DOCUMENT RESUME

ED 412 086 SE 060 598

AUTHOR	Cai, Jinfa
TITLE	An Investigation of U.S. and Chinese Students' Mathematical Problem Posing and Problem Solving.
PUB DATE	1997-04-00
NOTE	29p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, March 24-28, 1997).
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Comparative Education; *Computation; Elementary School Mathematics; Foreign Countries; Foreign Students; Grade 6; Intermediate Grades; Mathematical Applications; *Mathematics Achievement; Mathematics Instruction; Middle Schools; *Problem Solving; Thinking Skills
IDENTIFIERS	China (Zhejing Province); *Cross National Studies; *Problem Posing

ABSTRACT

Cross-national studies in mathematics have consistently reported that U.S. students do not perform as well as Asian students on tasks requiring the application of mathematical knowledge and skills routinely learned in school. Recent studies have shown, however, that for tasks assessing relatively novel and complex problem solving, performance differences between U.S. and Asian students are not so pronounced. This study explored 181 U.S. and 223 Chinese sixth-grade students' mathematical problem solving and problem posing abilities. It is part of a continuous effort to examine U.S. and Chinese students' performance by conducting a cognitive analysis of student responses to mathematical problem-solving and problem-posing tasks. The tasks included computational exercises, a problem-posing task in which students were asked to pose mathematical problems based on a given figural pattern situation, a division-with-remainder story problem, and a figural pattern problem. The Chinese students outperformed U.S. students on computation tasks but there were many similarities and differences between U.S. and Chinese students in performing relatively novel tasks. The findings suggest a generality with regard to the direct link between mathematical problem solving and problem posing from an international perspective. Contains 26 references. (PVD)

An Investigation of U.S. and Chinese Students' Mathematical Problem Posing and Problem Solving

Jinfa Cai

Department of Mathematical Sciences

523 Ewing Hall

University of Delaware

Newark, DE 19716

Running Head: U.S. AND CHINESE POSING AND SOLVING

An earlier version of this paper was presented at the annual meeting of the American Educational Research Association; Chicago, IL; March 24-28, 1997.

Preparation of this paper is supported, in part, by the National Academy of Education and Spencer postdoctoral fellowship. Any opinions expressed herein are those of the author and do not necessarily represent the views of the National Academy of Education and Spencer Foundation. The author acknowledges the assistance of John Moyer for collecting U.S. data and Yifang Mi for collecting Chinese data.

An Investigation of U.S. and Chinese Students' Mathematical Problem Posing and Problem Solving

ABSTRACT

This study explored 181 U.S. and 223 Chinese sixth-grade students' mathematical problem solving and problem posing. It is part of a continuous effort to examine U.S. and Chinese students' performance by conducting a cognitive analysis of student responses to mathematical problem-solving and problem-posing tasks. The findings of this study provided further evidence that Chinese students outperformed U.S. students on computation tasks, but there were many similarities and differences between U.S. and Chinese students in performing relatively novel tasks. Moreover, the findings of this study suggest a generality with respect to the direct link between mathematical problem solving and problem posing from an international perspective.

INTRODUCTION

Cross-national studies in mathematics have consistently reported that U.S. students did not perform as well as Asian students on tasks requiring the applications of mathematical knowledge and skills routinely learned in school[1] (e.g., Husen, 1967; Lapointe, Mead, & Askew, 1992; Robitaille & Garden, 1989; Stevenson, Lee, et al., 1990; Stevenson & Stigler, 1992; Stigler, Lee, & Stevenson, 1990, U.S. Department of Education and National Center for Education Statistics, 1996). However, recent cross-national studies have shown that for tasks assessing relatively novel and complex problem solving, the performance differences between U.S. and Asian students are not so pronounced (e.g., Becker, 1992; Cai, 1995; Cai & Silver, 1995; Silver, Leung, & Cai, 1995). For example, Cai (1995) conducted a cognitive analysis of a group of 250 U.S. and a comparable group of 425 Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. He found that Chinese students outperformed U.S. students on tasks involving computation and simple problem solving, but not on tasks involving complex problem solving. The detailed cognitive analysis of student responses to complex tasks revealed many subtle differences and similarities in their thinking and reasoning. For example, almost every strategy that was used by U.S. students was also used by Chinese students, and vice versa. U.S. students tended to use visual or pictorial representation more frequently than Chinese students, and Chinese students used symbolic or notational representation more frequently than U.S. students.

Silver et al. (1995) analyzed the responses of 206 Japanese fourth-grade students and 151 U.S. fourth-grade students to a task requiring multiple solutions of a single problem, in which a picture of a set of marbles arranged in a certain way was given to students and they were asked to figure out the number of marbles. They found that Japanese students performed better than U.S. students with respect to the proportions of correct solutions. However, many similarities in the solution strategies employed and the modes of explanation of the solution processes were noted. For example, both U.S. and Japanese students were able to solve the problem with multiple solution strategies.

