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ABSTRACT

Project FOCUS has developed seven mathematics Instructional Modules, each of which focuses on a specific mathematical concept and related skills. The modules are: "Integers and Computation with Integers"; "Fractions and Computation with Fractions"; "Decimals and Computation with Decimals"; "Ratio, Proportion and Percent"; "Graphs and Their Interpretation"; "Variables and Equations"; and "Problem Solving." For each of the modules, there is an "Instructor's Guide" that contains pretests and posttests, a reproduction of the student module with complete answers, and suggestions of ways to impplement the program. The FOCUS modules, developed by mathematics education faculty at Boston University (Massachusetts), are designed for college students in need of remediation in mathematics. This project report provides an overview of the project; a section tracing the project from problem definition to project conclusion with a discussion of administrative pitfalls; information on the background and origins of the project organized into phases with a focus on organization, policies, and funding; a full description of the project and its workshops; and an evaluation and project results. (DDR)

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Project FOCUS

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PROJECT FOCUS- EXECUTIVE SUMMARY

The problem of inadequate preparation for college mathematics is pervasive. The 1989 report of the National Research Council states that, "Each term nearly three million students enroll in post-secondary mathematics courses. About 60 percent study elementary mathematics and statistics below the calculus level." Since 1989, enrollment in remedial and/or basic mathematics courses at U.S. colleges and universities has increased, now topping more than two million each year.

At the same time that the number of students in need of remediation is growing, demands on college students to take more mathematics and/or mathematics related courses is increasing. These demands stem in part from the expanding number of career options in technology fields that require additional study of mathematics, and the fact that, at comprehensive universities, virtually all programs now require some university level mathematics. Unfortunately, there is limited help for college students who are having difficulties in mathematics. Colleges and universities generally offer one-semester or one-year remedial mathematics courses. The courses cover the same skills and concepts, generally arithmetic computations and algebraic manipulations, for all students regardless of their particular mathematical strengths and deficits. For many students, much of the content is a review of skills in which they are already proficient. For others, the remediation is a duplication of high school instruction with which they were previously unsuccessful.

Evidence from various research studies suggest students' primary mathematical difficulties are not computational in nature, but rather conceptual. These difficulties include data interpretation misconceptions, lack of understanding of mathematical relations, and limited knowledge of problem-solving heuristics. Thus, remedial courses, as they are currently comprised, are neither helping students overcome their difficulties nor preparing them for further study of mathematics and mathematics-related subjects. To be able to assist these students, there must be a systematic way of identifying their difficulties and an appropriate instructional program that targets their specific difficulties. This was the stimulus for the development of the Probe Assessment of Mathematical Abilities and for the FOCUS modules.

The Probe Assessment of Mathematical Abilities (PAMA), funded by the Fund for the Improvement of Post-Secondary Education (FIPSE) of the Department of Education (Greenes, 1985-87, #G008541041), assesses the mathematics needs of college students who would normally enroll in remedial or basic mathematics courses, and pin-points specific deficiencies. During the development of PAMA, greater insight was gained into the nature of college students' difficulties with mathematics. College students in remedial mathematics courses experience great difficulty with translating words to symbols and vice versa; understanding proportionality and using proportions to solve problems; interpreting mathematical relations presented in graphs; understanding the concept of variable and the uses of variables in expressions and equations; carrying out complex computations with fractions, decimals, and integers; and being able to use various strategies to solve mathematical problems. Not only are these skills and concepts not addressed in remedial courses, but there is a paucity of instructional materials that focus on these skills and concepts. Materials that do deal with these topics are generally poorly developed. For some of the topics, materials do not exist. For this reason, the FOCUS modules were developed.

FOCUS consists of seven mathematics instructional modules, each of which *focuses* on a specific mathematical concept and related skills. The seven modules, varying in length from 43 to 114 pages, are:

1) <u>Integers and Computation with Integers</u>

This module develops the concept of integer, ways in which integers can be represented, the comparing and ordering of integers, and computation with integers. The Chinese Red-Black Rod Model, where a red rod represents $^{+}1$ and a black rod represents $^{-}1$, is used to model addition and subtraction. Integers are compared and ordered on a number line. Throughout the module, real-life settings enhance understanding of ways in which integers are used.

2) <u>Fractions and Computation with Fractions</u>

Comparing and computing with fractions are the foci of this module. The relationship between fractions and their decimal equivalents is emphasized, and the technique for converting fractions to decimals for comparison and computation purposes is presented. Historical references to the use of fractions in real-world applications are included throughout the module.

3) <u>Decimals and Computation with Decimals</u>

The focus of this module is on the comparison of and computation with decimals. Writing very large and small numbers in scientific notation form and computing with numbers in this form using a scientific calculator are developed.

4) <u>Ratio, Proportion, and Percent</u>

This module focuses on developing an understanding of ratio, proportion, and percent and applying proportional reasoning to the solution of real-life problems. Applications from statistics, architecture, science, and mathematics are explored. The concept of percent is examined with percents less than one as well as with percents greater than 100. Problems involving percent increase, percent decrease, discount and markup are explored.

5) <u>Graphs and Their Interpretation</u>

This module focuses on mathematical relations presented in linear and curvilinear graphs. Graphs are compared with one another and matched with phenomena from the natural, physical, and social sciences. The concept of slope and the processes of interpolation and extrapolation are explored in detail. Typical graph interpretation misconceptions are identified, illustrated, and discussed.

6) Variables and Equations

The focus of the module is on the symbolic representation of mathematical relations presented in prose. The meaning of variables and the behavior of dependent and independent variables are described, followed by development of the concept of equality and procedures for the words to symbols translation of mathematical relations. Step-by-step directions are given for solving equations and systems of equations. All problems in the module use real data, and require formulation and solution of one or two equations.

7) Problem Solving

This module presents an overall plan for solving problems and detailed instructions for the use of six problem-solving strategies: 1) organizing information, 2) extending patterns and generalizing functions, 3) simplifying the problem by reducing the magnitude of the data or by decomposing the problem into component parts, 4) making cases, 5) modeling, and 6) working backward. Strategies are used either singly or in combination to solve problems from a variety of settings.

