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ABSTRACT

This six month study investigated the similarities and differences between 137 fourth graders' understanding of and reasoning about multiplication, division, and proportion tasks. All students were administered a pretest mid-year and a posttest at the end of year. A sample of 18 students was interviewed individually on tasks involving reasoning about multiplicative and proportional relationships. This study provides evidence that students who are encouraged to use invented strategies for multiplication and division based on number relationships have a better understanding of the meaning of those operations and are more successful in extending their knowledge to proportional reasoning tasks than are those students who are taught conventional procedures exclusively. Contains 42 references. (Author/NB)

A COMPARISON OF FOURTH GRADERS' PROPORTIONAL REASONING IN REFORM AND TRADITIONAL CLASSROOMS

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ABSTRACT

This six month-long study investigated the similarities and differences of 137 fourth graders' understanding of and reasoning about multiplication, division and proportion tasks. All students were administered a pretest mid-year and a posttest at the end of year. A sample of 18 students were individually interviewed on tasks involving reasoning about multiplicative and proportional relationships. The study provides evidence that students who are encouraged to use invented strategies for multiplication and division based on number relationships have a better understanding of the meaning of those operations and are more successful in extending their knowledge to proportional reasoning tasks than are students who are taught conventional procedures exclusively.

There is no question that understanding proportional situations is critical for future mathematics learning. The NCTM *Standards* (1989) state that proportional reasoning is "of such importance that it merits whatever time and effort must be expended to assure its careful development" (p. 82). Common issues seem to arise when children attempt proportional reasoning tasks. Research has shown that (1) Children often view ratio as an additive operation and, as a result, resist using multiplication strategies (Hart, 1981); (2) Some children use multiplication strategies periodically but avoid those methods if multiplying by a fraction is necessary (Hart, 1981; Luke, 1990); and (3) Even when multiplication strategies are part of a child's problem solving repertoire, there is a tendency to revert to more simplistic methods when problem situations are unfamiliar or numerically complex (Tourniaire, 1983). These findings suggest that many children do not develop the multiplicative thinking required to reason about proportion situations.

Some researchers (Fischbein et al., 1985; Luke, 1988) have suggested that fault may lie in the methods used to teach multiplication. Standard textbooks have focused almost exclusively on the repeated addition model for multiplication. Building on their understanding of addition, the repeated addition model of multiplication is readily understood by most children, but does not lead to full understanding of the operation. It may also encourage many children to continue to think about multiplication additively rather than develop ways to think about multiplicative relationships. In addition, most traditional textbooks teach the standard algorithm for multiplication, which strips the numbers of their magnitude by requiring operation on fragmented digits. However, students who are facile with standard procedures may not necessarily have an understanding of the operation or have a well-developed understanding of number relationships (Kamii, 1989; Lampert, 1992; Madell, 1985), both of which are critical for proportional reasoning.

There have been some important findings on reasoning approaches children use for multiplication. Two recent studies on multiplicative thinking (Clark & Kamii, 1996; Mulligan & Mitchelmore, 1997) describe how children develop naturally through various levels of reasoning and that their performance varies depending upon the type of multiplicative situation being modeled. Through interviews with children in grades 1-5, Clark and Kamii identified four distinct levels, beginning with a primitive "no serial correspondence" and advancing to "multiplicative thinking with immediate success." Mulligan and Mitchelmore identified the following strategies children implement for solving multiplication problems: direct counting, rhythmic counting, skip counting, additive counting, and multiplicative calculation.

Typical problems children have with multiplication have also been well documented. Those findings have shown that: (1) Children often add instead of multiplying (Hart, 1981); (2) When children do not know the product, many cannot figure it out from one they know. This difficulty is in contrast to children's facility in figuring out a sum they do not know (Kamii, 1989); and (3) Children who have little difficulty with computation often have problems with the meaning of multiplication (Lindquist, 1989). For example, when children were asked to write a story problem for 6 x 3 = 18, 37% of fourth graders and 44% of fifth graders formulated problems such as "There are six ducks swimming in the pond. Then a while later three more ducks come, so how many are there?" (O'Brien and Casey, 1983).

With the traditional treatment of multiplication in many programs, it is not surprising that many children do not develop the level of thinking necessary to reason about proportion situations. An alternative approach to instruction on computation, suggested by some to avoid many of the misconceptions and undeveloped ways of thinking about the operations, is the use of invented procedures (Burns, 1992; Kamii, 1989; Madell, 1985; Lampert, 1992). They suggest that students who invent procedures for multiplication and division based on their understanding of the operations have a better understanding of place value and number relationships.

This study set out to investigate whether different approaches to teaching multiplication and division would have an impact on the reasoning patterns and misconceptions students have about the operations and how this would effect their proportional reasoning. This involved a comparison of students in three different instructional programs, one reform program that encouraged the use of invented procedures, one reform program that taught a variety of conventional procedures, and a traditional program that taught standards algorithms. Specifically, this study investigated the differential effects in achievement and strategy use among the two reform groups and a control group for multiplication and proportion problems.

