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ABSTRACT

This project supported the creation of a computer-based interactive text for linear algebra using guided discovery in a laboratory-based course which emphasized active learning, collaborative learning, and the use of writing. This pedagogical approach had as its goal improved student understanding and retention of the concepts and methods of linear algebra, improved student confidence in using linear algebra in a variety of disciplines, and enhanced student ability to read and write about mathematics. The project report provides an overview of the project, a section that traces the project from problem definition to project conclusion and discusses lessons learned, details on the background and origins of the project categorized by phases, student and faculty reactions to the project, evaluations of the product by its reviewers. An appendix contains a "Mathcad Electronic Book" on a 3.5 computer diskette. (DDR)

HE

Cover Sheet

Grantee Organization:

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209 S 33rd Street
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Grant Number:

P116B20642-94

Project Dates:

Starting Date: August 7, 1992
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Number of Months: 36

Project Director:

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FIPSE Program Officer: Brian Lekander

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	Year 1:	$94,357
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	Year 3:	$5,579
	Total	$142,844

FINAL REPORT - FIPSE GRANT P116B20642-94

An Interactive Text for Linear Algebra

SUMMARIES:

This project supported the creation of a computer based interactive text for linear algebra using guided discovery in a laboratory based course emphasizing active learning, collaborative learning and the use of writing. This pedagogical approach had as its goal improved student understanding and retention of the concepts and methods of linear algebra, improved student confidence in using linear algebra in a variety of disciplines, and enhanced student ability to read and write about mathematics. The resulting electronic text together with a print version is being published under the title: Interactive Linear Algebra in Mathcad by Springer-Verlag and should be available early in 1996.

Project Director:

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Product:

Interactive Linear Algebra in Mathcad
Gerald J. Porter and David R. Hill
Springer-Verlag, Inc.
ISBN 0-387-94608-X

FINAL REPORT - FIPSE GRANT P116B20642-94
An Interactive Text for Linear Algebra
Executive Summary

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Project Director:
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A. Project Overview: The project funded Gerald J. Porter and David R. Hill to author and class-test a computer based interactive text for Linear Algebra.

B: Purpose: The proximal goal was, of course, to author the text; the ultimate goal is to improve student learning. In particular we believe that the use of the text can improve student

- understanding of the concepts of linear algebra,
- long term retention of the ideas and problem solving strategies of linear algebra,
- ability to apply linear algebra to problems in other disciplines,
- confidence in their own ability to read and use mathematics,
- realization that mathematics is an experimental science,
- knowledge about the use of technology to solve linear algebra problems,
- ability to write about mathematics.

C. Background and Origins: In recent years there has developed wide agreement among mathematics educators that the traditional lecture approach to teaching mathematics is not working. Students do not achieve mastery of the subject matter and retention of the material that is learned rarely lasts beyond the final examination. We know from our experience that lasting knowledge comes not from extraordinary teachers but from an active approach in which the student learns by doing. Implicit in this change is the empowerment of the student to control his or her learning environment.

Today's high powered personal computers have made possible a generation of computer algebra systems with symbolic, numerical, and graphical capacity formerly available only on high priced mainframes. Using these computer algebra systems one can create interactive mathematics texts that facilitate student explorations while freeing the student from the drudgery of hand computations. This presented us with the opportunity to create a computer based interactive linear algebra text with which we could teach linear algebra as a laboratory course with a focus on active learning, collaborative learning, and the use of writing.

D. Project Description: We proposed to achieve the goals listed above by authoring an Interactive Linear Algebra Text and developing a laboratory centered linear algebra course. We chose to do this in Mathcad, a commercially available mathematics-engineering computational computer environment. We view Mathcad as a smart electronic blackboard because it displays mathematical expressions the way they would appear in a `standard' mathematics text or as an instructor might write as part of a lecture, it provides numerical, symbolic, and graphical tools for the student to use to explore the mathematics, and it allows the student to annotate the text fully.

The student personalizes the text by their explorations and reports. This process facilitates the transfer of ownership of the text to the student. With ownership of the text the student also acquires ownership of the concepts in the text.

An integral component of the authoring process was the fact that we would use the draft versions of the text in our courses at Penn and Temple. This allowed us to estimate our success in achieving the goals articulated above and to receive feedback from our students on the strengths and weaknesses of the text "in process."

E. Project Results: As we write this final report in October 1995 the final version of the text is 98% complete. It exists as an Mathcad Electronic Book (included as an appendix to this report) and as a printed workbook that will accompany the electronic version. We have a contract with Springer-Verlag for publication and we anticipate that the text will be commercially available by January 1996.

We have each taught linear algebra as a laboratory course five or six times during the past three years using these materials or their precursors. In addition the text was used by David Smith at Duke University during the 1995 spring semester. A review of the text by Smith is included as an appendix to this report.

F. Lessons Learned:

For the students: In our course, students must learn that they are responsible for their own learning. This means that they

- have to actively engage the material. It is no longer possible simply to master templates and use them on exams. As one student put it, "plug and chug" doesn't work in this course. In particular, this requires the ability to read mathematics.
- must manage their time effectively. Discovery based learning takes longer than passive learning. Students need to spend about twelve hours per week (including class time) on the course. And this must be done every week. The time demand is too great to be made up if one falls significantly behind. Some students have time constraints that preclude them from taking this course.
- must develop time management skills that facilitate group learning.
- must develop verbal skills about mathematics that allow them to work in a group to solve problems.
- must know when it is appropriate to appeal for help from an outside authority (the instructor in this case).
- must realize that not every problem has a unique or even best answer. There may be a range of answers from which to choose.
- need to develop writing skills for mathematics. As students enter the work force they will need to master technical writing in order to communicate effectively.
- must learn to develop computer skills on a "need to know" basis.
- have to develop the confidence and ability to confront problems that they do not know how to solve and to investigate the problem through numerical experimentation. These skills are essential in the workplace.

For the instructor: The role of the instructor has also changed as learning has become student centered rather than instructor centered. In this new role the instructor should

- orient the students to the new learning environment. The instructor should discuss the rationale for the changed environment and the ways that the changes can improve student learning and prepare the student for the workplace.
- provide the course structure. It is essential that work be scheduled at the start of the term and that the students understand when work and examinations are due.
- carefully explain the reward structure (grades).
- provide timely feedback for the students.
- provide a time away from the computers to pull together the material and straighten out misconceptions. There are several ways that this can be done.

- interrupt the lab, when it is obvious that several students are encountering a similar difficulty, and have a brief discussion of the material causing the problem.
- be available by email to answer questions.
- be prepared to accept the fact that questions will arise for which he or she doesn't know the answer. Learning should not be restricted to the students.
- be knowledgeable but not necessarily expert about the use of the computer software. Technology can be enabling and it can be inhibiting. Certain common errors repeat themselves and the instructor needs to be aware of these.
- be prepared for student unhappiness. This is particularly true in environments where the students have been successful in traditional modes of learning. Don't get discouraged.

For the authors: Authoring an interactive text is a time consuming and intellectually challenging activity. Potential authors should be aware that

- collaboration is as valuable for the authors as it is for the learners and for the same reasons. Work with one or more partners.
- you are writing for students not for your colleagues.
- students may not develop concepts that are as deep and focused as ours.
- students need practice in making relationships and seeing the forest rather than the trees.
- if a section doesn't "work" it may be more effective to start over than to repair the current version.
- it is necessary to obtain student feedback on a regular basis.
- everything takes at least twice as much time as you anticipated.
- it is easier to prepare templates for student to use than to create materials for guided discovery learning. Ask don't tell. This is harder than it sounds.
- each section should have a goal . Try to stick to that goal.
- the students may not understand the instructions that you believed were the apogee of lucidity.
- it is necessary to test the material on different types of students.
- a rich authoring environment should provide both the mathematics and the word processing required for manuscript preparation.
- what the student sees on the screen is the most important aspect of the environment.
- the primary goal of the text is to help students learn mathematics not computing.
- one or two problems are usually all that is necessary for a student to retain a concept. "Drill and kill" is not required.
- it is difficult to read very much on a computer screen.
- it should be clear when an answer is required.
- the students needs constant feedback IN THE TEXT to ensure that they have not gone awry.
- modern technology and word processing have greatly increased the opportunities to prepare truly ugly materials.
- white space is cheap and should be utilized.
- pictures should be used frequently by both the authors and the students.
- a printed copy of the text allows students to prepare for class away from a computer, write notes on the material, and find material more quickly than they can on line.
- no matter how many typos you have corrected, you missed the most obvious one.
- just as you complete the final version of the text a new version of the software will appear.

FINAL REPORT - FIPSE GRANT P116B20642-94

An Interactive Text for Linear Algebra

Grantee:
Department of Mathematics
University of Pennsylvania
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Project Director:
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A. Project Overview:

The project funded Gerald J. Porter and Daivd R. Hill to author and class-test a computer based interactive text for Linear Algebra.

B: Purpose:

The goal was to improve student

- understanding of the concepts of linear algebra,
- long term retention of the ideas and problem solving strategies of linear algebra,
- ability to apply linear algebra to problems in other disciplines,
- confidence in their own ability to read and use mathematics,
- realization that mathematics is an experimental science,
- knowledge about the use of technology to solve linear algebra problems,
- ability to write about mathematics.