For each module, there is an Instructor's Guide that contains pretests and posttests, a reproduction of the student module with complete answers, and a description of the project and ways in which the modules may be used. Pretests or the Probe Assessment of Mathematical Abilities may be used to determine assignment of students to particular modules. Or, students themselves may choose modules with which they believe they need additional experience.

The modules are written using an "active" fill-in format that encourages students to participate in the development of a concept or skill, and to use the concept/skill to solve problems in the exercise sets. Each module contains articles designed to engage students' interest in the settings, to encourage them to read analytically for relevant information, and to enhance their understanding of the mathematical ideas by demonstrating application of these ideas. The elaborate and extended development of big ideas provides students with opportunities to see mathematical concepts in a variety of contexts and applications. Topics for the applications are drawn from the physical and natural sciences, the social sciences, the arts, and everyday living. Because of the use of real problems with real data, many of the computations are "messy." For this reason, a scientific calculator is recommended for use by students with all modules. A detailed development of the special functions of the scientific calculator is presented in the module, Decimals and Computation with Decimals.

Each module has been designed to be used independently of the others, and may serve as the content focus of a short-term mini-course, or to support existing remediation programs. Students, individually, may use the modules in a tutorial setting with a more advanced student or faculty member as a mentor, or the modules may be used by small groups of students in cooperative learning settings. Finally, FOCUS modules collectively may serve as the text for whole class instruction.

FOCUS modules have been used by students in remedial mathematics courses at two-year junior colleges and technical schools, at four-year colleges and universities, and in high schools by students in mathematics review support programs. FOCUS modules and Instructor's Guides may be obtained by writing to:

Dr. Carole E. Greenes
Boston University
School of Education
605 Commonwealth Avenue
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Project FOCUS

The FOCUS modules, developed by mathematics education faculty at Boston University, are instructional materials designed for college students in need of remediation in mathematics. Each of the seven modules *focuses* on a specific mathematical concept and related skills. The modules are: Integers and Computation with Integers; Fractions and Computation with Fractions; Decimals and Computation with Decimals; Ratio, Proportion and Percent; Graphs and Their Interpretation; Variables and Equations; and Problem Solving. For each of the modules, there is an Instructor's Guide that contains pretests and posttests, a reproduction of the student module with complete answers, and suggestions of ways to implement the program. By pinpointing specific difficulties and providing instruction in those areas only, the time devoted to remediation may be reduced substantially.

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Products: Seven FOCUS modules and seven Instructor's Guides.

PROJECT OVERVIEW

The problem of inadequate preparation for college mathematics is pervasive. The 1989 report of the National Research Council states that, "Each term nearly three million students enroll in post-secondary mathematics courses. About 60 percent study elementary mathematics and statistics below the calculus level." Since 1989, enrollment in remedial and/or basic mathematics courses at U.S. colleges and universities has increased, now topping more than two million each year.

At the same time that the number of students in need of remediation is growing, demands on college students to take more mathematics and/or mathematics related courses is increasing. Unfortunately, there is limited help for college students who are having difficulties in mathematics. Colleges and universities generally offer one-semester or one-year remedial mathematics courses. The courses typically cover the same skills and concepts, arithmetic computations and algebraic manipulations, for all students regardless of their particular mathematical strengths and deficits. For many students, much of the content is a review of skills in which they are already proficient. For others, the remediation is a duplication of high school instruction with which they were previously unsuccessful.

Evidence from various research studies including Project PROBE (Greenes, 1985-87) funded by FIPSE, indicates that college students' primary mathematical difficulties are not computational in nature, but rather conceptual. These difficulties include data interpretation misconceptions, lack of understanding of mathematical relations presented in prose and/or symbols, and limited knowledge of problem-solving heuristics. Thus, remedial courses, as they are currently comprised, are neither helping students overcome their difficulties nor preparing them for further study of mathematics and mathematics-related subjects. To be able to assist these students, there must be a systematic way of identifying their difficulties and an appropriate instructional program that targets their specific difficulties. This was the stimulus for the development of the Probe Assessment of Mathematical Abilities (PAMA) and for the FOCUS modules.

During the development of PAMA, the 2200 college students in remedial mathematics courses who participated in the project experienced great difficulty with translating words to symbols and vice versa; understanding proportionality and using proportions to solve problems; interpreting mathematical relations presented in graphs; understanding the concept of variable and the uses of variables in expressions and equations; carrying out complex computations with fractions, decimals, and integers; and being able to use various strategies to solve mathematical problems. Not only are these skills and concepts not addressed in remedial courses, but there is a paucity of instructional materials that focus on these skills and concepts. Materials that do deal with these topics are generally poorly developed. For some of the topics, materials do not exist.

Project FOCUS staff began with the mathematical topics with which college students have great difficulty as identified by PAMA. High School and college level textbooks that include these topics were then analyzed in terms of their development of the concepts and skills and the nature of the application problem settings. Some topics received little or no attention, and in the majority of college-level remedial textbooks, application problems were non-existent. Subsequently, FOCUS staff laid out a development for each mathematical topic, and then identified phenomena and problems from the real world which could be modeled and solved by using the mathematics. Because data were real, computations were messy, necessitating the use of a scientific calculator for all modules.

As sections of the modules were developed, they were pilot-tested at Boston University and refined. The same procedure was carried out for complete modules, and the pre- and post-tests. The Instructor's Guide for each module was developed based on feedback from instructors using complete modules in the field-testing. Field-testing of FOCUS modules and Instructor's Guides was carried out at New Hampshire College in Manchester, New Hampshire Technical College at Claremont, Northern Essex Community College in Massachusetts, Boston University School of Education, Wheelock College in Boston, Florida A&M in Tallahassee, Indiana University at Bloomington, and the University of Iowa, as well as at Culver City High School in California and Derby Academy in Massachusetts. In addition, copies of each of the FOCUS modules were sent to the following sites for review and possible use: University of Wisconsin at River Falls, University of Minnesota at Minneapolis, Concordia University in Montreal, University of Maine at Columbia Falls, University of Delaware, East Carolina University at Greenville, University of New Hampshire at Derry, Indiana University at Bloomington, Portland State University in Oregon, Virginia Technical School in Blacksburg, Pennsylvania State University at New Kensington, University of South Carolina, Pennsylvania State at University Park, University of Georgia, and the University of Oklahoma. In addition, materials were sent to coordinators of the College Board Equity 2000 Agenda Project at Providence, Rhode Island; Fort Worth, Texas; Nashville, Tennessee; San Jose, California; Milwaukee, Wisconsin; and Prince George's County, Maryland.

