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ABSTRACT

This paper reports on a project aimed at designing, implementing, and evaluating a hypermedia system, IPER-3, facing the three classical problems in the history of mathematics (i.e., trisection of the angle, quadrature of the circle, and duplication of the cube). The goal of the project is to study the opportunities offered by this kind of technology to the presentation of mathematical topics both in teacher training courses and in classroom work. Topics discussed include: (1) background on the choice of technology and content for the project; (2) the structure of the IPER-3 system, including information islands/viewpoints and the interface; (3) an analysis of use of the system by university students, including organization of the experiment, students' observations, and students' projects for completing IPER-3; and (4) possible future developments, focusing on hypermedia as a promoter of metacognition and as an environment for studying teaching and learning processes. A table provides an overview of students' observations on the technical nature of the computer and software, the mathematical content, the way the content is organized in IPER-3, hypermedia as a learning instrument, and their own work and reactions. Two figures present IPER-3 frames related to the quadrature of the circle. (Author/DLS)

Hypermedia As a Means for Learning and for Thinking About Learning

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Abstract: The paper refers to a project aimed at designing, implementing and evaluating a hypermedia system, IPER-3, facing the three 'classical' problems in the history of mathematics (trisection of the angle, quadrature of the circle, duplication of the cube). The aim of the project is to study the opportunities offered by this kind of technology to the presentation of mathematical topics both in teacher training courses and in classroom work.

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Introduction

In this paper we refer to a project aimed at designing, implementing and evaluating a hypermedia system, the IPER-3 system, facing the three 'classical' problems in the history of mathematics (trisection of the angle, quadrature of the circle, duplication of the cube) and some issues which have developed around them. The aim of our work is to study the opportunities provided by hypermedia systems in dealing with topics of interest for the training of teachers and/or for class work, exploiting the possibility of a non-linear presentation, representation of heterogeneous pieces of information, modularization, increment and flexibility of use, as discussed in the literature [Conklin, 1987; Tomek et al., 1991].
In the paper, first of all we motivate the choice of the history of mathematics and, inside this, the choice of the three classical problems. Then we present the structure of the hypermedia system IPER-3 and some design and implementation choices which are interesting from a teaching viewpoint. In the last section we analyze an experience of actual use of the system and delineate some possible developments of this type of activity.

Background

The field of knowledge of the history of mathematics appears particularly suited to a representation of a hypermedia type. At the level of both research and, with the due change of scale, work in class, tracing the history of mathematics is a complex activity in which there coexist two different investigation needs, the purely 'informative' one (characters, events, dates, places, etc.) and the 'conceptual' one of the analysis of ideas. Discovering history also means consulting sources including illustrations, manuscripts and various documents, and, since the same topic can be viewed in various milieus and in various epochs, it also means referring to history in general, art, geographical and ethnic situations, etc. In common texts of the history of mathematics, because of the very structure of the book, it is very difficult to handle the non-linearity of the historical development of ideas and hence, for the non-professional historian (for example, a student or a teacher) it may prove difficult to grasp the various connections or the influence of the context and, consequently, also the thread linking the developments of these ideas.
In making our choice to work with a hypermedia system in the history of mathematics we have taken into account, on one hand, these aspects of history and, on the other, the specific characteristic that hypermedia systems have of lending themselves to presentations of a different type than the book, being more dynamic and flexible in both spatial and temporal representation.
The association between the history of mathematics and computer technologies is not a new one. Apart from the massive use made of databases for the organization of historical material, which here does not concern us, it must be remembered that much information of a historical type is available on the Internet. Some students who have participated in the experience have used this resource for getting first information on certain historical facts, which they have then compared with other (more canonical) sources.
Apart from the reasons linked to the specific characteristics of the medium used, the choice of the history of mathematics is also connected to our opinion on the role that this discipline has in the teaching of mathematics [Furinghetti, 1997]. We believe that this role is central in the training of the mathematics teacher, in an outlook which has to do not only with knowing facts, which is certainly important, but above all with epistemological reflection as an integral part of the teacher's craft. In this sense history may be one of the contexts which can be