METHODOLOGY

Subjects

This study occurred during the 1996-97 school year. The subjects were 137 fourth grade students from a suburban school district. Six intact classrooms from three schools participated. This study used a pretest-posttest design with three treatment groups. One group of two classrooms (Reform1) received instruction from one NSF-funded reform curriculum. The second group of two classrooms (Reform2) used another NSF-funded reform curriculum. The third group of two classrooms (Traditional) served as a control by continuing with their traditional textbook program. Although both reform programs were based on the NCTM Standards (1989), they differed on several fronts. Reform1 consisted mainly of investigations facilitated by the teacher whereas Reform2 consisted primarily of teacher-directed instruction. In addition, Reform1 encouraged student invention of computational procedures, while Reform2 introduced to the students a variety of procedures, including the standard algorithms, and encouraged students to choose from among these procedures.

Instruments

A pretest/posttest was designed by the researchers and consisted of 24 questions organized into the following three sections: computation (written and mental), word problems, and conceptual understanding. The computation section included standard multiplication and division problems of varying degrees of difficulty. The word problem section contained standard word problems representing equal groups and area interpretations of multiplication and measurement and partitive interpretations for division. The conceptual understanding section included items requiring explanations of the meaning of multiplication and division, second solutions of problems, and explanations of how to use a known multiplication fact to figure out another. These three subsections were included in order to assess the strengths of the various programs so as not to weight results in favor of one program over another.

The interview tasks assessed students' ability to: (1) use known multiplication facts to figure out another by thinking multiplicatively or additively; and (2) use proportional reasoning to solve a variety of word problems. The word problems were taken from published research studies (Kamii, 1996; Lamon, 1989; Scarano & Confrey, 1996) on proportional reasoning. They were selected to examine children's thinking in varying circumstances because it is recognized that the methods children use change with the context of the situation (Hart, 1988). Hence, the tasks varied in familiarity, semantic type, and numerical complexity as well as in the nature of the quantities. The three semantic types used in the study were (1) well-chunked measures, (2) part-part whole, and (3) unrelated sets. Well-chunked measures refer to the fact that the relationship between the quantities being compared form a third, familiar quantity such as miles per hour or speed (Lamon, 1989). Comparing the number of lions to the number of tigers in a zoo is an example of a part-part whole task, and pizzas to people exemplifies unrelated sets. The numerical complexity of the tasks ranged from easy integer ratios to noninteger ratios. Three of the tasks involved discrete objects, and two involved continuous.

Procedures

The pretest and posttest were administered to all students in the study by classroom teachers, mid-year for the pretest and at the end of the year for the posttest. Based on posttest scores, a sample of students (three from each classroom) representing a range of performance were interviewed individually by the researchers. Students were as much time as needed to complete the pretest/posttest and the average time to complete was about 45 minutes.

Data for mean percent correct scores on the pretest/posttest were analyzed using students who completed both tests. All others were eliminated from this analysis. Mean percent correct scores were analyzed for each of the subsections of the test. Average mean percent correct scores of the three subsections were used to report overall test scores. Data were also analyzed separately according to Incoming Quantitative Background (IQB) based on pretest scores. The students who received higher than 72% correct for a composite score on the pretest were placed in the High IQB group. Those who received between 44% and 72% (inclusively) were placed in the Middle IQB group and students with lower than 44% correct were placed in the Low IQB group.

For the strategy analysis on the posttest, data were compiled from all students who completed it. Four items from the computation section, four of the five word problem items, and one item from the conceptual section were coded for strategies. Students' explanations of the meaning of multiplication and division were coded as well.

The interview lasted about one hour. Each task was read to the child by the researcher and the child was given as much time as was needed to complete the task. The child was asked to explain the approach he/she used for each task. All interviews were tape recorded and transcribed to analyze solution strategies. Mean percent correct scores were analyzed for four of the proportion tasks on the interview. Average mean percent correct scores were used to report scores on the computational reasoning items and a selection of related proportion tasks.

RESULTS

Total Scores

The pretest/posttest was comprised of computation, word problem, and conceptual items. Data from the pretest and posttest are presented in Table 1. The Reform1 and Reform2 groups outperformed the Traditional group on all sections of the posttest. Reform1 had substantial gains on each section of the test with the greatest gain in conceptual understanding (18%). Reform2 had modest gains on the computation and word problem sections, but displayed a decrease in conceptual understanding. The Traditional group had a loss in conceptual understanding as well, while achieving substantial gains on the computation and word problem sections. With a lower skill level at the time of the pretest and with a greater focus on developing computational skill and facility with standard word problems in the Traditional program, it is not surprising that there were substantial gains on those two sections. However, posttest results were not significantly different among the groups ($F=2.83$, $p=.063$).