These goals were consistent with the following report of the Linear Algebra Curriculum Study Group

> *"In recent years, demand for linear algebra training has risen in client disciplines such as engineering, computer science, operations research, economics and statistics. At the same time, hardware and software improvements in computer science have raised the power of linear algebra to solve problems that are orders of magnitude greater than dreamed possible a few decades ago. Yet it appears that in many courses, the importance of linear algebra in applied fields is not communicated to students and the influence of the computer is not felt in the classroom, in the selection of topics covered or in the mode of presentation. Furthermore, a worthwhile but sometimes overemphasis on abstraction may overwhelm beginning students to the point where they leave the course with little understanding or mastery of the basic concepts they will actually use in their careers."*
>
> *We believe that student use of computers . . . can reinforce concepts from lectures, contribute to the discovery of new concepts and make feasible the solution of realistic applied problems.*[1]

[1] Preliminary Report of the Linear Algebra Curriculum Study Group, January 17, 1991

C. Background and Origins:

In recent years there has developed wide agreement among mathematics educators that the traditional lecture approach to teaching mathematics is not working. Students do not achieve mastery of the subject matter and retention of the material that is learned rarely lasts beyond the final examination. We know from our experience that mathematics is not a spectator sport and that lasting knowledge comes not from extraordinary teachers but from an active approach in which the student learns by doing. Implicit in this change is the empowerment of the student to control his or her learning environment.

Lynn Steen described this phenomenon clearly in an article in Change magazine. He wrote:

> *"Learning takes place when students construct their own representation of knowledge. Facts and formulas will not become part of deep intuition if they are only committed to memory. They must be explored, used, revised, tested, modified, and finally accepted through a process of active investigation, argument and participation. Science (and mathematics) instruction that does not provide these types of activities rarely achieves its objectives."*[2]

To achieve our goals we chose to create a computer based interactive linear algebra text that would enable us to teach linear algebra as a laboratory course with a focus on

- active learning
- collaborative learning, and
- the use of writing.

Interactive texts provide an environment in which students can fruitfully engage and explore mathematics. An interactive text is a computer document from which symbolic, numerical, and graphic tools can be invoked. The results of these computations can be pasted into the document so that each learner has an individual record of his or her explorations. By using interactive texts students can easily explore mathematical concepts through numerical and graphical exercises without the drudgery required by hand computations.

Interactive texts are used in a setting which encourages collaborative learning. The interplay of ideas and the give and take of intellectual discussion are an important component of learning.

Writing is an important component in the use of interactive texts. The notebook interface not only permits the author to write but also encourages the student to make notes in his or her copy of the text and to include written comments and explanations with

[2] Steen, Lynn A. , Reaching for Science Literacy, *Change*, Vol.23, No. 4, July/August 1991, p. 13

assignments. Students must be able to communicate the results of their investigations to others.

D. **Project Description**:

We proposed to achieve the goals listed above by authoring an Interactive Linear Algebra Text and through the use of this text, to develop a laboratory centered linear algebra course. We chose to do this in Mathcad. Mathcad is a commercially available mathematics-engineering computational computer environment. We view Mathcad as a smart electronic blackboard because

- it displays mathematical expressions the way they would appear in a `standard' mathematics text or as an instructor might write as part of a lecture,
- it provides numerical, symbolic, and graphical tools for the student to use to explore the mathematics,
- it allows the student to write mathematics in the text.

The student personalizes the text by their explorations and reports. This process facilitates the transfer of ownership of the text to the student. With ownership of the text the student also acquires ownership of the concepts in the text.

An integral component of the authoring process was the fact that we would use the draft versions of the text in our courses at Penn and Temple. This would allow us to estimate our success in achieving the goals articulated above and to receive feedback from our students on the strengths and weaknesses of the text "in process."

Our original proposal anticipated that we would have an interactive text available for pretesting by the end of the summer of 1993 and a final version by January 1, 1994. We were, of course, much too optimistic. The amount of time needed to create the text was at least twice the amount we estimated and an additional four months were required to prepare the texts (paper and electronic) for publication.

E. **Project Results**:

As we write this final report in October 1995 the final version of the text is 98% complete. It exists as an Mathad Electronic Book (included as an appendix to this report) and as a printed workbook that will accompany the electronic version. We have a contract with Springer-Verlag for publication and we anticipate that the text will be commercially available by January 1996 only two years beyond our initial target date.

We have each taught linear algebra as a laboratory course five or six times during the past three years using these materials or their precursors. In addition the text was used by David Smith at Duke during the 1995 spring semester. A review of the text by Smith is included as an appendix to this report.

We have had three visits by teams of outside evaluators visit and meet with the students using the texts. We include their reports in the Appendix. Equally important were the comments of our students. One of us (Porter) includes a question on each term's final exam asking the students to write an essay on "what linear algebra is" or "give advise to a friend who is thinking about taking the course." We include a sample of these comments, sometimes edited, in the Appendix as well. We should point out that Porter's students are primarily engineering or finance majors.

F. Lessons Learned:

For the students:

In our course, students must learn that they are responsible for their own learning. This means that they

- have to actively engage the material. It is no longer possible simply to master templates and use them on exams. As one student put it, "plug and chug" doesn't work in this course. In particular, this requires the ability to read mathematics. Too often in introductory level mathematics courses the text is redundant and is used primarily as a catalog of templates and a source of problems.

- must manage their time effectively. Discovery based learning takes longer than passive learning. Students need to spend about twelve hours per week (including class time) on the course. And this must be done every week. The time demand is too great to be made up if one falls significantly behind. Some students have time constraints that preclude them from taking this course. A student who participates in extracurricular activities that demand twenty hours a week can not take this course and still maintain the same level of extracurricular activities. Athletes should take the course during their off-season.

- must develop time management skills that facilitate group learning. We have encountered students whose idea of collaborative learning is "You do this section and I'll do the next." for the most part this doesn't work. Occasionally we have encountered partners who were unable to find common time for group work. Unless these students were able to join other groups they were unsuccessful in this course.

- must develop verbal skills that allow them to work in a group to solve problems. This includes the ability to verbalize incomplete thoughts and to talk about problem solving techniques.

- must know when it is appropriate to appeal for help from an outside authority (the instructor in this case).

- must realize that not every problem has a unique or even best answer. There may be a range of answers from which to choose. This is true, for example, in choosing "best-fit" curves.

- need to develop writing skills for mathematics. Most mathematics homework assignments and examinations are computation intensive and verbally minimal. As students enter the work force they will need to master technical writing in order to communicate effectively with their colleagues.

- must learn to develop computer skills on a "need to know" basis. Our text introduces the features of Mathcad as they are required and encourages the student to use help files as needed.

- have to develop the confidence and ability to confront problems that they do not know how to solve and to investigate the problem through numerical experimentation. These skills are essential in the workplace.

For the instructor:

The role of the instructor has also changed as learning has become student centered rather than instructor centered. In this new role the instructor should

- orient the students to the new learning environment. In particular, the instructor should discuss most of the topics listed in the previous section. The instructor should discuss the rationale for the changed environment and the ways that the changes can improve student learning and prepare the student for the workplace.

- provide the course structure. It is essential that work be scheduled at the start of the term and that the students understand when work is due and when the examinations (whatever form they take) are. Effective time management is an essential requirement for the success of the course and this must be emphasized. One way of ensuring that work is handed in on time is to post (electronically) answers within a few hours of the due date. Students understand that late homework has no value if the answers have been posted.

- carefully explain the reward structure (grades). Appropriate weight must be assigned to the weekly (group) assignments to underline the importance of that work. Homework counts for 25% of the final grade in our courses.

- provide timely feedback for the students.

  In class, this means that the instructor should walk around the lab looking at what the students are doing on the screen and what they are writing in their books. If something looks too far off pose a question that relates to the item. Probe a bit to see why their response was 'off'. It may be that they are on a tangent that requires a bit of discussion to rectify. Try not to do this in a 'telling' mode. Use a question/response/example format that flows with the text style.

  Homework must be promptly graded and returned. The next assignment depends upon an understanding of the previous work. When necessary student attention should be directed to the posted answers.

- provide a time away from the computers to pull together the material and straighten out misconceptions. There are several ways that this can be done.

  Schedule a time each week to go over topics and discuss main ideas. This can be required or optional. It may be the last part of a lab or scheduled independently. Use a series of questions to engage the students rather than say 'who has questions' or 'what troubled you in the last assignment'.

  Every few weeks conduct a 'relationship' question and example session as part of an instructional period. By following their progress in the weekly assignments the instructor should be able to sense when the group needs such guidance.

  Give a self-quiz weekly that is designed to probe understanding of the concepts rather than computational aspects of the topics. This should be tuned to the weaknesses seen in the homework.

- interrupt the lab, when it is obvious that several students are encountering a similar difficulty, and have a brief discussion of the material causing the problem.

- be available by email to answer questions. A great deal of student work takes place between 10 PM and 3 AM. It is often a good idea to check for questions last thing at night and first thing in the morning. A simple misunderstanding, that can be easily corrected, may adversely affect student work. This is particularly true if the students are working on take-home exams.

- be prepared to accept the fact that questions will arise for which he or she doesn't know the answer. Learning should not be restricted to the students. The instructor should be prepared to learn from the students and from questions raised by the students.

- be knowledgeable but not necessarily expert about the use of the computer software. Technology can be enabling and it can be inhibiting. Certain common errors repeat themselves and the instructor needs to be aware of these. In other cases the instructor may have to examine the difficulty in more detail to deduce the cause of the problem. It can be very frustrating for the student who continually encounters computer problems. Remind the students to save their work often. A network crash toward the end of a class can wipe out the day's work and negate a positive learning environment.

