#### DOCUMENT RESUME

ED 417 956 SE 061 263

AUTHOR Silver, Edward A.

TITLE Improving Mathematics in Middle School: Lessons from TIMSS

and Related Research.

INSTITUTION Office of Educational Research and Improvement (ED),

Washington, DC. Office of Reform Assistance and

Dissemination.

REPORT NO ORAD-98-1107

ISBN-0-16-049532-6

PUB DATE 1998-03-00

NOTE 29p.

CONTRACT 43-31HA-7-10013

AVAILABLE FROM U.S. Government Printing Office, Superintendent of

Documents, Mail Stop: SSOP, Washington, DC 20402-9328.

PUB TYPE Reports - Evaluative (142)

EDRS PRICE MF01/PC02 Plus Postage.

DESCRIPTORS Educational Change; \*Equal Education; \*Instructional

Improvement; \*Mathematics Achievement; \*Mathematics
Curriculum; \*Mathematics Instruction; \*Middle Schools;

Secondary Education; Teaching Methods

IDENTIFIERS \*Third International Mathematics and Science Study

#### ABSTRACT

Results from the Third International Mathematics and Science Study (TIMMS) have recently captured the attention of the United States and once again focused the nation on matters of mathematics teaching and learning. In general, the TIMSS results indicate a pervasive and intolerable mediocrity in mathematics teaching and learning in the middle grades and beyond. At Grades 7 and 8 and also at Grade 12, U.S. students achieve poorly in mathematics compared to students in much of the rest of the world. Even the 12th-grade students who take advanced mathematics courses performed substantially below students in most other nations. The results also suggest that the demands made by the U.S. school mathematics curriculum and mathematics classroom instruction are not as challenging as in other countries. This document reports the major findings from TIMMS and makes some recommendations for enhancing mathematics teaching and learning in the middle grades. Contains 40 references. (ASK)

Reproductions supplied by EDRS are the best that can be made

\*\*\*\*\*\*\*\*\*\*\*\*\*







# Improving Mathematics in Middle School:

Lessons From TIMSS and Related Research

By Edward A. Silver University of Pittsburgh School of Education and Learning Research and Development Center



U.S. Department of Education Richard W. Riley Secretary

Marshall S. Smith Acting Deputy Secretary

Judy Wurtzel

Director, Mathematics Initiative

Office of Educational Research and Improvement Ricky T. Takai Acting Assistant Secretary

Office of Reform Assistance and Dissemination Peirce Hammond Director

Media and Information Services Cynthia Hearn Dorfman Director

This report, written under U.S. Department of Education Contract No. 43 31HA 7 10013, expresses the opinions of the author and does not necessarily reflect the views of the Department. This report is based on a longer, forthcoming study entitled Enhancing Mathematics Teaching and Learning in the Middle School Years: Lessons from TIMSS and Related Research by Edward A. Silver. References provided in this publication appear as received by the individual authors.

The full text of this public domain publication and the longer forthcoming study are available at the U.S. Department of Education's home page at http://www.ed.gov/inits.html#2 and in alternate formats upon request. For more information, please contact us at:

U.S. Department of Education
Office of the Deputy Secretary
Room 6251, FB10
Washington, DC 20202
FIRS 1-800-877-8339, 8 a.m.-8 p.m. ET, M-F



## Introduction introduction

Results from the Third International Mathematics and Science Study (TIMSS) have recently captured the nation's attention and once again focused it on matters of mathematics teaching and learning. In general, the TIMSS results indicate a pervasive and intolerable mediocrity in mathematics teaching and learning in the middle grades and beyond. At grades 7 and 8, and also at grade 12, U.S. students achieve poorly in mathematics compared to students in much of the rest of the world. Even our 12th-grade students who take advanced mathematics courses performed substantially below students in most other nations. The results also suggest that the demands made by the U.S. school mathematics curriculum and our mathematics classroom instruction are not as challenging as in many other countries.

