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ABSTRACT

This final report and appended conference proceedings describe activities of the Earth Math project, a 3-year effort at Kennesaw State University (Georgia) to broaden and disseminate the concept of Earth Algebra to precalculus and mathematics education courses. Major outcomes of the project were the draft of a precalculus textbook now being reviewed prior to field testing, and a series of independent study modules appropriate for use in mathematics education courses or as supplementary material in algebra and trigonometry courses. Earth Math materials focus on application of mathematics to real world problems and are intended to improve students' problem-solving and decision-making abilities. The materials are also intended to demonstrate how to implement the National Council of Teachers of Mathematics standards in school mathematics courses. Separate sections of the report describe the project's purpose, background and origins, description, evaluation/results, and conclusions. The final evaluation report, which is appended, includes the evaluation instruments, guidelines for grading, and sample administrative communications. Appendix B, the separate proceedings document, includes a keynote address by Tina Straley, summaries of workshops on technology and hydrology, and a talk by Ben Fusaro. Also included are reports from test institutions and a summary of the project evaluation. Most of the proceedings document consists of several chapters of the draft textbook, which conference participants worked through in a workshop. (DB)

FROM EARTH ALGEBRA TO EARTH MATH:

An Expansion And Dissemination Of The Methods Of Earth Algebra

Summary

The goals of the Earth Math project are to broaden and disseminate the concept of Earth Algebra to precalculus and mathematics education courses. New materials were designed which are suitable for precalculus algebra and trigonometry. These materials form the skeleton of a complete precalculus course and can be used in teacher education courses. Incorporation of Earth Math materials into precalculus courses will improve students' problem solving and decision making ability and offer insight and increase interest and appreciation of the role of mathematics in society. Incorporation of Earth Math materials and methods in mathematics education courses will provide pre-service teachers with a model of how the NCTM Standards can be implemented in school mathematics courses. A precalculus textbook incorporation these materials will be published by Addison Wesley in 1999.

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Year 1	40,540
Year 2	59,600
Year 3	53,949
Total	154,097

EXECUTIVE SUMMARY

FROM EARTH ALGEBRA TO EARTH MATH: An Expansion And Dissemination Of The Methods Of Earth Algebra

Nancy Zumoff and Christopher Schaufele
Kennesaw State University

PROJECT OVERVIEW

The goals of the Earth Math project are to broaden and disseminate the concept of Earth Algebra to precalculus courses, both algebra and trigonometry, and to mathematics education courses. New materials, in the form of modular studies, were designed which are suitable for precalculus algebra and trigonometry. These studies serve a dual purpose: not only do they form the skeleton of a complete precalculus course, but together with selected studies from Earth Algebra, they can be used in teacher education courses. Specifically, the goals of the incorporation of Earth Math materials into precalculus courses are to improve students' problem solving and decision making ability and to offer insight and increase interest and appreciation of the role of mathematics in society. Second, the goal of the incorporation of Earth Math materials and methods in mathematics education courses is to provide pre-service teachers with a model of how the NCTM Standards can be implemented in school mathematics courses.

Materials were developed which are appropriate for precalculus courses and mathematics education content courses after research and consultation with environmental experts and faculty from ten test institutions. Then these materials were class tested at designated test sites and a formal evaluations were conducted at selected test-sites. Following this, representatives from the test institutions, consultants, high school teachers, potential class testers, and individuals from around the country active in mathematics reform attended the Earth Math Conference and Workshop at Kennesaw State University in April of 1996. Technology workshops were conducted, reports were delivered on tests, and synopses of other innovative projects were delivered.

In response to all these activities, the materials were modified and have been incorporated into a textbook which is under contract with Addison Wesley, to appear in 1999. Numerous presentations, workshops, and short-courses have been conducted by the project directors; in addition, extensive dissemination activities are scheduled for the future.

The audience served by the project includes both precalculus students and preservice teachers. An additional audience consisting of in-service secondary teachers emerged during the course of the project.

The major outcomes of the project are the draft of a precalculus textbook which is now being reviewed and will be field-tested, and a series of independent studies appropriate for use in mathematics education courses or as supplementary material for algebra and trigonometry courses.

PURPOSE

The purpose of the EarthMath project is to broaden the Earth Algebra concept so that it can be easily used in a wider range of courses, both precalculus and mathematics education, in a variety of different institutions.

First, the goals of the precalculus component of the EarthMath project are to improve students' problem solving and decision making ability, and to offer insight and increase interest and appreciation of the role of mathematics in society. The introduction of resulting precalculus materials is also intended to enhance the learning of the mathematical skills and concepts prerequisite to calculus.

Second, the goal of the education component of the EarthMath project is to provide preservice teachers with a model of how the Standards can be implemented in mathematics courses by educating future teachers using the methods mandated by the NCTM Standards. Indirectly, future students at the secondary level will receive the same benefits as the precalculus students at the college level.

BACKGROUND AND ORIGINS

Until a few years before this project began, precalculus courses were neglected in the major reform efforts. Earth Algebra was one of the first truly radical approaches at this level. Other projects incorporated some of the components of Earth Algebra, but few projects existed which made significant change in the curriculum and pedagogy.

Some newer projects concentrated on modeling techniques to solve applied problems. None of these projects took the EarthMath approach of an extended in-depth study of a relevant, real world problem which drives the study of the mathematics. Neither did any of the current texts, supplements, or projects fully implement the spirit of the NCTM Standards at the college level. Further, the use of in-class group work and the use of graphing calculators are pedagogical components of the EarthMath approach that arise naturally from the curriculum; they are not add-ons to modernize a course.

PROJECT DESCRIPTION

The EarthMath project is an extension of the Earth Algebra project. The major components of the project are:

- development of materials designed in the spirit of Earth Algebra, which can be used in precalculus, both algebra and trigonometry, and in teacher education courses;
- consultation with numerous experts and with faculty at a wide variety of institutions, followed by piloting and testing of materials designed to satisfy varied curricula;
- assessment of the suitability of the materials and subsequent revision according to results;
- dissemination through workshops, conferences, lectures, and short courses nationally, an EarthMath Conference and two regional workshops;
- publication of a reform textbook on precalculus mathematics and a collection of short studies adapted from materials developed.

We feel that our extensive dissemination efforts have been as effective as those of the Earth Algebra project. It should also be noted that feedback from the workshops and presentations was incorporated into the revision of the materials.

EVALUATION / PROJECT RESULTS

The project evaluation was conducted by Dr. Pamela Drummond, Kennesaw State University, with consultant Dr. Mary Linquist of Columbus College, then president of the NCTM. The evaluation results are summarized below.

The evaluator examined the following query: Will the *EarthMath Studies* materials be successful in pre-calculus and mathematics teacher education courses? The following listing provides the major conclusions of the investigation.

1. Using *EarthMath Studies* in precalculus and mathematics teacher education courses is enticing.
2. Students in precalculus and mathematics teacher education courses dramatically improved their genetic mathematical prowess.
3. Precalculus students improved their views toward mathematics.
4. Mathematics teacher education and precalculus students experienced significant different problem-solving and decision making competence.
5. *EarthMath Studies* are remarkably aligned with the NCTM *Curriculum and Evaluation Standards.*
6. The EarthMath Project can be judged successful. This conclusion is justified after viewing the previous five conclusions.

Thus, in the final analysis, *EarthMath Studies* modules can enhance undergraduate precalculus and mathematics teacher education courses.

The major step for continued dissemination will be the publication of the textbook, *Earth Angles: precalculus mathematics with applications to environmental issues*, Addison Wesley, 1999. A compilation of studies resulting from the project will be prepared for use in mathematics education courses or as a supplement to algebra or trigonometry courses. One of the studies will be published in a volume of the MAA notes featuring mathematics and the environment at the precalculus level. We will continue to conduct workshops and presentations on this project. Project evaluation will continue as the complete course is integrated into the curriculum.

SUMMARY AND CONCLUSIONS

The major activities during the grant period are listed below.

1. A new course was developed to replace the traditional precalculus course.
2. The course was piloted at Kennesaw State University and selected modules were tested at KSU and other institutions around the country.
3. A text book under preparation by the project directors is scheduled for publication by Addison Wesley in 1999.
4. A compilation of modules will be prepared for use in mathematics education courses or as a supplement in college algebra or trigonometry courses.
5. The project evaluation was designed and directed by Dr. Pamela Drummond who was not involved in development.
6. Numerous workshops, presentations, and minicourses have been given by the project directors.

FROM EARTH ALGEBRA TO EARTH MATH:
An Expansion And Dissemination Of The Methods Of Earth Algebra

Nancy Zumoff and Christopher Schaufele
Kennesaw State University

PROJECT OVERVIEW

The goals of the Earth Math project are to broaden and disseminate the concept of Earth Algebra to precalculus courses, both algebra and trigonometry, and to mathematics education courses. New materials, in the form of modular studies, were designed which are suitable for precalculus algebra and trigonometry. These studies serve a dual purpose: not only do they form the skeleton of a complete precalculus course, but together with selected studies from Earth Algebra, they can be used in teacher education courses. Specifically, the goals of the incorporation of Earth Math materials into precalculus courses are to improve students' problem solving and decision making ability and to offer insight and increase interest and appreciation of the role of mathematics in society. Second, the goal of the incorporation of Earth Math materials and methods in mathematics education courses is to provide pre-service teachers with a model of how the NCTM Standards can be implemented in school mathematics courses.

Beginnings

Insight gained from reviews of Earth Algebra and presentations and workshops nationwide indicate that the course is being well received as either a terminal mathematics course, or as a prerequisite to "business" calculus or statistics. However, many colleges and universities felt that Earth Algebra does not cover in sufficient depth the topics or skills necessary for students who intend to enter a standard calculus course. Such colleges and universities for the most part are still offering the traditional precalculus because they have not found a viable alternative. The project began in response to these concerns.

Happenings

Materials were developed which are appropriate for precalculus courses and mathematics education content courses after research and consultation with environmental experts and faculty from ten test institutions. Then these materials were class tested at designated test sites and a formal evaluations were conducted at selected test-sites. Following this, representatives from the test institutions, consultants, high school teachers, potential class testers, and individuals from around the country active in mathematics reform attended the Earth Math Conference and Workshop at Kennesaw State University in April of 1996. Technology workshops were conducted, reports were delivered on tests, and synopses of other innovative projects were delivered.

In response to all these activities, the materials were modified and have been incorporated into a textbook which is under contract with Addison Wesley, to appear in 1999.

Numerous presentations, workshops, and short-courses have been conducted by the project directors; in addition, extensive dissemination activities are scheduled for the future. (Note: The National Science Foundation has funded a significant portion of the dissemination of this project.)

Audience

The audience served by the project includes both precalculus students and preservice teachers. An additional audience consisting of in-service secondary teachers emerged during the course of the project.

Outcomes

The major outcomes of the project are the draft of a precalculus textbook which is now being reviewed and will be field-tested, and a series of independent studies appropriate for use in mathematics education courses or as supplementary material for algebra and trigonometry courses. In addition, an all-algebra version of some the studies is being used in the beginning algebra course at Navajo Community College. One study will be published in a volume of the Mathematical Association of America Notes devoted to mathematics and the environment.

PURPOSE

The purpose of the EarthMath project is to broaden the Earth Algebra concept so that it can be easily used in a wider range of courses, both precalculus and mathematics education, in a variety of different institutions.

Problems Addressed

Response to the Earth Algebra Project indicated that many institutions were interested in using the Earth Algebra approach but only offer a single pre-calculus track. Since Earth Algebra was not designed for this purpose, we addressed this problem by developing a similar course which can replace traditional precalculus. Further, future mathematics teachers need to be trained in effective techniques to improve the quality of mathematics education for future generations. By extending the concepts of Earth Algebra to a wider variety of courses and providing easily accessible materials to a wide variety of institutions, we attack these problems on a much broader basis.

The problems addressed in this project were much the same as those identified in the Earth Algebra project. The problems in mathematics education in the United States are well known: students drop out of the pipeline at the rate of fifty percent per year from ninth grade through graduate school; the course content is viewed as irrelevant to one's real life needs and experiences; mathematics is considered too difficult for all but an intellectual elite; it is acceptable to not like mathematics and to not do well; and, finally, it is believed that the country will not suffer if mathematics is left to those who have some special gift enabling them to master the material. We do not believe this is true. The general populace will need greater mathematical skills in the work place and at home in the years ahead, and more mathematicians and scientists will be needed for the health of the economy.

Goals

First, the goals of the precalculus component of the EarthMath project are to improve students' problem solving and decision making ability, and to offer insight and increase interest and appreciation of the role of mathematics in society. The introduction of resulting precalculus materials is also intended to enhance the learning of the mathematical skills and concepts prerequisite to calculus.

Second, the goal of the education component of the EarthMath project is to provide preservice teachers with a model of how the Standards can be implemented in mathematics courses by educating future teachers using the methods mandated by the NCTM Standards. Indirectly, future students at the secondary level will receive the same benefits as the precalculus students at the college level.

Problems Encountered

By working with a variety of test institutions, one problems which emerged is that content mathematics education courses range in level from intermediate algebra to post-calculus. Due to the scope of the project, it was decided after consultation with test institutions to focus on the development of modules not requiring calculus. However, some of these modules are readily adaptable to the use of techniques from the calculus.

The logistics of dealing with many different test institutions for both testing and evaluation were greater than we expected. Also, individual faculty committed to the project don't necessarily end up teaching relevant courses. We learned to be very flexible.

BACKGROUND AND ORIGINS

Context

Until a few years before this project began, precalculus courses were neglected in the major reform efforts. Earth Algebra was one of the first truly radical approaches at this level. Other projects incorporated some of the components of Earth Algebra, but few projects existed which made significant change in the curriculum and pedagogy.

Some newer projects concentrated on modeling techniques to solve applied problems. None of these projects took the EarthMath approach of an extended in-depth study of a relevant, real world problem which drives the study of the mathematics. Neither did any of the current texts, supplements, or projects fully implement the spirit of the NCTM Standards at the college level. Further, the use of in-class group work and the use of graphing calculators are pedagogical components of the EarthMath approach that arise naturally from the curriculum; they are not add-ons to modernize a course.

We quote from the Mathematical Association of America, A Call for Change : "Such substantive changes in school mathematics as envisioned by the NCTM Standards will require corresponding changes in the preparation of teachers. Teachers need opportunities in their collegiate courses to do mathematics: explore, analyze, construct models, collect and represent data, present arguments, and solve problems. The content of collegiate level courses must reflect the changes in emphases and content of the emerging school curriculum and the rapidly broadening scope of mathematics itself." A student enrolled in a precalculus course which has been designed and is taught in the spirit of the NCTM Standards will likely teach in that spirit since we tend to model our teaching on our experiences as students. For these reasons, we decided to design materials specifically for the preservice mathematics teachers

Institutional Information

Kennesaw State University is a comprehensive and progressive regional university in the University System of Georgia, enrolling over 12,000 undergraduate and graduate students. The University offers opportunities for concentrated study in the arts, humanities, the sciences, and the professional fields of business, education, health and social services. During the grant period, the school has undergone significant changes, including a change in mission reflected by the name change from college to university, and major curriculum revision in preparation for conversion from quarter to semester system to be implemented in the Fall of 1998.

The Department of Mathematics has 28 full-time faculty, shares four faculty with the Computer Science Department, four to five faculty with the Developmental Studies Department and employs part-time faculty for fifteen to twenty sections per quarter. The Department offers a Bachelor of Science degree in mathematics, a joint BS degree in mathematics education, and support graduate courses for a master's degree in K-8 education. The University system has a ten hour requirement in core mathematics courses for all undergraduate degrees, and therefore a primary role of the department is service.

Test Sites

The test sites included both public and private, large and small, two-year to graduate level institutions; test instructors ranged from experienced instructors to graduate students. The involvement of test institutions was of varying degrees. Developed materials were used for classroom testing at each of the following institutions: Louisiana State University, Baton Rouge, Louisiana; Salish Kootenai College, Pablo, Montana; Portland State University, Portland, Oregon; Montana State University, Bozeman, Montana; San Juan College, Farmington, New Mexico; Navajo Community College, Shiprock, New Mexico and Tsaile, Arizona; Emory University, Atlanta, Georgia; and Kennesaw State University, Kennesaw, Georgia. Representatives from each of these institutions reported results of their tests to the Earth Math conference which was held at Kennesaw State University in April, 1996. Copies of the proceedings have been widely distributed and are available from the project directors.

PROJECT DESCRIPTION

The EarthMath project is an extension of the Earth Algebra project. The major components of the project are:

- development of materials designed in the spirit of Earth Algebra, which can be used in precalculus, both algebra and trigonometry, and in teacher education courses;
- consultation with numerous experts and with faculty at a wide variety of institutions, followed by piloting and testing of materials designed to satisfy varied curricula;
- assessment of the suitability of the materials and subsequent revision according to results;
- dissemination through workshops, conferences, lectures, and short courses nationally, an EarthMath Conference and two regional workshops on innovative approaches in precalculus teaching, and publication of the Earth Math Conference Proceedings (the National Science Foundation provided additional funds for the project dissemination.);
- publication of a reform textbook on precalculus mathematics and a collection of short studies adapted from materials developed.

Efforts

The most difficult and time-consuming aspect of this project was the development of new applications using real situations. We basically had to start from scratch in gathering data and developing methods. After these modules were developed and refined it was still a major effort to integrate them into a complete course. Even the "traditional" material had to be rewritten and reordered to fit into our approach.

The project consultation was both difficult and rewarding. On our site visits to test institutions to consult about curricula, we, on many occasions, were introduced to faculty doing research on environmental problems. From these contacts, we were able to get a broader range of environmental expertise than we had anticipated. However, due to schedule and personnel changes, it was difficult to maintain continuous communication with some test sites.

After class testing modules, it was discovered that modules simply inserted in a traditional course were not effective. Our response was to design a complete precalculus course with these studies as the backbone. The textbook is under contract with Addison Wesley. Originally our contract was with HarperCollins, the publisher of Earth Algebra, but its college division was sold to Addison Wesley. The first draft was submitted to the publishers in May, 1996, but due to the sale, there has been a delay in the review and revision process and as a result the publication date has been extended to 1999.

Initial evaluation has been completed; a summary is included elsewhere in this report and a complete copy is appended.

We feel that our extensive dissemination efforts have been as effective as those of the Earth Algebra project. Following is a partial list of workshops and presentations.

1994

- Farmington, New Mexico - Presentation, San Juan College;
- Atlanta, Georgia - Presentation to graduate teaching assistants, Emory University;
- Santa Fe, New Mexico - Presentation, College of Santa Fe;
- Salisbury, Maryland - Presentation to Mathematics faculty, Salisbury State University;
- Shiprock, New Mexico - Workshop, Navajo Community College

1995

- San Francisco, California - Presentation at national MAA meetings;
- Emmitsburg Maryland - Workshop/presentation on EarthMath NSF sponsored program for high school teachers
- Farmington, New Mexico - presentation, Navajo Studies Conference
- Newport, Oregon - Presentation, Oregon AMATYC meeting
- Newport News, Virginia - Presentation, Virginia-Maryland MAA/AMAYTC Meetings
- Atlanta, Georgia - Presentation, University System of Georgia Deans of Schools of Business

1996

- Orlando, Florida-MAA and AMS Joint Winter Meetings:
  Mini-course - Earth Math Project
  Presentation - Earth Algebra and Earth Math

- Emmitsburg Maryland - Workshop/presentation on EarthMath NSF sponsored program for high school teachers
- Baton Rouge, Louisiana, - presentation at the Mississippi and Louisiana Section Meeting of the MAA
- Billings, Montana - workshop at American Indian Higher Education Consortium;
- Big Stone Gap, Virginia - Workshop/presentation on EarthMath NSF sponsored program for high school students;
- Kennesaw, Georgia - Earth Math Conference and Workshop (2 days);
- Belcourt, North Dakota - High Plains Rural Systemic Initiative Workshop (6 days) for teachers at tribal high schools
- Portland State University - regional EarthMath conference

1997

- San Diego, California - Presentation, MAA winter meetings;
- Phoenix, Arizona - Two workshops for the Maricopa Mathematics Consortium;
- Emmitsburg Maryland - Workshop/presentation on EarthMath NSF sponsored program for high school teachers

It should also be noted that feedback from the workshops and presentations was incorporated into the revision of the materials.

EVALUATION / PROJECT RESULTS

The project evaluation was conducted by Dr. Pamela Drummond, Kennesaw State University, with consultant Dr. Mary Linquist of Columbus College, then president of the NCTM. The evaluation results are summarized below; the complete report is Appendix A.

Evaluation Summary

The evaluator examined the following query: Will the *EarthMath Studies* materials be successful in pre-calculus and mathematics teacher education courses?

To answer this question, the evaluator posed three research questions. These questions centered on the following constructs, each operationally defined by an associated variable: (a) generic mathematical prowess, as measured by the Calculus-Readiness Test (CRT), (b) view toward mathematics, as measured by the View of Mathematics Inventory (VMI), and (c) mathematical problem-solving and decision making competence, as measured by the Final Examination Questions (FEQ).

