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ABSTRACT

This paper presents the methodology for performing and interpreting second-order factor analysis. Procedures for extracting and rotating solutions are presented. Critical issues of interpretation, such as interpreting second-order factors are discussed. Two methods for accomplishing this are explained, including multiplying the first- and second-order factor pattern matrices and the Schmid-Leiman (1957) orthogonalized solution. Methods and interpretation are discussed for both exploratory and confirmatory second-order models. Results of example heuristic analyses are presented to aid understanding of both approaches. An appendix presents the correlation matrix for selected ability tests. (Contains 9 tables and 19 references.) (Author/SLD)

RUNNING HEAD: SECOND-ORDER

Second-Order Factor Analysis: Methods and Interpretation

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ABSTRACT

The present paper presents the methodology for performing and interpreting second-order factor analysis. Procedures for extracting and rotating solutions are presented. Critical issues of interpretation, such as interpreting second-order factors from variables rather than first-order factors are discussed. Two methods for accomplishing this are explained, including multiplying the first- and second-order factor pattern matrices and the Schmid-Leiman (1957) orthogonalized solution. Methods and interpretation is discussed for both exploratory and confirmatory second-order models. Results of example heuristic analyses are presented to aid understanding of both approaches.

Second-Order Factor Analysis: Methods and Interpretation

As Gorsuch (1983) noted, one of the common goals of all scientists is to "summarize data so that the empirical relationships can be grasped by the human mind" (p. 2). In this regard, factor analysis is often quite useful. Specifically, when a researcher is faced with scores on a large number of variables, conceptualization of the relationships between those variable is extremely difficult, if not impossible. Factor analysis can aid in the conceptualization since one of its primary functions is to explain the "maximum amount of information from the original variables in as few derived variables, or factors, as possible to keep the solution understandable" (Gorsuch, 1983, p. 2).

While many researchers are familiar with the methods of factor analysis, probably not as many are familiar with second-order factor analysis (Kerlinger, 1984). The basic concept of second-order factor analysis derives directly from that of ordinary factor analysis. As many researchers know, factor analysis involves extraction of factors from a matrix of associations between the variables (or other factored entities) under study, usually a correlation matrix or variance-covariance matrix. Then, the extracted factors may be subjected to one of the many available rotation procedures, which redistributes the variance contributed by the variables to the factors in a way that yields a more understandable structure. However, if an oblique rotation is used, this leads to factors that are themselves correlated. In that case, there would be a factor correlation matrix, which could itself, in turn, be factor analyzed. As Gorsuch (1983) put it, "...if we have a correlation matrix, we can factor it" (p. 239). The factors that are extracted from such an analysis are called "higher-order" or "second-order" factors.

The present paper has three purposes. One is to provide a conceptual explanation of second-order factor-analysis, including the rationale for its use. The second purpose is to explain the method for performing a second-order factor analysis. The third purpose is to describe how to interpret the results of a second-order factor analysis. Methods and interpretation will be discussed for both exploratory and confirmatory second-order models, highlighting their similarities and differences. Results of example analyses using actual data will be presented as an to aid understanding the methods and interpretive issues for both exploratory and confirmatory models.

The present paper does not present first-order analysis in detail, and it is assumed that the reader is familiar with the basic concepts of ordinary factor analysis. Those who are not are directed to other sources for a detailed treatment of first-order factor analysis (cf. Gorsuch, 1983; Kim & Mueller, 1978).

Hetzel (1996) provides an excellent chapter-length review of these same issues.

Conceptual Overview and Rationale

To start, a conceptual overview and rationale for using second-order analysis is presented. Just because one can factor analyze a factor correlation matrix does not imply that there is good reason to do so. However, when factors are correlated, there is, in fact, a broader level of generalization that is not captured by the first-order analysis alone (Gorsuch, 1983), and thus a second-order analysis can be conducted to obtain a different perspective on the data. A useful analogy was offered by Thompson (1990): "The first-order analysis is a close-up view that focus on the valleys and peaks of mountains. The second-order analysis is like looking at the mountains at a greater distance and yields a potentially different perspective on the mountains as constituents of a range" (p. 597). McClain (1996) expands on this analogy, proposing that that first-order analysis is like examining the mountains (the "observed variables") with zoom-lens binoculars, while the second-order analysis uses wide-angled lens.