TIMSS is a particularly rich data source about the middle grades because it includes not only achievement data and a curriculum analysis but also a classroom instruction videotape study.[1, 20, 29, 39] Teachers, principals, parents, policy makers, and others wishing to improve mathematics education in the middle grades can learn much from TIMSS by: reviewing some major TIMSS findings related to grades 7 and 8; considering these findings in light of other relevant research on mathematics curriculum content, classroom instruction, and student achievement; and then pondering the lessons from TIMSS and related research about what must be done to ensure that U.S. students have access to better mathematics education that will prepare them for the challenges of today and

tomorrow.



#### \_

### A Summary of Some Major Findings From TIMSS Major Findings From TIMSS

Among the major findings reported from TIMSS related to student achievement at grades 7 and 8, curriculum content in grades 4 and 8, and classroom instruction in grade 8 are the following:

#### Student Achievement

- U.S. 8th-grade students' mathematics achievement is below average internationally, and it is lower than that of students in many countries that are our economic competitors.
- U.S. 8th-grade students perform relatively better in some mathematics content topic areas than in others. Relative to international averages, U.S. students are about average in the areas of algebra; fractions; and data representation, analysis, and probability; and below average in geometry, measurement, and proportionality.

#### Curriculum Content

- ◆ The U.S. school mathematics curriculum is unfocused—"a mile wide and an inch deep." More topics are included in the U.S. curriculum at each grade level than are found in the curricula of most other countries.
- The U.S. school mathematics curriculum is excessively repetitive. Many topics appear in the U.S. curriculum at more grades than in the curricula of most other countries.



◆ The U.S. school mathematics curriculum is not sufficiently demanding. Compared to many other countries, the content taught at grade 8 in the United States is similar to the content taught at grade 7 elsewhere, and the performance expectations are lower in the United States.

#### Teachers and Teaching

- ◆ At grade 8, U.S. instruction is quite uniform and it is oriented neither toward understanding (e.g., U.S. teachers tend to "state" ideas rather than to "develop" them) nor toward intellectual challenge (e.g., U.S. teachers tend to use tasks that engage students with low-level cognitive activity, such as memorizing and recalling, rather than high-level thinking, such as reasoning and problem solving).
- More than in many other countries, teachers in the United States lack structured, sustained opportunities to improve their practice.



# Putting TIMSS in Context: Considering Related Evidence Evidence

In general, the major findings of TIMSS reveal few surprises. Most of the findings regarding student achievement, curriculum content, and classroom instruction confirm or are consistent with many other research studies.

#### Student Achievement

The TIMSS findings are consistent with other analyses of U.S. student achievement from an international perspective. [15, 16, 17] For example, the relatively poor overall achievement and differential performance across content areas by U.S. 8th-grade students resonate with those of the Second International Mathematics Study (SIMS), which was conducted in 1981–82. In that study, U.S. students also performed poorly overall and well below the international average in geometry and measurement; they were at or near the international average for the topic areas of arithmetic, algebra, and statistics, although well below students in the top performing countries.

Although the U.S. TIMSS data have not yet been analyzed to see whether the performance of white, black, and Hispanic students varies, results from the National Assessment of Educational Progress (NAEP) provide ample reason to suspect that such variation exists. There are longstanding performance differences among white, black, and Hispanic students at all grade levels tested by NAEP.[4] Although the performance gap among black, Hispanic,



and white students has been closing over time on tasks that assess basic procedural knowledge and skills, substantial performance differences remain on tasks that assess conceptual understanding, mathematical reasoning, and problem solving. [30, 33]

#### Curriculum Content

The TIMSS analysis found that the K-8 mathematics curriculum in the United States is unfocused and repetitive. SIMS also noted that the "spiral curriculum" seemed to have gone awry. Mathematical ideas were treated repetitively over the years rather than in successively more sophisticated ways. This analysis also pointed to excessive fragmentation of knowledge—taking "big ideas" and decomposing them into tiny bits—as another key reason for the lack of focus and repetitive character of the U.S. curriculum.