In short, the statistical evidence showed that students who participated in this study increased significantly with regard to their general mathematical prowess. Mathematics teacher education students significantly enhanced their problem-solving and decision making competence. Because the *EarthMath Studies* modules were incorporated into the courses less that 35% of the time, it is inappropriate to maintain that their use provides the single cause of these changes. Nevertheless, the materials themselves are particularly noteworthy. The extent to which the *EarthMath Studies* modules respond to the challenge of the NCTM *Standards* is particularly noteworthy.

Assessment Conclusions

To assess the extent of the success of the EarthMath Project, the results of testing six hypotheses were analyzed to determine if the materials enhanced general mathematics ability, views toward mathematics, and problem-solving and decision making proficiency. And, the materials themselves were analyzed to determine the extent to which they are aligned with the recommendations of the NCTM *Standards*. Although the results of these analyses have been discussed in detail and summarized above, the following listing provides the major conclusions of the investigation.

1. Using *EarthMath Studies* in precalculus and mathematics teacher education courses is enticing.

2. Students in precalculus and mathematics teacher education courses dramatically improved their genetic mathematical prowess.

3. Precalculus students improved their views toward mathematics.

4. Mathematics teacher education and precalculus students experienced significant different problem-solving and decision making competence.

5. *EarthMath Studies* are remarkably aligned with the NCTM *Curriculum and Evaluation Standards.*

6. The EarthMath Project can be judged successful. This conclusion is justified after viewing the previous five conclusions.

Thus, in the final analysis, *EarthMath Studies* modules can enhance undergraduate precalculus and mathematics teacher education courses. Students in these courses did experience significant gains in generic mathematical competence and precalculus students made notable gains in attitude toward mathematics. There was a notable difference in problem-solving and decision making ability among both groups of students. Beyond these results is the superb manner in which the materials respond to the challenge of the NCTM *Standards*.

The acquisition of innovative courseware appropriate for undergraduates mathematics students at the precalculus level which are also suitable for use with aspiring mathematics teachers is very refreshing. That the materials allow students to persist in their quest toward becoming mathematically empowered is noteworthy. The above report appears to indicate that the principal goal of the project has been attained.

Plans For Continuation And Dissemination.

The major step will be the publication of the textbook, *Earth Angles: precalculus mathematics with applications to environmental issues*, Addison Wesley, 1999. A compilation of studies resulting from the project will be prepared for use in mathematics education courses or as a supplement to algebra or trigonometry courses. One of the studies will be published in a volume of the MAA notes featuring mathematics and the environment at the precalculus level.

We will continue to conduct workshops and presentations on this project. Currently we have the following workshops scheduled:

Michigan MAA/AMATYC - May '97
Project Kaleidoscope - June, '97
Three workshops for teachers at tribal schools - June and July, '97
Arizona AMATYC - July, '97
ICTCM national conference - November, '97
National AMATYC - November, '97

Support for additional workshops in 1997 and l998 will be provided by Addison Wesley. A small workshop for class testers is tentatively planned for May, '97.

Project evaluation will continue as the complete course is integrated into the curriculum.

SUMMARY AND CONCLUSIONS

The major activities during the grant period are listed below.

1. A new course was developed to replace the traditional precalculus course.
2. The course was piloted at Kennesaw State University and selected modules were tested at KSU and other institutions around the country.
3. A text book under preparation by the project directors is scheduled for publication by Addison Wesley in 1999.
4. A compilation of modules will be prepared for use in mathematics education courses or as a supplement in college algebra or trigonometry courses.
5. The project evaluation was designed and directed by Dr. Pamela Drummond who was not involved in development.
6. Numerous workshops, presentations, and minicourses have been given by the project directors.

Even though Earth Algebra was a radically different course and was subject to a fair amount of criticism, it was widely accepted by many institutions. However, we have discovered that precalculus is much more sacred than college algebra. The resistance to change in this course is much more entrenched. Many of those who have resisted the reform efforts in calculus feel that calculus and precalculus go hand in hand. The natural audience for this precalculus course consists of those institutions willing to experiment with reform calculus. Our regional workshops, mini-courses and presentations have been well attended and well received. Even though there are disagreements with the Earth Math approach, the positive results and excitement it generates in the classroom by far overshadow the objections. Once again, our advice to the practitioners is the advise we received from an early reviewer, "If you're going to be radical, be radical".

APPENDIX A
Evaluation Report

EARTH ALGEBRA TO EARTHMATH PROJECT

A FINAL EVALUATION REPORT

by

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The purpose of this evaluation is to assess a project whose purpose is to broaden and disseminate the philosophy of Earth Algebra, a college algebra course, to a wider range of courses in a variety of institutions. In this spirit, mathematical concepts will be motivated using technology intensive modules (EarthMath Studies) which center around a single environmental issue and require mathematical modeling and the analysis of real data. Hence, this evaluation centers around two issues: the effectiveness of the use of the Studies and the integrity of the materials themselves. Specifically, the question is:

Will the *EarthMath Studies* materials be successful in pre-calculus and mathematics teacher education courses? In assessing the success of the *EarthMath Studies*, the evaluator will address first the extent to which the students exhibit gains calculus-readiness skills and positive attitudes toward mathematics. Further, the study will seek to determine the extent to which the students are empowered to solve problems. Lastly, the results of this evaluation will ascertain the degree to which the *EarthMath Studies* themselves embrace the "spirit" of the NCTM *Curriculum and Evaluation Standards*.

Research Questions

To respond this question the evaluator will answer the following research questions:

1. Does the use of the *EarthMath Studies* modules in a precalculus or mathematics course for pre-service teachers enhance a student's calculus-readiness and/or view toward mathematics?

2. Does experience with *EarthMath Studies* modules enhance a student's problem-solving and/or mathematical decision making ability?

3. To what extent are the *EarthMath Studies* aligned with the

recommendations of the National Council of Teachers of Mathematics (NCTM) as delineated in the 1989 *Curriculum and Evaluation Standards*?

Instrument Selection

In light of the aforementioned research questions, the evaluator selected and/or designed three instruments to be used in this study: Calculus-Readiness Test (CRT), View of Mathematics Inventory (VMI), and Final Examination Questions (FEQ). These instruments may be found in Appendix A.

Calculus-Readiness Test (CRT)

In an effort to assess the extent to which students have appropriate precalculus skills to be successful in a calculus course, the evaluator received permission from the Mathematical Association of America to use the MAA's Calculus-Readiness Test. This test contains twenty-five multiple choice questions that assume the use of calculator technology. The same instrument was used for both pretest and posttest assessments. Questions were marked either correct or incorrect with scores of 1 or 0 accordingly. Partial credit awarded was not awarded.

View of Mathematics Inventory (VMI)

Originally developed in 1972 by W. L. Retig, Sr. in his doctoral dissertation at the Ohio State University, the VMI assesses possible changes in attitudes toward mathematics. The evaluator and the evaluation consultant wrote and/or selected the 38 items which form the instrument used as both a pretest and posttest in this study. Possible choices for responses to the items were: 1) Strongly Disagree, 2) Disagree, 3) Neutral, 4) Agree, and 5) Strongly Agree. Students received 0 points to 4 points for each of the items depending upon how closely aligned the given response was to the expected response. For example, it the correct response was Strongly Disagree, given responses would be awarded points in the following way: Strongly Disagree, 4 Points; Disagree, 3 Points; Neutral, 2 Points; Agree, 1 Point; Strongly agree, 0 Points.

Final Examination Questions

In an attempt to determine the extent to which students are willing to attempt and able to solve problems in the spirit of the *EarthMath Studies*, the project directors prepared two questions for the evaluator to use as instruments in the study. These questions were furnished to the instructors who, in turn, used them as questions on either the final test or the final examination. In this way, the students were careful that their solutions were their 'best work.' Note that these questions were not used as a pretest. Students received a maximum of 4 points for each of the items. Partial credit was awarded using a schema developed by R. Charles, F. Lester, and P. O'Daffer. (1987. *How to evaluate progress in problem solving*. Reston, VA: National Council of Teachers of Mathematics.) A copy of the partial credit delineation is included in Appendix B.

Experimental Design

This study uses a variation of the pretest-posttest design. Initially, fourteen mathematics professors at nine universities, four in the Atlanta Metropolitan Area and others throughout the United States, expressed interest and were invited to participate in the study. Of these, four professors at two universities actually took part. Some sections were excluded because they did not use the *EarthMath Studies* materials. (There was confusion over the distinction between *Earth Algebra* and *EarthMath*.) In others, the instructor simply lost interest or (s)he no longer had "an appropriate class" to teach.

Four instructors in five classes (116 students) completed all phases of the evaluation. These included four sections of precalculus. Of these 101 students, 45 were at Louisiana State University and 56 were at Kennesaw State University. The study also involved one section of 15 Mathematics for Middle Grades Teachers from Kennesaw State University.

Two of the previously described instruments, CRT and VMI, served as both pretest and posttest. All students completed the battery of these two tests as a pretest and, at the end of the course, as a posttest. The FEQ was written as part of the testing in each course at the end of the term.

Hypotheses

To assess the effectiveness of the use of these materials in mathematics courses to effect change in preparedness for calculus and/or views toward mathematics the evaluator examined the following hypotheses:

1. There is no significant gain in scores among precalculus students on the CRT.

2. There is no significant gain in scores among mathematics teacher education students on the CRT.

3. There is no significant gain in scores among precalculus students on the VMI.

4. There is no significant gain in scores among mathematics teacher education students on the VMI.

5. There is no difference in the problem solving and/or decision making scores of precalculus students as measured on the CRT posttest and the FEQ.

6. There is no difference in the problem solving and/or decision making scores of mathematics teacher education students as measured on the CRT posttest and the FEQ.

The results of the six hypotheses will provide information useful in answering the first two research questions. In particular, the results of Hypotheses 1-4 will be central in responding to Research Question 1: Does the use of the *EarthMath Studies* modules in a precalculus or mathematics course for pre-service teachers enhance a student's calculus-readiness and/or view toward mathematics? Research Question 2 asks: Does experience with *EarthMath Studies* modules enhance a student's problem-solving and/or mathematical decision making ability? The response to this query will involve the results of Hypotheses 5 and 6.

Subjects

Students participating in this study were enrolled in Kennesaw State University (KSU), a member of the Georgia University System located 20 miles north of Atlanta, GA, and Louisiana State University (LSU), the only Land Grant University in public state system located in Baton Rouge, LA. During the time of this study the enrollment at the LSU of 23,000 students was approximately double that of KSC with 12,000 students. One other notable difference between the two institutions is that KSU is an urban commuter university and LSU is considered an urban residential institution.

Precalculus students in this study were enrolled at both institutions and were expected to complete (at least) one module. In general, 20% - 25% of the class time in each of the four sections was devoted to *EarthMath Studies* modules. The prerequisite for the particular precalculus course at KSU is Precalculus (Algebra) or Advanced Algebra & Trigonometry in high school with a grade of B. At LSU there is no prerequisite for the precalculus course. KSU is on the quarter system and courses are 10 weeks long for 5 hours. LSU is on the semester system which last 15 weeks for 3 hours.

It should be noted that at the time of registration, students had no indication that the precalculus class for which they were registering would be using *EarthMath* materials. Also note that students in each of these sections at both institutions were expected to use a graphing calculator.

In addition to precalculus students, one section of pre-service middle grades mathematics students participated in the study and completed one *EarthMath Study* module. These students were enrolled in the course for 10-weeks at KSU. Prerequisites for this course include two 'math for teachers' courses and at least one precalculus course. Note that technology is a major component of this course and students were expected to use a graphing calculator.

The number of students in this investigation was 116. The precalculus group contained 101 students from four class sections at two universities. There were three instructors (two at KSU and one at LSU). The mathematics teacher education group contained 15 students and were taught by one instructor at KSU.

Data Gathering

This study occurred during 1995. The mathematics education group and one of the KSU precalculus classes took place during spring quarter and the other precalculus groups participated during the Fall.

Pretests

Prior to the beginning of the course, the evaluator contacted each of the instructors who would be teaching the classes involved in the study to explain the purpose of the evaluation and to delineate the procedures to be followed. In short, the mathematics instructors were to administer the pretest during the first week of the course. Students were to have 60 minutes to respond to the items. The instructors were to encourage the students to cooperate and make an earnest effort. A copy of the Memorandum, Instructor's Information Sheet, and Time-Line are included in Appendix C.

Posttests

Course instructors administered the posttests during one of the final class meetings of the course. As previously noted, the students in both groups completed both the CRT and the VMI as posttests. Note that the results of any testing were not examined until after the posttests were completed. With the direction and supervision of the evaluator, a student assistant assisted in scoring these instruments.

Statistical Results and Discussion

This pretest posttest part of this study entails the use of two variables that will be partitioned into two groups. The pretest and posttest instruments are the Calculus-Readiness Test (CRT) and View of Mathematics Inventory (VMI). The corresponding variables are generic mathematical prowess and attitude toward mathematics. In order to determine appropriate statistical models for data analysis, the scores from the variables listed above will be classified as to type of data and type of distribution. The instrument denoting mathematical problem-solving and decision making competence is the Final Exam Questions (FEQ).

For parametric statistical tests, scores should represent interval data and should be approximately normal. Four of the scores used in the statistical analyses are gain scores, the differences between the posttest and pretests, and one score is the difference between the FEQ and CRT posttest. Hence we will refer to them as CRT Pre-Cal Gain, CRT MathEd Gain, VMI Pre-Cal Gain, VMI MathEd Gain, and FEQ Diff. It should be noted that percentages were used to represent FEQ scores and CRT Posttests before the differences were computed.

The distributions were examined for the sets of scores generated by precalculus and teacher education separately. To determine which distributions are approximately normal, each distribution was compared to the standard or Gaussian distribution using the Shapiro Wilk W Test for Normality. The results are reviewed in the following table:

Table 1

Tests for Normality

Distribution	Mean	StdDev	N	W	Prob<W
CRT Pre-Cal Gain	1.1386	3.2864	101	0.968354	.1037
CRT MathEd Gain	2.6667	3.65800	15	0.935006	.3172
VMI Pre-Cal Gain	1.40217	11.86653	92	0.928977	.0001
VMI MathEd Gain	1.15385	6.90225	13	0.950862	.5725
FEQ Diff Pre-Cal	7.96429	33.40136	98	9.952851	.0055
FEQ Diff MathEd	24.58333	13.82501	15	0.946617	.1456

Four of the distributions, CRT Pre-Cal Gain, CRT MathEd Gain, VMI MathEd Gain, and FEQ Diff MathEd are approximately normal, indicating that parametric statistical models are appropriate. The other distributions, VMI Pre-Cal Gain and FEW Diff Pre-Cal do not appear normal. Tests involving the use of these scores will be treated using nonparametric (distribution free) statistical models.

Results of Hypothesis Testing

This section presents a restatement of the six hypotheses, along with the relevant data used either to reject or not reject each stated hypothesis at the .01 level of significance. These results will provide information useful in answering the first research question. A summary, following the outcomes, is provided in Table 2.

Hypothesis 1. There is no significant gain in scores among precalculus students on the CRT.

Results of the testing shows that the mean gain score in calculus readiness of precalculus students is 1.1386. Because the scores are normally distributed a t-test ($t(101) = 3.4819$,

$p < .0001$) confirms that this gain is highly significant. Therefore, we reject Hypothesis 1.

Hypothesis 2. There is no significant gain in scores among mathematics teacher education students on the CRT.

An analysis of the CRT MathEd Gain scores indicates that the scores are normally distributed. The mean of these scores is 2.6667. This indicates that, on average, students correctly answered between two and three questions more on the posttest than on the pretest. A t-test ($t(15) = 2.8234$, $p < .01$) confirms that this is a significant gain and we reject Hypothesis 2.

Hypothesis 3. There is no significant gain in scores among precalculus students on the VMI.

Because the VMI Pre-Cal Gain scores are not normally distributed we need a nonparametric test to analyze the results. The mean for this group (mean(VMI Pre-Cal Gain) = 1.4) suggests that there was a positive change in student's beliefs about mathematics during the time of the study. However the distribution-free Wilcoxon / Mann-Whitney Test (Signed-Rank) shows that this change (WMW = 444.500, $p > .04$) is not significant using the criteria of this study. Therefore, even though these results are conspicuous, there is insufficient evidence to reject Hypothesis 3.

Hypothesis 4. There is no significant gain in scores among mathematics teacher education students on the VMI.

Since the VMI MathEd Gain scores meets the assumptions for a parametric statistical test, a t-test will be used in response to this hypothesis. The mean of this group is 1.15385. Although positive, the t-test ($t(13) = 0.6027$, $p > .30$) indicates that the VMI MathEd Gain is not significant. Therefore, we fail to reject Hypothesis 4.

Hypothesis 5. There is no difference in the problem solving and/or decision making scores of precalculus students as measured on the CRT posttest and the FEQ.

The FEQ Diff Pre-Cal scores are not normally distributed, but it looks like there is a positive difference. An analysis of the data using the distribution-free Wilcoxon / Mann-Whitney Test (Signed-Rank) shows that there is a difference (WMW = 583, $p < .01$) between the two sets of scores. There is sufficient evidence and we reject Hypothesis 5.

Hypothesis 6 There is no difference in the problem solving and/or decision making scores of mathematics teacher education students as measured on the CRT posttest and the FEQ.

Results of the Shapiro-Wilk W Test indicate that the FEQ Diff MathEd scores are normally distributed, so a t-test will be used to verify that the scores are significantly different. The test statistic, $t(25) = 6.8869$ was highly significant ($p < .0001$). Therefore, we reject Hypothesis 6.

Table 2

Summary of Hypothesis Testing

	Test-Statistic	Prob > Test Stat
Hypothesis 1	t = 3.4819	< .0001*
Hypothesis 2	t = 2.8234	.0068*
Hypothesis 3	WMW = 444.5	.037**
Hypothesis 4	t = 0.6027	.2790
Hypothesis 5	WMW = 583	.0019*
Hypothesis 6	t = 6.8869	< .0001*

*$\underline{P} < .01$
**$\underline{p} < .05$

Research Questions

Research Question 1. Does the use of the *EarthMath Studies* modules in a precalculus or mathematics course for pre-service teachers enhance a student's calculus-readiness and/or view toward mathematics? The results of the four hypotheses provide information useful in answering this question.

With regard to calculus-readiness students in both precalculus and mathematics teacher education courses realize positive increases during the study that are statistically significant. Mathematics teacher education students realized positive gains in their view of mathematics that were highly significant. And, even though the gains made by precalculus students were not high enough to be statistically significant for this study, they were very high.

Although these results are very positive, and in no way detract from the credibility of the *EarthMath Studies* modules, one must be cautiously in assigning tributes. Before giving all the credit to the EarthMath Project, recall that only a **small** portion of the class time of each course that devoted to the use of these materials. It is unreasonable to attribute any change to a treatment that was used as little as 20% -

25% of the time. It does seem permissible to allege that beliefs about mathematics and mathematical knowledge should (and apparently does) increase with additional study of mathematics. Furthermore, using the *EarthMath Studies* modules has no negative effect on precalculus or mathematics teacher education students' view of mathematics. Neither do they de-enhance calculus-readiness. It appears that we continue to add evidence that increased study of mathematics strengthens mathematical attitudes and power.

Research Question 2. Does experience with *EarthMath Studies* modules enhance a student's problem-solving and/or mathematical decision making ability?

With regard to problem-solving and mathematics decision making, the results of several hypotheses are valuable. The evidence suggests that both groups of students were significantly more mathematically empowered at the end of the courses than at the beginning. Hypotheses 5 and 6 imply that their mathematical problem-solving, and decision making competence were even stronger.

Given the small amount of class time that was devoted to using the *EarthMath Studies* modules, one needs to be as cautious answering this question as with the question above. However, it does seem reasonable to assert that students are highly unlikely to attempt to solve 'big' word problems like the ones found on the FEQ and in the *EarthMath Studies* modules without any prior experience. Therefore, even with the short amount of time allotted to these materials in class, it appears that these highly favorable results may be attributed to the EarthMath Project.

Research Question 3. To what extent are the *EarthMath Studies* aligned with the recommendations of the National Council of Teachers of Mathematics (NCTM) as delineated in the 1989 *Curriculum and Evaluation Standards*?

Because these materials are suggested for use in precalculus courses, it is entirely fitting to judge them on the basis of the NCTM's *Curriculum and Evaluation Standards*. Inherent in the answer to the aforementioned question is whether or not these materials are aligned with the particular standards. In the *Standards* document, NCTM presents a vision for mathematics curriculum relevant for the years leading toward the 21st Century. These "standards are value judgements based on a broad, coherent vision of schooling derived from several factors: societal goals, student goals, research on teaching and learning, and professional experience." (p.7) Note that these standards call for a curriculum that "emphasizes conceptual understandings, multiple representations and connections, mathematical modeling, and mathematical problem solving." (p.125)

Curricular materials that embrace these notions aspire to empower students mathematically. Thus, the curriculum (and the instruction) meets the challenge of the *Standards*. Table 3 depicts the results of a comparison to the four process standards and Table 4 presents an alignment of *EarthMath Studies* with the following four curriculum standards: Standard 5, Algebra; Standard 6, Functions; Standard 9, Trigonometry; and Standard 13, Conceptual Underpinnings of Calculus. The six remaining content standards entail topics and concepts which are not germane to *EarthMath Studies* modules.