Table 1: Mean Percent Correct Scores by Program

	Reform 1 (n = 50)		Reform2 (n = 47)		Traditional (n = 28)	
	Pre	Post	Pre	Post	Pre	Post
Computation (n=14)	71.9 (16.5)	77.9 (14.9)	77.0 (17.0)	81.4 (15.9)	58.7 (17.3)	72.2 (17.5)
Word Problems (n=5)	57.2 (23.6)	68.4 (24.9)	57.4 (23.7)	60.9 (27.6)	35.6 (27.4)	52.6 (27.3)
Conceptual Understanding (n=5)	56.4 (26.1)	78.4 24.5	69.1 (27.9)	55.2 (26.7)	48.2 (31.0)	46.7 (33.3)
Composite Results[1]	61.8 (17.6)	74.9 (16.1)	67.9 (19.3)	65.8 (18.0)	47.5 (21.5)	57.2 (21.9)

[1] Represents average of three subsections

Results by Incoming Quantitative Background (IQB) Levels

The means of the scores from the three subsections of the pretest were used to partition the students into low, middle and high IQB groups. The mean number of correct items and standard deviations are presented in Table 2. It is noteworthy that the achievement of the low Reform1 group was higher than that of the middle Traditional group on the conceptual section. Furthermore, the middle Reform1 students outperformed the high Reform2 group on the conceptual items.

No significant differences existed among any of the IQB groups on the pretest. However, significant differences on the posttest occured on the conceptual subsection for all three IQB groups. Tukey follow-up comparisons of the three groups indicated that for the high IQB group, Reform1 scored significantly higher on the conceptual items than Reform2 ($F=8.87$, $p < .0006$). For the middle IQB group, Reform1 scored significantly higher on the conceptual items than Reform2 and Traditional ($F = 6.79$, $p < .0017$). Finally, for the low IQB group, Reform1 also significantly outperformed Reform2 and Traditional ($F = 8.72$, $p < .001$).

Table 2: Mean Correct Scores on Pretest and Posttest by IQB Levels

	Reform1		Reform2		Traditional		F	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
High IQB	(n=15)		(n=23)		(n=5)			
Computation[1]	11.73 (1.53)	12.40 (1.92)	12.48 (1.47)	12.39 (1.31)	11.00 (1.23)	12.20 (1.30)	2.59	0.04
Word Problem[2]	4.00 (.85)	4.00 (1.00)	3.74 (.62)	3.70 (1.19)	3.80 (.45)	4.20 (.84)	0.65	0.63
Conceptual[3]	4.20 (.77)	4.80 (.56)	4.43 (.73)	3.48 (1.16)	4.20 (.84)	4.40 (.89)	0.52	8.87***
Middle IQB	(n=27)		(n=17)		(n=10)			
Computation	10.00 (1.88)	10.37 (2.00)	9.65 (1.70)	11.00 (2.29)	8.60 (1.96)	9.90 (2.78)	2.12	0.74
Word Problem	2.63 (.74)	3.30 (1.30)	2.18 (.81)	2.47 (1.33)	2.00 (.94)	2.90 (.99)	2.98	2.26
Conceptual	3.00 (.87)	3.52 (1.16)	2.48 (.94)	2.41 (1.06)	3.10 (.88)	2.50 (1.08)	2.59	6.79***
Low IQB	(n=8)		(n=7)		(n=13)			
Computation	7.13 (1.89)	9.88 (.99)	7.50 (1.38)	8.67 (2.50)	6.75 (2.05)	9.42 (2.19)	0.33	0.63
Word Problem	1.50 (1.07)	2.75 (1.17)	1.50 (1.23)	2.17 (1.17)	0.75 (.75)	1.75 (1.14)	1.92	1.81
Conceptual	1.38 (.74)	3.63 (1.51)	1.00 (.63)	1.67 (.75)	1.08 (.99)	1.33 (1.50)	0.41	8.12***

*** $p < .001$
[1] Computation section includes 14 items
[2] Word Problem section includes 5 items
[3] Conceptual section includes 5 items

Item Analysis

The item analysis presents the mean percent correct on each test item. Results for each subsection of the test are shown in Table 3. The easiest items for all groups were the multiplication items and the division items which were basic facts. On the pretest and the posttest, the hardest item, 301 ÷ 5, involved division with a remainder and a zero placeholder in the dividend. Striking gains were made by the Traditional group on three items, 204 x 6, 301 ÷ 5, and 112 ÷ 8. Those items involved multiplication with a zero place holder, division with a zero place holder as well as a remainder, and division with a remainder, respectively. All of those items entailed dividing by a single digit. The additional time the Traditional group devoted to procedural practice may account for these gains.

