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ABSTRACT

Constructivist approaches to mathematics instruction based on the standards of the National Council of Teachers of Mathematics (NCTM) have been widely advocated and are expanding in use. However, many educators express a need for constructivist approaches that provide specific student materials, assessments, teachers' manuals, professional development, and other supports to enable a broad range of teachers to succeed with a broad range of children. MathWings, part of a comprehensive school reform effort funded by New American Schools, was designed to accomplish this goal. In grades 3 through 5, MathWings provides a practical, comprehensive approach based on the NCTM standards. Three evaluations have examined the impact of MathWings. One, involving four rural Maryland schools, found substantially greater gains on the mathematics sections of the Maryland School Performance Assessment Program for MathWings students than for the rest of the state. The four pilot schools, which were more impoverished than schools in the state as a whole, started far below state averages, but ended up above the state average. The second study, in one urban school in San Antonio (Texas), also found substantial gains on the Texas Assessment of Academic Skills math scale in grades 3 through 5 from the year before the program began to the end of the first implementation year. The third study found substantial gains on the California Test of Basic Skills mathematics concepts and applications scale for grades 4 and 5, but not 3, in a Palm Beach County (Florida) school. An appendix contains sample MathWings curriculum and assessment materials. (Contains 4 figures and 21 references.) (Author/SLD)

CRESPAR

# MATHWINGS

# Early Indicators of Effectiveness

Nancy A. Madden • Robert E. Slavin • Kathleen Simons



Report No. 17 / September 1997

CENTER FOR RESEARCH ON THE EDUCATION OF STUDENTS PLACED AT RISK

Johns Hopkins University & Howard University

# MATHWINGS
## Early Indicators of Effectiveness

**Nancy A. Madden, Robert E. Slavin, and Kathleen Simons**
**Johns Hopkins University**

**Report No. 17**

**September 1997**

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# The Center

Every child has the capacity to succeed in school and in life. Yet far too many children, especially those from poor and minority families, are placed at risk by school practices that are based on a sorting paradigm in which some students receive high-expectations instruction while the rest are relegated to lower quality education and lower quality futures. The sorting perspective must be replaced by a "talent development" model that asserts that all children are capable of succeeding in a rich and demanding curriculum with appropriate assistance and support.

The mission of the Center for Research on the Education of Students Placed At Risk (CRESPAR) is to conduct the research, development, evaluation, and dissemination needed to transform schooling for students placed at risk. The work of the Center is guided by three central themes — ensuring the success of all students at key development points, building on students' personal and cultural assets, and scaling up effective programs — and conducted through seven research and development programs and a program of institutional activities.

CRESPAR is organized as a partnership of Johns Hopkins University and Howard University, in collaboration with researchers at the University of California at Santa Barbara, University of California at Los Angeles, University of Chicago, Manpower Demonstration Research Corporation, University of Memphis, Haskell Indian Nations University, and University of Houston-Clear Lake.

CRESPAR is supported by the National Institute on the Education of At-Risk Students (At-Risk Institute), one of five institutes created by the Educational Research, Development, Dissemination and Improvement Act of 1994 and located within the Office of Educational Research and Improvement (OERI) at the U.S. Department of Education. The At-Risk Institute supports a range of research and development activities designed to improve the education of students at risk of educational failure because of limited English proficiency, poverty, race, geographic location, or economic disadvantage.

# Abstract

Constructivist approaches to mathematics instruction based on the standards of the National Council of Teachers of Mathematics (NCTM) have been widely advocated and are expanding in use. However, many educators express a need for constructivist approaches that provide specific student materials, assessments, teachers' manuals, professional development, and other supports to enable a broad range of teachers to succeed with a broad range of children. MathWings was designed to accomplish this goal. In grades 3-5, MathWings provides a practical, comprehensive approach based on the NCTM standards.

Three evaluations have examined the impact of MathWings. One, involving four rural Maryland schools, found substantially greater gains on the mathematics sections of the Maryland School Performance Assessment Program for MathWings students than for the rest of the state. The four pilot schools, which were much more impoverished than the state as a whole, started far below state averages but ended up above the state average. The second study, in San Antonio, Texas, also found substantial gains on the Texas Assessment of Academic Skills math scale in grades 3-5 from the year before the program began to the end of the first implementation year. The third study found substantial gains on the CTBS mathematics concepts and applications scale for grades 4-5 (but not 3) in a Palm Beach County, Florida school.

# Acknowledgments

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# Introduction

The teaching of mathematics in the early elementary grades is in the midst of a revolution. This revolution goes under many names, but the name most often attached to it is constructivism. Constructivist mathematics teaching emphasizes *understanding* rather than algorithms (Carpenter, et al., 1994; Davis, Maher, & Noddings, 1990). It begins with problem-solving and "authentic" complex tasks, rather than building up from arithmetic. For example, children in kindergarten can figure out how many busses are needed to get the class to a picnic long before they learn any division algorithm. They can figure out how to share a pizza fairly long before they learn formal representations of fractions. Constructivist methods make extensive use of cooperative learning, projects, and integrated thematic units. They use many external representations of mathematical ideas, such as base-ten blocks, pictures, and stories. Constructivist theories see the learner as active, intrinsically motivated, and possessing background knowledge and experience that can and must be taken into account in instruction (Paris & Byrnes, 1989). In this view, the task of mathematics instruction is more to introduce students to symbolic representations of concepts they already possess than to teach completely new ideas. For example, children arrive in kindergarten knowing a great deal about combining and separating, more and less, halves and wholes, and so on. Constructivist teaching methods recognize and build on this knowledge, emphasizing discovery, reflection, multiple solutions, and explanation of learning processes by children themselves (Resnick, 1992).

The broad influence of standards developed by the National Council of Teachers of Mathematics, the acceptance of closely related standards by many states, and the development of performance tests increasingly used for state accountability purposes, have all added significantly to the press for more constructivist teaching in elementary schools. The stakes for schools and students are rising. State-of-the-art state mathematics assessments require students to solve complex, non-routine problems and to explain their thinking processes.

These new standards and assessments create significant opportunities for reform in the teaching of mathematics at all levels. Yet they also create a serious danger. Studies of new performance assessments are finding that poor and minority students are scoring worse on these assessments (relative to middle-class students) than they do on traditional standardized measures (Shavelson, Baxter, & Pine, 1992). If anything, this problem is likely to become worse. Moving from traditional to constructivist teaching requires a substantial investment in top-quality professional development. Middle class school districts are likely to be able to make such investments and to be able to hire elementary teachers who are already reasonably proficient in mathematics. Impoverished schools are less likely to have teachers who are up-to-date in new conceptions of mathematics or to be able to provide the months of inservice often required to enable even good teachers with strong interests and backgrounds in mathematics to internalize learner-centered teaching in mathematics.

A number of new approaches to mathematics curriculum and instruction have been developed for elementary schools to help them move toward constructivist conceptions of learning. Examples include Conceptually Based Instruction (Hiebert & Wearne, 1993), Cognitively Guided Instruction (Carpenter & Fennema, 1992), Supporting Ten-Structured Thinking (Fuson, 1992), and QUASAR (Stein & Lane, 1995). These and other methods are expanding in use in elementary schools. Yet there is still a need for further development and research directed at creating practical constructivist methods capable of being used on a large scale by all teachers, not only those with particular interests and backgrounds in mathematics. Many projects have shown success on a limited scale at introducing constructivist methods in elementary schools, including those serving many students placed at risk (see, for example, Fuson, 1992; Jamar, 1995; Stein & Lane, 1995; Campbell, Cheng, & Rowan, 1995; Carpenter Fennema, Peterson, Chiang, & Loef, 1989). However, mathematics instruction, especially in urban elementary classrooms, remains overwhelmingly algorithmic, teacher-centered, and traditional.

The goal of reform in elementary mathematics must be to provide deep understanding of mathematical ideas for *all* students, not just for those fortunate enough to have teachers with extraordinary skills and interests in mathematics. Mathematics for *all* will require approaches very different from those needed to demonstrate on a small scale that students can learn in new ways. It will require the development of new curricula and school support structures capable of ensuring that every elementary teacher, even those in high-poverty, underfunded schools, can enable students to be strategic, flexible, self-aware, and motivated problem solvers in mathematics.

This report describes three evaluations of a program designed to make constructivist mathematics instruction practical and successful for a broad range of children and teachers in high poverty schools. This program, called MathWings, is part of a comprehensive school reform approach called Roots and Wings (Slavin, Madden, & Wasik, 1996), the development of which was funded by New American Schools. Roots and Wings adds MathWings as well as social studies and science programs to a reading, writing, and language arts program called Success for All (Slavin, Madden, Dolan, & Wasik, 1996).

The development of MathWings drew heavily on the experience of developing Success for All, although the curricular approaches are quite different. Both strategies emphasize well-structured student materials, frequent assessment, cooperative learning, effective classroom management methods, and extensive teacher training and followup. The idea is to improve the instructional strategies of all teachers, whether or not they are experts in mathematics, but to build in flexibility that allows the best teachers to go further.

## MathWings: Program Description

The *NCTM Curriculum and Evaluation Standards* (1989) advocate emphasizing problem-solving rather than rote calculation with algorithms. MathWings lessons involve the students in problem-solving in "real" situations to give validity and purpose to their mathematics explorations, and in daily problem-solving as part of the routine of math class. MathWings lessons also make connections to literature, science, art, and other subjects as well as the students' world and personal experiences to provide this real world problem-solving context.

Another strand of the Standards is mathematical reasoning. Students develop their ability to think through and solve mathematical problems when they use manipulatives to develop concepts and then represent what is actually happening with symbols. MathWings units are constructed to develop concepts from the concrete to the abstract so that each step of the reasoning is clarified.

The Standards also promote the use of calculators for developing concepts and exploring advanced problem-solving situations rather than for checking answers or replacing skills and mental math. MathWings students use calculators to increase both their mathematical reasoning skills and the scope and complexity of the problems they can solve, and to focus their energy on mathematical reasoning rather than mere mechanical calculation.

The Standards emphasize communication, both oral and written, to clarify, extend, and refine the students' knowledge. MathWings students constantly explain and defend their solutions orally throughout the lessons, and write regularly in their individual Logbooks. This emphasis on communication extends to assessments as well. The Standards suggest the use of alternative assessments which incorporate communication as well as calculation. MathWings units involve the students in many different types of assessment. The students complete concept checks in which they explain their thinking as they solve problems after every few lessons. They work on performance tasks at the end of each unit to use the skills they have learned to solve practical real world situations and explain and communicate about their thinking. Teacher observations of students using manipulatives, collecting data, and carrying out other activities, as well as their written and oral communications, are all used to assess their understanding.

The use of cooperative learning in MathWings is based on years of research regarding effective strategies for classroom instruction. This research has shown that the cognitive rehearsal opportunities presented by cooperative learning, as well as the opportunities for clarification and reteaching for students who do not catch a concept immediately, have positive effects on academic achievement. Research has also shown that using cooperative learning in the classroom can have positive effects on inter-ethnic relationships, acceptance of mainstreamed academically handicapped students, student self-esteem, liking of others, and attitudes toward school and teachers (Slavin, 1995). In cooperative learning, students work together to learn; the team's work is not done until all team members have learned the material being studied. This positive interdependence is an essential feature of cooperative learning.

Research has identified three key components which make cooperative learning strategies effective: team recognition, individual accountability, and equal opportunities for success. In MathWings, as in other Student Team Learning strategies (Slavin, 1994), students work in four-member, mixed-ability teams. Teams may earn certificates and additional means of recognition if they achieve at or above a designated standard. All teams can succeed because they are working to reach a common standard rather than competing against one another. The team's success depends on the individual learning of all team members; students must make sure that everyone on the team has learned, since each team member must demonstrate his or her knowledge on an individual assessment. Students have an equal opportunity for success in MathWings because they contribute points to their teams by improving over their own individual performance, by bringing in their homework, and by meeting particular behavior goals set by the teacher. Students who are typically seen as lower achievers can contribute as many points to the team as high achievers.

The MathWings program is designed to use the calculator as a tool, not a crutch. Calculators enable the students to explore and demonstrate concepts in an appealing way. Students discover that they need to check their calculator answers for accuracy since the calculator is only as accurate as the information and process that is keyed into it. Thus, students develop their skills in estimating and predicting outcomes. Students also spend more time actually thinking about math and the processes that will most efficiently solve a given problem rather than focusing completely on tedious and lengthy calculations. Because of the speed of calculators, students are more willing to try several approaches to solving a problem situation or to reevaluate their answers and try a different method of solution. Finally, calculators build students' confidence in mathematics as they receive much positive reinforcement from generating solutions. This leads, in turn, to a greater willingness to tackle more challenging mathematical situations in the belief that they have the ability and the tools to solve them.