Using an inspection of the tables as a basis for the decision, it seems plausible to allege that the *EarthMath Studies* are congruent with the recommendations of the *Curriculum Standards*. Specifically, 86% of the pertinent recommendations of NCTM (85% of the process standards and 87% of the content standards) are included in the *EarthMath* modules. For courseware that was designed for undergraduate mathematics courses, this is a very notable citation indeed. In the experience of the evaluator, finding such innovative materials designed for undergraduates is rare.

Table 3

Comparison of Recommendations: NCTM Process *Standards* with *EarthMath St*

Recommendation

Included Not Included

Recommendation			
Mathematics as Problem Solving			
Use problem-solving approaches to investigate and understand content		X	
Apply integrated problem-solving strategies to solve problems within and outside mathematics	X		
Recognize and formulate problems from situations within and outside mathematics		X	
Apply the process of mathematical modeling to real-world problem situations		X	
Mathematics as Communication			
Reflect upon and clarity thinking about mathematical ideas and relationships		X	
Formulate mathematical definitions and express generalizations discovered though investigations			
Express mathematical ideas orally and in writing			X
Read written representations of mathematics with understanding		X	
Ask clarifying and extending questions related to mathematics		X	
Appreciate the economy, power, and elegance of notation and its role in mathematical development		X	
Mathematics as Reasoning			
Make and test conjectures			X
Formulate counterexamples			X
Follow logical arguments			
Judge the validity of arguments			X
Construct simple valid arguments			
Construct proofs for mathematical assertions, including indirect proofs and math'ical induction			
Mathematical Connections			
Recognize equivalent representations of the same concept			X
Relate procedures in one representation to procedures in an equivalent representation	X		
Use and value the connections among mathematical topics			X
Use and value he connections between mathematics and other disciplines		X	

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Table 4

Comparison of Recommendations: NCTM Curriculum *Standards* with *EarthMath* S

Recommendation			Includ
Algebra			
Represent situations involving variable quantities with expressions, equations, inequalities, & matrices	X		
Use tables & graphs as tools to interpret expressions, equations, & inequalities		X	
Operate on expressions and matrices, and solve equations and inequalities		X	
Appreciate the power of mathematical abstraction and symbolism			X
Use matrices to solve linear systems			X
Demonstrate technical facility with algebraic transformations		X	
Functions			
Model real-world phenomena with a variety of functions			X
Represent and analyze relationships using tables, verbal rules, equations, and graphs	X		
Translate among tabula, symbolic, and graphical representations of functions		X	
Recognize that a variety of problem situations can be modeled by the same type of function		X	
Analyze the effects of parameter changes on the graphs of functions		X	
Understand operations on, and the general properties & behavior of, classes of functions		X	
Trigonometry			
Apply trigonometry to problem situations involving triangles			
Explore periodic real-world phenomena using VXK sine and cosine functions			X
Understand the connection between trigonometric and circular functions		X	
Use circular functions to model periodic real-world phenomena		X	
Apply general graphing techniques to trigonometric functions		X	
Solve trigonometric equations and verify trigonometric identities			
Understand the connections between trigonometric functions and polar coord, complex numbers, & series			
Conceptual Underpinnings of Calculus			
Determine maximum & minimum points of a graph and interpret the results in problem situations	X		
Investigate limiting processes by examining infinite sequences and series and areas under curves		X	
Understand conceptual foundations of limit, the area under a curve, the rate of change, & the slope of tangent line, and their applications in other disciplines		X	
Analyze the graphs of polynomial, rational, radical, and transcendental functions		X	

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Summary

The evaluator examined the following query: Will the *EarthMath Studies* materials be successful in pre-calculus and mathematics teacher education courses?

To answer this question, the evaluator posed three research questions. These questions centered on the following constructs, each operationally defined by an associated variable: (a) generic mathematical prowess, as measured by the Calculus-Readiness Test (CRT), (b) view toward mathematics, as measured by the View of Mathematics Inventory (VMI), and (c) mathematical problem-solving and decision making competence, as measured by the Final Examination Questions (FEQ).

The testing instruments have been previously described. In addition, the six hypotheses explicitly describing the relationships among these three variables have been described, as well. The results which emanated from these hypotheses yielded information which aided the evaluator in answering the first two research questions. Comparing the materials to the NCTM *Standards* provided information for answering the third research question. Ultimately, these answers resolve the foremost question of this study which focuses on the success of the *EarthMath Studies*.

Regarding the first research question, the evidence indicated that students involved in the study experienced significant gains in generic mathematics prowess. The precalculus students made modest gains in their attitudes toward mathematics. It is not surprising that the preservice teachers did not realize remarkable gains in their attitudes toward mathematics. For these particular students, this was (at least) their fourth undergraduate mathematics course. Views of mathematics tend to level-off over time of continued study of mathematics. Most of the precalculus students were in their initial mathematics course. But what caused the change in the students?

Because the class time that was given to the use of these materials was meager, it is presumptuous to proclaim that the credit for any significant result belongs solely to the EarthMath Project. However, it is reasonable to claim that the *EarthMath Studies* modules did **not** reduce students generic mathematical ability nor diminish their potential for success in the Calculus or subsequent courses. That, in and of itself, is notable.

An analysis of Research Question 2 showed that problem-solving and decision making competence was stronger at the end of the course. For mathematics teacher education students the difference was highly significant and for precalculus students, even though slightly less significant, the difference was notable. Again, as cautioned stated above, it is difficult to assign credit for the success solely to the EarthMath Project. Given the modest amount of time that the *EarthMath Studies* modules were actually used in class many other factors may have contributed to the differences among students at the end of the term. It does seem fair to allege that the *EarthMath Studies* modules did not reduce students proficiency in problem-solving or decision making. In other words, the use of these materials in courses for precalculus and/or mathematics teacher education students may enhance students ability to attempt and solve problems from situations requiring the use of real data. This is a noteworthy result.

To answer Research Question 3 the evaluator partitioned the 1989 NCTM *Curriculum and Evaluation Standards.* Eight curriculum standards in this highly acclaimed model for 'state of the art' mathematics included concepts which are germane to the *EarthMath Studies*: the four process standards (Problem Solving, Communication, Reasoning, and Communications), Algebra, Functions, Trigonometry, and Conceptual Underpinnings of Calculus. A comparison of the individual components for each standard with the *EarthMath* materials revealed that the modules were highly congruent with the *Standards*.

This may be the most exciting result of the study. Certainly, it is the most conclusive. It may be difficult to substantiate changes made in student behavior, but written curriculum is hard evidence. The *EarthMath Studies* modules are highly innovative undergraduate courseware which are very responsive to the recommendations of the mathematics community for instructional materials.

In short, the statistical evidence showed that students who participated in this study increased significantly with regard to their general mathematical prowess. Mathematics teacher education students significantly enhanced their problem-solving and decision making competence. Because the *EarthMath Studies* modules were incorporated into the courses less that 35% of the time, it is inappropriate to maintain that their use provides the single cause of these changes. Nevertheless, the materials themselves are particularly noteworthy. The extent to which the *EarthMath Studies* modules respond to the challenge of the NCTM *Standards* is particularly noteworthy.

Conclusions

To assess the extent of the success of the EarthMath Project, the results of testing six hypotheses were analyzed to determine if the materials enhanced general mathematics ability, views toward mathematics, and problem-solving and decision making proficiency. And, the materials themselves were analyzed to determine the extent to which they are aligned with the recommendations of the NCTM *Standards*. Although the results of these analyses have been discussed in detail and summarized above, the following listing provides the major conclusions of the investigation.

1. Using *EarthMath Studies* in precalculus and mathematics teacher education courses is enticing.

2. Students in precalculus and mathematics teacher education courses dramatically improved their genetic mathematical prowess.

3. Precalculus students improved their views toward mathematics.

4. Mathematics teacher education and precalculus students experienced significant different problem-solving and decision making competence.

5. *EarthMath Studies* are remarkably aligned with the NCTM *Curriculum and Evaluation Standards.*

6. The EarthMath Project can be judged successful. This conclusion is justified after viewing the previous five conclusions.

Final Remarks

Thus, in the final analysis, *EarthMath Studies* modules can enhance undergraduate precalculus and mathematics teacher education courses. Students in these courses did experience significant gains in generic mathematical competence and precalculus students made notable gains in attitude toward mathematics. There was a notable difference in problem-solving and decision making ability among both groups of students. Beyond these results is the superb manner in which the materials respond to the challenge of the NCTM *Standards.*

The acquisition of innovative courseware appropriate for undergraduates mathematics students at the precalculus level which are also suitable for use with aspiring mathematics teachers is very refreshing. That the materials allow students to persist in their quest toward becoming mathematically empowered is noteworthy. The above report appears to indicate that the principal goal of the project has been attained.

Appendix A

Evaluation Instruments

FINAL EXAMINATION QUESTIONS: Precalculus

In a certain region in the US, the maximum monthly precipitation t 2.56 inches occurs in the month of March and the minimum of 0.46 inches occurs in September. Assume that precipitation is cyclic on annual basis and can be approximated for month t using a sine function.

1. Using $t = 1$ for January, determine the function of the form $P(t) = a + b\sin(pt)$ which can be used to approximate monthly precipitation in inches in this region, (Round coefficients to two decimal places.)

2. Use the function you derived in #1 to determine the annual precipitation in inches for this region. (Round answer to one place.)

FINAL EXAMINATION QUESTIONS: Mathematics Teacher Education

1. The table shows the average temperature per month for several months of a year.

Month	t	Temperature (degrees Fahrenheit)
January	1	26.69
March		31.08
April		32.57
June		34.16
September		33.03
October		31.72
December		27.70

Plot the data points on the graph. Use a t, $F(t)$ coordinate system. Label the axes and show VXK scale.

Write the equation of the parabola which describes the temperature data in the table above; let t denote VXK month with $t = 1$ for January, $t = 2$ for February, etc., and use the points which correspond to the months March, June, and October. Denote the resulting quadratic function f, $F(t)$, the average temperature in month t. (Round coefficients to two places. Sketch the function on the graph.

2. In the Final Water Project, you found the relationship of the average Annual Temperature (AT) and the average Annual Rainfall (AR). The function that relates these is:

$$AR(AT) = -1.56AT + 70.48.$$

Sketch this function on the graph. Use an AT, AT(AT) coordinate system. Label the axes and show the scale.

Write a verbal interpretation of the slope of the linear function AR(AT).

Describe the general impact of Global Warming on rainfall and the water supply <u>downstream</u> from River City.

APPENDIX B:

Partial Credit Delineation

Partial Credit Delineation

Award points for free-response items in the following way:

0 points:
(1) left blank, or
(2) recopied, but nothing was done with the data, or there was work but no apparent understanding of the problem,
(3) contained an incorrect answer and no other work was shown.

1 point:
(1) a start toward finding the solution beyond just copying the data that reflected some understanding, but the approach used would not have led to a correct solution, or
(2) an inappropriate strategy that is started but not carried out, and there is no evidence that the student turned to another strategy, or
(3) the student tried to reach a subgoal but never did.

2 points:
(1) used an inappropriate strategy and got an incorrect answer, but the work showed some understanding of the problem, or
(2) used an appropriate strategy but it was not carried out far enough to reach a solution or it was implemented incorrectly, or
(3) indicated the student had reached a subgoal but went no further, or
(4) had a correct answer but the work was not understandable or no work was shown.

3 points:
(1) the student implemented a solution strategy that could have led to the correct solution but (s)he misunderstood part of the problem or ignored a condition in the problem, or
(2) the student applied appropriate solution strategies but then answered the problem incorrectly for no apparent reason, or the correct numerical part of the answer was given and the answer was not labeled or was labeled incorrectly, or no answer is given, or
(3) the correct answer is given, and there is some evidence that appropriate solution strategies were selected.

4 points:
(1) the student made an error in carrying out an appropriate solution strategy, but this error does not reflect misunderstanding of either the problem or how to implement the strategy, but rather seems to be a copying or computational error, or
(2) the student selected and implemented appropriate strategies. The answer is given in terms of the problem.

APPENDIX C:

Administrative Communications

APPENDIX C:

Administrative Communications

MEMORANDUM

TO: EarthMath Instructors, Spring '95

FROM: Pam Drummond

RE: Evaluation

DATE: December 19, 1994

Thank you for agreeing to participate in the evaluation of the EarthMath materials, written by Chris Schaufele and Nancy Zumoff and funded by FIPSE and NSF. Assessment of these materials will provide feedback to the funding agencies in terms of student outcomes, curriculum evaluation, and instructional aspects. Your help with the first and third of these is essential, so I am providing a brief overview of these areas.

I hope that this information is sufficient and succinct. If you have any questions or comments, please contact me! The most efficient way is to use e-mail or leave a message on my answering machine at home.

<u>Student Outcomes</u> Three constructs will be assessed: "traditional" Content, dispositions, and processes.

"Traditional" content. Pre- and post-test in multiple choice form. Students will need to use scientific calculators. I will grade the tests; you need to administer them, along with the disposition survey, during one of your first three sessions.

Dispositions. Pre- and post disposition survey.

The pre-assessments with instructions are included in the package.

Process. Assessed through problem sets and a capstone problem during the term. To accomplish this I ask that you maintain a folder for each student which contains solutions to a problem set from the first 25% of the material, solutions to a problem set in the material during the third 25% (i.e., middle to three-quarters of the way through), solutions to a "final" problem, and solutions to final examination question. (If you require a project, I would like to see it, as well.)

The "final" (or capstone) problem will be mailed to you approximately one month before the end of the term. I expect students to work on solutions to this problem cooperatively. However, on the final exam each student will write her or his own solution. At that time, we will ask some additional questions which will use and extend their understanding of the solution. [Note that any expense you incur in copying and/or mailing this student work may be reimbursed from the grant.]

Instructional Aspects I will ask you to report your instructional practice during the class and reflect upon your students' growth. Generally speaking, this means: How was your course organized? How was class time used? How was the grade determined? Was there a project? How do you assess student learning? etc. Specific questions will be sent to you toward the end of January. However, initially I ask that you send me copies of your syllabus and class roll when you return the completed pre-tests. I am interested in your feelings about the student reactions because of the high correlation between instructor"s opinions and reality.

I look forward to working with you on the evaluation of this exciting project. Again, thank you for all the help you have agreed to give.

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e-mail: pdrummon@kscmail.kennesaw.edu

INSTRUCTOR'S INFORMATION

Institution:____

Instructor's Name:____________________________________

Telephone Numbers: School:________________________

Home:________________________

Fax:________________________

e-mail:____

Date Materials Received:______________________________

Number of Evaluation Instruments Received:__________

Date Instructor's Information Returned:_______________

Anticipated Number of Students in Course:______________

Name of Course:____________________________________

Prerequisites:__________________________________

____

Class Meeting Days & Times:______________________

Course Beginning Date:__________________________

Course Ending Date:_____________________________

Please return this paper *immediately* in the small envelope provided. Thank you.

Pam Drummond
Mathematics Department
Kennesaw State College
P.O. Box 444
Marietta, GA 30061

Pre-Test INFORMATION

Institution:______________________________

Instructor's Name:______________________________

Date Pre-Test Given:______________________________

Anticipated Date of Post-Test:______________________________

Anticipated Dates EarthMath Materials will be used:______________________________

____

Have you included: 1) A copy of your syllabus? Yes or No

2) A copy of your class roll? Yes or No

To Be Returned with Pre-Tests. Thank You!

Pam Drummond
Mathemtics Department
Kennesaw State College
P.O. Box 444
Marietta, GA 30062

Time-Line: EarthMath Evaluation

Immediately: Return *Instructor's Information*

By 3rd Class Meeting: Administer Pre-Test (included in packet)
Complete *Pre-Test Information*
Mail in postage-paid envelope:
- Pre-Tests
- Pre-Test Information Sheet
- Course Syllabus
- Class Roll

Early February: Receive *Instructor's Survey*

Processes:

During 1st 25% of EarthMath Materials: Take-up Problem Set from students

During 3rd 25% of EarthMath Materials: Take-up Problem Set from students

Early April: Receive "Final" Problem

Late April: Receive Post-Tests

Last three weeks of course: Assign "Final" Problem

One of three last class meetings: Administer Post-Test

Final Exam: Students will give his/her solution to the "Final" Problem. Additional questions will be provided for student to answer.

End of Course: **Mail to Pam Drummond:**
- Post-Tests
- Final Exam Questions
- *Instructor's Survey*
- Students' Problem Sets

Pam Drummond
Mathematics Department
Kennesaw State College
P.O. Box 444
Marietta, GA 30061

Home: 404-255-2468
Work: 404-423-6327
Fax: 404-423-6629
email: pdrummon@kscmail.kennesaw.edu

PR#:_______

Name:________________________

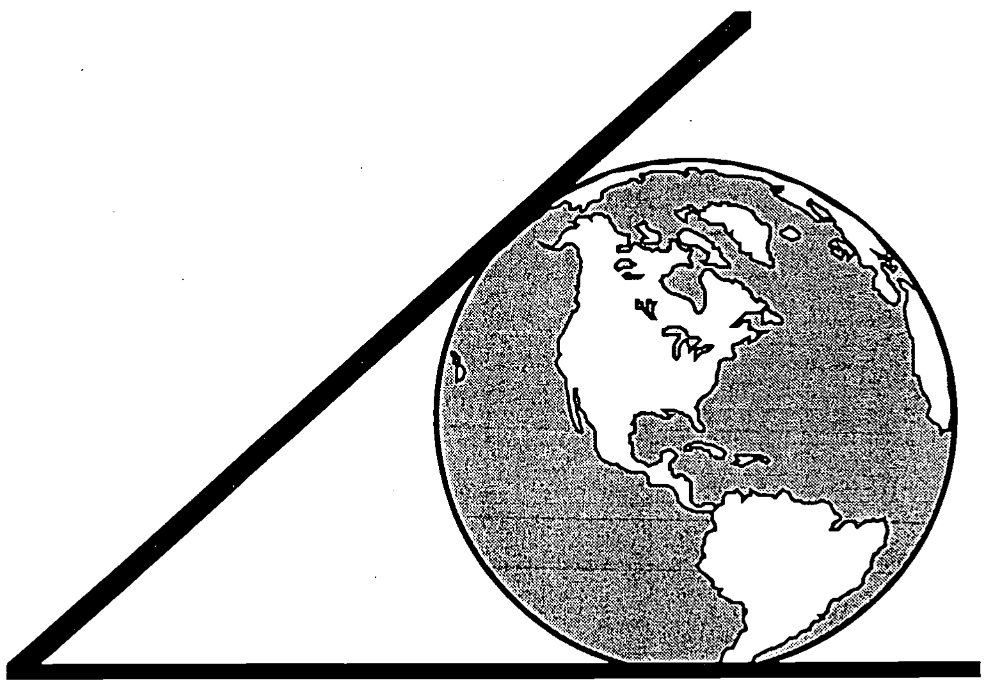
SSN:_________________________

The students in your class have been asked to participate in a study funded by The U.S. Department of Education and The National Science Foundation. Attempt to answer each of the questions to the best of your ability. Circle the one best answer to each question. You are encouraged to use the calculator of your choice in arriving at your answers. Note that a scientific calculator will be needed for some of the questions. Do not spend too much time on any one question. You will have 60 minutes to complete your responses.

Please know that **your solutions to these problems will in no way affect your grade**. Your complete cooperation and an earnest effort on your part is expected and appreciated.

APPENDIX B
Proceedings of the Earth Math Conference

PROCEEDINGS
EARTH MATH CONFERENCE
APRIL 19 - 20, 1996
KENNESAW STATE UNIVERSITY



The Earth Math Conference is supported by
U.S. Dept. of Education, FIPSE, grant #P11A30555
and
National Science Foundation grant # DUE-9354647

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Overview

The EarthMath Conference was held at Kennesaw State University on April 19 and 20, 1996. It was attended by representatives of test institutions, consultants, high school teachers, potential class testers and individuals from around the country active in mathematics reform. A conference schedule is included at the end of this section for your convenience.

EarthMath is a project which broadens and disseminates the concept of Earth Algebra. Earth Algebra was a successful project, funded in 1991 by both US Department of Education: FIPSE and the National Science Foundation (pilot project, Undergraduate Course and Curriculum Development Program) with funding ending in August. 1993. The EarthMath project is also funded by both of these agencies. It is an extension of the Earth Algebra concept to precalculus courses, both algebra and trigonometry, and to mathematics education courses.

