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ABSTRACT

A case study approach was used to explore the relationship between subject matter knowledge and the acquisition of pedagogical content knowledge in the preparation of preservice elementary mathematics teachers. A preservice elementary teacher was interviewed prior to her mathematics methods course to ascertain her subject matter background and prior pedagogical content knowledge. Both a mathematics educational biography and a structured task interview based on concepts of fractions were employed. Changes in the teacher's subject matter knowledge and pedagogical content knowledge were documented through observations in the mathematics methods course and her field mathematics teaching, and a final structured interview. The study showed that the teacher wanted to do a good job in teaching mathematics but faced many problems that were largely due to her mathematics preparation. Four appendices provide: the Preservice Elementary Mathematics Teaching Information Sheet, the Mathematics Education Biography Interview, the interview task #1 protocol, and the final task interview protocol. (Contains 15 references). (Author/SM)

Learning for teaching: A case of constructing the bridge between subject matter knowledge and pedagogical content knowledge.

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Abstract

A case study approach was used to explore the relationship between subject matter knowledge and acquiring pedagogical content knowledge in the preparation of preservice elementary mathematics teachers. A preservice elementary teacher was interviewed prior to a mathematics methods course to ascertain her subject matter background and prior pedagogical content knowledge. Both a mathematics educational biography and a structured task interview based on concepts of fractions were employed. Both subject matter knowledge and pedagogical content knowledge are described using Skemp's (1976) relational and instrumental understanding. Changes in these bodies of knowledge are documented through observations in both the mathematics methods course and the preservice teacher's field mathematics teaching, and a final structured-task interview. This study should inform policy decisions for mathematics educators.

The National Council of Teachers of Mathematics (NCTM), in the Curriculum and Evaluation Standards document, has called for improved teaching of mathematics by changing the definition of what it means to understand mathematics (NCTM, 1989a). Rote recitation of arithmetic facts is not to be considered proof of mathematical competence. Rather, it is assumed that students will construct an understanding of mathematical concepts through the use of representations and experiences which encourage discovery. To accomplish the transformation from rote learning to conceptual understanding, the NCTM has established both curriculum standards and professional standards for teaching mathematics (NCTM, 1989a, 1989b).

Creating curriculum standards is a laudable goal until we consider the teachers who are going to be implementing them. Cohen (1990) demonstrated the dangers of assuming that experienced teachers implement curriculum in the way policymakers intended. He detailed the problems of a teacher attempting to adopt the California Mathematics Framework via Mathematics Their Way. This teacher attempted to teach for understanding but was unable to get past her own limited mathematical understanding. Ball (1988) demonstrated the lack of depth of subject matter knowledge that prospective elementary and secondary teachers bring to teacher education programs. The effects of this lack of knowledge seem obvious. One can only teach for understanding what one knows.

In elementary preservice teacher education programs, subject matter specific methods courses are offered to demonstrate current ways of teaching particular school subjects. In an elementary mathematics methods course, preservice teachers can have to cope with learning both subject matter and pedagogy for mathematics simultaneously. While it seems logical that teachers should have the subject matter knowledge required to teach a subject, Ball (1988) has shown that this cannot be taken for granted in mathematics. How does prior subject matter knowledge affect the learning of pedagogical content knowledge in a mathematics methods course at the preservice level? This study will explore an instance of the relationship between knowledge of mathematics and learning pedagogical content knowledge in a mathematics methods course. This illumination is needed so that effective policies can be shaped to enhance elementary teacher education in mathematics.

Conceptualization of the Research Problem

Shulman (1986) defined pedagogical content knowledge as "subject matter knowledge for teaching" (pg. 9). The definition of pedagogical content knowledge varies from author to author (e.g. Marks, 1990; Grossman, 1989; and Shulman, 1986). As used here, pedagogical content knowledge includes knowledge of students' conceptions and misconceptions about a topic, instructional strategies which include alternative forms of representation, knowledge of the curriculum for the subject matter, and conceptions of what it means to teach a topic.

This study was conducted to serve as a bridge to demonstrate the resulting ties between subject matter knowledge and pedagogical content knowledge. By considering the variations of what it means to have knowledge of a subject and its pedagogy, we can get a better picture of what it means to prepare teachers for their profession.