Table 3: Mean Percent Correct Scores by Item

	Reform1		Reform2		Traditional	
	Pre	Post	Pre	Post	Pre	Post
Computation						
4 x 5	100	100	100	100	96	96
7 x 8	76	90	91	96	67	85
4 x 10	100	100	100	100	100	100
12 x 5	94	88	91	96	85	100
20 x 6	78	98	98	93	85	100
26 x 4	78	80	72	89	52	74
204 x 6	64	64	70	72	44	70
24 ÷ 3	90	90	87	93	81	93
48 ÷ 6	72	76	85	83	70	78
475 ÷ 25	52	52	48	41	7	22
301 ÷ 5	26	36	35	39	11	44
120÷10	76	82	74	87	59	56
300÷50	74	70	78	91	41	44
112÷ 8	26	64	50	59	22	48
Word Problems						
14 x 5	78	88	80	80	67	78
12 x 11	52	66	35	43	37	63
52 ÷ 8	62	82	63	74	22	41
90 ÷ 6	54	66	61	63	33	63
36 lions, # tigers[1]	40	40	48	43	19	19
Conceptual Understanding						
Meaning of 6 x 7	70	88	83	78	74	74
Meaning of 28÷4	54	86	70	46	41	59
26 x 4 2nd way	54	84	70	50	52	30
Use 3x8 for 6x8	66	76	70	65	41	44
Use 3x12 for 5x12	38	58	54	37	33	26

[1] Proportional reasoning word problem.

It is notable that the Reform1 and the Reform2 groups had a higher percent of correct responses than the Traditional group on all division items where the divisor was a factor of one hundred and the dividend was a multiple of the divisor. Reform1 and Reform2 groups also outperformed the Traditional group on division items involving a two-digit divisor. On the item, 120÷10, 82% of the Reform1 group and 87% of the Reform2 group answered correctly. In contrast, only 56% of the Traditional group produced the correct answer. More than twice the number of students from the Reform1 group and nearly twice the number form the Reform2 group responded correctly to the item, 475 ÷ 25, than from the Traditional group.

On the word problem section, the Reform1 group had a higher percent of correct responses on all of the word problems except the complex proportional reasoning task: "The Detroit Zoo has 3 lions for every 4 tigers. If there are 36 lions, how many tigers are there?" All groups found this to be a difficult item. By the posttest, the achievement of the Reform1 and Reform2 groups declined slightly; however, both groups outperformed the Traditional group.

On the conceptual understanding section, explaining the meaning of 6 x 7 was the easiest item for all groups. The Reform1 group explained the meaning of multiplication and division with equal facility. Whereas, describing the meaning of division was more difficult than that of multiplication for the Reform2 and the Traditional groups. This may be due to the fact that, multiplication and division are treated as isolated topics in Reform2 and the Traditional program. However, Reform1 emphasizes the relationship between and meaning of the operations.

On the posttest, showing a second way to multiply 26 x 4 was the most difficult item for the Traditional group and, at the same time, the easiest item for the Reform1 group. Although half of the Traditional group was able to show a second way to multiply on the pretest, only 29% could do so by the posttest. By the posttest, more than ninety percent of the Reform1 group showed a second way to multiply, a gain of nearly forty percentage points.

Less than half of the Traditional group were able to use the fact that 3 x 8 = 24 to help solve 6 x 8 on the posttest. Explaining how to use the fact that 3 x 12 = 36 to help find 5 x 12 was difficult for all groups; however, in comparison to the Reform2 and Traditional groups, the Reform1 group had twenty percentage points higher of correct responses on the posttest.

Understanding of the Operations

On the pretest/posttest, students were asked to explain what 6 x 7 (28 ÷ 4) means to an alien who knows nothing about multiplication (division). Responses were coded for the interpretation expressed or model used to describe the meaning of the operation. The results for the multiplication item are shown in Figure 1 and those for division in Figure 2. Repeated addition was the predominant interpretation for multiplication for all groups. It is striking that almost a third of the Traditional group provided no meaningful explanation. Rather, they either provided no explanation or described 6 x 7 as "6 times 7," which did not indicate whether or not the meaning of the operation was understood. The inability to provide a meaningful explanation for division was also quite evident in the Reform2 and Traditional groups. Nearly one half of all students in both of those groups could not describe division adequately.