The problems addressed by this project are much the same as those identified in the Earth Algebra project. The difficulties in mathematics education in the United States are well known: students drop out of the pipeline at the rate of fifty percent per year from ninth grade through graduate school; the course content is viewed as irrelevant to one's real life needs and experiences; mathematics is considered too difficult for all but an intellectual elite; it is acceptable to not like mathematics and to not do well; and, finally, it is believed that the country will not suffer if mathematics is left to those who have some special gift enabling them to master the material. We do not believe this is true. The general populace will need greater mathematical skills in the work place and at home in the years ahead, and more mathematicians, scientists, and teachers will be needed for the health of the economy. Future mathematics teachers need to be trained in effective techniques to improve the quality of mathematics education for future generations. By extending the concepts of Earth Algebra to a wider variety of courses and providing easily accessible materials to a wide variety of institutions, we attack these problems on a much broader basis.

Major efforts to improve the teaching of mathematics and the preparation of the teachers who will be teaching it are under way. In 1989, the National Council of Teachers of Mathematics published the Curriculum and Evaluation Standards for School Mathematics (The Standards), a blueprint for the reform of mathematics education in kindergarten through twelfth grade. The publication of the Professional Standards for Teaching Mathematics, by the NCTM in 1991, serves to "establish a broader framework to guide reform in school mathematics in the next decade." The principles incorporated in both of these reform movements, as well as the many calculus reform projects, are: mathematics education should be relevant, interesting, attainable, and modernized with the use of technology; it should be interdisciplinary and use communication skills and cooperative learning techniques. The current Earth Algebra course addresses these issues in the context of a college algebra course preliminary for those students not intending to continue in the traditional calculus sequence. The EarthMath precalculus courses extends the Earth Algebra concept to precalculus, both algebra and trigonometry, and teacher training.

Earth Algebra is being and will be well received as either a terminal mathematics course, or as a prerequisite to "business" calculus or statistics. However, many colleges and universities feel that Earth Algebra does not cover in sufficient depth the topics or skills necessary for students who intend to enter a standard calculus course. Such colleges and universities for the most part are still offering the traditional precalculus because they have not found a viable alternative. However, almost all mathematics departments recognize the need for change throughout the introductory curriculum, a change which would generate interest in and appreciation of mathematics and its role in modern society. The solution cannot be to simply add-on to the precalculus topics in present courses. Indeed, the appeal of Earth Algebra to students and faculty alike is the integration of the modeling of the environmental data throughout the course. All of the mathematics studied flows from the study of the environmental problems in a natural way.

The preservice teacher education component of this project addresses similar problems. We quote from the Mathematical Association of America, A Call for Change : "Such substantive changes in school mathematics as envisioned by the NCTM Standards will require corresponding changes in the preparation of teachers. Teachers need opportunities in their collegiate courses to do mathematics: explore, analyze, construct models, collect and represent data, present arguments, and solve problems. The content of collegiate level courses must reflect the changes in emphases and content of the emerging school curriculum and the rapidly broadening scope of mathematics itself." A student enrolled in a precalculus course which has been designed and is taught in the spirit of the NCTM Standards will likely teach in that spirit since we tend to model our teaching on our experiences as students. However, many K-12 teachers will never see such a course, and in many colleges and universities, the only math course taken by K-8 preservice teachers is one specifically designed for this audience, either content or methods oriented. Also, Secondary Education mathematics majors will never see a Standards based approach in the courses they take in a mathematics department. For these reasons, EarthMath combined with Earth Algebra materials are appropriate for the audience of preservice teachers of mathematics at all levels K-12.

First, the goals of the use of EarthMath materials in precalculus courses are to improve students' problem solving and decision making ability, and to offer insight and increase interest and appreciation of the role of mathematics in society. These goals are achieved by incorporating mathematics into a focused study of important environmental concerns, use of technology, group work, oral and written reports, and student research. The introduction of these materials is also expected to enhance the learning of the mathematical skills and concepts prerequisite to calculus.

Second, the goals of the use of EarthMath materials and methods in mathematics education courses is to provide preservice teachers with a model of how the Standards can be implemented in mathematics courses by educating future teachers using the methods mandated by the NCTM Standards. Indirectly, future students at the K-12 level will receive the same benefits as the precalculus students at the college level.

In addition to Kennesaw State University, a number of institutions around the country have cooperated with us in the development, testing and revision of these materials. A list of these institutions is included in these Proceedings.

A first draft of a textbook has been completed. The book, entitled *Earth Angles: precalculus with applications to environmental issues,* will be published by Addison Wesley.

The formal evaluation of this project, conducted by Dr. Pamela Drummond, has been completed; a summary is included in these proceedings and a complete copy is available upon request.

Earth Math Conference Schedule
April 19 - 20
Kennesaw State University
Kennesaw, Georgia

Friday, April 19

1:15	Welcome
1:30 - 2:20	Keynote Address, Dr. Tina Straley
2:20 - 2:30	Coffee break
2:30 - 4:30	Reports from Test Institutions
6:00 - 7:00	Cash Bar - Northwest Marriott Hotel
7:00 - 9:00	Dinner and Invited Speaker, Dr. Mary Linquist, NCTM

Saturday, April 20

9:00 - 9:30	Continental Breakfast
9:30 - 9:45	Earth Math Overview
9:45 - 10:30	Technology Workshop, Dr. John Kenelly
10:30 -10:45	Coffee Break
10:45 - 12:15	Earth Math Workshop - Part I
12:30 - 2:00	Lunch and Invited Speaker, Dr. Ravindra Srivastava
2:15 - 4:15	Earth Math Workshop - Part II
4:15 - 4:45	Invited Speaker, Dr. Ben Fusaro
4:45 - 5:00	Closing

All activities except for the cash bar and dinner will be held at the Jolley Lodge, lower level, Kennesaw State College.

Keynote Address: Dr. Tina Straley

We meet at a critical time in the history of mathematics and mathematics education. At the turn of the last century Hilbert listed the big questions that faced mathematicians. There were great expectations that these would be solved in the twentieth century. It was a great intellectual time, a flowering of knowledge and an appreciation of knowledge for its own sake.

A century later, the future of mathematics as an integral part of our educational fabric is uncertain. This comes at a time when there are tremendous advances in mathematical research being done by a record number of experts all over the world, a time of technological advances that make mathematics doable that was never before possible, a time of almost universal access to education, including advanced education in the Western World as well as great parts of the East.

Yet math is starting to struggle to maintain its position as one of the basics. This place was never questioned before as one of the three R's. We face many challenges in mathematics and education which include appropriate use of technology, declining numbers of majors, preservation of computational skills, relevancy, and selling mathematics. Each of these has their own characteristics.

Technology is endemic in society and we can't ignore it. It opens new possibilities, enhances doing mathematics and leads to advances in mathematics. There is a danger, however, of the "black box" mentality being developed. Along with this is defining the role of computational skills. These skills are declining in the population. How do we integrate computation and teaching for understanding?

The issue of Relevancy is very important. Mathematics is currently divorced from its applications and there is a lack of interest in the general population. Teaching mathematics in context lends meaning and purpose to the mathematics.

The selling of mathematics should be a priority. Currently there is a back to the basics mentality and a belief that there is less need for mathematics given the role of technology. Furthermore, there is a distrust of the math/science and government/corporate conspiracy. We should sell the pre-college experience as a combination of computational skills and use of technology. The constructivist approach should be phased in and real world uses emphasized. At the undergraduate level programs should be strongly connected to using mathematics in a significant way and should be designed for particular, identifiable, existent career opportunities. There should be real world uses in all math courses and significant uses of math in other courses. At the graduate level programs should prepare students for careers outside of academic research and teaching. and they should do a better job of preparing students for the academic profession. How do we sell such things? At the pre-college level we should work *with* local teachers, and educate campus career advisors of the usefulness of mathematics. At the other levels we work with advisory councils which represent local business and industry, and design internships and co-ops, and work at job placement.

Earth Math in this agenda combines skills and the use of technology. It incorporates mathematics into real-world uses of mathematics and sells the importance of mathematics in the real-world. It also provides an appropriate start for the undergraduate program and complements the pre-college program.

I conclude by stressing that we have no time to waste in turning the tide. Changing individual courses while preserving the old paradigms is not enough.

Last weekend at the SE Section- MAA meeting we celebrated the tenth anniversary of calculus reform. In a panel discussion concluding the special session, the moderator asked the audience if they were teaching reformed courses: some reform, major reform, none. The "nones" won.

Those of us who have been involved in changing curriculum are aware of the difficulties and the progress to date. This is not a complacent group. But we cannot rest on our laurels. Calculus reform was just the tip of the iceberg, and the NCTM Standards have not yet solved all of our problems.

The upside of all of this is the excitement of change of re-inventing ourselves and making mathematics again mainstream and essential. Even better if we can do it without turning people against us and our discipline.

Technology Workshop: Dr. John Kenelly

The workshop began with a brief introduction of Dr. Kenelly's involvement with using calculators in teaching mathematics. He was given a graphing calculator classroom set and from that time forward has required them in his class. It forever changed the way he looks at teaching. The goal is to have students move seamlessly from numbers to graphs to algebraic expressions. This is in opposition to the old custom of teaching an algebraic trick and then drilling the students on that trick. Students learned the tricks, were graded on the tricks, but were never told why they needed the tricks. And so today people feel mathematics is divorced from their own lives. It is mathematics teachers' obligation to show people how useful mathematics is in their daily lives.

Two situations with periodic data were investigated with a sine curve model. The TI-83 comes equipped with sine regression, which is a recently solved problem. The first model involves a levelized gas billing procedure. The data is previous months gas bill amounts and one wishes to decide when to enter the program. The second model involves a method to help electric companies forecast their load for the next few days based on the temperature forecast for the next five days. Both illustrate how modeling helps in decision making.

A third model focused on the impact of long term bonds on one's retirement money. It points out that fundamental ignorance of mathematics can result in financial loss. It also points out why people need this type of mathematics problem.

Hydrology: Dr. Ravindra Srivastava

Ravi Srivastava is a hydrologist who has worked in the Southwestern U.S. and has collaborated with Chris Schaufele in creating water models for use in teaching algebra to Navajo students. His talk was structured around the topics of water law, why people study water, basic hydrology and water allocation, and an algebra module for modeling the water cycle. His summary of water law also contained some history of the water rights of the Native Americans in the West in order to place the current situation in context.

Historically, water law deals with surface water such as rivers and lakes, and does not address the question of ground water. The essence of water law, and in particular the assignment of water rights, is encapsulated by the concepts of beneficial use, priority in time, and a "use it or lose it" test for retainment of water rights. These concepts, combined with the scarcity of water in the Southwest and the complicated history of Native American-Government relations, cause major tension in the area. The situation is further complicated by the legal status of Native American tribes as sovereign nations which must deal directly with the federal government, while it is the state governments which assign groups their water appropriation. Since water is over-appropriated, water law is a key component in understanding the milieu in which the students live.

In addition to the legal issues surrounding water, there are also the scientific issues concerning why and how people study water. In general, those who study water concentrate on the utilization of water, the control of excess or unwanted water, and water quality management. The main tool used by scientists is hydrology, an earth science encompassing all aspects of water. An understanding of the hydrologic cycle is the primary requirement for studying water. Briefly, the cycle describes how water moves from the ocean via evaporation to atmospheric water and then to the land via precipitation. It is at his point that humans begin dealing with water, and this is also the point where the algebraic models begin.

The standard hydrologic concept of the watershed is used to narrow the focus of the model. The Navajo reservation is in two subbasins of the Colorado River basin, so that all the data used to create the algebraic models concern these two watersheds. Since there is an abundance of data, many different models, at different levels of complexity, can be created. For example, the flow into a river basin can be partitioned into the various contributors and connected with the precipitation to create a model which involves many different mathematical concepts. Other models using temperature, snowmelt, runoff, population growth, air quality and global warming can be created in such a way that each model builds on the previous one. Thus, a fairly complex and realistic model. which is tailored specifically to the Navajo nation, is created.

The students tend to be very interested in such an algebra course because the water cycle is an application which is real, completely local, and of paramount importance. It also entices them into more advanced math courses. Finally, data about every watershed across the U.S. are available from a variety of sources, including the World Wide Web, and this makes it fairly easy to adapt to different locales.

Invited Talk: Dr. Ben Fusaro

Dr. Fusaro is an environmental mathematician who is Chair of the MAA Committee on the Environment. He is committed to making and keeping environmental issues central in the mathematics community. There are currently two crises, or you could call them challenges, in the mathematics community - the condition of the environment and the public perception of mathematics. With regard to the environment he discussed a number of critical problems such as the condition of Chesapeake Bay, the loss of diversified forests, the extinction of species, the destructive power of oil spills, and the warming effect in Atlanta caused by local deforestation. On the subject of public perception he quoted people both inside and outside of America who feel mathematicians are divorced from the reality of their environment. For our own selfish reasons we need to change this perception. The gravy train of the past is over and it is not coming back because the policy makers are not mathematics majors. The environmental mathematics movement can help in both these areas.

During an outline of milestones in environmental mathematics, he pointed out that areas such as ecological mathematics and biomathematics are too specialized to accomplish the goals of promoting environmental awareness and mathematical awareness in young people. The beauty of environmental mathematics lies in the facts that it is broad, interdisciplinary, at an elementary level and engaging. The Earth Algebra textbook provides all of these characteristics and, most importantly, it allows the concern for the environment to show. It is hoped that this will break the mold that scientists are supposed to be objective and not let their emotions show. It is important that mathematicians who are concerned about the environment be aware that they are not alone.

Turning to his view of modeling, Dr. Fusaro said he too has a rule of three which he came up with long before he heard of the Harvard one - solve things computationally, qualitatively and visually. The computational method may be changing faster than even reformers know with the possibility of graphing calculators and laptops converging. To solve something qualitatively can be described as knowing the answer to your problem before you solve it using whatever is available. Finally, visual can mean graphs but any other visual method is fair game.

He has a liberal arts approach to modeling which blends well with the fact that liberal arts majors are the leaders of the future. He disagrees with some people because he feels algebra is a language of modeling but it is not the only one. A definition for a model might be a representation that preserves relevant properties of relations. It is important to validate a current model but recognize that it can be reformulated and reinterpreted. The value of a model can be measured by its output. If you can get great output with nothing more than arithmetic that is great - a goal is to push the level of the input down while elevating the level of the output.

At this point there was a discussion about when mathematical modeling became important to him and he pointed to the change in Clemson's mathematics department name to the Department of Mathematical Sciences. This set a tone which inspired him to change the name of his own department. One high school teacher pointed out the difficulty in changing colleagues minds about the formalistic teaching of mathematics. Colleagues maintain that colleges still teach mathematics the traditional way and students must be prepared for it. This workshop allows him to go back and say, yes, but some colleges teach it another way.

The presentation ended with a five stage process for determining a model. The stages are: create an energy diagram, do some qualitative analysis, construct a flow equation, do the computation, and then describe it quantitatively.

He did several examples, including leaf decomposition and financial, to demonstrate this five stage process. He pointed out that this requires a change in teaching habits because you must learn to let the students do the work.

Reports from Test Institutions

A number of testers spoke to the advantages and disadvantages of using the material. This can be broken down into two categories - materials used in mathematics education courses and those used in mathematics courses.

In mathematics education the material was used at different levels: those used with teachers in graduate level courses who were actually out in the schools and those with future teachers. Those already teaching applauded the context of the material though there was skepticism that all of mathematics could be developed contextually. Often they review traditional material at the same time as working on the Earth Math material, and they liked the flexibility of the book which allows this. The consistency of the topic was useful because it unifies it. Also, it is useful for integrating science and mathematics, especially when real data is collected with CBL's and then analyzed. On the other hand, students who have not taught in the field seem to have a preconceived notion as to what mathematics is and how they are going to teach it. This creates some resistance to the material. Students had to adjust to problems which went on for days at a time. But it was noted that just tacking the material on to a traditional class made students more resistant to the change. By integrating the modeling approach throughout the course the students were more accustomed to the process and did better on the particular problems dealing with water. Most students were interested in the water problems, though those in the West seemed more interested than those in the East where water is a more plentiful resource. There also seemed some resistance to solving a problem in more than one way such as graphically and algebraically. A possibility for a solution to the preservice teachers is to invite local math supervisors to speak about the qualities of people they are hiring - this should be very influential in convincing students of the importance of mathematics in context.

In the regular mathematics courses the same problem occurred with tacking the material on to the traditional course - the students had a difficult time adjusting to the modeling approach. Those students who had been previously exposed to this approach were most enthusiastic. Students who had come from a more traditional class had a harder time adjusting. Otherwise, students greatly enjoyed seeing the usefulness of mathematics and working on involved projects. Negative comments usually involved not being able to meet groups outside of class because of busy schedules. Yet, they often cited the group work as a very important and useful part of the course.

The overall response from both students and testers was positive. Feedback from colleagues who had previously tested the material was particularly important for later successes. It was universally acknowledged that integrating the material throughout made for a much smoother course. One thing that is still uncertain is how will students of these precalculus classes do in a traditional calculus class. There is some evidence to support that they do just as well. or in some cases better, than traditionally prepared precalculus students.

Evaluation: Dr. Pamela Drummond

The evaluator examined the following query: Will the *EarthMath Studies* materials be successful in pre-calculus and mathematics teacher education courses?

To answer this question, the evaluator posed three research questions. These questions centered on the following constructs, each operationally defined by an associated variable: (a) generic mathematical prowess, as measured by the Calculus-Readiness Test (CRT), (b) view toward mathematics, as measured by the View of Mathematics Inventory (VMI), and (c) mathematical problem-solving and decision making competence, as measured by the Final Examination Questions (FEQ).

The testing instruments have been previously described. In addition, the six hypotheses explicitly describe the relationships among these three variables and have been described in the complete report. The results which emanated from these hypotheses yielded information which aided the evaluator in answering the first two research questions. Comparing the materials to the NCTM *Standards* provided information for answering the third research question. Ultimately, these answers resolve the foremost question of this study which focuses on the success of the *EarthMath Studies*.

Regarding the first research question, the evidence indicated that students involved in the study experienced significant gains in generic mathematics prowess. The precalculus students made modest gains in their attitudes toward mathematics. It is not surprising that the preservice teachers did not realize remarkable gains in their attitudes toward mathematics. For these particular students, this was (at least) their fourth undergraduate mathematics course. Views of mathematics tend to level-off over time of continued study of mathematics. Most of the precalculus students were in their initial mathematics course. But what caused the change in the students?

Because the class time that was given to the use of these materials was meager, it is presumptuous to proclaim that the credit for any significant result belongs solely to the EarthMath Project. However, it is reasonable to claim that the *EarthMath Studies* modules did **not** reduce students generic mathematical ability nor diminish their potential for success in the Calculus or subsequent courses. That, in and of itself, is notable.

An analysis of Research Question 2 showed that problem-solving and decision making competence was stronger at the end of the course. For mathematics teacher education students the difference was highly significant and for precalculus students, even though slightly less significant, the difference was notable. Again, as cautioned stated above, it is difficult to assign credit for the success solely to the EarthMath Project. Given the modest amount of time that the *EarthMath Studies* modules were actually used in class may other factors may have contributed to the differences among students at the end of the term. It does seem fair to allege that the *EarthMath Studies* modules did not reduce students proficiency in problem-solving or decision making. In other words, the use of these materials in courses for precalculus and/or mathematics teacher education students may enhance students ability to attempt and solve problems from situations requiring the use of real data. This is a noteworthy result.

To answer Research Question 3 the evaluator partitioned the 1989 NCTM *Curriculum and Evaluation Standards*. Eight curriculum standards in this highly acclaimed model for "state of the art" mathematics included concepts which are germane to the *EarthMath Studies*: the four process standards (Problem Solving, Communication, Reasoning, and Communications), Algebra, Functions, Trigonometry, and Conceptual Underpinnings of Calculus. A comparison of the individual components for each standard with the *EarthMath* materials revealed that the modules were highly congruent with the *Standards*.

This may be the most exciting result of the study. Certainly, it is the most conclusive. It may be difficult to substantiate changes made in student behavior, but written curriculum is hard evidence. The *EarthMath Studies* modules are highly innovative undergraduate courseware which are very responsive to the recommendations of the mathematics community for instructional materials.

In short, the statistical evidence showed that students who participated in this study increased significantly with regard to their general mathematical prowess. Mathematics teacher education students significantly enhanced their problem-solving and decision making competence. Because the *EarthMath Studies* modules were incorporated into the courses less than 35% of the time, it is inappropriate to maintain that their use provides the single cause of these changes. Nevertheless, the materials themselves are particularly noteworthy. The extent to which the *EarthMath Studies* modules respond to the challenge of the NCTM *Standards* is particularly noteworthy.

Conclusions

To assess the extent of the success of the EarthMath Project, the results of testing six hypotheses were analyzed to determine if the materials enhanced general mathematics ability, views toward mathematics, and problem-solving and decision making proficiency. And, the materials themselves were analyzed to determine the extent to which they are aligned with the recommendations of the NCTM *Standards*. Although the results of these analyses have been discussed in detail and summarized above, the following listing provides the major conclusions of the investigation.