Implied by this analogy is that higher-order factors offer a different perspective because they offer more generalizability. It is important to keep in mind, however, that even though this is true, the greater generalizability is gained at the expense of detail. Alternatively, the first-order factors offer more detail but with less generalizability. These concepts can also be related back to Thompson's (1990) analogy. Since first-order factors are a close-up view (zoom lens), we can see minute details, but we also have only a limited, narrow view. Conversely, when we examine second-order factors, we are viewing the mountains from a much greater distance, where we have a much broader view of the mountains, allowing a view of how they cluster together to make up a mountain range. At this distance, however, it becomes difficult to discern details from any particular mountain. The close-up and distance views both yield important information and there is no reason why anyone would only want one view if both were available. This, then, is the rationale for using a second-order factor analysis. Indeed, both McClain (1996) and Gorsuch (1983) argued that when factors are correlated, a second-order analysis should always be performed to obtain as much understanding of the data as possible.

Exploratory Second-Order Factor Analysis

A second-order factor analysis must always begin with a first-order analysis. Any extraction procedure (for example, principal components or principle axes) may be used for the first-order analysis. When the first-order factors are rotated to do a hierarchical factor analysis, an oblique rotation must be used so that the factors are allowed to be correlated; if orthogonal rotation were used, factors would be perfectly uncorrelated and a higher-order analysis would be impossible.

Promax rotation (Hendrickson & White, 1964) is a commonly used oblique rotation method that generally yields good simple structure solutions (Gorsuch, 1983).

An oblique rotation of the first-order analysis will yield a first-order factor correlation matrix which is then the subject of the second-order analysis. As in the first-order analysis, any extraction procedure may be used. The factors extracted from the first-order factor correlation matrix are called the second-order factors.

Rotation of Second-Order Factors

At this point, the second-order factor solution can and usually needs to be rotated to make the solution more interpretable. For rotation of the second-order solution, however, the researcher is not constrained to an oblique rotation, but rather may make the decision to perform either an oblique or an orthogonal rotation. Just as with first-order factors, the researcher may have reason to believe the factors should be correlated. However, if the factor correlations are nonexistent or deemed trivial, the researcher may use an orthogonal solution instead (Gorsuch, 1983).

Third-Order Analyses

If an oblique solution is used for the second-order analysis, the second-order factors are correlated. If so, then yet another factor analysis could be conducted, extracting "third-order" factors from the second-order factor correlation matrix. Theoretically, this process of factoring higher and higher could continue indefinitely until (a) orthogonally-rotated (uncorrelated) factors are identified or (b) only one facotor is extracted.

Interpretive Guidelines

Whenever factors are correlated, it is important to interpret both pattern and structure coefficients (Thompson, 1997; Thompson & Borrello, 1985). Pattern coefficients indicate the weight given to the variable in calculating scores on the latent factors while the structure coefficients are the correlations between the variables and the factors.

There is at least one more step before higher-order factors can be interpreted. Since a second-order analysis is a factor analysis of the correlations between first-order factors, the analysis yields a factor-pattern matrix indicating the weights given to the first-order factor scores in determining the second-order factors. The problem with interpreting the analysis using this matrix is that one has become so far removed from the original variables that it becomes very difficult to determine the meaning of the higher-order factors. As Gorsuch (1983) put it, one is "basing interpretations upon interpretations of interpretations" (p. 245). This has also been referred to as "interpreting shadows of the shadows of mountains rather than the mountains themselves" (McClain, 1996, p. 233).

The answer to this problem is to interpret second-order factors in terms of the original variables rather than in terms of the first-order factors. There are two useful strategies for accomplishing this (see also Wherry, 1959).

First, Gorsuch (1983) recommends postmultiplying the first-order factor pattern matrix by the second-order factor-pattern matrix. This operation yields a product matrix (first-order by second-order) containing the pattern coefficients for the second-order factors in terms of the original variables.

