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ABSTRACT

This report includes all of the presentations from the fifth annual workshop on the Current State of Research on Mathematical Beliefs held in Helsinki, Finland, on August 22-25, 1997. The papers, all of which were presented in English, are as follows: "Between Formalism and Creativity: Teachers' Conceptions of a Good Computer Science Student" (Peter Berger); "Mathematical Views of Seventh-Graders in Bielefeld and Aschaffenburg" (Gunter Graumann); "Teacher as an Enactivist Researcher" (Markku Hannula); "Change in Mathematical Views of First Year University Students II" (Iris Kalesse); "Metaphor and Teaching" (Ingrid Kasten); "The Development of Prospective Teachers' Math View" (Sinikka Lindgren); "Can Gender, Language and Regionality Affect Choices in Upper Secondary School Mathematics?" (Marja Nevanlinna); "Teachers' Conceptions on Mathematics Teaching" (Erkki Pehkonen); "Pupils' Beliefs on Mathematics Teaching in Ukraine" (Sergy A. Rakov); "Students' Mathematical Beliefs as Predictors of Mathematical Performance at Entering College Level" (Martin Risnes); "Discontinuities of the Mathematical World Views of Teachers during Pre-Service Education ("Referendariat")" (Christiane Romer); and "Mathematical Beliefs and Their Impact on the Students' Mathematical Performance: Questions Raised by the TIMSS Results" (Gunter Torner). (All papers contain references.) (SM)

Research Report 184

Markku Hannula (ed.)

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Proceedings of the MAVI-5 Workshop
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Research Report 184, 1998

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Current State of Research on Mathematical Beliefs V
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Abstract

The fifth workshop on Current State of Research on Mathematical Beliefs took place in the Department of Teacher Education at the University of Helsinki from the 22nd of August to the 25th of August 1997. The conference language was English. There was no plenary talks, but every presentation had a time slot of 30 minutes with a follow-up discussion of another 30 minutes. The concept 'belief' was seen in a wide meaning and presentations in this workshop dealt also with conceptions, views and attitudes.

Although the workshop has originated as Finnish-German, there has always been contributions from other countries as well. This time the visitors were Sergey A. Rakov from Ukraine and Martin Risnes from Norway. This report contains all the papers given in the workshop.

The presentations were mainly on empirical research although three were theoretical (Kasten, Rakov and Törner) and one was on methodology (Hannula). Out of the empirical research four (Graumann, Nevanlinna, Pehkonen and Risnes) fall under the quantitative paradigm and another four (Berger, Kalesse, Lindgren and Römer) under the qualitative paradigm. This distinction however is not exclusive, while many of the works use both kinds of approaches.

This time the focus of only two studies (Graumann and Nevanlinna) was beliefs of the pupils at secondary school, another two studies (Kalesse, Risnes) focus at post-secondary education, another two (Lindgren and Römer) at teacher education and yet another two (Berger and Pehkonen) at teachers in service.

Keywords: mathematics, beliefs, conceptions, views, attitudes

HELSINGIN YLIOPISTO
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Tutkimuksia 184, 1998

Markku Hannula (toim.)

Matemaattisten uskomusten tutkimuksen nykytila V
Raportti MAVI-5 kokouksesta 22 - 25.8.1997. 92 s.

Tiivistelmä

Viides matemaattisten uskomusten tutkimuksen tilaa käsittelevä kokous pidettiin Helsingin yliopiston opettajankoulutuslaitoksella elokuun 22. päivästä elokuun 25. päivään 1997. Kokouskieli oli englanti. Kokouksessa ei ollut erikseen pääesitelmiä, vaan kaikille esityksille oli varattu aikaa 30 minuuttia, jonka jälkeen seurasi 30 minuutin keskustelu. Käsite 'uskomus' oli ymmärretty laajassa merkityksessään ja kokousesitelmät käsittelivät myös käsityksiä, näkemyksiä ja asenteita.

Vaikka kokoukset on alunperin aloitettu suomalais-saksalaisena yhteistyönä, on osallistujia ollut myös muista maista. Tällä kertaa vierainamme olivat Sergey A. Rakov Ukrainasta sekä Martin Risnes Norjasta. Tämä raportti sisältää kaikki kokousesitelmät.

Esitelmät perustuivat pääosin empiiriseen aineistoon. Mukana oli kolme lähinnä teoreettiseksi katsottavaa esitystä (Kasten, Rakov ja Törner) sekä yksi menetelmiä käsittelevä esitys (Hannula). Empiirisistä töistä neljää (Graumann, Nevanlinna, Pehkonen ja Risnes) voidaan pitää luonteeltaan lähinnä kvantitatiivisina ja neljää (Berger, Kalesse, Lindgren ja Römer) laadullisina tutkimuksina. Tämä jaottelu ei ole jyrkkä, sillä useissa töissä oli käytetty molempia lähestymistapoja.

Tällä kerralla ainoastaan kaksi tutkimusta (Graumann ja Nevanlinna) kohdistui yläasteen ja lukion oppilaiden uskomuksiin, kaksi (Kalesse ja Risnes) lukion jälkeiseen opiskeluaikaan, kaksi opettajakoulutukseen (Lindgren ja Römer) sekä samoin kaksi (Berger ja Pehkonen) ammatissa toimivien opettajien uskomuksiin.

Avainsanat: matematiikka, uskomukset, käsitykset, näkemykset, asenteet

Contents

Preface

The fifth Finnish-German workshop on Current State of Research on Mathematical Beliefs, the so-called MAVI-5 workshop took place in the Department of Teacher Education at the University of Helsinki from Friday the 22nd of August to Monday the 25th of August 1997. There were 19 participants of whom almost everybody had a presentation. This volume contains the abstracts of all of the talks given at the workshop.

In this report, every author is responsible for his / her own text. These are neither proof-read by the editor, nor their language is checked. Addresses of the contributors can be found in the appendix.

The Finnish-German research group MAVI (MAthematical VIews) is initiative of my colleague Günter Törner and myself, and its aim is to study and examine those mathematical-didactical questions which arise through research on mathematical beliefs. Three earlier workshops (MAVI-1, MAVI-2 and MAVI-4) were organised at the University of Duisburg on October 1995, March 1996 and April 1997 respectively and one workshop (MAVI-3) at the University of Helsinki on August 1996. Their proceedings are published in the pre-print -series of the corresponding institute. The next workshop is planned to take place at the University of Duisburg on March 1998.

In this place, I want to thank our graduate students for the help I got when organising the workshop: Mr. Markku Hannula had major part of the organisatorial work, from organising the Sunday excursion to Seurasaari to the editorial responsibility of this report. In addition, Ms. Marja Nevanlinna helped him e.g. by taking care of the coffee breaks. Furthermore, I like to express my gratitude to the Finnish Academy and the German DAAD for the financial support of our Finnish-German co-operation. My thanks are also due to our Head of Department, prof. Irina Koskinen for publishing these proceedings in the Research Report -series of the Department of Teacher Education.

Helsinki, December 1997

Erkki Pehkonen

Between Formalism and Creativity

Teachers' Conceptions of a Good Computer Science Student

Peter Berger, University of Duisburg

Background

This paper summarises some results of a research project which was undertaken in order to analyse the computer and computer science 'world views' of teachers (cf. Berger 1997a). The study is methodologically based on instruments of modern qualitative social research (cf. Lincoln & Guba 1984, Tesch 1990, Giddens 1993, Lamnek 1995, Berger 1997b), which are, apart from participant observation, mainly characterised by detailed analyses of exemplary single cases with the help of intensive in-depth interviews. The analysis of the interviews followed the principles of modern hermeneutics (cf. Titzmann 1977, Oevermann et al. 1979, Oevermann 1986, Beck & Maier 1994). In this paper, I will focus on one aspect of that study, i.e. on the teachers' preferences for certain types of (successful) students.

Having certain educational conceptions, teachers want to reach certain pedagogical aims. Logically enough, they can achieve those aims only in co-operation with their students. Teachers, of course, know that as reflections about aims and students belong to their every day life. So, shouldn't we reasonably assume that there is a correlation between a teacher's educational conceptions on the one hand, and his or her conceptions of students – particularly of good students – on the other hand? However, even reasonable assumptions are not necessarily true to reality.

In my investigation of the computer science world views of teachers, I ran a preliminary study with open interviews with nine teachers. In those interviews, the teachers came up on their own account with descriptions of good computer science students. Some respondents simply gave descriptions, some showed their personal likes or dislikes of certain types of students. Essentially, there were two different types of students which were contrasted in those descriptions.

To gain a deeper insight into the teachers' views of those types of students, I formulated a question for the standardised interviews of the main study (which contained 21 interviews). This question was based on the descriptions given in the pre-study interviews.

The research questions had been:

1. Do the teachers accept those two types of students, named by the respondents of the first series? Do they modify the descriptions? Will they give descriptions of other types?
2. Do teachers really have certain preferences for types of students? And if so, will they admit that they have these preferences? (Doesn't it belong to the role of a teacher not to be influenced by personal likes and dislikes?)
3. How do those preferences – if there are any – correlate with the interviewees' educational concepts and computer science beliefs?

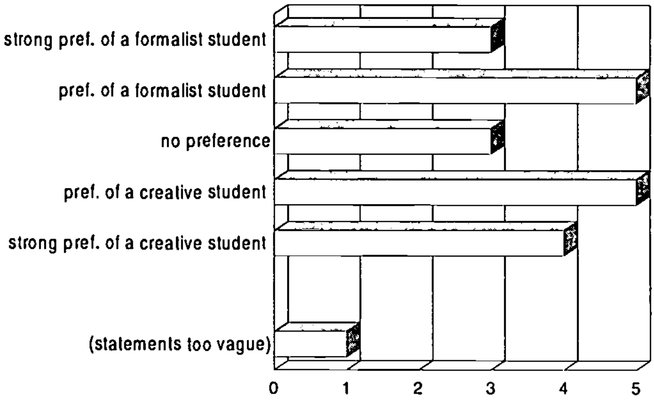

Diagram 1. Teachers' preferences for student types (N=21).

The question was:

"There seem to be two extreme types of good computer science students: The 'creative type' likes problem solving, finds unexpected solutions, but may sometimes work in a somewhat lax manner and may dislike team-work and explaining his/her ideas to others; the 'formalist type' likes accuracy, works in a more disciplined way, gives exact explanations, is co-operative, but may sometimes fail to have good ideas. Which type do you prefer?"

On a merely phenomenological level, the outcome can be described by the following simple observations:

- All interviewees give detailed answers, based on their own experiences.
- Only one interviewee questions (implicitly or explicitly) the existence of both types.

- The interviewees give more (and more detailed and vivid) characterisations of creativity than of formalism.
- 17 of 21 interviewees (80%) have a clear preference for one of the types.
- None of both types is significantly preferred more often than the other one (cf. Diagram 1).

Teachers' characterisations of creativity

The respondents made very few contributions to the description of the formalist student type. Their statements mostly stick to the characterisation of the formalist type given in the question. In contrast to this, the comments on the creative student type are altogether vivid, detailed, and frequently coloured by individual assessments. These assessments are often emotionally charged, covering a wide range from euphoric agreement to vehement disapproval. As an illustration, we quote from different interviews:

Teachers preferring a formalist student

- "The problem is, that I have to get along with this chaotic creative student who can hardly be persuaded to co-operate. I wish the good students would stop being obstinate and follow the conventions we made."
- "Well, why should I change a formally correct student? Because he has no good ideas? But perhaps he is not able to have any. [...] The creative student may be enthusiastic about his marvellous ideas; however, we have to put it into his head that he just causes troubles to himself this way."
- "I don't like those 'single combatants' in the class room who call themselves 'cracks'. [...] I think, the student who asks 'which tools are available to solve the problem?' is more successful than the creative student [...] who is creative, but unproductive."
- "If it works in an egocentricy, creativity will simply be useless, especially if you are to produce something in a team."
- "Creativity is in a way only a preliminary stage; it must be revised later on in a clean and formalistic way."
- "I definitely prefer the formalist type. I think, the reason is that I myself fall into that category. [...] I see difficulties in integrating the creative type into the class and to motivate him to co-operate. [...] However, something formally correct appeals more to me."
- "Well, I am against those solitary, reclusive [creative] students who are not able to use their resources economically."
- "The creative type is much more problematic [...] and requires a terrific lot of teacher's care."

Teachers with no preference

- "I do really like this kind of sound creativity, creativity based on learned stuff. But not, however, this chaotic creativity, which is chaos pure, a relapse into anarchy."
- "Creative phases are oozing with ideas."
- "It is useless to develop great ideas if you are unable to write the correct program, because you find it too silly to pay attention to details."

Teachers preferring a creative student

- "Creative students are constantly electrifying the others with good ideas. [...] Certainly creativity is a quality which is somehow encoded in the genes. You just have to offer it the right topics."
- "Well, to a student who is creatively running away I have to make clear that he has to slow down now and then on his way to ecstasy. [...] Of course I prefer a student who must be bridled and curbed. He makes my job much easier."
- "The creative, let's say do-it-yourself man is the more interesting one – I like such students."
- "I think it's great if someone is able to be creative. {Commenting on a certain creative student:} In the sixties he would have become a hippie, however, today times have changed and we now have the computer, and so he is realising his 'flower power ideas' this way."
- "A student who likes problem solving – a creative puzzle type – is a very good basis. [...] Whereas a formalist ... You don't know if he will be able to manage the other things ... if he is able to be creative. The first type [creative] has in any case proved to be talented – he will able to learn the rest somehow."
- "Certainly, the creative type is nearer to my heart. [...] In computer science classes, those creative people dominate over the others [...] who are waiting to be supplied with ideas. [...] Nevertheless, I must say it is fun. They are nearer to my heart."
- "I have a liking for the creative type. Naturally, they are more difficult to treat. [...] {Which type has a better success prognosis?} Well, I might almost think: the formalist - it's a pity."
- "Well, this sloppy and spontaneous creative type is closer to me. I don't know how far creativity can be learned. In any case, I think that it is more difficult to learn creativity than to learn discipline. [...] Actually, discipline means to cut down on one's innate behaviour. Strictly speaking, that's nothing positive."

- "Naturally, we all like that creative student more than mediocre people. It's great if there is such a student in the class."

The effect of the teachers' educational level

In a first approach, we may compare the respondents' preferences of a certain student type with their own educational level in computer science. German computer science teachers have not always graduated from a university. Some participated in a two year in-service-training and there are even some teachers with a merely autodidactic training.

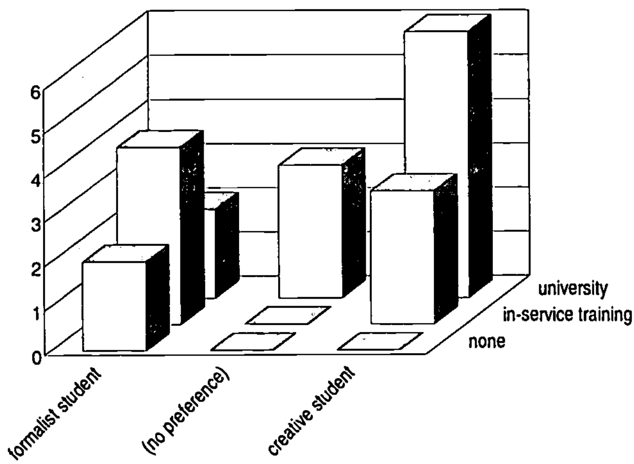

Diagram 2. Teachers' degrees in computer science and their preferred student type (N=20).

Among the 20 teachers of the main study (leaving out the one respondent whose comments on the student types are too vague to be evaluated), there are 11 with a university degree, 7 with in-service-training, and 2 without any degree. This roughly corresponds to the general situation in North-Rhine-Westphalia, i.e. the Federal State of Germany the respondents originate from.

The outcome will be described in the following statements (cf. Diagram 2):

- Teachers with a university degree in computer science are more likely to prefer the creative type.
- Teachers preferring the creative type are more likely to have a university degree in computer science.

- Within the group of teachers with in-service training degrees the preferences of both types are balanced.

What do the teachers' preferences for student types correlate with?

The most interesting question was: How do the teachers' preferences for a student type correlate with their educational concepts of computer science? In the questionnaire, one question aimed at the interviewees' preference concerning the substance of a student's solution:

"A student's solution of a programming task should show ...

- *reflection of obstacles* and difficulties encountered in the solution process;
- *creative ideas* in the process of problem solving;
- knowledge and application of *standard techniques*.

Please, distribute 100 points to the three items according to your personal assessment."

At this point, one could formulate the hypothesis that there should be a clear correlation between the answer to this question and the preference for a certain type of student. Such a qualitative correlation should be represented in Diagram 3, which shows the result in barycentric co-ordinates, as forming a design where the relevant symbols form groups. We should expect the white points to be oriented towards the left corner and the black ones to the right. If there were really a correlation between the teachers' pedagogical concepts and their preferences for creative or formalist students, it should simply be that teachers preferring creative students should prefer creative ideas. It is quite remarkable that no such design shows up.

We get similar outcomes from the further questions concerning the teachers' preferences for the *form of a student's solution* and concerning their concepts of the *central topics* and the *educational aims* of computer science as a school subject. The analysis did not show up any correlation between the respondents' interview statements regarding their preferences for a certain student type and their answers in the questionnaire concerning their pedagogical conceptions.

All in all, we found only one correlation of the sort we might have expected, i.e. the respondents' self-concepts as computer users. In the preliminary study, the interviewees came up with multi-faceted and partly contradictory comments on their attitudes as computer users. These comments have been crystallised into three 'typical statements' forming the basis of another question of the questionnaire:

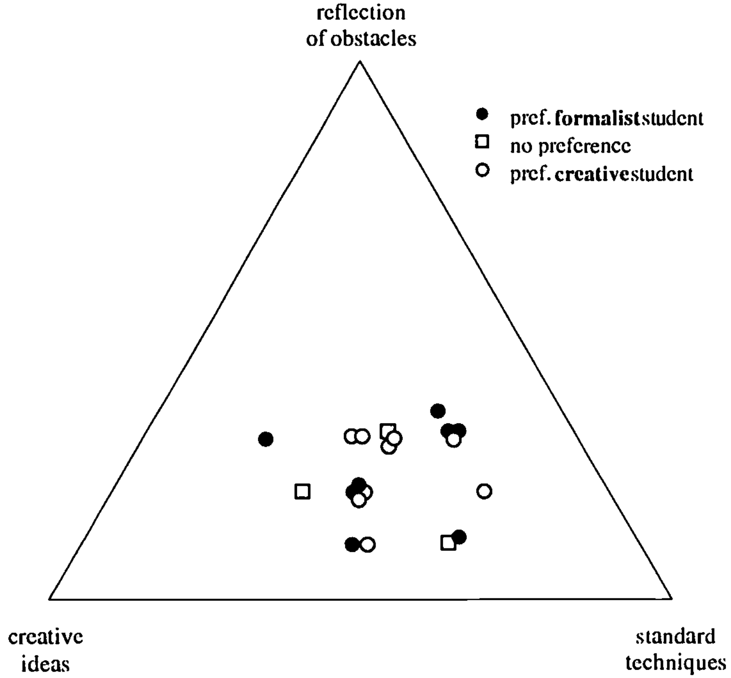

Diagram 3. Teachers' preferences for the substance of a student's solution.