1. Using *EarthMath Studies* in precalculus and mathematics teacher education courses is enticing.

2. Students in precalculus and mathematics teacher education courses dramatically improved their generic mathematical prowess.

3. Precalculus students improved their views toward mathematics.

4. Mathematics teacher education and precalculus students experienced significant different problem-solving decision making competence.

5. *EarthMath Studies* are remarkably aligned with the NCTM *Curriculum and Evaluation Standards*.

6. The EarthMath Project can be judged successful. This conclusion is justified after viewing the previous five conclusions.

Final Remarks

Thus, in the final analysis, *EarthMath Studies* modules can enhance undergraduate precalculus and mathematics teacher education courses. Students in these courses did experience significant gains in generic mathematical competence and precalculus students made notable gains in attitude toward mathematics. There was a notable difference in problem-solving and decision making ability among both groups of students. Beyond these results is the superb manner in which the materials respond to the challenge of the NCTM *Standards*.

The acquisition of innovative courseware appropriate for undergraduate mathematics students at the precalculus level which are also suitable for use with aspiring mathematics teachers is very refreshing. That the materials allow students to persist in their quest toward becoming mathematically empowered is noteworthy. The above report appears to indicate that the principal goal of the project has been attained.

Summary of Discussions

Several participants discussed projects with which they have been working. They were Darrell H. Abney from Maysville Community College in Maysville , KY; Chris Allgyer from Mountain Empire Community College in Big Stone Gap, VA; Lynn Darragh from San Juan College in Farmington, NM; and Alan Jacobs of the Maricopa Mathematics Consortium in Tempe, AZ.

Darrell H. Abney began with a discussion of the AMATYC Standards and the workshops and conferences that the society have. His school has been using the Earth Algebra text for a long time and made the decision to go with this approach because of the poor success rates in the course. They hope that the Earth Angles text will help with the precalculus courses. Of course, at a community college many students are not ready for precalculus, so classes begin with arithmetic and continue through elementary algebra and intermediate algebra. His particular project deals with Intermediate Algebra. They have written a text which deals with functions through applications. It has been tested for three semesters and Addison Wesley is going to publish it in a preliminary edition form in the Fall of 1996. They are hoping to test it outside of Kentucky to get some feedback. The guiding principle of the text has been if it doesn't have an application don't put it in the book. This has been hard to be faithful to. They are also going with the idea that less is best and thus a whole lot of standard intermediate topics are missing.

Chris Allgyer's project began in 1985 when the community college received a windfall from the state legislature to cooperate with the Regional Governor's School in their area. This school wanted help with teaching mathematics and science to grades 10 through 12. What began as a couple of weeks course became a four week course involving teamwork between mathematics and science teachers. They wanted the course to be hands on, non-traditional, and use regional topics to spark interest. For the Appalachian country of Virginia this meant coal mining and forestry. They went out with professionals to the mines and forests. He learned that in thirteen years of teaching he had been going at it backwards. The chemist took them to the site and did dissolved oxygen tests, streamflow, pH and other samples. These scientists were using special graph paper to analyze their results - graph paper which was never used in mathematics courses. It was a revelation that something so fundamental to the chemist's job was never mentioned in mathematics classes. A very valuable part of this project has been the cooperation between mathematicians and scientists. Now students do a chemistry experiment using CBLs in their lab course and proceed to their mathematics class where they do the analysis of the data. He recommends that every mathematics teacher spend time with a practicing scientist.

Lynn Darragh of San Juan College is involved with many projects. She had words of encouragement for those currently struggling with reform issues. They started three or four years ago with the developmental courses and now these students are arriving in calculus with real world problem solving skills and technology skills. The professors are excited by what the students understand and they do not feel that the technology is a detriment to learning. She points out that the faculty who constantly fought the reform are now, several years later, just as excited and happy about the changes as those who began the reform movement. She believes in pulling the rug out from under the traditional courses completely and doing significant change all at once.

Alan Jacobs of the Maricopa Mathematics Consortium said that there are challenges to pulling the rug out from under the whole thing at once. They have tried to redesign curriculum from the ground up by throwing out every topic and every course and then putting back things that they think every student must learn. Now the materials are developed but the materials must be tested and the only place to test them is in the traditional courses. He doesn't feel comfortable changing everything at once as Lynn suggested because he doesn't want to use untested curriculum. He feels these types of issues will be with us for some time.

Earth Math Workshop

As part of the Conference, a workshop was conducted by Nancy Zumoff and Christopher Schaufele on the content and methodology of the precalculus course, Earth Angles. Participants worked through a portion of the notes which are included in these Proceedings. This was followed by a discussion of topics included in precalculus and mathematics education courses. The workshop closed with brief presentations of other reform projects under development by participants, Alan Jacobs, Chris Allgyr, and Darryl Abney.

The notes provided herein consist of chapters which include the environmental applications: the complete draft of the text also includes detailed exposition of mathematical concepts from precalculus.

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CHAPTER TWO
U.S. WATER USAGE AND POPULATION

2.1 Introduction

There is a limited amount of fresh water available in the United States, and this water must meet the needs of the growing population. Water is withdrawn from lakes, streams and underground aquifers for basic human needs such as drinking, cooking, and sanitation; it is also used extensively for irrigation as well as industrial and commercial uses. There are also important "instream" uses for water: maintenance of water quality, providing habitat for fish and wildlife, navigation, recreational use, and hydroelectric power generation. Also, free flowing rivers and unpolluted lakes are beautiful.

Water which is withdrawn may be returned to the river (usually after treatment, possibly in degraded form), or consumed (by evaporation, incorporation into products, discharged into the ocean, or otherwise made unavailable for use). The demand for water is the amount needed for instream use plus the amount withdrawn for all purposes. In this chapter we will examine water withdrawal and see how water use relates to population. We will examine the following components:

total population;
total water withdrawal for public use;
per capita water withdrawal for public use;
water withdrawal for all purposes, both total and per capita.

The water withdrawn for public use includes both urban and rural domestic and commercial purposes and accounts for about 8 - 12% of all US withdrawals. Other uses are irrigation (about 33% of the total) and industrial uses, including utilities (55 - 60%).

2.2 Population

The following table (Statistical Abstract of the United States) gives resident U.S. population (in millions), for selected years.

Year	Resident U.S. Population (in millions)
1950	152
1960	180
1970	204
1980	227
1990	249
1995	262

Table 2.1

In order to predict water requirements for the future we need to predict the population for future years; i.e., we need to find a formula which can be used to estimate the population for any given year, or at least for any year over some reasonable period of time. **(Omitted in these notes)**

* * * *

Group Work

Determine the linear function passing through the data points for the years:

i) 1980 and 1995;

ii) 1950 and 1990;

iii) 1960 and 1950.

For each of these functions answer the following questions.

1. What is the predicted rate of change in population?
2. Predict the population in the year 2000.
3. When will the population reach 300,000,000?

* * * *

2.3 Choosing the Model

(Details omitted here)

Group work

1. Answer the questions below for each of the three functions derived in section 2.2. Use the table or the appropriate graph to determine the answers.

i) For which years does the function over-estimate the actual population? What is the sum of the over-estimate?

ii) For which years does the function under-estimate the actual population? What is the sum of the under-estimate?

2. If we decide that the "best" function is the one for which the over-estimate plus the under-estimate is least, which function is best? (Note: use a positive number for both the over-estimate and the under-estimate.)

* * * *

2.3.1 Linear Regression

We have only considered three possible linear functions. There are many other possibilities, in fact infinitely many possibilities if we don't require that the line passes through any of the data points. Statisticians, mathematicians and scientists using statistics have a definition of "best" function and derived formulas which can be used to determine which of all possible lines is the best. Their definition of "best" is very close to the one we used above, but instead of taking the absolute value of the differences between predicted and actual values, the square of the differences is used. The technique used for determining this line is called *linear regression*, and the line which is determined is called the *regression line*. Any graphing calculator or software with statistical capabilities can be used to find the regression line. For more information, read the manual, ask your instructor, or experiment. We illustrate the procedure in the next example.

Example 3

Determine the regression line for the population data in Table 2.1. Use this function to predict the population in 2000 and to determine when the population will reach 300,000,000.

Solution (Omitted in these notes)

* * * *

Group Work: Use the regression equation to answer the following questions.

1. Predict the population for this year.
2. At what rate is the population increasing?
3. When will the population reach 275 million?
4. Predict the population in 100 years. Do you think this is reasonable? Explain your answer.
5. When was the population 0? Is this reasonable? Explain.

Your answers in questions 4 and 5 are dealing with what would be a reasonable restriction for the domain of the function. In general, the further away you move from the actual data used, the less accurate your prediction will be. The trend for population in the last 50 years is fairly stable, but it is very different from the trends exhibited earlier; in the future there might be other changes. When you use a function to make a prediction, be aware of the fact that your prediction is valid only if the current trend continues.

2.4 Water Withdrawal

How much water, on the average, does a person in the United States use each year? The answer depends on how we define the usage. There is direct usage, for needs such as drinking water, bathing, etc..., which could be estimated by looking at your water bill. But much of the water use is indirect, for example, for irrigation, power generation, and industrial usage. In this section we will look at direct personal use as measured by water withdrawal for public supply.

Table 2.3 provides data for selected years on daily United States water withdrawal for public supply in billions of gallons per day. This includes withdrawal for domestic and commercial use, but it excludes water withdrawn for irrigation, industrial and rural use.

United States Water Withdrawal	
Year	Daily Public Supply ($\times 10^9$ gallons)
1950	14
1960	21
1970	27
1980	34
1990	41

Table 2.3

Group Work

1. Plot the points corresponding to the data provided. Use a (t,Y) coordinate system with

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Y = daily withdrawal for public supply ($\times 10^9$ gallons) in year 1950 + t.

2. Determine the regression line for these data. Round coefficients to two decimal places. This gives a function W(t) which can be used to predict the daily withdrawal for public supply ($\times 10^9$ gallons) in year 1950 + t.

3. Give a verbal interpretation of the slope.

4. Predict the annual increase in withdrawal for public supply.

5. Estimate the current daily and annual withdrawal.

6. Predict when the daily withdrawal will be 50 billion gallons.

CHAPTER THREE
PER CAPITA WATER USE

3.1 Per Capita Water Withdrawal

In the last chapter we saw that water withdrawal for public supply is increasing. This isn't too surprising since the population is increasing. What isn't yet clear is whether each person is, on the average, using more water, less water or the same amount of water each year. The measure which will answer this is the *per capita water withdrawal* (for public supply), i.e., the withdrawal per person which is

$$\textit{per capita water withdrawal} = \frac{\textit{withdrawal}}{\textit{population}}.$$

We let C(t) designate the per capita daily water withdrawal for public supply in year 1950 + t. The units are important,

$$\frac{10^9 \textit{ gallons}}{10^6 \textit{ persons}} \textit{ per day} = 10^3 \textit{ gallons per person per day.}$$

Thus

$$C(t) = \frac{W(t)}{P(t)} \times 10^3 \textit{ gallons per person per day in year } 1950 + t.$$

C(t) is a *rational function*, that is, the quotient of two polynomial functions, in this case two linear functions. In the exercises below we will examine the function C(t) and determine how much more or less water each person has been using each year.

Group Work

In the work below, use the functions (the regression equations) for P(t) and W(t) developed in section 2.3 and 2.4. Give answers for C(t) rounded to the nearest gallons per person per day.

1. Determine the per capita daily withdrawal for 1968, 1983 and 1996.
2. Graph the function $C(t) = \frac{W(t)}{P(t)} \times 10^3$. The graph should show per capita withdrawal for the years 1950 to 2010.
3. For the period 1950 to 2010, when was per capital withdrawal greatest? least? What were these greatest and least withdrawals?
4. What trends do you see?

If the trend shown by the 1950 - 1990 data continues, not only will total water withdrawal for public supply increase, each person will withdraw more water. In the next section we see what the effect would be if the per capita withdrawal levels off or decreases. Finally, we will look at total withdrawal for all uses.

3.2 Change in Withdrawal Rates

Recall from the previous section the current (1996) per capita water withdrawal for public use is $C(46) = 169$ gallons per person per day. Use this figure and the population function $P(t) = 2.40t + 154.31 \times 10^6$. In each **Group Work** in this section we will see the consequences if various changes in per capita withdrawal occurred.

Group Work

Suppose that the per capita withdrawal remains at the 1996 level of 169 gallons per person per day.

1. Determine a new function $W_1(t)$ for withdrawal for public supply for $t \geq 46$. The units are the same as $W(t)$.
2. According to this function, what is the rate of increase in withdrawal? How does this compare to the rate determined in section 2.4?
3. Graph $W_1(t)$ and the original function $W(t)$ on the same coordinate system.

Group Work

In the original model the predicted per capita withdrawal rate in 2010 is $C(60) = 182$ gallons per person per day. Suppose that the trend is reversed and that between 1996 and 2010 the per capita withdrawal actually decreases from 169 to 164. (This is actually just 10% less that the projected value of 182.) We will assume that the per capita withdrawal falls at a constant rate, i.e., that the function is linear.

1. Determine a new function $C(t)$ to describe the per capita withdrawal between 1996 and 2010. (According to our assumption, this should be a linear function.)
2. Determine a new function $W_2(t)$ to describe withdrawal for public supply for $t \geq 46$. The units are the same as $W(t)$.
3. Graph $W_2(t)$ and the original function $W(t)$ on the same coordinate system.

4. Determine the vertex of the parabola and give a verbal interpretation of the this information.

3.3 Total Water Withdrawal

The pattern of total water withdrawal is less clear than domestic withdrawal. Table 3.1 below provides data of daily per capita withdrawal for all sources (Statistical Abstracts).

Year	Total Daily Per Capita Withdrawal (gallons per day)
1940	1027
1950	1185
1960	1500
1970	1815
1980	1953
1985	1650
1990	1620

Table 3.1

You can see that there was a fairly regular increase from 1940 - 1980, but then the pattern changed. Data past 1990 is not yet available. The decrease in water withdrawal was primarily reflected in withdrawal for irrigation and industrial uses. In this section we will look at what would happen to total water withdrawal under various scenarios. Since most of the water withdrawal in the United States is for industrial use (55 - 60% of total) and irrigation (about 33%), these amounts are very significant. Moreover, much of the water withdrawn for these uses is either consumed or returned in a severely degraded form.

Group Work

Based on the data above, we make the following assumptions:

* *the per capita withdrawal decreased at a constant rate from 1650 gallons per person per day in 1985 to 1620 gallons per person per day in 1990,*

** the above trend continues;*

** the population trend determined in section 2.3 continues.*

1. Determine a function which describes the daily per capita withdrawal in year 1950 + t. We continue to have t = 0 in 1950 since we will use this function together with the population function derived in section 2.3. Note, however, that the domain of this function consists of all $t \geq 35$. Use the notation

   TC(t) = Total per capita withdrawal in year 1950 + t gallons per person per day.

2. Predict the per capita withdrawal in the year 2010.
3. Give a verbal interpretation of the slope.
4. Determine the function which describes the total daily water withdrawal ($x10^9$) gallons per day; denote it TW(t), where t = 0 in 1950.
5. According to your model, in what year will the total withdrawal be greatest? What is the predicted total withdrawal in that year? per capita withdrawal? population?

3.3.1 Further Investigations

In the previous sections we have examined some possible outcomes regarding water withdrawal under various assumptions. Here are some possibilities for additional investigations.

1. If each person in the United States cut personal water withdrawal gradually for the next ten years, is it possible to effect an immediate decrease in total domestic withdrawal? If so, how much would the per capita withdrawal rate need to be cut each year?

2. Data shows that although water withdrawals for irrigation and industrial uses fell during the first part of the eighties, the amounts stabilized for irrigation and rose for industrial uses in the last half of the eighties. The per capita rates still declined due to population growth. The total daily withdrawal actually increased from 403 billion gallons to 411 billion gallons from 1985 to 1990. Assume that this increase in total withdrawal continues at a constant rate until at least 2010. What does this tell you about the per capita daily withdrawal?

3. Suppose that beginning in 1996 the population growth rate decreases by 10%. What is the new growth rate? Write a new population function using this growth rate which will be valid for $t \geq 46$. What would be the impact on water withdrawal, both for public supply and total withdrawal? Use the assumptions regarding per capita withdrawal made previously.

4. Try to find the following information for your city or municipality. How much water per day does your local water system supply, and how many people use this water? If you can find both current and past information, you can try to model local withdrawals. If you use well water, skip this question.

CHAPTER FOUR
CONSTANT FLOW RIVERS

4.1. Introduction

Water allocation is a very important and often controversial issue. In the early days of this country, during the "discovering" of the American West, ranchers and farmers would often dam streams and rivers so that they could have enough water for their cattle and for irrigation of their crops. This would understandably anger other ranchers and native peoples who lived downstream and were also dependent upon the stream for their water supply. Violent fights over water rights occurred in many instances like this, so government regulation of water allocation became necessary. However, even today many legal battles are being fought over water usage, and as our population increases so does the number of such cases. Industry requires more and more water to produce consumer products, more is needed for irrigation of crops and watering of livestock for food production, and of course, more simply for human consumption.

The issue of water allocation is more crucial in the southwestern United States than in other parts of the country. Indeed, the Colorado River which, together with all its tributaries supplies almost all of the southwest with water, never reaches its natural destination, the Pacific Ocean. It's as if the human population served by the Colorado basin were to increase by just one person, then everyone else would get just a little less water.

Easterners enjoy much more water affluence than westerners. Many southwestern rivers are either dry or run at very low volume during certain times of the year whereas most eastern rivers, during a normal year, carry a fairly constant volume if not regulated by a dam. But even in the east, court battles regarding rights to river water are springing up more and more as demands grow.

In order to determine water allocations, one piece of information is obviously necessary: the amount of water available. For rivers and streams, the measure of available water is streamflow. *Streamflow* is the volume of water which passes through a cross-section of a river in a specified interval of time. In the U. S. streamflow is usually measured in cubic feet per second (*cfs*). If the streamflow for Turkey Creek at the highway bridge is 25 *cfs*, this means that 25 cubic feet of water in Turkey Creek runs under the bridge each second. Other units of measure can also be used, for example, cubic meters per minute, or cubic feet per month, but the measurement of streamflow must always be volume per unit of time. Streamflow will clearly vary at different times of the year, at different locations on the stream, and from year to year. Because of these variations and diverse demands for water, the prediction of streamflow is a very important business. A major factor in this prediction is the amount of precipitation over the stream's watershed. Precipitation amounts have been recorded for years in many regions of the U. S. and average precipitation data can be used along with other factors to predict streamflow. Records of streamflow have also been kept; these data are gathered by actual measurement at various points along the stream.

In this chapter we will look at a simplistic method of predicting streamflow along a small river with an assumed constant flow. This analysis of streamflow will use certain algebraic functions known as *polynomial functions*. The polynomial functions needed for this study will be introduced in the discussion, then will be studied in more generality and depth in later sections of this chapter.

4.2. Predicting streamflow for Chinle River

The Chinle River has its headwaters in a warm climate and derives all its water from precipitation in the region and is unaffected by snow or ice. The precipitation is fairly constant throughout the year and therefore the Chinle River has a constant flow. A coal-burning power plant operates on the river and draws its water from the river for cooling purposes. Just below the village, Chinle River flows into a lake which is a popular fishing and boating place. In order to support these recreations, it is necessary that the lake be kept at a certain level. It is important to know the amount of water that Chinle River can supply so that a fair balance among these requirements can be maintained.

We begin by predicting streamflow at any point on the river from its origin to its entrance into the lake. The prediction is based primarily upon the regional precipitation and the size of the watershed; the relevant information is listed below (these are our assumptions).

i) Annual precipitation is 4 feet uniformly distributed throughout the year.

ii) The watershed from the origin of the river to the lake is shaped roughly like a triangle with vertex at the headwaters. The river flows in a straight line through the triangle for a distance of 10 miles and empties into the lake at the side opposite its origin; the distances from the entrance of the river into the lake to each of the other vertices are, respectively 5 miles and 3 miles. (See Figure 4.1.)