Thompson (1990) argued that this product matrix can itself then also be orthogonally rotated by the varimax procedure to make the results more interpretable (Thompson, 1990). Of course, if the second-order factors are rotated obliquely (allowing them to be correlated) it is important to interpret both pattern and structure coefficients, just as with the first-order solution. To obtain the structure matrix for the second-order factors in terms of the original variable, one simply has to postmultiply the product matrix described above by the second-order factor correlation matrix (Gorsuch, 1983).

Second, another useful procedure for interpreting second-order analyses was outlined by Schmid and Leiman (1957). This solution is also referred to as an orthogonalized solution, because it produces a pattern matrix of both higher- and first-order factors in terms of the original variables, but with the common variance accounted for by the second-order factors residualized from the first-order factors. In other words, the pattern coefficients of the variables for the first-order factors represent the unique variance accounted for by the first-order factors which is not accounted for by the higher-order factors. Therefore, first-order factors are orthogonal to the second-order factors, and thus the term orthogonalized solution.

It is important to note, however, that the name refers to orthogonality between factors across successive levels of analysis, and not between factors within a given level of analysis. As discussed previously, orthogonality between factors within a given level of analysis can only occur at the highest level of analysis, or else higher-level factoring would not be possible. Since an available computer program (described later) computes a Schmid-Leiman (1957) solution, the mathematics of the solution will not be presented. The interested reader is directed to Schmid and Leiman (1957) and Gorsuch (1983) for a mathematical treatment of the topic.

Heuristic Example of Exploratory Second-Order Factor Analysis

This section presents a heuristic example of an exploratory second-order factor analysis using 11 ability measures selected from of group of 24 that were administered to 301 junior high children (Holzinger & Swineford, 1939). The raw data for the complete set of 24 measures can be found in Holzinger and Swineford (1939). The correlation matrix for the 11 selected measures can be found in the Appendix so the interested reader can replicate the example analyses of the present paper. A description of the selected measures appear in Table 1.

INSERT TABLE 1 ABOUT HERE

The analysis was performed using the FORTRAN program SECONDOR (Thompson, 1990). For information about the availability and use of SECONDOR, see Thompson (1990). The program begins by extracting first-order factors using principal components analysis. In the present example, 3 factors were extracted and rotated obliquely using the Promax procedure. The factor-pattern coefficients are presented in Table 2. Since structure coefficients are also important aids to interpretation, these appear in Table 3. [Pattern and structure coefficients differ for a given data set whenever factors are correlated, i.e., rotated obliquely (see Hetzel, 1996).]

The first factor was labeled Verbal Ability, given the salient pattern and structure coefficients for the following variables on that factor: General Information (V5), Paragraph Comprehension (V6), Sentence Completion (V7), and Word Meaning (V9). The second factor was called Speed, given the salient pattern and structure coefficients for the following variables on that factor: Speeded Addition (V10), Speeded Coding (V11), Speeded Counting of Dots (V12), and Speeded Discrimination of

Letters (V13). Factor III was called Memory ability, with high correlations on Memory of Target Words (V14), Memory of Target Numbers (V15), and Memory of Object-Number Associations (V17).

INSERT TABLE 2 AND TABLE 3 ABOUT HERE

Since an oblique rotation was used with the first-order solution, the factors were correlated, in this case ranging from .155 to .273. Thus, second-order factors were implied and a higher level of generalization could be obtained with a second-order analysis. One second-order factor was extracted from the first-order factor correlation matrix, which was given the label General Intelligence. The second-order factor pattern matrix is presented in Table 4. Again, this matrix consists of the weights given to the first-order factors to determine the second-order factor.

INSERT TABLE 4 HERE

As noted previously, interpretation should not be based on this matrix, but rather should proceed from an analysis of the original variables making up the second-order factor, using the two previously mentioned methods. The first method, postmultiplying the first-order pattern matrix by the second-order pattern matrix, results in the second-order product matrix, which is presented in Table 5 along with the communalities (h^2). From this matrix it can be seen that all of the variables make a noteworthy contribution to the second-order factor, as evidenced by the magnitudes of their pattern coefficients, which range from .413 to .709.