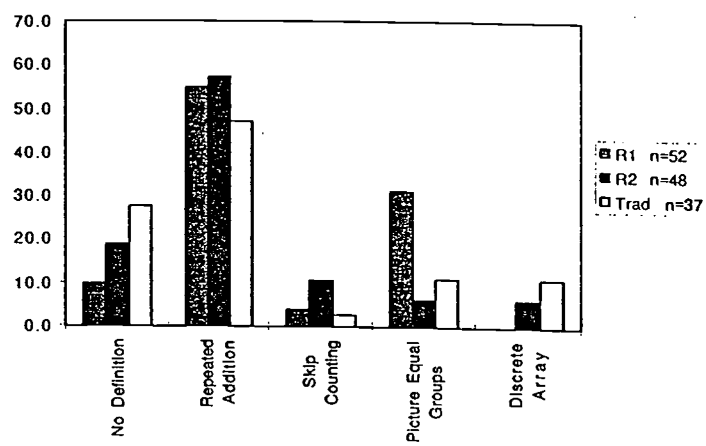


Figure 1: Cumulative Percentage of Interpretations of Multiplication by Program

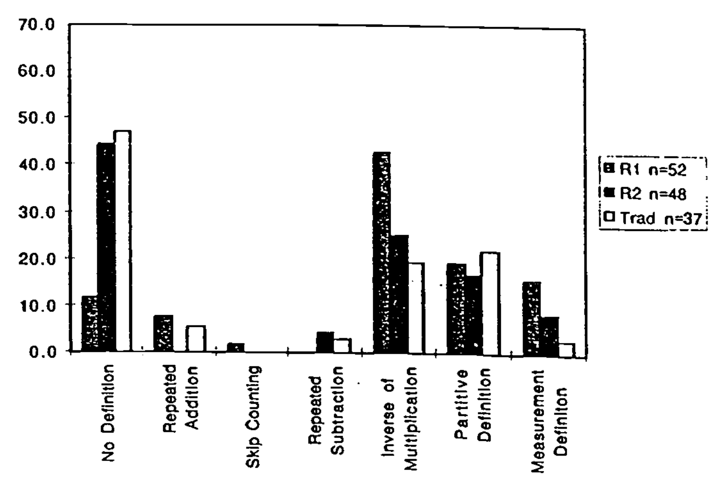


Figure 2: Cumulative Percentage of Interpretations of Division by Program

Almost half of the Reform1 students described division as the inverse of multiplication whereas less than a third of the students in the Reform2 and Traditional groups did so. This may suggest differences in instruction among the programs in which multiplication and division are attended to at the same time or as isolated operations.

Strategies

Data from the computational strategies on selected posttest items are shown in Table 4. Strategies were coded for five computational items. Two of the computational items, 204 x 6 and 475 ÷ 25, were selected for their potential to elicit a variety of strategies and because they were the most challenging computational items for each of the operations. The remaining three computation items, 26 x 4, find 26 x 4 using a second method, and 112 ÷ 8, were embedded in the conceptual section.

The strategies children used fell into the following six categories: (1) No Strategy/Meaningless; (2) Conventional Procedures; (3) Direct Model-Pictorial; (4) Direct Model-Numerical; (5) Invented Procedures; and (6) Mental Math. Conventional procedures represented those procedures that could only be accomplished in one way and included the standard algorithms for multiplication and division, partial products, and the lattice method. Direct Model-Pictorial included any kind of pictorial representation of the problem, such as an equal groups picture or a discrete or continuous array. Direct Model-Numerical was used to describe those strategies that directly modeled the problem yet involved numerical representations, such as repeated addition and skip counting and repeated subtraction. Finally, Invented Procedures represented those procedures that displayed the usage of number relationships.

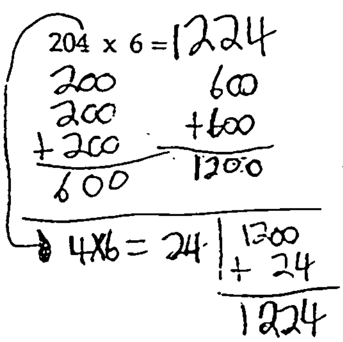
There were striking differences between the multiplication and division strategies used by each of the groups. The predominant method for the Traditional group was to use conventional procedures with very little flexibility. In addition, the Traditional group provided no work or meaningless responses 21% of the time for multiplication and an astonishing 43% of the time for division. Reform2 also relied heavily on conventional procedures, but displayed some flexibility

Table 4: Percent Usage of Strategies for Multiplication and Division Computation and Word Problem Items

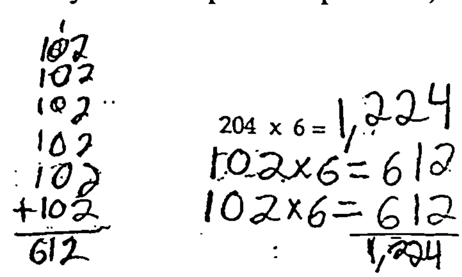
Item	No Strategy/ Meaningless			Conventional Procedures			Direct Model- Pictorial			Direct Model- Numerical			Invented Procedures			Mental Math		
	R1	R2	T	R1	R2	T	R1	R2	T	R1	R2	T	R1	R2	T	R1	R2	T
Multiplication																		
204 x 6	5.9	8.5	2.8	29.4	72.3	88.9	0.0	0.0	0.0	13.7	4.3	0.0	39.2	2.1	0.0	11.8	12.8	8.3
14 x 5	5.9	8.6	22.2	17.6	46.8	66.7	23.5	19.1	2.8	21.6	4.3	2.8	17.7	6.4	0.0	13.7	14.9	5.6
12 x 11	2.0	4.3	13.9	21.6	57.5	61.1	29.4	25.5	13.9	23.5	2.1	8.4	17.7	10.6	0.0	5.9	0.0	2.8
26 x 4	2.0	2.1	13.9	31.4	78.7	80.6	0.0	0.0	0.0	23.6	10.6	5.6	43.2	8.6	0.0	0.0	0.0	0.0
26 x 4 again	15.7	40.4	52.8	7.8	12.7	27.7	15.8	14.9	8.4	33.4	27.7	11.1	27.5	4.2	0.0	0.0	0.0	0.0
Multip. Total	6.4	12.9	21.1	22.1	54.1	65.0	12.9	11.2	5.0	23.7	9.9	5.6	28.5	6.4	0.0	6.4	5.6	3.3
Division																		
475 ÷ 25	31.4	46.8	69.4	2.0	14.9	13.9	2.0	0.0	0.0	17.6	2.1	0.0	21.6	8.5	5.6	25.5	27.7	11.1
52 ÷ 8	4.0	6.4	33.3	0.0	12.8	19.4	51.0	53.2	19.4	17.6	2.1	0.0	2.0	2.1	0.0	25.5	23.4	27.8
90 ÷ 6	11.8	27.7	30.5	3.9	21.3	50.0	33.3	14.9	8.3	7.8	0.0	2.8	29.5	17.0	2.8	13.7	19.1	5.6
112 ÷ 8	17.6	36.2	38.9	9.8	27.7	52.8	15.7	10.7	0.0	9.8	2.1	0.0	45.1	19.2	5.6	2.0	4.3	2.8
Division Total	16.2	29.3	43.1	3.9	19.1	34.0	25.5	19.7	6.9	13.2	1.6	0.7	24.5	11.7	3.5	16.7	18.6	11.8