The FOCUS Modules

FOCUS consists of seven mathematics instructional modules, each of which *focuses* on a specific mathematical concept and related skills, and may be used by students without instructor intervention. The seven modules are:

1) Integers and Computation with Integers

   This module develops the concept of integer, ways in which integers can be represented, the comparing and ordering of integers, and computation with integers. The Chinese Red-Black Rod Model, where a red rod represents $^{+}1$ and a black rod represents $^{-}1$, is used to model addition and subtraction. Integers are compared and ordered on a number line. Throughout the module, real-life settings enhance understanding of ways in which integers are used.

2.) Fractions, and Computation with Fractions

   Comparing and computing with fractions are the foci of this module. The relationship between fractions and their decimal equivalents is emphasized, and the technique for converting fractions to decimals for comparison and computation purposes is presented. Historical references to the use of fractions in real-world applications are included throughout the module.

3.) Decimal and Computation with Decimals

   The focus of this module is on the comparison and computation of decimals. Writing very large and small numbers in scientific notation form and computing with numbers in this form using a scientific calculator are developed.

4.) Ratio, Proportion, and Percent

   This module focuses on development of an understanding of ratio, proportion, and percent and the application of proportional reasoning to the solution of real-life problems. Applications from statistics, architecture, science, and mathematics are explored. The concept of percent is examined with percents less than one as well as with percents greater than 100. Problems involving percent increase, percent decrease, discount and markup are explored.

5.) Graphs and Their Interpretation

   This module focuses on mathematical relations presented in linear and curvilinear graphs. Graphs are compared with one another and matched with phenomena from the natural, physical, and social sciences. The concept of slope and the processes of interpolation and extrapolation are explored in detail. Typical graph interpretation misconceptions are identified, illustrated, and discussed.

6.) Variables and Equations

The focus of the module is on the symbolic representation of mathematical relations presented in prose. The meaning of variables and the behavior of dependent and independent variables are described, followed by development of the concept of equality and procedures for the words to symbols translation of mathematical relations. Step-by-step directions are given for solving equations and systems of equations. All problems in the module use real data, and require formulation and solution of one or two equations.

7.) Problem Solving

This module represents an overall plan for solving problems and detailed instruction for the use of six problem-solving strategies: 1) organizing information, 2) extending patterns and generalizing functions, 3) simplifying the problem by reducing the magnitude of the data or by decomposing the problem into component parts. 4) making cases, 5) modeling, and 6) working backward. Strategies are used either singly or in combination to solve problems from a variety of settings.

Each module has been designed to be used independently of the others, and may serve as the content focus of a short-term mini-course, or to support existing remediation programs. Students may use the modules individually in tutorial settings, in small study- group sessions, or as textbooks for an entire course.

For each module, there is an Instructor's Guide that contains pretests and posttests, a reproduction of the student module with complete answers, and a description of the project and ways in which the modules may be used. Beginning September, 1992, and until a publisher can be found, FOCUS modules and Instructor's Guides will be sold at cost by Boston University.

PURPOSE

There are more than two million students in United States colleges and universities who are enrolled in remedial mathematics courses, and trends suggest that this number is increasing. Regardless of their specific mathematical difficulties, students are generally assigned to generic-type remedial courses, which review arithmetic computations and algebraic manipulations. During the development of PAMA, the Probe Assessment of Mathematical Abilities, (FIPSE funded, 1985-87, Greenes), greater insight was gained into the types of difficulties college students have with mathematics, and in most cases, the difficulties were not computational in nature. Subsequent examination of high school and college-level mathematics textbooks revealed either limited or no attention to these difficult topics (e.g., interpretation of graphs, problem solving heuristics), or developments (e.g., variables, proportionality) that relied solely on rote memorization without understanding. Further, in almost all instructional materials for college-level students, there was a paucity of application problems.

The mission of Project FOCUS was to develop instructional modules that focus on the major mathematical concepts and skills deemed prerequisite to the study of higher level mathematics and other math-related courses, and which were identified by PAMA as topics of great difficulty for the majority of college students enrolled in remedial mathematics courses. The combination of settings with real data and the fill-in "interactive" format engages student interest in the text, encourages students to read analytically for relevant information, and promotes student participation in the instructional development of a concept or skill and its application to the solution of real-world problems. The required use of scientific calculators with all modules permits the use of real data, relieves the tedium of computation with these data and retains the emphasis on the problem solving process.

When Project FOCUS began in 1989, the plan was to use performance on PAMA to assign students to modules. Unfortunately, PAMA runs on Apple II computers which are no longer being supported by the Apple Computer Company. As a consequence, most colleges and universities have switched to Macintosh computers or have moved to an IBM platform. Thus, use of PAMA has declined. For this reason, pretests were developed for each module as a means of assessing a student's need for the module.

FOCUS modules may also be used at the high school level to help students with difficult topics before they enter institutions of higher learning. FOCUS modules may reduce the need for remedial courses in mathematics at colleges and universities nationwide.

BACKGROUND AND ORIGINS

For several years, mathematics education faculty at Boston University were concerned about the increasing number of students in need of remediation in mathematics, and the fact that after completing remedial courses, students generally did not continue to study mathematics or math-related subjects.

In 1985, with funding from FIPSE, the mathematics education faculty began work designing the Probe Assessment of Mathematical Abilities (PAMA), a computer-based instrument that assesses the computational, linguistic and cognitive factors that contribute to college students' difficulties with mathematics. During the development of PAMA, project staff gained insight into those mathematical skills related to the topics of arithmetic, algebra, graphs, and applications, that give students little or no difficulty, moderate difficulty, or great difficulty.

Little or No Difficulty:

Arithmetic

- Compute with whole numbers.
- Compute with integers.
- Add and subtract decimals.
- Find the percent of a number.

Algebra

- Substitute values for variables.
- Solve equations with one variable.

Graphs

- Identify the coordinates of a point that lies on the intersection of grid lines.
- Interpret bar graphs.

Applications

- Read, understand, and obtain data from prose.
- Solve word problems with action sequences.