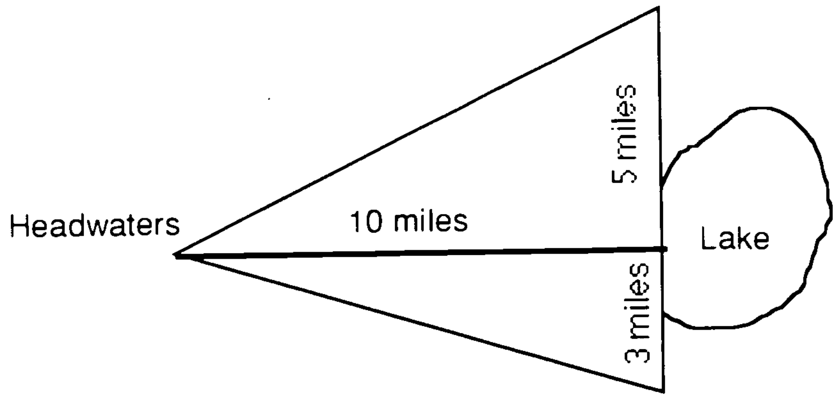

Figure 4.1

iii) The drainage in the watershed is in a direction perpendicular to the river bed; thus the watershed for a section of the river extending from its origin to a point x miles downstream will be the sub-triangle which is the corresponding portion of the entire watershed. (See Figure 4.2)

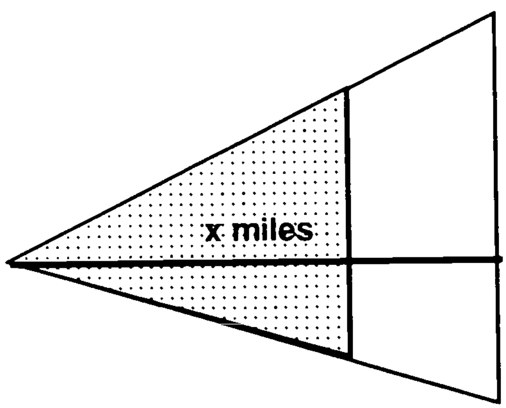

Figure 4.2

iv) Not all of the precipitation which falls on the watershed actually reaches Chinle River; much of it is taken by evaporation and evapotranspiration. As the distance downstream increases, the percentage of precipitation from the corresponding watershed which reaches the river bed decreases. It is estimated that this percentage varies linearly from 60% at the headwaters to 20% at the lake entrance.

Based on the above assumptions, we can now predict the streamflow for Chinle River at any point.

4.2.1. Area of the watershed

Let x denote the number of miles downstream from the origin of the river. Streamflow is determined by the amount of precipitation over the watershed which actually enters the river bed. Therefore the first thing which must be determined is the total volume of water which falls over the watershed for any portion of the river from 0 miles to x miles. We know the depth of the rainfall; the volume of water is the depth multiplied by the area of the watershed, so we must determine the area of the watershed. We place the watershed for Chinle River on a coordinate system with the headwaters at the origin and with the river flowing along the positive x-axis over the interval [0, 10]. Thus the entrance to the lake is at the point (10, 0), and the watershed extends vertically 5 miles up and 3 miles down from this point. The watershed for the portion of the river from 0 miles to x miles is the sub-triangle shown in the figure. Follow the steps below to find the area of the watershed for the portion of the river from 0 miles to x miles, then answer the related questions.

Group Work

1. Write the equations of the lines which are the northern and southern boundaries of the watershed.
2. Use these equations to determine the area A(x) of the watershed for the river from 0 to x miles.
3. What is the area of the watershed for the river from 0 to 3 miles?
4. What is the area of the watershed for the river from its origin to the lake?

* * * *

4.2.2. Percentage of precipitation which enters the river bed

We must determine the percentage of water which contributes to the streamflow for Chinle River. Recall that this decreases linearly from 60% at the origin to 20% at the lake.

Group Work

1. Determine the function which gives the percentage D(x) of water from precipitation which reaches the portion of the river from 0 to x miles.

2. What percentage of precipitation reaches the river at 7.2 miles?

* * * *

4.2.3. Total volume of precipitation over the watershed

Recall that the volume of water from precipitation is the depth multiplied by the area of the watershed. We must be careful about the units of measurement when determining this. Since we will measure streamflow in cubic feet per second, it will be convenient to measure the volume of precipitation in cubic feet.

Group Work

1. Determine the function V(x) which gives the annual volume of precipitation in cubic feet which falls over the watershed for the river from 0 miles to x miles. Note that the amount of rainfall is already given in feet but the area of the watershed as determined by the function A(x) is in square miles. Therefore you must first convert this area to square feet.

2. What is the volume of precipitation which falls over the watershed for the portion of the river from 0 to 5.1 miles?

3. What is the volume of precipitation which falls over the entire watershed for the river?

4. Graph the function V(x).

* * * *

4.2.4. Streamflow

Now we are ready to achieve our goal, i.e., to determine the streamflow for Chinle River. Recall that the streamflow is predicted by the amount of water which actually reaches the river bed.

Group Work

1. Use the percentage function D(x) and the volume function V(x) to determine the function F(x) which gives streamflow for the river at any point x miles downstream for its origin. The measure for streamflow should be cubic feet per second; the volume V(x) is in cubic feet per year so you must first convert this to ft^3/sec. Round coefficients to two decimal places.

2. What is the predicted streamflow for Chinle River at 2.2 miles?

3. What is the predicted streamflow for Chinle River at the entrance to the lake?

4. What is the amount of water which flows from Chinle River into the lake each year?

5. Graph the function F(x).

* * * *

4.3. Electricity and Hot Water

As the population of our country grows, so does its need for electricity. A coal burning power plant is put into operation on Chinle River at its entrance to the lake. When coal is burned steam is produced, causing turbines to rotates and generate electricity. Only a portion (about 40%) of the heat produced is converted to electrical energy; the rest is waste heat which must be removed from the generator. Some of this heat is removed through the smokestack as hot gases (about 15%) and the remainder must be removed by other processes. In a common process, cooling occurs when water from a river or lake flows through the turbine condenser. The water is warmed by the waste heat and then is discharged back into the river. A large dependable flow of water must be available for this process, and the water used returns to the stream at a higher temperature. creating potentially harmful effects. In this section we will study the water flow necessary for this cooling process, how much the process increases the temperature of the stream from which it is drawn, and then determine what capacity electricity generating plant the Chinle River can support.

When heat is removed by water flowing through the condensers, the stream of water leaving has a higher temperature than the entering stream. If waste energy must be removed by cooling water at a rate of R megawatts, abbreviated MW, where 1 MW = 10^6 Joules per second then the energy discharged is R x 10^6 Joules. The specific heat of water (at 17 °C) is 4.184 $^{J}/_{g\ °C}$; this means that the energy required to raise the temperature of 1 gram water 1° C is 4.184 Joules. Therefore it takes $4.184 \times M \times d$ Joules to raise the temperature of M grams of water d° C. Then R x 10^6 Joules will raise the temperature of

$$\frac{1}{4.184} \times \frac{R}{d} \times 10^6 \text{ grams} = .239 \frac{R}{d} \times 10^6 \text{ grams}$$

of water at d^o C. Therefore if the energy discharge is R megawatts and the temperature increase is to be d^o C, then the required flow of water is $.239 \frac{R}{d} \times 10^6$ grams per second. Since

10^6 grams = 10^3 kilograms = 1 cubic meter,

this is $.239 \frac{R}{d}\, m^3/\text{sec}$.

If we convert this cubic feet per second and degrees Fahrenheit, (using the conversion factors, 1 cubic meter = 35.31 cubic feet and 1^o C = 1.8^o F) then the relationship is

$$F = .239 \times 35.31 \times 1.8 \frac{R}{T} = 15.2 \frac{R}{T}\, cfs,$$

or

$$F = 15.2 \frac{R}{T}\, cfs,$$

where

F = streamflow of the cooling water (in cubic feet per second),

R = rate at which waste heat is removed (in megawatts),

T = temperature change (in degrees Fahrenheit).

Example 1

A large coal burning power plant has the following characteristics:

*40% of the power generated is converted to electricity;

*15% of the waste heat removed through the smokestack;

*85% of the waste heat is removed by cooling water;

*the temperature of the cooling water is increased 18^oF in the process.

If the generating capacity is 1000 megawatts, what streamflow is required for cooling? We use the following information in solving this problem.

1000 MW = 10^6 kW = 10^9 x 3.412 Btu/hr =36x3.412 metric ton coal per hour.

2500 MW = 307 metric tons per hour.

Solution

We begin answering some easy questions.

1. How much power does it take to produce 1000 MW electricity?

Let P be the total rate of power production in megawatts. The efficiency of the plant tells you what percent of the energy is converted to electricity. Since this plant has 40% efficiency and generates 1000 MW electricity. to answer that question just solve the equation .40P = 1000 to get P=2500. The energy input (from burning coal) must be 2500 MW.

2. At what rate is waste heat discharged to cooling water?

Since the total energy input is 2500 MW and 60% of that is converted to waste heat, then waste heat is produced at a rate of 1500 MW (60% of 2500). Eighty-five percent of the excess is removed by the cooling process, so the rate of discharge to cooling water is 85% of 1500 MW, which is 1275 MW.

3. Determine the streamflow required to remove the waste heat, given that the temperature change is 18°F.

We have R = 1275, T = 18; therefore

$$F = 15.2 \times \frac{1275}{18} = 1077 \text{ cfs (rounded)}$$

This generating plant requires a streamflow of 1077 cubic feet per second.

* * * *

This example shows that to generate 1000 MW electric power, at least 1077 cubic feet water per second must be diverted from the river. When the water is returned to the river, assuming no loss of water due to evaporation and no cooling, it will be 18°F warmer than the undiverted water in the river. If this warmer water contributes too substantially to the total flow, the river water temperature will increase to an unsatisfactory level. Excessive temperature changes will kill fish living in the water, increase evaporation, and have adverse long-term effects on the ecosystem. For example, the optimal temperature for brook trout is 58° F; the maximum average temperature desirable for sustaining grown is 65° F and a temperature of 78° F is lethal. For these and other reasons power plants are limited as to how much warm water can be returned to the river. In the next example we will see how much the water temperature downstream will increase from this warm water if the river flow is 3000 cfs.

Example 2

Assume sufficient water is withdrawn for cooling the generating plant described above and the warmed water is returned to the river. If the river has a flow of 3000 cfs, how much will the downstream river temperature increase? Assume the warm water is fully mixed with the undiverted water.

Solution

We have the same heat discharge, 1275 MW, but the flow is now considered to be the water flow of the entire river, 3000 cfs. Use the basic relationship $F = 15.2\frac{R}{T}\ cfs$ and solve for T,

$$T = \frac{15.2 \times 1275}{3000} = 6.46.$$

The downstream water temperature will increase 6.46° F. Figure 4.3 illustrates the process.

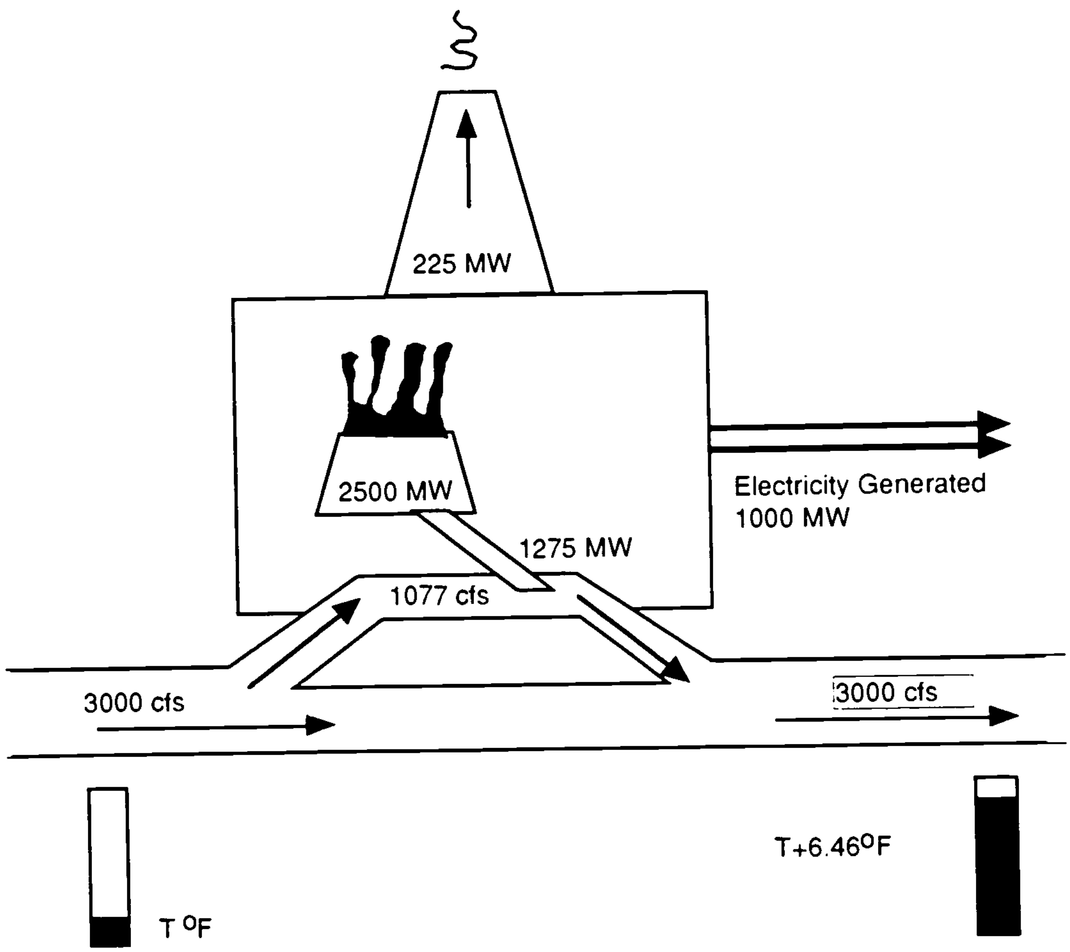

Figure 4.3

Now we return to the Chinle River where the generating plant has been built ten miles downstream from the headwaters of the river. We will determine the maximum capacity for this plant and estimate the number of people that can be supported.

Group Work

The coal-fired generating plant has the same characteristics as that described in the examples above:

*40% of the power generated is converted to electricity;

*15% of the waste heat removed through the smokestack;

*85% of the waste heat is removed by cooling water;

*the temperature of the cooling water is increased 18°F in the process.

Use the streamflow of the river at the power plant determined in the previous section.

1. Determine the total rate of power production and the electric power generating capacity of the plant if all of the river water is used for cooling.

2. If all of the river water is used for cooling, the river temperature will increase 18° F, causing serious damage to the fish, plants and microorganisms in the river. To avoid this damage, a decision is made that only 30% of the flow can be diverted. With this restriction, determine the capacity of the plant and determine how much the river temperature will be increased after the warmed water is returned to the river.

3. It is estimated that a generating plant with capacity 1000 MW will support 100.000 people and we will assume that the ratio of power to people is constant; i.e. that one megawatt supports 100 people. How many people can be supported by the generating plant with the restrictions described in part 2? Without the restriction.

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CHAPTER FIVE
THE PRICE OF POWER

5.1 Introduction

In this chapter we will see other effects of the power plant on Chinle River. Considerable amounts of sulfur (S) are locked up in coal and when coal is burned, the sulfur is released into the air in the form of sulfur dioxide (SO_2) at which stage the SO_2 acquires an additional oxygen molecule to become sulfur trioxide (SO_3). The precipitation absorbs SO_3 and produces sulfuric acid (H_2SO_4), which is deposited along with the precipitation onto the ground and into streams. The presence of excessive amounts of sulfur in the atmosphere means more acid falls with the precipitation. When this happens, the precipitation is called *acid rain*. Acid rain can not only harm vegetation but when rivers and lakes become too acid, fish and other aquatic life die out and drinking water obtained from these sources becomes foul.

The annual emission of sulfur dioxide from the plant is $10^{7.81}$ grams per year. This chapter is devoted to the study of the effect of this emission on the water of the Chinle River and hence on the lake (known as Lonesome Lake), or, the cost of having ample electricity. In order to do this, we look at the annual emission of sulfur from the power plant together with the annual precipitation. The chemical reactions of the sulfur, oxygen and water create hydrogen ions; the acidity of a liquid is measured numerically by a pH scale which is a measure of concentration of hydrogen ions (a formal definition is given below). The greater the amount of sulfur, the greater the number of hydrogen ions; this makes the liquid more acid. A low pH value indicates acid liquid, whereas alkaline, or less acid. water has a higher pH value. A pH value of 7 means the water has neutral acidity. The goal of this chapter is to determine the pH of the water in Lonesome Lake.

5.2. Sulfur Dioxide Emission

The first step in determining the effect of sulfur emission on Lonesome Lake is to determine the percentage of sulfur which actually winds up over its watershed. The dispersion of the SO_2 is dependent upon weather conditions and the effective height of the smokestack. We assume a fixed weather pattern with prevailing winds in the area out of the east, so that the watershed is downwind from the power plant. The *effective height* of the smokestack is the distance from the ground to the horizontal centerline of the smoke plume (see Figure 5.1).

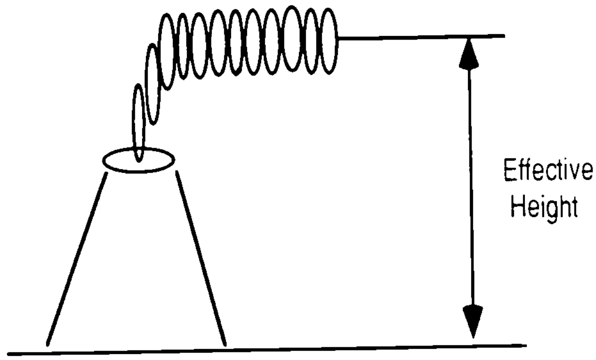

Figure 5.1

Class Work

If the horizontal distance d is 100 meters and the angle of elevation is 50.2° determine the effective height of the plume (round to the nearest meter).

* * * *

The tangent is one of the trigonometric functions which will be studied in chapter six. At the end of this chapter we will study the use of trigonometry to analyze right triangles.

The steps below lead you to a function which estimates the ground level concentration of sulfur dioxide in the air at a specified distance downwind from the stack. This function is used to approximate the percentage of sulfur emissions which lies over the watershed. As might be expected, the ground-level SO_2 concentration increases from the point of emission to its maximum relatively near the stack, then drops off rather rapidly approaching 0 at a distance. **Graph omitted here)**

Such a pattern can be approximated by a rational function with a denominator of degree larger than that of its numerator. In this case, a function of the form

$$C(x) = \frac{Q}{u}\left[\frac{Ax}{x^3 + B}\right]$$

will give a reasonable approximation. Here,

C(x) = ground-level concentration of sulfur dioxide in μg/m³ (x10^{-4}) at a point x km downwind from the stack,

Q = emission rate of SO_2 in μg/sec from the top of the stack,

$u =$ wind speed in m/sec,

and

A and B are constants.

The effective height of the smokestack determines both the point x_0 km downwind where the maximum ground-level concentration occurs and the maximum concentration $C_0 = C(x_0)$. We use observed data for these factors to first find functions which give x_0 and C_0 in terms of effective stack height H and then use these functions to derive the important function $C(x)$.

IMPORTANT: The data provided in the tables that follow are for $Q = 1$ μg/sec. and $u = 1$ m/sec.

Group Work

1. The data shown in Table 5.1 provide a relationship between stack height and the point of maximum concentration. Make points (H, x_o) out of these data where

$H =$ effective height of the smokestack in meters (x10),

and

$x_o =$ distance in kilometers downwind from the smokestack where the maximum ground-level concentration occurs.

Set up a coordinate system and plot these points.

Effective Stack Height (x 10 m)	Point of max SO_2 conc. (km)
2	0.15
3	0.32
4	0.42
5	0.55
7	0.80
10	1.30
15	1.80
20	2.50
25	3.00
30	4.00

Table 5.1

2. After observing the pattern of the plotted points, determine the linear function which can be used to approximate these data; use points corresponding to effective heights of 20 and 300 meters. The resulting function should define x_0 in terms of H.

3. Next, the maximum value C_0 ($\times 10^{-4}$) of the ground-level SO_2 concentration will be determined as a function of stack height H. Table 5.2 provides data relating these two factors; here H is in meters x10 and C_0 is in $\mu g/m^3$ ($\times 10^{-4}$).

H (x10)	C_0 ($\times 10^{-4}$)
2	3.500
3	1.500
4	0.900
5	0.580
7	0.300
10	0.150
15	0.064
20	0.035
25	0.022
30	0.015

Table 5.2

Make points (H, C_0) out of these data, set up a coordinate system and plot your points.

4. The pattern formed by the plotted points suggests a simple rational function. Fit the data with a function of the form $C_0 = k/H^2$ using the point corresponding to an effective stack height of 100 meters, i. e., H = 10.

At this point, you should have two functions each of which is defined in terms of the stack height H. For a particular stack height, these functions determine the point x_0 kilometers downwind where the maximum ground-level concentration of SO_2 occurs and the SO_2 concentration C_0 at that point. Using these, it is possible to determine the constants A and B for the concentration function,

$$C(x) = \frac{Q}{u}\left[\frac{Ax}{x^3 + B}\right],$$

when $Q = 1$ μg/sec. and $u = 1$ m/sec.

5. Determine the values of x_0 and C_0 for the effective height of this smokestack (recall your answer to #1).

6. Using methods from calculus, it can be determined that the maximum value for $C(x)$ occurs when $x = (B/2)^{1/3}$. Use this information to determine the constants A and B, and hence the concentration function $C(x)$ when $Q = 1$ μg/sec. and $u = 1$ m/sec.