This would be facilitated if there were a common way of describing both subject matter knowledge and pedagogical content knowledge. Skemp (1976) could be used to provide this connection. He defined two types of mathematical understanding: relational understanding and instrumental understanding. Relational understanding, knowing what to do and why, is markedly different from instrumental understanding which means knowing rules without reasons. I propose to extend these two ways of mathematical understanding to pedagogical content knowledge. Instrumental pedagogical content knowledge would mean a rule-based knowledge of teaching a subject. For instance, a sort of "if it's fractions, it must be pattern blocks" system would be apparent. Understanding why pattern blocks might be an appropriate representation would not be a part of an instrumental pedagogical content knowledge. Relational pedagogical content knowledge would include understanding why, how, and when pattern blocks are appropriate and having that same type of understanding for the myriad of representations available for fractions. Does the type of subject matter understanding influence what the preservice teacher learns in mathematics methods course and therefore influence the depth or character of pedagogical content knowledge? Can a generalist with instrumental mathematical knowledge understand enough content-specific pedagogy to help students' construct their own mathematical understandings? How does a mathematics methods course change subject matter knowledge for students with instrumental understandings? Do their instrumental mathematics understandings translate into instrumental pedagogical content knowledge? A case study approach can explore aspects of this phenomenon.

The line between pedagogical content knowledge and subject matter knowledge can be blurry. Marks (1990) acknowledged that "pedagogical content knowledge, by its nature, contains elements of both subject matter knowledge and general pedagogical knowledge. . ."(pg. 8). While it has been demonstrated that lack of subject matter knowledge in mathematics exists for preservice elementary school teachers (e.g. Ball, 1990) and that this extends to experienced elementary teachers (e.g. Stein, Baxter & Leinhardt, 1990), the tie to subsequent pedagogical content knowledge should be further explored. Carpenter Fennema, Peterson, & Carey (1988) explained teachers' pedagogical content knowledge of students solutions of addition and subtraction word problems. The teachers' knowledge was not organized into a coherent network that related distinctions between problems, children's solutions, and problem difficulty. How much of this can be attributed to the teachers' subject matter understanding at the preservice level? If there is interference when the learning of this important pedagogical content knowledge in subject matter specific methods courses is taking place, educators need to be shown it exists and how it effects the subsequent teaching.

Understanding the pedagogical content knowledge and subject matter knowledge gains made via a mathematics methods course will help inform the education community to make appropriate decisions about elementary mathematics education. The NCTM is already being faced with a call for subject matter specialists at the 5-8 grade level. This study could help inform that decision.

Methods

First-quarter elementary preservice teachers currently enrolled in a large research university's teacher education program were asked to volunteer for this study. The rationale for choosing this particular program is that the program meets the NCTM guidelines for the preparation of teachers of mathematics (Institutional Folio, 1985). Students are admitted to the program based on grade point averages and the competition for admittance usually means a 3.0 or higher grade point average. The participants were purposefully selected for diverse preparations in mathematics, using a self-report information sheet for data (See Appendix A). While the design of the study contrasted several cases of differing subject matter knowledge, for this paper, I am purposefully describing the participant who began the program with an instrumental knowledge of mathematics.

Several data collection techniques were employed: a mathematics educational biography interview, two in-depth fraction task interviews, two classroom observations of the preservice teacher, and the observation of the mathematics methods course. The participants were initially interviewed during their first quarter in the teacher education program to ascertain their mathematics educational biography. The protocol for this interview (See Appendix B) was based on the initial interview done by Ball (1988) to ascertain both subject matter knowledge and beliefs about mathematics.