The use of manipulatives is a basic building block of the MathWings program at all levels. Students construct understanding and develop original methods for solving problems using manipulatives. As they work with manipulatives and discuss and defend their thinking, they gradually make the concepts their own. Once a problem can be solved with manipulatives, students draw a picture and then write a number sentence to represent what was happening with the manipulatives as they solved the problem. This gradual progression from concrete to pictorial to abstract provides a solid foundation of understanding upon which the students can build. Every method or algorithm can be understood, and even reinvented, with manipulatives, thus replacing rote learning of algorithms with understanding of concepts and ways to efficiently apply them. Once the concepts have been firmly established and students understand how the algorithms work, they move away from using concrete manipulatives. However, manipulatives can be revisited at any time to reinforce or extend a concept as needed.

Most MathWings whole-class units have a literature connection which is an integral part of the concept development. Literature provides a wonderful vehicle for exploring mathematical concepts in meaningful contexts, demonstrating that mathematics are an integral part of human experience. The use of literature incorporates the affective elements and demonstrates the aesthetic aspect of mathematics. Finally, the use of literature encourages students to pose problems from real and imaginary situations and to use language to communicate about mathematics.

MathWings involves students in daily routines for skill practice and reinforcement to facilitate efficiency in calculation and application. Once the students have mastered mathematics facts and basic algorithms, they become tools for the students to use as they develop concepts and problem-solving. These daily routines include practice and weekly timed tests to encourage mastery of the basic facts, and then daily practice on problems at varying difficulty levels to provide for fluency in the use of the essential algorithms.

## Daily Schedule

MathWings is composed of two kinds of units: Action Math units, which are whole class units, and Power Math units, which are individualized units. Action Math units are 2 to 6 weeks in length and explore and develop concepts and their practical applications. One-week Power Math units are interspersed between whole class units and provide time for students to individually practice previous skills or explore more accelerated skills.

There is a frame around the lesson every day. This frame is provided by the daily routines, which are efficient ways to provide for team management, problem-solving, and fluency of skills. Daily routines are part of every Action Math or Power Math lesson.

Every day, students spend at least 60 minutes in their mathematics class. Daily lessons consist of three components: Check-In, Action Math or Power Math, and Reflection.

The first 15-minute segment is Check-In. Check-In is an efficient class start-up routine in which team members regularly complete one challenging real world problem individually and then come to a team consensus about their problem-solving strategies and solutions. They also complete a facts study process twice a week, and check homework briefly every day.

The next 40 minutes in either Action Math or Power Math is the heart of the lesson. When the class is doing an Action Math unit, the lesson involves the students in active instruction, team work, and assessment. During active instruction the teacher and students interact to explore a concept and its practical applications and related skills. The teacher may present a challenging problem for students to explore with manipulatives to construct a solution, may challenge the teams to use prior knowledge to discover a solution, or may ask the teams to find a pattern to develop a rule.

During team work the students first solve a problem together to develop their understanding of concepts. A team member is chosen randomly to share his or her ideas with the class. Then students individually practice similar problems, with teammates available for support as needed. The team members check answers and explanations with each other and rehearse to be sure every team member can explain them.

At the end of the team work, there is another brief feedback opportunity. The teacher randomly chooses a team member to share the ideas or solutions of the team, and to explain their thinking. This enables the teacher to assess the understanding of the group as a whole and ensures that teammates are invested in making sure that all members of the team are mastering the concepts.

The final portion of an Action Math lesson is assessment. One or more brief problems are used as a quick individual assessment of mastery of the concept or skill explored in the lesson.

When the class is doing a one-week Power Math unit, the 40-minute heart of the lesson involves each student in reinforcing, refining, or accelerating his/her skills. Students work at their own pace on the skill which they need to practice, completing check-outs and mastery tests successfully to move to another skill they need to practice. Students who have mastered the basic skills explore accelerated units at their own pace. The teacher teaches mini-lessons to small groups of students (working on the same skills) gathered from various teams while the other students continue to work individually.

The last five-minute segment of class is Reflection. This is an efficient routine used to bring closure to the class time. During Action Math units, reflection involves a quick summary of the key concepts by the teacher. During both Action Math units and Power Math units, a short entry is written in the MathWings Logbook in response to a writing prompt about the lesson, and homework sheets are passed out.

All students should not only be given the opportunity to establish a solid foundation in mathematics, but also the opportunity to extend and stretch their knowledge and experience in mathematics. Thus, a program of mathematics should include a structure to accommodate a diversity of abilities and background mathematical knowledge, while ensuring that *all* students experience the depth, breadth, and beauty of mathematics. The MathWings curriculum incorporates this philosophy in its development.

## Research on MathWings

Data on the achievement outcomes of MathWings are available for the intermediate program, grades 3-5, for six early pilot schools, four in rural Maryland, one in San Antonio, Texas, and one in Palm Beach County, Florida.

### St. Mary's County

The pilot schools for the Roots and Wings program, including MathWings, are four schools in and around Lexington Park, a rural community in Southern Maryland. The four schools are by far the most impoverished schools in the district; on average, 48% of their students qualify for free lunch. The schools began implementing the reading aspects of Roots and Wings in 1993-94, and then began to phase in MathWings in grades 3-5 in 1994-95. By 1995-96, all teachers in grades 3-5 were using MathWings.

Because there were no schools in St. Mary's County comparable in poverty or prior achievement to the Roots and Wings schools, an experimental-control comparison within the district could not be carried out. Instead, test score gains over time in the Roots and Wings schools were compared to those in the state as a whole. The test is the Maryland School Performance Assessment Program, or MSPAP, a state-of-the-art performance assessment in which students are asked to solve complex problems, set up experiments, write in several genres, and so on. The MSPAP uses matrix sampling, which means that different children take different parts of a broad, comprehensive test. Scores are reported in terms of percentages of children achieving at high levels labeled "satisfactory" and "excellent" in each school. In elementary schools, only third and fifth graders are assessed.

The results for the MSPAP mathematics scales are summarized in Figures 1 and 2. Figure 1 shows that third graders in the four MathWings pilot schools started off far below the state average. By 1995, they had essentially caught up to the state average and, in 1996, exceeded it. Exactly the same pattern was found in fifth grade, where by 1996 the MathWings pilot schools also had higher scores than the state average.

**Figure 1**
**Mathematics Performance, Maryland School Performance Assessment Program, 1993-96**
**MathWings Pilot Schools vs. State of Maryland**
**Grade 3**

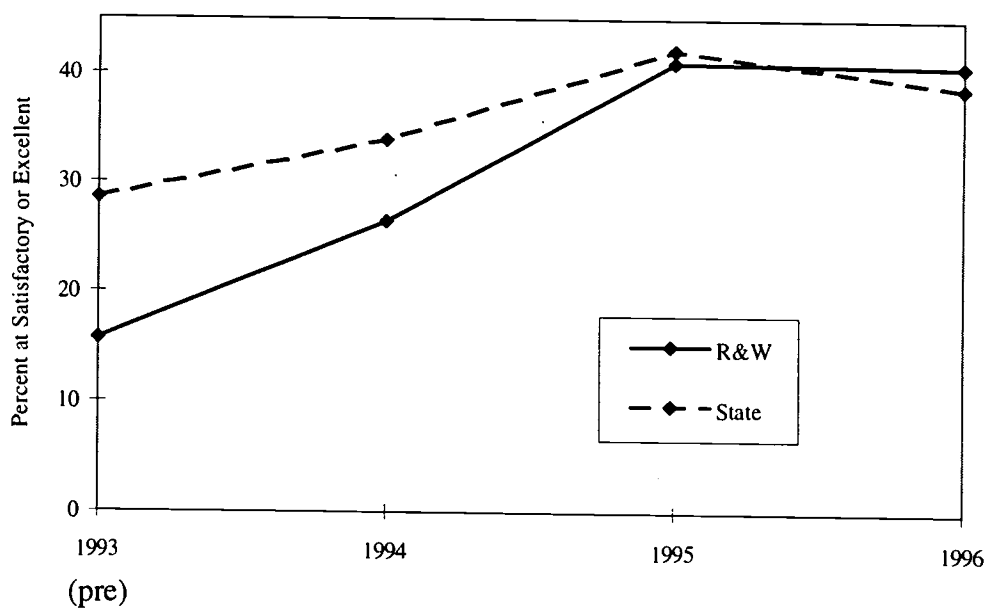
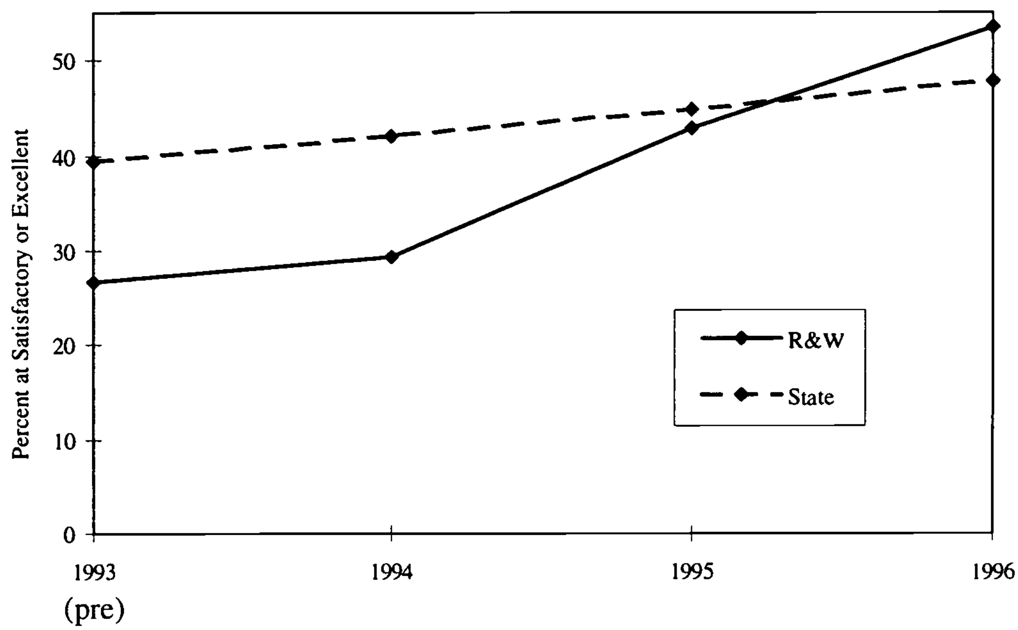


**Figure 2**
**Mathematics Performance, Maryland School Performance Assessment Program, 1993-96**
**MathWings Pilot Schools vs. State of Maryland**
**Grade 5**

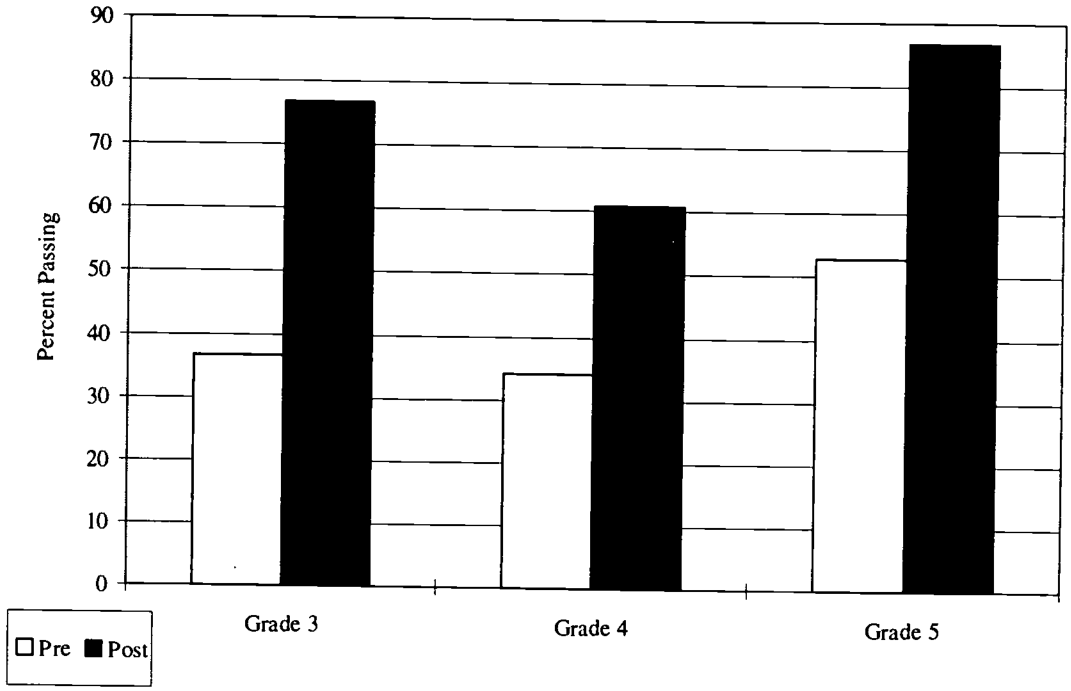



## San Antonio, Texas

One of the first pilot schools for MathWings outside of Maryland was Lackland City Elementary School in the Northeast Independent School District in San Antonio, Texas. Lackland City had implemented the Success for All reading program in 1994-95, and then began to pilot MathWings in 1995-96. Lackland City is one of the most impoverished schools in its district; 86% of its students qualify for free lunch. A majority of its students are Latino (78%), with a high proportion categorized as Limited English Proficient.