7. Convert the annual emission of SO_2 from the power plant to micrograms per second and write your answer as a power of 10; this will be the value for Q.

8. Use your answer to #7 and a wind speed of $u = 1$ m/sec. to determine the function

$$C(x) = (Q/u)[Ax/(x^3 + B)]\ (\times 10^{-4}).$$

This will be the concentration function we will use throughout the remainder of this and sections 5.3 - 5.4.

9. Graph the function $C(x)$ and determine its maximum value. (Use the calculator or computer.)

Now we will determine the ground level SO_2 which actually is distributed over the watershed. Recall your answer to #4 in 4.2.1, the area of the watershed. Also recall that the watershed is 10 miles long extending in the downwind direction from the power plant. At distances over 20 kilometers from the plant, SO_2 concentrations are insignificant, so we will find the percentage of SO_2 distributed over the basin relative to the amount which is distributed from the point of emission (smokestack) to the point 20 kilometers downwind.

10. The units of measure for $C(x)$ are micro-grams per cubic meter of SO_2. For example if $C(3) = 52$ μg/m^3, this means that one cubic meter of air on the ground at 3 km from the smokestack contains 52 micro-grams of sulfur dioxide. Now imagine lots of cubic meters of air lined up on the ground from the smokestack to 20 km downwind. This would form a "box" of air one meter wide, and 20 km long. (See Figure 5.4.)

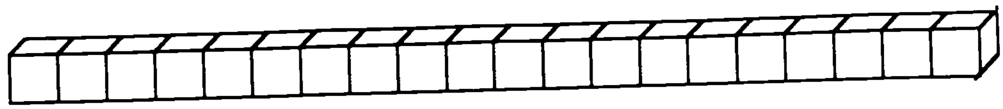

Figure 5.4

A) Approximate the amount of SO_2 in this "box" of air. Assume that the SO_2 concentration is C(x) for each kilometer from x - 1 to x. (Note: using the above example, each cubic meter of air in this box from 2 km to 3 km contains 52 micrograms of SO_2.)

B) The lake is on the eastern end of the watershed, the watershed itself extending lengthwise 10 miles west from there. Approximate the total amount of SO_2 in the "box" of air through the watershed. Use the conversion 10 miles = 16 kilometers.

11. Use your answers from #10 to approximate the percentage of SO_2 which lies over the watershed. (Although this percentage does not take into account sulfur dioxide over the entire region, it does give an approximation which can be used to compute the amount of sulfur deposited annually over the watershed.)

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5.3. Acid Rain

You are now ready to begin work on the determination of the overall effect of SO_2 emission on Lonesome Lake. The next step is to figure the acidity of precipitation in the region.

The acidity of a liquid is measured by its hydrogen ion concentration, and is denoted by pH. If z denotes the hydrogen ion concentration in moles per liter, then

$$pH = -\log z.$$

The pH varies from 0 to 14; a graph is shown in Figure 5.5. **(Graph omitted here.)**

Note that as the hydrogen ion concentration z increases, the pH decreases. Since the larger the hydrogen ion concentration the more acidic the liquid, then small values of pH indicate an acid liquid, whereas large pH values indicate alkalinity. Water with pH = 7 is neutral and considered "pure" whereas water with pH smaller than 7 is considered acidic. Thus for pure water, the hydrogen ion concentration is 10^{-7}, and

$$pH = -\log(10^{-7}) = -(-7) = 7.$$

The steps below will lead you to the determination of the pH of precipitation over the watershed. For simplicity, the only sulfur dioxide source we consider in these calculations is the emission from the power plant.

Group Work

1. Recall the total annual emission of SO_2 from the plant is $10^{7.81}$ grams per year. Our primary concern is the amount of sulfur emitted since sulfur is the component which forms the sulfuric acid, and sulfur must be measured in *moles*. Determine the number of moles of sulfur in the annual SO_2 emission from the power plant. You need this information. Sulfur is approximately one-half the weight of sulfur dioxide; the atomic weight of one sulfur molecule is 32.066 awu (atomic weight units) whereas one oxygen molecule has atomic weight 16 awu. One mole of substance is equal to its molecular weight in grams; so 32.066 grams of sulfur equal one mole.

2. Recall your answer to # 3 in 4.2.3. This is the total amount of water (in cubic feet) from precipitation over the watershed. Use this to answer A and B.

A) Convert the total amount of water from precipitation over the watershed to cubic meters. (Note: 1 ft = .3048 m.)

B) How many liters of water result from precipitation over the basin? (Note: one liter of water occupies 10^{-3} cubic meters.)

3. Determine the concentration of sulfur (in moles per liter) in precipitation over the watershed. Use your answers to #1 and #2 in this section and assume that approximately one-fourth of the airborne sulfur is deposited in precipitation.

4. Now you can determine the pH of the precipitation. For every molecule of sulfuric acid, two hydrogen ions are produced; i.e. for each molecule of H_2SO_4 which contains one sulfur molecule, two hydrogen ions (H^+) result. Determine the concentration (in moles per liter) of hydrogen ions in precipitation over the basin.

5. What is the pH of the precipitation?

5.4. Acidity of the Reservoir Water

Two things happen which affect the pH of the precipitation before it reaches the reservoir. Recall from Chapter 4 that at the point of entry of Chinle River into Lonesome Lake only 20% of the precipitation over the watershed actually enters the stream. Next, significant evaporation of the surface water in the lake occurs; both of these events further concentrate the sulfuric acid. In this process the sulfuric acid remains so that there is an even higher concentration in the lake water. Thus the concentration of H^+ ions is higher and the pH is lower. In the steps below, you will determine the acidity of the water in the reservoir; we assume that no other chemical reactions take place which will further affect the pH of the water.

1. Determine the hydrogen ion concentration of the water from precipitation as it enters the lake.

2. What is the pH of the water as it enters Chinle River?

After the precipitation finally reaches the lake, more evaporation takes place. Evaporation from a large surface of water occurs at a great rate; one-fourth of the lake water is evaporated. This, once again, increases the sulfuric acid (and hence the hydrogen ion) concentration.

3. Determine the hydrogen ion concentration after this second evaporation occurs.

4. What is the pH of the water in Lonesome Lake?

CHAPTER SEVEN
SNOW-FED RIVERS

7.1 Predicting Streamflow

We begin a study of water supply for a small but growing village, River City, on a mountain stream, Nizhoni River, which is fed primarily by snow. The village is located near the high elevation mountainous region which is the origin of the stream, and the entire water supply for the village comes from this stream. The situation is fictitious. but is designed as a sort of scaled-down version of the La Plata River which originates in the San Juan Mountains of southwestern Colorado, and a scaled-up version of the small town of Hesperus, Colorado, which is located very near the source of the La Plata. Due to the arid nature of this country it is very important to be able to predict the amount of water which will be available each year and when it will be available because during periods of high streamflow, it is necessary to store water for use during drier periods.

The forecasting of annual water supply is based primarily on the amount of precipitation, particularly in the form of snow, and temperatures in the region. Average monthly temperatures and precipitation generally are cyclic on an annual basis, and of course, snowfall is dependent upon both these factors. The patterns for our region are: most of the precipitation comes in winter and spring, tapers off to a low point in fall and then increases after that; temperatures follow the known pattern. lowest in winter and highest in summer. For this study data describing temperature and precipitation will be provided. Based on these data we will derive functions which describe both of these, and together with a function which estimates the amount of snow melt at a particular temperature, use these to predict streamflow, the amount of water which goes down the river in some interval of time.

In the following sections we will derive functions which will be used to determine the streamflow for the Nizhoni River. The functions are all functions of the variable t, $0 \leq t \leq 12$, where t represents time (in months) from the beginning of the year.

$A(t)$ = temperature at time t (degrees Fahrenheit)

$R(t)$ = rate of precipitation at time t (inches per month)

$W(t)$ = rate of precipitation over the watershed at time t (x 10^6 ft^3 per month)

M(t) = rate of snow melt over the watershed at time t ($\times 10^6$ ft^3 per month)
Note that R(t) measures the rate of precipitation at a particular point while W(t) measures the rate of precipitation over the entire watershed. From these we will derive a function F(t) which describes the streamflow of the Nizhoni River in million cubic feet per month.

7.2 Temperature

Air temperature is a cyclic phenomenon with a cycle of 12 months. We derive a function A(t) to estimate the temperature at time t, based on an average taken over a number of years. Note that this function of course is not a very accurate indicator of temperature at a specific time since it is normally cool in the morning, warmer during the day, and cooler again at night. Also, for any particular year and at different times of day the temperature will vary from the average. However it will allow us to approximate average temperature for certain times. For example, suppose A (2.5) = 27. This means that at time 2.5 (some time on March 16th) the temperature will be approximately 27 degrees.

Group Work

The Nizhoni River originates high in the mountains and even during the warmest times of year the temperature is quite low. Based on average temperature data, we make the following assumptions:

the minimum temperature of 10.0° occurs at the end of January (t = 1);
the maximum temperature of 49.7° occurs at the end of July (t = 7);
temperature is cyclic on an annual basis.

Determine a function which describes the average daily temperature at time t; use a cosine function of the form $A(t) = a + b \cos[c\,(t - d)]$ which satisfies all of these assumptions. Follow the steps below (round coefficients to two decimal places).

1. Sketch a rough graph of how the function should look. Label the maximum and minimum points.
2. What is the period?
3. Determine the amplitude. Is b positive or negative?
4. Determine the phase shift and the vertical shift.
5. Write the function A(t) satisfying the properties in the previous steps, and graph a complete cycle of the function.

Use A(t) to answer the remaining questions about temperature in this chapter.

6. What is the temperature at the end March? When does the temperature again reach that level?

7. When will the temperature be 32°?

8. We assume that when the temperature is above 32°, precipitation means rain; otherwise precipitation is in the form of snow and it sticks. Determine the period during which temperatures are above freezing, i.e., when precipitation is in the form of rain and snow melts. (Round your answers to one decimal place.)

9. Indicate on the graph when A(t) = 32.

* * * *

7.3 Precipitation

The Nizhoni watershed is quite dry; the wettest times of year are the end of March (t = 3) when the precipitation rate is 2.5 inches per month, and the end of September (t = 9) when it is 3.0 inches per month. The driest times are late June (t = 6) and the end of December (t = 0 or t = 12). Although in fact it is a bit drier in June than December, to simplify this model we will assume that the rate is the same at both times, 1.65 inches per month. Figure 7.1 below shows a possible precipitation graph.

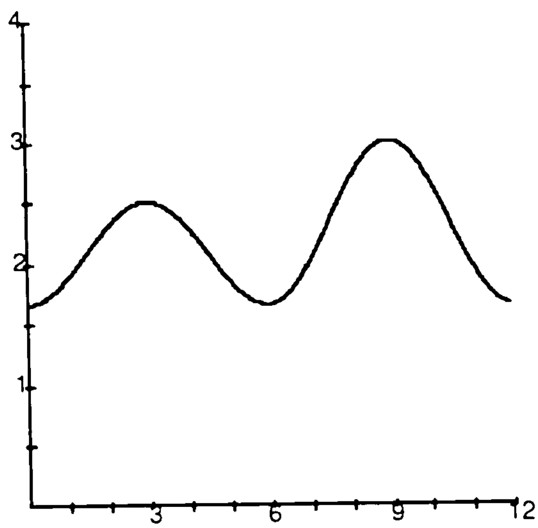

Figure7.1

In the group work which follows you will derive a function R(t) which describes the rate of precipitation (in inches of water per month) at time t. The sketch above might suggest two functions, one for the first six month and another for the next six month. We will derive two trigonometric functions which together will make up a piece-wise function which will give the rate of precipitation at any time. The steps below will lead you through this procedure.

Group Work

We will determine the equation for each of the two pieces, corresponding to the intervals $0 \leq t \leq 6$ and $6 \leq t \leq 12$. Since these will agree for $t = 6$, they can be pieced together to give a function defined for $0 \leq t \leq 12$.

1. Sketch a graph of the first piece, $0 \leq t \leq 6$. Recall that the first period has maximum precipitation rate of 2.5 inches per month when $t = 3$ and minimum rate of 1.65 inches per month when $t = 0$ or $t = 6$.

(a) Indicate the maximum and minimum values.

(b) Determine the period, amplitude, vertical shift and phase shift.

(c) Determine an equation which has the properties as shown on the graph. Use the form $y = a + b \cos[c (t - d)]$.

2. Repeat the process for the second piece, $6 \leq t \leq 12$. For this interval the maximum precipitation rate is 3.0 inches per month when $t = 9$, with minimum rate of 1.65 inches per month when $t = 6$ or $t = 12$. If the phase shift is more than one period, you might be able to simplify the function. Do so if you can.

3. Write the function R(t) in the form

$$R(t) = \begin{cases} \text{first expression} & \text{if } 0 \leq t \leq 6 \\ \text{second expression} & \text{if } 6 < t \leq 12 \end{cases}$$

and graph the function. Use this function (and its graph) to answer the remaining questions.

4. Predict the precipitation rate on December 15.

5. When will the rate be greater than 2 inches per month?

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6. We can estimate the average precipitation for any particular month or portion of a month by determining the rate at the mid-point of the time period. For example, the inches of precipitation for the month of May can be approximated by

[R (4.5) inches/month] x [1 month];

the number of inches of precipitation for the first half of June is approximated by

[R (5.25) inches/month] x [.5 month].

Estimate the precipitation for:

(a) January;

(b) May;

(c) the last half of October;

(d) the period from t = 2.4 to t = 2.7.

* * * *

7.4 Rain and Snow

We now can determine at any time t how cold it will be and the rate of precipitation. In this section we will determine how much of this precipitation will fall as rain and how much as snow. This will be important in determining the streamflow since the rain enters the stream almost immediately but the snow doesn't enter the river until the spring snow melt. First we will modify the precipitation function R(t), which describes precipitation rate in terms of inches per month to get a related function, W(t), which describes the precipitation rate over the watershed as million cubic feet per month.

In order to do this, we first need to determine the area of the watershed.

Group Work

The shape of the watershed can be approximated by Figure 7.5 shown. (Dimensions are given in miles.) Use the method illustrated above to determine the area of the watershed. Round all intermediate calculations to four decimal places, and round the final answer to the nearest square mile.

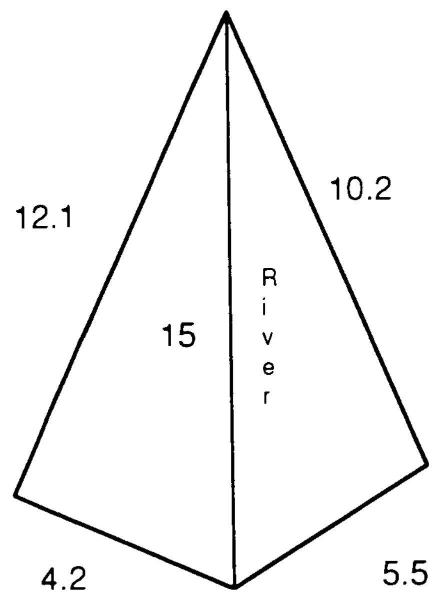

Figure 7.5

* * * *

Now that you have determined the area of the watershed it is possible to determine the precipitation rate over the entire watershed as cubic feet per month. Before you do this, it will help to think for a minute about volume. Imagine that one inch of rain that falls uniformly over one square mile. The volume of rain is the volume of a box of water with base 1 square mile and height one inch. The volume of that box in cubic feet is

$$
\begin{aligned}
1 \text{ square mile} \times 1 \text{ inch} &= 52802 \text{ square feet} \times 1 \text{ inch} \\
&= 27.8784 \times 10^6 \text{ square feet} \times \frac{1}{12} \text{ foot} \\
&= \frac{27.8784}{12} \times 10^6 \text{ cubic feet} \\
&= 2.3232 \times 10^6 \text{ ft}^3.
\end{aligned}
$$

Thus one inch of rain produces 2.3232×10^6 ft^3 of water over each square mile of the watershed.

Group Work

Follow the steps below to determine a function W(t) which describes the precipitation rate at time t in terms of million cubic feet per month.

1. The area of the drainage watershed for the river was determined in the previous group work. Use this information to convert the function R(t), which measures inches of rainfall per month, to a new function W(t) which measures cubic feet of water per month over the entire watershed at time t. Write coefficients $x10^6$ and round to four decimal places.

2. Graph the function W(t).

3. Now use W(t) to approximate the total monthly precipitation in cubic feet ($x10^6$) of water over the entire watershed, and sum these figures to determine the total annual precipitation. It will be useful for future work to make a table with two columns, the first indicating month t and the second showing precipitation ($x10^6$ ft^3) in month t. (Round your answer to four decimal places.) Use the techniques described in #6 in the previous section.

4. Some of the precipitation that falls comes down as rain and some as snow. Use the function W(t), the answer to #8 in the temperature section, and the techniques in #3 above to determine the number of cubic feet of water annually from snow over the watershed. (Round your answer to four decimal places.) Again, a table will be helpful, this time with four columns, the first indicating the interval of time, the second the midpoint, the third the length of the interval and the fourth the precipitation from snow.

* * * *

7.5 Snow Melt

The rate at which snow is melting, measured in inches per day, depends on temperature as well as other factors. The rate can be approximated by a linear function of surrounding air temperature,

$$M_o = C(T - 32^o),$$

where M_o denotes the number of inches of water from snow melt per day at the surrounding average air temperature T degrees Fahrenheit, and C is called the melt factor (Hydrology, by Wisler and Brater, John Wiley, N.Y., 1959). Generally the melt factor can vary from .02 to .13; in order to accurately reflect what really happens in the La Plata watershed we choose C = .03 so

$$M_o = .03\ (T - 32\ ).$$

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In order to determine streamflow, it will be necessary to know the rate of snow melt over the entire watershed in cubic feet of water at time t. Follow the outline below to obtain this function.

Group Work

1. The function M_0 is given above in inches of water per day. Convert this to a new function M_1 which describes the rate of snow melt over the entire watershed in cubic feet ($x10^6$) of water per month at surrounding air temperature T^o F. Round coefficients to four places; use $\frac{365}{12}$ days per month and follow the techniques of #1 in the previous section.

2. You now have enough information to determine the rate of snow melt at time t. Recall the function A(t) for temperature derived in the preceding section. Form a composite function by substituting this for T in the function M_1 from #1 above; then simplify. The result should be a function M(t) which describes the rate of snow melt in million cubic feet of water per month for the entire watershed at time t. (Round coefficients to four places.)

The next two parts enable you to determine the domain of the function M(t).

3. Note that $M_0 = .03(T - 32)$ only makes sense if $T \geq 32^o$, so recall your answer to #8 from the **Temperature** section and use this to specify the left-hand end point of the domain of M(t). (Round values of t to one decimal place.)

4. Recall your answer to #6 from the **Rain and Snow** section and use this and the function M(t) to determine when all of the snow in the watershed will be melted. First approximate snow melt for each appropriate month or portion of a month; use the mid-point of the interval to estimate the rate of snow melt for that interval and round answers to one place. Your answer is obtained when the cumulative amount of water from melting snow approximately equals the amount of water from snow determined in #4 in the previous section. This will determine the right-hand end point of the domain of M(t). Again it may help to make a table, this one with five columns, the first indicating the interval of time, the second the

mid-point, the third the length of the interval, the fourth the corresponding snow melt and the fifth the cumulative amount.

5. What is the interval which is the domain of M(t)?

6. Graph the function M(t) and indicate its domain.

* * * *

7.6 Streamflow

Streamflow is defined as the volume of water which passes a cross-section of a stream in a specified unit of time. Units are usually cubic feet per second (cfs) but could be otherwise, such as cubic inches per minute or cubic meters per day, etc. Streamflow will be different at various points on a stream and at different times of the year, so point of measurement as well as time of year must be designated. For example, to say that the streamflow of the La Plata River at Hesperus, Colorado, on April 15 is 83 cfs means that on that day at Hesperus, 83 cubic feet of water passed down the La Plata in one second. It is important to be able to predict flow for periods during the year in order to monitor water storage or to prepare for floods or droughts and other things of this nature. In this section of the study, we will predict streamflow for Nizhoni River. Our prediction will be based on information about precipitation, temperature and snow melt derived previously.

We consider three ways in which precipitation is dispersed, *runoff, ground water*, and *evaporation*. Runoff refers to water which runs over the surface of the ground directly into the stream; ground water is water which seeps into the ground: and evaporation refers to water which, well..., evaporates. So runoff is a direct and immediate contribution to streamflow; ground water eventually seeps into the stream thus being an indirect contributor to streamflow. Evaporation, of course, contributes nothing. We make the following assumptions regarding precipitation and the water from melting snow:

- 80% is runoff;
- 15% is ground water;
- 5% is evaporated.