To gain an understanding of the subject matter knowledge and pedagogical content knowledge both before and after a mathematics methods course two structured task interviews were used. A task that involves explanations of how certain mathematics properties work is commonly used in this type of research (e.g. Marks, 1990; Ball, 1988; Carpenter, Fennema, Peterson, & Carey, 1988). The mathematics topic explored was fractions because of the abundance of research in this area. Fractions are often a difficult subject for mathematics teachers so it was easier to differentiate the participant's subject matter understanding. Marks's (1990) pedagogical content knowledge research is in this area. Concrete representations that teachers use to teach seem to be an excellent vehicle for approaching this question. These representations afford the possibility of getting past the manipulation of symbols and into the understanding of the mathematics by the teacher. The task involved the use of both manipulatives and diagrams to demonstrate the teachers' subject matter knowledge and prior pedagogical content knowledge for fractions. At the initial task interview, pedagogical content knowledge was not expected to be substantial. This task is similar to one used by Mack (1990) to show elementary students' understanding of fractions (See Appendix C). Both the symbol manipulation and concrete manipulation of fractions are required along with explanations of why procedures are completed. The tasks occurred in a think-aloud interview situation so that responses were probed. Two questions concern the approaches for teaching the material and call for a variety of representations. The initial task interview was conducted prior to the beginning of the mathematics methods course and the final interview was conducted two weeks prior to the end of the mathematics methods course, after the fraction lessons. The final task interview contained similar problems to the initial task but the numbers were changed and creating a word problem was required for all operations and not just division.

During the second quarter, the participant was observed two out of the three times she taught mathematics in the third-grade elementary school classroom that quarter. The lesson plan for the third class was made available and did not appear to vary from the classes observed. She was interviewed after each mathematics lesson. Participant-written lesson plans for all of the mathematics lessons taught also served as data. The university mathematics methods course the participant was enrolled in was also observed. These observations were done for the entire quarter up until the final interview. The class met twice a week for an hour and a half for ten weeks.

The analysis was done on a case basis using the variety of data collection methods as a means of triangulation. Marks (1990) noted that a more integrated view of "particular teachers' patterns of knowledge" (pg. 53) would probably be in the form of a case study. The unit of analysis was the interaction of the participant's subject matter knowledge and acquisition of pedagogical content knowledge and/or subject matter knowledge during the enrollment in a mathematics methods course and fieldwork. Interviews and observations were transcribed and coded using the two variables of instrumental and relational knowledge for subject matter knowledge and pedagogical content knowledge. Coding was done based on a thought unit that conformed to complete thoughts expressed by the participant. Interrater reliability for the coding was 92% after training.

Jeannie

Jeannie chose to go back to college for a teaching credential after her own children started school. She had worked as a teacher's aide in a local school and enjoyed it enough to realize her thoughts of becoming an elementary school teacher. An English major, she fulfilled the university mathematics requirement for all elementary school applicants to the fifth year credential program by taking a single course in the mathematics department. A capable individual, her college grade point average is above a 3.0. She passed the required mathematics course, which is designed for future elementary school teachers, the quarter prior to starting the program. She describes her changing view of this course:

> I thought it was going to be easy. It seemed as though it was supposed to be something you were going to do in preparation for teaching in elementary school. . . . I guess I was looking back on my elementary school math and thinking, what's so hard about adding, subtracting, multiplying and dividing--I can do this. . . . I thought we were going to do something like--This is how you teach a child to do long division. . . . I certainly didn't think it was going to be what it was. It was throw me down and mix me up. . . . We went in there, and I guess I could say that it was story problem after story problem, after story problem. . . . It was very difficult for me. [Jeannie, MB, p. 8]

Jeannie remembers being fairly successful in grade school mathematics and enjoyed high school mathematics until the second year of algebra and trigonometry. She went back and took this course at a community college a few years prior to beginning her teaching credential program. Jeannie's elementary school mathematics learning seemed to rely on the textbook. So much so that when she was asked what experience in mathematics stood out, she replied, "The textbook. Especially with math. The teacher would stand up and give us a short lesson, and then we'd do problems from the book" (Jeannie, MB , p. 1). Her stated goals for teaching are dissimilar from this traditional mathematics background. "I want them to be able to use manipulatives in math" (Jeannie, MB, p. 18). "I think that [manipulatives] helps them to understand things better, to make it more than just a textbook thing" (Jeannie, MB, p. 18).