Students in Texas are tested annually on the Texas Assessment of Academic Skills, or TAAS. Scores are reported in terms of the percentages of children passing the TAAS in each subject. Figure 3 shows the TAAS mathematics gains for Lackland City in grades 3, 4, and 5 from 1994-95 (just before the program began) to 1995-96, the first implementation year.

**Figure 3**
**Percent of Students Passing TAAS Math in 1995 (Pre) and 1996 (Post)**
**Lackland City Elementary School**




As Figure 3 illustrates, students in all three grades made substantial gains on the TAAS, in comparison to the previous cohorts of students in the same school. Percent passing more than doubled in third grade, from 36.7% to 76.7%. Fourth graders did almost as well, increasing from 34.2% to 60.9% passing, and fifth graders increased from 52.9% to 86.8% passing. Although Lackland City is far more impoverished than its district average (86% free lunch vs. 42% for the district), its students had TAAS passing rates higher than the district average in third grade (76.7% to 73.7%) and fifth grade (86.8% to 81.5%), although not in fourth grade (60.9% to 75.5%).

## Palm Beach County, Florida

Another early pilot site for MathWings was Lincoln Elementary School in West Palm Beach, Florida. Lincoln began implementation of Success for All in 1993-94, and began its MathWings pilot in 1996-97. Like Lackland City, Lincoln is one of the most impoverished schools in its district; 100% of its students are African American, and more than 90% qualify for free lunch.

Florida elementary schools take the Comprehensive Test of Basic Skills mathematics concepts and applications scale in grades 3-5. Percentage scores for the three years preceding MathWings implementation and for the first year following are shown in Figure 4.

**Figure 4**
**CTBS Math Concepts & Applications, Mean Percentile Rank (PR), 1994-97**
**Lincoln Elementary School**

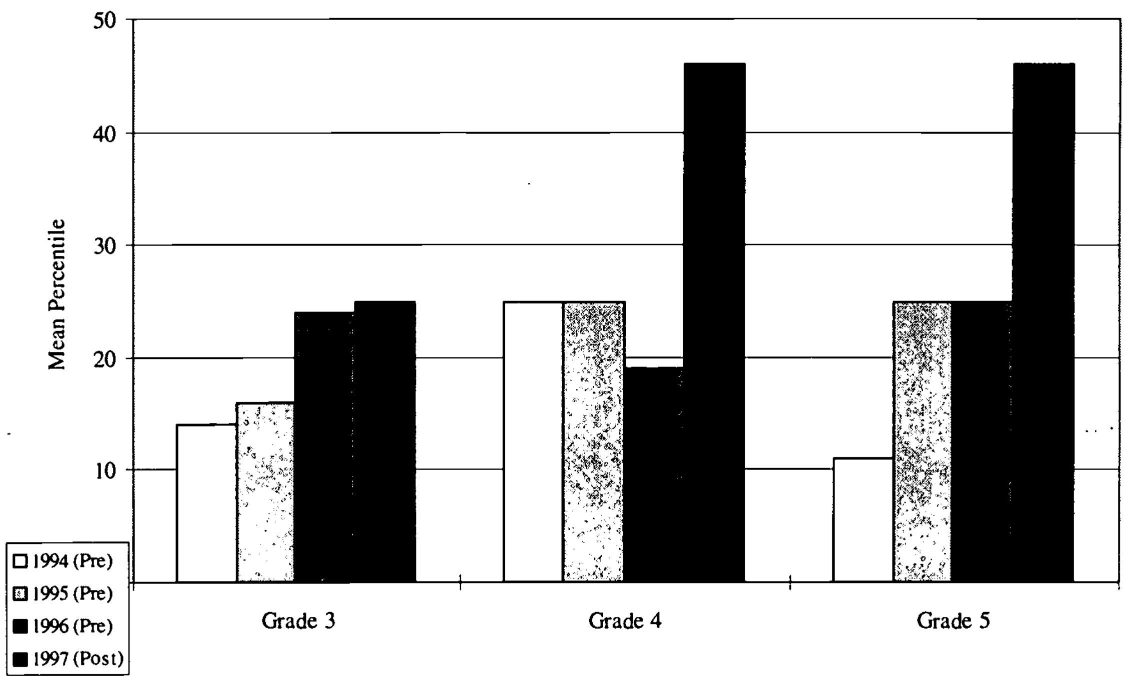



Figure 4 shows substantial gains in grades 4 and 5, a gain of 27 percentile points in grade 4 and 21 percentage points in grade 5. These gains put this very impoverished school nearly at grade level (46th percentile), and ahead of the district's own math-science-technology magnet school. Grade 3 gains, however, were slight (only one percentage point).

# Conclusion

Trends on state accountability measures for six pilot schools in three districts show substantial gains due to implementation of MathWings. In all three districts, high-poverty schools which were initially performing significantly below district or state averages reached or exceeded these averages after implementing MathWings.

There is much more to be done in the evaluation of MathWings. None of the evaluations reported here used control groups, and it is possible in all three cases that at least part of the gains in mathematics performance is due to implementation of the Success for All reading program, implemented in all of these MathWings pilot schools 1-3 years earlier. Experimental-control comparisons among schools that have all implemented Success for All are currently under way. However, the dramatic gains seen in the six pilot schools are unlikely to have been entirely due to the reading program or to other factors; in all districts assessed, baseline scores reported here would already reflect the reading implementations, and the largest gains were seen in the year when MathWings was implemented.

These results demonstrate that schools serving many children in poverty can substantially accelerate the mathematics achievement of their students using an approach tied to NCTM standards but developed to be practical for a broad range of teachers and effective for a broad range of students.

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# APPENDIX

# SAMPLE MATHWINGS MATERIALS

**MATHWINGS™ MATHWINGS™ MATHWINGS™ MATHWINGS™**

Dear Parents:

Your son or daughter is involved in an innovative mathematics program at our school. This mathematics program, called **MATHWINGS™**, is an exciting opportunity for your child to learn mathematics in an active setting with a balance of problem solving and skills. The mathematics that you see your child doing at home and talking about from the classroom is different than what you may have experienced in your schooling. A program to prepare students for mathematics in the twenty-first century needs to actively involve students in the conceptual development and practical application of their mathematics skills. The **MATHWINGS™** program reflects a balance of solid mathematical conceptual development, problem solving in real world applications, and a maintenance of necessary mathematics skills.

You will hear from your child about working with others on teams, using the calculator, discovering math through hands-on methods, finding mathematics in stories, solving real life problems, practicing skills, problem solving, discovering the algorithms, and much, much more.

An algorithm is a rule about how to write a problem out in numbers and symbols and solve it. Your child will learn expanded or adapted algorithms for addition, subtraction, multiplication, and division which are different from the traditional algorithms most of us were taught. Both the traditional and expanded forms of the algorithms are acceptable. We use the expanded or adapted form because it helps children understand the process better. We hope that you will enjoy some of the homework activities that will involve the family. We want the children to learn to enjoy mathematics and not be afraid of it. We welcome your support and interest as we help our students become better mathematicians.

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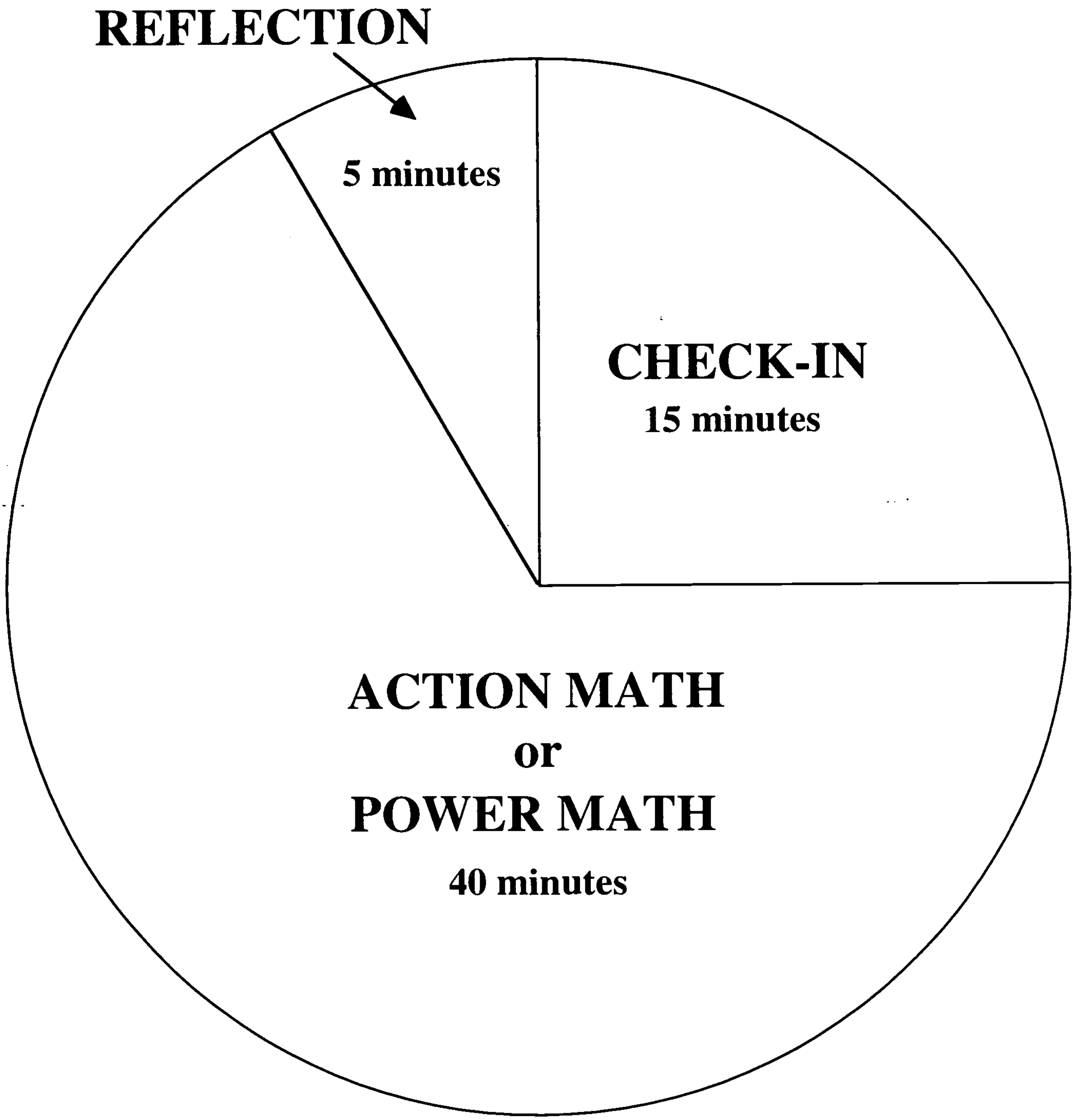
# TIME MANAGEMENT IN MATHWINGS UNITS



NUMBER SENSE C

LESSON 8

**GOAL: Students will bc able to determine whether numbers in written material, answers to math problems, and calculator answers are reasonable.**

**OUTLINE**
**I. CHECK-IN**
**II. ACTIVE INSTRUCTION**
Have students analyze the numbers in written material to see if they are reasonable.
Demonstrate the need for using estimates to check calculator answers.
**III. TEAM WORK**
**Team Consensus**
Have each team member use number sense to estimate answers and explain her thinking.
**Team Mastery**
Have each team member use number sense to estimate answers and explain his reasoning.
**IV. ASSESSMENT**
Have each student use number sense to estimate 539 + 192 and explain her reasoning.
**V. REFLECTION**

**Introduction for Teacher:**

Estimation is the math we use most often in daily life. We estimate at the store, doing projects, and evaluating written materials to see if they make sense. Your students need to be able to **estimate** (make a good guess at) answers to determine if the actual answers they find are reasonable. You need to help your students analyze problems, written materials, or other data to determine if the numbers used are reasonable. You want them to become thoughtful citizens and critical consumers. It is important for your students to estimate answers when solving written problems and also when using the calculator. It is easy to make a careless mistake or hit the wrong key on the calculator. If they have no idea what the answer should be, they might not realize if their answer or the calculator answer is a mistake.