"Please consider to what extent you can agree with the following statements and distribute 100 points according to the degree of your agreement:

- I like to work with a computer. I am able to use it in a creative way. It saves time and work, and where it does not, I enjoy doing the work with a computer in the same time that it would take me without it – but with a much better result.
- I regard the computer as a necessary (and sometimes troublesome) evil. It is relevant, and as a teacher I therefore feel it important to know my way with computers. However, I regard them with some scepticism.
- I think the computer is a means to an end. I do work with it that I previously did without it. Sometimes it bothers me when I realise that the computer costs more time than it saves."

The diagram reveals the respondent group preferring a creative student as being oriented towards the position of assessing the computer as creative and motivating, whereas the formalist group tends to take a more utilitarian perspective (cf. Diagram 4). The correlation between the respondents' preferences for a student type and their self-concepts as computer users, as it is manifested here, is also corroborated by the explicit comments of the interviewees during the interviews.

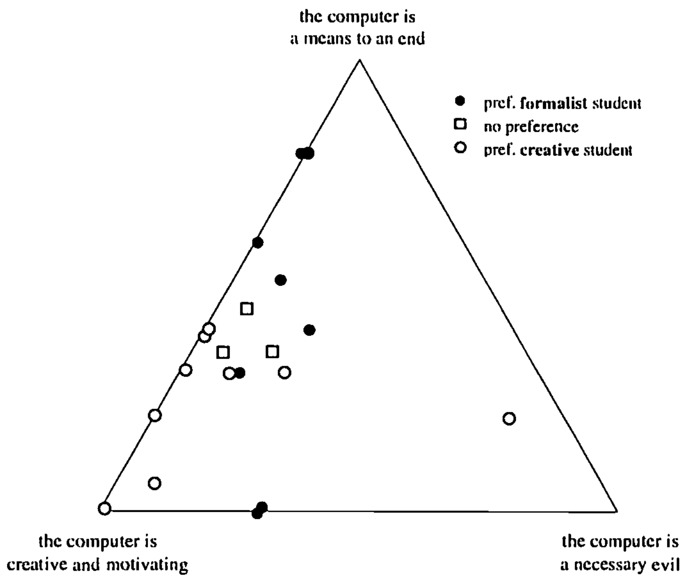

Diagram 4. Teachers' self-concepts as computer users.

Summary

On the basis of a thorough analysis of the interview statements and the outcomes of the questionnaire, we can draw the following conclusions.

1. The question of the preferred type of a good student in computer science polarises the group of teachers. The teachers preferring a creative student form a more homogeneous group than the teachers preferring a formalist student – both with regard to the teachers' self-concept as computer users (the majority sees the computer as creative and motivating) and with regard to the teachers' educational level in computer science (they dominantly have a university degree).
2. The formalist teacher's view of a creative student is characterised by keywords such as "chaotic", "anarchic", "obstinate", "unproductive", "egoistic". He or she sees a creative student as a "single combatant" which "causes troubles to himself". In contrast to that, a creative teacher depicts a

creative student as "sloppy", "spontaneous", "hippie", "electrifying", "oozing with ideas", "on the way to ecstasy", "more difficult to treat", "above mediocrity".

3. There is a significant difference between the ways teachers describe, substantiate, or justify their preferences for the creative and the formalist student type. The teachers preferring the creative type refer to the positive aspects of a creative student, whereas the majority of the teachers who prefer the formalist type do not refer to the positive aspects of a formalist student, but rather to the negative aspects of a creative student.
4. The teachers are likely to have a *'central' attitude towards the creative type*. This attitude conforms to (is induced by?) the teacher's self-concept. It is this central attitude towards the creative type which determines the 'satellite' attitude towards the formalist type.
5. It is not the teacher's educational concepts of computer science which makes him or her prefer a creative student or a formalist student. Those preferences rather originate in the teacher's personal likes or dislikes of persons, i.e. in the teacher's attitudes towards people. With other words, the preference is *socially determined,* not conceptionally. Teachers prefer a student type fitting their own type of personality, not fitting their pedagogical conceptions.

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Mathematical Views of Seventh-graders in Bielefeld and Aschaffenburg

Günter Graumann, University of Bielefeld

Some years ago Erkki Pehkonen and Bernd Zimmermann developed a questionnaire relevant for beliefs about mathematics education. It has been used already in several countries for example 1989 in Helsinki (Finland) and 1991 in Bielefeld (North-Rhine-Westphalia). In February 1997 this questionnaire was used again by Heidrun Schneider (a student of mine) with seventh-graders in Aschaffenburg (Bavaria). Thus I want to report here about some interesting results of Aschaffenburg from 1997 in comparison with those results of Bielefeld from 1991.

But before reporting I would like to make a methodical hint. I think there are first two different ways for interpreting results of such questioning. Either you look at the data and some obvious peculiarities and try to find interpretations of them. Or you give classifications of the items with relations to didactical conceptions at first, sort the data in respect to this classification and find interpretations then.

One classification of our items given by Erkki Pehkonen is those in views about mathematics, mathematics learning and mathematics instruction or a little bit more differentiated in: views about 1a) mathematical contents 1b) mathematical working method 2) mathematical learning 3a) the pupils role in mathematics instruction and 3b) the teachers role in mathematics instruction.

This classification I use during the presentation of the means of all items but I will not use it during the presentation of peculiarities and comparisons, i.e. I use the first named method. Then at the end, I would like to discuss classifications for the second method, especially in connection with didactical aspects you will find e.g. in Grigutsch (1996) namely: application aspect, process aspect, formalism aspect, scheme aspect and rigid scheme orientation.

Now I first will give you an overview of the means of Aschaffenburg (BAY) and Bielefeld (NRW) so you can get your own impression of the results. The population of the questioning in Aschaffenburg (BAY) exists of 137 pupils with the following distribution: Main-school (HS) 34 [18 male + 16 female], middle-school (RS) 59 [17 male + 42 female] and gymnasium (Gy) 44 [8 male + 36 female] . The population in Bielefeld was originally about 250 but for comparison with Aschaffenburg we dropped out the pupils of the comprehensive school (Gesamtschule) because such type of school does not exist in Bavaria. The so got population of Bielefeld (NRW) exists of 151 pupils with the following distribution: Main-school (HS) 51 [26 male + 25 female], middle-school (RS) 58 [27 male + 31 female] and gymnasium (Gy) 42 [27 male + 15 female].

In addition to these quantities I should mention a speciality of the Bavarian school system: At the end of the primary school the pupils can choose between main school and gymnasium depending on their achievement. After sixth grade the middle school starts for good pupils of the main school and the poor pupils of the gymnasium. The first year (the seventh grade) is valid as test year. The final exams especially of the gymnasium (the Abitur) are done with central chosen tasks equal for all students.

For better categorising this data I have added the already published data of Finland (FIN) and Germany (D) [see Graumann & Pehkonen 1993] as well as the ranges of all special means of NRW and BAY. Under the column "remarks" you find some hints for the later classifications and the significant differences between Finland and Germany discussed in 1993.

In order to understanding all named details I put down the legend of the following table.

FIN = Helsinki 1989, D = Bielefeld 1991, NRW = Bielefeld 1991 without Gesamtschule, BAY = Aschaffenburg (Bavaria) 1997

Mean range BAY/NRW = smallest and largest mean within the group of specific means
(BAY/Gy/f), (NRW/Gy/f), (BAY/Gy/m), (NRW/Gy/m),
(BAY/RS/f), (NRW/RS/f), (BAY/RS/m), (NRW/RS/m),
(BAY/HS/f), (NRW/HS/f), (BAY/HS/m), (NRW/HS/m)

HS = main school (Hauptschule), RS = middle school (Realschule), Gy = gymnasium,
f = female, m = male

\+ + = full agreement (means of FIN, D, NRW, BAY < 2.05) ,
\+ = agreement (means of FIN, D, NRW, BAY < 2.75) ,
o = neither-nor (2.51 < means of FIN, D, NRW, BAY < 3.35
\- = disagreement (3.30 < means of FIN, D, NRW, BAY < 3.99
\- - = full disagreement (4.08 < means of FIN, D, NRW, BAY < 4.27
F = significant differences (on 0.01% level) between FIN and D
B = mean NRW minus mean BAY > 0.3 , B' = mean NRW minus mean BAY < - 0.3

The 32 items are arranged as already mentioned in the aspects of 1a) mathematical contents 1b) mathematical working method 2) mathematical learning 3a) the pupils role in mathematics instruction and 3b) the teachers role in mathematics instruction.

Results (Means) of the Pupil-Questioning

No	short text	FIN N≅250	D N≅250	NRW N=151	BAY N=137	mean range BAY/NRW	remarks
1	mental arithm.	1.76	1.81	1.91	1.74	1.22 - 2.50	+ +
3	written comp.	2.00	1.53	1.52	1.69	1.46 - 1.88	+ + , F
6	drawing figures	2.31	2.07	2.26	1.91	1.38 - 2.73	+ , B
9	word problems	2.20	2.07	2.01	2.61	1.67 - 3.45	+ , B'
14	use of calculators	2.04	2.98	2.99	2.56	2.12 - 3.36	+ , F , B
17	topics sep.taught	2.77	2.02	2.02	1.63	1.47 - 2.33	+ , F , B
19	practical benefits	1.52	2.02	1.99	1.64	1.41 - 2.42	+ +, F,B
22	calc. of area/vol.	2.09	2.20	2.33	2.07	1.53 - 2.63	+
28	concrete objects	2.91	3.07	3.19	2.86	2.44 - 3.63	o , B
2	right answer imp.	3.60	3.78	3.70	3.94	3.00 - 4.36	-
5	express exactly	3.22	3.23	3.35	3.01	2.25 - 392	o , B
10	scheme for result	3.07	2.57	2.59	2.51	2.44 - 3.27	o , F , B
16	reasoned exactly	2.59	2.69	2.71	2.57	1.94 - 3.07	+
20	math. talent nec.	4.27	4.12	4.14	4.08	3.00 - 4.47	- -
21	not always fun	2.46	2.45	2.47	2.43	2.16 - 3.24	+
23	a lot of effort	2.87	3.18	3.16	3.21	2.63 - 3.47	o
24	not only one way	1.85	1.75	1.64	1.61	1.25 - 1.86	+ +
11	all understand	1.85	1.47	1.44	1.23	1.06 - 1.56	+ + , F
12	memorising rules	3.30	3.70	3.95	3.99	3.39 - 4.47	- , F
18	a lot of repetition	2.45	2.48	2.47	2.01	1.56 - 2.78	+ , B
29	a lot of exercises	2.34	1.99	1.93	1.78	1.56 - 2.80	+ , F
30	must understand	2.01	1.42	1.42	1.21	1.12 - 1.54	+ + , F
4	make guesses too	1.98	2.63	2.65	2.75	2.41 - 3.45	+ , F
7	right answer nec.	3.74	3.30	3.37	3.22	2.47 - 3.76	- , F
8	strict discipline	1.51	2.40	2.51	2.96	1.78 - 3.73	+, F,B'
13	work by oneself	2.00	2.17	2.26	2.29	2.00 - 2.50	+
27	solve by oneself	2.05	1.51	1.95	1.80	1.33 - 2.21	+ + , F
31	working i. groups	1.86	1.82	1.97	1.77	1.25 - 2.24	+ +
15	t. helps at once	2.04	1.81	1.81	1.83	1.13 - 2.44	+ +
25	math. games	2.18	2.02	1.97	2.23	1.79 - 2.78	+
26	exact explanation	2.08	1.51	1.42	1.47	1.28 - 1.58	+ + , F
32	t. tells what to do	2.58	2.36	2.39	2.66	1.94 - 3.14	+

The first thing you can notice is that there are only a few **disagreements**, namely with full disagreement only No 20 (math. talent necessary) and with disagreement No 12 (memorising rules), No 2 (right answer important), No 7 (right answer necessary).

Items with full agreement all over (++) as

No 1	(mental arithmetic)	[no differences between BAY/NRW/D/FIN]
No 15	(teach. helps at once)	[only small differences BAY/NRW/D < FIN]
No 31	(working in groups)	[only small differences BAY < D/FIN < NRW]
No 19	(practical benefits)	[FIN < BAY < NRW/D, sign. diff. D-FIN/BAY]
No 3	(written computation)	[NRW/D < BAY < FIN, sign. diff. D-FIN]
No 27	(solve by oneself)	[D < Bay < NRW < FIN, sign. diff. D-FIN]
No 26	(exact explanation)	[BAY/NRW < D < FIN, sign. diff. D-FIN]
No 24	(not only one way)	[BAY < NRW/D < FIN, BAY]
No 11	(for all understandable)	[BAY < NRW/D < FIN, sign. diff. D-FIN]
No 30	(you must understand)	[BAY < NRW/D < FIN, sign. diff. D-FIN]

Noticeable is also the very high agreement with No 11 and 30 in BAY

Differences between the mean of NRW and BAY

No 9	(word problems)	[NRW-BAY = -0.60, agreement NRW] *
No 8	(strict discipline)	[NRW-BAY = -0.45, neither-nor BAY] *
No 28	(concrete objects)	[NRW-BAY = +0.33, neither-nor both]
No 5	(express exactly)	[NRW-BAY = +0.34, neither-nor both]
No 6	(drawing figures)	[NRW-BAY = +0.35, agreement BAY]
No 19	(practical benefits)	[NRW-BAY = +0.35, agreement both]
No 17	(topics sep. taught)	[NRW-BAY = +0.39, agreement both]
No 14	(use of calculators)	[NRW-BAY = +0.43, neither-nor NRW]
No 18	(a lot repetition)	[NRW-BAY = +0.46, agreement both]

*the items No 9 and 8 have more agreement in NRW

These results must be differentiated a bit because within one city there are sometimes exceptions to identify if we look on the special means. Therefore we will also have a look at the

Differences NRW-BAY differentiated between school-types.

No 9	(word problems)	HS = +0.05	RS = **-0.40**	Gy = **-1.35** *
No 8	(strict discipline)	HS = **-0.66**	RS = **-0.37**	Gy = -0.04 *
No 28	(concrete objects)	HS = **+0.87**	RS = +0.09	Gy = +0.08
No 5	(express exactly)	HS = **+0.62**	RS = -0.37	Gy = **+0.95**
No 6	(drawing figures)	HS = +0.28	RS = +0.29	Gy = **+0.56**
No 19	(practic. benefits)	HS = **+0.74**	RS = +0.24	Gy = +0.06
No 17	(topics sep.taught)	HS = +0.25	RS = **+0.42**	Gy = **+0.50**
No 14	(use of calculators)	HS = -0.08	RS = **+1.01**	Gy = +0.07
No 18	(a lot repetition)	HS = **+0.67**	RS = +0.16	Gy = **-0.67**
No 25	(math. games)	HS = **-0.81**	RS = +0.12	Gy = **-0.30** *
No 32	(t. tells what to do)	HS = +0.28	RS =-0.27	Gy = **-0.69** *
No 10	(scheme for results)	HS = **+0.99**	RS =**-0.59**	Gy = +0.09
No 16	(reasoned exactly)	HS = **+0.80**	RS =-0.21	Gy = +0.05
No 29	(lot of exercises)	HS = +0.11	RS =-0.15	Gy = **+0.60**

*the average means in NRW of the items No 9, 8 and 25, 32 indicate more agreement in NRW

In the following, this picture will be deepened by the **results of the open questions** of the pupils in NRW and BAY

Good experiences with mathematics instruction

mentioned keywords	NRW	BAY
fun, amusing, relaxing, sometimes joking	21 %[1]	11 %
understood a lot, learnt a lot, was interesting	17 %[1]	18 %
teacher explained fine, good teacher	15 %[2]	17 %[2]
nice, good tempered and sympathetic t., t. helped	9 %	7 %
playful, vivid	8 %	3 %
geometry	5 %	4 %
good marks	0 %	16 %[3]

[1] from it nearly half HS [2] "expl. fine" most HS and RS, "good t." most BAY/Gy [3] 70% RS

Bad experiences with mathematics instruction

mentioned keywords	NRW	BAY
no fun, too serious, too much dry theory	6 %	12 %
unnecessary/useless, illogical and meaningless things	0 %	10 %[4]
too difficult, too fast, too much topics, so little time	13 %[5]	40 %[5]
bad or complic. explanations, too much terms, bad t.	7 %	25 %[6]
rigorous/humourless t., no sympathy for poor pupils	18 %	9 %
too much homework	12 %	5 %
word problems and percentage	6 %	7 %
bad marks	5 %	10 %

[4] mostly Gy (no HS) [5] mostly RS and Gy [6] mostly RS (esp. pupils that changed from Gy)

Wishes for mathematics instruction

mentioned keywords	NRW	BAY
more fun/more change, loose and interest. instruction	13 %	25 %[7]
useful and meaningful things for practical benefits	2 %	9 %[8]
good and more explan., everybody should understand	18 %	27 %[9]
easier tasks, more exerc. And repetition, better marks	10 %[10]	8 %[10]
slower/more time, not so fast changes, less topics	2 %	9 %
nice and well-tempered t., helpful t., symp. for poor	9 %	15 %
not so much homework	11 %	3 %
math. games, group work, mental arithmetic	30 %	10 %
more geometry, more drawings	8 %	1.5 %

[7] mostly RS and Gy [8] mostly Gy 9 very often RS [10] mostly Gy

From the results of these remarkable differences between NRW and BAY in the concerned classes - and only for them - we can get the following impression:

- fun and loose instruction appear more in NRW than in BAY
- useful and meaningful things appear more in NRW than in BAY
- better explanations are wanted in BAY (RS and Gy)
- less homework is wanted in NRW
- more geometry is wanted in NRW
- playful mathematics appear more in NRW but are also wanted more in NRW
- good marks have great importance in BAY (RS)

[for the last point see the mentioned speciality of the Bavarian school system]

In addition to these hints of possible differences between the sets of the two cities of Germany I like to show you some remarkable differences in NRW and BAY between male and female. You can not find out them from the given data but from all specials means I already mentioned (which can not be put down here). I see remarkable differences if the specific mean for the female minus that for the male is larger than 0.4 or smaller than - 0.4 .