Jeannie's cooperating teacher, Mrs. R, uses manipulatives to teach mathematics and they are readily available in the classroom. "She [Mrs. R] calls it tubbing. She says there is a marked difference from those kids who haven't done the tubbing. That they seem to catch on quicker, what it really is and there's more enthusiasm" (Jeannie, MB, p. 20). Mrs. R's use of "tubbing" means using manipulatives which are stored in plastic tubs. Mrs. R is interested in elementary mathematics and attends the regional mathematics conferences of the NCTM. This same professionalism is evident when Jeannie looks at a variety of sources to plan her own mathematics lessons. Her lesson plans use materials and ideas from Mathematics Their Way and Activities in Math and Science (Baratta-Lorton,1976; AIMS, 1979) Jeannie is willing to take the risks of teaching hands-on activity lessons because "they'll get more out of it. I realized that this was going to be something that wasn't going to be easy. It's important for them to see that math is more than just writing numbers on a piece of paper" (Jeannie, SCO, p. 11).

During both the initial task interview and the mathematics background interview, Jeannie's understanding of mathematics was instrumental. Jeannie was confident that there was a correct answer in mathematics, in fact she held it as her favorite part of mathematics.

> I like the idea that there is always an answer. That always seemed . . . nice and tidy. . . . There is no gray area. It's right or it's wrong. You can see where you made your mistake. Even the long, involved problems that require you to do a lot of steps, you could trace those. You can go back and you can see. . . . You know, it's cut and dried, really. There is a right answer, and I like that. The security of knowing that you are either going to get it or you're not--the answer. (Jeannie, MB, p. 10-11)

She seems to have the idea that there is a way to solve mathematics problems and that there is always an answer. Another example of her instrumental understanding of mathematics is her solution to the problem two and a third divided by one-half.

> I'm going to take this mixed number and make it into an improper fraction or whatever you'd call them. Three times two is six plus one is seven. Seven thirds divided by one half and now I have no idea why (laughs), but I always learned that you invert and multiply. So that's what I'm going to do. [Jeannie, ISM, p. 4-5]

Even with probing for the reasons why the procedures worked, Jeannie followed rules without having explanations.

Manipulatives

Jeannie tried to use the manipulatives on two out of the ten questions during the initial interview (pattern blocks and cuisenaire rods were available during both task interviews). "These were never made available to me when I was learning math" (Jeannie, ISM, p. 8). The manipulatives were not used to solve the problems but to show the process to imaginary students who were having difficulty. Her first use of pattern blocks was to use them as discrete objects.

> I have the green triangles. I'm using those for fourths. So I'd get four of them and say that that was fourths and here's three fourths and if I was going to take away two fourths, which is what this problem would be, three fourths take away two fourths, and then I would just strictly remove them. How many fourths do we have left? We have one fourth left. (Jeannie, ISM, 17)

She repeated the explanation in the same discrete fashion using four yellow cuisenaire rods for students who might not have understood her first explanation. Her third explanation of the same problem finally equated the four yellow rods with two orange rods. She labeled the orange rods as one whole and pointed out that "four fourths equals one whole" (Jeannie, ISM, 21).

O		O	
y	y	y	y

This was the only instance where the manipulatives were used to show part of a whole and not as discrete objects in part of a set of objects. Her diagrams were all pie-shaped circles.

Jeannie's Methods Course Observations

The mathematics methods course challenged several of her methods of using manipulatives and diagrams. Pattern blocks were used from the beginning of the first day of the fractions unit in the mathematics methods course and cuisenaire rods were also available at each table. Most of the two class sessions were spent with students moving the pattern blocks explaining concepts to fellow students in groups of two or three. Jeannie was in a group of three who remained on task, but she did not take the opportunity to perform the requested operations. She observed the manipulations and was actively commenting, but she did not demonstrate to her peers. The concepts stressed during this unit were that students' understanding of fraction was important and that the definition of the unit or whole was important. Jeannie asked a question about using pattern blocks for teaching fractions and whether this would be on the final. The instructor replied to Jeannie's query:

> Pattern blocks, I'm just using as an example. So the pattern block itself is not what's important. I could have used squares and circles. I could have used fraction bars. And, in fact, if we could get to it, I'd like to use cuisenaire rods. So the media, you know, the object--the object in one sense is not important, but in another sense, it is. In that, right now, we're using area to help understand fractions. But that's not the only way to think about area. Now the answer to your question: 'Why would anybody ask this on the test?' is because you all are getting ready to teach

mathematics. You've moved beyond a user of mathematics to a teacher of math. So you concern has to be: 'How do I help children gain an understanding of fractions?' And my concern is that I'd like you to at least entertain the notion that to understand fractions can partly be reflected in my ability to give representations for that idea. [MMCO, 5/15, p. 2]