- be prepared for student unhappiness. This is particularly true in environments where the students have been successful in traditional modes of learning. "Just tell us the methods and let us solve the problems." Positive reinforcement is needed in these cases. Don't get discouraged. One Duke student wrote on the evaluation and we paraphrase: "The text was disorganized, this was much more work than the standard course and I would never do it again. By the way, I learned a lot." Many students are risk adverse and this course is threatening. If a student is terribly unhappy about the course early on in the semester, you may do the student and yourself a favor by making it possible for the student to transfer to a more traditional course even though you ***know*** the student will learn more in this course.

For the authors:

Authoring an interactive text is a time consuming and intellectually challenging activity. Potential authors should be aware that

- collaboration is as valuable for the authors as it is for the learners and for the same reasons. Work with one or more partners.
- you are writing for students not for your colleagues. Write in an informal style and go slowly so that students see the step-wise development of ideas.
- students may not develop concepts that are as deep and focused as ours.
- students need to practice in making relationships and seeing the forest rather than the trees.
- if a section doesn't "work" it may be more effective to start over than to repair the current version.
- it is necessary to obtain student feedback on a regular basis. Ask students to rephrase portions they had particular difficulty with. Listen carefully to their suggestions and coordinate what they say with their homework performance to see where the 'rough' spots are.
- everything takes at least twice as much time as you anticipated.
- it is easier to prepare templates for student to use than to create materials for guided discovery learning. Ask don't tell. This is harder than it sounds. Divide the problem into small steps and provide help along the way.
- each section should have a goal . Try to stick to that goal but revise it when needed. Keep the goals short and connected or organize the material into multiple sections.
- the students may not understand the instructions that you believed were the apogee of lucidity.
- it is necessary to test the material on different types of students. Get opinions from colleagues who teach at other schools and when possible have them try the materials with their students.
- a rich authoring environment should provide both the mathematics and the word processing required for manuscript preparation.
- what the student sees on the screen is the most important aspect of the environment.
- the primary goal of the text is to help students learn mathematics not computing.
- one or two problems are usually all that is necessary for a student to retain a concept. "Drill and kill" is not required.
- it is difficult to read very much on a computer screen. Explanations or instructions should be short and there should be many opportunities for the reader to interact with the text.
- it should be clear when an answer is required. One way to do this is to have the word "ANSWER" follow each question and precede white space where the answer should be entered.

- the students needs constant feedback IN THE TEXT to ensure that they have not gone awry. One way to do this is by regularly providing mini-summaries of the active work, possibly in disguise. These can be of the form: "If you did the previous exercise correctly your answer should be ..." or "You should have discovered that ... ."
- modern technology and word processing have greatly increased the opportunities to prepare truly ugly materials. There is no need to use five fonts in a single paragraph.
- white space is cheap and should be utilized.
- pictures should be used frequently by both the authors and the students.
- a printed copy of the text allows students to prepare for class away from a computer, write notes on the material, and find material more quickly than they can on line. It takes much less time to turn to a section in a book than it does to open a file.
- no matter how many typos you have corrected, you missed the most obvious one.
- just as you complete the final version of the text a new version of the software will appear.

G. Appendices

1. Reprint of Learning Software column from May 1995 UME Trends
   (review of Interactive Linear Algebra in Mathcad)

2. Reports by outside evaluators

   a: April 1995
   Alan Tucker
   Nancy Baxter-Hastings
   David Smith

   b: March 1994
   Alan Tucker
   Eugene Herman
   Nancy Baxter

   c: April 1993
   Horacio Porta
   Alan Tucker
   Eugene Herman

3. Student essays from final exams at Penn.

4. **Interactive Linear Algebra in Mathcad**
   the electronic version of the text is enclosed on a disk.

Learning Software

Edited by David A. Smith

Interactive Linear Algebra

As I write, I am half-way through my first experience teaching a course from a fully interactive textbook: *Interactive Linear Algebra in Mathcad*, by Gerald J. Porter and David R. Hill. In spite of my experience using interactive laboratory materials in a variety of courses, I wasn't fully prepared for student reaction to this course. The concept of "interactive text" will be bandied about in coming years, and I want to share some early observations.

What's an interactive mathematics text? At a minimum, it is material for discovery-based active learning (ranging from a single lab to an entire course) that is delivered by computer. It must enable the student (or team of students) to change examples *ad infinitum* and to use the computer for visualization and for numerical and symbolic computation. Ideally, it should have hypertext capabilities that allow different routes through the material, possibly including some not anticipated by the authors. Students must be actively engaged in *doing*, not just reading, observing, turning pages. And the *sine qua non* is that students must participate in actually *writing* text.

We are already seeing the first trickles of an anticipated flood of CD ROM-based texts called "interactive"—most of which do not fit the definition in the preceding paragraph. In fact, in addition to the text under review, I am aware of only one other for a full lower-division mathematics course: *Calculus & Mathematica*—see my column in *UME Trends* January, 1995. Both *C&M* and *ILAM* are quite linear in structure. *Mathematica* doesn't have real hypertext capabilities, and the hypertext features now available in *Mathcad* have not been fully exploited in the Porter-Hill text. But the authors of both texts fully expect students to become co-authors—and that expectation has a dramatic impact that I didn't fully appreciate until this semester.

Let's start with the book itself. It has a brief Chapter 0 that introduces features of *Mathcad* and its front end to *Maple*, then lays out three models that are used as examples throughout the text: an input-output model for oil refinery products, a Markov chain model for weather prediction, and a Leontief model for a simple economy. The authors credit Alan Tucker both for the models and the idea of threading them through almost every section. The subsequent chapters have familiar titles:

1. Vectors and Matrices
2. Systems of Linear Equations and their Solution Sets
3. Determinants and their Applications
4. Lines and Planes
5. Applications of the Dot Product
6. Eigenvalues and their Applications
7. Linear Transformations
8. Vector Spaces

The section titles within these chapters are equally bland and familiar, but the titles hide a richness of interacting concepts and applications that is certainly unusual, if not unique. For example, early introduction of the Markov chain example leads—in Chapter 1—to the concepts of n-th powers approaching a limit as $n \to \infty$ and of eigenvectors. The first two chapters—which comprise about 40% of the course—include such concepts as correlation coefficient (dot product), growth rate of the number of n-paths in a graph (matrix powers), traffic flow on city streets (convex combinations of solutions), the diet problem (linear programming), and linear regression (normal equations). None of this is daunting, because students have access to sufficient computing power to handle all the little details, including symbolic ones.

The strategy of the book is simple. Students work through carefully constructed activities (e.g., moving the hands of a clock by applying certain matrices) that lead them to formulate concepts in their own words. Within a screen or two, the same concepts are summarized and the standard terminology is explained. At the end of each section, students write a paragraph or two to summarize the mathematics learned in that section.

My class meets in a computer lab for two hour-and-a-quarter periods per week, not enough time to work through the text. Students need another 6 to 8 hours on their own time for the same kind of work they do in class. They work in teams of two, which I assigned on the basis of their schedules to facilitate teamwork outside of class. When they work alone, they are expected to share their work with their partners as soon as practicable.

At first, all the students were apprehensive about whether they could operate in a mode so different from their previous experience with mathematics courses. (Only one student came from our laboratory calculus program—the rest avoided it by advanced placement.) They were afraid they wouldn't learn anything if I wasn't telling them the "right stuff." Within a week or two, they started getting annoyed if I interrupted them during class to say anything to the whole group—even when it was about a mistake in the final exam schedule that affected them.

Almost right away, I was hearing students say—to each other as well as to me—"Hey, I really understand this stuff!" My own observation of their conversations and of the questions they asked was that they were understanding concepts better and earlier than any previous linear algebra class I had ever seen. To the question "How are we going to study for a test?" I could only answer "You're doing it." They didn't believe that, but it turned out to be true. On my first test—a week-long take-home with questions they had never seen before—there was a near-perfect correlation between steady, active participation and B-or-better success.

Please see Learning Software on page 10

Learning Software from page 8

My syllabus sets out a pace of about four sections of text per week. Each team answers essentially all the questions and exercises in every section and submits their work by saving "annotated book" files on our network server. I respond with my own annotations, plus a weekly letter grade. In addition, I place filled-in copies of the book files in a public directory so they can look up anything they miss. That saves me response time—for common problems, I use a boiler-plate reference to the public book. Tests (in weeks 6 and 12 of a 14-week semester, plus a final exam) work the same way, except everyone works alone—and they have the option of doing some or all of the test with other technologies, including paper. The final grade will be 45% tests and final exam, 55% team submissions.

Is this the wave of our future? Maybe, maybe not. I have only ten students—an appropriate number for an experiment. It takes me at least an hour a week per team to respond to their work and several more hours to prepare the public book. (The latter is my class preparation time as well.) After the first time through, I will be able to concentrate more on student submissions and thereby handle more teams. I'm learning some shortcuts that might make it possible to take on a class of 30—but it's a daunting prospect. I averaged about an hour per test paper—some of which ran to 15 pages when printed.

Several questions on the first test amounted to solving systems of equations, but they had to interpret what was being asked to figure out the right system. I asked them to generate square matrices with random entries and keep track of how many were invertible—then explain what they observed. I asked if CBC^T must be symmetric when B is symmetric—if yes, prove it; if no, give an explicit counterexample. I asked them to advise a car rental agency on the number of parking spaces needed at each of its locations, given transitional probabilities from rental location to return location. Most of my questions were ones I would previously have asked much later in the course and in a less open-ended way.