No 9 (word problems)	BAY = **- 1.02**	NRW = - 0.01
No 2 (right answer imp.)	BAY = **- 0.89**	NRW = - 0.16 *
No 7 (right answer nec.)	BAY = **- 0.70**	NRW = - 0.25 *
No 16 (reasoned exactly)	BAY = **- 0.52**	NRW = - 0.24
No 20 (math. talent nec.)	BAY = **- 0.46**	NRW = - 0.30 *
No 15 (t. helps at once)	BAY = **- 0.48**	NRW = +0.09
No 1 (mental arithm.)	BAY = **- 0.43**	NRW = +0.39
No 6 (drawing figures)	BAY = +0.23	NRW = **- 0.46**
No 32 (t. tells what to do)	BAY = +0.15	NRW = **- 0.42**
No 21 (not always fun)	BAY = **+0.42**	NRW = - 0.07 **

* topics with disagreement ** female agree more than male in BAY

Because of the special distribution of male and female in the questioned middle school and especially the questioned gymnasium in Aschaffenburg we can use these results only as hints and the possible interpretation shown below as stimulus for following research.

- in BAY *male* agree more or disagree less to topics responding traditional instruction (with the exception of "mental arithmetic" the male in NRW have the same trend)
- in NRW *male* agree more to "exact instruction" and "drawings"
- in BAY *female* see mathematics instruction as more severe than male.

Conclusions

The differentiation between HS on one the hand and RS/Gy on the other hand is mostly necessary. But it also seems that there are some cultural differences between NRW and BAY which lead us to look nearer at the circumstances which may influence the beliefs. Thus different types of questionings - like questionnaires, open questions, interviews of pupils as well as interviews of their teacher - have to complete the view (open questions and the interviews have the chance to lead to beliefs not only on the surface). Also teacher and researcher who know the circumstances should be included in the interpretations.

In a second or third step the research on beliefs should focus on special questions and be connected with different conceptions and objectives. Therefore a discussion about classifications of the items of a questionnaire is helpful. Varying the didactical aspects of Grigutsch I finally bring up for discussion the following classification (or better cluster) of our 32 items.

- aspect of scheme and exercise orientated learning (No 2, 3, 7, 8, **10**, 12, 15, 17, 18, 26, 29,32)
- aspect of application and profit for everyday life (No 1, 3, 9, 14, **19**, 22, 28)
- aspect of process orientated aims and open work (No 2, 4, 6, 7, 13, 21, 24, 25, 27, 28, 31)
- aspect of exact mathematics and mental demands (No 5, 8,11, 16, 20, 21, 23,30)

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Teacher as an Enactivist Researcher

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In our research project on development of pupils' mathematical beliefs (Hannula, Malmivuori & Pehkonen, 1996) I have combined roles of teacher and researcher. In this article I discuss some methodological points concerning teacher as a researcher of mathematical beliefs. I am not trying to present complete theory of teacher as a researcher. Rather I try to present a theory *for* teacher as a researcher. Ontological questions about the nature of beliefs I base on Poppers idea of three worlds. For epistemological questions I present the enactivist theory for cognition, which allows the widest rage of possibilities in inquiring beliefs. I continue presenting the enactivist methodology and finally conclude with some practical guidelines for enactivist teacher-researchers.

Three worlds of Popper

Popper divided reality into three worlds (Figure 1). World 1 consists of physical objects and phenomena. World 2 consists of subjective experiences and mental states. World 3 is created by humans, and there you find texts, theories and art. Human brain belongs to World 1, but mind to World 2. A paper belongs to World 1, but the story written on it to World 3. A thought (World 2 item) becomes a part of World 3 when it is spoken out. (Popper & Eccles 1977 36–50, 451).

In this framework mathematics is part of the World 3. Subjective mathematical knowledge and mathematical beliefs belong to World 2. Any inquiry on World 2 objects is very difficult because we have no direct access to those subjective mental states – except our own. When these mental states are expressed (in words or other ways) they become accessible – as they turn into World 3 objects.

Beck & Meier (1994) come to similar conclusion that "every reality, which is relevant for research on didactic of mathematics, is formed like a text." They quote Ricour's (1985, p. 83) definition of text as "written document of our culture", which includes "also all kinds of documents and monuments that will be fixed in a form similar to writing". A videotape, a photography and a drawing are texts in this sense.

Unlike Beck and Meier I see that also such mental states that can not be fixed in a text can and should be researched. But how do we handle observations of a researchers in action? They may be richer than one could ever put into words. The enactivist methodology gives a way to grasp these impressions.

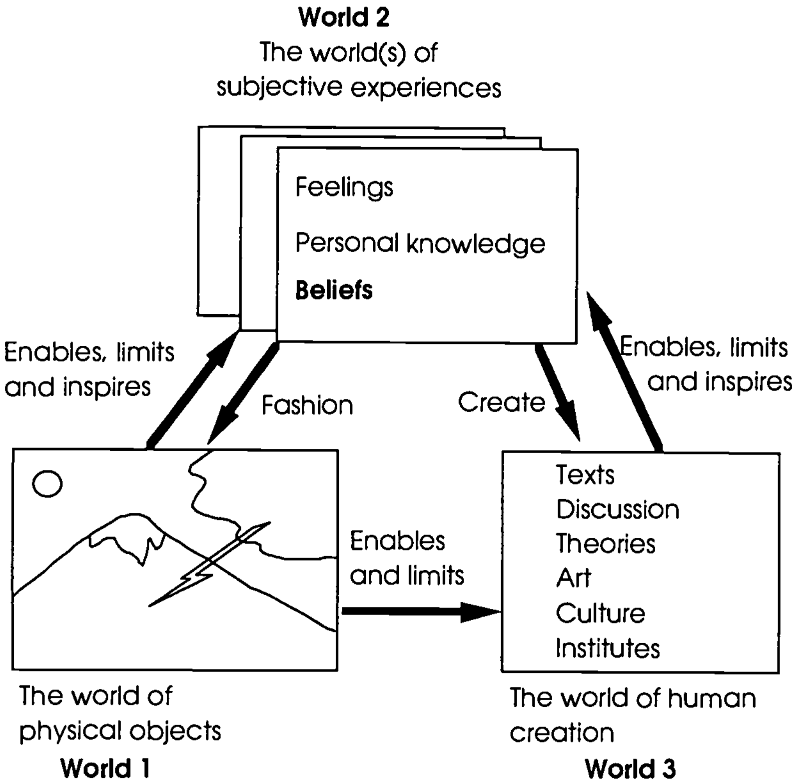

Figure 1. The three worlds of Popper.

Enactivism

To understand enactivist methodology we need to have some idea of the underlying general theory for cognition, enactivism. The key concepts in enactivism are autopoesis, structure determinism, structural coupling and coemergence. (Maturana & Varela, 1992; Reid, 1996)

Autopoesis is the spontaneous self-organisation of complex, dynamic systems. For example cells and animals are autopoetic entities but so are also individuals, institutions, ideologies and cultures.

Structure determinism refers to the idea, that the structure of an autopoetic system determines action (and not an external stimulus). The structure does not mean only the cognitive structure of an individual but the mind and body together.

Autopoetic entities tend to organise themselves into networks of interaction and thus form new autopoetic systems. This is called structural coupling. Individuals organise themselves into institutions (for example nations) which live their own lives far beyond the lives of the individuals.

Coemergence refers to structurally coupled autopoetic entities developing together. For example cells in a embryo affect each other's development. Similarly humans in a culture develop together - coemerge.

Learning in enactivism is the continuous structural change of an autopoetic system. Learning takes place, when an autopoetic system adapts to change in its milieu. Learning is a structural change that allows new kinds of actions for the system. The learned knowledge is embodied and not necessary conscious. Learning can take place in any autopoetic system like an institution or an idea. Embodied knowledge means that all components of the system together carry on the cognition. (Maturana, 1987, p. 74–75; Reid, 1996)

Enactivist methodology

Enactivist methodology sees research as a learning process. To put it simple, it means that to gain knowledge you just need to interact. In this interaction there are two sides that need to be considered. Firstly is the research topic, which has to be connected somehow to the researcher. Most methodologies set clear rules for this connection. Enactivism leaves this quite open and subordinate to the other side, the researcher. In enactivism the structure of an autopoetic entity determines what actions are possible. In other words: what we can learn is determined by our theories, beliefs, biases and even our feelings in the research situation. Enactivist methodology seeks ways in which learning is least restricted. (Reid, 1996)

The enactivist methodology gives some guidelines for research. Reid (1996, p. 207) writes that "[the] two key features of enactivist research [are] the importance of working from and with multiple perspectives, and the creation of models and theories which are good enough *for*, not definitively *of*."

You can reach the goal of multiple perspectives in various ways. You could work on a common collection of data together with researchers with different theories and even different research questions. Each researcher's own perspective would provide a new view for others. You could also have a wide rage of data. Here wide means large variance, not necessarily large quantity. You could also review the same data several times in different situations and with different people. The creation of models and theories provides also new perspectives. These models and theories need not be presentations of reality. It is enough if they clarify our understanding of the data and help answering our research questions.

Enactivist methodology in belief research

Let me now go back to the three worlds of Popper, and the problem of doing research on World 2 items. Enactivist methodology gives us the way to gain such knowledge on beliefs that is not necessarily possible to write down as a text. Think of a teacher who has been teaching a group of pupils for several years. He/she knows the pupils well, and is able to tell a lot about them. There is also knowledge that he/she will not be able to put into words. A researcher (especially a teacher-researcher) can gain such embodied knowledge. The problem remains how this knowledge can be used in research.

A researcher in enactivism is seen structurally coupled with a research community. Enactivist methodology claims that in such coupling embodied knowledge can spread out to the whole community. To clarify this idea I refer to the theory of truth by Peirce (figure 2).

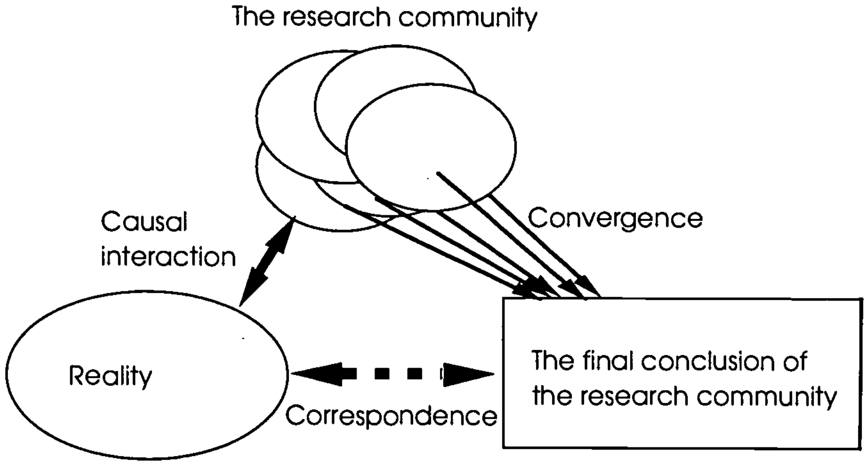

Figure 2. The theory of truth by Peirce (Haapasalo 1997, p. 57; Niiniluoto 1987, 46).

The interaction between researcher and reality enables the researcher to gain some knowledge on reality. Each researcher has a different view and some views may be quite far from reality. Through its discussion the research community will converge towards one conclusion[1]. This conclusion is not the same thing as the reality, but it has a correspondence with it.

Enactivism claims that the conclusion is determined by the structure of research community, which includes the researchers with their embodied knowledge. So this knowledge has an influence in the discussions even if it was never spoken out. The conclusion is not determined by the arguments alone, but also by the discussioner's activity and persistence that are based on their intuitions and commitment.

Teacher as an enactivist researcher

Finally I would like to conclude from all this philosophical pondering some practical guidelines for teacher-researchers. Let me begin with Hopkins' (1985, p. 41–43) five criteria for classroom research by teachers:

1. "The teachers primary job is to teach, and any research method should not [] disrupt the teaching commitment."
2. "[T]he method of data collection must not be too demanding on the teacher's time."
3. "The methodology must be reliable enough to allow teachers to formulate hypotheses confidently and develop strategies applicable to their classroom situation."
4. "[T]he research problem undertaken by the teacher should be one to which he or she is committed."
5. "[The teacher researchers need] to pay close attention to the ethical procedures surrounding their work."

The first criterion is easily achieved within enactivist methodology. Good teacher learns to know his/her pupils and is constantly sensitive for any feedback. Enactivist methodology only underlines this side of good teaching.

There are some problems with the second criterion. A teacher has much interaction with the pupils teaching them and data collection could be limited to copies of pupils works and field notes by teacher. So the amount of data would be sufficient, but enactivist methodology stresses also the need to work from and with multiple perspectives. Talks with colleagues and parents serve some new perspectives. An enactivist teacher-researcher should also use every possibility to meet the pupils outside the class: at lunch breaks, school parties and excursions. For a form master of the class this will come naturally.

[1]Convergence towards one conclusion may be an overoptimistic assumption. We should rather talk about coverging, where ideas continue to be different, but in relationship (Kieren 1997).

The third criterion is probably the most problematic. Enactivist methodology uses the researcher as a tool, and the reliability is dependent on how well the teacher is aware of his/her biases and shortcomings. The need for multiple perspectives is stressed again because with too few perspectives it is possible to come to very biased results. The teachers can not rely on methodology, but vice versa the methodology relies on the teacher-researcher.

The fourth criterion of commitment has two sides. Commitment increases the amount and intensity of interaction. On the other side it may also lead to biases, if the teacher-researcher is committed to certain practice and is only looking for evidence to support the conclusions made earlier. So the commitment should be for a question, not for desired results. An experienced teacher should have an intuition (embodied knowledge) which topics are important for improving his/her own practice.

The fifth criterion is an issue that can not thoroughly be discussed here. I will point only to one specific characteristic of enactivist methodology. Even if enactivist methodology does not necessarily affect the teaching practice in any ways and it even allows the research project to remain invisible, those involved should be informed and asked permission for. One advantage enactivism has over other methodologies is that it is not dependent on any particular piece of data. The pupils may want to keep any of the material unpublished and yet it would not destroy the project (as long as there would be some material available).

There is one more question left, reporting. What I have gone through gives hopefully an idea how teachers can learn from and about their pupils. Many teachers have actually done that throughout centuries. How should we bring this subjective knowledge to the research community? Mason (1994, p. 181) has contrasted research in education with mathematical research by stating that "in education it is impossible to build upon a proposed 'result' without testing it in one's own experience and situation. If it checks out, sheds light, sharpens awareness, or extends the range of actions, it will be taken as valid for that individual, otherwise it will fade into the background." He also spoke about "not-seeing and coming to see" (ibid. p. 188). The researchers task is to help reader to see something. For this he/she can use for example accounts, metaphors and even narrative stories. The final responsibility for validity and reliability is left for the reader. In Masons words: "The test of validity is whether it generates convincing stories about the past, and whether it informs actions in the future" (ibid. p. 184).

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Change in Mathematical Views of First Year University Students II

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Intention of the study

The education of future mathematics teacher is oriented by science and research. Also an other aspect has to be mentioned: The attitude to mathematics and the way of teaching can be formed by the education at university, but this attitude is also influenced by earlier - for example in school lessons - acquired attitudes and prejudices, which should be known in order to educate the students at university.

Concerning the efficiency of the teacher education the students' attitudes, their origins and the process of their development are too little explored.

The intention of our study is to find out something about the students' attitudes to mathematics and the process of their development:

- during their school days,
- after their first month studying mathematics at university,
- and after six month when they did their first exams at university.

In doing so not only the students' attitude to 'pure' mathematics were considered to be important, but also the general attitude of former schoolchildren and future teachers towards mathematics as a subject taught at school.

Theoretical framework

Since there is no commonly accepted definition of beliefs we assume that beliefs are attitudes constituting themselves through at least three components: an affective, a behavioural and a cognitive component. Each component is measured by different ways of behaviour. In other words, we are in favour of the "three-components-approach" according to Rosenberg & Hovland (1969) regarding attitudes as a system of cognition, affection, and behavior (or conation).

Methodology

The study is based on two series of videotaped interviews with seven mathematics teacher students. The first series of interviews took place at the end of October 1995. The second series was recorded at the end of April 1996. The students were chosen by random.

The creation of the questionnaires was based on the "three-components-approach". Special emphasis was put on cognition, affection and behaviour. Furthermore we tried to collect some information on attitudes to mathematics as science and to mathematics as a subject taught at school. We were also interested in attitudes towards teacher education in mathematics at university as well as possible ways of improving this education, and in attitudes towards being a future teacher of mathematics at school.

When working out the interviews we were lead to a great extent by works of Sander (1995), Reichel (1991, 1992) and Ernest (1989).

Collection and Analysis of Data

During both interviews the students were asked to rank fifteen items which are associated with the nature of mathematics, its teaching and learning. The highest rank was assigned to 15, the lowest rank to 1.

Each item was represented by a card; the students were asked to define some ranking of these cards by ordering them. The students' comments throughout this process were recorded.

We decided to link these items primarily with the cognitive (c), the affective (a) or the behavioural (conative) (b) dimension of the attitudes.

Furthermore, in the first series of interviews the following points were taken into account:

1. What are their subjective impressions concerning their own lessons in mathematics at school?
2. What do they consider to be a 'good' teacher of mathematics?
3. What were their attitudes towards mathematics taught at university before going to university?
4. What are their attitudes to mathematics in general?

In the second series of interviews in addition to ranking of the items the following point were considered:

1. How do they see their first months at university?
2. How do they consider their attitudes and changes in their attitudes after their first months at university?
3. How do they consider their attitudes and changes in their attitudes towards mathematics in general?

4. In how far do they see any possibilities of improving teacher education at university?

Some results about the ranking of the cards have been presented in the Proceedings of MAVI-3 (Törner & Kalesse, 1996). In the following other results of our investigation in respect to the students' attitudes and their attitude changes will be presented.

a. Subjective impressions concerning their own lessons in mathematics

It is to mention that the students' attitudes are very much depending on their own experiences at school. These experiences are sometimes significantly different. Generally, it can be noticed that - by having received efficient lessons in mathematics at school - all students have been motivated firstly to become teachers and secondly to study mathematics.