in using other strategies, especially for division problems. However, they too provided no work or meaningless explanations at a high frequency, especially for division. When asked to multiply a second way, over half of the Traditional group and 40% of the Reform2 group did not respond. It appears that, if the standard algorithm could not be recalled, then students in the Reform2 and Traditional groups were more likely to give up than those in the Reform1 group.

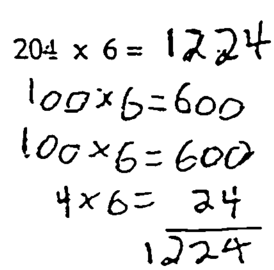
The Reform1 group was more varied in their usage of strategies, and predominantly relied on procedures that displayed their conceptual understanding of problems. In the following example, PW20 appears to interpret 204 x 6 as the total of six groups of 200 and six groups of 4. The six groups of 200 were first decomposed into three groups of 200 and then doubled before adding on six groups of 4.



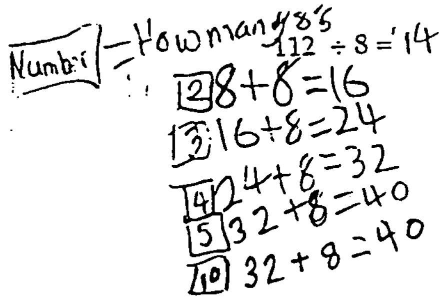
PW27 presented an alternative way to decompose the problem, doubling 102 groups of 6.



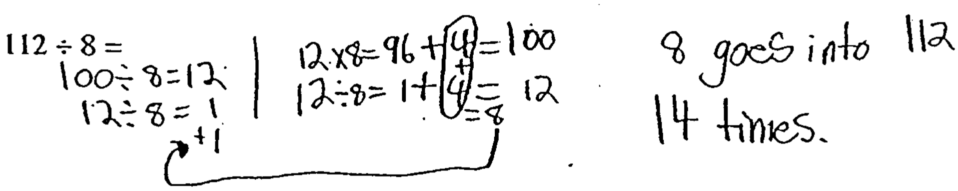
Finally, PG18 added six groups of 100 to another six groups of 100 to get 1200 and then added on six groups of 4.



Students in the Reform1 group displayed an intuitive understanding of the properties underlying the number system by using invented procedures that displayed the proportional relationship between factors and products and composing and decomposing numbers according to the distributive property. Consider the following invented procedure for 112 ÷ 8. PW19 realizes from a systematic list that five eights is 40 and then doubles that to figure out that ten eights is 80. He then realizes that ten eights and four more eights will make 112; thus the answer to the problem is 14.



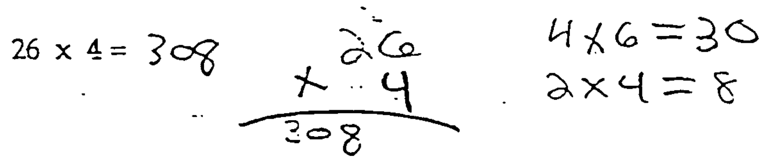
PW1 decomposes the 112 into 100 and 12 and realizes that there are 12 eights in 100 (with a remainder of four) and one eight in 12 (with a remainder of four). He then recognizes that the two remainders make one more eight; thus the answer is 14.