Moderate Difficulty

Arithmetic

- Multiply with decimals.
- Compute with fractions.
- Find decimal equivalents for whole number percents greater than 100 and less than 10.

- Compute with measurement units.

Algebra

- Translate words to symbols: Use a variable to express a direct relation.

Graphs

- Distinguish between two lines given a prose description of the relation.
- Recognize the function of the scales and title of the graph.

Applications

- Solve special case word problems taught algorithmically (mixture, distance-rate-time).

<u>Great Difficulty</u>

Arithmetic

- Divide with decimals.
- Find decimal equivalents for percents with fractions.
- Compute with percents.

Algebra

- Translate words to symbols: Use a variable to express an inverse relation.
- Recognize and write equivalent equations to solve equations.
- Solve proportions.
- Write equations to express relations.
- Solve equations with two variables.

Graphs

- Interpolate.
- Extrapolate.

Applications

- Solve word problems for which data must be obtained from a graph through interpolation or extrapolation.
- Solve variations of special case word problems taught algorithmically (e.g.,break-even analysis).
- Solve word problems in unfamiliar settings.

An examination of college level mathematics textbooks used in remedial/basic mathematics courses revealed that much of what was being taught addresses those skills with which students have little or no difficulty. Furthermore, in studies with students enrolled in remedial courses at Boston University and the University of New Hampshire, it became apparent that there are many students whose difficulties in mathematics are very specific and, if identified, could be remediated with short-term remediation sessions. This would be a benefit not only to Boston University and the University of New Hampshire, but also to colleges and universities nationwide in which resources have been diverted from regular programs to remediation efforts. This would also benefit students who are in need of short programs of remediation but who currently must endure a full semester or a year of review. For these students, the experience of being classified as "remediation students" as well as having to take a full course in which much of the content is already known, is sufficient to discourage any future study of mathematics and mathematics-related topics. Based on the data collected during the development of PAMA, it was evident that the mathematics curriculum of remedial courses needed to be redesigned to focus on the difficulties students actually experience with mathematics. And, the remedial materials must be in the form of modules, each of which focuses on a different topic, in order that students can be assigned to only those topics with which they need additional work. These were the factors that prompted the design of the FOCUS modules.

Development of Instructor's Guides

Based on comments from instructors, tutors, and students, the Instructor's Guides were developed in the third year of the project. Each module has an Instructor's Guide, which contains detailed description of the content of the sections of the module, identification of the prerequisite knowledge and skills, a reproduction of the student module with complete and alternative answers, and pre- and posttests with answers. In addition, general information about the program is presented, as well as synopses of all modules, methods for implementing the modules, and a suggested time frame.

Methods of Utilization

During the pilot-testing phase, the modules were used in a variety of settings. Modules were incorporated into peer tutoring sessions, whole class instruction, small group exploration, and individualized study. For each of these settings, and based on the needs of the students, the level of teacher involvement varied as did the amount of time students needed to complete the modules and the number of supplementary materials they used.

Teachers in middle schools, high schools and two- and four-year colleges used the FOCUS modules to teach and remediate concepts through whole class, small group and individualized instruction. In the whole class instruction model, the teacher explained concepts perceived to be particularly difficult or unfamiliar to students in advance of students' work on the modules. Students then worked alone or in pairs to complete the modules. This was done both in and out of class. Small group instruction included having students work on the modules in groups of three or four. Students worked on their own, then discussed problems and questions together as the need arose.

Some instructors reported that students who displayed or described a weakness in an area that was not part of the course syllabus, were directed to use one or more FOCUS modules on their own. Students who worked alone with the modules were offered the answer key for checking their work, then encouraged to meet with the teacher to discuss any difficulties.

At Boston University, undergraduate education majors are required to pass the Probe Assessment of Mathematical Abilities (PAMA) with a score of 60% or better in order to be certified as teachers. Students scoring less than 60% are paired with mathematics education majors who serve as peer tutors. The PAMA assessment tool is used to pinpoint a student's area(s) of difficulty. The student then works together with the tutor and the appropriate FOCUS modules to remediate only those areas in which there have been demonstrated difficulties. Students and tutors usually work together one or two hours per week discussing concepts, or working through sections of a module together. Students take the modules home, then return to the tutor for feedback on the completed sections. The peer tutoring program has been very successful. Both students and tutors have been positive about both the peer tutoring experience and the FOCUS materials themselves. With one or two exceptions, students have passed the PAMA examination immediately after completing the tutoring with the FOCUS modules, scoring 10 to 20 points higher than before.

Focus Module Descriptions

I. INTEGERS AND COMPUTATION WITH INTEGERS

This module develops the concept of integer, ways in which integers can be represented, the comparing and ordering of integers, and computation with integers. The Chinese Red-Black Rod Model, where a red rod represents $^{+}1$ and a black rod represents $^{-}1$, is used to model addition and subtraction. Integers are compared and ordered on a number line. Throughout the module, real-life settings enhance understanding of ways in which integers are used.

Prerequisite Knowledge

- Compute with whole numbers and decimal numbers.
- Find information in multi-column tables.

Module Content

I. Representing Integers

The Chinese Red-Black Rod Model was used as early as 500 B.C. to perform calculations with positive and negative numbers. In this model, red rods represent positive numbers and black rods represent negative numbers. Using the rods, every integer can be represented in a variety of ways. For example, the number $^{+}3$ can be shown with 3 red rods, or 4 red rods and 1 black rod ($^{+}4$ and $^{-}1$), or 5 red rods and 2 black rods ($^{+}5$ and $^{-}2$). The additive inverse property, used in the addition and subtraction algorithms, is illustrated as the sum of one red rod and one black rod ($^{+}1 + {}^{-}1 = 0$).

II. Comparing and Ordering Integers.

Some quantities have both magnitude and direction. Such quantities are described by integers, and are best compared and ordered using a number line. In application settings, positive direction is described as *above, gain, win,* and negative direction is described as *below, loss, lose.* Application problems draw attention to these directional terms.