INSERT TABLE 5 ABOUT HERE

As discussed earlier, the Schmid-Leiman (1957) solution is another method of interpreting the second-order factors from the perspective of the original variables. The Schmid-Leiman solution for the present example, along with communalities, is presented in Table 6. Again, the solution presents the first-order solution in terms of the variables not accounted for by the second-order solution. Inspection of this solution reveals that while there is a meaningful General Intelligence factor made up of noteworthy contributions from all 11 variables, there is also a noteworthy amount of unique variance accounted for by the first-order factors that is not captured by the General factor. This is evidenced by the fact that the pattern coefficients for all the variables in the first-order factor retain noteworthy magnitudes (ranging from .419 to .671) after being residualized of the variance accounted for by the second-order factor. One can also see that the traces for the first-order factors are meaningful (ranging from .98 to 1.76).

INSERT TABLE 6 ABOUT HERE

The pattern of results found in this example is only one of many that are possible. It may be the case that the first-order factors do not explain any variance above and beyond that of the second-order factors. In such a case, all of the pattern coefficients for the first-order factors in the Schmid-Leiman solution would be approximately zero. Alternatively, the pattern coefficients for the second-order factors may be relatively small in magnitude relative to those of the first-order factors, even though this analysis does inherently emphasize the second-order factors over the first-order factors.

Confirmatory Second-Order Factor Analysis

So far, the analyses discussed have all been exploratory factor analysis (EFA). Another type of factor analysis, called confirmatory factor analysis (CFA), is often more appropriate than EFA when there is an explicit theory or a strong empirical base suggesting a specific model. CFA is notable for its ability to test specific hypotheses, and has less of a tendency to capitalize on chance than EFA (Stevens, 1996; Thompson & Borrello, 1992). While EFA examines all possible relationships between all variables, in CFA the researcher specifies one or more particular models of relationships and examines how well the models fit the data at hand. The present discussion will present an overview of second-order CFA within the framework of LISREL (Joreskog & Sorbum, 1986). However, other software packages, such as EQS (Bentler, 1983), are available that utilize graphic model specification rather than specification of models via matrix programming.

In the LISREL framework, the model is specified by indicating hypothesized patterns of relationships in a series of matrices, corresponding to factor pattern and factor correlation matrices. This is accomplished by "fixing" certain parameters in the matrices while "freeing" other parameters, allowing them to be estimated in the analysis. As in most CFA models, the Lambda-Y matrix is used as the first-order factor pattern matrix. This matrix consists of as many rows as measured variables and as many columns as the number of hypothesized first-order factors. Within this matrix, the only parameters freed to be estimated are those parameters relating measured variables to latent factors with which the measured variables are hypothesized to be associated. Parameters relating measured variables to latent factors with which they are not hypothesized to be related are fixed to zero. In order for there to be a unique solution to the model (a situation in which the model is said to be mathematically "identified"), one parameter within each factor is constrained to be one. This also creates a scale for the parameter estimates. [Although this is one common method of identifying the model, another method is described later.]

The second matrix, called Gamma in LISREL, is the second-order factor pattern matrix. The number of rows will be equal to the number of first-order factors and the number of columns equal to the number of second-order factors. Parameters are freed to be estimated based on the hypothesized model of first-order to second-order relationships. Also, since the model must be identified at both the first- and second-order levels, the Gamma matrix can be used to identify the model at the second-order level, by fixing one parameter for each second-order factor to one.

The third matrix, called Psi, is the first-order factor variance-covariance matrix. Often the diagonal elements of the matrix are fixed to be one, which turns Psi into a correlation matrix. Also, this is another way of identifying the model at the first-order level. Thus, this is a common alternative to the previously discussed method of model identification in which one parameter for each first-order factor in the Lambda-Y matrix is fixed to be one.

Since Psi is the factor variance-covariance matrix, it is with this matrix that the researcher indicates whether or not correlated first-order factors are hypothesized. This highlights a major difference between EFA and CFA, because when using EFA, a second-order factor analysis is not possible unless the first-order factors are correlated. This is not true with CFA. In a CFA model, second-order factors can be made up of uncorrelated first-order factors. If uncorrelated first-order factors are hypothesized, then all the off-diagonal elements of Psi involving the first-order factors are fixed to zero. If not, they are freed to be estimated, allowing the first-order factors to be correlated.