TIMSS also noted that the U.S. mathematics curriculum was not very demanding, based on an examination of basal texts, pre-algebra texts, and algebra texts commonly used in grade 8. This lack of demand manifested itself in two ways. First, especially in the non-algebra courses, there was a preoccupation with arithmetic at the expense of algebra, geometry, and measurement. Second, in all courses—even the algebra courses—there tended to be excessive attention to low-level knowledge and skills without sufficient attention to conceptual understanding or complex problem solving. Similar findings were reported in SIMS and in several other studies of U.S. curriculum. [10, 11, 25]

9



#### Teachers and Teaching

Striking findings of the TIMSS study have emerged from the videotape analysis of instruction in mathematics classrooms in the United States, Germany, and Japan. This analysis has revealed that mathematics instruction in 8th-grade classrooms in the United States emphasizes low-level rather than high-level cognitive processes, and it engages students with mathematical ideas in a superficial rather than deep way. Across different teachers, classroom settings, and lesson topics, this form of instruction was consistently found in the TIMSS study. The absence of attention to high-level processes, such as problem solving and reasoning, in typical mathematics instruction has also been noted in numerous other research studies over the past three decades.[12, 36, 40]

Another TIMSS finding related to teaching is the lack of structured support that American teachers have for improving their pedagogical practice. Unlike teachers in Japan and Germany, the TIMSS data indicate that U.S. teachers do not have much time or structured opportunity for interaction with colleagues about instructional issues. In the United States, teachers are also more likely to work in isolation than are teachers in many other countries. Similar findings regarding the isolation of U.S. teachers have been reported elsewhere.[2, 8, 12] A lack of focused support for teachers, along with limitations in the preparation of many individuals who are assigned to teach mathematics, no doubt suppresses innovation and improvement in U.S. teaching.



# Recommendations for Enhancing Mathematics Teaching and Learning in the Middle Grades in the Middle Grades

From TIMSS and other research we gain insights into characteristics of the U.S. school mathematics curriculum and typical classroom instruction that affect student learning. It is important to note, however, that many of the TIMSS findings are tied to deeply rooted education practices in the United States. Thus, the task of upgrading U.S. mathematics education is challenging and is quite likely to require a substantial commitment of time and resources, including human and financial capital. The findings of TIMSS, combined with those of related research, suggest some pathways along which progress can be made. First, there needs to be a serious national commitment to improved mathematics learning by all students. Second, the mathematics curriculum and instructional practices utilized in the middle grades need to be enhanced. Third, a substantial investment needs to be made in teacher professional development in order to support a more ambitious curriculum and more intellectually challenging mathematics instruction, which will in turn lead to greater student achievement.

# Recommendation 1: Make a serious national commitment to improved mathematics learning by all students

To the casual observer it would certainly appear that school mathematics holds a place of importance in U.S. society. Almost all students study mathematics every year in grades K–10, and many continue to study mathematics in grades 11–12. In grades K–8, mathematics receives more instructional attention than any other school subject, except reading. Moreover, students' mathematics achievement is regularly monitored, and mathematical competence is often used to determine access to educational and employment opportunities at the postsecondary level. Yet, there is also an ambivalence about our national commitment to mathematics education.

Alongside the affirmation of its importance, there is a widely held belief that most people are "just not very good at math" and that only a few have the ability to be successful with mathematics. A view that most people have limited ability to learn mathematics may be tied to the widely held belief that students must master all the "basics"—typically defined to be sets of arithmetic facts and procedures-before attempting to solve challenging mathematical problems or studying other areas of mathematics (e.g., algebra or geometry). Contrast this view of mathematics with the way that society views literacy. There is a core belief that everyone can learn to read and write in English, and a failure to do so is viewed as socially unacceptable. But few would argue that students must master all of the basics of grammar,



punctuation, and spelling before ever reading a novel or writing a poem.