Interactive Linear Algebra in Mathcad will be published commercially, but is not yet under contract. For more information on acquiring and site testing, contact Jerry Porter at University of Pennsylvania or Dave Hill at Temple University. The book is distributed on a single compressed floppy disk and requires Mathcad 5.0 or Plus 5.0. You will also need a master paper copy and permission to make copies available to students—my students all insisted on buying a printed copy, even though I made that optional.

Reader responses and comments are welcome, as are suggestions or contributions for future columns. Contact David A. Smith, Department of Mathematics, Duke University, Box 90320, Durham, NC 27708-00230, email: das@math.duke.edu

Interactive Linear Algebra Text

Advisory Group Report on Visit of March 14, 1995, to Temple University and University of Pennsylvania.

Committee members: Nancy Baxter Hastings, Dickinson College; David Smith, Duke University; Alan Tucker, SUNY-Stony Brook.

The committee visited linear algebra interactive text class/laboratory sessions at both universities. In each class, we spent the first part of the session observing the students while they worked on computer activities. We then held a group discussion with the students for about 30 minutes (with the instructors and lab assistants out of the room). The committee spoke at length with the instructors, David Hill and Gerald Porter, discussing progress we had observed over the three years development period, sharing the observations we had made during this visit and making some final recommendations. During these conversations, we reported orally most of the comments contained in this report.

Overall Observations

Professors Hill and Porter have undertaken ground breaking developmental work that has required an enormous amount of time and energy. The good news is that the project is really coming along nicely. A lot of progress has been made and this is reflected in the reactions of the students.

• Students comfort level with the text is way up. The text and students seem more in synch. Portions of the text that were rough and crude have been smoothed out after several iterations of rewriting. Consequently, an ill-posed question is a rare event and the discovery-based activities are more palatable than before. Moreover, the need for discussion groups has declined as the text has become more refined.

• Student frustration with the amount of time demanded by the course is way down. Students don't perceive the course to have as much busy work as before. Students' resentment about the amount of time demanded by the course way down. They recognize that it's time well spent since it helps them learn, and they are willing to spend the time since they are engaged in the learning process.

• Students know they are learning and feel they are developing a deeper understanding than those in a traditional course. This positive attitude is exemplified by their feelings that they do not need to cram for a test or to have discussion sessions. Students applaud the writing portion of the course as something that helps them think and understand.

Temple University Student Responses

Understanding: Students were unanimous in expressing the feeling that they were actually learning in this class. They claimed they were developing a better understanding of the ideas than traditionally taught students and that the students who had dropped out of the class had done so because they didn't want to spend the time it took to really learn the material.

Time element: Students felt that they were spending a lot more time on the course than they would be spending if they were in a traditional course. Instead of griping, however, about the amount of time the course required, they pointed out that the class forces them to learn since they have to meet deadlines and cannot put things off until just before a test. A few students said they wasted some time trying to figure out what a question wants or how to use the software.

Instructional materials and software: Students felt the material was readable, and that having a hard copy of the material was useful. As one student remarked, the hard copy enables them to look ahead at the exercises before trying to do them on the computer and consequently saves them from having to use class time figuring out what they needed to do. Students who were able to purchase MathCad for their home computers found this to be extremely useful as far as helping them manage the time required by the course. In general, students felt that the software enabled them to do more complicated problems and that it was useful in other courses.

Writing component: At the end of each section students are asked to describe in their own words the concepts they have learned and how they connect with ideas they have studied previously. Students uniformly praised this aspect of the course, claiming that it makes us think; it's helpful to refer back to; it helps us understand ideas better; it helps us be aware of the things that we do not understand before we go on; it makes us verbalize mathematical ideas.

Tests: The students claimed a test doesn't seem like a test, in the traditional sense, but more like just another homework assignment. They said there was less anxiety and less stress. They felt they didn't need to prepare for a test the way they ordinarily would, since they were preparing all along, and the test was just a continuation of the work they were already doing.

University of Pennsylvania Student Responses

The Penn students echoed many of the things the Temple students had said earlier in the day. Their overall attitude was quite positive. The general feeling was that things were going smoothly, and although they were happy to chat with us, they were pleased to get back to work.

Understanding: They felt the focus on how to solve problems, rather than on how to do manipulations by hand, helps them develop a better understanding of the underlying ideas. They liked being able to figure things out for themselves.

Time element: They acknowledged that the course was a lot of work, but added that working in teams helps make the workload bearable. They liked being able to go at their own pace, although they found it was sometimes frustrating to be so much on their own, especially when they encountered an ambiguous question.

Instructional materials: They liked having a hard copy of the text, but felt it would be more helpful if the examples in the text were filled in--that is, if the commands in the examples were executed and the output was included in the hard copy. They expressed a little frustration about the fact that they didn't know if their answers were right until after they were graded. When asked if they found the posted answer sheets useful, they said they usually look at their own solutions first, only using the posted sheets as a last resort.

Tests: They admitted that they spent very little time preparing for exams. Instead they said they simply compared the exam questions to problems they had done previously and then went on from there.

Recommendations

Assessment: The committee encourages the developers to gather at least some attitudinal information from students who have completed the course concerning the impact of the course on their (1) confidence level and (2) development of critical thinking skills.

Help file: The addition of a help file would be very useful. (We realize that this might not be feasible given the amount of time it would take for development.)

Presence of the instructor: Even though the class seems to run by itself and students do not seem to want to be interrupted, the instructor (and the grader for the course, if there is one) need(s) to be in the classroom during scheduled class times, interacting with the students. The instructor serves to point students in the right direction and to minimize frustration. Having the grader/student assistant present enables him or her to hear the kinds of questions students raise as they do the exercises and then to grade in the context of the student experience.

Notes to the instructor: A set of notes to the instructor needs to be written that explicitly states the ground rules for the course and describes the psycho-dynamics and social aspects of the pedagogical approach. For instance, the notes to the instructor should include discussions pertaining to:

- the role of the instructor in the interactive laboratory environment.
- the pedagogical reasons for encouraging students to work in groups.
- the philosophy of testing, the place of examinations in this type of course, and the types of questions that might be used.
- the importance of the end-of-section summaries with regard to student understanding.
- how to deal with the expectation that every student completes every exercise.
- topics which follow naturally from the materials but are not in the text.
- how to handle the fact that completed copies of the materials will be available the second time the course is offered.

In addition, the notes might contain additional exercises for both the weaker students and the stronger students.

While the notes need to be carefully thought out and explicitly stated, it is also important to allow for flexible use of the materials.

Exploration: The text uses a guided-inquiry approach. Evaluators questioned if there might be a way to encourage more exploration on the part of the student.

Dissemination: The time has come for Professors Hill and Porter to launch a comprehensive dissemination program. Their next major task is to get the materials from a small group of believers to a broader audience.

Interactive Linear Algebra Text Project

Advisory Committee Report on Visit of March 21-22, 1994, to Temple University and the University of Pennsylvania.

Committee members: Nancy Baxter, Dickinson College; Eugene Herman, Grinnell College; Alan Tucker, SUNY-Stony Brook. (Baxter replaces Horacio Porta, University of Illinois, who is in ill health.)

The committee visited linear algebra interactive text laboratory sessions at both universities, talking for about 30 minutes with each of the classes. (Instructors and lab assistants were out of the room during this time.) We also talked with the instructors afterwards and reported orally to them most of the comments contained in this report.

General Overview

Professors Hill and Porter have made exceptional progress in developing their interactive text since we last visited them. They have combined the best features of their individual approaches and now have a single version of their interactive text. The first three-plus chapters of the text are complete and quite polished. The students understand quite well the innovative nature of the course and were remarkably insightful, forthright, and voluble in our discussions with them. Although they expressed some concerns, the general attitude they conveyed was that the course forced them to become active learners who are responsible not only for assimilating material but developing much of it as well. They expressed confidence that they were acquiring a much greater depth of understanding than they would in a conventional course.

In the remainder of our report we focus on "recommendations concerning current work" and "recommendations concerning future work." Professors Hill and Porter will not be surprised by these comments; they are well aware of the many challenges posed by their innovative approach. But perhaps our comments will help them focus on a few of the most promising challenges, and perhaps some of our suggested alternatives will be of use to them.

Recommendations Concerning Current Work

A concern expressed repeatedly by students during both this and our previous visit was the amount of time required by the course. While it is not certain that this concern is anything more than the usual student exaggeration of how many hours they are required to work (and was explicitly contradicted by some students), we suggest that it is worth investigating. One method that might be particularly fruitful would be to have students keep a detailed log of their work during a few selected weeks. This would show not only how much total time they are using, but it might show whether they are apportioning it sensibly or using too much time for less important tasks.

A number of small adjustments might also be considered that could, when accumulated, save students significant amounts of time. Store in a file many of the matrices needed in the exercises, so a student could merely call up a matrix rather than typing it in. Encourage students to use paper and pencil for some of the shorter non-repetitive exercises that might take longer to do on the machine. Encourage students to write notes to themselves as they carry out a long series of experiments; this might help them remember the earlier examples in the series and make their search more efficient. We suspect that Professors Hill and Porter could come up with even better ideas for helping students use their time more efficiently, once they take a systematic examination of their course from this point of view.