The fourth matrix, Phi, is the second-order factor variance-covariance matrix. Again, it is the researcher's decision whether or not to hypothesize correlated second-order factors. If the model posits uncorrelated second-order factors, then the off-diagonal elements of Phi are fixed to zero. The diagonals can then be fixed to be one, or be freed to be estimated. This decision must take into account the identification of the model. Just as certain elements of the first-order factor matrices must be fixed for identification, certain elements of the second-order matrices must also be fixed. If the diagonal elements of Phi are set to one, then the model identification for the second-order matrices is accomplished. However, if the researcher allows the second-order factor variances to be estimated (the diagonal of Phi), then one coefficient for each second-order factor in the second-order factor pattern matrix (Gamma) must be fixed to one, as discussed previously.

The last matrix, Theta-epsilon, consists of the error/uniqueness components of each measured variable. This matrix may either take the form of a one-dimensional array, with one element for each measured variable (Thompson & Borrello, 1992), or a symmetrical matrix with the diagonal elements as the error/uniqueness estimates and the off-diagonal elements as the error/uniqueness covariances (Marsh & Hocevar, 1985). However, error/uniqueness covariances are typically not estimated unless there is a specific reason to believe correlated errors to be plausible. If such is the case, the diagonal elements as well as specific off-diagonal elements in the matrix are freed to be estimated. Using a symmetric matrix with all the off-diagonal elements constrained to be zero would be analogous to a one-dimensional array in which only error/uniqueness are estimated.

Interpretive Guidelines

Just as in first-order CFA, the first step to interpretation is to determine how well the data fit the specified model, for which purpose several "goodness of fit" statistics are available. Three of the more commonly used are the chi-square, the goodness of fit index (GFI), and the adjusted goodness of fit index (AGFI). However, dozens of other fit statistics are available (e.g., the comparative fit index (CFI), and the root mean square error of approximation (RMSEA)). Fan, Thompson, and Wang (in press) and Fan, Wang, and Thompson (1997) provide a review. It is usually desirable to consult more rather than fewer fit statistics when evaluating model fit.

The chi-square statistic tests the null hypothesis that the model is consistent with the observed covariation in the data. Therefore, contrary to conventional statistical significance testing, one does not want to reject the null hypothesis when testing model fit with the chi-square statistic. The problem with this statistic is that it is very sensitive to sample size (Stevens, 1996), and if a large sample size is obtained, as is desirable in CFA, the null hypothesis will quite frequently be rejected. However, it is worthwhile to examine the value of the chi-square statistic without regard to statistical significance testing, where lower values are indicative of better model fit. However, other statistics should also be used for further confirmation.

One alternative to the chi-square is the GFI. The GFI is an indicator of the amount of variance in the data that can be explained by the model, and is analogous to the R^2 in multiple regression (Stevens, 1996). Since this statistic indicates more variance accounted for simply by adding more parameters to the model, the AGFI was developed to adjust for degrees of freedom such that lower values are obtained as more parameters are added to the model (Stevens, 1996). When interpreting goodness of fit, a value above at least .90 is desirable. It is important to keep in mind, however, that a good fit does not rule out the possibility that alternative models may also fit the data (Thompson & Borrello, 1992). Therefore, it is desirable to fit several competing models to the data.

Once the overall fit of the model has been determined to be adequate, the values of individual parameter estimates are examined, just as the pattern and structure coefficients are interpreted in EFA. In addition to interpreting the magnitudes of the parameter estimates, the standard errors of the estimates should also be examined. Not only should parameter estimates have noteworthy values, but they should also be at least twice their respective standard errors (Marsh & Hocevar, 1985; Thompson & Borrello, 1992).

Example Confirmatory Second-Order Analysis

Two example confirmatory analyses were conducted, again using the 11 selected measures from the Holzinger and Swineford (1939) data. The first example tested a first-order model with correlated factors. The second example tested a second-order model with uncorrelated first-order factors. In the second example, the first-order factors were hypothesized to be uncorrelated, primarily to show that this is possible with CFA and to highlight this as a major difference between second-order EFA and CFA approaches.

First-Order Example. The first matrix, Lambda-Y, consisted of 11 rows and 3 columns, corresponding to the 11 measured variables and 3 hypothesized factors: Verbal, Speed and Memory. Corresponding to the model of correlated factors, the off-diagonal elements of Psi were freed to be estimated. The diagonal elements of this matrix were fixed to be ones, which identifies the model at the first-order level and also turns Psi into a correlation matrix. Since no correlated error variance was hypothesized, only the diagonal elements of Theta-Epsilon (error variances) were freed to be estimated and all off-diagonal elements (error covariances) were fixed to zero.