A belief that high-level mathematical performance expectations are not appropriate for all students also appears to undergird forms of instruction that have generally been found in research studies of schools serving students from low-income communities, many of whom are assigned to "lower track" instruction. Accentuating the worst of what was observed more generally in TIMSS, research has found that students in lower track mathematics classes receive less instruction than do their peers in regular or higher track classes, and the instruction they do receive almost exclusively emphasizes low-level knowledge and skills.[22, 24, 27] Moreover, students from low-income communities are far more likely to be taught by unqualified or underqualified teachers. [19, 21, 23]

It is clear that performance expectations for all our students need to be raised. The TIMSS findings at grade 12 show that even our advanced students—those taking pre-calculus and calculus—perform significantly below advanced students in other nations. However, we are unlikely to see substantial overall improvements in the mathematics achievement of U.S. students unless and until we as a nation adopt the belief that all students can learn mathematics, and then act on that belief. Research-based examples exist to demonstrate the validity of the view that all children can learn mathematics. [5, 13, 32] This belief needs to be clearly reflected in the expectations we set and the support we provide to students and teachers as they



engage in serious academic work. One small but telling finding of the TIMSS video study was that U.S. classrooms were the only ones in which mathematics lessons were frequently interrupted for nonacademic reasons; such interruptions were not found in German and Japanese classrooms. Of course, more than commitment to and belief in students is required. Attention also needs to be paid to mathematics curriculum, teaching, and teachers.

Recommendation 2: Make the school mathematics curriculum more ambitious and enhance classroom instruction

A key element in any effort to improve the mathematics achievement of U.S. students is curriculum. Almost all states and many school districts have a curriculum framework, or set of objectives, for mathematics. Such a framework that is clearly tied to standards that set high expectations for all students can serve as a guide for teachers as they plan, implement, and evaluate their instruction and their students' learning. The TIMSS findings suggest a need for states and districts to examine their mathematics curriculum frameworks to see if they are sufficiently ambitious, especially in grades 5-8. What is needed is a reconceptualization of basics for middle grades mathematics. These basics must include much more than arithmetic; they must also include geometry and measurement, proportionality, and algebra. Any instructional approach that focuses narrowly on arithmetic facts and skills is unlikely to improve

the relative performance of U.S. 8th-grade students within the international community.

Students need to be introduced to core concepts of algebra and geometry in the middle grades so that they are better prepared to tackle more advanced work in these content areas in high school.[31] But the solution to creating a more ambitious curriculum for the middle grades is not simply requiring students to take a standard high school algebra course in seventh or eighth grade.[7, 37] The poor performance of grade 12 students in NAEP on rudimentary algebra skills calls into question the adequacy of the typical algebra course in developing real algebraic competence, and there is little reason to think that taking this course 1 or 2 years earlier in one's school career will make it more efficacious. In fact, it may lead to even more "tracking" in the middle grades, with all the attendant negative consequences. Moreover, a narrow focus on algebra ignores the need to improve students' performance in geometry and measurement.

Curriculum materials for the middle grades—materials in which algebraic ideas develop in concert with knowledge in other mathematics content areas—have been created, many with support from the National Science Foundation (NSF), and are now being made available for use on a large scale. These new curricula differ significantly from the textbooks analyzed in TIMSS. In general, the new materials are more focused, often devoting a unit to the careful development of two or three key mathematical ideas. Moreover, they include novel and demanding mathematical topics, many of which



are contextualized in interesting and engaging ways. These curricula set ambitious goals for students, and they attend to an appropriate mix of important mathematics topics, including algebra. These curricula also engage students in cognitively demanding activity quite unlike that observed in U.S. mathematics classrooms in the TIMSS study. In the new curricula, students are challenged to think and reason, to solve complex problems, and to communicate their ideas.

Schools, school districts, and states need to get these and similar curricula into the hands of teachers. This effort will require attention to curriculum revision and textbook adoption procedures in order to ensure that the right questions are asked as decisions are being made. The involvement of teachers in these processes is crucial. We also need to attend to the kinds of support teachers need to use such curriculum materials well.

Recommendation 3: Invest in teacher professional development and capacity building to support improved mathematics achievement

Once materials representing a more ambitious curriculum are available for use in schools, we need to take steps to ensure their informed and effective use in order to promote increased student achievement. More ambitious curriculum is closely tied to more complex teaching, and this presents challenges for teachers of mathematics in the middle grades.