All probationers similarly commend the possibility of asking questions, the large number of explanations, and the friendly relationship to the teachers.

b. Idea of a 'good' teacher

All of the students expect a 'good' teacher of mathematics

- to be competent in all matters of mathematics as a basis of their teaching,
- to strictly avoid frightening the pupils,
- to inspire independent working and self-responsibility in the pupils,
- to show much patience to the pupils' problems in understanding mathematics,
- to use clear examples, practical exercises, vivid models and so on.

Some students prefer teachers involving games and jokes in their lessons, others want them to include more creativity and fantasy.

c. The first six months at university

The first six months of the probationers at university can be described as follows:

In spite of early warnings the students' expectations have not been fulfilled; they have even been passed in a negative way. Those students, who did not pass their first exams at university, have more fear than before. This is due to the fact that the subject matters of the lecture were too concentrated, it was not possible to ask questions like in school, varied explanations were missing.

The students expected the contents taught at school to be consolidated and further expected to gain first insight into some teaching practice.

d. Attitudes and change in attitudes towards mathematics taught at university

There have been no fundamental changes in the attitudes of the students towards mathematics taught at university during the first six months at university. When doing the second series of interviews the students declared that their negative expectations have been surpassed by far.

During the *first interview* the probationers suspected being forced to learn subject matters, which are not essential for 'good' teaching. The students already believed that their studies at university and their future profession did not have very much in common. They mainly criticised the rather abstract lessons, which are the same for future teachers as well as for future mathematicians working on their diploma, and the lecturers' terrible teaching methods.

During the *second interview* the students remarked that during the lessons there has not been enough time for taking notes or even for understanding the subject matters. This is not only criticised by the 'losers' but also the 'winners' among the students. Even rather successful students considered the lessons to be boring, stereotyped, and following the motto „do or die". They also regard their time at university as an intermediate stage, which is a necessary evil on their way of becoming teachers of mathematics.

e. Attitudes and change in attitudes towards mathematics in general

During the *first series of interviews* the probationers judged mathematics in general positively. This tendency was not as strong in the *second series of interviews*.

Some students emphasised the practical application of mathematics, others enclosed theoretical aspects, proofs, and definitions being typical of mathematics. Creativity and fantasy were only connected with mathematical riddles.

In the *second series of interviews* emphasis was put on abstract subject matters, which are not taught at school. The more successful students became more interested in creative and uncommon ways of solving mathematical problems.

f. students' suggestions of improving the situation

1. All students demand a pedagogical organisation of the studies at university:

- Practical exercises at school should be introduced and they should be accompanied by university lecturers and school teachers.
- Teaching practice at school should be an integral part of university lectures from the very beginning.
- School teacher should be invited regularly to give an insight into the students' future profession.

2. The methods of teaching mathematics at university have to be improved essentially:

- The students should be offered more possibilities of asking questions during the lectures.
- There should be vivid lectures in a relaxed atmosphere.
- Illustrative examples and comprehensible explanations should be integrated into the lectures.
- There should be special support for future teachers at university.

3. The education of future teachers should be separated from the training of future mathematicians working on their diploma.

Conclusions

Our conclusions mainly concentrate on two points:

The first point deals with the universities internal possibilities of improving the shortcomings mentioned above. The second point refers to external factors determined by political trends and administrative regulations.

1. internal possibilities:

To establish an image of mathematics as an art and as a creative science it is important to start teaching mathematics as being creative and inventive as well as getting deeper insight into fundamental interrelations.

This cannot be achieved by using rather dated methods of teaching (for example 'chalk and talk'). But not only practical exercises at school and modern teaching methods are required but also that the lecturers at university themselves become more creative and innovative in their way of teaching.

2. external factors:

The external factors are determined by three competing necessities, namely:

Professional competence on a level as high as possible.

The mastering of modern teaching methods not only concerning educational theory in general but also subject oriented knowledge allowing the future teacher to disclose unknown mathematical interrelations.

For most universities it is compulsory to save money.

We basically agree with Weitz's (1996) suggestion (referring to Huber) presented in an article in the Süddeutsche Zeitung on the 8th and 9th of June 1996. Weitz is in favour of 'one subject teachers'. This would be a possible way of reducing the costs without neglecting theoretical education. Furthermore a certain scope for innovative teaching (both for university lecturers and future teachers) can be established.

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Metaphor and Teaching

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Abstract

The contemporary views about the role and importance of metaphor have a long history that goes back to the times of Aristotle. For centuries metaphors were especially associated with the field of poetics, literary ornaments and linguistic refinements. Today there are different issues in the voluminous literature on metaphor which are of interest for my purposes. For the discussion of metaphor in (mathematics) learning and teaching some valuable trails have already been open by the works of others like Petric (1979), Munby (1986), Lopez-Real (1989, 1990), Thomson (1992). Any attempt to escape from metaphors in education may be condemned to failure: metaphors would seem to have an important place in the provision of explanation. This paper addresses the use of metaphor describing teaching and teachers' beliefs.

Introduction

Metaphors would seem to have an important place in the provision of explanation using nature of metaphor as a speech act and a linguistic tool for overcoming certain cognitive limitations. As a tool for efficient communication metaphor can transfer large chunks of information: A way of understanding or describing phenomena in familiar terms so that the understanding may be deepened and gave us hints about the speakers' beliefs.

There are different perspectives on metaphors in educational research:

- Issues of generality and how categories embrace specific instances are themes of Lakoff and Johnson (1980 and later). They have contributed to the understanding of teaching and learning the imaginative aspects of reason, among which they list „metaphor, metonomy and mental imagery "are central to reason.
- The role of metaphor in mathematics learning (and metaphorical thinking that takes place in classes)and their contributions on the pedagogical status of metaphor were given by works of others like Pimm (1987), Sierpinska (1994), Carreira (1997): Connecting mathematics with real phenomena will involve the creation of nets of metaphors; mathematical models were seen as the formal surface of metaphorical matrixes.
- Ernest (1988) distinguished three conceptions of mathematics using metaphorical descriptions (instrumentalist, platonic, problem-solving view).

- Tobin and LaMaster (1992) gave an interpretation of teaching based on metaphors, which were used to conceptualise teaching roles.

To be aware of the intellectual heritage should then places us on a position to capture different insights of the use of metaphors.

Metaphors in history and literature

Types of metaphor which are relevant from the perspective of education are especially („root") metaphors, which are deeply embedded in our culture. They influence the way we act, think and speak. There are a few metaphors in history which determined and indicates different conceptions of teaching and mathematics. Some examples are given in table 1 (quotations found in Ballauf (1969)).

Ongoing research (Tobin (1990, 1992)) suggests that metaphors used to capitalise particular teaching roles guide many of the practices adopted by teachers; Tobin gave following examples of science teaching roles: INTIMATOR; PREACHER; POLICEMAN MOTHER HEN; ENTERTAINER; CAPTAIN OF THE SHIP; FACILATOR; COMEDIAN; SOCIAL DIRECTOR; RESEARCHER; MENTOR; MANAGER; ASSESSOR; REWARDER; PUNISHER.

Teacher as AMPLIFIER and EDITOR : metaphor based on some dynamics in communication used by Hewitt (1997).

The german poet Heinrich Heine (1797-1856) used the metaphor „Pauker" (DRUMMER).

Metaphorical description of mathematics can be also found by Freudenthal(1973): Geometry between the Devil and the Deep Sea.

Nature of metaphors

The word „metaphor" is derived from Greek ´metaphora´ (= transfer, carry over).

Since man began to philosophise the special place of metaphor in language and cognition have been recognised: Aristoteles' „Rhetoric" is devoted to discussion of metaphor: a substitution theory whereby any metaphor can be replaced by a series of literal statements.

In such „proportional" metaphor two seemingly disparate domains are shown to be related to one another. The similarities may be found in structure, function ore some derived relations. (see Leino, Drakenberg (1993))

METAPHORS

GARDENER

Children are the promising plants of a public welfare.
(Gaius Plinius, 23-79)

At least as a child man(kind) should be educated like different plants in a garden: under the prudent cultivation of a gardener they develop to their perfection and characteristics.
(Friedrich Fröbel, 1782-1852, established the first „Kindergarten")

MIDWIFE

In this I feel like a midwife: God forces me to attend at a birth.
(Socrates, 470 -399 a. Chr.)

SHEPHERD

I am the "Good Shepherd"- this describes in form and assignment the educator, who guides children like a shepherd leads his flock.
(Clemens of Alexandria, ca 215)

GENERAL, STEERSMAN

Like a general straightens up the line of battle in order to keep his soldiers save and sound, and like a steersman navigates his ship with the attention to keep his passengers alive, thus an educator leads children in loving care to a healthy way of life.
(Clemens of Alexandria)

FATHER

Above all things the teacher assumes the meaning of a father to his pupils.
(Marcus Fabius Quintilianus, 35-96 n. Chr.)

DOCTOR/ PHYSICIAN

Similar to physician, who establishes health in his patient through the impact of nature, one can say that one human being evokes knowledge in another human being through the impact of natural reason.
(Thomas of Aquino, 1225-1274)

(MAGICIAN)

The Good Christ has watch out for mathematicians and for those, who are in the habit of predicting science especially when these predictions come true. Because there is a danger that mathematicians together with the devil might becloud our mind and entangle man with the bonds of hell.
(Aurelius Augustinus, 354-430.)

Table 1.

Different aspects of metaphors were pointed out by Richards. (1936). His first theme is based on the transferring nature of language to reality; it claims that all language is fundamentally metaphorical. The second theme is concerned with the way metaphor works: „when we use a metaphor we have two thoughts of different things active together and support by a single word of phrase whose meaning is a resultant of their interaction."

This interactionist position of Richard has been developed by Black (1979, 1988). He sees an active metaphor as having two distinct subjects, primary and secondary or vehicle and topic, which interact through a system of relationships. This description as an active behaviour describes not only the part of the authoror producer of the metaphor but also the interpretative act on the part of the listener.

Petrie (1979) sees the function of metaphor as being to „provide a rational bridge from the known to the radically unknown" and metaphor being „epistemologically necessary" for „learning something that is radically new"

In this example (teacher as gardener) established an analogical, proportional relationship that the teacher is to children's mind what a gardener is to plants; that is, a stimulant to growth.

In Richards (1936) view the „teacher" would be the tenor, „gardener" the vehicle, what those two share in their work would be called ground and the dissimilarities the tension of the metaphor.

On the other hand it pointed out, that metaphor is a process by which we view the world and the heart of how we think and learn. We can also consider metaphor to go beyond the level of words to a shared body of knowledge and assumptions that are associated with the words. Metaphor is often used as a powerful tool to describe phenomena or domains that are vague and difficult to define.

Metaphors and interpersonal diagnosis

Reflective thinking may enable teachers to consider specific beliefs in relation to the manner in which the curriculum is implemented (Tobin, LaMaster 1992). What a teacher believes about knowledge and learning can have no direct effect on how students actually learn, but it is important to know teachers' belief. Beliefs about teaching and learning are associated with teaching roles and metaphors are used to conceptualise teaching roles. Metaphors can probably be at the basis of most concepts, so that the metaphors used to make sense of the main teaching roles (attitudes, beliefs, behavioural intention and behaviours) can be the focus for reflection and change.

Different aspects of teachers' beliefs were pointed out by Pope and Keen (1981): In a view of „cultural transmission" the teachers' view is mainly a TRANSMITTER of information. The learner can acquire absolute truth by a process of iterative accumulation. The view of the learner is that of a machine, with the teacher being the ENGINEER (see: „helmsman"). A romantic view pointed out a discovery of the natural and innerself. A metaphor would be that of „organic growth", the learner being the plant and the teacher the HORTICULTURALIST. (see: „gardener", „midwife", „doctor"). The deschooling view according to which knowledge should not to be seen as a purely intellectual concern and thus the emotional as well as the intellectual life is seen. (see: „father", „shepherd").

Different models of mathematics teaching was given by Kuhs and Ball (1986): 1. learner-focused view, 2. content-focused view with an emphasis on conceptual understanding, 3. content-focused view with an emphasis on performance and 4. classroom-focused view.

Ernest (1988) distinguished three conceptions of mathematics teaching: „INSTRUMENTALIST view" (Mathematics is a set of unrelated but utilitarian rules and facts), „PLATONIC view" (Mathematics is discovered not created) and „problem-solving view" (Mathematics is not a finished product, for its results remain open to revision).

Brown (1978) identified different forms of explanation used by teachers: the „interpretative type" uses questions of the form 'What is...?' to lead on discussion about a posed problem. The „descriptive type" uses especially questions 'How is...?' to lead to discussion of how a solution to problems can be found. The third type called „reasoning- giving type" leads to causes and reasons with questions 'Why is...?'

Leary (1957) gave a model to map interpersonal teacher behaviour using contrary dimensions: The „proximity-dimension" (co-operation/opposition) and the „influence-dimension" (dominance/submission).These dimensions can already extended to more different behaviour aspects (for example: friendly behaviour, way of guidance).

Thomson (1992) distinguishes three conceptions of mathematical teaching: At „Level 0" the instructional focuses on facts, rules and procedures. The role of the teacher is perceived as a DEMONSTRATOR. At „Level 1" the conception of mathematics is broadened to include an understanding of principles behind the rules. At „Level 2" the view of teaching for understanding grows out of engagement in the very processes of doing mathematics.

I think a realistic view about teaching and teachers' belief lie somewhere in the middle using several or all of these types or adaptations of these types.

Aspects of research

Prototypes of teaching and classroom behaviour lie at the centre in the radial structure of categories. Metaphors of teaching may help to find out specific categories, which are useful to describe teaching of individual persons.

My first aim is to find few categories to give an accurate picture of different teaching roles.

The given examples of metaphors pointed out, that not only cognitive dimensions of teaching are relevant; thinking, feeling, doing and emotional, social, practical behaviour must be considered in the same way.

Such descriptions of teaching roles can probably help to examine teachers' beliefs and practices and consider whether or not alternatives would lead to improvement in the classroom. (Tobin 1990).

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The Development of Prospective Teachers' Math View

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My interest has been in the structure of beliefs and conceptions of teaching mathematics and in the possibilities of changing student teachers' math views and instructional practices. I define an individual's mathematical *beliefs* (*math view*) as his or her subjective implicit knowledge about mathematics and its teaching and learning. *Conceptions* are conscious beliefs. The conscious and unconscious beliefs form a *belief system.* (Törner & Pehkonen 1996.) *Instructional practices* I define operationally as the scores the teacher attains on certain evaluation scales. Such scales may be based on Thompson's levels and classification on relevant questions concerning the teaching of mathematics (Thompson 1991).

Background

I began my study of beliefs of prospective primary school teachers in the fall 1993. I have been gathering quantitative and qualitative data from the students I am teaching at the Department of Teacher Education , the University of Tampere. The quantitative data was collected with two Likert type questionnaires from students in the years 1993 and 1994. Factor analyses were conducted in order to analyse the structure of beliefs about teaching mathematics. On the basis of the correlation matrix items with weak correlations with the other items of the data were dropped. The remaining 25 items were then analysed with the Principal Component Method. In the factor analyses the following factors came up. (For a more detailed description see Lindgren 1995.)

Open - Approach	Discussions & Games	Rules & Routines	Content - Oriented - Approach	Student - Oriented - Approach

Figure 1. The five factors extracted from quartimax-rotated data from 105 Finnish prospective primary school teachers.

The three first of these extracted factors seem to refer to Alba Thompson's levels 0, 1 and 2. I have chosen these as new variables and I call these: level OA (Open Approach), level DG (Discussions and Games), and level RR (Rules and Routines). The values for the levels OA, DG, and RR were obtained as means from items with high loadings on the corresponding factors. This factor analysis and the distribution of these three new variables gave me a hypothetical model of three partly overlapping levels. (See Lindgren 1996a.)

I used the whole target group of 163 students for analysing the distribution of the standardised levels over the last school grade in mathematics. I also analysed in greater detail those prospective teachers who had a high value and those with a low value on the level OA. Diagrams of these matters and some interview statements of these students can be found in *Zentralblatt für Didaktik der Mathematik* (Lindgren 1996a).

It is of great importance that teachers in pre- and in-service training could be aware of their own beliefs about learning and teaching mathematics. It seems evident that the spectrum of the variety of math teachers in the prospective teachers' educational history plays major role in the inception, development, and manifestation of the student's math view. (Kupari 1996.) The student teachers' own experiences as learners are often re-enacted in their own teaching practices in class. The educational memories of the 12 prospective teachers interviewed are discussed in my report for MAVI-3 (Lindgren 1996b).

Method

I was interested not only in the structure of beliefs but also in how the beliefs and conceptions about mathematics teaching develop during the four years in teacher education. In order to answer this question, 12 students were selected from my first target group of 72 students for closer follow-up. They had filled in the Likert type questionnaire twice: at the beginning of September 1993 and at the end of April, 1994. The 12 were selected on the basis of their answers to the questionnaire and a math exam during their first academic year. During their second year teaching practicum they were interviewed and one lesson from each was videotaped. In the fourth year final practicum six of these 12 were teaching mathematics. Their lessons were observed and one was videotaped for each student. Their lesson plans and portfolios were analysed. Some of their reflections are given at the end of this paper. At the end of the spring semester 1997 all 12 filled in the same Likert-scale questionnaire for the third time, now complemented with some open-ended questions.

In the spring of 1997 the supervising class teachers of the six practising student teachers were asked to fill in a questionnaire. It included five questions by Thompson and the sixth: what is problem solving? (See Pehkonen 1994, 64.) Each of the six units obtained included six statements referring to Thompson's levels 0, 1, and 2. The teacher was asked to choose from these statements two which best fitted the practising teacher's view of mathematics and teaching mathematics. The best choice was to be marked with the number 1 and the

1. MATH AS A SUBJECT	Rules and routines	______	Understanding the concepts
	A useful tool in everyday life	______	Interesting, beautiful, and challenging
	Finding the right answer	______	Processes and generalizations
2. TEACHING MATH	Go through the problems in the book	______	Uses many games
	Memorize the rules	______	Asks for ideas and proposals
	Teacher illustrates everything him/herself	______	Allows pupils to search and to use manipulatives
3. LEARNING MATH	Finishing the problems correctly and neatly	______	Do you understand?
	Memorizing the rules	______	What do you think about this?
	Did you get the answer?	______	What can you conclude from this?
4. TEACHER'S ROLE	Keeps order	______	Encourages thinking and drawing conclusions
	Manages pupils' work	______	Takes into account pupils' proposals
	Demonstrates rules and routines	______	By explaining tries to get the pupils to understand
5. EVALUATION	Tells/shows the right answer	______	Pupils are responsible for the correctness of the solutions.
	Checks the problems	______	Common assessment
	Teaching is successful when the pupils understand the matters in hand.	______	Teaching is successful when the process is a good one
6. PROBLEM SOLVING	Going through story problems	______	Working with concrete problems
	Going through problem cards or duplicated copies	______	Almost all work in the math class
	Finding of right strategies or algorithms	______	A method to learn something new in math

In addition to the videotape the reflections by the practicing teacher were used for items 1, 5 and 6.