**Key Concepts:**
**An estimate** is a good guess or an educated guess at an answer, <u>not</u> a wild guess.
**Estimates** are used to **decide** whether numbers in written materials, answers to math problems, and calculator answers are **reasonable.**

**PREPARE:**
**Transparencies:**
- REASONABLE NUMBERS PARAGRAPH
- blank transparencies

**Team Materials:**
- **Student**
  calculator

## CHECK-IN:
Team Set-up
Problem Solving / Facts or Fluency
Homework Check

## ACTIVE INSTRUCTION:
**Reasonable Numbers in Written Material:**
Explain:

**An estimate is a good guess or an educated guess at an answer, <u>not</u> a wild guess.**

Point out:

**Estimates are used to decide whether numbers in written materials, answers to math problems, and calculator answers are reasonable.**

Note:
**In fact, we use estimation far more often in real life than we use paper and pencil to find exact answers!**

Show the transparency of the REASONABLE NUMBERS PARAGRAPH and read it through with the class underlining the numbers as you read. Explain that some of the numbers in the paragraph make sense and are reasonable and some of them don't make sense.

REASONABLE NUMBERS PARAGRAPH

Four friends were having a party at 9:00 A.M. on Friday afternoon after school. After they had played games for 600 minutes, they were hungry. They decided to have their snack of 180 raisin cookies, 120 mystery cookies, and 140 chocolate cookies. They ate all 340 cookies and were still hungry, so they decided to go home for supper at 6:00 P.M.

**T** Have the team members discuss each number in the paragraph and decide if it makes sense and is reasonable in the context. Have them be sure every team member can explain which numbers are or are not reasonable and how they know.

✔ Randomly choose team members to share the teams' ideas with the class.

(Four friends might easily have a party, so that is reasonable.
9:00 A.M. is in the morning not in the afternoon.
600 minutes is ten hours, which is not a reasonable amount of time to play games.
180 raisin cookies, 120 mystery cookies, and 140 chocolate cookies is 440 cookies not 340 cookies.
440 cookies means that each person would eat 110 cookies, which is too much to eat at one sitting.
6:00 P.M. is a reasonable time for supper.)

Point out:
**It is a good idea to analyze numbers in things you read to see if they make sense.**

Explain:
**It is always a good idea to think about the numbers in advertisements, because advertisers often use numbers to get you to buy something. For example, a car will be advertised for $9,995 because the advertisers know that people will think $9,000, because they see the 9 first, rather than $10,000, even though $9,995 is almost $10,000.**

**Reasonable Calculator Answers:**
Point out:
**It is especially important to estimate the answer when using a calculator because it is easy to hit the wrong key, and if you have no idea what the answer should be, you might not realize if the answer on the calculator is a mistake.**

Write "8 + 11" on a blank transparency and ask the students to use their number sense to estimate the answer. ✔ Randomly choose a student to share the estimate. (8 is about 10 and 11 is about 10, so the answer is about 20.)

Say:
**That's right. 8 is about 10 and 11 is also about 10. Did you know that there is a short cut for writing that? There is a symbol that means "is about equal to." Write "8 ≈ 10 and 11 ≈ 10" on a blank transparency.**

Now have the students use their calculators to solve the problem. Tell them to pretend that they looked too quickly and accidentally hit the x key instead of the + key because they are very similar, and see what answer they get. (88)

**T** Have team members compare their actual answers and their estimates and discuss whether they think their answer is reasonable. ✔ Randomly choose a team member to share his idea with the class. (No; it is way too big. We must have done something wrong, so let's try again.)

**T** Now have them use their calculators very carefully to solve the problem and compare their new answer with their estimate to see if it is a reasonable answer.

✔ Randomly choose a student to share his answer and whether it is reasonable. (Yes; 19 is close to 20, so it is reasonable.)

## TEAM WORK:

### Team Consensus:

Tell the teams that they can also use their number sense to decide on a reasonable answer for a number problem. Write 123 + 589 on the blank transparency. Explain:
**I might think 123 is about 120 and 589 is about 600, so the answer is about 720. Or I might think 123 is about 100 and 589 is about 600, so the answer is about 700. Since estimates are not exact, people can have different estimates that are all reasonable.**

Quickly find the answer on the calculator and note that 712 is close to both 700 and 720 so it is a reasonable answer.

Write the following problems on a blank transparency:
"117 + 479
896 - 148"

**T** Have team members use their number sense to estimate answers to these problems without actually solving the problems. Have team members take turns sharing their thinking out loud so everyone in the team can think about it and see if it makes sense. Have them be sure every team member rehearses how she finds estimates and explain her thinking. (117 is about 100 and 479 is about 500, so 117 + 479 ≈ 600. 896 is about 900 and 148 is about 150, so 896 - 148 ≈ 750.)

**T** Have the team members quickly find the answers with their calculators, compare their answers and their estimates, and come to consensus about whether the answers are reasonable. ✔ Randomly choose a team member to share the estimates, how she got the estimates, the answers, and whether the answers are reasonable. (117 is about 100 and 479 is about 500, so 117 + 479 is about 600. 596 is about 600, so it is reasonable. 896 is about 900 and 148 is about 150, so 896 - 148 is about 750. 748 is about 750, so it is reasonable.)

### Team Mastery:

Write the following problems on a blank transparency:

1. 93 + 12
2. 89 - 48
3. 214 + 898
4. 896 - 108
5. 1394 + 978
6. 1505 - 387
7. 456 + 149
8. 795 - 508

Have team members use their number sense to estimate answers to these problems.

**T** Have every team member work the first four problems. Then have him check in with his team members to come to consensus on their answers. If a team member has incorrect answers or can't explain how he got them, have other team members explain the process, and then have him do the last four problems. If a team member has the correct answers and can explain how he got them, have him help other team members who need more practice.

Ask them to find the answers with their calculators and decide whether they are reasonable. Have them be sure every team member rehearses how to estimate to decide if the answers are reasonable and how to explain his thinking.

✔ Circulate as they work to observe their understanding of number sense estimation. When teams have demonstrated understanding, move on to Numbered Heads Together.

**✔ Numbered Heads Together:**
Use Numbered Heads Together, or another strategy, to randomly choose a team member to explain how the team got the estimate and give the answer explaining whether it is reasonable for 214 + 898. (214 is about 200 and 898 is about 900, so the answer is about 1,100. 1,112 is about 1,100, so the answer is reasonable.)

Answers and Possible Estimates:

| | | |
|---|---|---|
| **1.** 93 | ≈ 100 | Answer: 105 |
| + 12 | ≈ 10 | Estimate: about 110 |
| | 110 | Reasonable |

| | | |
|---|---|---|
| **2.** 89 | ≈ 100 | Answer: 41 |
| - 48 | ≈ 50 | Estimate: about 50 |
| | 50 | Reasonable |

| | | |
|---|---|---|
| **3.** 214 | ≈ 200 | Answer: 1,112 |
| + 898 | ≈ 900 | Estimate: about 1,100 |
| | 1,100 | Reasonable |

| | | |
|---|---|---|
| **4.** 896 | ≈ 900 | Answer: 788 |
| - 108 | ≈ 100 | Estimate: about 800 |
| | 800 | Reasonable |

| | | |
|---|---|---|
| **5.** 1,394 | ≈ 1,400 | Answer: 2,372 |
| + 978 | ≈ 1,000 | Estimate: about 2,400 |
| | 2,400 | Reasonable |

| | | |
|---|---|---|
| **6.** 1,505 | ≈ 1,500 | Answer: 1,118 |
| - 387 | ≈ 400 | Estimate: about 1,100 |
| | 1,100 | Reasonable |

| | | |
|---|---|---|
| **7.** 456 | ≈ 450 | Answer: 605 |
| + 149 | ≈ 150 | Estimate: about 600 |
| | 600 | Reasonable |

| | | |
|---|---|---|
| **8.** 795 | ≈ 800 | Answer: 287 |
| - 508 | ≈ 500 | Estimate: about 300 |
| | 300 | Reasonable |

## ✔ ASSESSMENT:

Write the following problem on a blank transparency:
"539 + 192."

Ask each student to use number sense to estimate the answer, find the answer with the calculator, decide whether it is reasonable, and write a sentence to explain her thinking.

Collect the assessments as you point out that 539 is about 550 and 192 is about 200, so the answer is about 750. Explain that 731 is about 750, so the answer is reasonable. Remind the students that they may have estimated differently, but their estimate should be somewhere in that ballpark.

## REFLECTION:

**Key Concepts Summary:**

- **An estimate** is a good guess or an educated guess at an answer, not a wild guess.
- **Estimates** are used to **decide** whether numbers in written material, answers to math problems, and calculator answers are **reasonable.**

**✔ Logbook:**
Write about whether you think you can count to a thousand by ones in one minute and how you know.

**Homework:**
Lesson 8 Homework

**Team Wrap-up**

## NUMBER SENSE C - LESSON 8 HOMEWORK ANSWERS

### SKILL PROBLEMS

1. Estimate the answer to 345 + 126.
**345 is about 350, 126 is about 125, 350 + 125 = 475, so the answer is about 475.**

2. 568 + 125 = 443. Is the answer reasonable and how do you know?
**No, because 5 + 1 = 6, so the hundreds place is definitely too small.**

3. Estimate the answer to 527 - 216.
**527 is about 500, 216 is about 200, 500 - 200 = 300, so the answer is about 300.**

4. 672 - 431 = 241. Is the answer reasonable and how do you know? **Yes because there is no regrouping and 6 - 4 = 2.**

5. Five dozen donuts is 15 donuts. Is this answer reasonable and how do you know?
**No, because one 12 is almost 15, so 5 12's is much more than 15.**

### MIXED PRACTICE

1. $75 + 25 = \mathbf{100}$
2. $75 - 25 = \mathbf{50}$
3. $77 + 15 = \mathbf{92}$
4. $77 - 15 = \mathbf{62}$
5. $34 + 56 = \mathbf{90}$

### REAL WORLD PROBLEMS

1. Tom traded half of his collection of 100 baseball cards and kept 50 for himself. Are these numbers reasonable? Why or why not? **Yes, because $^1/_2$ of 100 = 50 cards, so each would have 50 cards.**
2. Nancy ate half of a candy bar. She gave her sister half of the candy bar and her cousin the last half of the candy bar. Are these numbers reasonable? Why or why not?
**No, because $^3/_2$ is more than one whole candy bar.**
3. Samantha baby-sat for two hours at $2 an hour and earned $4. Is this amount reasonable? Why or why not? **Yes, because $2 x 2 is $4.**

### PROBLEM SOLVING

Barry had three baseball hats (Orioles, Blue Jays, Angels) and 2 team T-shirts (Tigers and Giants). How many different outfits could he make out of them?
**He could make 6 different outfits:**
**Orioles/Tigers, Orioles/Giants, Blue Jays/Giants, Blue Jays/Tigers, Angels/Tigers, Angels/Giants.**

## PARENT PEEK

Your child will be exploring estimation in Lessons 8, 9, and 10. **Estimation is one of the most important math skills.** In fact, we use estimation far more often in real life than we use paper and pencil to find exact answers!

We often estimate almost without thinking about the strategies we are using to get approximate answers in real life situations. We **use different estimation strategies** depending on the problem and which strategies we are most comfortable using.

Remember, an **estimate** is a good guess, or an **educated guess,** at an answer, **not a wild guess. Answers in estimation will vary depending on the strategy and thinking used, and any answer that is in the right ballpark is acceptable.**

**One of the estimation strategies is number sense.** In this lesson your child used number sense to estimate answers. We use our number sense to decide whether numbers in written materials, answers to math problems, and calculator answers are **reasonable.** Your child can use number sense to estimate an answer for a number problem or to check whether an exact answer makes sense.

$\approx$ **is the symbol** that means one amount **"is about as much as"** another amount. For example: $8 \approx 10$ means that 8 is about 10 and $11 \approx 10$ means that 11 is about 10.