III. Adding Integers.

Addition of integers is modeled as the joining of sets of red (positive) and/or black (negative) rods. For example, to find the sum, $({}^{-}3 + {}^{-}3)$, 3 black rods are joined with 3 black rods for a total of 6 black rods, or 6 negatives $({}^{-}6)$. To find the sum, $({}^{-}3 + {}^{+}4)$, 3 black rods are joined with 4 red rods. Since 3 black rods together with 3 red rods add to 0, they are removed, leaving one red rod, or ${}^{+}1$.

IV. Subtracting Integers.

Subtraction is modeled as take-away, or the removal of a subset of rods. To find the difference, $({}^{-}6 - {}^{-}2)$, a set of 6 negatives (6 black rods) is formed. From this set, 2 negatives are removed, leaving 4 negatives. Thus, $({}^{-}6 - {}^{-}2 = {}^{-}4)$. The case in which the two integers, A and B in A - B have different signs, and the case in which the absolute value of B is greater than the absolute value of A, as for example, $({}^{+}5 - {}^{-}8)$ or $({}^{-}2 - {}^{-}4)$, are developed in detail.

V. Multiplying and Dividing Integers.

No model is provided for the multiplication and division of integers. Rather, directions are given to multiply and divide integers as with whole numbers, and then determine the sign of the answer (i.e., third number). If the two numbers have the same sign, the sign of the third number is positive; if the two numbers have different signs, the sign of the third number is negative. A Signed Number Rap assists memory and recollection of the rules for determining the sign of the third number.

The second part of the section focuses on computations with exponents. Both positive and negative numbers are used as base numbers, but only positive exponents are used.

The third part of the section deals with order of operations. There is evidence that students have limited knowledge of the Fundamental Order of Operations which identifies the order in which computations should be performed (i.e., (1) computations in parentheses, (2) powers, (3) multiplications and divisions from left to right, and (4) additions and subtractions from left to right). Detailed guidance for following the Fundamental Order of Operations to compute the value of numerical expressions is provided.

In the final part of this section, the order of operations on a calculator is explored. While some calculators are programmed to perform computations following the Fundamental Order of Operations, others are not. A test is provided by which it can be determined if the calculator is programmed to follow this fundamental order. Ways to use the *change sign* key to enter positive and negative numbers in the calculator, and ways to use the exponent key, are discussed.

II. FRACTIONS AND COMPUTATION WITH FRACTIONS

Comparing and computing with fractions are the foci of this module. The relationship between fractions and their decimal equivalents is emphasized, and the technique for converting fractions to decimals for comparison and computation purposes is presented. Historical references to the use of fractions in real-world applications are included throughout the module.

Prerequisite Knowledge

- Compute with whole numbers and integers.
- Convert fractions to decimals.
- Use a scientific calculator.

Module Content

I. Fractions and Equivalence Classes

A fraction is defined as the quotient of two integers. The zero property of multiplication is employed to explain why fractions cannot have divisors of zero. The multiplicative property of one is used to define equivalent fractions, to generate equivalence classes, and to reduce fractions. Fractions are compared by renaming them with common denominators and then comparing their numerators, or by comparing their decimal equivalents. A technique for converting fractions to decimals is presented. Fractions that have finite and infinite repeating decimal equivalents are differentiated.

II. Multiplication

A technique for computing products of fractions is presented, followed by methods for estimating the magnitude of products based on the values of the factors. The section concludes with the development of the multiplicative inverse relationship.

III. Addition

Addition of fractions is carried out using a common denominator approach in which the common denominator is always the product of the denominators of the addends. Addends involve fractions less than, equal to, and greater than one.

IV. Subtraction

The same common denominator approach as used with addition is employed for subtraction, and estimation before computation of differences is stressed. Once again, fractions less than, equal to, or greater than one are used.

V. Division

Division of fractions is also carried out using a common denominator approach. Once the common denominator is determined and fractions are renamed with the same denominator, the quotient is found by dividing the numerators. Techniques for estimating quotients are developed.

III. DECIMALS AND COMPUTATION WITH DECIMALS

The focus of this module is on the comparison and computation of decimals. Writing very large and small numbers in scientific notation form and computing with numbers in this form using a scientific calculator are developed.

Prerequisite Knowledge

- Compute with integers, exponents, and fractions.

Module Content

I. Comparing, Ordering and Rounding Decimals

In this section, techniques for comparing and ordering decimals are presented. Decimals are compared by examining them digit-by-digit in terms of place value. Decimals are rounded to tenths, hundredths, and thousandths place using the "5 or greater" rounding rule.

II. Adding and Subtracting Decimals

Addition and subtraction of decimals is introduced through an application in forestry. The algorithms for both operations direct students to compute as with whole numbers then place the decimal point in the answers.

III. Multiplying Decimals

In this section, the multiplication algorithm is developed. Emphasis is placed on estimating products by considering the magnitude of the product in relation to the size of the factors. Scientific notation is introduced and techniques for converting very large and very small numbers from decimal to scientific notation form and vice versa are presented. Computations are performed both by hand and with a scientific calculator.

IV. Dividing Decimals

Techniques for dividing decimal numbers are developed. Division with positive decimal numbers is checked by noting relationships between quotients and divisors when divisors are less than or greater than one. Very large and very small numbers are converted to scientific notation form before computations are performed.

IV. RATIO, PROPORTION, AND PERCENT

This module focuses on development of an understanding of ratio, proportion, and percent and the application of proportional reasoning to the solution of real-life problems. Applications from statistics, architecture, science, and mathematics are explored. The concept of percent is examined with percents less than one as well as with percents greater than 100. Problems involving percent increase, percent decrease, discount and markup are explored.

Prerequisite Knowledge

- Compute with fractions and decimals.

Module Content

I. Ratio and Proportion

In this section the concepts of ratio and proportion are developed with real-life applications. Ratios are used to describe part-to-whole relationships (e.g., number of left-handed students to total number of students) and part-to-part relationships (e.g., number of left-handed students to number of right-handed students). Variables and phrases are used to represent proportional relationships.

II. Ratios and Rates

A rate is a special kind of ratio in which the two quantities being compared are of different types (e.g., number of *miles* to number of *hours*). Since rates are ratios, rate and ratio problems may both be solved using proportions. Unit ratios and unit rates (i.e., comparing many to one) are given special attention because of their common use in unit pricing and in measurement (e.g., miles per hour, population per square mile, and feet per second.)