activated both for constructing concepts and for attaining awareness of the epistemological obstacles lying behind the difficulties met. These considerations concern not only the training of teachers, but also classroom practice.
Within history the choice fell on the three classical problems for a 'structural' reason: they are a multifaceted subject, a catalyst of many theories and many cultural contexts but one which for this very reason is mortified by the traditional presentation. Moreover, we were interested in the teaching implications of the three classical problems. These implications are linked to their cultural importance which is due to the fact that mathematicians attempted to solve them using the rule and the compass. Only in modern times it was proved that this way of solving them is not possible. The importance of this subject, from the teaching viewpoint, is founded on the belief that it is necessary to account for the reason why algebraic structures were introduced.
In the way of presenting the historical content we bore in mind the exploratory character of this work and we did not choose to consider a single type of user. For this reason, on one hand advanced developments are suggested, while on the other the language and form of presentation are conceived in such a way as also to be accessible to secondary school students, and some space has been devoted to topics of an elementary type (e.g. the detailed explanation of certain geometrical constructions).

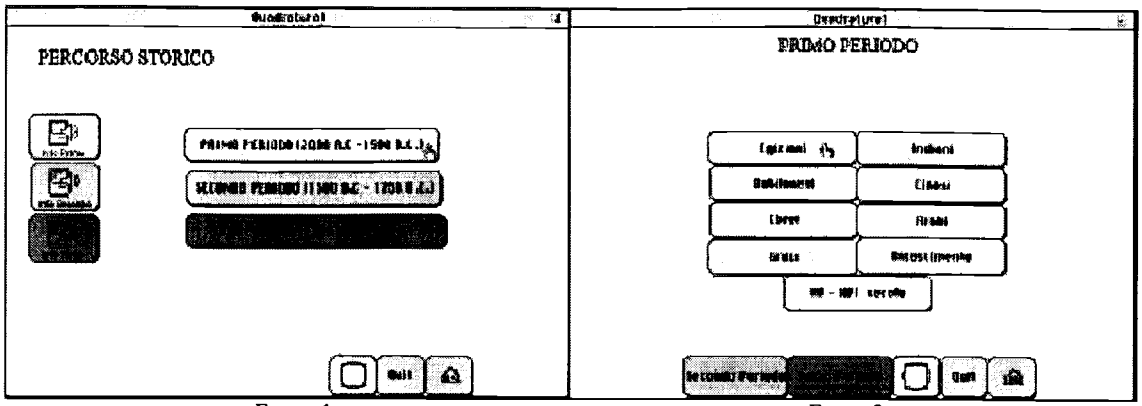
The Iper-3 System

As already stated, the IPER-3 system deals with the quadrature of the circle, the trisection of the angle and the duplication of the cube. The present prototype only gives full development of the quadrature of the circle. Work is still in progress in relation to the other two problems.