Try using your number sense to estimate 123 + 589. You might think $123 \approx 120$ and $589 \approx 600$, and $120 + 600 = 720$, so the answer $\approx 720$. Or you might think $123 \approx 100$ and $589 \approx 600$, and $100 + 600 = 700$, so the answer $\approx 700$. **Remember, since estimates are not exact, people can have different estimates that are all reasonable.** The exact answer is 712, which is close to both 700 and 720, so it is a reasonable answer.

You might like to have your child explain his answers to some of the Real World problems in this Homework to see how his number sense is growing.

# DAILY ROUTINES

## CHECK-IN: (ACTION MATH & POWER MATH)

### Team Set-up:

- Go-Getter gets all needed team materials.
- All homework is out on desk and recorded by Recorder on Team Score Sheet.
- All needed materials should be ready (pencils, etc.).
- Team members should be ready to start class.

### Facts or Fluency:

- Tuesday -- Partner FACTS study or individual FLUENCY section/s worked
- Thursday -- timed test on section/s
- Students' progress is observed weekly and assessed once a unit.
- For FACTS or FLUENCY: Facts Bonus Points are recorded on Team Score Sheet -- 5 points if one section is passed and 10 points if more than one section is passed **for the first time only**.

### Problem Solving:

- This is a gradual process of developing problem solving skills:
  - Day 1: SKILL ACTIVITY
  - Day 2: ONE-STEP PROBLEM
  - Day 3: MULTIPLE STEP PROBLEM
  - Day 4: PROCESS PROBLEM
  - Day 5: PROCESS PROBLEM
- One problem is collected each week to assess student progress. The problem is scored using scoring tool.

### Homework Check:

- Answers are stated by teacher or provided on the board or a transparency.
- Individual students check their own HOMEWORK for correct answers.
- Team Recorder checks for homework completion or attempt, and marks the Team Score Sheet accordingly.
- Teacher briefly discusses a few chosen problems.
- Teacher collects HOMEWORK daily and scores one homework weekly.
- 1 homework bonus point is earned each day if all team members complete HOMEWORK for a possible weekly total of 5 to be recorded on the Team Score Sheet.

## **TEAM WORK: (ACTION MATH ONLY)**

### Team Consensus:

- Team will work together to understand what has been explored in class.
- Each team member needs to help and encourage all of the team members.
- Team members should make sure everyone on the team understands the math and **rehearses** how to explain the team's answers.

### Team Mastery:

- Each team member practices the math individually.
- Team members compare answers to come to consensus on the correct answers.
- Team members make sure everyone on the team understands the math and **rehearses** how to explain the team's answers.

### Numbered Heads Together:

- One team member will be called on to give the team's answer and explanation.

## **ASSESSMENT: (ACTION MATH ONLY)**

- Individuals are assessed on lesson concept practiced with team.

## **REFLECTION: (ACTION MATH & POWER MATH)**

### Key Concepts Summary:

- Teacher facilitates a quick summary review, emphasizing key concepts.

### Logbook:

- A logbook entry is included with each lesson. Logbooks should be used at least twice a week.
- Students individually write in designated part of math notebook or journal.
- Teacher examines student written communication progress weekly, assessing with a "✓" or "✓+". Provide student with individual feedback, challenging the student to develop her mathematical thinking.

### HOMEWORK:

- All students have HOMEWORK Book and assignment.

### Team Wrap-up:

- Teamwork Points are recorded.
- Team go-getter returns necessary material.
- Team members check that everyone has their HOMEWORK.
- Team members organize for next class.

**CONCEPT PRACTICE:** Lessons 8 - 11 **Answers**

1. Are the numbers in the sentence below reasonable? Explain why or why not.
   Pam didn't have enough to buy an ice cream cone,
   because she only had $5 and 2 quarters.

**Possible answer: This is not reasonable because an ice cream cone costs less than $1 and Pam had more than $5.**

2.
a. Use your **number sense** to estimate the answer.

$$\begin{array}{r} 342 \\ +\ 129 \\ \hline \end{array}$$

**Possible estimate: about 475**

b. Explain how you got the estimate.
**Possible explanation: 342 is about 350, and 129 is about 125, so the sum is about 475.**

3. Use **front end** estimation to estimate the answer.

$$\begin{array}{r} 783 \\ +\ 321 \\ \hline \end{array}$$

**Possible estimate: 700 + 300 = 1,000, 83 ≈ 80, 21 ≈ 20, 80 + 20 = 100, 1,000 + 100 = 1,100, so about 1,100**

4. Use **flexible rounding** to estimate the answer.

$$\begin{array}{r} 92 \\ +\ 16 \\ \hline \end{array}$$

**Some possible estimates: 92 → 90, 16 → 20, 90 + 20 = 110, so about 110, OR 92 → 100, 16 → 20, 100 + 20 = 120, so about 120**

5. Use **flexible rounding** to estimate the answer.

$$\begin{array}{r} 349 \\ -\ 107 \\ \hline \end{array}$$

**Some possible estimates: 349 → 350, 107 → 100, 350 - 100 = 250, so about 250, OR 349 → 300, 107 → 110, 300 - 110 = 190, so about 190, OR 300 - 100 = 200, so about 200.**

**Partner Initials:** ____________________

**CONCEPT HOMEWORK I:** Lessons 8- 11 **Answers**

1. Are the numbers in the sentences below reasonable? Explain why or why not.
   Tara had a quick breakfast in the ten minutes before the school bus came.
   She ate 1 egg and 2 pieces of toast.

**Possible answer: This is reasonable because a person could eat 1 egg and 2 pieces of toast in 10 minutes.**

2.
a. Use your **number sense** to estimate the answer.

$$\begin{array}{r} 845 \\ +\ 219 \\ \hline \end{array}$$

**Possible answer: about 1,050**

b. Explain how you got the estimate.

**Possible explanation: 845 is about 850, and 219 is about 200, so the sum is about 1050.**

3. Use **front end** estimation to estimate the answer.

$$\begin{array}{r} 603 \\ +\ 279 \\ \hline \end{array}$$

**Possible estimate: 600 + 200 = 800, 03 ≈ 0, 79 ≈ 89, 0 + 80 = 80, 800 + 80 = 880, so about 880.**

4. Use **flexible rounding** to estimate the answer.

$$\begin{array}{r} 47 \\ +\ 12 \\ \hline \end{array}$$

**Possible estimate: 47 → 50, 12 → 10, 50 + 10 = 60, so about 60.**

5. Use **flexible rounding** to estimate the answer.

$$\begin{array}{r} 502 \\ -\ 295 \\ \hline \end{array}$$

**Possible estimate: 502 → 500, 295 → 300, 500 - 300 = 200, so about 200.**

**CONCEPT CHECK:** Lessons 8 -11 **Answers**

1. Are the numbers in the sentences below reasonable? Explain why or why not.
   Colin bought a fruit drink for \$0.69. He paid with a \$1 bill. He got \$0.50 change.

**Possible answer: This is not reasonable because \$0.69 is a bit more than $^1/_2$ dollar. If he paid more than $^1/_2$ dollar, he can't get $^1/_2$ dollar back in change.**

2.
a. Use your **number sense** to estimate the answer to the problem below.

$$\begin{array}{r} 386 \\ \underline{+\,411} \end{array}$$

**Possible answer: about 800**

b. Explain how you got the estimate.

**Possible explanation: 386 is about 400 and 411 is about 400, so the sum is about 800.**

3. Use **front end** estimation to solve the problem below.

$$\begin{array}{r} 568 \\ \underline{+\,342} \end{array}$$

**Possible estimate: 500 + 300 = 800, 68 ≈ 70, 42 ≈ 40, 70 + 40 = 110, 800 + 110 = 910, so about 910.**

4. Use **flexible rounding** to estimate the answer.

$$\begin{array}{r} 323 \\ \underline{+\,697} \end{array}$$

**Some possible estimates: 323 → 320, 697 → 700, 320 + 700 = 1,020, so about 1,020, OR 323 → 300, 697 → 700, 300 + 700 = 1,000, so about 1,000.**

5. Use **flexible rounding** to estimate the answer.

$$\begin{array}{r} 67 \\ \underline{-\,14} \end{array}$$

**Some possible estimates: 67 → 70, 14 → 10, 70 - 10 = 60, so about 60, OR 67 → 100, 14 → 10, 100 - 10 = 90, so about 90.**

**CONCEPT HOMEWORK II:** Lessons 8 -11 **Answers**

1. Are the numbers in the sentences below reasonable? Explain why or why not.
   Annette claimed that she could walk 2 miles in 5 minutes.

**Possible answer: This is not reasonable because 2 miles is much too far to walk in 5 minutes.**

2.
a. Use your **number sense** to estimate the answer to the problem below.

$$\begin{array}{r} 234 \\ +\ 743 \\ \hline \end{array}$$

**Possible estimate: about 1,000**

b. Explain how you got the estimate.

**Possible explanation: 234 is close to 250 and 743 is about 750, so the sum is about 1,000.**

3. Use **front end** estimation to solve the problem below.

$$\begin{array}{r} 377 \\ +\ 487 \\ \hline \end{array}$$

**Possible estimate: 300 + 400 = 700, 77 ≈ 80, 87 ≈ 90, 80 + 90 = 170, 700 + 170 = 870, so about 870.**

4. Use **flexible rounding** to estimate the answer.

$$\begin{array}{r} 622 \\ +\ 377 \\ \hline \end{array}$$

**Some possible estimates: 622 → 620, 377 → 400, 620 + 400 = 1,020,**
**OR 622 → 600, 377 → 400, 600 + 400 = 1,000,**
**OR 622 → 600, 377 → 380, 600 + 380 = 980,**
**OR 622 → 620, 377 → 380, 620 + 380 = 1,000.**

5. Use **flexible rounding** to estimate the answer.

$$\begin{array}{r} 84 \\ -\ 27 \\ \hline \end{array}$$

**Some possible estimates: 84 → 100, 27 → 30, 100 - 30 = 70,**
**OR 84 → 80, 27 → 30, 80 - 30 = 50.**

# PERFORMANCE TASK - NUMBER SENSE C

## How Many Flyers Do We Need?

## Teacher Directions / Procedures

This task will involve your students in applying many of the concepts they have learned in the number sense unit. It may take more than one day to complete the whole Performance Task.

**Materials needed:**
**a calculator per student**
**data collection transparencies**

Provide calculators for each student as materials for this task. Tell the students that they may use their calculators when they feel they are an appropriate tool for the task, but, in particular, for Activity 2 and Activity 3. Students may want to use a calculator to get an <u>actual</u> value for Activity 2 rather than to use the calculator to help find an estimate. When scoring their responses, make sure there is evidence of the use of estimation strategies.

Activity 2 is a team activity. All the other activities are to be done individually.

Before you begin this task, initiate a brief discussion about the situation described in the opening paragraph. Explain that the principal often needs to run off flyers for each student in the school to take home and that they are going to help the principal find out how many flyers will be needed.

Fictitious school data is included (Everett Elementary) that can be used for Activity 2. However, you may want to use the actual data of your own school for this activity. You can get this information from your school office or send student representatives to each classroom to collect the data. A data collection transparency has been provided to help you organize this information.

EVERETT ELEMENTARY SCHOOL DATA

| TEACHER | CLASS SIZE |
|---|---|
| PRE-KINDERGARTEN | |
| Mrs. Smith | 15 |
| Mrs. Jones | 19 |
| KINDERGARTEN | |
| Mr. Frederick | 19 |
| Mrs. Nichols | 24 |
| GRADE 1 | |
| Mr. Hobbs | 25 |
| Mrs. Baker | 23 |
| Miss Williams | 28 |
| GRADE 2 | |
| Mr. Ford | 31 |
| Mrs. Robinson | 22 |
| Mrs. Hanson | 28 |
| GRADE 3 | |
| Mrs. Brown | 26 |
| Miss Lawrence | 29 |
| GRADE 4 | |
| Mr. Watkins | 36 |
| Mrs. Knight | 32 |
| GRADE 5 | |
| Mrs. Fortune | 24 |
| Mrs. White | 28 |

SCHOOL DATA FORM

| TEACHER | CLASS SIZE |
|---|---|
| PRE-KINDERGARTEN | |
| | |
| | |
| KINDERGARTEN | |
| | |
| | |
| GRADE 1 | |
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| GRADE 2 | |
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| GRADE 3 | |
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| GRADE 4 | |
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| GRADE 5 | |
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**Please read the following aloud to the class as an introduction for this task:**

The principal has asked your class to be in charge of giving out flyers to advertise the next Parent-Teacher meeting. Enough flyers need to be given to each class in the school so that every student in each class can take one home. You will need to find out about how many flyers will need to be printed to be able to give one to each student in the school.