III. Applications: Sampling

Sampling in which a small group, the sample, is selected and analyzed as representative of the whole group, or population, is used in many fields. In this section, the technique of sampling is developed and used to make predictions.

IV. Applications: Similarity

When two plane geometric figures are similar, the ratio between any pair of corresponding sides is the same as the ratio for any other pair of corresponding sides. Similar triangles are used to determine lengths of shadows, and heights of buildings, trees and people.

V. Applications: Scaling

A scale is a ratio or a comparison of actual size to, for example, drawing or model size. Applications include blueprints, models of the solar system, sculptures, life-drawings, and movie props.

VI. Applications: Simple Machines

In this section, proportional relationships involving pulleys, levers, and gears are examined. Pulleys change the direction of an applied force. There is a relationship between the distance a rope is pulled and the distance an object is raised. Likewise, there is a relationship between weight of an object raised and effort applied. The lever is designed such that a force applied at one point of a lever produces a useful action at another point. There exists a proportional relationship between effort applied and weight moved. Bicycle gears are used to vary speeds of wheels which in turn produce changes in the force needed to propel the bicycle. Proportions are used to compare the speeds of interlocking gears, and to calculate and compare distances wheels and pedals travel.

VII. Percent

Percent is introduced as a ratio in which a number is compared to 100. All forms of percent problems are solved by setting up proportions where one of the ratios is the *percent ratio* (i.e., the percent compared to 100). Percents less than one, from one through 100, and greater than 100 are used in the application problems dealing with Neilson Ratings, the amount of alcohol in various beverages, and intoxication levels.

VIII. Percent Change

Percent increase and percent decrease as well as mark-ups and discounts are discussed in this section. Proportions are formulated and solved for all application problems. One- and two-step methods for solving mark-up and discount problems are analyzed.

V. GRAPHS AND THEIR INTERPRETATION

This module focuses on mathematical relations presented in linear and curvilinear graphs. Graphs are compared with one another and matched with phenomena from the natural, physical, and social sciences. The concept of slope and the processes of interpolation and extrapolation are explored in detail. Typical graph interpretation misconceptions are identified, illustrated, and discussed.

Prerequisite Knowledge

- Interpret the title, labels, key, and scales of a graph.
- Identify the (x,y) coordinates of a point on a graph.
- Locate a point on a graph given the (x,y) coordinates of the point.
- Compute with integers, decimals, and fractions.

Module Content

I. Graph Reading

The section begins with the identification and interpretation of points on a line graph. A technique for locating the interval of greatest and least change is presented. Double line graphs are investigated and data and relationships represented by the lines are compared. The section concludes with explorations of slope with lines that do and do not contain the origin. Through applications, variable and fixed costs are related to a line's slope and y-intercept respectively.

II. Linear Relations

Through examination of tables of data, relationships between data and graphs of these data are uncovered. Characteristics of data sets that produce lines and curves are identified. Slopes of lines (both positive and negative) are calculated from tables of data, and the relationship between the magnitude of the slope and the steepness of a line is explored. Applications involve direct and inverse relations from real-life situations.

III. Extrapolation

Extrapolation is the process of computing values beyond the range of values shown in the graph. In this section, extrapolation is investigated with step-functions and graphs of lines that do and do not contain the origin. Applications involving the computation of extrapolated values include taxi and electricity rates, and paint costs.

IV. Interpolation

Interpolation is the process of identifying the coordinates of a point on a graph that lies between two points whose coordinates are known. Techniques are presented for interpolating directly from the graph, as well as by using the slope of the line. Applications focus on appreciating and depreciating costs.

V. Curves

The relationship between real situations and their representations in tabular and graphical form is the focus of this section. Both direct and inverse relations are examined, and situations involving decreasing and increasing rates of change are distinguished.

VI. Graph Interpretation

The final section examines relationships among pictures, paragraphs, and graphs of an event, and typical misconceptions associated with relationships among the representations. Applications relate distance and time, distance and speed, and the filling of containers and time.

VI. VARIABLES AND EQUATIONS

The focus of the module is on the symbolic representation of mathematical relations presented in prose. The meaning of variables and the behavior of dependent and independent variables are described, followed by development of the concept of equality and procedures for the words to symbols translation of mathematical relations. Step-by-step directions are given for solving equations and systems of equations. All problems in the module use real data, and require formulation and solution of one or two equations.

Prerequisite Knowledge

- Compute with whole numbers, integers, decimals and fractions.
- Use a scientific calculator.

Module Content

I. Using Variables and Equations

This section begins with a discussion of variables. A variable always represents a *number* of things as for instance, tickets, booklets, dollars, inches, and people. A $\square$, to "hold" a number, is used to represent a variable. Dependent and independent variables are differentiated. Independent variables are those that may vary in value. By contrast, the value of a *dependent* variable is the result of computations involving the number(s) chosen for the *independent* variable(s). Application problems deal with precipitation, blood pressure, and the skid distance of automobiles.

An equation is defined as a mathematical sentence that shows balance, and is represented using a two-pan balance scale model. Equations involving *more than, less than, N times as many as* relationships are developed using the pan balance. Combined relationships are also presented. Applications involve shoe sizes, temperature, piano keys, body measurements, and forensic medicine.

II. Solving One Equation

A technique for solving an equation with one variable is developed with emphasis on the need to "balance" the equation at all times. Applications involve mountain climbing, gold mine and temperature relationships; the expansion of bridges; the flowering of fruit trees; and stock market averages.

III. Solving Systems of Equations

The technique of substitution is the method developed for solving systems of two equations with two variables. Applications involve sports, fruit baskets, and temperature conversions.

IV. Special Applications: Cost

All of the application problems in this section deal with cost: cost of car rentals, brokerage fees, and book royalties. All application problems require one or two equations to be formulated and solved. Fixed costs and variable costs are highlighted and distinguished. Fixed cost is then related to the constant term in an equation and the point where a line intersects the vertical axis of a graph. Variable cost, as its name suggests, changes depending on the value of the variable. The variable unit cost appears as the coefficient of the variable in a linear equation, and as the slope of the line in a graph of the equation.