Using their time efficiently when away from the supervised laboratory sessions was a related concern expressed by several students. Some felt they occasionally spent undue amounts of time when stuck on a hard problem and help was not at hand. Unlike the exercises in a conventional course, where missing a few exercises is normal, the exercises in this course are seen by students to be almost always essential; in a way, the students are developing the course in the exercises. Perhaps a lab assistant could be available for help during selected "office hours" in the lab.

An interactive text is so different from the ordinary textbook that many basic questions of organization and content must be considered anew. Professors Hill and Porter have indeed done so, but we see a few places where such questions might still be reexamined. For example, the text contains no detailed expositions of the basic theory, since students are expected to develop the theory themselves. An excellent device the authors have used to provide this missing structure is to require the students to write a summary at the end of every section of the text. They might also consider distributing a model summary, perhaps a good one written by one of the students, to help the students see more clearly the structure behind their work. Also, the organization of the interactive text is so expansive and so dominated by the details leading up to the students' discoveries that it is difficult to use as a reference. Thus, in addition to an index, a glossary of definitions could be particularly useful to students, both on-line and on paper.

Clearly a hallmark of the approach taken by Professors Hill and Porter is that students actively discover the ideas and results of the subject rather than passively hear and read about them. This has the great advantage that students understand and remember the subject unusually well and have an unusual degree of confidence in their ability to use it. On the other hand, less material is covered than in the usual course. We do not suggest a fundamental reexamination of the style of the course, but we put forward, tentatively, two ways in which the course might do more with relatively little cost. The course instructor might give an occasional lecture on related material -- an interesting application, a surprising connection with ideas in other courses, a pleasing extension of ideas the students have just finished exploring -- which could have the effect of broadening the students' understanding, preparing for a new round of student discoveries, or simply creating a welcome break and rejuvenating their desire for discovery making. Also the authors might reexamine their text to see whether a few of the topics that students currently study by a slow process of discovery receive relatively little benefit from this approach. They might decide that some topics could be grasped almost as well by being presented rather than by being discovered. If these are less crucial or less subtle topics, the consequent loss of depth in student understanding might be more than compensated for by the fact that valuable additional material could be covered. An alternative approach might be to identify (in an instructors' guide) those topics which the authors suggest could be treated by some instructors as lecture material and thus skipped by students when encountered in the interactive text.

Recommendations Concerning Future Work

We suggest that now is the right time for Professors Hill and Porter to begin writing about their work on an interactive linear algebra text. In particular, we suggest that they write an instructors' guide and an expository article for a general audience of mathematics educators. Such writing will benefit not only the readers but the authors as well. Writing can have the wonderful effect of forcing one to think through and explain clearly the purposes and results of one's work. Here are some examples of issues and questions that Professors Hill and Porter might address in their writing. What are the goals of the interactive linear algebra text project? How does the project meet these goals? What are the central topics of an introductory linear algebra course, and how does your text develop them? Who is the intended audience for a course based on your text? What steps should an instructor who uses your text take in order to help students see the structure of the subject and be aware of its key results? Is this an issue of sufficient concern that you would recommend regular weekly discussion periods? What steps should an instructor take in order to insure that students do discover what you intend them to discover and that they do so in a reasonable amount of time? Is this an issue of sufficient concern that you would recommend particularly thorough and skilled grading of their work and the availability of a particularly well-trained lab assistant? Does the interactive text tend to encourage students to work through the material in an overly rigid order and thus incline them not to work ahead when stuck on minor matters? If so, how do you recommend that instructors handle such circumstances?

Now that their interactive text is reaching maturity, Professors Hill and Porter will want to consider more thorough and penetrating evaluations of their work. This is a difficult task whose conclusions seem often more confusing than enlightening. We know of no completely reliable mechanisms. However, there are several that we think are worth serious consideration. Introducing other instructors to one's materials can have the dual benefits of disseminating a valuable work and getting useful feedback from serious users of that work. Specifically, we recommend that Professors Hill and Porter give workshops on how to use their interactive text and that they work closely with a few selected faculty who are willing to use the text in their own classes. A variety of follow-up studies of students who have taken their course should also be considered. For example, the students could be interviewed to find out whether they have observed any long-term impact on their mathematical maturity, their retention of the material, and their self-confidence in using the material. Instructors in the students' later courses could be interviewed to see whether they observe these same effects on the students. We noted that several students had studied some linear algebra in our earlier course. These students could be given pretests and posttests on material they presumably already knew. Current students could be asked to contribute to the evaluation process on a regular basis. For example, one might ask the students (at the completion of each section) to describe the positive aspects of the materials and the learning environment and to make suggestions for improvements (what took too long, or was too repetitive, or was unclear, etc.).

Concluding Remarks

We are quite excited about the promise of Professors Hill and Porter's interactive linear algebra text project. It offers a very different approach to learning than the standard course, one that may well transform many students from passive learners into active ones, from regurgitators of unassimilated facts into reflective analysts, from math-fearing students into mathematical thinkers. Two students' comments during this visit made especially strong impressions on us. One student was an older, returning student who is preparing herself for a new career as a secondary school mathematics teacher. She felt that this course, unlike any before, truly made her learn to think like a mathematician and stimulated her to want to understand deeply. Another student had studied some linear algebra earlier and reported on the striking differences between her two experiences. She felt that she was understanding the subject for the first time and consequently learning much more and enjoying learning much more. We hope these experiences prove to be transferable to many more students.

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Interactive Linear Algebra Text Project

Advisory Committee Report on Visit of March 29-30
to Temple University and University of Pennsylvania

Committee Members: Eugene Herman, Grinnell College; Horatio Porta, University of Illinois-Champagne/Urbana; Alan Tucker, SUNY-Stony Brook.

The committee visited linear algebra interactive text laboratory sessions at Temple University and the University of Pennsylvania, talking for about 30 minutes with each of the classes (with instructor and assistants out of the room). The committee spoke at length with the instructors and also talked with the course assistants Hans Johnston (Temple) and Nina Edelman (UPenn). The committee reviewed written student evaluations of the two courses. Most of the comments presented below address points that are well known to Professors Hill and Porter, who are working to address them in future development of the interactive text.

General Overview:

In general, the interactive text courses at Temple and UPenn seem to be off to a very constructive start. While the project is only 7 months old, workable although rough interactive texts already exist. Professors Hill and Porter are taking slightly different tacks, but both seem to be progressing well at this early stage. Students at both institutions were giving a substantial amount of helpful feedback to the instructors; students were also quite forthcoming in their discussions with the advisory committee.

The Temple University course

Professor Hill is working with an interactive text for the first time this semester, modifying the materials created by Professor Porter last semester. Hill seemed to be making good use of the capabilities of interactive text. Although his material is somewhat traditional in the way it initially presents topics, the interactive mode is used fully to develop consequences of basic definitions and operations and to lead students into insightful explorations of the topics. The course has two 1 1/2-hour laboratory sessions and one 1 1/2-hour discussion session each week. The students used terms like "refreshing" to describe the interactive text and overwhelmingly liked the course format. There was extensive cooperative learning going on. Virtually all the students said that they would recommend this course to a friend. The students' main concern was with support. They wanted a reference text that followed closely the interactive text; they wanted a bound printed copy of the interactive text (for easy reference); they wanted more discussion time with the instructor and most of all they wanted more technical and academic support. The innovative nature of this course means that fellow students and computer laboratory support staff are not familiar enough with the course materials and software to assist them. When students were not clear exactly what a question wanted them to do, they could waste many hours if they were working outside of the scheduled laboratory sessions and the assistant's help times in the laboratory. Except for expanding the discussion time and distributing bound copies (which first requires that the interactive text be completed and revised a little), these support problems do not have easy answers- this situation is faced by all innovative teaching projects. A short 'manual' of Mathcad, with pointers about common missteps would help a little with the technical support problem. The technical and academic support is exacerbated by having the computer laboratory in a different building from the offices of the mathematics department. It is essential that as soon as possible an undergraduate computer laboratory, with at least a dozen PC's, be set up in the mathematics department. Without such a nearby lab, mathematics courses are greatly handicapped in integrating computers into their instruction. Some of the Temple students said that the interactive text course took more time than a typical mathematics course, but they felt that the additional time was accompanied by greater learning. Several spoke of feeling under greater pressure here than in a traditional course because the homework is graded so carefully (really a plus) and because there was an insistence that students stay in step (not fall behind in the pace that units are completed). The sense of pressure is a pedagogical challenge for the instructors.

The University of Pennsylvania course

Professor Porter is teaching the interactive text course for the second time. His interactive text materials use directed discovery learning much of the time, although some topics are presented in a more traditional, although quite terse, style. Porter has chosen to give more attention to geometric topics than is found in typical linear algebra courses. The course meets for laboratory two 1 1/2-hour laboratory sessions each week. Students put in 5 to 8 hours outside of official laboratory sessions- a typical amount for a course at Penn. There are no discussions although Professor Porter occasionally stops the class for brief presentations of new topics using an overhead projector. There was extensive cooperative learning in the class. All but one of the students thought the interactive text course was better than a traditional math course and that it forced them to learn the material better. Students liked applied approaches, e.g., developing a new topic around a concrete applied example, rather than from a theoretical viewpoint, and volunteered that they wished more material was presented this way. The students are required to write a summary of what they learned in each section and several students remarked that this is valuable practice in mathematical writing. Like the Temple students, the U Penn students wanted a printed copy of the text and felt that they needed more interaction with a human. Several students said that the interactive text is not sufficiently helpful in the sense that it cannot answer questions and it has only one programmed line of reasoning. If students cannot follow this line of reasoning, the interactive text is useless. In contrast, an instructor can offer a variety of ways of thinking about a concept (implicitly the instructor is reading students' faces and trying different approaches until she/he sees satisfied expressions). One student complained that the lessons often seemed disconnected from one another. Understanding how ideas fit together seemed harder with the interactive text. Students agreed that some lessons were very successful, some not at all. One suggested that maybe some topics are just not well suited to the interactive text style.