First-Order Results. Table 7 presents the model fit to the data. The chi-square statistic, with 41 degrees of freedom, was 104.30. The goodness of fit index was .941, while the adjusted goodness of fit index was .904. These results support the plausibility of the hypothesized model. However, as mentioned previously, a good fit does not rule out competing models, since many different models may fit a given data set (Thompson & Borrello, 1992). This idea will also be made more clear when the second-order example is presented.

INSERT TABLE 7 ABOUT HERE

Second-Order Example. For the second-order example, the same measured variables were again hypothesized to make up the same three first-order factors. Further, it was hypothesized that the first-order factors were uncorrelated and that they all contributed to a second-order factor of general intelligence.

The first matrix, Lambda-Y, consisted of 11 rows and 3 columns, corresponding to the 11 measured variables and 3 hypothesized factors: Verbal, Speed, and Memory. Corresponding to the hypothesis of uncorrelated factors, the off-diagonal elements of Psi were fixed to zero. Also, in order to identify the model, the diagonal elements of this matrix were set to one. The second-order factor-pattern matrix, Gamma, consisted of 3 rows (first-order factors) and 1 column. All three of these elements were freed to be estimated, corresponding to the hypothesis that there would be one second-order factor (General Ability) that would be made up of all three of the first-order ability factors. Since there was only one second-order factor, Phi consisted of only 1 element, which was fixed to one to allow for model identification at the second-order level. Since no correlated error variance was hypothesized, only the diagonal elements of Theta-Epsilon (error variances) were freed to be estimated, and all off-diagonal elements (error covariances) were fixed to zero.

Second-Order Results. Table 8 presents the unstandardized second-order model fit to the data. The reader will not that the Psi matrix is depicted in this table even though no element in this matrix were estimated. This is primarily to highlight the fact that it is possible to have uncorrelated first-order factors in a second-order CFA model and also to show that the factor variances were fixed to be one in order to identify the model at the first-order level.

INSERT TABLE 8 ABOUT HERE

The chi-square statistic, with 41 degrees of freedom, was 104.30. The goodness of fit index was .941, while the adjusted goodness of fit index was .904. These fit indices suggest that the hypothesized second-order model is plausible. The reader will also note that these are exactly the same fit statistics for the first-order model with correlated factors, meaning the data fit both the first and second-order models equally well. Second-order CFA expresses the first-order model used here in a different structural model, but the CFA second-order model had no more (or less) ability to reproduce the measured variables than did the first-order model used here.

Since the overall fit of the second-order model was acceptable, the parameter estimates can be evaluated for further interpretation. As in the previous example, the standard errors of the estimates were first consulted. Most of the standard errors for the Lambda-Y estimates ranged from .052 to .074 for the Verbal and Memory factors. However, the standard error for the Lambda-Y estimates of the Speed factor suggested a possible problem for interpretation. The parameter estimates (ranging from .249 to .290) were almost of the same magnitude as their standard errors (ranging from .228 to .265). The same problem occurred for the second-order pattern coefficients (Gamma estimates). Here, too, the estimates for the Verbal and Memory factors were several times as large as their standard error (.115 and .164, respectively) while the estimate for the Speed factor was actually smaller than its standard error (2.499). These findings further suggest a problem with the Speed factor in the second-order model. Possible sources of this problem will be discussed later.

Since it is important to use the standardized solutions for interpreting higher-order CFA models, the standardized solution for the present example is presented in Table 4. The only difference between the standardized and unstandardized solution is that the variables are standardized before parameter estimations to obtain the standardized solution. Although the estimates for the Speed factor appear more stable for the standardized solution, this is actually misleading because the standard errors of the estimates also change for the standardized solution, preserving the same ratio of parameter estimate to standard error of the estimate.