For any stream, *base flow* is water obtained from ground water, and since ground water seeps slowly into the stream, we consider base flow for our river to be constant year round, i.e., the same each month. Therefore base flow is sort of a minimum flow for the stream, rain and snow melt are like "extra" water.

You now have all the information needed to write a function which describes streamflow. This will be a piecewise function; follow the steps below to obtain its definition.

Group Work

1. Determine the periods of the year when base flow is the only water in Nizhoni River. (See Figure 7.6.)

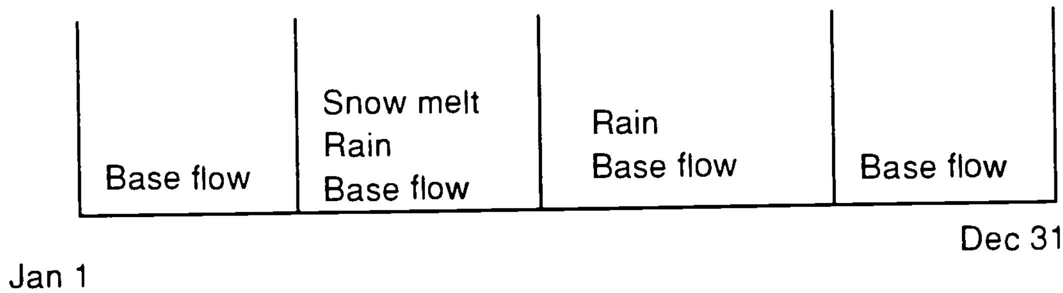

Figure 7.6

2. Determine each of the following (see Figure 7.6):

a. the period of the year during which, in addition to base flow, the stream receives water from both rain and snow melt;

b. the period of the year during which the only source of water other than base flow received by the stream is from rain.

3. We consider the base flow to be constant each month. This amount can be determined if we assume the total base flow is 15% of the annual rainfall, the amount that seeps into the ground. Determine monthly base flow for Nizhoni River. (Write your answer x10^6 cubic feet and round to four decimal places.)

4. Now put the pieces together. Use the assumptions made in this section to write the piecewise function F(t) which describes streamflow for Nizhoni River in cubic feet (x10^6) in month t, $0 \leq t \leq 12$. When simplifying, round coefficients to four decimal places. (See Figure 7.6.)

5. Graph the function F(t) and indicate the interval over which each piece is defined.

6. Use the function F(t) to approximate monthly flows for the river, i. e., the total amount of water which flows through each month. This must be done separately for each piece of the function F(t): make a table with four columns, the first indicating the interval of time, the second the mid-point, the third the length of the interval and the fourth indicating the streamflow for the month or the portion of the month. Now make a table which indicates total streamflow for each month of the year. (Round answers to four decimal places.)

* * * *

CHAPTER EIGHT
THE TOWN OF RIVER CITY

Introduction

In this chapter we study the population growth of a village and the resulting increase in water demand. In order to accommodate this increase, it will be necessary to construct a reservoir, and details of this project are dependent upon both population growth and availability of water.

8.1 Population and Water Usage

The small village of River City, located high in the mountains on the Nizhoni River, depends solely on the river for its source of water. The settlement was started some years ago by a small group of people in search of solace. They were joined by others of like mind and presently the population is 3000 and has been growing at the rate of 6% per year. In order to predict water consumption, we must first create a model that predicts the village population in future years. Since the population is now 3000 with a projected growth rate of 6%, then next year there should be 3000 + .06(3000) people. Rather than calculate the numerical value of this expression we work to obtain a general function which will give the population in any year. So factor out the common term 3000 to get

$$3000(1 + .06) = 3000(1.06)$$

people in the village 1 year from now. In 2 years there will be

$$3000(1.06) + .06[3000(1.06)] = 3000(1.06)[1 +.06] = 3000(1.06)^2$$

people.

Now follow this example in the first three problems below to obtain the desired function.

Group Work

1. Write the expression which gives the village population in 3 years; leave it as a product of terms as in the above example.

2. Repeat problem #1 for 4 years from now.

3. Generalizing from the examples and problems #1 and 2, write the function which gives the village population $P(x)$ in x years from now (note that $x = 0$ in the

current year). This function is called an *exponential function*; you will see more about exponential functions in the latter part of this chapter.

4. Graph the function P(x), $0 \leq x \leq 35$.

5. Predict the population of River City 5 years from now: 15 years from now.

* * * *

Upon first settling, the people of the village daily withdrew the amount of water necessary for their personal use. However, as the population grew the residents who lived further downstream noticed that at certain times during the year less and less water was actually getting to them. This was obviously a cause for concern. They observed that in spring and fall, there was ample water flowing through but that in late fall and early winter, there was not much at all getting downstream. It became clear that the increasing population was drawing more and more water from the Nizhoni River, thus decreasing the water flowing downstream and eventually depriving the downstream settlers of sufficient water in the latter part of the year. To protect the rights of downstream users (other villages and towns, farmers, ranchers, etc.) regulations were put on the maximum amount of water that can be withdrawn from the river. Regulations applied to River City are: a maximum of 10% of base flow, no more than 15% of the flow which exceeds base flow can be withdrawn monthly, and a maximum monthly withdrawal of 30 million cubic feet (this is just over seven million gallons a day). Sometimes the Nizhoni River provided more than enough water, sometimes barely enough. Hence, it would be desirable to save the excess water which flows through in spring and fall for use in the drier months of the year, i. e., build a reservoir. But how big should it be?

In the early days of River City, its residents' water needs were small; water for drinking, cooking, cleaning, etc., was all that was necessary. But as the town grew, people began trades, businesses were started, the need for community support developed, and some people began farming and raising livestock. With all of this, their per capita daily water consumption increased to about 195 gallons which is approximately 26 cubic feet. If indeed the residents are to assume the seemingly necessary task of constructing a reservoir, in light of the increasing population it would be wise to build it so that it would serve the town for some years to come. The town decides to build a reservoir provided that it is possible to build one large enough to handle the water needs of the community for at least the next thirty years. In order to make the final decision some important questions needed answering.

* What is the projected population for the next 30 years?
* How many people can the river support if no reservoir is built?
* How many people can the river support if a reservoir is built?
* When must the reservoir be completed in order that the residents not be without water?
* If the reservoir is built to capacity, for how many years will it serve the needs of the community?

These questions are addressed in the steps below. Work through each of these steps (you will need answers obtained from previous work in this study) to reach your conclusions about provision of water for future residents of River City. Recall the regulations on the amount of water that can be withdrawn:

** a maximum of 10% of base flow;*

** no more than 15% of the flow which exceeds base flow can be withdrawn monthly; and*

** a maximum monthly withdrawal of 30 million cubic feet.*

When you have completed the steps below, write a summary report which answers each of the above questions.

Group Work

We will determine projected water needs based on the population model.

1. Use the population function to predict the village population in 30 years.

2. Assuming a constant monthly amount of water is used each month, what is the total amount of water used monthly by the residents presently? Can enough water be withdrawn each month from Nizhoni River to provide for this usage?

Next we determine how many people the river can support, with and without a reservoir.

3. Determine how many people the river can support if no reservoir is built. Assume that the residents use a constant amount of water each month. When will the population reach this amount?

4. Determine the maximum amount of water that can be withdrawn from the reservoir each month if the regulations are followed. The table shown will help you answer this question. The first two columns summarize the information on flow derived in the streamflow section. To complete the table follow the steps below.

i) Determine the maximum amount which can be withdrawn annually from base flow.

ii) Determine the annual amount of water flowing through Nizhoni River which exceeds base flow and then determine the maximum amount which can be withdrawn annually from flow which exceeds base flow.

iii) Determine the maximum amount of water which can be withdrawn by River City annually from the river under the imposed regulations.

5. How many people can be supported by the river if all the water which can be withdrawn is saved for use when needed.?

6. When will the population reach this level?

7. Summarize your work in a report which answers the questions posed above. Should the town build the reservoir? If so, when must it be completed and for approximately how many years will it supply enough water for all of the residents.

	FLOW			ALLOWABLE WITHDRAWAL		
Month	Total	Base	Flow over base	From Base	From flow over base	Total
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
Totals						

8.2 Reservoir Capacity

Follow the steps outlined below to determine the size for the reservoir which will serve the citizens of River City for as many years as possible. The reservoir should support the population which you determined in #5 in the preceding section.

Group Work

Fill in the table below to determine how big the reservoir should be and how much water it will hold at different times of the year. The first column in this new table comes from the table above.

1. i) Determine the monthly usage when the population is at capacity. Assume the water usage is constant each month.
   ii) Determine what the surplus or deficit is for each month; i.e. what is the difference between the amount of water used and the amount withdrawn from the river each month. Use positive numbers to denote surplus (when more water is withdrawn than used) and negative numbers to denote deficit.

2. Determine the minimum volume of the reservoir which must be constructed to hold the amount of water necessary to store in order to support the maximum number of people of River City. Since you want to have a little extra room, round up to the nearest 5 million cubic feet.

3. The last column of the table constructed indicates the volume of water in the reservoir at the end of the indicated month. The volume of the reservoir is slightly more than the amount needed, so assuming the reservoir is filled to capacity there will always be a certain amount of water remaining (this will be the minimum volume of water, or the difference between the actual capacity and the required capacity you determined in #2). Complete the table beginning with the first month with surplus water and assume that the volume of water is minimum at this time.

4. Summarize your work in the form of a short report. Be sure to designate the volume of the reservoir, the number of people it will support under current

usage, and the number of years before the reservoir will become inadequate due to population increase.

Month	Allowable withdrawal	Monthly Usage	Surplus or Deficit	Volume of Water
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
Total				

* * * *

CHAPTER NINE
ARSENIC

9.1 Introduction

Development, agriculture, mining and industrial activity not only increase the amount of water used, these activities also effect the quality of the water. Mining is particularly hard on surrounding water sources. In this chapter we examine the effect of a gold mine near the Nizhoni River. Many substances are used in the processing of gold, and we will examine the effect of one of these, arsenic, on the water supply of River City.

A gold mine built near the river upstream from River City produces over 6000 metric tons of waste for each kilogram of gold produced, and a portion of that waste seeps into the river and downstream into the water supply system. The mine discharges 15 pounds of arsenic each day in the waste that enters the river and mixes with the water before flowing into the reservoir. Although a small amount of arsenic has no noticeable effect, if the level gets too high the water quality suffers. Since the reservoir is used for drinking water and since the residents eat fish from the river, the people of the town are concerned about the degradation of the water supply. Government regulations limit the concentration of arsenic in drinking water to .05 milligrams per liter of water. The residents of River City begin monitoring the quality of water, testing for arsenic to ensure that the level remains within the government guidelines.

9.2 Arsenic Concentration

The concentration is measured in weight per volume. The regulations use milligrams per liter but we can use any convenient units of weight and volume. In Chapter 8 you computed the volume of water in the reservoir in million cubic feet, and since the amount of arsenic entering the river is given in pounds, we will first measure the concentration in pounds per cubic feet. The concentration varies with time; arsenic enters the river, mixes with the water, flows through the reservoir and flows out. The concentration depends not only on the amount of arsenic entering the reservoir and the volume of water in the reservoir but also on the amount of time it takes the water together with the polluting substance to flow out of the reservoir.

9.2.1 Average Flow, Average Volume

We start with a simplified case, where the volume and flow are constant. Although the actual volume of water in the reservoir and flow of water through the reservoir varies over time, we will begin with a simplified study. Assume that the volume of water in the reservoir is constant, 37.5 million cubic feet. and the flow of water into and out of the reservoir is constant 180 million cubic feet per month. (These are "averages" for the volume and flow from chapter X.) Assume also that arsenic enters the reservoir at a constant rate, 15 pounds per day. Let c(t) denote the concentration (pounds per 10^6 cubic feet) and m(t) the amount (pounds) of arsenic in the reservoir at time t. We will answer a series of questions to determine whether the water quality meets the EPA standards.

Question 1 How much water flows into (and out of) the reservoir each day?

The flow of water is given per month so we convert this to daily flow,

$$180.0 \times 10^6 \text{ cubic feet per month} =$$
$$180 \div \frac{365}{12} \times 10^6 \text{ cubic feet per day} =$$
$$6 \times 10^6 \text{ cubic feet per day (rounded).}$$

Question 2 How much arsenic flows in each day?

This is easy. fifteen pounds per day.

Question 3 What portion of the arsenic in the reservoir flows out each day?

The amount flowing out depends on the amount present, but a fixed portion flows out each day. The amount of arsenic flowing out each day is the concentration of arsenic in water times the amount of water flowing out (6 x 10^6 cubic feet). Since the concentration is $\frac{m(t)}{37.5}$ pounds per 10^6 cubic feet, arsenic flows out at a rate of $6 \times \frac{m(t)}{37.5} = .16 \times m(t)$ pounds per *day*. That is, 16% of the arsenic present flows out each day and 84% remains.

Question 4 How much arsenic is there in the reservoir at any time t? Determine a formula for the function m(t) which describes this.

To determine a formula first notice that each day 15 pounds of arsenic is added to the 84% of "old" arsenic that remains, $m(t+1) = 15 + .84m(t)$.

$m(1) = 15 + 84(0) = 15$

$m(2) = 15 + 84(15) = 15(1 + .84)$

$m(3) = 15 + .84\{15(1 + .84)) = 15(1 + .84 + .84^2)$

...

At the end of t days the amount of arsenic in the lake is given by the formula

$m(t) = 15(1 + .84 + .84^2 + \ldots + .84^{t-1})$;

notice that this is a geometric series and recall the formula for the sum (see section 9.4 for this derivation and more on geometric series):

$$a\left(1+r+r^2+\ldots+r^{n-1}\right) = a\cdot\frac{1-r^n}{1-r}.$$

Therefore after t days the amount of arsenic present is

$$15\left(1+.84+.84^2+\ldots+.84^{n-1}\right) = 15\cdot\frac{1-.84^n}{1-.84} = 93.75(1-.84^n),$$

i. e.,

$m(t) = 93.75(1 - .84^t)$.

Question 5 What is the concentration on day t? Determine a formula for this.

The concentration is simply the amount divided by volume,

$$c(t) = \frac{93.75(1-.84^n)}{37.5} = 2.5(1-.84^n) \text{ pounds per } 10^6 \text{ cubic feet.}$$

Question 6 What happens to concentration of arsenic in the water over a long period of time? Use the graph of c(t) and interpret its asymptote. **(Graph omitted here.)**

The graph appears to be leveling off, and this apparent trend can be verified if we take a closer look at the function $c(t) = 2.5(1-.84^n)$. This is an exponential function with base less than 1. The graph has a horizontal asymptote y = 2.5 and the range consists of all y < 2.5 and hence the concentration is always less than 2.5 pounds per million cubic feet.

Question 7 What is the highest level (in milligrams per liter) that the concentration reaches?

A conversion is required,

$$\frac{1 \text{ pound}}{10^6 \text{ cubic feet}} = \frac{.016 \text{ milligram}}{\text{liter}}.$$

Then the highest level of concentration reached is 2.5 x .016 = .040 mg per liter, so the concentration stabilizes at a rate within the federal guidelines.

9.2.2 Change in Concentration.

In the example above we determined the concentration of arsenic using the average volume and average flow for the reservoir. In the group work that follows examine what would happen during a low volume, low flow month.

Group Work

Repeat the study above, now assuming that the volume and flow are close to those which you determined in chapter 8 for March; the actual volume and flow you got depended on round-off but the answers should be close to a volume of 13.5 million cubic feet and flow of 28.4 million cubic feet per month. Assume:

the volume of water in the reservoir is constant, 13.5 million cubic feet;

the flow of water is constant, 28.4 million cubic feet per month;

arsenic enters the reservoir at a constant rate, 15 pounds per day.

Again, let c(t) denote the concentration (pounds per 10^6 cubic feet) and m(t) the amount (pounds) of arsenic in the reservoir at time t and follow the steps below.

1. How much water flows into (and out of) the reservoir each day?

2. How much arsenic flows in each day?

3. What portion of the arsenic in the reservoir flows out each day?

4. Determine a formula for the function m(t) which describes the amount of arsenic in the reservoir at time t.

5. Determine a formula for the concentration of arsenic on day t. Graph the function and determine any asymptotes.

6. What is the highest level (in milligrams per liter) that the concentration reaches?

7. What happens in the long run? In particular, does the water ever fail to meet the federal guidelines of .05 milligrams per liter? If so, when?

* * * *

9.2.3 A General Formula

In the work above we assumed that there was no arsenic present initially and that the volume of the reservoir (V), the daily rate of water flow (F) and the rate of arsenic flowing into the water (A) were all constant. We can now easily derive a general formula for the concentration of arsenic in the water. To do this, we again answer questions 1 - 4 above.

1. F is the daily flow.
2. A pounds of arsenic flow in each day.
3. The portion which flows out each day is F/V and r = 1 - F/V is the portion remaining.
4. m(t + 1) = A + r m(t); the pattern is the same as before.

That is.

$$m(1) = A,\ m(2) = A + rA,\ m(3) = A + rA + r^2A,$$

etc.. and as before this gives us

$$m(t) = A(1+r+r^2+\ldots+r^{t-1}) = A\times\frac{1-r^t}{1-r} = \frac{A}{1-r}\left(1-r^t\right).$$

Note that

$$r = 1-\frac{F}{V}$$

and so

$$\frac{A}{1-r} = \frac{A}{F/V} = A\times\frac{V}{F}$$

and

$$m(t) = A\times\frac{V}{F}\times(1-r^t).$$

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Note that this interpretation makes sense provided that F < V; if the flow is greater than the volume, change the unit of time. For example, if the flow is 4.8 million cubic feet per day and the volume is only 4 million cubic feet, then change to hours. Then the flow is .2 million cubic feet per hour, the volume is still 4 million cubic feet, .2/4 = .05 is the portion flowing out and 95% of the water remains each hour.

If there is already arsenic present in the water, say that the amount present is p pounds, then a slight modification is necessary in the last step.

$m(0) = p$, $m(1) = A + rp$, $m(2) = A + rA + r^2p$, $m(3) = A + rA + r^2A + r^3p$, etc., and this gives us

$$m(t) = A(1 + r + r^2 + \ldots + r^{(t-1)}) + r^t p.$$

The term $r^t p$ has been added and so

$$m(t) = A \times \frac{V}{F} \times (1 - r^t) + r^t p.$$

If t is very large, this new term is negligible.

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9.3. Arsenic And Risk

The risk from toxic substances in the water depends on many factors: the concentration of the substance; the extent of the exposure; the use of the water; etc.... Water that might be safe for boating could be unsafe for drinking water: water acceptable for swimming might contain fish with high levels of toxins in their tissues. The Environmental Protection Agency provides information to help determine the risk. The Table 9.1 below provides information about average human size and intake of water and fish.

	Weight (kilograms)	Daily water intake (liters)	Daily fish consumption (grams)
Adult	70	2	6.5

Table 9.1

Toxic substances in water can be dangerous to humans either directly through the water they drink or indirectly if they accumulate in the tissue of fish which are then eaten by humans. The measures defined below help determine the risk of toxic substances in water.

The measure of the tendency for a substance to accumulate in the tissues of fish is called the *bioconcentration* factor and is measured in liters per kilogram. The concentration of the substance in fish is the concentration in water (mg per liter) times the *bioaccumulation* factor (liters per kilogram). The *average daily dose* measures the milligrams of substance per kilogram of body weight consumed per day (mg/kg/day). The *potency factor* is the risk produced by a lifetime daily dose of 1 mg/kg/day and provides a measure for determining the *lifetime risk*. This is the probability of getting cancer over an average (70 year) life span,

Lifetime Risk = (average daily dose) x (potency factor).

Table 9.2 provides:

- the maximum acceptable concentration level (MCL) for arsenic (mg per liter of water);
- the bioconcentration factor (liters per kilogram; and
- the potency factor.

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	MCL (mg per liter)	Bioconcentration (liters per kg)	Potency
Arsenic	0.05	44	1.75

Table 9.2

Use this information to complete the following examination of the long-term effects of arsenic in the drinking water of the residents of River City.

Group Work

To study the long-term effects of arsenic on the residents of River City we will assume that the volume of water in the reservoir is constant, the flow is constant, the influx of arsenic is constant, and therefore we can use the information above and assume that the concentration has stabilized at .040 mg per liter.

1. Fill in the table below to determine risks from arsenic from drinking water and eating fish out of the reservoir.

Source	Concentration	Daily intake mg/day	Ave. daily dose mg/kg/day	Risk
Water				
Fish				

2. Use the risks determined in the table above to predict the number of cancer cases per thousand residents of River City.

3. Unfortunately for the inhabitants of River City, the reservoir volume is not constant. During months when volume is low and residence time is high the concentration is greatly increased. To see the significance of this, determine the concentration at the end of the month of March (as determined in the group work above) and assume that the initial amount of arsenic present is zero. What would be the lifetime risks if the concentration remained at this level?

4. Determine the maximum daily inflow of arsenic so that the March concentration does not exceed the acceptable level of .05 mg/L.

* * * *

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