Concluding Remarks

The most immediate task for Professors Hill and Porter is to refine the interactive text materials developed to date. The larger challenge facing the instructors is to give adequate coverage to all the basic topics in linear algebra while properly utilizing the potential of interactive text for exploratory learning. The choice of whether to have a fairly uniform style for all material or to explore a lot in some places and have more traditional textual material in other places is something that the developers are having to grapple with. The advisory committee encourages the instructors to experiment with a mixture of types and levels of interaction. For some topics, it may be appropriate to engage the students in a lengthy process of discovery in which different students may take quite different paths. For others, it may be appropriate to give the students much more guidance in directing them toward a desired "discovery," which would be difficult or too time consuming for them to make if left too much on their own. Yet other topics may be either too similar to ones already covered or may be too minor to warrant much of a discovery process at all; these might be presented quite briefly with some quick confirmations carried out by students or might be exploited later in an engaging application in which the topic is used but is assumed to be known rather than discovered. The proper level of human interaction and use of traditional forms of instruction, like brief presentation by the instructor of new topics or standard written quizzes, also is evolving. The human interaction gives students many important but subtle forms of guidance in learning and is much more adaptive, e.g., adding an illustrative example when appropriate. Current interactive text software is limited, lacking extensive branching capabilities (to skip forward when students quickly show mastery or to turn to a more in-depth presentation of elementary aspects of a topic when students are having trouble). Further, it lacks the artificial intelligence to give helpful feedback to students when they are stuck. These comments are meant to be taken in the context of the overall assessment that the interactive linear algebra text project is off to an excellent start. The overwhelming approval given to the interactive text mode and the current courses, even though parts of the current texts are in very rough form, is most impressive.

Sample student response:

On the final exam, one of us (Porter) asks the students to write an essay that describes the course to another student. This provides feedback on the student's understanding of the concepts, overview of the course, and what the student viewed as important. We present excepts from these essays below. We have not corrected typos or spelling and grammatical errors.

Student 1:

The topics are presented in different ways so as to help the student have a better grasp of the concepts. When appropriate, both geometric and linear algebraic interpretations of problems are investigated (in such topics as projections and solving a system of equations). This also helps in case you dont understand one interpretation of a problem since youll be able to visualize it in other ways. The Mathcad environment also helps this visualization with graphical tools that let you plot/graph/display mathematical equations and sets of data.

Instead of working through endless problem sets involving simply changing a set of numbers plugged into a theorem (plug and chug) in order to force you to memorize it through shear numbers, the course actually makes you think, as oftentimes the main concepts are derived by the students. This means that you actually know how a theorem works as well as how to use it.

The concepts are also illustrated with real world examples, which include applications into topics some students may be interested in such as probability problems (such as predicting long-term behavior like in Leslie population models), curve fitting, computer graphics (rotating, translating, scaling and reflecting images), and finding the "best" solution to possibly unsolveable problems (least square solutions). These are just the tip of the iceberg since I've managed to apply what Ive learned to my other classes in my mechanical engineering classes. Topics like dynamics and kinematics require changes of reference frames (change of bases, with rotation and translation maps) as well as having to solve a large number of equations for unknowns.

The computer section allows you more time to think about the problem, after youve spent time getting the hang of Mathcad, rather than spend it laboriously working it out by hand and then having to worry about having committed a careless mistake in any of the computations. Since the computer is doing all the dirty work, topics such as higher-dimensional spaces are possible (imagine computing the inverse of a 10 by 10 matrix by hand!). I think the time spent learning how to use Mathcad is a valuable investment that pays off when one has to deal with more complex topics which demand a higher degree of computation (such as methods which require iteration such as the Gramm-Schmidt process or when one has to deal with Markov Chains).

Student 2:

It is five o'clock in the morning and I am glad that I just finished my Math 312 exam. The course was real fun and I would strongly recommend you take this course (this section!) in the next semester.

What I enjoyed the most is the computer session. I am not sure whether I have the biased opinion of a typical computer science major, but I found that Math is much more fun when you can play with the symbols interactively. And with the computer it also means you can do more things. I can concentrate on the interesting part of a problem without having to get bogged down doing tedious manipulations. Using the computer is definitely a big plus for solving linear equations. How often have you inverted a 10x10 matrix by hand? I really hated it in Math 240 when I always made mistakes solving linear equations. When the help of the computer, you not only can solve your problems faster, you will also have the time to learn more!

A bonus for taking the computer session is, of course, you get to learn how to use the computerized mathemtical tools like MathCad and Maple. These tools are very important for scientists, engineers or even economists.

The way Dr Porter arranges the materials in this course also makes it possible to relate linear algebra knowledge with other math knowledge. I am taking a statistics and a calculus class simulatenously with Math 312, and often in the assignments and exams I found nice connections bewteen what I do here and what I learn in other classes. For example, the markov chain probability model and legendre polynomials

Of course, apart from being a happy student in this class, I do see many things that can be improved. For example, Dr Porter does not give lengthy class instructions like other profressors do. He'd leave us to explore the knowledge by ourselves. While I find much freedom doing that, sometimes, especially at the beginning of the semester, I found myself stranded in the lecture notes and didn't know where I was heading to.

Also, I really appreciate the efforts that Dr Porter has spend putting together the computerized lecture notes. However, sometimes I found it difficult to locate the information that I was looking for. If you want to take this course, you definitely should buy the bulkpack, because it will be faster to look up something from the bulkpack than from the computer. I think this can be improved if there are more indices in the lecture notes. A better index.mcd file will certainly help. Something like Mosaic will be ideal. However, I don't think this is possible with MathCad, because MathCad is very slow, if you want to switch to another document via a link, it taks a long time. Well, I guess we'll just have to put up with poorly designed PC programs.

In short, if you want to take a math course, take this one. There is a lot to learn and the computer is fun. Of course, you have to do some hard-work, but the things you learn will be perhaps be more useful than any other Math course that has a higher number than 312, that is, at least for non-math majors.

Student 3:

First of all, Math 312 is not a repeat of Math 240. Math 312 both in depth and in broad teaches much more stuff in the area of system of linear equations, vector space, and linear tranformation. It teaches you how to map functions into vector space, how to use one function to approximate another function by using mapping, how to do graphics translation... It really teaches you how to use matrix, many problems when translate into matrix space, can be solved very easily. It not just teach you math formula, the course combines the linear algebra theory and its applications, giving people a vivid view of what they can use it for. Like the course teaches you the weather model, population model, many of these application can be used in economic study. I am taking finance right now, I feel many of stuff learned in this course can be used in economics to construct math model. Overall I feel the material I learned in this course is very pratical and useful.

We all know math is the foundation of the modern science, and this course I feel is particularly related with other fields. When I took an electrical engineering course, the professor taught us Fourier series function, but he did not teach us how the Fourier functions was invented, I am always curious about it. And I learned this in math 312. I was so surprise when some problems are translate into another space, how easy it is to solve them. That's what you will learn in math 312. Math 312 also teaches a lot of graphics stuff, and to how to use matrix to solve them, which is directly related to one of my course I took last semester. At that time, I haven't learned linear algebra yet, but I need to write a big program to do graphics animation. Eventually I solve this problem by using a stupid method, but now I feel I can solve it very easy by using matrix. I just hate myself why not take this course earlier. If you are an econ major, then you will defintely like this course. A big portion of this course will teach you how to construct matrix, which can used as an math model in economics. Many of my finance classmates don't understand why without productivity increase, the country economy will eventually go into a steady state. I now understand this problem thoroughly from both math and econ points of view.

I am especially like the computer session of math 312. You know in math 240, you need to do a lot of calculation by hand. That can be greatly relieved in the computer session. You don't need to memorize derivative formula, integration formula... MathCad has many building functions thelp you to do these calculation. This is also a chance for you to practise your computer kill.

I can tell you there is a lot of homeworks in this course. You are expected to spend a lot of time on the homework each week. But I feel it is worthy, at least you won't forget these stuff quickly after the final, because you spend a lot of time on it each week. I am sure you will see the returns in the future.

At last, I really happy you registered the computer session. It is really worth while. I hope you will enjoy it as I do.

Student 4:

The computer section of Math 312 is very different from any other course that you could possibly take at PENN. The applications of Math 312 stretch all across the university. I have applied concepts that I learned in 312 to my finance, management and statistics courses this semester. There is no doubt that the computer section of Math 312 is a lot of work, but once it is finished you should feel a sense of accomplishment.

The many applications of the ideas taught in Math 312 are what make the course so interesting and useful. Computers really helped make this possible, as we took matrices and multiplied them by themselves many, many times, which would have been an impossible feat by hand.

The course is also taught in a logical and sequential method, whereby we add new information and methods to the previous lessons information each time.

The method of discovering these properties was more of an experiment and discover process than a outward presentation of the material. For example, the addition and subtraction of matrices was presented, and instead of stating that the dimension of the matrices involved had to be the same, various matrices were presented and it was up to the student to discover which were possible and which were not. The ease of the process using the computer was manipulated in order to allow a different level of learning. Time was not wasted with simple addition and subtraction on scratch paper, but rather those skills were assumed so that a broader range of experimentation was possible.