One challenge is mathematical. Teachers in grades 5–8 often have the same mathematics background as teachers in grades K–4, yet they are expected to teach more complex content. The additional challenges inherent in the more ambitious curriculum material will require them to be more like mathematics specialists than their original training may have prepared them to be. These teachers often have had little exposure to some of the mathematical ideas that ambitious curricula will require them to teach their students. Moreover, teaching mathematics in ways that make it understandable by students requires deep, flexible knowledge on the part of the teacher.

A second challenge is pedagogical. Most teachers have had limited experience teaching more ambitious curricula, helping students deal with complex mathematical tasks, or learning complex mathematics themselves in settings in which innovative pedagogy is used. As research has shown, teachers often struggle, at least initially, when they use ambitious curricula and cognitively complex mathematics tasks in the classroom. [26, 28, 38] These kinds of tasks require a style of engagement and interaction that is quite different from the pattern of low-level cognitive activity noted in the TIMSS video study and in many other classroom-based research studies.

These challenges call for increased attention to building the capacity of teachers to enhance their mathematics instruction. Both initial preparation and ongoing professional development support are critical to the successful implementation of a more ambitious



mathematics curriculum in the middle grades. We need not only to enhance teachers' knowledge of mathematics content and pedagogy but also to provide specific support for their efforts to teach mathematics in better ways in the classroom. Research has shown that effective support for teachers to enhance their mathematics instruction can come in many different forms, including collaborating within teacher networks or with knowledgeable persons outside the school; attending summer institutes or related courses; reflecting on one's own experience as a mathematics learner; tying innovations in pedagogy to research-based knowledge of student learning, and linking professional development closely to ambitious curriculum materials.[3, 6, 9, 34] Most of these effective approaches to professional development address directly or indirectly the pervasive isolation of American teachers noted in TIMSS and in other research.

The power of professional communities to support instructional innovation has been seen clearly in schoolwide, districtwide, and regional improvement efforts. Thus, it is critical that special attention be paid to nurturing and sustaining professional communities in which enhanced practice can be developed and supported. Otherwise, research suggests that the ambient culture of isolation and resistance to change that typically pervades mathematics departments in schools is likely to inhibit progress. [14, 18, 35]



#### Moving Beyond TIMSS: Moving Beyond TIMSS: Concluding Thoughts Concluding Thoughts

TIMSS provides important information about U.S. students and their teachers, and about the school mathematics program that influences what they learn and teach. Although the comparison with other nations is what captures media attention, it is the power of TIMSS as a lens through which to view U.S. mathematics education that is most valuable. perspective and the knowledge generated in TIMSS, when combined with other studies of curriculum, teaching, and learning, form a base for needed action to enhance U.S. mathematics education. If we as a nation adopt the belief that all students can learn mathematics; if we act in consistent, coordinated ways to effect that goal; and if we make the requisite commitment of human and financial resources, there is good reason to think many more students will succeed. Our children deserve nothing less than the best mathematics education in the world.

#### References References

- 1. Beaton, A. E., Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1996). Mathematics achievement in the middle school years: IEA's Third International Mathematics and Science Study. Chestnut Hill, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College.
- 2. Black, P., & Atkin, J. M. (1996). Changing the subject: Innovations in science, mathematics, and technology education. London: Routledge & Kegan Paul.
- 3. Brown, C. A., & Smith, M. S. (1997). "Supporting the development of mathematical pedagogy." *Mathematics Teacher*, 90 (2), 138-143.
- 4. Campbell, J. R., Reese, C. M., O'Sullivan, C., & Dossey, J. A. (1996). *NAEP 1994 Trends in Academic Progress*. Washington, DC: National Center for Education Statistics, U.S. Department of Education.
- 5. Campbell, P. (1995). Project IMPACT: Increasing mathematics power for all children and teachers (Phase I, Final Report). College Park, MD: Center for Mathematics Education, University of Maryland.
- 6. Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). "Using knowledge of children's mathematical thinking in classroom teaching: An experimental study." *American Educational Research Journal*, 26 (4), 499-531.



