) that are visually recognizable. The

Figure 3

"Kinds" of Cards and Symmetries Prior to 19th Century Advances



Caption for Figure 3.

Where the four suits are identified by shape of pip, the thirteen cards in each suit are identified by thirteen different pictures: ten being distinguished by an arrangement of pips, and three by pictures of different human figures. The pictures on the thirteen cards in any suit are "isomorphic up to pip-shape" to the pictures on the cards of any other suit (almost). The four cards that are related by this (almost) isomorphism, are called four of a kind; there are thirteen kinds.

Well-defining this (almost) isomorphism would make a non-trivial classroom investigation into geometric formatting. A study of symmetry could focus on the shapes of the pips, or patterns on the cards. What makes the seven of diamonds so special? What did the late 19th-century introduction of corner pips do to the symmetry of the cards?

(Hint: compare this picture to a modern deck.)

patterns are amenable to algebraic notation for written analyses, which can be interpreted easily through visual inspection. Abstract notation relates naturally to the concrete and tangible cards, where actions on the things yield the same results as obtained by operations on the symbols.

Card game activities offer a physiological and psychological edge over traditional didactic methods in the math classroom. Unlike many text and work-sheet assignments, they provide a comfortable and interactive setting for drill and practice. Successfully learning to play a well-defined card game gives a foundation for self-confidence that many students lack when faced with well-defined ventures in formal mathematics.

Introducing students to the structure and form of games with a deck of cards enables their facility with probabilistic concerns of both formal and informal natures. Card games provide natural motivation to improve at pattern recognition and strategy development. Knowledge of the deck and its structure will support, exemplify, and motivate many diverse units of study, beyond the standard counting problems used in most extant probability texts. Classroom use of such knowledge may also enable families and friends to access and support student work in mathematics.

Conclusion

The deck can be used through the years of "mathematics class" as a consistent, and familiar, setting in which to model and demonstrate mathematical concerns. It is a visual and social manipulative useful in lessons from simple pattern recognition, to lessons in formal probability and set theory, to lessons on the relationships among sub-groups of a finite group. The deck can provide an interactive experience to allow students' sense memories to support their mathematical studies, and to show that mathematical methods are valuable, even in casual interactions, even to those not bound for work in the science laboratory.

Cards and card games are powerful tools for teaching modern mathematics. They are not ends in themselves, but their use in the mathematics classroom is completely justifiable.

With this article I hope to break the 20-year academic silence regarding this inexpensive and already-popular manipulative. I have demonstrated some technical mathematical applications for the standard deck of cards in the mathematics classroom, and have suggested more. I concur with AMATYC and NCTM standards; teaching mathematics is a complex practice and not reducible to recipes. Herein are some tools and ingredients for the pantry. It is the individual teacher's job to decide which activities might be useful for their class, and when.

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Author(s): Robert N. Baker

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Printed Name/Position/Title: Robert N. Baker / Assistant Professor Mathematics

Organization/Address: Univ. of Alaska Southeast - Ketchikan Campus / 2600 Seventh Ave. Ketchikan, AK. 99901

Telephone: 907-228-4502

FAX: 907-225-3624

E-Mail Address: robert.baker@uas.alaska.edu

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