Table 1. A evaluating scale for a videotaped math lesson.

second best with the number 2. At the end there was an open-ended question regarding the supervisor's general view of the student. From the sum of the attained points I got a value (**IP**) for assessing each student's instructional practices. This questionnaire can be found in the report for MAVI-4 (Lindgren 1997, 81).

Before studying the supervising teachers' evaluations I carefully studied the videofilm of each student. To get a more precise view of the student teachers' behaviour and professed beliefs I had prepared an assessment scale for evaluation of a math lesson (Table 1). This had the same units as the questionnaire for the supervising teachers. For each unit the six items formed three bipolar axes. This scale is presented below. Studying the videofilm, I drew lines on the axes referring to what happened, how intensively, and how often. The line was scaled from 1 to 5. By counting the means for each unit and the whole scale I got a value referring to my opinion of the student's instructional practice.

Results

It was quite astonishing how close the values obtained from this scale came to the IP values obtained through the supervising teachers' evaluations. These values were also in harmony with the overall evaluation the supervising teachers gave in their questionnaires. The following tables show both values of IP, the one assessed from the videofilm and the one which is counted from the teachers' questionnaire.

The tables also summarise data obtained from the students beliefs as exposed in the questionnaires in September 1993, April 1992 and April or May 1997. The data is given in two tables indicating to whether the student had mathematics in the final practicum or not.

Table 2 shows that during the first year the values of OA for all students rose or (for Beth) were unchanged. But after that year only the values of Beth and David have risen. For the other four student teachers the OA values decreased. During this period the students did not have any math education or didactical guidance for teaching mathematics. The changes in the RR values are not so congruous but mostly there is a decrease during the period of four years. The conclusion can be drawn that the positive changes during the first academic year have been partly temporary.

	OA	DG	RR
Anna			
1.	3.25	2.75	4.00
2.	3.88	3.50	2.83
3.	3.63	3.00	2.83
Beth			
1.	3.50	4.25	2.50
2.	3.50	3.50	2.60
3.	4.25	3.25	2.13
David			
1.	3.88	.00	3.33
2.	4.13	2.00	3.33
3.	4.25	3.25	2.67
Fanny			
1.	4.00	3.25	2.50
2.	4.13	3.50	2.83
3.	3.88	3.50	2.67
Hanna			
1.	3.75	3.75	2.67
2.	4.63	3.25	2.83
3.	4.38	2.75	2.50
Jan			
1.	4.75	4.00	2.33
2.	5.00	3.75	2.83
3.	4.63	3.75	4.83

Table 2. Likert scale (5 full agreement) belief inventories 1. 9/93, 2. 4/94, 3. 5/97 for those students who did not teach mathematics in their final practicum.

P Instructional Practices: * My evaluation and comments, ** Supervising teacher's evaluation scale form, **OA** Open Approach, **DG** Discussions and Games, **RR** Rules and Routines.

For those students who were teaching mathematics in their final practicum (Table 3) the changes on levels OA, DG and RR are more coherent. For the RR value there is an increase only for Laura, who was teaching a difficult multigrade class. For the OA values there was decrease only for Ivan and Laura. In all cases the OA value was at least 4.13.

	OA	DG	RR	IP	COMMENTS
Carl					A time gap of half a year between IP assessments. A very positive attitude, but unsure in teaching.
1.	3.75	3.50	3.33	* 2.6	
2.					
3.	4.13	3.00	3.33	** 1.3	
Eva					
1.	4.00	4.24	3.17	* 3.3	A very dedicated teacher
2.	4.13	4.00	3.00		
3.	4.50	3.50	2.83	** 3.2	
Greetta					
1.	3.88	3.75	3.50	* 4.0	Very good teaching.
2.	4.25	4.00	3.17		
3.	4.50	3.50	2.83	**3.8	
Ivan					
1.	4.63	3.75	4.00	* 2.8	A very positive attitude, but unsure in teaching.
2.	4.88	3.75	4.00		
3.	4.13	3.00	3.17	** 2.7	
Karin					A time gap of half a year between IP assessments. A very good teacher.
1.	4.63	3.00	3.17	* 4.4	
2.	4.88	3.50	2.50		
3.	4.63	3.75	2.33	** 3.9	
Laura					
1.	4.88	3.25	2.83	* 3.5	A difficult multigrade class (3.&4.).
2.	4.63	2.75	2.83		
3.	4.50	4.25	3.17	** 4.4	

Table 3. Likert scale (5 full agreement) belief inventories 1. 9/93, 2. 4/94, 3. 4/97 for those students who taught mathematics in their final practicum.

I counted that for a mathematics lesson this kind of a evaluating scale with six units built up on the groupings of Thompson and Pehkonen (Thompson 1991, Pehkonen 1994) is useful. At this moment my collection and ordering of items which form the dimensions of each unit needs specification and refinement. I assess the work done so far in evaluating the final practicum lessons as a pilot study before using the scale for evaluating the lessons of the in-service phase. At that stage it would be useful to have two observers for independent evaluations of the same lesson.

Conclusions

According to Ernest three factors determine the autonomy of the mathematics teacher and the outcome of his or her teaching: 1) the teachers mental contents or schemes, particularly the system of beliefs, 2) the social context of the teaching situation and 3) the teachers level of thought processes and reflections (Ernest 1989.) While following the instructional practices of the prospective teachers mentioned above I could mirror the relevance of these Ernest's factors.

As I was reading the teacher students' portfolios after the final practicum I found a great variety in the profundity of the reflections of the math lessons. For an example in her reflections Laura wrote: "The teaching should start from the pupils' earlier experiences. I hope to find out the relationship between, on the one hand, practising with the help of games and playing, and, on the other hand, mechanical practising, in cases where learning is most effective and motivated." Greetta gives special attention to problem solving, and she seems to identify important elements and features in teaching mathematics. Greetta's teacher evaluates her math period: "It was one of the best math periods I ever saw."

A good teacher can never be like a machine. Choice is a basic human prerogative. Making choices is an ever-present reality in teaching. The good teacher not only chooses between various alternatives but also reflects on the meaningfulness, integrity and the validity of each decision made.

To be is to choose

to accept

or reject

the possibilities of life.

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Can Gender, Language and Regionality Affect Choices in Upper Secondary School Mathematics?

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Philosophical reasons for this inquiry

Heidegger (1988, 4-7) represents an idea of worlds we live in that shape us. According to him we are part of the world we live in and all of our behaviours are learnt from the culture of that world. The social practices of a culture make or form the world of that culture. Local traditions and subcultures are worlds in Heidegger's sense of meaning just as well as boys' and girls' cultures can be seen as worlds. In Finland we have our Swedish speaking culture or a world just as we have our Finnish speaking culture or a world.

According to constructivists (Sintonen 1996, 128, 130) an individual constructs his or her own reality in the context and situation he or she lives in. To me the constructivistic situation and context and the Heideggerian world describe the same thing, they describe how an individual is a product of the tradition and customs of the region he or she was brought up in .

This phenomenological - constructivistic frame of thought makes it intriguing to unveil what possible worlds could be found among Finland's Finnish and Swedish speaking population in the provinces of Uusimaa and Vaasa, worlds that reveal themselves even in upper secondary school mathematics choices.

The language situation in chosen areas

There is a culturally vital and active Swedish speaking minority (5,8% of population) in Finland. Most of Finland's Swedish speaking population reside in the province of Uusimaa and the province of Vaasa. In Uusimaa about 10,5% of population speak Swedish as their native language and in the province of Vaasa the corresponding figure is about 22,3%. The chosen areas have a prominent local tradition and culture and they differ clearly from each other. (Statistical Yearbook of Finland 1995)

Realisation of the research

In this paper we study how female and male upper secondary school pupils from two different language groups from two separate parts of Finland chose advanced or general mathematics for their first mathematics course during fall 1993 and whether they passed their final exams in advanced or general mathematics three years later in spring 1996. The statistics from the

Matriculation Examination Board (Ylioppilastutkintolautakunta) in Finland and the statistics from the Statistics Finland (Tilastokeskus) about upper secondary school mathematics choices have been used as the material for this research.

The figures from the Matriculation Examination Board consist of:

1. The number of pupils from every Finnish and Swedish upper secondary school in the provinces of Uusimaa and Vaasa who passed the finals in 1996.
2. The number of females who passed the advanced mathematics exam in 1996.
3. The number of females who passed the general mathematics exam in 1996.
4. The number of males who passed the advanced mathematics exam in 1996.
5. The number of males who passed the general mathematics exam in 1996.

The second material consists of the statistics from the Statistics Finland concerning the upper secondary school pupils' first year's first choice of either advanced or general mathematics in every Finnish and Swedish upper secondary school in Uusimaa and in the province of Vaasa in 1993.

These figures consist of:

1. The number of pupils in each upper secondary school in 1993.
2. The number of female pupils who chose advanced mathematics.
3. The number of female pupils who chose general mathematics.
4. The number of male pupils who chose advanced mathematics.
5. The number of male pupils who chose general mathematics.

In this study we are not concerned with received grades. We are only interested in the number of pupils who studied advanced or general mathematics in upper secondary schools and the number of pupils who passed the final exam in either advanced or general mathematics.

The upper secondary schools were divided according to the popularity of advanced mathematics in final exam in 1996 into five separate categories.

Category 1. Upper secondary schools, where less than 20% of pupils passed the final exam in advanced mathematics.

Category 2. Upper secondary schools, where 20-30% of pupils passed the final exam in advanced mathematics.

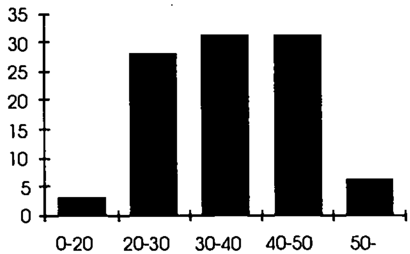
Category 3. Upper secondary schools, where 30-40% of pupils passed the final exam in advanced mathematics.

Category 4. Upper secondary schools, where 40-50% of pupils passed the final exam in advanced mathematics.

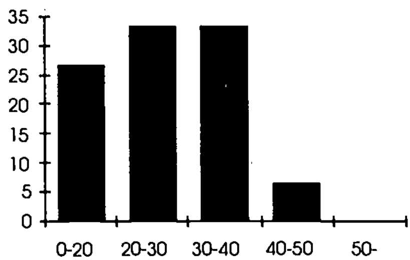
Category 5. Upper secondary schools, where more than 50% of pupils passed the final exam in advanced mathematics.

1. The province of Uusimaa: upper secondary schools according to the popularity of advanced mathematics in final exams.

Finnish upper secondary schools

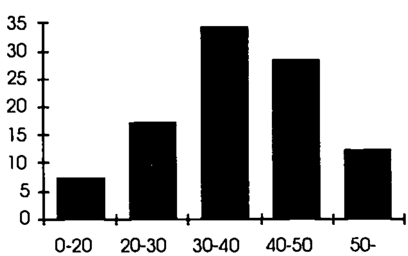


Swedish upper secondary schools

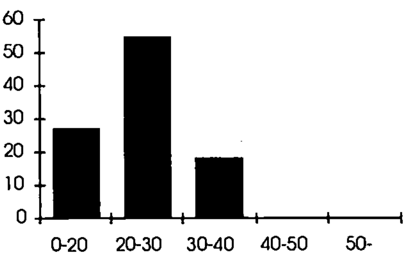


2. The province of Vaasa: upper secondary schools according to the popularity of advanced mathematics in final exams.

Finnish upper secondary schools



Swedish upper secondary schools



The categorising above is presented as columns. We can see that there is not one Swedish upper secondary school where more than 50% of pupils pass advanced mathematics finals. Instead, in about a quarter of Swedish upper secondary schools less than 20% of pupils passed the advanced mathematics finals. We can also see the prominent regional profiles.

The material yields us evidence that in Finland along with sex also the language is a discriminating factor, because in both areas the Swedish studied less mathematics.

2 A. Advanced mathematics final exam 1996: all graduates.

	FINNISH	SWEDISH
UUSIMAA	38,07%	28,86%
VAASAN LÄÄNI	35,07%	25,08%

2 B. Advanced mathematics choice 1993: all students.

	FINNISH	SWEDISH
UUSIMAA	53,00%	41,82%
VAASAN LÄÄNI	51,38%	35,72%

2 C. Advanced mathematics: the dropout percentage 100 * (1 - # of passed/ # of first choice)

	FINNISH	SWEDISH
UUSIMAA	37,46 %	36,73%
VAASAN LÄÄNI	38,91%	42,21%

3 A. Advanced mathematics final exam 1996: males.

	FINNISH	SWEDISH
UUSIMAA	64,82%	59,55%
VAASAN LÄÄNI	64,68%	64,07%

3 B. Advanced mathematics: the dropout percentage of males.

	FINNISH	SWEDISH
UUSIMAA	32,29%	36,90%
VAASAN LÄÄNI	33,08%	37,79%

4 A. Advanced mathematics final exam 1996: females.

	FINNISH	SWEDISH
UUSIMAA	35,18%	40,45%
VAASAN LÄÄNI	35,32%	35,93%

4 B. Advanced mathematics : the dropout percentage of females.

	FINNISH	SWEDISH
UUSIMAA	45,17%	36,47%
VAASAN LÄÄNI	47,32%	48,72%

There is to be seen considerable differences between schools. This, of course is a well known fact. One discrepancy was that in some schools the amount of pupils who chose advanced mathematics was much bigger than the amount of pupils who passed the advanced mathematics final exam. There seems to be two possible explanations. One explanation offered suggests that pupils often come from lower secondary schools with unrealistically high grades and they swap to easier mathematics after finding the advanced courses too difficult. There are also schools where pupils with good grades in advanced mathematics rather do the finals in easier general mathematics just to make sure they get the highest grade. These schools seem to be the exclusive ones with highest pressure on their pupils.

We don't have figures from single schools about their spring 1996 finals sex distribution. However, the nation-wide sex distribution of upper secondary school graduates in 1996 was 59% females and 41% males.

5 . The amounts of upper secondary schools and pupils who graduated

	FINNISH	SWEDISH
UUSIMAA	81/ 6654	15 / 925
VAASAN LÄÄNI	32 / 2301	11/ 666

6. Percentage of females of all pupils

	FINNISH	SWEDISH
UUSIMAA	55,5%	55,5%
VAASAN LÄÄNI	56,59%	62,19%

7. Amount of schools with a male majority

	FINNISH	SWEDISH
UUSIMAA	17 / 81	4 / 15
VAASAN LÄÄNI	6 / 32	0 / 11

Some of the male majority schools were specialised in science or sport but most of them were weak schools.

Matched samples

There are more Finnish than Swedish upper secondary schools in the chosen areas. About 11,1% of upper secondary school graduates in Uusimaa are Swedish speaking and the corresponding figure in Vaasan lääni is 22,1%. The size difference of the language groups could be thought to have a biased influence on results. I tried to prevent this by matching Finnish upper secondary schools with the Swedish upper secondary schools both in the province of Uusimaa and in the province of Vaasa. Schools included in samples are, when possible, from same or similar locations as the Swedish schools. When this was not possible, schools of same size were found.

8. The Uusimaa sample compared with Finnish Uusimaa and Swedish Uusimaa

	SAMPLE	FINNISH	SWEDISH
ADVANCED	41,65%	38,03%	28,86%
GENERAL	36,60%	34,25%	43,57%
NO MATH EXAM	21,75%	27,72%	27,57%
FEMALES OF ADV.	36,56%	35,15%	40,45%
MALES OF ADVANCED	63,44%	64,85%	59,55%
CHOICE OF ADVANCED	59,35%	53,00%	41,82%
CHOICE OF GENERAL	45,87%	47,00%	58,18%

9. The province of Vaasa sample compared with the Finnish speaking province of Vaasa and the Swedish speaking province of Vaasa.

	SAMPLE	FINNISH	SWEDISH
ADVANCED	35,25%	35,07%	25,08%
GENERAL	48,57%	47,54%	50,45%
NO MATH EXAM	16,18%	17,90%	24,47%
FEMALES OF ADV.	36,63%	35,32%	35,93%
MALES OF ADVANCED	63,37%	64,68%	64,07%
CHOICE OF ADVANCED	55,91%	51,38%	35,72%
CHOICE OF GENERAL	44,09%	48,62%	64,28%

The figures indicate that in the province of Uusimaa and in the province of Vaasa the Swedish speaking study less advanced mathematics than the Finnish speaking but instead in the province of Uusimaa they study more general mathematics. The differences between the Uusimaa sample and the Finnish speaking population indicate that in equally attractive neighbourhoods the mathematical activity of the Finnish speaking pupils is even bigger.

10. The amount of the students in the sample compared with the Swedish speaking

	FINNISH SAMPLE	SWEDISH
UUSIMAA	1366	904
VAASAN LÄÄNI	976	666

The figures indicate that Finnish schools are bigger, they have about 30% more pupils than the Swedish schools.

Afterwords

The results about the differences between the two language groups in Finland were different from what was expected. It is generally assumed that those upper secondary school pupils who study advanced mathematics often have an educated middle class background. The Swedish speaking population in Helsinki according to Allardt & Starck (1981, 241-242) is better educated and more affluent than the Finnish speaking population. So it was only natural to assume that especially the Helsinki Swedish upper secondary school pupils would be more into mathematics than they turned out to be.

When trying to explain this I even contacted professor Allardt, a sociologist , who has studied minorities. He suggested that like all minorities also the Swedish speaking in Finland might be more outward oriented and due to this they just may study more languages instead of mathematics.

Because of the minority quotas it is considerably easier for Swedish speaking upper secondary school graduates to be accepted in a university or a college than it is to Finnish speaking upper secondary school graduates in Finland. This might imply that the Swedish speaking pupils don't have to bother with troublesome advanced mathematics because they find their way into colleges anyway.

There does not seem to be any obvious explanation for the regional differences, especially concerning the Swedish speaking province of Vaasa. What seems obvious, is that within languages there lie traditions which affect population's behaviour, even at the level of choosing extensive or general mathematics courses in high school.

Finding the underlying differences of thought and world-view is my next project. The above is a sort of map of the worlds that are waiting for exploring.