- 7. Chambers, D. L. (1994). "The right algebra for all." *Educational Leadership*, 51 (6),85-86.
- 8. Choy, S. P., Bobbitt, S. A., Henke, R. R., Medrich, E. A., Horn, L. J., & Lieberman, J. (1993). America's teachers: Profile of a profession. Washington, DC: National Center for Education Statistics, U.S. Department of Education.
- 9. Cohen, D.K., & Hill, H.C. (1997 March). "Instructional policy and classroom performance: The mathematics reform in California." (paper presented at the American Educational Research Association annual meeting, Chicago, IL).
- 10. Flanders, J. (1987). "How much of the content in mathematics textbooks is new?" *Arithmetic Teacher*, 35 (1), 18-23.
- 11. Freeman, D. J., & Porter, A. C. (1989). "Do textbooks dictate the content of mathematics instruction in elementary schools?" *American Educational Research Journal*, 26 (3), 403-421.
- 12. Goodlad, J. I. (1984). A place called school: Prospects for the future. New York: McGraw-Hill.
- 13. Griffin, S.A., Case, R., & Siegler, R.S. (1994). "Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure." In. K. McGilly (Ed.), Classroom lessons: Integrating cognitive theory and classroom practice. Cambridge, MA: MIT Press.



- 14. Grossman, P., & Stodolsky, S. (1994). "Considerations of content and the circumstances of secondary school teaching." In L. Darling-Hammond (Ed.), Review of Research in Education, Volume 20 (pp. 179-221). Washington, DC: American Educational Research Association.
- 15. Lapointe, A. E., Mead, N. A., & Askew, J. M. (1992). *Learning mathematics*. Princeton, NJ: Educational Testing Service.
- 16. Lapointe, A. E., Mead, N. A., & Phillips, G.
  W. (1989). A world of differences: An international assessment of mathematics and science.
  Princeton, NJ: Educational Testing Service.
- 17. McKnight, C. C., Crosswhite, F. J., Dossey, J. A., Kifer, E., Swafford, J. O., Travers, K. J., & Cooney, T. J. (1987). The underachieving curriculum: Assessing U. S. school mathematics from an international perspective. Champaign, IL: Stipes Publishing Co.
- 18. McLaughlin, M., & Talbert, J. (in press). High school teaching in context. Chicago: University of Chicago Press.
- 19. National Center for Education Statistics. (1997). *America's teachers: Profile of a profession, 1993-94* (NCES report 97-460). Washington, DC: U.S. Department of Education.
- 20. National Center for Education Statistics. (1997). Pursuing excellence: A study of U.S. eighthgrade mathematics and science teaching, learning, curriculum, and achievement in international context.

Initial findings from the Third International Mathematics and Science Study (NCES report 97-198). Washington, DC: U.S. Department of Education.

- 21. National Commission on Teaching and America's Future. (1997). *Doing what matters most: Investing in quality teaching.* New York: Author.
- 22. Oakes, J. (1985). Keeping track: How schools structure inequality. New Haven, CT: Yale University Press.
- 23. Oakes, J. (1990). Multiplying inequalities: The effects of race, social class, and tracking on opportunities to learn mathematics and science. Santa Monica, CA: The Rand Corporation.
- 24. Page, R.N. (1991). Lower-track classrooms: A curricular and cultural perspective. New York: Teachers College Press.
- 25. Porter, A. (1989). "A curriculum out of balance: The case of elementary school mathematics." *Educational Researcher*, 18 (5), 9-15.
- 26. Prawat, R. S. (1992). "Are changes in views about mathematics teaching sufficient? The case of the fifth-grade teacher." *The Elementary School Journal*, 93 (2), 163-177.
- 27. Raudenbush, S. W., Rowan, B., & Cheong, Y. F. (1993). "Higher order instructional goals in secondary schools: Class, teacher, and school influences." *American Educational Research Journal*, 30 (3), 523-553.