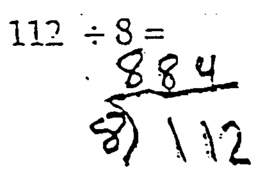
Error Analysis

An analysis of the trends in errors provided additional information on the differences among groups. While using the standard algorithms for multiplication and division, the numbers operating or being operated on were fragmented into separate digits, stripping the original numbers of their magnitude. Therefore, many of the examples of typical errors had unreasonable answers that went unnoticed. However, errors with invented procedures or those that directly modeled the problem generally yielded a range of answers that were close in magnitude to the correct answer. This is possibly due to the fact that invented procedures require thinking about number relationships and the meaning of the operation being employed.

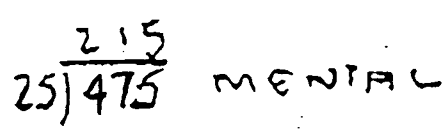
A zero placeholder in a multiplication item was the source of difficulty for many students. A typical error was to confuse multiplication by zero with that by one. Another common error was to misapply a "rule" for multiplication. For example, in addition to a basic fact error, Reform2 student DP15 appeared not to have mastered the multiplication algorithm. Each digit of the multiplicand was multiplied by the multiplier separately and then those products were annexed without regard to place value.



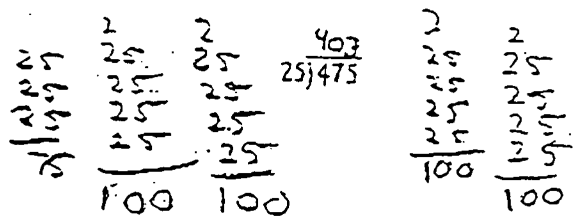
Errors also occurred when the division algorithm was incorrectly applied. Traditional student TC6 reversed the role of the dividend and the divisor and, upon completion, failed to recognize that the answer was unreasonable.



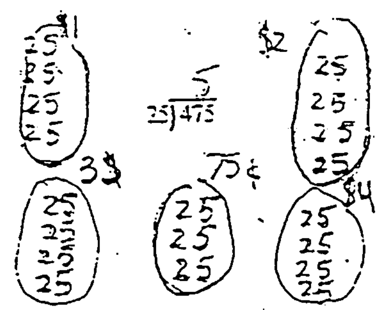
A two digit divisor increased the difficulty of correctly applying the standard algorithm. Traditional student TC7 divided the first digit of the dividend (4) by the first digit of the divisor (2) and then divided the remaining two digits of the dividend (75) by the ones digit of the divisor (5) to arrive at fifteen.



On occasion, incorrect counting schemes used in invented procedures resulted in implausible answers as well. Reform 1 student PG13 had nineteen 25s but lost track of the fact that the number of 25s contained in 475 is the solution. Instead, he took the sum of sixteen of the 25s and then added that to the remaining number of 25s (3).



Another example is that of Reform1 student PG14, who initially thought of the problem in terms of money. The nineteen 25s represented the number of quarters in $4.75. After circling all groups of quarters equivalent to a dollar, he circled the remaining three quarters. It appears that division was interpreted as the number of groups of four 25s contained in $4.75, hence, the answer 5.



While these errors resulted in seemingly unreasonable answers, the thought involved at reaching these solutions was qualitatively different than the thought involved in most of the errors made with conventional procedures. PG13 and PG14s solutions display an understanding of the operation of division and what it means to divide. They also display an understanding of number relationships in grouping four 25s to make 100. The errors made by those students using conventional procedures, on the other hand, did not provide any such information into the students' understanding.

Interview Results

The interview contained computational reasoning tasks and extension tasks dealing with proportional reasoning. (See Appendix for the interview protocol.) The mean scores are summarized in Table 5. The three most difficult items were identified based on combined scores. Reform1 performed the best on all three of the most difficult items, followed by the Reform2 and Traditional programs. On the most difficult item, the bicycle trip, a correct solution was provided by a third of the students in Reform1, whereas not one student was able to provide a correct solution in the Reform2 and Traditional programs.

Table 5: Performance of Each Group on Interview Tasks by Percent Correct

Item	Reform1 (n=6)	Reform2 (n=6)	Traditional (n=6)
Computational Reasoning[1]	72	44	57
Proportional Reasoning			
Little, Middle, Big Fish[2]	77	75	62
More Pizza	83	83	67
Bicycle Trip	33	0	0
Detroit Zoo	67	50	50
Number of Words Read	50	17	17

[1]Percentages for this item represent an average for 3 individual tasks.
[2]Percentages for this item represent an average for 10 individual tasks.

Data from solution strategies are presented in Table 6. The strategies were classified as (1) misunderstanding, (2) pattern, (3) additive error, (4) qualitative, and (5) composite unit (Ito-Hino, 1996). Misunderstanding includes the inability to respond to the task. The pattern category contains responses based on the construction of a numerical pattern without taking number relationships into account. The category, additive error, refers to the practice of using a constant difference rather than a ratio. A response that only recognizes the increase or decrease in the magnitude of a quantity is categorized as qualitative. The final category, composite unit, is one in which the student interprets the problem in terms of two pairs of composite units and uses the relationship between those units to produce an answer.