Learning in this class depends greatly on self motivation and willingness to spend extra time, more than in other classes. It is good to have the independence to work at your own pace, but when there is nothing to fall back on, such as in a typical lecture-recitation class, you can easily start to fall behind. It is difficult sometimes to learn completely from the on-line text, even though you do have a partner. The computer sessions definitely are beneficial and necessary, considering that the whole course is structured on-line. However, I feel that in order to achieve the full effect of hands-on experience there must also be some type of conventional teaching employed. For instance a discussion of each lesson before the students begin for themselves. But taking everything into consideration, I think that this course does offer what I expected to gain. I didn't expect to completely understand every single topic covered, nor to remember everything covered. But I have been able to pick up on the things that I feel will be necessary in other applications and familiarize myself with them through this learning format.

However do expect to work hard and work a lot for the homeworks and the take home exams are very difficult and very tedious. Yes, I said take home exams. However they take work to do well. Do not expect to blow this course off.

The things that you use in this course can definitely be used in other courses. Of course some of the material is very theoretical and is hard to use it to other subjects. However much of the material learned is practical and can be used even in real life. One very good example of this is reflection where you study the reflection of a line or a plane. I'm sure that such things can be useful in laser beams or in space projects, etc.

I must be honest though that the course can get lost in it's own depth at times and you will be too. Therefore you may lose some interest at times, especially considering that the homeworks are very long and tedious. However, the course keeps interesting and follows a pattern where you build upon previous concepts that were learned.

I thought that the computer section was very rewarding for many reasons. First of all, you get to do more firsthand learning than you do in other classes. This is because all of the chapters teach you by examples and by theory, and try to make you piece together all of the information that you need to know. Also, having the course taught on a computer is very helpful for doing the calculations, as well as checking your work and experimenting with it to discern the correct answer. The course also allows you to work at your own pace since you can work on the chapters and the exams all week before they are due and not have to worry about having enough time to finish. A final description of this section is to compare it to a brain teaser. Although it can be very annoying sometimes when you're having trouble figuring things out, it is a very satisfying and rewarding learning experience when you do figure things out.

Mathcad is an awesome program! It is very user-friendly compared to some of the other math programs out there. Just think of it like an electronic blackboard. The structure of the course is built around the computer, so you end up with a solid grasp on how to use Mathcad. In the short time since I learned how to use it, I have employed Mathcad in many of my other classes and find in indispensable on long, messy mathematical problems.

From what Ive heard, the computer section of Math 312 is somewhat harder than the other section and involves substantially more work. The main aspect which I liked about it was the fact that I learned how to use Mathcad. I also support the idea of the take-home test since it allowed me to answer the questions at my own pace which improved the quality of my answers and of my learning experience. Some unfavorable aspects of the computer section is that the assignments are often very long, so make sure you dont fall behind. Also, I found that some questions and certain theoretical concepts were difficult to understand.

Student 5:

Well, I'd like to start by saying that you should only take the computer section of the course. I know a person who is taking to "normal" part of the course and in my opinion he has not learned anything. If you want to take math 312 as a gutt course, take the other section, because the computer section requires a considerable ammount of work. Basically the course is structure in a way that you have a constant load, meaning that you are going to spend x number of hours each week doing various problmes, it's not going to be a lot, but it's a faire amount of work.

Well, in the first part of the course, a couple of weeks you are oging to go trhought the material you partially covered in 240. It's basically going to refresh your memory and for people who have not had math 240 previously, which is not a prerequisite, the'll eb learning new materila. Some of the material that you've had in 240 such as vectors, planes, spaces, solution set is gong to be used here as well.

Well the next step is going ot be the introduction to the solutions space: how to solve a set of linear equations by row reduction. Then we go into eigenvalues and eigenvectors, I think you've heard of them in 240 and how they relate to probaility.

Oh, I think the most interesting part of the course, it was for me is probably goign to be a new method for finding different probability series ie. if you have a set of probabilties of going from state one to state 2 what's the probability you are going to end up in state x after x steps. It was interesting to me because i learned in before, in statistics, how to solve these kinds of problems and now I was tought a basically new approach tto the old problem.

Well, ok after that you are going to study linear transformations- functions that preserve linear properties.

I think the biggest difference you are going to encounter in this course as opposed to other science and math courses is that you are not going to have formulas which you;'ll have to plug into a problem to get an answer: you'll actually have to think about what you are doing and why it works the way it works; you have to derive all the formulas.

Student 6:

X+Y = 5
2X+5Y = 16
Solve for X and Y.
Have you ever been asked to solve this sort of a problem? Pretty easy, eh? Well what if you were given 10 equations and 10 unknowns? Would that be so easy? The answer is "YES, using Linear Algebra!". Linear Algebra, typically a college level mathematics course, allows a mathematician to simplify an extremely complex problem into a relatively simple one. Linear Algebra does so by making use of what is called a matrix (perhaps you've heard a bit about them in your advanced calculus courses?) For example, to solve the problem described above would simply require transforming the system of equations into a matrix, and then reducing the matrix into what is known a row-reduced echelon form using a techniqe called Gaussian Elimination. From this the solutions can be very easily read off the matrix. It is simple, and quick. In fact some software packages can solve such a system at the punch of a single button. But wait, there's more...

Let's say you had the same two set of equations as above for various sets of right hand side constants (i.e. the numbers to the right of the equal sign were the only things that were different). If we set up the matrix appropriately, using Linear Algebra, one could calculate the inverse of the matrix and use that result to calculate the various answers corresponding to various right hand side constants. And all this in a matter of seconds. What does this mean to you? It means you do not have to go through all the tedious algebraic simplifications (subtracting 2 times the first row from the second, etc., etc.) again and again just because you have a different set of constants. Clearly, this is especially useful when there are a large number of unknowns. Linear Algebra saves time and energy. Oh but wait again, 'cause that's NOT all folks...

Linear Algebra has many other applications in addition to solving a system of equations. It is extensively used in Statistical Analysis as well. Calculating covariance and correlation coefficients bocomes a piece of case. No more breaking nails while punching a zillion numbers into your calculator.

Well, by now you must be asking, "Gosh, is there anything that Linear Algebra cannot do?" Probably not. It may have some limitations when delving into nonlinear processes, but even then through the use of linear transformations and techniques such as Legendre polynomials Linear Algebra can usually offer an adequate apporoximation. Linear Algebra is an excellent tool for simplifying and solving very complex problems. I must warn you, however, that there are some growing pains associated with such good things. One must learn some difficult and what one somethimes considers useless concepts (such as Euclidean Vector spaces, the basis, spanning sets, orthogonality, orthonormality, rank, dimension, ...NEED I SAY MORE?) to finally get to the "good stuff". But all in all, it's well worth it in the end and I definitely recommend the course highly to all students, mathematicians or not.

Student 7:

If someone asked me to describe calculus with two words, I would say derivatives and integrals. If someone asked me the same question about Linear algebra I would say matrices and fun! It's even more fun if you use computer software like Mathcad to help you solve some of the problems. Of course, there are some tedious and complicated concepts that you must learn but in general you will discover the power of a matrix and it's multiple uses. You will also encounter that after performing a few simple operations on a matrix, you can get very important and interesting information that will solve a problem, relate or compare data, or even predict the future!!!!

For example, you can use a matrix to solve a system of linear equations. You might say to yourself, "Big deal, all I need to do is solve for one variable in one equation and substitute it into another one and find the answer." Well, fine, that will work if you have two or three equations with the same number of unknowns, but what about if you have a system of ten equations and ten unknowns? Go ahead and try the same method and I'll see you in three and a half years when you get the solution. On the other hand, if you set up your equations in matrix form, it would take you only a few minutes to get the solution and even less if you use Mathcad. This is just a small example of the amazing power of matrices.

If you are one of those people that love statistics and like to know if something like batting average is related to the weight of a player and his age, for example, then take Linear Algebra and you'll love something called regression. By getting your data together you can figure out a linear equation that will relate the players weight and age to his batting average and you'll be able to see if the relationship really exists or if they totally have nothing to do with each other. This, of course, is done by using matrices.

Another pretty interesting use of matrices deals with probability. To use a simple example, you can set up a matrix that contains the probability of getting a head or a tail when you toss a coin. You might be interested in what the probability of getting two, three, or more heads in a row is after a number of tosses. This is very simple to figure out as long as you set up your matrix right (which is very easy to do, by the way). Let's say you want to know the probability of getting four heads or four tails in a row after n tosses. All you need to do is raise your matrix to the n-1 power and you'll get your answer. This same method can be used to play cards or to predict the weather!

As you can see, linear algebra is not something that you use in class and then forget about it. You can use it for every day things or in your other courses regardless of whether they are in the field of engineering, science, business or liberal arts. I've already proven to you that matrices are indeed an extremely powerful tool, but these are only a few examples. If you really want to get the whole picture, then register for Linear Algebra when you are in college.

Student 8:

"Solve the following set of simultaneous equations by reducing the matrix to row echelon form." Every math student dreads reading instructions like this, but linear algebra is not a tedious process that one must go through, instead it is a very useful and practical method of solving problems. Linear algebra can even be applied to topics which are considered by most to be other fields of mathematics, such as geometry, trigonometry, and probability.