V. Graphic Solution Method

The *break-even point* in business occurs when total cost and total income are equal. This point can be identified on a graph as the intersection of the lines representing cost and income. This section describes a method of graphing the lines for cost and income and identifying the *break-even point*. Income less than the amount identified by the *break-even point* will result in a loss; income greater than the break-even amount will result in a profit. This section also relates the steepness of the line in a graph to the magnitude of the variable unit cost.

VI. Special Applications: Mixture

A variety of mixture problems are presented and require the formulation of systems of two equations with two variables for their solutions. Mixture application problems deal with investments, food, and alcoholic beverages.

VII. Special Applications: Distance-Rate-Time

Distance-rate-time application problems are of three types: 1) opposite direction problems involving two vehicles who start at the same point and travel in opposite directions; 2) closure problems involving two vehicles starting at different points and traveling toward one another until they meet or pass; and 3) overtake problems involving two vehicles traveling the same route where one vehicle leaves later and travels at a faster rate than the other vehicle. The equations that represent the relationships must be formulated and solved in order to produce the solutions.

VIII. Special Applications: Work Together

For all of these problems the amount of work done is calculated using the equation, *Amount of work = (rate of work) x (time worked).* In work together problems, the rate of work per unit of time for each worker is established. These rates are then used to compute the amount of work completed by each worker. The total amount of work accomplished is the sum of the workers' performances.

IX. Puzzle Problems

This section presents *consecutive number problems*, *age problems*, *coin problems*, and *bottle-and-cork* problems. For each problem, subjects must write the equation or equations that represent the mathematical relationships presented in the text, and then solve the equations to find the solutions to the problems.

VII. PROBLEM SOLVING

This module presents an overall plan for solving problems and detailed instructions for the use of six problem-solving strategies: 1) organizing information, 2) extending patterns and generalizing functions, 3) simplifying the problem by reducing the magnitude of the data or by decomposing the problem into component parts, 4) making cases, 5) modeling, and 6) working backward. Strategies are used either singly or in combination to solve problems from a variety of settings.

Prerequisite Knowledge

- Compute with integers, decimals, and fractions.
- Use variables to write functions.
- Manipulate expressions containing exponents.

Module Content

I. Solving Problems: Introduction

The introduction presents George Polya's four-step model for solving problems. The four steps, 1) understand the problem, 2) make a plan, 3) carry out the plan, and 4) look back, define the non-linear process. Each step of the process may be revisited several times during the solution of a problem.

II. Organizing Information

When a set of possibilities or a set with no replications must be generated for the solution to a problem, the identification of all elements of the set may be cumbersome if not properly organized. Methods for organizing data including the labeling of regions, the creation of tables, and the construction of tree diagrams are illustrated. The multiplication principle is developed as a means of computing the total number of possibilities or elements in a set when the specific identity of each element is not required. Logic tables are presented as a method for solving problems in which elements from two or three sets must be matched based on constraints (e.g. clues) or inferences drawn from problem conditions.

III. Extending Patterns and Generalizing Functions

The identification of numerical and geometric patterns, the verbal description of the generalizations of the patterns, and the words to symbols translation of the generalizations are the foci of this section. Functional relationships among pairs of data arranged in tables are explored and the Technique of Finite Differences is developed as a method for generating functions from tables of ordered pairs.

IV. Simplifying the Problem

Some problems appear so complex that it is often difficult to know how to begin to solve them. In this section, two simplification techniques are developed: 1) reducing the magnitude of the data (i.e., the "size" of the problem) which permits analysis of the reduced data, the identification of relationships, and the application of that information or technique to the solution of the original "big" problem; and 2) decomposing the problem into its component pieces or sub-problems, thereby facilitating application of known techniques to the sub-problems. The original problem is then recomposed and the solution is the "sum of the parts."

V. Making Cases

Certain types of problems may be solved by using some of the problem's conditions to establish sets of candidates for the solutions, and then testing the candidates against the remaining conditions of the problem in order to home in on the answers. Three types of problems are used to develop this logical reasoning strategy: 1) number problems in which strings of digits must be identified, 2) incorrectly labeled boxes that must be relabeled, and 3) situations in which statements are assumed to be true or false, and contradictions are identified.

VI. Modeling Problems

In this section, action diagrams and Venn diagrams are used to model, analyze and solve application problems. Action diagrams are used with compass and direction problems; logic problems dealing with river crossings and bears climbing trees; and problems with footsteps and strides, bouncing balls, and filling containers. Venn diagrams are used to represent and analyze complex relationships among overlapping sets.

VII. Working Backward

The work-backward technique is used with problems in which information about the end state is given, and information about the initial state must be found. The technique requires the reversal and "undoing" of steps in the solution process. In the case of numeric problems, the undoing involves the use of inverse operations (e.g. multiplication to "undo" division). The technique is developed with problems about mystery numbers and flow charts, wills and benefactors, the pouring of liquids, and the sharing of money. The section concludes with ways in which the work-backward technique can be used to analyze games and identify winning strategies.

Difficulties Encountered During the Development of the FOCUS Modules

Identifying real settings with real data from the natural and physical sciences, economics, business, history, the arts , and sports, that could be used as the context for application problems and concept development in the modules, required a massive amount of research and a great deal of time. In some cases, as with integers, finding application settings was extremely difficult. Further, conflicting data were often found for the same phenomenon. When this occurred, project staff consulted content experts who either settled the dispute or identified an authentic source for the data.

The use of real situations with real data requires lengthier text presentations than are typically found in existing mathematics textbooks at the high school and college levels. During the formative and summative evaluation stages of the modules, project staff found that the increased amount of reading was praised by the vast majority of students who when asked what they liked best about the modules, said, "The real problems are "interesting" and the modules "explained the math better than did other mathematics textbooks." By contrast, some of the instructors who reviewed the modules before agreeing to test them, said that there was "too much reading" and that the modules would be "too hard" for their students. We suspect that many instructors of remedial mathematics have preconceptions about their students' reading abilities that are inaccurate and that prevent them from using quality instructional materials.

One final problem: From comments made by some instructors in their review of the mathematical developments in the modules, we suspect that remedial courses in mathematics at the college level are often being taught by instructors who themselves are ill-prepared in the content of mathematics.