INSERT TABLE 9 ABOUT HERE

Given the problem with the Speed factor in the second-order model, a possible explanation is warranted. One interpretation of the parameter estimates is that Verbal is a very strong first-order factor, while the Speed and Memory factors are not quite as strong, as evidenced by their lower standardized Lambda-Y estimates. However, based on inspection of the standard errors, it is really hard to interpret Speed as even a coherent factor. Speed appears to be more or less dominating the second-order factor, but again, there is almost a one to one ratio between its Gamma estimate and its standard error. Based on this, the best explanation would probably be some kind of model specification error. Specifically, it may be the case that the Speed factor should not contribute to the second-order factor. This model could be tested by fixing the Gamma estimates only for Speed to zero. This model would also be theoretically plausible given the traditional concurrence among intelligence theorists that speed is not nearly as related to general intelligence as other factors such as memory and verbal ability. Such a model would say that speed exists at the first-order but not at the second-order level.

Summary

In summary, second-order factor analysis is a useful, if not necesary, aid to the interpretaton of factor structure when first-order factors are correlated. An EFA approach is most useful when the researcher as no a priori hypotheses about the expected relationships, while CFA is a powerful tool for testing specific hypotheses. Ultimately, second-order analyses allow for a more complex view of the often highly complex reality under study in the social sciences.

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TABLE 1
Listing of Selected Ability Tests from the Holzinger and Swineford (1939) Data

Variable Label	Ability Test
V5	General Information - Verbal
V6	Paragraph Comprehension
V7	Sentence Completion
V9	Word Meaning
V10	Speeded Addition
V11	Speeded Coding (code shapes with letter)
V12	Speeded Counting of Dots
V13	Speeded Discrimination of Letters
V14	Memory of Target Words
V15	Memory of Target Numbers
V17	Memory of Object-Number Association

TABLE 2
Promax Rotated Factor Pattern Matrix

Variable	Verbal	Speed	Memory	h^2
V5	**.905**	-.006	-.044	82.1%
v6	**.888**	-.013	.030	79.0%
v7	**.861**	-.009	.086	74.9%
v9	**.876**	.039	-.073	77.4%
v10	-.075	**.780**	.022	61.5%
v11	-.069	**.838**	-.125	72.3%
v12	.072	**.715**	-.053	51.9%
v13	.183	**.651**	.135	47.6%
v14	.118	-.111	**.808**	67.9%
V15	-.087	-.070	**.801**	65.4%
V17	-.077	.312	**.608**	47.3%
Trace	3.19	2.36	1.72	7.27

TABLE 3
First-Order Factor Structure Matrix

Variable	Verbal	Speed	Memory
V5	.896	.230	.095
V6	.889	.237	.164
V7	.872	.248	.217
V9	.875	.260	.073
V10	.141	.765	.213
V11	.140	.787	.083
V12	.259	.721	.144
V13	.382	.736	.333
V14	.212	.132	.798
V15	.018	.115	.769
V17	.102	.449	.677

TABLE 4
Second-Order Factor
Pattern Matrix

Factor	General
Verbal	.670
Memory	.765
Speed	.655
Trace	1.460

TABLE 5
Second-Order Product Matrix

Variable	General	h^2
V5	0.574	32.9%
v6	0.605	36.6%
v7	0.626	39.2%
v9	0.570	32.4%
v10	0.560	31.3%
v11	0.513	26.3%
v12	0.560	31.4%
v13	0.709	50.2%
v14	0.523	27.4%
v15	0.413	17.1%
V17	0.585	34.2%

TABLE 6
Schmid-Leiman Orthogonalized Solution

Variable	General	Verbal	Speed	Memory	h^2
V5	.574	**.671**	-.004	-.033	78.1%
V6	.605	**.659**	-.008	.023	80.1%
V7	.626	**.639**	-.006	.065	80.5%
V9	.570	**.650**	.025	-.055	75.1%
V10	.560	-.056	**.503**	.016	56.9%
V11	.513	-.051	**.540**	-.094	56.6%
V12	.560	.054	**.461**	-.040	53.1%
V13	.709	.136	**.419**	.102	70.7%
V14	.523	.087	-.071	**.611**	66.0%
V15	.413	-.064	-.045	**.605**	54.3%
V17	.585	-.057	.201	**.459**	59.6%
Trace	3.59	1.76	.98	.98	7.31

Note. The column after the orthogonalized matrix presents the sum of the squared entries in a given row. The first 1 column represents the second order factors. The next 3 columns represent the first order solution, based on variance orthogonal to the second order (Gorsuch, 1983, pp. 248-254).