Another imbedded feature of this unit was the notion that students should construct their own understanding. A student explained to the class, his approach for teaching the comparison of fractions with the beginning, "With students I guess I would demonstrate" and received immediate instructor feedback:

> You wouldn't demonstrate. You would have them show you. I'm going to try to get the word out of you that says, I'm going to demonstrate. There is a lot of research out there that tells you 'You can demonstrate until you are blue in the face, but if the kid doesn't construct, forget it.' You've got a lot of nice demonstrations. <u>You</u> understand it. [MMCO, 5/15, p. 8-9]

The idea that the definition of the unit was important began at the start. Pattern blocks were defined using the 'cover' terminology where a variety of examples such as "If the yellow hexagon is one, what'll I call the green triangle?" (MMCO, 5/15, 2). The response suggested and affirmed was, "Since it takes six green triangles to cover it. We call each one one-sixth. That's the concept behind one sixth. When I use area as my representation. Now this is not the only representation for fractions" (MMCO, 5/15, 2).

This line of reasoning occurred for most of the two class sessions, so it is reasonable to assume that Jeannie would be prepared to use pattern blocks for fractions in a different way from her initial task interview. Unfortunately, this did not occur in the final task interview. Jeannie used pattern blocks, and attempted cuisenaire rods on one out of a possible eleven questions, again for demonstrating to the imaginary student who didn't understand. After a similar attempt with cuisenaire rods, she used the pattern blocks as discrete objects to show why two-thirds plus one-fifth does not equal three-eighths. She took out five triangles and pointed out one of them as one-fifth. In a separate pile, she put three green triangles and pointed out two of them as two thirds (See figure 1).

$$\frac{2}{3} \quad + \quad \frac{1}{5} \quad \neq \frac{3}{8}$$

Figure 1.

She realized that this explanation might be confusing but never discussed creating an area representation. This figure when looked at in totality is indeed very confusing. Because the child could look at the picture and see three-eighths. This is contrary to what was taught in the mathematics methods course.

Jeannie's Teaching of Fractions

Observing the two lessons made it obvious that Jeannie feels manipulatives are important. All of the lessons she planned for the quarter required the students to be actively involved in constructing their own understanding. Her first mathematics lesson had the students constructing multiples of six with a variety of representations found in her cooperating teacher's "tubs". The second and third lesson required the students to get experience with measurement, graphing, classification, and estimation using real apples.

Jeannie's subject matter knowledge made her understanding of pedagogical content knowledge erratic. She believes that manipulatives are important but she had trouble understanding how to use them for fractions. She did not change her discrete approach to pattern blocks after being shown differing ways in the methods course. She could not recreate her area representation of cuisenaire rods. Jeannie is in line with the standards for using physical models to teach mathematics, but the end result could be as confusing as the rule-based teaching that NCTM is seeking to avoid.

Diagrams

Jeannie's use of diagrams was also challenged with mixed results. A direct attack was made on the use of only pie-shaped diagrams for displaying fractions before the fraction unit began when the instructor asked

> What's the first picture you see when you think of a fraction? Pies! Kids learn to think that fractions have to do something with circles and pies because they are overexposed to that. They don't view fractions in terms of a half a glass of milk, or in terms or rectangles, or in terms of areas like out in the quad. They start to view fractions as pie. In other words, you don't want the concept to become attached to the manipulative. [MMCO, 5/1, p. 6]

Jeannie incorporated this view immediately and she expressed her dismay that this was her only view of fractions in the mathematics methods course during the fraction unit and in the final interview.