Some of your students will need help reading and understanding the directions. **Please read and explain the directions before each activity and discuss what they mean before they work on the activity individually.** Be available to assist them with the reading if necessary.

**Optional extension:** An appropriate extension to this activity would be to have the students design a flyer that the principal could use to send out advertising this meeting. Students could brainstorm in their teams for the necessary information that would go on this flyer, then the students would make their own flyer design. One design could be chosen as the one that would be run off for the school.

The following procedures may help organize this option:

**Team Work:**

1. After the students have the necessary information, distribute blank pieces of $8^1/_2$" x 11" paper to every team, at least one for each team member.

2. The team will share ideas on creating a flyer using the information developed in class. They need to come to consensus on a flyer design, including all the information discussed in class

3. Each team member should make a flyer using the same design. When all team members have finished, the team needs to choose one of the team's flyers to submit as the team's entry in the competition.

**Judging the entries:** (This can be going on as the rest of the project continues.)

1. The judging of the teams' flyers may be done in a manner that you feel is fair and positive. Some suggestions are: ask the principal to pick the winning flyer; ask a panel of faculty members to choose the winner; or involve the entire class in the selection process with a secret ballot.

2. Remind the students that those flyers not chosen as the winner will be used as posters around the school to advertise the event (if the event is an actual event).

## PERFORMANCE TASK - NUMBER SENSE C

### How Many Flyers Do We Need?

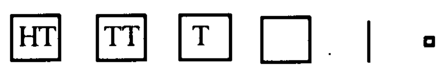
The principal has asked your class to be in charge of giving out flyers to advertise the next Parent-Teacher meeting. Enough flyers need to be given to each class in the school so that every student in each class can take one home. You will need to find out about how many flyers will need to be printed to be able to give one to each student in the school.

**Activity 1**
Part A: Make a reasonable guess of how many flyers you will need for every student in the school.
Write your guess on the line below.
Write a sentence to explain how you made your guess.
Part B: To make a **good estimate** of how many flyers will be needed, what other information do you need? Write your answer on the lines below.

**Activity 2**
Use the information given to you by your teacher to work together with your team to come up with a <u>better estimate</u>. Calculators may be used to help you figure out the estimate.

Write your new estimate:
Explain how you and your team got this new estimate.

**Activity 3**
Part A: Use a calculator to find the actual number of flyers needed. Write that number on the line below.
Part B: Draw the number you found in Part A on the place value mat below.
(Use the symbols shown below on your place value mat as you need them.)




| Hundred Thousands | Ten Thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|
| | | | | | |

Part C: Write the **word name** on the line below for the number shown on the place value mat:
Part D: Write the number shown on the place value mat **in expanded notation** on the line below.

**Activity 4**
Write your guess from Activity 1.
Write your estimate from Activity 2.
Write the actual number found in Activity 3, Part A.
Use <, >, or = to order the above numbers from least to greatest.

**Activity 5**
Would you tell the principal to use your **guess** or your **estimate** or the **actual number** to make enough copies for every student in each class to get a flyer?

Write a sentence to explain why you chose the one you did.

## SCORING RUBRIC: Activity 1 -- Part A

| SCORE POINT 2 |
| --- |
| • Guess is reasonable.<br>• Explanation of guess is clear and reasonable. |

| SCORE POINT 1 |
| --- |
| • Guess is somewhat reasonable.<br>• Explanation of guess is somewhat reasonable but unclear or not complete. |

| SCORE POINT 0 |
| --- |
| • Guess is not reasonable.<br>• Explanation of guess is not clear or is missing.<br>• No response or unscorable. |

## SCORING RUBRIC: Activity 1 -- Part B

| SCORE POINT 2 |
| --- |
| • Information listed as needed will help make a good estimate.<br>• Response is clear. |

| SCORE POINT 1 |
| --- |
| • Some information listed as needed may help make a good estimate.<br>• Response is partially clear. |

| SCORE POINT 0 |
| --- |
| • Information listed as needed will not help to make a good estimate.<br>• Response is not clear.<br>• No response or unscorable. |

## SCORING RUBRIC: Activity 2

| SCORE POINT 2 |
| --- |
| • Estimate is reasonable. (For Everett Elementary School data, an estimate between 300 and 500 is reasonable.)<br>• Explanation of how estimate was figured out is clear and reasonable.<br>• There is evidence that estimation strategies were used. |

| SCORE POINT 1 |
| --- |
| • Estimate is reasonable.<br>• Explanation of how estimate was figured out is somewhat reasonable but unclear or not complete.<br>• There is little evidence that estimation strategies were used. |

| SCORE POINT 0 |
| --- |
| • Estimate is not reasonable.<br>• Explanation of how estimate was figured out is not clear or is missing.<br>• There is no evidence that estimation strategies were used.<br>• No response or unscorable. |

## SCORING RUBRIC: Activity 3 -- Part A

**SCORE POINT 1**

If Everett Elementary School data is used:
- 409 flyers

If own school data was used:
- Correct sum of data given.

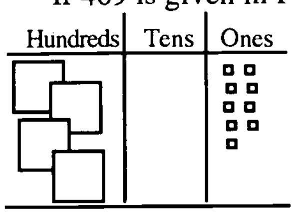
**SCORE POINT 0**

- Any other answer.
- No response or unscorable.

## SCORING RUBRIC: Activity 3 -- Part B

**SCORE POINT 1**

- If 409 is given in Part A:




- If any other number (even an incorrect answer) is given in Part A: number is correctly put on place value mat.

**SCORE POINT 0**

- Answer given in Part A is not correctly placed on place value mat.
- No response or unscorable.

## SCORING RUBRIC: Activity 3 -- Part C

**SCORE POINT 1**

- If 409 is given in Part A: Four hundred nine
- If any other number (even an incorrect answer) is given in Part A: number is correctly written as a word name.

**SCORE POINT 0**

- Answer given in Part A is not correctly written as a word name.
- No response or unscorable.

## SCORING RUBRIC: Activity 3 -- Part D

**SCORE POINT 1**

- If 409 is given in Part A: 400 + 0 + 9
- If any other number (even an incorrect answer) is given in Part A: number is correctly written in expanded notation.

**SCORE POINT 0**

- Answer given in Part A is not correctly written in expanded notation.
- No response or unscorable.

## SCORING RUBRIC: Activity 4

| SCORE POINT 2 |
|---|
| • Guess, estimate and actual values are listed correctly.<br>• The three numbers are correctly ordered from least to greatest. |

| SCORE POINT 1 |
|---|
| • Did not list all of the values (guess, estimate or actual) correctly, but ordered the numbers they did list correctly.<br>• Had one of the three numbers out of the correct order.<br>• Ordered the numbers correctly from greatest to least. |

| SCORE POINT 0 |
|---|
| • Did not list the values (guess, estimate and actual) correctly, and did not order the numbers they did list correctly.<br>• Incorrectly ordered the numbers.<br>• No response or unscorable. |

## SCORING RUBRIC: Activity 5

| SCORE POINT 2 |
|---|
| • Explanation of choice is clear and reasonable. |

| SCORE POINT 1 |
|---|
| • Explanation of choice is somewhat reasonable, but unclear or not complete. |

| SCORE POINT 0 |
|---|
| • Explanation of choice is not reasonable or complete.<br>• No response or unscorable. |

# ADVANCED NUMBER SENSE

Teaching Lesson A: Understanding Place Value up to 999,999,999 and Ordering Fractions and Decimals on a Number Line

**GOAL:** Students will review place value, comparing, and ordering numbers up to 999,999,999 and order positive and negative fractions, mixed numbers, decimals and mixed decimals on a number line.

**OUTLINE**

**I. WARM-UP**

Have students compare 2 sets of 5 digit numbers.

**II. ACTIVE INSTRUCTION**

Review numbers on PLACE VALUE MAT transparency, and name them with words, numerals, and expanded notation.

Review comparing numbers and then comparing and ordering numbers with numerals on PLACE VALUE MAT transparency.

Model locating negative and positive fractions, mixed numbers, decimals, and mixed decimals on a number line.

**III. GUIDED PRACTICE**

Have students then work with 5 place value, comparing, and ordering situations and 3 naming and ordering of fractions and decimals situations.

**IV. INDEPENDENT PRACTICE**

Have students begin work on Lesson 1 and continue through their individual packets taking tests when ready.

**Introduction to the Teacher:**

**In Introduction to Number Sense and Intermediate Number Sense, the students reviewed place value up to 999,999. You will only need to do a very quick review in this lesson to explore the millions before you concentrate on ordering fractions and decimals on a number line.**

Have your students use a place value mat to understand **place value** and **expanded notation** and to **compare numbers** up to 999,999,999. **Expanded notation** is when a number is written as the sum of the amount of each place value. The students will use >, <, = to compare numbers and to order numbers from least to greatest and greatest to least.

Challenge your students to locate positive and negative fractions, decimals, mixed numbers and mixed decimals on a number line. **Fractions and decimals** are numbers whose values are less than one whole. Fractions and decimals are found between -1 and 0 and 0 and 1 on the number line. Fractions and decimals can both be used to name the same amount. For example, the fraction 3/5 = the decimal 0.6 because 3/5 = 6/10 and 0.6 is another way to represent 6/10.

**Mixed numbers** are numbers that are a combination of a whole number and a fraction and are found between the whole numbers on the number line. **Mixed decimals** are numbers that are a combination of a whole number and a decimal and are also found between the whole numbers on the number line.

**Key Concepts:**

- Our number system is a **base ten system** and each place is ten times as big as the place to its right.
- We **compare** numbers starting at the left and comparing each place until we find a place where the digits are different.
- **Negative numbers** are numbers whose value is less than zero. They are found to the left of zero on the number line.
- **Proper fractions and decimals** are numbers whose value is **less than one whole.** They are found **between -1 and 0 and 0 and 1** on the number line.
- **Mixed numbers and mixed decimals** are numbers that are a combination of a whole number and a fraction or decimal. They are found between the whole numbers on the number line.

**PREPARE:**

**Transparencies:**
- PLACE VALUE MAT, master included (Page 8 )
- NUMBER LINES, master included (Page 9)

**Copies:**
- PLACE VALUE MAT, one per student, master included (Page 8)
- NUMBER LINES, master included (Page 9)

**Group Materials:**
- a calculator for each student
- a copy of the PLACE VALUE MAT sheet for each student
- a copy of the NUMBER LINES sheet

**WARM-UP:**

The purpose of the warm-up problems is to review prior skills on which your students will build in this lesson. Write the following pairs of numbers on the blank transparency.

1. 43,509 and 43, 059    2. 65,732 and 65,723

Have your students compare each pair of numbers and use <, >, or = to write a true number sentence for each pair. ✔ (Answers: 43,509 > 43,059 or 43,059 < 43,509 and 65,732 > 65,723 or 65,723 < 65,732)

**ACTIVE INSTRUCTION:**

**Review of Place Value:**

**Remember, you should be able to review place value very quickly.**

Show the PLACE VALUE MAT transparency, point out each period or number family from the ones to the millions. Quickly review how each place is ten times as great as the place to its right and tens times smaller than the place to its left.

Ask how many tens make a hundred, how many hundreds make a thousand, how many ten thousands there are in one hundred thousand, how many hundred thousands there are in one million, etc..

Remind them:

**Our number system is a base ten system and each place is ten times as big as the place to its right.**

Say: **An accountant was entering numbers in his ledger. When I tell you the amounts, put the standard form of each number on your calculator and hold it up. Here we go: seven hundred eight (708); nine hundred thirty-two thousand, five hundred sixty-three (932,563); four hundred seventy-six million, two hundred thirty-nine thousand, twenty-eight (476,239,028).**

✔ Quickly check that they know the standard form of the numbers and take careful note of whether the students understand place value and the use of zeroes as place holders.

Next ask the students to write the number 876,543,210 on their place value chart paper. ✔ Check for accuracy and go over each place value. (8 hundred millions, 7 ten millions, 6 millions, 5 hundred thousands, 4 ten thousands, 3 thousands, 2 hundreds, 1 ten, 0 ones)

Remind the students that **expanded notation** means that a number is written as the sum of each place value so that 75,346 is written as 70,000 + 5,000 +300 + 40 + 6.