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TABLE 7
Confirmatory First-Order Solution
(n = 301, v = 11)

LAMBDA Y Variable	VERBAL	SPEED	MEMORY
V7	.859	.000	.000
V9	.858	.000	.000
V6	.825	.000	.000
V5	.834	.000	.000
V10	.000	.626	.000
V12	.000	.627	.000
V13	.000	.636	.000
V11	.000	.731	.000
V14	.000	.000	.604
V15	.000	.000	.527
V17	.000	.000	.637

PSI

	VERBAL	SPEED	MEMORY
	1.000		
SPEED	.381	1.000	
MEMORY	.211	.466	1.000

Note: Entries of ".000" were all fixed to be zeroes; entries of "1.000" were all fixed to be ones; all other results were maximum-likelihood estimates that were considered "free" in the model fit to the data.

TABLE 8
Confirmatory Second-Order Solution (Unstandardized)
(n = 301, v = 11)

LAMBDA Y Variable	VERBAL	SPEED	MEMORY
V7	.781	.000	.000
V9	.780	.000	.000
V6	.750	.000	.000
V5	.759	.000	.000
V10	.000	.249	.000
V12	.000	.249	.000
V13	.000	.252	.000
V11	.000	.290	.000
V14	.000	.000	.521
V15	.000	.000	.454
V17	.000	.000	.549
PSI			
	VERBAL	SPEED	MEMORY
VERBAL	1.000		
SPEED	.000	1.000	
MEMORY	.000	.000	1.000
GAMMA			
	GENERAL		
VERBAL	.457		
SPEED	2.311		
MEMORY	.589		

Note: Entries of ".000" were all fixed to be zeroes; entries of "1.000" were all fixed to be ones; all other results were maximum-likelihood estimates that were considered "free" in the model fit to the data.

TABLE 9
Confirmatory Second-Order Solution (Standardized)
(n = 301, v = 11)

LAMBDA Y Variable	VERBAL	SPEED	MEMORY
V7	.859	.000	.000
V9	.858	.000	.000
V6	.825	.000	.000
V5	.834	.000	.000
V10	.000	.626	.000
V12	.000	.627	.000
V13	.000	.636	.000
V11	.000	.731	.000
V14	.000	.000	.604
V15	.000	.000	.527
V17	.000	.000	.637
PSI			
	VERBAL	SPEED	MEMORY
VERBAL	1.000		
SPEED	.000	1.000	
MEMORY	.000	.000	1.000
GAMMA			
	GENERAL		
VERBAL	.415		
SPEED	.918		
MEMORY	.508		

Note: Entries of ".000" were all fixed to be zeroes; entries of "1.000" were all fixed to be ones; all other results were maximum-likelihood estimates that were considered "free" in the model fit to the data.

APPENDIX

Correlation Matrix of Selected Ability Tests from the Holzinger and Swineford (1939) Data

	V7	V9	V6	V5	V10	V12	V13	V11	V14	V15
V9	0.7199800	1.0000000								
V6	0.7331900	0.7044920	1.0000000							
V5	0.7160260	0.7393030	0.6571840	1.0000000						
V10	0.1020490	0.1211050	0.1738330	0.1745920	1.0000000					
V12	0.1386750	0.1496160	0.1069010	0.1648580	0.4867740	1.0000000				
V13	0.2274740	0.2141670	0.2078520	0.2052470	0.3406550	0.4490280	1.0000000			
V11	0.2997760	0.2902510	0.3415890	0.3161250	0.4472790	0.3977150	0.4776630	1.0000000		
V14	0.1535580	0.1720540	0.2224220	0.1203480	0.0929304	0.0385054	0.1360860	0.2248100	1.0000000	
V15	-0.0188893	0.0519092	0.0694560	0.0264116	0.1089590	0.0778617	0.0720519	0.1400420	0.3967490	1.0000000
V17	0.0923058	0.1455430	0.1452770	0.0349765	0.3309080	0.2299290	0.1981170	0.3443500	0.3546960	0.3052400

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