At the University of Pennsylvania, Professor Gerald Porter teaches a computer-oriented linear algebra course. In this course, the normal "textbook" teaching method has been replaced by the use of MathCad, a math software package. This eliminates the need for "grinding out" the answer since the computer does the actual calculation of the problem. Since this greatly eliminates the time involved in solving the problems, the focus of the course can be directed to more applications of the concepts.

After introducing the "basics" of linear algebra, most importantly vectors and matrices, vector geometry and matrix operations are covered. Vectors have direction and magnitude and this information can be written in matrix form. The matrices can then also be manipulated to find other valuable information by using dot and cross products; for instance, the projection of one vector onto another can be found with the use of the dot product and a normal vector can be found with the cross product. Systems of equations can also be written into matrices, and matrix operations can be applied. For example, to find a solution to $Ax=b$, you can use the inverse of A to rewrite it as $x=A^{-1}b$. But, this can also be solved by row reduction by using simple elementary matrices. The fact that most problems can be solved by more than one means is what makes linear algebra so versatile

Lines, planes, areas and volumes, topics normally thought of to be geometry, can be solved using linear algebra. Lines and planes can be written in many forms, and although the general equations are typically used, they can also be written in vector equations. The area mapped by two vectors can be found by simply taking the absolute value of the determinant of the (2 x 2) matrix. This idea can be expanded to find the space spanned by a (3 x 3) matrix which is the volume.

Topics of correlation coefficients and regression, typically covered in probability, can also be found by linear algebra. The correlation coefficient of two vectors having the same dimension can be found by taking the cosine of the angle between (**U**-mean(**U**)) and (**V**-mean(**V**)). Regression can be used to find the relationship between a set of data; so, the idea of the "best fit" comes into play.

These examples are only a small amount of work, normally thought of as other fields of mathematics, that can be solved with the help of linear algebra. So, although linear algebra sounds like a separate entity, it is in fact a component in the wide field of mathematics.

Linear algebra is only a branch of mathematics, but its applications are limitless. If you want to learn little formulas and plug in numbers, then linear algebra is *not* the way to go. If you want to learn a new way to approach problems and to discover alternate methods to solving problems, then linear algebra a course that will not disappoint you.

Student 9:

LINEAR ALGEBRA ISN'T JUST FOR MATHEMATICIANS

The property of a linear function is that the output is proportional to the input by a coefficient and a constant. Linear algebra is the study of such functions. Many important and practical phenomena can be modeled linearly, therefore this field of study provides many useful applications. A very convenient mathematical system in which to investigate linear problems is vector algebra. Because vector algebra is well-suited for computer-based investigations, a modern class in linear algebra, such as the one given by Dr. Porter of the University of Pennsylvania, is most effectively taught in a computer environment.

The mathematics application software used in our class is MathCad. MathCad allows entry of mathematical formula in standard notation and readily performs any vectorial analysis required. Using such a system greatly reduces the amount of time spent by the student performing non-insightful, repititious calculations. More importantly, because of the speed and power available via the computer, many experiments can be performed in a short amount of time on large amounts of data so that the student can glean patterns and gain insight into the structure of linear algebra.

Fields of applied studies in linear algebra include probability, regression, and optimization of production functions which involve the solutions to systems of linear equations. Many everyday events, such as the weather and card games, are modelled probabilistically due to uncertanties and variances in their behaviour. Linear algebra offers a very convenient technique for analyzing the behaviour of these systems. The method is known as Markov chains. The methodology requires the construction of a square matrix whose components are the individual probabilities that any of the possible outcomes will occur, given the fact that a certain event has previously occured. By structuring the characteristics of the system in such a matrix, long-run or intermediate probabilities can be found simply by raising the markov chain to a higher power. Additional information, such as length of time or number of trials until a certain outcome occurs and the expected payoff of a probabilistic game can be found through simple manipulations of the Markov chain.

Another important concept described by linear algebra is regression. Regression is a method of finding a linear model which best explains the relationship between the independent variables of a system and the dependent parameter. Regression provides the factor loads or coefficients for each of the independent variables investigated and a constant term, the inherent value of the dependent variable. The resulting equation provides the best fit line for the plotted data by minimizing the squares of the differences between the observed nature of the system and the hypothesized behaviour. Additionally, non-linear structures for the modeled data, such as polynomial or exponential, can be linearized and solved using the same techniques.

Linear equations can often approximate with much reliability the operations of a manufacturer. There are certain criteria which must be satified by the set of linear equations in order to be assured of unique solutions to the parameters. Using techniques of linear and vector algebra, these conditions can be verified, and when a unique solution does not exist, an optimal solution can be found.

Student 10:

Before Linear Algebra: What am I doing taking an algebra course in college? Didn't I learn algebra in elementary school? How to solve polynomials and equations? What happened to the calculus and diffential equations? What about the Fourier and Laplace Transforms. Now those are some challenging concepts in math.

After Linear Algebra: Oh my goodness, so that is what is meant by a set of orthogonal functions. This is how a matrix relates to my electrical circuits equations. Now I see what a line or a plane really is.

As a third year electrical engineer at the University of Pennsylvania, I thought that algebra was a simple concept that probably would never challenge me, intellectually again. After all, I've already taken 4 semesters of College Calculus and have seen integrals that would blow your mind. So why should Linear Algebra pose a threat? Well to make a long story short, my course in Linear Algebra has not only taught me new concepts in math that I never knew of before, but it also explored previous concepts with to such a stringent degree that it solidified my knowledge of them. Let me give a few examples to demonstrate this.

Suppose you have a huge electrical circuit complete with resistors, capacitors, and inductors that we will call devices. We can analyze the currents going through circuits by deriving equations relating voltage across our devices to current through our devices. This will result in a large number of equations with a large number of unknowns, the solution of which will be the current and voltage values of our circuit. Now, if there were only 2 equations and 2 unknowns, it would probably be easy to solve. But what happens if you have 140 equations and 140 unknowns? Yes, you better consider this, because in the real world, you are more likely to see the latter than the former. If you were to attempt to solve this gigantic problem using the elimination method, it would take forever, and there would be a lot of room for error. However, using linear algebra, we could insert our equations into a huge matrix (a rectangular array of numbers) and proceed to solve this very simply. (As a matter of fact with the latest software out, it can be done even easier.) In addition to our solution, we now have a structure that allows us to take that same circuit and solve for different resistor values, or voltages with very minor adjustment. For engineers who need to be able to solve equations quickly and efficiently, matrices and row reduction certainly has the edge on the elimination method.

In addition to its powerful solving capability, Linear Algebra has powerful geometric capabilities as well. Linear Algebra creates concise definitions of lines and planes in all dimensions. To see a plane in 4-D is still hard to visualize because we line in a 3-D world, however we can mathematically visualize this using linear algebra. In linear algebra, yhou can set up a matrix to visualize whatever dimension you wish.

This is but a brief explanation of the many capabilities of Linear Algebra. It is a challenging, yet highly stimulating course which I encourage all of those interested in math, science or engineering to take. Years from now, when you are faced with some decisions about upper level courses to take in mathematics, remember that Linear Algebra is one course that will open up a whole new world of mathematics to you.

Student 11:

I used to think linear algebra is one of those plug and chug courses that if you are careful and meticulously neat, you will get the A you wanted. After taking LinearAlgebra 312 at UPenn, I stand corrected. All those number crunching ways of calculating different matrices are actually based on very logical interpretations of number analysis.

For example, solutions we obtain from a set of equation actually stems from the some linearly independent basis. From these basis, a span set can be created (with span properties) which give rise to different equations that will lead to the independent basis which are the "solutions" we seek. Set of basis are the subspace. These subspace are contained in 1, 2 or 3 dimensions as we know it. With some inferences, we can expand subspaces into dimensions that are unfamiliar to us, such as 4th or more. All the abstract thinking involved in these reasonings horns my analytical ability, which is always is always a plus.

All the thereoms are not invented to make life difficult for non-math majors, they do have a practical usage. As we have seen in Markov Chains, eigenvalues, eigenvectors being applied to population and probabilities problem. Probabilities that can have pratical purposes such as betting (the coin toss we did can be tried on ignorant friends with cash overflows). In case of population, we can determine how long before population doubles, triples etc by using matrices.

We also used projections, least Squares, pseudo-inverse, regressions to do some error analysis. The solution space, row space, subspaces are all teminology in which mathematics can be communicated. Regression can be used to infer how different things have in common, the most well know case is the SAT and GPA.

Orthogonal basis, Row reduction, invert matrices, psedo-iverse, all of which is what makes all the number crunching process possible. Without which, a lot of problems cannot be solved. We have also seen many familiar powerful mathematical theorems such as the Law of Cosine and proving it, although hard at first, proves quite satisfying after all the "decoding".

Even though, if wasn't for the pressure of a grade, I can even say that I enjoyed the class. Not to mention that it has helped my analytical ability. Since most of the plug and chug is done by the computer, we are free to do more conceptual discoveries. I used to think of Math as a tool to be used in professional fields such as engineering, business etc and why people would major in math is beyond me. This class makes me realize math in itself holds more facinating concepts and usage that made it a interesting field to study in. To think linear algebra is only a branch of math!!!

Interactive Linear Algebra in Mathcad

To view the text you must have Mathcad installed on your computer.

The text is compatible with either Mathcad 5.0 or 6.0 and runs under the Windows operating system.

To install the text type:

a:linalg -d c:\winmcad

where a is the drawive where the disk is located and c:\winmcad is the mathcad directory.

Interactive Linear Algebra in Mathcad

David R. Hill

November 1995

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