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Teachers' Conceptions on Mathematics Teaching

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Abstract

In the spring 1995, the National Board of Education ran a national grade 9 mathematics test in Finland. In connection to the test, a sample of 50 classes were selected all around Finland, in order to investigate pupils' mathematical knowledge and teachers' instructional organisation. Among other things, the aim of the research based on the test results was to find out "What are teachers' conceptions on mathematics teaching and learning?" And this is the topic of this paper.

Summarising the results, the main findings are, as follows: A teacher's mathematics view contains 20–40 % calculation aspect, 10–30 % system aspect, 10–30 % process aspect, and 30–60 % application aspect in average. Teachers seem to have very different views on optimal learning. Some teachers discussed on learning situations very actively from the view point of pupils, whereas some teachers seemed to stick strongly to such descriptions which are compatible with behaviouristic view of learning.

1. On the realisation of research

The National Board of Education in Finland decided in the beginning of the 1990s that the development of the teaching reform realised in schools will be followed with the aid of an unofficial and voluntary examination (test). The aim of the research in the spring 1995 was to find out how successful mathematics teaching has been in the area of "everyday mathematics". As an additional task, the Board was interested in the topic of teachers' organisational management of mathematics teaching. The entire evaluation is published in Finnish (Pehkonen 1997a). This paper will focus on the question of teachers' conceptions on mathematics teaching in the light of the results of the mathematics evaluation study. For the terminology used here – conception, view, belief, etc. – see e.g. the earlier MAVI papers (or Pehkonen & Törner 1996).

Indicators

The national grade 9 examination (test) was arranged on the 25th of April 1995. The point of emphasis in the test was: mathematics in everyday life. Beside pupils' mathematics test, there was a questionnaire for teachers.

The teacher questionnaire was a compound of ordinary background questions as well as a teacher's evaluation on his own teaching, his conception on mathematics and assessment. Most of the items expected open answers, but a part of the statements was readily structured.

Furthermore for checking the reliability of results, interviews were made in five schools from the Helsinki region in the beginning of May. As a rule the teacher and a group of four pupils from each of the five school were interviewed separately. In connection to the interview, one mathematics lesson in that class was usually observed.

Practical realisation

The test subjects are from a sample of 50 schools taken at random by the National Board of Education in January 1995 to cover the whole Finland. Since in many schools, there are several parallel grade 9 groups, the schools were asked to send from each school only the group 9A material.

In March 1995, the sample schools were sent a letter from the National Board of Education explaining the forthcoming research, and the tests were mailed in the beginning of April.. Full material (pupils' test papers, questionnaires) was received only from 44 sample schools; therefore, the percentage of responding was 88 %. Thus, the group of the test persons was a compound of 739 pupils, and their 44 teachers.

Methodology used

The statistics used are mainly percentage tables, as well as means and standard deviations of items. The chi square test is used to check a possible statistical significance of differences between the distributions. The StatView-program on the MacIntosh computer was used for the data analysis.

In the text, usual abbreviations and language for the significance levels are used: Three stars (***, very significant) means that the error percentage p is smaller than 0.1 %, two stars (**, significant) that $0.1\ \% \leq p < 1\ \%$, and one star (*, almost significant) that $1\ \% \leq p < 5\ \%$.

2. Teachers' conception on mathematics teaching and learning

In the research of last twenty years, it has become clear that there is no unique conception on mathematics, but that also mathematics teachers have several different views (or subjective philosophies) on mathematics. According to research results (cf. Grigutsch & al. 1995), there might be possibly four different dimensions which can be differentiated: (A) *the calculation aspect* (≈

the meaning of mathematics lies in calculations, i.e. the "tool box" aspect), (B) *the system aspect* (≈ the meaning of mathematics lies in rigid proofs), (C) *the process aspect* (≈ the meaning of mathematics lies in its continuous development), (D) *the application aspect* (≈ the meaning of mathematics lies in its practicability). It is important to notice that every teacher has all these aspects with some weight in his mathematics view.

According to the results of Grigutsch & al. (1995) study, some statistically significant correlations between the four dimensions A–D (Figure 1) were found with the help of factor analysis. The correlation between the toolbox and system aspects as well as between the process and application aspects are positive. Whereas the correlations between the process and toolbox / system aspects are negative. Thus instead of four dimensions, there seems to be only two main factors: toolbox/system and process/application.

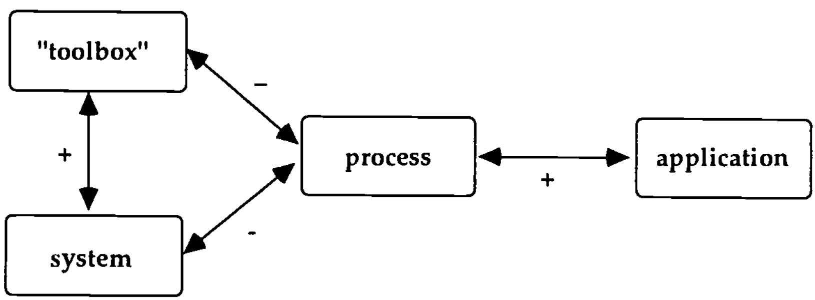

Figure 1. Correlations between the different aspects of teachers' mathematics view revealed with a questionnaire and factor analysis (Grigutsch & al. 1995, 43).

Teachers' mathematics view

The four dimensions of the Grigutsch & al. (1995) study was used as a basis for one question in the teacher questionnaire (question 20). Here, the teachers were asked to describe with percentages the amount of each aspect in their own mathematics view with the sum of 100 %. The results gained are rounded to the nearest ten percentage, and arranged into Table 2.

Table 2. The teachers' own estimations in percentages on their mathematics views (in each case the lower limit is contained into the interval, but in the last column also the upper limit).

	percentage limits	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
A	calculation aspect	-	9	13	15	4	1	-	-	-	-
B	system aspect	6	16	17	3	-	-	-	-	-	-
C	process aspect	1	20	16	5	-	-	-	-	-	-
D	application aspect	-	-	3	12	12	11	2	2	-	-

According to Table 2, a teacher's mathematics view contains 20–40 % calculation aspect, 10–30 % system aspect, 10–30 % process aspect, and 30–60 % application aspect in average.

In addition, the teachers were asked here to describe with own words their view of mathematics. The following beginning of a sentence was used as a starting point: "According to my opinion, mathematics primarily is ...", and after that there were some empty lines. Most of the teachers' responses could be classified into the four categories given above, but additionally there was one more category – the aesthetic aspect: (A) learning of basic calculations (or a tool, a tool subject), (B) a waterproof logical system (or logical thinking and its development), (C) problem solving, (D) an application aspect (or development of the skills needed in everyday life), (E) a nice hobby.

Teachers' view of learning

Nowadays we are considering different views when discussing on learning. The terms active and passive learning are used as well as static and dynamic view of knowledge. In the next question of the teacher questionnaire (question 21), their conceptions on learning were asked using some typical teaching situation in the lower secondary school mathematics. The situations in question were, as follows: computing with negative numbers, concepts connected with solids, and calculations with polynomials.

In each part, a sentence beginning was given, and there was some empty lines for the answer. In the following, the beginning of a sentence is given and the teachers' responses in a densed form.

<u>Negative numbers</u>. The beginning of a sentence was, as follows: "According to my opinion, one learns best computing with negative numbers, when ...". Here, the following responses were given: comparing with a thermometer, concretising the situation with a number line, comparing with the situation "money in the pocket / debt", bank account "withdrawals and inputs", with plenty of exercises, learning the sign rules properly, thinking the calculations with point models.

Solids. Here, the beginning of a sentence "According to my opinion, one learns best the concepts connected with solids, when ..." resulted the following answers: dealing with concrete solids, cutting and clueing, constructing and opening of solids, drawing and naming of solids, giving models to pupils to hand and consider, reveal solids in the environment.

Polynomials. Now the beginning of a sentence was "According to my opinion, one learns best calculations with polynomials, when ...". It gave the following responses: practising much, giving a meaning for terms, learning the calculation rules properly, taking examples from common units (addition and subtraction), after learning basic topics via playing different polynomial games.

Summarising one may state that the teachers seem to have very different views on optimal learning. Some teachers discussed on learning situations very actively from the view point of pupils, e.g. *"a pupil correlates negative numbers with frosty grades of a thermometer and uses the thermometer scale instead of a number line"* (teacher nr. 34) and *"pupils will construct solids, measure with water and cut out nets"* (teacher nr. 20). Whereas some teachers seemed to stick strongly to such descriptions which are compatible with behaviouristic view of learning, e.g. *"a pupil exercises much"* (teacher nr. 22).

Comparison with the Helsinki results

During the spring 1995, a similar study was realise with a sample of 20 schools in the City Helsinki. The same research design was used, in order to be able to compare the results from Helsinki with those from the whole country. The whole study is described in a published report which is written in Finnish (Pehkonen 1997b). In Table 3, there is the comparison of the teachers' own estimations on their mathematics views in the whole county (N = 44) and in the City Helsinki (N = 20).

When the Helsinki distribution was compared with a khi square test to the results of the whole country, one noticed that in all four aspects differences were statistically significant: the calculation aspect A (**, χ^2 = 15,8); the system aspect B (***,χ^2 = 19,0); the process aspect C (***,χ^2 = 23,4); the application aspect D (*,χ^2 = 14,3). The differences were largest in the system aspect which the teachers from Helsinki supported less than the sample of teachers representing the whole country, and in the process aspect where the views of the Helsinki teachers were more scattered than those in the whole country. Some teacher from Helsinki seem to be ready to emphasise the process in mathematics which reflects the modern view of learning.

Table 3. The teachers' own estimations in percentages on their mathematics views (in each case the lower limit is contained into the interval, but in the last column also the upper limit).

	percentage limits	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
A	calculation aspect										
	WHOLE FINLAND	-	9	13	15	4	1	-	-	-	-
	HELSINKI	-	5	5	4	6	1	-	-	-	-
B	system aspect										
	WHOLE FINLAND	6	16	17	3	-	-	-	-	-	-
	HELSINKI	4	12	-	4	-	-	-	-	-	-
C	process aspect										
	WHOLE FINLAND	1	20	16	5	-	-	-	-	-	-
	HELSINKI	3	8	4	1	2	2	-	-	-	-
D	application aspect										
	WHOLE FINLAND	-	-	3	12	12	11	2	2	-	-
	HELSINKI	-	1	2	4	4	8	-	1	-	-

As a next step, we will calculate the mean values for the distributions of Table 3, and thus resulted Table 4. When looking at the mean values of the teachers' estimations, we will observe the same phenomena more clearly: The differences are largest in the system aspect and process aspect.

Table 4. The mean values of the teachers' percentage estimations on their mathematics views.

	calculation aspect	system aspect	process aspect	application aspect
Finland	30,0	19,0	21,0	45,7
Helsinki	31,7	17,0	23,5	45,0

Showing Table 4 as a radar chart (Chart 5) will make the overall emphasis of the application aspect more visible. In both samples, it clearly dominates. But the dominance of the application aspect in teachers' mathematics view is understandable, since the National Board of Education and teacher educators at all Finnish universities have emphasised applications in school mathematics more than ten years.

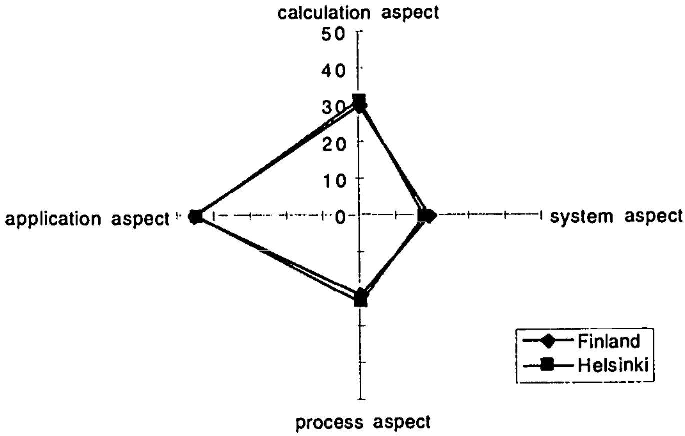

Chart 5. Teachers' percentage estimations on their mathematics views.

3. Discussion

All five interviewed teachers considered the national mathematics test as a successful one, none of them said any negative on it. The general view from the teacher questionnaire was that most of the teachers experienced it very large and difficult to respond. The following quotations demonstrate teachers' negative feelings toward the questionnaire: *"it was a little too long"*, *"there occurred such parts which I didn't understand at all"* (teacher nr. 2); *"it was rather large and terribly difficult"* (teacher nr. 3); *"according to my feelings, it was terribly difficult"* (teacher nr. 5).

Responding difficulties were not due to the whole questionnaire, but only a part of it. In the case of mathematics view, some teachers reacted very negatively. Obviously these questions used such a language to which they were not accustomed. For example, *"in the item 21, it is dealt with passive ... huh, huh"* (teacher nr. 1); *"yee, this was me difficult, this view of mathematics, although it was explained it was difficult to answer"* (teacher nr. 2); *"[in question 21] we were wondering a little this 'static and dynamic view of knowledge', I didn't really understand"* (teacher nr. 3).

In a couple teachers' interview responses, one might "read between the lines" that they have not problemised the concept "mathematics" – their mathematics views were still implicit. Only one teacher of five have a positive overall view on the questionnaire: *"it was OK to answer"* (teacher nr. 4). Therefore, when considering the reliability of the results, one should take into account the problems some teachers had with understanding of the language in the questionnaire.

Furthermore, it is good to notice that teachers' conceptions on mathematics are gathered with a questionnaire. Therefore, the results in the first place reflect teachers' conscious conceptions (cf. surface beliefs in Kaplan 1991). What kind of thoughts they have in their mind (usually unconsciously) is not able to be investigated with questionnaire technique. In order to do that, we are compelled to use time consuming individual interviews.

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Pupils' Beliefs on Mathematics Teaching in Ukraine

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The aim of mathematics education

The majority of educators can agree that the aim of mathematics education is development of mathematics culture which is double-sided including two mutually connected components:

- knowledge and skills in solving standard classes of mathematics problems;
- understanding the matter of mathematics in mankind knowledge, understanding the mathematics method and consequently - ability to apply mathematics to solve real life problems;

The first component is more technological (in controlling as well as in teaching and learning), especially in contemporary time of wide use of computer technologies in education. As a consequence, school mathematics becomes more and more routine and boring, the alive spirit vanishes from school mathematics curricula. Nevertheless first component of mathematics education attracts many of teachers and not only weak teachers because of it takes less efforts from teacher in preparation and may be easy reproduced with a new audience in new environment, gives most valuable result - good testing of graduates and their success in entering the universities (because the majority of tests can be used only for measuring just the first component).

Concentration only on the first mentioned above component was bad even in previous less dynamic time, but now it became really awful. Nowadays need creative persons in all areas of life and ability to use mathematics for solving real life problems - ability to formalise the real situation, to build mathematical model, solve it, interpret obtained results and so on. Modern high qualified specialist in any area must be good mathematics educated in the sense of not only ability to solve the standard mathematics problems but understanding of all the circle of questions concerning the place of mathematics in real life and ability to apply mathematics to real life problems. In other words every high qualified specialist in any subject must be in small a real mathematician (at least on primitive level, but with adequate understanding what Mathematics could give (accordingly what couldn't give today or even nevermore) and in what way). In this paper we will use the notions of *mathematics knowledge* for the first component mentioned above and *mathematics beliefs* for the second.

The matter of pupils' mathematics beliefs and their role in teaching and learning mathematics, the current state in their research are described e.g. by

Pehkonen and Törner (1996), and continued in a series of the MAVI workshops.

It could be mentioned to this point that the second component of mathematics culture (mathematics beliefs) is more difficult maybe in all aspects of teaching and learning and are hard for measuring. Even the results in Mathematics Olympiads are not in a good correlation with mathematics culture - the further success of Olympiads winners in mathematics differs, although there are a lot of good mathematicians who had very weak results in Mathematics Olympiads.

How to measure the mathematics culture?

It is really a very hard question to measure the mathematics culture, since there are no difficulties to measure the mathematics knowledge - one or other type of tests but there are no simple direct methods to measure the mathematics beliefs. The only real correct way to verify mathematics beliefs - measuring the ability to solve the real life problems but it is very expensive and hard method.

One of the possible indirect method of measuring Mathematics Beliefs is to formalise the Ideal Mathematics Beliefs (IMB) and measure correspondence of Mathematics Beliefs (MB) to IMB. Unfortunately, there are a lot of difficult questions in this case as well:

- What is IMB (any talented mathematician can easily answer this question but the answers have a lot of differences)?
- How to measure the correspondence of the MB to IMB?

The approach in an international project «Pupils' beliefs on Mathematics Teaching» (Pehkonen 1995) consists of using specialised questionnaire for testing pupils in different countries in this concern. This test with corespondent answers (canonical or ideal in our terminology) can be regarded as *IMB by Pehkonen.*

Data gathered by Prof. Pehkonen are really a treasure and can be used for investigations of various kinds (not only for finding national traditions in Mathematics education or differences in different countries). For example it could be interesting to compare answers in the same country for a long time or explore the correlation between different kinds of pupils' beliefs on Mathematics and their success in Mathematics and so on. The force of such questionnaires is two-sided: its rich content and its canonicity (invariability, with conscious that any changes (alteration) in it is sacrilege).

Nevertheless there are a lot of aspects of the nature of Mathematics itself and its role in science and life which are important in mathematics beliefs and which couldn't be outlined by any questionnaire. Some statements of great mathematicians which are not reflected in Pehkonen's questionnaire are given below as examples:

- «Mathematics is a language of science» (Kholmogoroff A.N.);
- «Mathematics is a science which deals with infinity» (Alexandrov P.S.);
- «Mathematics is a theory of abstract structures» (Bourbaki N.);
- «Mathematics is the ability to see similarities» (Banach S.);
- «Mathematics is the only discipline which use the deductive method, it means that the only criteria of truth of any fact is its logical reducing to axioms» (Mathematical folklore).

Emphasise the last mind: statistic material can be interpreted in many ways and be used for various investigations and the statistic material itself has the greatest and most objective value. So it could be of great interest to provide researchers an access to this data bank. Of course all materials are gathered for analysis. All kinds analysis is interesting and important and first of all the analysis maid by the authors of the questionnaires because of they are the most suitable for their original approach. Nevertheless the gathered material is richer than its interpretations because of the interpretations include only conclusions (as usual - any object is richer that its model). Access to this materials must be independent from any commentaries and interpretations (because they could influent on other researches). The mechanism for access to such kind of data base can be simplified with the information technologies.