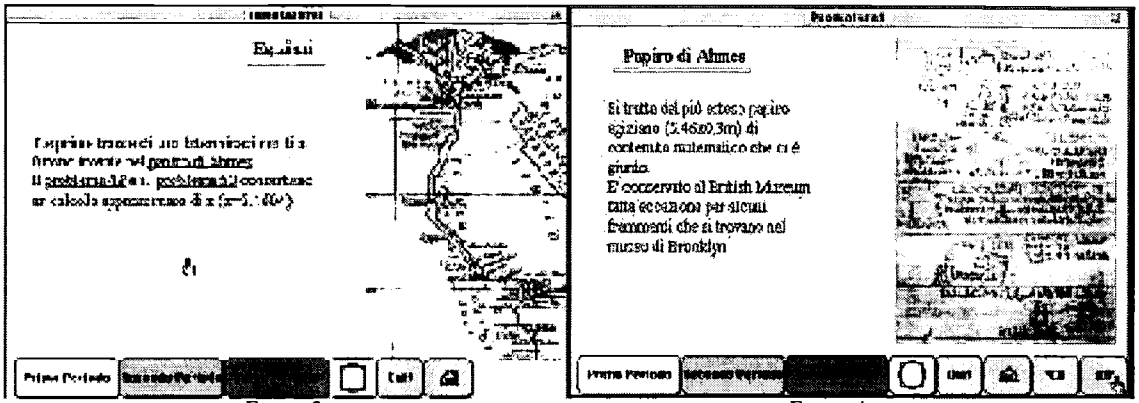
Information islands and viewpoints

IPER-3 consists of a structured set of information islands. Here the term 'information island' is used to mean an organized set of pieces of information which can be accessed by means of hot-words, buttons and sensitive areas. The information islands are six: 'introduction', 'classical problems' (one for each problem), 'rule and compass', 'glossary'. It is to be observed that the pieces of information thus organized are interconnected.
The 'introduction' allows one to obtain general information on the system and on the three problems dealt with in it. From this information island one can get to the specific ones for each of the three classical problems. The latter are structured in accordance with four "viewpoints". Viewpoints give the possibility to deal with each classical problem according with a different perspective: "general information", "historical development", "solutions", "tools".
The "general information" viewpoint identifies information of a general character (presentation of the problem, different formulations of the problem, cultural references, anecdotes, etc.). The "historical development" viewpoint identifies a section presenting some of the main contributions to the study of the specific problem in various periods. The "solutions" viewpoint is structured in accordance with the type of approach followed for the solution (geometrical, analytical). The "tools" viewpoint is closely linked to the previous one: it concerns the instruments used to arrive at the solutions proposed (mechanical, geometrical, etc.).

Each perspective has an internal pathway (see Fig. 2 for an example of an historical pathway concerning the problem of the quadrature of the circle) and also allows access to information, referred to the same topic at hand, from different viewpoints, as shown in Figure 3. For example, the quadratrix was one of the tools used to face the problem of the quadrature of the circle (see Frame 1 of Fig.3); by means of this tool a geometrical solution to the problem was obtained (see Frame 2 of Fig.3) by Dinostratus (see Frame 3 of Fig.3). It is also possible to see the dynamic construction of the quadratrix from both the tools and the solutions viewpoints (Frame 4 of Fig.3).