So I'm going to draw a little picture and I'm going to fool you this time. Because the last time that I did this, I drew circles. . . . I'm trying to practice not using pies. Because that's all I ever did, I think, was have pies when I was doing fractions. . . . That shape is imbedded into me, as how you do fractions, and I don't want to do that to the kids. [Jeannie, FSM, p. 28]

Jeannie was able to attempt a variety of diagrams that usually evoked the word problem that she had created during the final interview. However, the diagrams confirmed that she had a basic problem with the definition of unit for fractions. The first time I observed this was when she was asked to illustrate her story problem for five-sixths plus seven-eighths. Her story problem was "I have five-sixths... of a pan of brownies, and you have seven-eighths of a pan of brownies and [when] put altogether, how may of a pan of brownies do we have?" (Jeannie, FSM, p. 26). She correctly used her rule-based methods for solving this problem. When asked to diagram it (See figure 2), she described her procedure:

I'm drawing a rectangle and I'm splitting it up, making five lines here. So I have six pieces, because I'm working with five-sixths. So the shaded part is how many brownies I have. Because my husband ate one sixth of the brownies. He didn't know that I wanted to take it to the party. . . . Then Linda's husband came along. She had a longer pan of brownies, and he ate one-eighth of hers. . . . I'll shade in seven-eighths, and that's all we get to take. [Jeannie, FSM, 26]

6. Create a story problem for $\frac{5}{6}+\frac{7}{8}=$

$\frac{20}{24}+\frac{21}{24}$

Draw a picture to illustrate your story.

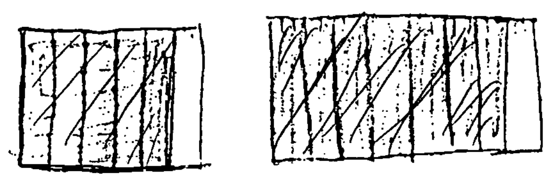

Figure 2

To ascertain her understanding of common units, the following discussion ensued

(I= interviewer, J= Jeannie)

I. Did Linda's pan of brownies have to be bigger?

J. No, it doesn't. I drew it bigger, but it doesn't have to be.

I. Should it be the same size?

J. Probably should be the same size.

I. Why?

J. So that you could see that you can use the same unit and divide it in different fractions?

I. Well, does it have to work that way? Can the pans be the same sizes?

J. No. They could be different sizes. Because I could have seven-eighths of this unit and five-sixths of this unit. [Jeannie, FSM, p. 27]

A similar discussion occurred later in the interview on other problems where her diagram was not a circle. Jeannie did not understand at this point that the same unit of reference for the fractions was necessary. This is an important concept for understanding fractions that was covered repeatedly in the methods course but was not a part of Jeannie's repertoire. This would not be made obvious by her use of pattern blocks as discrete objects because other explanations could be applicable. It was not until Jeannie began to diagram with rectangles that this misconception was apparent. So her use of non-pie diagrams made other

Implications

It is obvious that we need to consider subject matter background as a crucial part of teacher education. Jeannie wanted to do a good job in teaching mathematics but faced many problems (not entirely of her own making) due to her mathematics preparation. She was able to pick up pieces of pedagogical content knowledge but her understanding was hampered by her subject matter knowledge. Can we expect teachers who have instrumental mathematics understandings to teach using tools that require relational mathematics understandings? Do we need mathematics specialists to teach at the elementary levels? It cannot be assumed that x-number of mathematics courses will provide the multiple representations needed to explain fractions. Can we raise our expectations of what it means to learn mathematics and expect preservice teachers will be able to teach at the level required for this type of understanding given the current NCTM institutional standards for education programs?

We know what type of knowledge preservice teachers bring to elementary mathematics education programs and what type of pedagogical content knowledge we want teachers to have. Now we need to further explore how subject matter knowledge can filter the acquisition of pedagogical content knowledge. Mathematics methods educators can use this information to guide their construction of methods courses and information that is needed by preservice teachers. Mathematics instructors at the college level can use this information to plan appropriate courses for adequate subject matter knowledge. By improving the education for future elementary mathematics teachers, we can improve elementary mathematics instruction.

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Interview references

MB- Mathematics background interview; See appendix B, 3/91

MMCO- Mathematics methods course observation.