Preliminary comments on pupils' mathematical beliefs in Ukraine

It is difficult yet to say something reasonable about question given in header for many reasons. First of all it is the consequence of the fact that the phenomena of beliefs and in particular beliefs on Mathematics and on Mathematics education does not discussed actively on the pages of journals in former USSR. All was conscious in methodology and pedagogy in ideological state.

There are a lot of good scientists in psychology and pedagogy of Mathematics teaching and learning as well as practitioners in Ukraine and former USSR. They have many interesting and useful results in theory and practice of developing teaching, activation of the learning, activity approach in education, introduction of advanced mathematics courses, using of information technologies in Mathematics education and so on. The theory and practice of education in general and Mathematics education in particular developed in isolation in former USSR. Thereby the current moment is very interesting and beneficial for mutual information and establishing working contacts between educators, scientists and practitioners in mathematics education at both sides of the former iron curtain.

The only source for conclusions about pupils' beliefs on mathematics and on mathematics teaching in Ukraine is the questioning of the about 500 pupils with the Prof. Pehkonen questionnaire. The detailed analysis of its results will be done in future in comparison with the results of Finnish pupils in our future work with Prof. Pehkonen. The main rather obvious result of this questioning is that the pupils of majority of specialised mathematical schools are oriented on active forms of education: problem solving, solving real life problems and are boring in repetition and solving problems with the given algorithm. By contrary the majority of pupils of humanitarian and ordinary schools are oriented on reproductive methods of teaching and learning, as a consequence fond of solving standard problems by given algorithms and wonder as many repetitions as possible.

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Students' Mathematical Beliefs as Predictors of Mathematical Performance at Entering College Level.

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It is a common observation that students' beliefs about mathematics and beliefs about themselves as learners of mathematics is of central importance for student learning and performance in mathematics. Pehkonen and Törner (1995) is discussing how beliefs can be seen as indicators of prior exposure to mathematics and at the same time beliefs may be considered to be predictors for students' performances. McLeod (1992) is arguing for more studies on the relation between the affective domain and the cognitive domain as part of mathematical learning. This paper reports some preliminary results from a study on business college students' mathematical beliefs and their relationships to prior exposure to mathematics and to students performance.

Method and data collection.

Data were collected by two written self-report questionnaires, in August at the first week of college and in November at the end of the fall semester of 1996. The questionnaire consisted of three parts, one part for affective variables relating to students' beliefs about mathematics and about self as learner of mathematics, one part for background information about study tracks and mathematics courses in upper secondary school and one part including a test on mathematical performance.

In Risnes (1997a) we identified based on the August sample, 8 beliefs factors using Structural Equation Modelling (SEM) in the LISREL program implementation. We also presented some structural models relating beliefs variables, background variables and test result for the August sample. The paper also gives a short discussion on the theoretical background of our beliefs constructs relating them to expectancy-value theory and social cognitive theory.

In this paper the focus is on using regression analysis techniques to explore some possible relations between beliefs, school background and performance.

Instruments

The affective domain is described using five beliefs constructs:

Anxiety . Variable in the tradition of studies on mathematics test anxiety.

Sample item: "I feel anxious at mathematics tests".

Ability. Variable named 'self-perceived ability to learn' in Skaalvik and Rankin (1995). This variable is analog to 'self-concept of mathematics ability' in Pokay (1996).

Sample item: "I can learn mathematics if I work hard".

Interest. Variable for mathematics as an interesting and enjoyable subject analog to 'intrinsic motivation' in Skaalvik and Rankin (1995).

Sample item: "I like mathematics".

Motivation. Measures self-efficacy as part of motivational beliefs from Pintrich and De Groot (1990).

Sample item: "I'm certain I can understand the ideas taught in this course".

Regulation. Measures self-efficacy for self-regulated learning adapted from Zimmerman, Bandura and Martinez-Pons (1992).

Sample item: "How well can you concentrate on school subjects?"

Students background from upper secondary school are characterised by:

program=general and academic oriented "gymnasium" track or vocational oriented commercial/ business track

years = 1,2 or 3 years of mathematics courses

grade = 1, 2, 3, 4, 5, 6 with 6 as highest grade in mathematics

Students performance are measured by one test in August and one in November.

Results

Examining the scores for the beliefs variables, we find that the mean values and standard deviation are rather stable from August to November. Running a t-test at significance level 5 % , we find that females are scoring higher than males on the belief variable interest and self-efficacy of regulation in the August sample and on regulation in November. We do not find significant differences by gender for the variables anxiety, self-perceived ability and self-efficacy of motivation.

To study the relationship between our variables, we start by presenting Pearson correlation coefficients between some of the variables in the August sample.

Table 1. Correlation coefficients between variables in the August sample, split by gender into female group F below and *male group M* above the diagonal.

	years	grade	test	anx	abil	inter	mot	reg
years	1	*0.10*	*0.42*	*0.22*	*0.33*	*0.33*	*0.33*	*-0.01*
grade	0.04	1	*0.35*	*0.23*	*0.29*	*0.33*	*0.33*	*0.07*
test	0.49	0.35	1	*0.32*	*0.29*	*0.39*	*0.38*	*0.09*
anx	0.13	0.31	0.41	1	*0.38*	*0.24*	*0.34*	*-0.17*
abil	0.17	0.26	0.26	0.45	1	*0.47*	*0.57*	*-0.03*
inter	0.38	0.45	0.63	0.35	0.47	1	*0.63*	*0.12*
mot	0.38	0.15	0.35	0.29	0.55	0.64	1	*0.10*
reg	0.07	-0.16	-0.16	-0.05	0.13	0.03	0.23	1

Table 1 indicates that the correlation between the variables are rather high, with 38 coefficients being greater than 0.20 and 12 coefficients being greater than 0.40. The variable interest correlates strongly with the other variables. For the female group the correlation between doing well on the test and expressing a positive interest in mathematics is as high as 0.63 compared to 0.39 for males. For both females and males there is a high correlation between the two variables for interest and motivation (0.64/0.63). The test score for both females and males correlates rather strongly with the belief variables (except regulation) and with the variables for years and grades.

To investigate possible influences of our 5 belief variables as predictors of performance, we use a linear regression model estimated by proc reg in SAS. In table 2 we also include by the forward selection method, the F statistics for each of the independent variables contribution if it is included into the model.

Table 2. Linear regression analysis for dependent variable test as a function of school background variables and belief variables for female group F (n=62) and *male group M.* (n=178). August sample.

independent variable	parameter estimate		Pr > T		standardised estimate		F to enter		partial R-square	
	F	*M*	F	*M*	F	*M*	F	*M*	F	*M*
program	-1.67	*-3.59*	0.26	*0.000*	-0.12	*-0.29*	0.67	*23.97*	0.01	*0.10*
years	3.15	*3.16*	0.001	*0.000*	0.40	*0.38*	8.61	*36.75*	0.08	*0.17*
grade	0.75	*1.58*	0.28	*0.000*	0.12	*0.27*	1.04	*28.06*	0.01	*0.10*
anxiety	1.79	*0.74*	0.09	*0.20*	0.19	*0.09*	5.16	*1.51*	0.04	*0.01*
interest	4.66	*1.29*	0.002	*0.07*	0.47	*0.15*	39.25	*4.11*	0.40	*0.01*
regulation	-1.36	*-0.89*	0.20	*0.08*	-0.12	*-0.11*	4.13	*4.13*	0.03	*0.01*

The regression coefficients for the variables ability and motivation do not seem to be different from zero in this model and these variables are not shown in table 2. For the female group a total of 58 % of the variance in test score is explained within the model, compared to 41 % for the male group. The standardised regression coefficients for the school background variables are high for the male group, indicating that the test result is heavily influenced by the program chosen, the number of years and the grades in mathematics at upper secondary school. The belief variables seems to be of minor importance for males, while for females a positive interest in mathematics strongly influences the test score.

The November data based on an analog self report questionnaire, were analysed by using the same regression model as in table 2. In table 3 we show the explained variance given by the sum of partial R-squares for the school background group with variables program, years and grade, and the belief group with variables anxiety, ability, interest, motivation and regulation.

Table 3. Explained variance for dependent variable test in August and November by school background and beliefs for female F and *male M.*

	school background		beliefs	
	F	*M*	F	*M*
August	0.09	*0.37*	0.49	*0.03*
November	0.21	*0.19*	0.17	*0.25*

In the August sample the school background variables for the male group are explaining 37% of the variance in performance and the belief variables are contributing only 3%. For the female group the belief variables count for 49 % of the variance in test score with only 9 % for the background variables. In November we do not find this asymmetry by gender, for both groups the background variables and the belief variables seem to explain an equal amount of the variance in test result. From the school background variables we find that the program chosen at school is important for males in both samples but not important for females. The number of years taking mathematics has an influence on both females and males, this influence is stronger in August than in November. The influence of grades in school is strongest in August.

The belief variables seem to have a rather strong influence on the test results. For the female group the greatest influence is with the variable for interest explaining 40 % of the variance in August and 15 % in November. For the male group the most important belief variable in November is motivation explaining 15 % of the variance in performance.

To further explore some possible relationships between our beliefs variable, we studied regression models with each of the belief variables anxiety, ability, interest and motivation as a dependent variable with background, test result and other beliefs as independent regressors.

Table 4. Linear regression of a belief variable as function of background, test result and other beliefs. Standardised regression coefficients split by gender F and M. August sample.

Independent variables									**R square**	
Depend.var	**anxiety**		**ability**		**interest**		**motivation**			
	F	*M*	**F**	*M*	**F**	*M*	**F**	*M*	F	*M*
anxiety			**0.30**	***0.22***	**-0.09**	***-0.04***	**0.04**	**0.14**	**0.36**	**0.23**
ability	**0.27**	***0.17***			**0.14**	***0.14***	**0.37**	**0.37**	**0.43**	**0.40**
interest	**-0.05**	***-0.03***	**0.09**	***0.13***			**0.43**	**0.46**	**0.66**	**0.46**
motivation	**0.03**	***0.08***	**0.29**	***0.29***	**0.55**	***0.39***			**0.56**	**0.54**

In the model presented in table 4 the school background variables program, years and grade, the August test variable and the belief regulation, are included as regressors even though their regression coefficients are not shown in the table. The model indicate that the interest variable for females are closely connected to grades and test score. For both females and males, motivation is a strong predictor for interest. The variable motivation is predicted by the variables interest and ability for both females and males. For females the anxiety variable is influenced by the test score and for both groups also by ability. This relation is reciprocal with anxiety influencing ability level for both groups. For both groups the level of ability is strongly influenced by judgement of self-efficacy of motivation. The analysis indicate that the belief variables in table 4 to some extent can be predicted by the school background variables and other belief variables. For females 66% of the variance in interest is explained within our model.

Discussion

We will interpret the asymmetry in table 3 to indicate a somewhat different approach by males and females to the selection of academic programs in upper secondary school. The test taken in August is mainly based on topics from mathematics covered in the comprehensive school and everybody in our sample should by their school background be reasonable prepared to answer the problems. In the August sample we find that the test result for males depends heavily on school background but this seem to be less important for females. For both groups the most important school variable is the number of years taking mathematics at school. For the female group the beliefs variables anxiety, interest and regulation seem to be predictors for performance with interest as the dominant variable. We could interpret this to indicate that males are choosing their career through upper secondary school more according to their potential in mathematics than females are doing. Males who judge themselves to be rather weak in mathematics are more inclined to choose a vocational study track in upper secondary school. They will elect mathematics courses and get grades according to their academic abilities. We could say that the males who are scoring high on the August test are those who have chosen to elect more mathematics courses in upper secondary school. For the females their career through upper secondary school is influenced by a lot of other factors. The test result for the females are not related to choosing an academic or vocational study track and the grades are of minor importance. The dominant factors influencing performance, relate to females beliefs about mathematics and about themselves as learners of mathematics. We might say that the females are choosing to go into study program with little mathematics even if they would have the potential to take more mathematics courses. This is in agreement with the observation that females are selecting themselves away from mathematics and science in schools.

In the November test the emphasis is on the concept of derivative and its interpretation as the slope of the curve. For a short presentation of this test, see Risnes (1997b). Students with only one year of mathematics will not have worked with the concept of derivative at school, while students with three years of mathematics are supposed to know the basics of elementary calculus. We now find that the school background relate as expected to the performance test, students from the more academic program are scoring higher as are students with more years and better grades in mathematics. For both females and males the belief variables are relevant predictors for the November test result, accounting for about half of the explained variance of approximately 40 %. The most important predictor for females are the variable for interest in mathematics with a partial R-square of 15%. For males the variable for self-efficacy of motivation is explaining 15 % of the total variance in test result.

Conclusion

The analysis in this paper indicates that the scales for anxiety, ability, interest, motivation and regulation can be used to give an adequate description of parts of students' beliefs system. These variables seem to be acceptable predictors for students' performance level in mathematics and at the same time they are mutually influencing each other. We find that the beliefs variables are more important for females than for males, with interest as a dominant variable. The variable for interest is closely related to self-efficacy of motivation for both gender groups. For the male group the variable motivation seems to have a stronger predictive power than the interest variable.

An examination of students' performance scores at entering college level, indicate that the test score for males is predominately influenced by prior exposure to mathematics in upper secondary school while for females these variables are of less importance compared to the belief variables which strongly predict their performance test results. We interpret this as an indication of a tendency within the female group to avoid mathematics in upper secondary school even though they would be equally prepared to take more mathematics courses as the males. The achievements in the November test did not show the analog difference by gender. This observation fits nicely into the situation where the November test more directly is dependent on students exposure to mathematics in upper secondary school.

This paper is reporting on an exploratory study to find some possible relationships between belief variables, school background and performance. We plan to use these preliminary results for further investigations into more structural questions on how these variables are related as hypothesised by expectancy-value theories and social cognitive theories.

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Discontinuities of the Mathematical World Views of Teachers during Pre-service Education ("Referendariat")

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The education of teachers in Germany is divided into two parts. The first part is a theoretical education at a university, in form of subject specific studies and studies in the pedagogical science. In addition there are two years of practical education, the so called "Referendariat". During this time the future teacher attends and teaches at a school. He also attends a "Hauptseminar" for his pedagogical education as well as seminars -so called "Fachseminare" - with regards to his educational subjects.

The time of pre-service education is an important phase in the development of any teacher. And we are interested in the development of teachers mathematical beliefs. So we tried to find out something about the mathematical beliefs of pre-service teachers. And we were interested in the changes that took place in this period of teacher-education.

For getting some information about the mathematical beliefs of pre-service teachers, we interviewed five of them at the beginning of their "Referendariat" and a second time three month later. Both interviews were video-taped. The interviews were based on open-formulated questions and in addition we gave them questionnaires to fill in before each interview.

The results of this investigation can be departed into several groups. There is information about the role of mathematics in society and about mathematics as a science. And there are also statements about the education of mathematics teachers and the teaching of mathematics in school. Here I would like to report only about the aspect of teaching mathematics in school.

We divided the results about the teaching of mathematics into three parts, the beliefs about learning mathematics, the beliefs about teaching mathematics and the factors that influence both of them.

Beliefs about the learning of mathematics

All five pre-service teachers talked about some kind of mathematical talent. They were sure that some students have a natural understanding of mathematical subjects. Because of this there are also students who have difficulties to work with mathematical problems. One of the pre-service teachers said that this pupils have only the chance to learn by heart. In her opinion only 50% of the students have the chance to learn something in mathematics lessons by understanding. The rest have to learn by heart only to survive his time at school.

Another one of the interviewees saw the mathematical talent only as a level of understanding. Talented students have in her opinion a higher level of understanding. But also the pupils who are not so talented can understand a part of the subject.

Learning of mathematics meant for the pre-service teachers not only the ability of counting an the knowing of formulas. The pupils should also learn to think logically and to abstract. In addition meant learning of mathematics learning to think and speak exactly.

All five pre-service teachers preferred discovering and problem-solving learning. On this ideas the beliefs about teaching mathematics are based.

Beliefs about the teaching of mathematics

The pre-service teachers tried to give the pupils realistic tasks. They wanted to have a connection between the life of the students and the objects of the tasks. The pupils should have enough freedom to discover their own solutions.

The pre-service teachers described the teaching of mathematics as very pleasant. They said that it was easier for them to teach mathematics than their other subject. For this they gave two reasons. First that there are a lot of books to use for preparing a mathematics lesson. And second that it would be easier to react on the statements of the pupils. The pre-service teachers saw there more surance to decide if an answer of a student is right or wrong.

We found no hints, that the beliefs about learning and teaching mathematics had changed from the first to the second interview. But during the second interview three of the pre-service teachers talked about difficulties to practice conform with this beliefs. They still had the wish to orientate the tasks on the students but the have had problems to find such tasks. They wanted the students to discover their own solutions, but they had not enough time for this.

Here was the point where the idea of learning and teaching mathematics was confronted with the reality of school. The work of the pre-service teachers is based on their beliefs about teaching and learning mathematics, but there are a lot of factors which influence them. We were interested in these influence-factors.

Influence-factors on the teaching of mathematics

We used two methods to examine the influence-factors. The first method was the search for influence-factors in the interviews. We counted all factors, which were mentioned in connection with the teaching of mathematics. Factors which seamed to have some influence. Here we used a more quantitative method.

The second method was a qualitative one. In the second group of interviews we gave the pre-service teachers ten cards with possible influence-factors. They get the task to make a ranking with regards to the importance of the factors for their mathematics lessons. The selection of the factors were based on the first group of interviews and two investigations about the situation of pre-service teachers (Schmidt-Kern, 1977; Scholz, 1977).

This table shows the rankings of all five interviewees:

	AS	LK	ASC	MK	CK	average
A students	10	10	10	7	10	9,4
B own experiences as a student	8	8	9	8	8,5	8,3
C mentor	9	6	6	6	6,5	6,7
D stress	5	4	3	5	0	3,4
E marks	2	1	7	3	6,5	3,9
F fun	7	3	8	10	8,5	7,3
G other pre-service teachers	1	9	1	4	5	4
H fear	4	7	2	1	0	2,8
I studies	3	2	4	9	3,5	4,3
J "Fachleiter"	6	5	5	2	3,5	4,3

Table 1. Results of the cards for a 'normal' mathematics lesson

The average gives a positive impression. The pre-service teachers felt strong influenced by the "students". They enjoyed teaching ("fun") and gave "fear" and "stress" a quite low status. In addition it is shown, that the education in the seminar represented by the "Fachleiter" and in a broader sense also by the "other pre-service teacher" is not important.