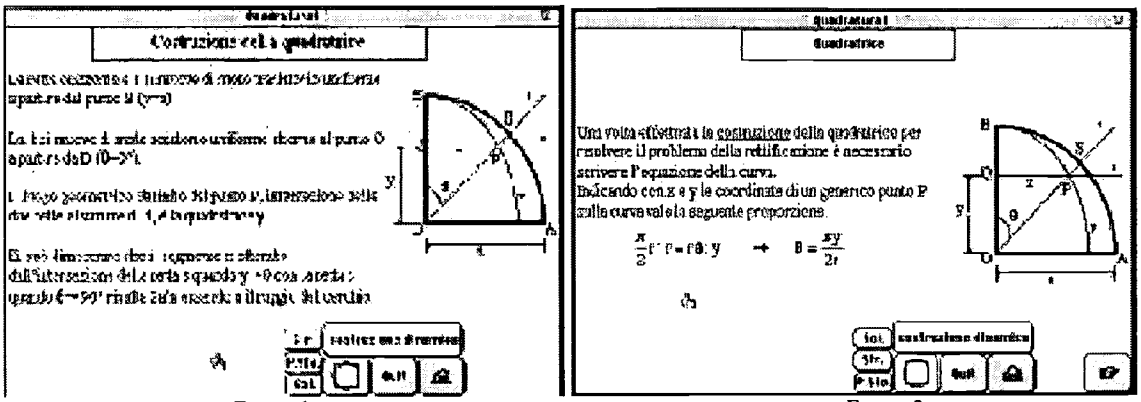


Frame 1 Frame 2

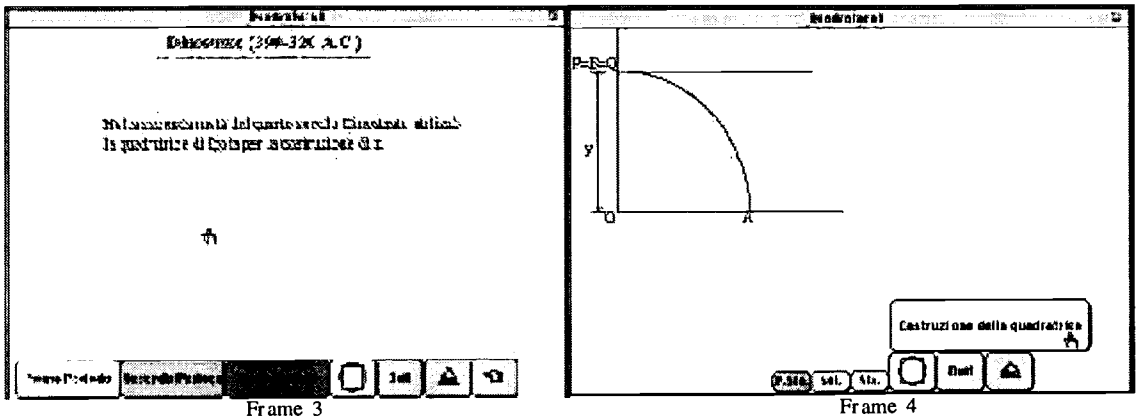


Frame 3 Frame 4

Fig. 2: Four IPER-3 frames referring to the historical development of the problem of the quadrature of the circle



Frame 1 Frame 2



Frame 3 Frame 4

Fig. 3: Four IPER-3 frames relating to the quadrature of the circle

It was felt to be useful to devote a specific information island to the rule and the compass in order to accentuate the link between the subject we are dealing with and Euclidean geometry.
The information island relating to the glossary is made up of information sheets briefly explaining some important terms. The user accesses the glossary by selecting the words or phrases which in the text are boxed (e.g.: geometrical constructions, absolute field of rationality, etc.).
For the mathematical and/or historical contents to put into the system use was made not only of the specific literature but also of notes on teaching experiences in classes at upper secondary schools, and of the Internet. The latter constitutes a work instrument supplying an enormous information patrimony which is continually expanding, though with some limits from the historical viewpoint (frequent lack of bibliography, approximate notices, etc.).

The interface

The system's interface has been designed in such a way as to be as user-friendly as possible, even for people who are not expert computer users, and for this reason reference was made to some of the principles and guidelines of human-computer interaction relating to user-centered design discussed in [Norman, 1986]. The interface is based on an icon paradigm: from the implementation viewpoint, this corresponds to the creation of buttons which are often semantically evocative. The icons are distinguished into two categories: those for handling the system, and those related to the topics faced. In creating the buttons, to guide people in using the system and reducing the risks of confusion often linked to the use of a hypermedia system, use was made of colors, voice and pictures. Color takes on three fundamental functions: it characterizes the perspective, characterizes the different sectors within the perspective (for example, the historical development is subdivided into three periods, each marked by a different shade of green), and distinguishes buttons according to their function (e.g. light blue ones allow management of the system). The voice is used to give an instruction or to suggest a cognitive pathway to follow. The pictures provide a visual support for information (maps, or photos or drawings of mechanical devices, etc.). In the system it is also possible to activate dynamic constructions and step-by-step constructions. The former are used to visualize the generation of curves obtained by means of movements of geometrical entities (e.g. two points on two different lines, the quadratix of Dinostratus, etc.), the latter to visualize in successive steps the solving procedure for geometrical problems (e.g. the construction of a segment whose length is the square root of x).

Experimentation With The System

Organization of the experimentation

The first experimentation of the system was carried out with university students in the mathematics degree course who were studying for a specialization in teaching. The experimentation, carried out during the last month of the course, was divided into the following phases:

- Introduction to hypermedia. This phase provided an introduction to hypermedia systems as a means of representing and accessing information. The theoretical basis of such systems was presented; concepts, definitions and explanatory examples referring to their main characteristics were given.
- Presentation of IPER-3.
- Use of IPER-3 by students. They were initially assisted in using the system and then left free to explore.
- Writing of a report by the students (individually) on their impressions regarding the use of IPER-3 and the way they had explored the knowledge incorporated in the system, as well as what they had learnt about hypermedia systems.
- Creation of a project (only on paper, not implemented) by the students, divided into two groups, to complete IPER-3 as regards the problems of the duplication of the cube and the trisection of the angle.