PROJECT RESULTS

Project FOCUS has developed seven instructional modules, each with an Instructor's Guide, that focus on mathematical topics which are considered to be pre-requisite to the study of higher-level mathematics, the sciences and the social sciences, and with which students at the college level have great difficulty. Each of the modules, Integers and Computation with Integers; Fractions and Computation with Fractions; Decimals and Computation with Decimals; Ratio, Proportion, and Percent; Graphs and Their Interpretation,; Variables and Equations; and Problem Solving has been designed to be used independently of the others. The Instructor's Guide for each module contains pretests and posttests, identification of prerequisite knowledge and skills, descriptions of the sections of the module, a reproduction of the student module with complete answers, summaries of the contents of all of the other modules, alternative instructional methods, and a suggested time frame.

Brochures describing the FOCUS modules are being sent to two-year and four-year colleges in the United States. Until a publisher is found, FOCUS may be obtained by writing to:

Dr. Carole E. Greenes
Associate Dean, and
Professor, Mathematics Education
School of Education
Boston University
605 Commonwealth Avenue
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(617) 353-3289

During the development of the FOCUS modules, Project staff have given presentations that describe PAMA and FOCUS at the Research Pre-session (AERA) of the National Council of Teachers of Mathematics annual meeting in Salt Lake City, Utah, in 1990; the annual meeting of the Education Commission of the States in Denver, Colorado, in 1991; the annual meeting of the Psychology of Mathematics Education - North America in Blacksburg, Virginia, in 1991; and the Spring meeting of the New England Educational Research Organization in Portsmouth, New Hampshire, in 1992. The paper presented at the Psychology of Mathematics Education meeting was published in the conference proceedings. Currently, the FOCUS modules are being used as resource materials by the Equity 2000 Program of the College Board, which is focusing on the enhancement of instruction in mathematics at the grades 8 through 10 levels with the ultimate goal of increased access to higher level mathematics by minority students.

Because FOCUS topics are those generally introduced and studied at the pre-algebra and algebra levels, project staff are exploring use of the modules as supplements to the regular curriculum for these students. Use of FOCUS modules will enhance student understanding of the concepts and skills from their introduction, and thereby increase success with mathematics at the high school level and eliminate the need for remediation.

SUMMARY AND CONCLUSIONS

During the development of the FOCUS modules, project staff gained greater insight into the types of application problems that engage student interest, and the particular algorithms that are easiest for students to understand and remember. These were incorporated into the modules.

With regard to application settings, project staff found that the greater the amount of information presented about the context of a problem, the more likely students were to want to solve the problem and were able to solve the problem. The relationship between problem context, its topic and amount of detailed information, and student success in solving problems, should be investigated further.

With regard to algorithms, project staff found that some procedures (e.g., the addition and subtraction of integers) could be modeled with concrete materials and therefore easily visualized, internalized, recalled and applied. In other cases, as for example with the algorithms for computing with fractions, the use of models did not facilitate understanding nor memory. Further, the traditional approach to addition and subtraction of fractions, of renaming fractions using the <u>least</u> common denominator, confounded rather than simplified the process. By contrast, using the common denominator obtained by finding the product of the denominators, simplified the renaming and computational steps, and was applied successfully in an alternative algorithm for dividing fractions.

During Years 1 and 2 of the project, one of the staff members, Deborah Callahan, a grade 8 teacher of mathematics at Derby Academy in Hull, Massachusetts, used the modules, <u>Graphs and Their Interpretation</u>, <u>Ratio, Proportion and Percent</u>, and <u>Integers and Computation with Integers</u>, with her pre-algebra students as introductions to the topics. Feedback from the students was amazing in light of the differences between the FOCUS modules and grade 8 mathematics textbooks. Typically, grade 8 textbooks have little reading, two- or three-page lessons, few exercises that require any lengthy responses or explanations, and multiple colorful pictures and photographs. By contrast, FOCUS modules are monochromatic. There are no "lessons," but rather, multi-page sections. The prose is abundant with no pictures breaking up the text development. And, the exercises often call for essay rather than numeric-type responses. Commenting on the integers modules, students said (original text with no editing):

> "I liked the realistic questions. This module was fast moving and precise. Most books I use do not explain the concept well and have to many exercises."

> "I liked the way it taught how to add and subtract integers. This (book) is different because it teaches better. In a text book they just have a couple of pages that have to do with adding. In this module it devotes a whole section to it."

"I liked the facts on wind chill and I also liked being able to work with a different system beside adding and subtracting that I have done for the last 5 years. It (book) is shorter and I feel it explains the material alot less confusing than most math text books. Overall I thought the book explained itself well and made sure we understood everything. The one thing I got lost on was page 3 when you asked to explain. There were too many possibilities. You could go on and on." In response to the question, "Do you feel that your understanding of the content of the module has improved as a result of using the module?", the same student responded, "Yes. All the material seems to be sticking in my head."

"I liked the Chinese rod method. It made it much easier to learn how to add, subtract, divide and multiply. It explained what to do clearly and took you threw it step by step. Overall I thought it was a good program and the only thing I would change would be the explaination of the windchill problems."

Commenting on the graphs module, students said:

"I liked the fact that it explained how to do everything very well. The module differs from other books I've had because it explains things better. Its easyier to understand than the textbooks. I feel my understanding of graphs has improved."

"I liked the different ways of finding answers to problems by looking at graphs differently. What I liked least was the vocabulary. It is smaller page wise. It does not have as many problems on one pg. It gives better ways of explaining examples of doing diff. kinds of probs."

"The module was thorough and easy to understand, unlike other mathematics books I have used."

"I liked the true facts you added in the book. It made it more fun to learn. In a text book, there would not be as many pages and problems on graphing. " In response to the question, "Do you feel that your understanding of the content of the module has improved as a result of using the module?", the same student responded, "Yes, because I was able to spend alot of time on things I didn't understand. Now I have them down pact - I think?"

"I liked the true facts in the graph. This book is much clearer and makes the ideas easyer to grasp."

"I liked the graphs w/explainations the most. It (book) was easier and explained itself better."

"Easy to understand. There are more explanations and facts. It explained everything very well."

Because of the success of the modules as introductions to key mathematical concepts and skills, we are considering producing versions of the modules for pre-algebra and algebra-level students. With comprehensive explanations and investigations as presented in the FOCUS modules, learning may be enhanced and the need for remediation, at the high school and college levels, reduced.