The "studies" have got a quite low ranking as well. Here we should have a look at the single ratings. While four of the pre-service teachers positioned their studies in the last third of their lists. Ms. MK gave them a very high position. But this is a clear contradiction to her statements in both interviews. There she said that her studies gave her almost no help for her job at school. So we have here a common trend for all five interviewees.

If we have a look at the other single results we also see some differences. Mr. LK's ranking includes the strongest deviations. He positioned "fear" very high and "stress" has more influence on him than "fun". This fits to the general impression we have got from Mr. LK during the interviews. He talked a lot about stress and pressure and seamed to dislike the whole pre-service education. Another interesting deviation in Mr. LK's ranking is the position of the factor "other pre-service teacher". He was the only one who experienced his colleagues as very important. This is especially interesting because he had the most contact with Ms. AS and Ms. ASC. But they positioned this card both on the very end of their lists.

Three of the pre-service teachers made two rankings. They distinguished the ranking of the factors for a 'normal' mathematics lesson and for an examination lesson.

		AS	LK	ASC	MK	CK	average
A	students	-	8	-	8	10	9,3
B	own experiences as a student	-	5	-	7	7,5	6,5
C	mentor	-	1	-	6	5	4
D	stress	-	4	-	5	0	3
E	marks	-	9,5	-	2	7,5	6,3
F	fun	-	2	-	10	7,5	6,5
G	other pre-service teachers	-	7	-	1	4	4
H	fear	-	6	-	4	0	3,3
I	studies	-	3	-	9	3	5
J	"Fachleiter"	-	9,5	-	3	7,5	6,7

Table.2. Results of the cards for an examination lesson

Expectionary the position of the "Fachleiter" is here much higher. Also the ranking of the "marks" is higher. The pre-service teachers understood marks here in a different way. They understood them as the marks they get for their examination-lesson. In the other table marks mean the marks of the pupils.

It might be interesting that Mr. LK positioned "stress" in the same way and "fear" even one point lower than for 'normal' lessons. He seams to be stressed during the whole time of the investigation. An examination-lessons meant not more pressure for him than the other parts of the pre-service education. Except Ms. CK all interviewees reported about stress, but only Mr. LK emphasised it that much.

The results of the quantitative method support the results of the qualitative method in the most cases. But there are also some deviations which must be observed.

A very often mentioned influence-factor is the work as a private tutor. All five pre-service teachers have had some experiences as private tutors. Four of them mentioned it in connection with their teaching in school. Ms. ASC gave her work as a private tutor as the main reason for her mathematical knowledge. It would be much more important for her work in school than her studies at university. Mr. LK described private tutoring as a possibility to try teaching without pressure.

	1. interview	2. interview
work as a private tutor	3	1
the own mathematics teacher	5	-
mathematics school books	1	4
"Fachseminar"	1	2
studies (in fachlicher Hinsicht)	-	1
other pre-service teachers	-	2
mentors	-	4
students	-	2
fun	-	2

Table.3. Explicitly named influence-factors

Another often mentioned influence-factor is the own mathematics teacher. All five pre-service teacher talked about his role. But the kind of influence he had is different. For three of the five he was an ideal. They wanted to become like him. But for two pre-service teachers he was right the opposite.

Here we found a correspondence between the number of namings and the results of the cards. There is surely a close connection between the factor "own experiences as a student" and the factor "the own mathematics teacher". Both represent the time where the pre-service teacher had been a student himself. And the experiences during that time were very important for their role as a teacher.

Another correspondence is the quite low importance of the other pre-service teachers. One of them mentioned in an interview that it was caused by organisational problems. The pre-service teachers who were together in one seminar were living in a large area. And also the distance between the schools for the practical education were often in different cities. A further problem was that the pre-service teachers had often only one teaching subject in common. So it was not easy for them to work together or to influence each other.

This was also an explanation for the high importance of the mentor. He was there where the problems were. When the pre-service teachers had some problems with their teaching, the mentor was there. He could directly give answers and he knew the students.

If we have a look at the teacher-education we can differ two parts. On the one hand there is a quite theoretical part. This part is represented by "studies", "Fachleiter" and the "Fachseminar". On the other hand there is the practical education in school, which is represented by the "mentor" but also by the "students" and "mathematics school books". The pre-service teachers gave the second part a extremely higher importance than the first one.

But there is in my opinion one problem. The pre-service teachers preferred practical help. Their teaching is not based on their education that means studies and seminars. But it is based on a somehow traditional kind of teaching. Experienced teachers, school books, private tutoring and their own experiences as a student built up their teaching style. So innovations and reforms have no chance to influence the future teachers. This is a well known problem in literature. Andelfinger talked about a "closed circle of acting schemes". Here might help more co-operation between university and school. The confrontation with the reality of school would set in earlier and could be reflected in university.

The interviewed pre-service teachers demanded also a closer connection between university and school. But only one of them saw the danger to teach in exact the same way the own teacher did.

One of the results of our investigation is that there is the necessity to talk about a reform of the teacher education in Germany. Four of the interviewed pre-service teachers would like a combined education. They wanted to have a close co-operation between university and school and more practical training during the whole teacher education.

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Mathematical Beliefs and Their Impact on the Students' Mathematical Performance

Questions Raised by the TIMSS Results

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TIMSS has played a major role in political discussions regarding education in the past few months due to the fact that German students only within the middle range pertaining to classroom performance. Unfortunately, the present discussion over language reforms in Germany seems to have pushed us and this most interesting topic aside. It is obvious, however, that one should pursue this question to its furthest possible extent in which the direction of our research may deliver a explainable contribution.

The following representations are to be understood as a preliminary listing; they describe more the actual state of didactics research as it is presented in the literature, and they contain annotations to the few TIMSS results which have been accessible to all up to now.

The confinements of brevity prevent us from providing detailed descriptions of definitions of beliefs at this time. The reader is referred to Thompson (1992), Pehkonen & Törner (1996) and Törner & Pehkonen (1996).

1. Some Fundamental Remarks

Of course one can basically have second thoughts about discussing the priority of teaching mathematics from the point of view of efficiency. I would like to quote the mathematics philosopher Reuben Hersh (1986):

> *Anyone who has even been in the least interested in mathematics, or has even observed other people who were interested in it, is aware that mathematical work is work with ideas. Symbols are used as aids to thinking just as musical scores are used as aids to music. The music comes first, the score comes later. Moreover, the score can never be a full embodiment of the musical thoughts of the composer. (p. 18 - 19)*

Hersh's comparison of mathematics with music more likely raises the question, to which extent one can do justice to the teaching of mathematics by the successful measuring of beliefs when concerning aspects of "virtuosity" of mathematics in the classroom. Furthermore Thompson (1992) expands on the idea by saying:

> *Yet, as noted in the Standards (NCTM, 1989), traditional teaching emphases have been on the mastery of symbols and procedures, largely ignoring the processes of mathematics and the fact that mathematical knowledge often emerges from dealing with problem situations. Indeed, the converse of Hersh's statement can be used to*

characterize typical school mathematics - first comes the score, but the music never follows.

In this respect many results are clouded by fundamental doubts concerning the success of teaching mathematics, especially those conclusions ascertained by the TIMSS. Yet the measurability of success by schooled learning is a political axiom. Only measurable success is actual success!?...

2. Mathematical Performance and Beliefs

And why should the innerlying beliefs of the teachers and the students be an important part to the character of the explanation? It is shown that the question why problem solving associated with individual cases has produced only minimal success refers directly to mathematical beliefs. And yet I have become more cautious (reserved) through my investigations as I will subsequently reveal.

2.1 Conceptual Frame for Interrelated Curricula

If one looks for didactical papers on mathematics which provide evidence for a connection between beliefs on mathematics and mathematical performance, one shall come across various articles in the literature of the early 80's in the area of problem solving (e.g. Buchanan, 1987; Cobb, 1984; Garafalo, 1989; Grover & Kennedy, 1994; Schoenfeld, 1983; Wheatley, 1984). The timely frequence of such articles is not astonishing, however, due to the fact that the style and character of teaching mathematics through problem solving was a main theme of didactics during those years. On the other hand informed persons were not astonished by the observations made at the time originating in a more open teaching situation.

The shortcomings described in these papers highlight fundamental limitations in when approaching mathematics. Attitudes were criticised which, for example, reduced the teaching of mathematics to the simple memorising of facts and algorithms. Such an opinion would naturally produce a negative effect on the will to memorise formulas if one believes that mathematics could be subdivided in isolated sections and that the material need only be retained until the next classroom test. He who favours formulas exclusively and depreciates their derivations as such, only supports a mechanical approach to mathematics. As representative of other quotations we refer to the following:

Recent research in mathematics education has shown that success or failure in solving mathematics problems often depends on much more than the knowledge of requisite mathematical content... Other factors, such as decisions one makes and the strategies one uses in connection with the control and regulatio of one's actions (...), the emotions one feels while working on a mathematical task (e.g. anxiety, frustration, enjoyment), *and the beliefs one holds relevant to performance on mathematical tasks, influence the direction and outcomes of one's performance. ([Ga89])*

If one takes these observations seriously, it is not so much the specific views of mathematics which are to be criticised, but it is rather the implementation of the mathematics curricula in the classroom; it is the reduction of the Intended Curriculum to the Implemented Curriculum in the daily lessons. These reductions occur yet again at further points, for example reduction from the Implemented Curriculum offered in the classroom to curriculum attained by the individual. In analogy of the considerations from the framework of the TIMSS (see [Ro93]), the following diagram describes the pattern of the transformation:

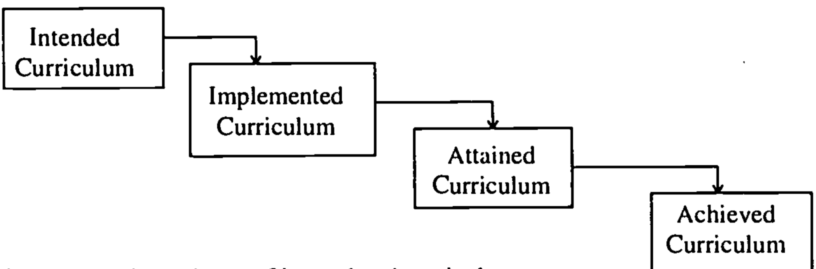

Figure: The dependence of interrelated curricula

These connections and indirect influences can rarely be altered through curricular modifications. Moreover, they can only be described to a certain degree and conditionally brought into question. Each teacher would attest to the indispensable compromises and inherent necessities which actually lead to a rather grey representation of real mathematics teaching in his or her classes originating from a colourful illustration of ideal mathematics teaching.

Let us assume for the sake of simplicity of our exposition a linear input-output model. The influencing effect of beliefs should be of interest. The box 'Intended Curriculum' serves as the input variable. Robitaille understands Intended Curriculum to be the following:

The question of who makes curriculum decisions is a fundamental and timeless issue... The array of participants who officially designated or who function through default to make curriculum decisions is complex enough, but the question centers around not only who makes them, but also what type of curriculum decision is under discussion.

The intervention of influence results from the many different players and interest groups, with which among others teacher training programs at the university are associated. These involvements crystallise into a concrete framework regarding content by school administrators. The curricula having originated as such may be monolithic in character, but they can reflect also inconsistent majority votes (see Ernest [Er93]). In this respect the intended curricula are never neutral concerning a world view; in fact, they are full of beliefs. In general, however, the curricula are organised in a plural fashion on the grounds of a necessary and widespread suitability. They are exemplary mostly of the idea of 'compromise.' Here there is room for a view of mathematics as an art form as well as addressing the toolbox aspect of mathematics. In addition, it allows for the idea which describes mathematics as a system. Finally the relevance of the applications will also not be concealed. In this regard one can assume that generally the official curricula can be classified as tolerant in respect to world views.

At the point between 'Intended Curriculum' and 'Implemented Curriculum' the teacher takes on a major role of importance:

> *Teachers fulfill a variety of functions regarding the creation and implementation of curriculum materials, their curriculum 'texts'... The interpretation of curriculum materials allows teachers to express their individual approaches to teaching, as well as their responses to the needs of their specific classroom situation.*

There are social and cultural stipulations surrounding this concept which decide the coupling gradients. It can be assumed, that the transition must generally be understood as a process of adaptation, which levels off peaks and fills in holes. Everything is being smoothed out and simplified as well as nonessentials being left out and avoided, whereby overdrawn requirements and expectations in the curricula may not be made responsible. In specific cases attention may be observed regarding additional, individual profiles and emphases with the conversion of the 'Intended Curriculum.' However, refinements will be used, in the author's opinion, quite seldom. Comparable processes can be assumed when forming and changing beliefs.

The phenomenon occurring at the interfaces between 'Implemented Curriculum - Attained Curriculum' and 'Attained Curriculum - Achieved Curriculum,' respectfully, shall not be dealt with at this time. On the whole it must be assumed that this coupling process specifically relating to mathematics is rarely dealt with in the literature. The same is true of the formation of beliefs induced at the time.

Back to the observations quoted above about limited mathematical beliefs: basically two clarifications must be admitted. Deficits can be assigned to the parties immediately involved such as teachers, school administrators, students, etc. Because the couplings described above (see Figure) are not, as a rule, fully fitting, it can be presumed with equal justification that the deficits are immanently induced from the transformation process (in a constructivist view: construction process, respectfully). For this reason the conclusions drawn from input variables (the role of specific beliefs), are flawed with a very large amount of uncertainty; hastily drawn conclusions cannot be scientifically responsible.

Independent of this, however, the question arises to which extent varying mathematical world views are resistant to transformation at the given segments (see Figure).

2.2 The Role of Self-Concepts

The self-concepts of the students play an important role in the literature in connection with beliefs and their prognosa character (Cooper & Robinson, 1991; Evans, 1987; Hannula & Malmivuori, 1996; Kloosterman, 1991; Malmivuori & Pehkonen, 1996). The effect of gender-specific factors would be suitable to be discussed here. The total immediate connections to the detailed mathematical beliefs appear unclear and require a more exact analysis within the structure of case studies.

3. Mathematical Beliefs and TIMSS' performance

3.1 The Philippou-Hypothesis

At the present time there is not enough data collected from the results of the TIMSS to draw key conclusions from the full spectrum of the performance of the individual countries. It will still take a fairly long time until the material in this point of view can be made accessible. In the first attempt, the work of Philippou represents the world views of 7th grade teachers. The author distinguishes the grouping of TIMSS countries into three categories. He characterises a top group, to which Singapore, Hong Kong, Korea, and Japan belong. The middle group comprises the countries of England, Germany, Belgium and Sweden. The bottom group is represented by Greece, Cyprus, Colombia and Iran. These groups are chosen with respect to the performance of the students in these countries. The responses of teachers in each group of countries (drawn from the report made available by Cyprus' national representative) were combined together (weighed average) to form one entity i.e. the agreement / disagreement proportion for each group of countries. Using some kind of Median Polishing Analysis he is able to determine significant differences concerning the conceptions about the nature of mathematics. The following table is deduced from data from the paper (Philippou, 1997) and only contains tendencies (++ / + / - / --) regarding items from the international questionnaire which we further quote below. In respect to the particular details we refer to the original work:

Countries / Item	N1	N2	N3	N4	N5
Top group		+	+	--	
Middle group	--	+	-	++	++
Bottom group	-	-	-	++	
Total	--	--	--	--	++

	Mathematics conceived as a set of rules
N1	Mathematics should be learned as sets of algorithms that cover all possibilities.
N2	Basic computational skills are sufficient for teaching primary school mathematics.
N3	Mathematics is primarily an abstract subject.
	Mathematics conceived as a connected and coherent body of knowledge
N4	Mathematics is primarily a formal way of representing the real world.
N5	Mathematics is primarily a practical and structured guide for addressing real situations.

The table above makes clear various view of mathematics. For brevity we shall not pay mention to obvious interpretations and will refer to the actual work

3.2 Are the TIMSS questions suitable to determine world views?

A major point of interest are the teacher questionnaires, whose results are only partly known in the public. The following questions obviously touch on the aspects of beliefs:

	In order to be good in mathematics in school, how important is it for students to...according to your opinion? (not important, somewhat important, very important)
15a	To be good in math is important to remember formulas and algorithms
15b	To be good in math is important to think logically and consistently
15c	To be good in math is important to understand mathematical concepts, fundamentals and strategies.
15d	To be good in math is important to be able think creatively.
15e	To be good in math is important to understand how mathematics is used in the real world.
15f	To be good in math is important to be able to provide reasons to support their conclusions.
	To which extent do you agree with the following statements? (strongly disagree, disagree, agree, strongly agree)
16a	In the first place, mathematics is an abstract field.
16b	Mathematics is primarily a formal way of representing the real world
16c	Mathematics is primarily a practical and structured guide to approach a real situation.
16d	If students are having difficulty, an effective approach is to give them more practice by themselves during the classes.

16e	Some students have a natural talent for mathematics and others do not
16f	More than one representation (picture, concrete materials, symbol, etc.) should be used in teaching a mathematics topic.
16g	Mathematics should be learned as systems of algorithms and rules which uncover all possibilities.
16h	Basic computational skills on the part of the teacher are adequate to teach the primary level of mathematics.
16i	Attachment to and understanding for the students are essential for teaching mathematics

Some of the results can be found in corresponding (see Beaton[2] et al.). The complete factor analysis has not been published and - as far as the author knows - does not reveal some convincing pattern.

Thus, the German TIMSS-team developed additional items which we list below. For brevity, however, we must omit a presentation of the results and their interpretation.

1	Mathematics affects each one of us everyday.
2	Mathematics helps in explaining economic occurrences.
3	Mathematics helps in solving technical problems.
4	Mathematics contains the aesthetics associated with formulas.
5	What I learn in mathematics I can use in other courses of study.
6	A mathematical theory is similar to a work of art because both are the result of creativity.
7	The goal of mathematical theories is to make life more comfortable.
8	Proof or derivation of a formula is not important. The important thing is that I can apply it.
9	One day the mathematicians will have discovered all of which mathematics can provide.
10	In mathematics there is always only *one* way to solve problems.
11	Mathematics is the remembering and application of definitions and formulas, all from mathematical facts and procedures.
12	Mathematics is just a game with figures, pictures and formulas.
13	Mathematics is of no use to me in other courses of study.
14	Most of the mathematical problems have already been solved.
15	Mathematics is used in many tasks in everyday life.
16	Almost all mathematical problems can be solved by the direct application of known rules, formulas and procedures.
17	Mathematics is essentially a game.
18	The goal of mathematical theories is to solve practical problems.
19	Doing mathematics means applying general laws and procedures to specific tasks.
20	Mathematics is a language which provides its own stimulation.

The upcoming papers will present and interpret the results.

[2] http://wwwcsteep.bc.edu/timss

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