We consider the population chosen as suited to this type of experience. Thanks to the course they were attending, the students were wholly used to dealing with mathematical topics and hence it was possible for them to focus their attention on the means of transmission and the type of mediation. Furthermore, many of them had attended the course on the history of mathematics.
The considerations made in the pages which follow are based on the students' reports and on their projects for the completion of IPER-3.

The students' observations

The students' reports were analyzed by means of a reading grid in which observations were collected in accordance with the following main parameters: observations of a technical nature on the computer and the software, observations on the mathematical contents and how it is organised in the system, observations on hypermedia as learning instruments, observations on their own work and reactions.
Table 1 shows the most significant observations which emerged from the students' reports. In general, we notice curiosity and appreciation in relation to the technological instrument, but also some diffidence regarding its teaching potential.
The students' observations show up a rather generalized need for guidance in the exploration of a new subject; hence the suggestion to include features in the interface (some suggest a sound-based guide) to which one can have recourse in case of uncertainty about which way to go.
The request made by many that in the opening pages there should be a statement regarding the impossibility of solving the three classical problems in an elementary fashion, may indicate some resistance towards the 'open' and exploratory approach which the system would allow. This impression may be linked to the fact that the majority of the students consider the technological instrument as useful for revision, but not for a first introduction to the subject. By contrast, some students emphasize that the possibility of following personal pathways is the aspect they perceived as most positive in the use of IPER-3.
The content chosen is judged very positively in that it makes it possible to deal with mathematical contents in a way which is not static but follows a historical evolution lending itself to different readings (solutions, instruments, epochs) and permits connections between topics often dealt with separately from one another.

Students' projects for completing IPER-3

In the projects made by the students for the completion of IPER-3 we noticed both common tendencies and diversified ones. Both groups exploited certain elements (color, icons) and accepted the IPER-3 idea of distinguishing various approaches to the problem. Hereunder we report some observations on they way of working.
The "Trisection of the angle" group with respect to the prototype added the viewpoint "curiosity", but created no explicit links between this viewpoint and the others. It is interesting to observe that, thanks to curiosity, the students used the Internet source in addition to books. This source was also exploited to obtain figures, portraits, etc. The group realized the importance of dynamic constructions. The necessity of a user's manual was suggested. In the last part of the project they highlighted the links (and hence they stressed their importance) with some graphic schemata for describing their work, which in the other groups did not happen.
The "Duplication of the cube" group introduced the new viewpoint "demonstrations" which makes navigation more flexible, considering whether one is interested in a given theorem in itself or only in its enunciation in order to go on dealing with a given topic. However, subsequently the group did not make good use of this route with appropriate links, maintaining, by and large, a marked sequentiality in the treatment.
Summing up the behaviour of both groups, it can be stated that in one of them (Duplication...) the worry about the mathematical contents prevails over interest in the medium. In the other group greater interest was observed in hypermedia opportunities. Having worked on the basis of an existing prototype accelerated and facilitated both acquisition of knowledge of the specific problems studied and hypermedia design, but it did not prevent students' elaborations which were to some extent new and personal.

Prospects

The system presented is a prototype of a way of constructing a hypermedia system in the history of mathematics. Our way of working with it (in both the construction and utilization phases) can also be considered a prototype and we feel we can make some considerations on it which are transferable to other situations. The points on which we wish to focus our attention are the following:

- working with a hypermedia system can promote *meta-cognition;*
- a hypermedia system can be seen as an *environment which lends itself to the study of the dynamics of teaching and learning processes.*

The concept of meta-cognition is present in many theories of education, though with varying interpretations due to the different frameworks (psychology, mathematical education, etc.) and authors. We refer to a recent article [Robert & Robinet, 1996] for an overview of the studies which rotate, in a more or less broad sense, around this concept. We are particularly interested in the approach to meta-cognition, closely linked to practical teaching situations, found in [Schoenfeld, 1987]. In this paper the concept of meta-cognition is introduced explicitly and the author explains that it is necessary to interest the student in knowledge of knowledge ('thinking about thinking'). The author makes a classification of those elements which he considers most important in meta-cognition: knowledge of one's own thought; control of one's own thought in work situations; mental

representations. Schoenfeld describes four modes of developing meta-cognitive skills in the students: using videotapes on badly organized ways of working in mathematics on the students' part, highlighting the teachers' way of working in mathematics in front of the students (with constant questioning and changes of strategy), organizing discussions with the class in which the teacher has the controlling role, getting the students to work in small groups in which the teacher periodically intervenes on the solving strategies.