Ask the students to write the number from their place value charts (876,543,210) in **expanded notation** on their papers. (800,000,000 + 70,000,000 + 6,000,000 + 500,000 + 40,000 + 3,000 + 200 + 10 + 0)

Remind them that expanded notation can be changed to standard form (the usual way we write a number) by adding all the place values together as you quickly demonstrate this on the PLACE VALUE MAT transparency as shown below.

| Millions | | | Thousands | | | Ones | | |
|---|---|---|---|---|---|---|---|---|
| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 5 | 0 | 0 | 0 | 0 | 0 |
| | | | | 4 | 0 | 0 | 0 | 0 |
| | | | | | 3 | 0 | 0 | 0 |
| | | | | | | 2 | 0 | 0 |
| | | | | | | | 1 | 0 |
| | | | | | | | | + 0 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

Say: **The accountant wanted to compare 987,654,321 and 987,564,321 in his ledger.**

✔ Ask the students to write these amounts one above the other on their PLACE VALUE MAT sheets as you check for accuracy and observe their understanding of place value.

Ask the students to **compare** these two numbers and write a true number sentence about them using <, >, or =. Remind them that they start at the left (like reading) and compare until they find two different digits in the same place value column as shown below.

| Millions | | | Thousands | | | Ones | | |
|---|---|---|---|---|---|---|---|---|
| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 9 | 8 | 7 | 5 | 6 | 4 | 3 | 2 | 1 |

same same same 6 > 5, so 987,654,321 > 987,564,321.

Ask how they know 987,564,321 is smaller. (Because 500,000 < 600,000, so the whole number is less.)

Ask if there is another way they could write that to describe how the two numbers compare. (987,564,321 > 987,654,321)

Agree that both statements are true and that any comparison can be written either way.

Say: **The accountant now wants to compare three numbers and order them from least to greatest and then greatest to least.**

Write the numbers 324,589,061 and 324,598,061 and 324,589,601 one above the other on the PLACE VALUE MAT transparency and have the students do the same on their sheets. Remind them to start at the left and compare the three digits in each place value column until they find a column where the digits are different, and then write number sentences ordering these numbers as shown below.

| Millions | | | Thousands | | | Ones | | |
|---|---|---|---|---|---|---|---|---|
| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| 3 | 2 | 4 | 5 | 8 | 9 | 0 | 6 | 1 |
| 3 | 2 | 4 | 5 | 9 | 8 | 0 | 6 | 1 |
| 3 | 2 | 4 | 5 | 8 | 9 | 6 | 0 | 1 |

same same same same 9 > 8 (9 = 9) 6 > 0

9 > 8, so 324,598,061 is the largest number.
6 > 0, so 324,589,601 is the next to largest number,
and 324,589,061 is the smallest number.

least to greatest:. 324,589,061 < 324,589,601 < 324,598,061
greatest to least: 324,598,061 > 324,589,601 > 324,589,061

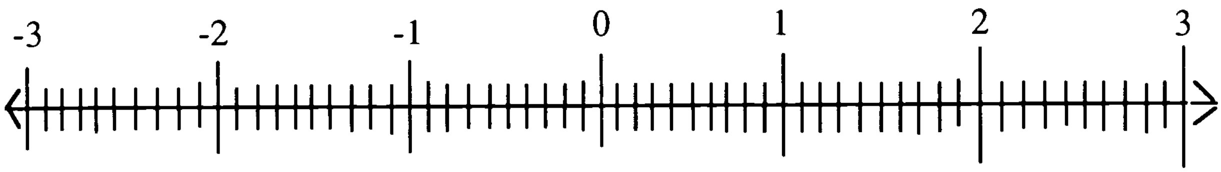
Summarize:

**We compare numbers starting at the left and comparing each place until we find a place where the digits are different.**

✔ As you check their papers for accuracy have the students discuss how they decided on the ordering of these numbers and how the placement of the numbers changed when they ordered from least to greatest and then from greatest to least .

**Ordering Fractions and Decimals on a Number Line:**
Show the NUMBER LINES transparency. Jot the following numbers on the transparency: -3, 2, -1, 3, -2.

Have the students write these numbers on one of their number lines in the correct positions. ✔ Then randomly choose a student to label the transparency number line as shown below and explain the reasons for the placements.




Emphasize that negative numbers are found to the left of 0 on the number line and positive numbers to the right of 0. Point out that the digits increase in size as they move away from the 0 in either direction.

Summarize:

| |
|---|
| **Negative numbers are numbers whose value is less than zero. They are found to the left of zero on the number line.** |

Ask the students what they call numbers that are less than one whole. (fractions, decimals)

Agree with the students that **fractions** are numbers that are less than one whole and ask them for some examples of fractions. (Answers will vary but might include numbers like 1/2 or -1/4.)

Agree with them again that **decimals** are another way of writing fractions whose denominators are 10, 100, 1,000, etc., and ask them for some examples of decimals. ( Answers will vary but might include numbers like 0.5 and -0.25.)

If no one gives negative examples, supply several as shown above and explain that fractions and decimals can be negative as well as positive. Give the example of something that is -1/2 ft or -0.5 ft of elevation, which means it is 1/2 or 0.5 ft below sea level.

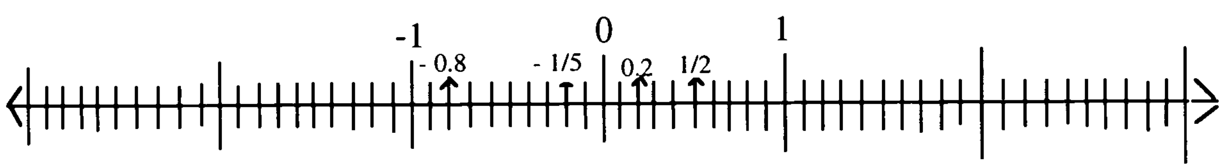
Ask if anyone knows where these numbers fit on the number line. (between -1 and 1)

Explain: **Positive and negative proper fractions and decimals are found between 0 and 1 and between -1 and 0 on the number line.**

Summarize:

| |
|---|
| **Proper fractions and decimals are numbers whose value is less than one whole. They are found between -1 and 0 and 0 and 1 on the number line.** |

Label several on the transparency as shown below.



Ask the students to label the fractions on the number line as decimals and the decimals as fractions. (-0.8 = - 8/10 or - 4/5; - 1/5 = -2/10 or -0.2; 0.2 = 2/10 or 1/5; 1/2 = 5/10 or 0.5)

Have the students work as partners to label several other points on their number lines as both fractions and decimals. ✔ Circulate to observe their understanding of the relative positions of positive and negative equivalent fractions and decimals on the number line.

Ask if anyone remembers what mixed numbers or mixed decimals are. (numbers with a whole number part and a fraction or decimal part)

Agree that **mixed numbers** are numbers with a whole number part and a fraction part, and that **mixed decimals** are numbers with a whole number part and a decimal part.

Ask the students to give some examples of each. (Answers will vary but might include numbers such as 1 1/4, 3.5, 45 1/2, 93.95, etc..)

Remind them that many money amounts are mixed numbers like a dollar and a quarter or mixed decimal amounts like $93.95.

Ask if the students think there can be negative mixed numbers and mixed decimals. (Yes.)

Agree that when people are in debt, they talk about being "in the red", which means that they have less than 0 money. For example, they might owe the bank $24.25, so they have -$24.25 or - 24 1/4 (a quarter) dollars.

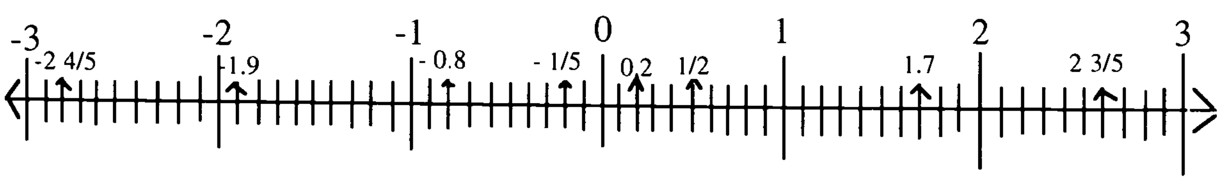
Remind them that the same amount can be described as either a mixed number or a mixed decimal.

Ask if anyone thinks they know where the mixed numbers and mixed decimals are found on the number line. (between the whole numbers)

Summarize:

| |
|---|
| **Mixed numbers and mixed decimals are numbers that are a combination of a whole number and a fraction or decimal. They are found between the whole numbers on the number line.** |

Label several on the transparency as shown below.




Ask the students to label the mixed numbers on the number line as mixed decimals and the mixed decimals as mixed numbers. (-2 4/5 = - 2 8/10 = -2.8; -1.9 = -1 9/10; 1/7 = 1 7/10; 2 3/5 = 2 6/10 = 2.6)

Have the students label several other points on their number lines as both positive and negative mixed numbers and mixed decimals. ✔ Circulate to observe their understanding of the relative positions of positive and negative equivalent fractions and decimals on the number line.

## GUIDED PRACTICE:

✔ The purpose of the guided practice is to have your students practice as you observe to be sure that they understand the process. Write the following problems on the blank transparency.

1. Write 900,000,000 + 0 + 5,000,000 + 800,000 + 30,000 + 2,000 + 600 + 40 + 2 in standard form.

2. Write the place value for the underlined digit in 654,609,321.

3. Compare 785,239,387 and 785,329,307.

4. Write 350,687,904 in expanded notation.

5. Compare and order 562,473,300, 562,433,073, and 562,437,300 from least to greatest.

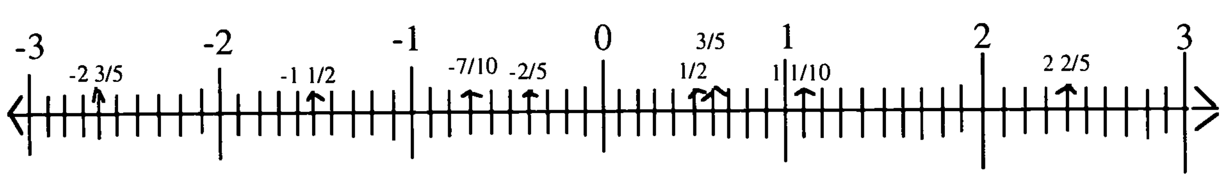
6. List the following numbers. Write the equivalent fraction or mixed number for each decimal or mixed decimal. Write the equivalent decimal or mixed decimal for each fraction or mixed number.
   1/2, 1 1/10, -2/5, -2 3/5, 0.6, 2.4, -0.7, and -1.5.

7. Locate all the fractions and mixed numbers on one number line.

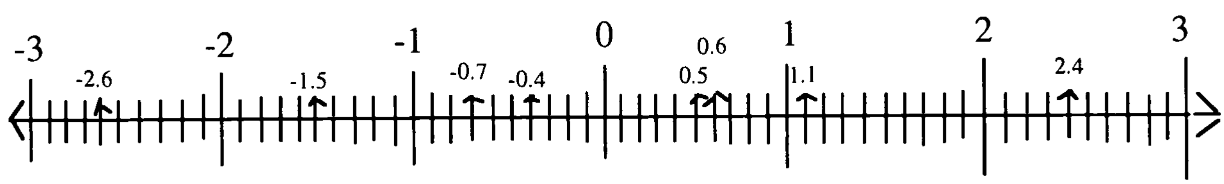
8. Locate all the decimals and mixed decimals on another number line.

(Answers:
1. 905,832,642
2. millions
3. 785,239,387 < 785,329,307
4. 300,000,000 + 50,000,000 + 0 + 600,000 + 80,000 + 7,000 + 900 + 0 + 4
5. 562,433,073 < 562,437,300 < 562,473,300
6. 1/2 = 0.5; 1 1/10 = 1.1; 2.4 = 2 4/10 or 2 2/5; -2/5 = - 4/10 = -0.4;
   -2 3/5 = -2 6/10 = -2.6; 0.6 = 6/10 or 3/5; -0.7 = - 7/10;
   - 1.5 = - 1 5/10 or - 1 1/2

7.




8.