ISM- Initial subject matter task interview; See appendix C, 3/91

FSM- Final subject matter task interview; See appendix D, 5/91

FCO- First classroom observation, 3/91

SCO- Second classroom observation, 5/91

Appendix A

Preservice Elementary Mathematics Teaching Information Sheet

Name______________________________________

Phone Number________________________________

I student teach at

____

(grade level, school and district)

College major(s)? ________________________

Minor(s)?________________________

<u>Mathematics background</u>

Did you take Mathematics 170 as a course?______,

If yes, When?_______ Taught by whom?____________

If no, circle the way you fulfilled this requirement:

test exemption correspondence course another course

Title and year of last mathematics course taken:

How do you feel about mathematics?

Is there anything else that I should know about you?

First interview- Mathematics Education Biography

Appendix B

The questions in this interview protocol are based on Ball's (1988) initial interview. Some used are verbatim.

1. I'm interested in your own past experience in school, with mathematics in particular. When you think back to your own experience with math when you were in elementary school, what stands out?

- What do you mean?
- Can you give me an example of that?
- Is there anything else you can remember?

Probe for a.) Material learned

b.) Your teachers

c.) How you felt about math class

d.) How you felt about yourself in relation to others in your class

2. What about at the junior high/middle school level?

- What do you mean?
- Can you give me an example of that?
- Is there anything else you can remember?

Probe for a.) Material learned

b.) Your teachers

c.) How you felt about math class

d.) How you felt about yourself in relation to others in your class

3. What did you take in high school?

•What do you mean?

•Can you give me an example of that?

•Is there anything else you can remember?

Probe for a.) Material learned

b.) Your teachers

c.) How you felt about math class

d.) How you felt about yourself in relation to others in your class

4. What mathematics courses have you taken at the college level? Since college courses vary so much would you describe the course you took briefly.

•What do you mean?

•Can you give me an example of that?

•Is there anything else you can remember?

Probe for a.) Material learned

b.) Your teachers

c.) How you felt about math class

d.) How you felt about yourself in relation to others in your class

5. Are there some things in mathematics or about mathematics that you especially like/enjoy?

•What do you mean?

•Why is that?

6. What about the other side of this, are there some things in mathematics or about mathematics that you especially dislike?

- What do you mean?
- Why is that?

7. As a teacher, there will be many things you'll be teaching, some of which you will probably understand better than others.
Can you think of something in mathematics that you feel that you yourself understand really well? Take your time to think about it.

- Can you tell me more about it?
- What about it makes you feel that you really understand this? When/where did you learn it?
- How do you think you came to understand it well?
- Is this something you think you'll be teaching?

8. What about the other side of this? Can you think of something in mathematics that you feel you really don't understand very well?

- Can you tell me more about it?
- What about it makes you feel that you really don't understand this? Did when/where you were supposed to have learned it play any role?
- Why do you think you didn't get it?
- Is this something you think you'll be teaching?

9. Are there things that you feel you especially need in order to be able to teach a math class?

- Why does that seem important?
- What do you mean by that?
- How/where do you think you could learn that?

10. What does "being good at math" mean to you? Think of someone you know who is good at mathematics.

- Why do you think of them as being good at mathematics?
- What does he/she do?
- Why do you think this person is good at mathematics?
    - What do you mean?
    - Can you give me an example?

11. What about the opposite? Do you know anyone who you would say is really not good at mathematics?

- Why do you think of them as not being good at mathematics?
- What does he/she do?
- Why do you think this person isn't good at mathematics?
    - What do you mean?
    - Can you give me an example?

If self is identified... Ask:

What's the explanation you give yourself about why you aren't so good at (or don't do so well at) math?

12. For this question, I'd like you to pick a grade you can imagine teaching. What grade is that? Imagine that you find yourself having a discussion with the principal early in the fall about your goals in mathematics for your students that year. What do you say that you want to accomplish in mathematics that year with your __________graders.

Probe for the sense of the important ideas in mathematics and the goals of school math instruction.

•What do you mean by that?

•Why is that important to you?

•What do you mean when you use that term?

•Are there important ideas that come to mind around that grade?

•Are there any things you'd say regardless of the grade you were teaching?

end of day one!

Interview Task #1 (SMK) Protocol **Appendix C**

(This interview is based on Mack (1990). These questions are revised questions that Monk, Stimpson, Hutchison, and Edwards used in conjunction with Teaching Mathematics to Middle School Students under a grant from the Washington Superintendent for Public Instruction.)