- 28. Romagnano, L. (1994). Wrestling with change: The dilemmas of teaching real mathematics. Portsmouth, NH: Heinemann.
- 29. Schmidt, W.H., McKnight, C.C., & Raizen, S.A. (1996). A splintered vision: An investigation of U. S. science and mathematics education. Boston: Kluwer.
- 30. Secada, W.G. (1992). "Race, ethnicity, social class, language, and achievement in mathematics." In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623-660). New York: Macmillan.
- 31. Silver, E.A. (1997). "Algebra for all: Increasing student access to algebraic ideas, not just algebra courses." *Mathematics Teaching in the Middle School*, 2 (4), 204-207.
- 32. Silver, E.A., & Stein, M.K. (1996). The QUASAR project: The 'revolution of the possible' in mathematics instructional reform in urban middle schools. *Urban Education*, 30 (4), 476-521.
- 33. Silver, E.A., Strutchens, M.E., & Zawojewski, J.S. (1996). "NAEP findings regarding race/ethnicity and gender: Affective issues, mathematics performance, and instructional context." In P. A. Kenney & E.A. Silver (Eds.), Results from the sixth mathematics assessment of the National Assessment of Educational Progress. Reston, VA: National Council of Teachers of Mathematics.

- 34. Simon, M.A., & Schifter, D. (1991). "Towards a constructivist perspective: An intervention study of mathematics teachers." Educational Studies in Mathematics, 22 (4), 309-331.
- 35. Siskin, L. (1994). Realms of knowledge: Academic departments in secondary schools. Washington, DC: Falmer Press.
- 36. Stake, R. E., & Easley, J. (Eds.). (1978). *Case studies in science education*. Urbana, IL: University of Illinois.
- 37. Steen, L. A. (1992). "Does everybody need to study algebra?" *Mathematics Teacher*, 85 (4), 258-260.
- 38. Stein, M.K., Grover, B.W., & Henningsen, M. (1996). "Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms." American Educational Research Journal, 33 (2), 455-488.
- 39. Stigler, J.W., & Hiebert, J. (1997). "Understanding and improving classroom mathematics instruction: An overview of the TIMSS video study." *Phi Delta Kappan*, 79 (1), 14-21.
- 40. Stodolsky, S. (1988). The subject matters: Classroom activity in mathematics and social studies. Chicago: University of Chicago Press.



## Acknowledgments Acknowledgments

I would like to acknowledge the valuable assistance in preparation of this brief report provided by Elizabeth Ann George. Numerous other colleagues contributed to my thinking in preparing the expanded paper, and so their debt should also be acknowledged here. The following individuals offered useful advice and suggestions early in the process: Jerry P. Becker, John Bruer, Daniel Chazan, Dwight Cooley, Frances Curcio, Francis (Skip) Fennell, Karen Fuson, Alice Gill, James Hiebert, Gerald Kulm, Steven Leinwand, Frank K. Lester, Mary M. Lindquist, Barbara Scott Nelson, Robert Orrill, Douglas T. Owens, Janice Petrovich, Andrew Porter, Lauren Resnick, Barbara Reys, Alan Schoenfeld, Walter Secada, Yoshinori Shimizu, Judith T. Sowder, Lynn A. Steen, Jacqueline Stewart, Kenneth Travers, Zalman Usiskin, Terry Wood, and Erna Yackel.

The following individuals provided critical feedback on an earlier draft version of the expanded paper and contributed to its improvement: Warren Crown, Frances Curcio, John Dossey, Karen Fuson, Jeremy Kilpatrick, Mary M. Lindquist, Tom Romberg, Harold Schoen, Alan Schoenfeld, Lynn A. Steen, Kenneth Travers, Norman Webb, Terry Wood, and Judith Zawojewski.

## Information information

For more information about the U.S. Department of Education's Mathematics Initiative and information and resources on improving mathematics education, visit the Mathematics Initiative Web site at: http://www.ed.gov/inits.html#2

Cover and Layout Design:

Kim Schied Silverman



For sale by the U.S. Government Printing Office
Superintendent of Documents, Mail Stop: SSOP, Washington, DC 20402-9328
ISBN 0-16-049532-6







United States
Department of Education
Washington, DC 20208 5645

Official Business Penalty for Private Use, \$300

Postage and Fees Paid
U.S. Department of Education
Permit No. G-17

Standard Mail (A)



ORAD 98-1107