We can deduce from the analysis of our experience that working with hypermedia serves precisely to promote a new mode of developing meta-cognition skills. From students' observations we can see two different levels of metacognition that the work have stressed: the reflection on how one faces an articulated content (see Points 3 and 5 of Table 1) and the reflection on how teaching a complex topic on the basis of one own experience as student (see Points 2 and 4 of Table 1). The idea of reflection on strategies is inherent in the very idea of 'non-random navigation' in the hypermedia. Becoming aware of one's own way of approaching a topic is detected in the phase of discussion on the use of IPER-3, and even more in the project completion phase, in which contents are organized in accordance with conceptual schemata differing from one group to the other (see the different viewpoints introduced by the two groups).

Hypermedia technologies, in an appropriate use context, can be a means for having information on the students' conceptual networks, since when they work on rendering their strategies explicit they provide the teacher with a key to their mental schemata. In this sense the work with an hypermedia system can offer a different perspective to the study of the dynamics of teaching and learning processes. This observation is an hypotesis we have done after analysing students' work. The study of this hypotesis was not an objective of our experience but it appears an interesting perspective for our future work.

We believe that, over and above the interesting practical aspect linked to easy access to information, seeing the hypermedia as a promoter of meta-cognition and as an environment for studyng teaching and learning processes is a significant approach to the analysis of the role that this instrument can play in the teaching/learning of mathematics.

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1. **Observations of a technical nature on the computer and software**
   There is appreciation for the use of different media, e.g. sound.
   There is appreciation for the interface and the use made of 'evocative' icons; it is seen as useful to always keep the buttons in the same position on the screen and to limit their number.
   It is considered useful to be able to keep more than one page open at a given moment with the same interface layout.
2. **Observations on the mathematical content**
   The history of mathematics is considered an interesting subject; in particular, the importance of being able to see the historical evolution of a mathematical problem is stressed. It is observed that usually mathematics is 'imposed' as something finished and bare.
   Students observed that it would be possible to utilize IPER-3 with secondary school students, marking out some possible itineraries to follow (e.g.: general information, simple line constructions, rule and compass constructions, demonstration of the transcendence of π).
3. **Observations on the way the content is organized in IPER-3**
   The need is pointed out to state, in the first phase of the system, the impossibility of solving the three classical problems in an elementary fashion. This is seen as important because the user by himself may not become aware of this impossibility.
   According to some, the hypermedia should be constructed after the users have been identified.
   The need is stressed for innovative instruments for school updating. Obviously, it is observed that in its present formulation IPER-3 is incomplete for any teacher desiring to use it for updating.
4. **Observations on hypermedia as a learning instrument**
   It is considered a useful instrument for looking deeper into a given topic and for revision.
   By contrast, it is considered a distracting instrument for the first approach to a subject for those who have no knowledge of the contents.
   Freedom of navigation is seen as useful, though it is pointed out that there is a need to guide focusing on the main topics.
   It is observed that a hypermedia instrument can permit the development of greater mental elasticity, favoring a linkup between different topics.
   It is observed that instruments of the type examined can be useful for accustoming students to using other technology-based learning instruments.
   There is positive consideration for the opportunity given by hypermedia to connect kindred topics with one another.
5. **Students' observations on their own work and reactions**
   A student observes that she feels the need to be able to construct the geometrical figures on her own, instead of finding figures already drawn (from this point of view step-by-step constructions are positive).
   Almost everyone notes the usefulness of the instrument in the revision phase (better than notes); in any case it is an instrument seen as a support to the book.
   Some stress that instruments of this type provide an opportunity to approach technology, and to promote a new work mentality.
   All express their own curiosity and interest.

Table 1: Overview of students' observations

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

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