I am going to ask you to solve some problems that involve fractions. While you are solving these problems I would like you to think aloud and tell me how you are solving them and what you are thinking. Please read the problem out loud and use the paper for all of the work.

(Give the first and second page of task--Problems 1-4 all operations)

Prompts:
What are you doing to solve that problem?
How does that work?
Please explain what you were thinking for that step.
How do you think about that problem?
Why did you do that step?

1. $\frac{7}{8} + \frac{5}{6} =$

2. $4 - \frac{7}{8} =$

3. $1\frac{3}{4} \times \frac{1}{2} =$

4. $2\frac{1}{3} \div \frac{1}{2} =$

Pass out problems 5,6, & 7
Probe: Why are these equivalent?
How did you figure out those were equivalent?
Is there any other way to show they are equivalent?

5. Write three fractions which are <u>equivalent</u> to:

$$\frac{2}{5}$$

Probe: How did you determine that it was bigger?
Would that method work for all problems?

6. Which is bigger, $2\frac{2}{3}$ or $2\frac{5}{6}$?

Draw a picture or explain in words why your choice makes sense to you:

Probe:
How did you solve that?
Would your students do it that way?
How would you have solved it in your school mathematics?

7. <u>About</u> how much do you have if you add $\frac{17}{20}$ to $\frac{35}{36}$?

Why does your answer make sense to you?

Pass out problems 8-10
Probe:
What other explanations would you give if the student still did not understand?
Would your diagram work for all subtraction problems involving fractions?
Would you expect sixth grade children to be able to explain it that way? Why or why not?

8. $1\frac{3}{4} - \frac{1}{2} =$

Suppose you were trying to explain your answer to this question to a student. What diagram would you use, or what explanation would you give, so that the student would understand what you are doing?

Probe:
Is your problem the type of problem that you would expect to find in a textbook? Why or why not?
How did you come up with that problem?

9. If I wanted to make up a story for the problem

$$2\frac{1}{2} + 3\frac{3}{4}$$

I might say:
"A family has 2 1/2 gallons of grape juice and 3 3/4 gallons of orange juice so they have 6 1/4 gallons of juice altogether."

<u>You</u> make up a story for the problem

$$2\frac{1}{2} \text{ divided by } \frac{3}{8}$$

Draw a picture to illustrate your story.

Prompt
How do you explain adding fractions?
What will you say if he replies, “I still don’t get it.”
If they say you need a common denominator
What do you mean by a common denominator?
What do you say if he asks why you need a common denominator?

10. Suppose you are helping a student with his math homework. He does this:

$$\frac{2}{3}+\frac{1}{5}=\frac{3}{8}$$

You tell him that he can’t do that, and he asks “Why not?”
What do you say?

Thanks for your time.

Final Task Interview **Appendix D**

I am going to ask you to solve some problems that involve fractions. While you are solving these problems I would like you to think aloud and tell me how you are solving them and what you are thinking. Please read the problem out loud and use the paper for all of the work.

1. $\frac{5}{6} + \frac{7}{8} =$

2. $3 - \frac{5}{6} =$

3. $2\frac{3}{4} \times \frac{1}{2} =$

4. $2\frac{2}{3} \div \frac{1}{2} =$

5. Write three fractions which are equivalent to: $\frac{3}{7}$

6. Create a story problem for $\frac{5}{6}+\frac{7}{8}=$

Draw a picture to illustrate your story.

7. Create a story problem for $3 - \frac{5}{6} =$

Draw a picture to illustrate your story.

8. Create a story problem for $2\frac{3}{4} \times \frac{1}{2} =$

Draw a picture to illustrate your story.

9. Create a story problem for $2\frac{2}{3} \div \frac{1}{2} =$

Draw a picture to illustrate your story.

10. Which is bigger, $2\frac{2}{3}$ or $2\frac{5}{6}$?

Draw a picture or explain in words why your choice makes sense to you:

11. Suppose you are helping a student with her math homework. She does this:

$$\frac{2}{3}+\frac{1}{5}=\frac{3}{8}$$

You tell her that she can't do that, and she asks "Why not?" What do you say?

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