Table 6: Percent Usage of Strategies for Proportional Reasoning Tasks

	Reform1	Reform2	Traditional
Misnderstanding	7.1	9.5	11.9
Pattern	8.3	6.0	3.6
Additive Error	3.6	15.5	19.0
Qualitative	2.4	1.2	7.1
Composite Unit	78.6	67.9	58.3

The trend in the data show that the Reform1 group used strategies based on number relationships more frequently than either Reform2 or Traditional groups. Reform1 used the relationship between composite units 79% of the time, 11 and 20 percentage points more than Reform2 and the Traditional groups, respectively. When using incorrect strategies, Reform2 and Traditional groups focused upon the difference between the magnitude of two quantities in a problem more frequently than the Reform1 group. This may suggest a reliance upon rules to solve problems on the part of Reform2 and Traditional groups. Whereas, the Reform1 group more frequently looked for patterns in the numbers to solve problems. When the problem was difficult 12% of the Traditional and 10% of the Reform2 groups did not attempt the problem or claimed to be confused. Consistent with the findings on the pretest, Reform2 and the Traditional groups appear less likely to try challenging problems.

DISCUSSION

We began this study to investigate the differential effects of instruction on students' achievement and strategy use on multiplication, division and proportion tasks. The evidence suggests that Reform1 students who were encouraged to invent procedures for multiplication and division had a qualitatively different way to think about those operations, to make sense of what they mean and how they are related, and were better able to transfer that understanding to proportional reasoning tasks. In addition, the fact that students from every level of incoming quantitative background in Reform1 had significantly more conceptual understanding than comparable Reform2 and Traditional groups suggests that learning invented procedures should benefit all students.

The findings also suggest that while some students have facility with the standard algorithms for multiplying and dividing, it is devoid of meaning for them. This was especially evident during the interview when students using that method were asked to explain why they placed a zero in the ones place when multiplying by the digit in the tens place of the multiplier. Most students were unable to correctly explain, with typical responses such as: "Because you have to carry the zero…down there. You have to carry that zero in every problem," "I'm not positive, you just do it," or "because that is how I was taught by my teacher." In addition, the majority of those students seemed unaware that the value of a number depends on the place it occupies. It appears that their place value knowledge is not connected with their understanding of multiplication.

In contrast, most students' explanations of invented procedures displayed an understanding of the meaning of the operation, of the structure of multiplication and division, and of place value. Reform1 students were much more flexible and varied in their approaches to solve problems as well in that many realized the multitude of ways to decompose quantities in order to multiply or divide.

Implications for Instruction

While our purpose was not to analyze instruction or how students actually learned to use any particular strategy or procedure in a specific program, it seems as though the approach used in the Reform1 program was effective in developing the conceptual understanding necessary to invent procedures for multiplication and division. It is interesting that students in Reform2 did not display the kind of conceptual understanding one would expect from a reform program. The basic instructional approach of the Reform2 program is to encourage students to invent strategies, but to teach a variety of conventional procedures, including lattice and partial products, if they do not invent appropriate procedures. The findings suggest that this may not be an appropriate message in that few Reform2 students actually used invented procedures and relied more heavily on conventional procedures.

The value of the partial products procedure came into question. It is possible to view the partial products procedure as a meaningful approach to multiplication in that place value knowledge is used to arrive at a solution. However, the Reform2 students who used this procedure in general did not display a higher level of conceptual understanding than Reform1 students. And many of those students did not have the requisite understanding to think appropriately about proportion situations. This may suggest that while this procedure may be considered to be more conceptual than the standard algorithm for multiplication, it is not as effective as encouraging children to invent procedures that are based on meaningful approaches that necessarily make sense to them.

Appendix:

Interview Protocol for Proportional Reasoning Tasks

1. How could you use the fact that 7 x 12 = 84 to help find 14 x 12?
2. How could you use the fact that 10 x 17 = 170 to help find 13 x 17?
3. How could you use the fact that 7 x 21 = 84 to help find 21 x 12?
4. A drawing of three fish was presented to each student. The student was told that the Middle Fish eats twice as much as the Little Fish, and the Big Fish eats three times as much as the Little Fish. Students were given various situations and asked what to feed the other fish.
   - A. Little Fish was given 1 cube of food.
   - B. Middle Fish was given 4 cubes of food.
   - C. Big Fish was given 9 cubes of food.
   - D. Little Fish was given 4 cubes of food.
   - E. Middle Fish was given 7 cubes of food.
5. A drawing presented the problem of 3 pizzas to be shared among 7 girls and one pizza among 3 boys. Students were asked: Who gets more pizza, the girls or the boys?
6. Samantha went on a long bicycle trip. In three hours, she had gone 20 miles. At 6 hours she had gone 40 miles. If she rode her bike at a constant speed during her trip, how many miles would she have gone in 4 hours?
7. The Detroit Zoo has 4 lions for every 5 tigers. If there are 32 lions, how many tigers are there?
8. How many words can you read in 18 minutes if you can read 540 words in 4 minutes?

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