)

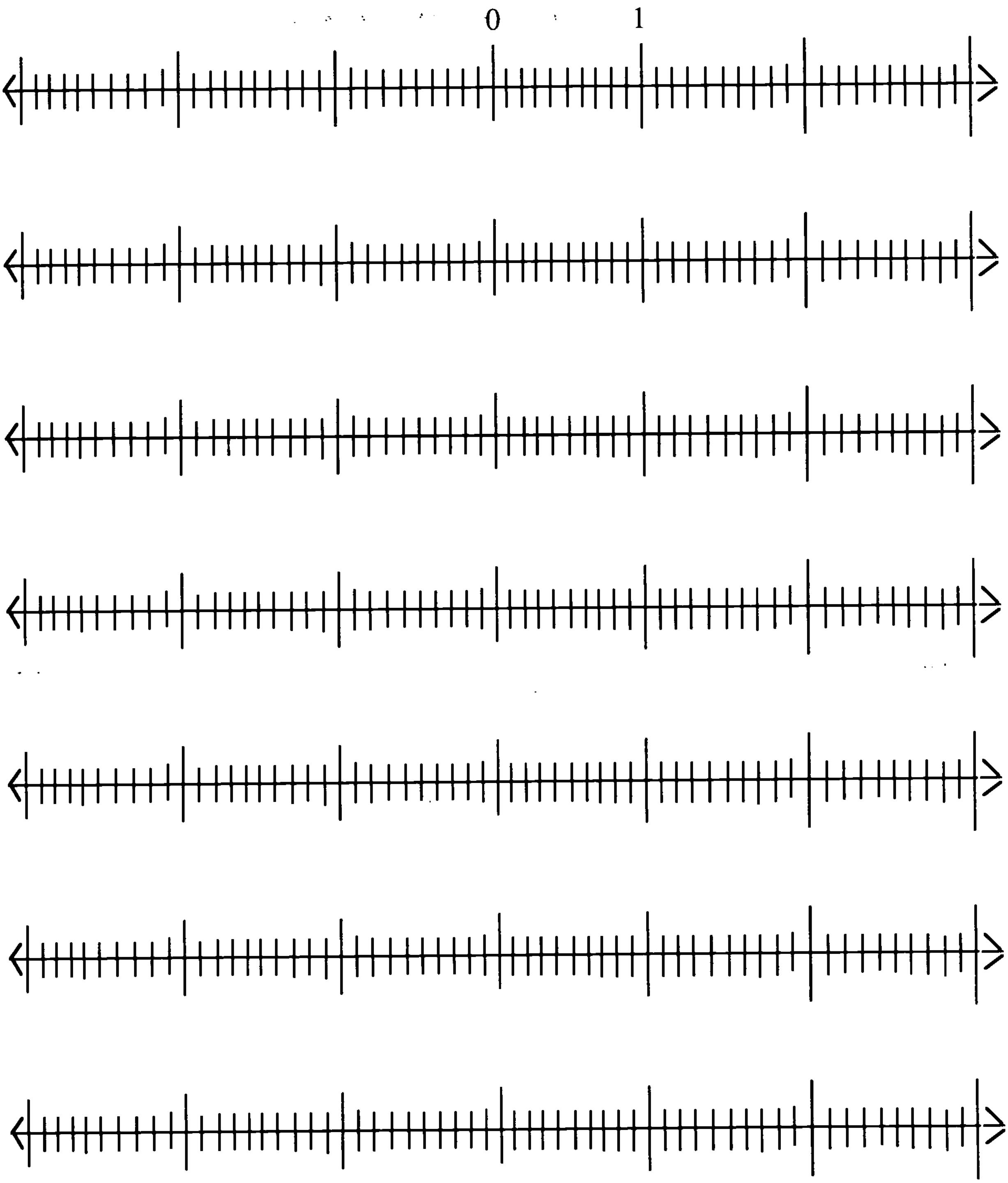
## INDEPENDENT STUDENT PRACTICE:

Students will begin work on Lesson 1 and continue through the lessons in their individual packets, taking tests when ready.

## PLACE VALUE MAT

| Millions | | | Thousands | | | Ones | | |
|---|---|---|---|---|---|---|---|---|
| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| | | | | | | | | |

## NUMBER LINES



Lesson 1: Understanding Place Value Up To 999,999,999

**GOAL: YOU WILL WORK WITH NUMBERS LIKE THESE.**

Write the standard form of the number six hundred ninety-two million, fifty-four thousand, six hundred thirty.

***692,054,630***

Write the expanded notation for 489,321,065.

***400,000,000 + 80,000,000 + 9,000,000 + 300,000 + 20,000 + 1,000 + 60 + 5***

Write the standard form of 500,000,000 + 60,000,000 + 700,000 + 10,000 + 4,000 + 900 + 20 + 3.

***560,714,923***

Write the place value for the underlined digit in the number 479,528,163.

***ten millions***

Compare.

871,909,090 ○ 871,090,909

871,909,090 (>) 871,090,909

Order from least to greatest.

954,672,810 and 954,627,801 and 954,762,018

***954,627,801 < 954,672,801 < 954,762,018***

**YOU NEED TO KNOW THAT:**

☞ There are several symbols which will help you in the Power Math activities:

☞ means MAKE SURE YOU SEE THIS!

💡 means THINK.

✍ means WRITE.

PRACTICE 1

Directions: Write the standard form of the numbers given below.

Example: eight hundred thirty-nine million, five hundred two thousand, four hundred sixty-seven

| Millions | | | Thousands | | | Ones | | |
|---|---|---|---|---|---|---|---|---|
| H | T | O | H | T | O | H | T | O |
| 8 | 3 | 9 | 5 | 0 | 2 | 4 | 6 | 7 |

✍ **839,502,467**

| |
|---|
| 1. six hundred two million, three hundred twenty-seven thousand, four hundred eighty-five |
| 2. seventy-nine thousand, five hundred seventy |
| 3. nine hundred eighty-three |
| 4. one million, three hundred ninety-one thousand, six hundred fifteen |
| 5. four hundred fifty-nine |
| 6. eight hundred fourteen million, five hundred thirty-six thousand, nine hundred seventy-four |
| 7. seven million, eight hundred fifty-two thousand, six hundred forty |
| 8. twenty-four thousand, seven hundred thirty-six |
| 9. nine million, two hundred seventeen thousand, five hundred eighty |
| 10. forty-eight thousand, six hundred ninety |
| 11. three hundred seven million, four hundred sixty-six thousand, two hundred thirty-four |
| 12. six hundred fifty-nine |

# PRACTICE 2

Directions: Write the expanded notation for each standard form given.
Write the standard form for each expanded notation given.

Example 1: 843,200,105

8 hundred millions + 4 ten millions + 3 millions + 2 hundred thousands + 0 ten thousands + 0 thousands + 1 hundred + 0 tens + 5 ones

**800,000,000 + 40,000,000 + 3,000,000 + 200,000 + 0 + 0 + 100 + 0 + 5**

Example 2: 50,000 + 1,000 + 900 + 40 + 4

| | | | | |
|---|---|---|---|---|
| 5 | 0 | 0 | 0 | 0 |
| | 1 | 0 | 0 | 0 |
| | | 9 | 0 | 0 |
| | | | 4 | 0 |
| + | | | | 4 |
| 5 | 1 | 9 | 4 | 4 |

***51,944***

| |
|---|
| 1. 5,000 + 800 + 50 + 9 |
| 2. 419,862 |
| 3. 60,000 + 7,000 + 0 + 20 + 2 |
| 4. 209,628,462 |
| 5. 6000 + 200 + 30 + 4 |
| 6. 236,934 |
| 7. 50,000 + 3,000 + 400 + 10 + 7 |
| 8. 567,848,312 |
| 9. 600,000 + 0 + 9,000 + 500 + 20 + 3 |
| 10. 987,456 |
| 11. 30,000,000 + 1,000,000 + 0 + 20,000 + 4,000 + 600 + 40 + 8 |
| 12. 345,987,123 |

# PRACTICE 3

Directions: Write the place value of the underlined digit in the number shown.

Example: 7<u>3</u>2,415,009

| Millions | | | Thousands | | | Ones | | |
|---|---|---|---|---|---|---|---|---|
| H | T | O | H | T | O | H | T | O |
| 7 | <u>3</u> | 2 | 4 | 1 | 5 | 0 | 0 | 9 |

***ten millions***

| | | | |
|---|---|---|---|
| 1. 1,62<u>0</u>,000 | 2. 5,4<u>3</u>2 | 3. 4<u>7</u>3,769 | 4. <u>2</u>19,171,613 |
| 5. 46,<u>8</u>93,467 | 6. 23,<u>0</u>06 | 7. 9<u>0</u>9,987,999 | 8. <u>3</u>,938 |
| 9. 32,8<u>2</u>5,001 | 10. <u>7</u>37,803,111 | 11. 4,5<u>4</u>5 | 12. 9<u>9</u>9,923,987 |

## PRACTICE 4

Directions: Compare the number pairs below.
Use >, <, = to make a true statement.

---

Example: 459,630,421 ○ 459,360,412

| Millions | | | Thousands | | | Ones | | |
|---|---|---|---|---|---|---|---|---|
| H | T | O | H | T | O | H | T | O |
| 4 | 5 | 9 | 6 | 3 | 0 | 4 | 2 | 1 |
| 4 | 5 | 9 | 3 | 6 | 0 | 4 | 1 | 2 |
| same | same | same | 6 > 3 | | | | | |

***459,630,421 (>) 459,360,412***

| |
|---|
| 1. 167,854,098 ○ 176,845,980 |
| 2. 1,640,230 ○ 1,460,230 |
| 3. 236,154,607 ○ 236,157,076 |
| 4. 7,865,423 ○ 7,856,432 |
| 5. 69,032,578 ○ 69,032,758 |
| 6. 45,678,999 ○ 45,678,999 |
| 7. 28,167,428 ○ 28,165,412 |
| 8. 45,456,456 ○ 45,654,456 |
| 9. 83,695,427 ○ 83,965,472 |
| 10. 34,567,889 ○ 78,787,879 |
| 11. 524,793,254 ○ 524,937,245 |
| 12. 111,333,222 ○ 113,332,221 |

# PRACTICE 5

Directions: Order from least to greatest or greatest to least. Use >, <, =.

---

Example: Order from least to greatest.
532,164,400 and 532,164,600 and 532,146,600

| Millions | | | Thousands | | | Ones | | |
|---|---|---|---|---|---|---|---|---|
| H | T | O | H | T | O | H | T | O |
| 5 | 3 | 2 | 1 | 6 | 4 | 4 | 0 | 0 |
| 5 | 3 | 2 | 1 | 6 | 4 | 6 | 0 | 0 |
| 5 | 3 | 2 | 1 | 4 | 6 | 6 | 0 | 0 |
| same | same | same | same | 4 < 6 | 4 = 4 | 4 < 6 | | |

***532,146,600 < 532,164,400 < 532,164,600***

| | | | |
|---|---|---|---|
| 1. Order from least to greatest. | 765,548 | 765,584 | 769,219 |
| 2. Order from greatest to least. | 497,645,325 | 497,654,532 | 497,465,523 |
| 3. Order from least to greatest. | 483,000,000 | 438,000,000 | 483,300,000 |
| 4. Order from greatest to least. | 324,320 | 323,680 | 326,436 |
| 5. Order from least to greatest. | 867,777 | 823,009 | 856,789 |
| 6. Order from least to greatest. | 148,375 | 148,975 | 148,576 |
| 7. Order from greatest to least. | 325,345,678 | 325,678,345 | 325,354,678 |
| 8. Order from greatest to least. | 609,278,435 | 609,278,598 | 906,652,857 |
| 9. Order from least to greatest. | 123,007 | 123,070 | 132,070 |
| 10. Order from least to greatest. | 685,476 | 685,576 | 658,675 |
| 11. Order from greatest to least. | 555,667,893 | 534,678,009 | 589,003,690 |
| 12. Order from greatest to least. | 213,498,231 | 213,478,312 | 213,496,321 |

## CHECKOUT A

1. Write the standard form.
   300,000,000 + 20,000,000 + 0 + 700,000 + 60,000 + 3,000 + 200 + 20 + 7

2. Write the expanded notation.
   4,311,128

3. Write the place value of the underlined digit.
   821,743,692

4. Compare.
   563,000,600 ○ 563,600,000

5. Order from greatest to least.
   251,418,111 251,814,111 251,481,444

## CHECKOUT B

1. Write the standard form.
   600,000,000 + 70,000,000 + 0 + 300,000 + 80,000 + 0 + 900 + 20 +4

2. Write the expanded notation for
   45,789,690

3. Write the place value of the underlined digit.
   567,893,303

4. Compare.
   112,334,565 ○ 112,334,656

5. Order from greatest to least.
   132,456,789 132,456,478 132,465,467

**Have your partner check and sign your paper.**

## ADVANCED NUMBER SENSE

Lesson 1: Understanding Place Value Up To 999,999,999

**Mastery Test**

1. Write the standard form.
   900,000,000 + 80,000,000 + 4,000,000 + 200,000 + 30,000 + 1,000 + 500 + 0 + 8

2. Write the expanded notation for
   3,609,885

3. Write the place value of the underlined digit.
   113,<u>9</u>09,334

4. Compare.
   234,123,543 ◯ 234,213,098

5. Order from least to greatest.
   560,896,430 560,869,340 560,986,037

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