---

[1] See Cai (1995) for a review of the mathematical performance differences and the contributing factors for the observed differences.

These few cross-national studies not only contributed to our understanding about students' problem solving in different nations, but also established the validity and feasibility for examining cognitive similarities and differences of students' mathematical performance. Yet, we are only beginning to reveal and understand the nature of these cognitive similarities and differences between U.S. and Asian students. Such examination is far more than enough to inform a broad spectrum of groups including classroom teachers, parents, educational policy makers, educational and psychological researchers, politicians, and public mediums, who are interested in the results of cross-national comparisons of mathematical performance (Bradburn, & Gilford, 1990; Cai, 1995). Continuous effort is needed to examine cognitive similarities and differences of U.S. and Chinese students' mathematical performance on a broad range of mathematical tasks. According to Bradburn and Gilford (1990), the information from such cognitive analyses can play an important role in educational research and policy development. Such analyses should be informative, guiding both the interpretation of cross-national performance differences and the policy recommendations that emerge from large-scale cross-national studies, such as the Third International Mathematics and Science Study (TIMSS).

This study was part of a continuous effort to examine U.S. and Chinese students' mathematical performance by conducting an analysis of student responses to mathematical problem-solving and problem-posing tasks. This study extends earlier work (Cai, 1995; Cai, & Silver, 1994; Cai & Silver, 1995) to focus on the examination of U.S. and Chinese students' mathematical thinking and reasoning, as related to complex problem solving and student-generated problem posing. The examination of U.S. and Chinese students' mathematical performance as related to complex problem solving and student-generated problem posing draws from their recognized importance in mathematics education (e.g., NCTM, 1989) and in cognitive psychology (e.g., Simon, 1989). Recent recommendations for the reform of school mathematics suggest an important role for problem solving and student-generated problem posing. For example, NCTM (1989) explicitly states that students should be given opportunities to solve mathematical problems using multiple solution strategies and to formulate and create their own problems from given situations.

This study also extended earlier work in mathematical problem posing (e.g., Silver & Cai, 1996) to examine the relatedness of mathematical problem solving and problem posing from an international perspective. Given the importance of problem-posing activities in school mathematics, some researchers have started to investigate various aspects of the problem-posing processes. One of the important investigations is to examine the link between problem posing and problem solving (e.g., Ellerton, 1986; Kilpatrick, 1987; Silver & Cai, 1996). Kilpatrick (1987) provided a theoretical argument that the quality of the problems subjects pose might serve as an index of how well they can solve problems. In addition to these theoretical arguments of the possible links between problem posing and problem solving, several researchers have conducted empirical studies to examine the link between problem posing and problem solving. For example, Ellerton (1986) compared the mathematical problems generated by eight high-ability young children with those generated by eight low-ability young children, by asking each to pose a mathematical problem that would be quite difficult for her or his friends to solve. Ellerton reported that the more able students posed problems that were more complex than those posed by less able students.

Silver and Cai (1996) analyzed the responses of more than 500 middle school students to a task asking them to pose three questions based on a driving situation. In particular, the students posed problems were analyzed according the types, solvability, and complexity of the problems they posed. Silver and Cai used eight open-ended tasks to measure the students' mathematical problem-solving performance. They found that students' problem-solving performance was highly correlated with their problem-posing performance. Compared to less successful problem solvers, good problem solvers generated more mathematical problems, and their problems were more mathematically complex.

As Silver and Cai (1996) indicated, understanding the depth and nuances of the relationship between problem posing and problem solving needs further scrutiny although these studies have provided evidence of a direct link between problem posing and problem solving. In previous investigations of mathematical problem solving and problem posing, students' problem-solving performance was measured using tasks which are rarely related to problem-posing tasks.

In this study, U.S. and Chinese students' performance on problem solving and problem posing are examined through related problem-posing and problem-solving tasks. Thus, this study not only provided an opportunity to examine cognitive similarities and differences between U.S. students and Chinese students in their complex problem solving and student-generated problem posing, but also provided an opportunity to examine if there is a generality with respect to the direct link between mathematical problem solving and problem posing from an international perspective.

METHODS

Subjects

A total of 181 U.S. sixth-grade students (87 boys and 94 girls) and 223 Chinese sixth-grade students (109 boys and 114 girls) participated in the study. The Chinese sample was from two average schools in Xiaoshan city (Zhejing province). The U.S. sample was from one private school (29 students) and four average public schools (152 students) in the Milwaukee metropolitan area.

Tasks and Administration

The tasks used in this study were from Cai & Silver (1994) with a minor modification. In particular, each student received a booklet containing the following tasks, shown in Figure 1: 1) four computational exercises, 2) a problem-posing task in which students were asked to pose mathematical problems based on a given figural pattern situation, 3) a division-with- remainder (DWR) story problem, and 4) a figural pattern problem.

Insert Figure 1 about here

The tasks were selected to reveal a range of students' mathematical performance, such as those connected with the computation skills in the computation exercises, the generative aspects of mathematical thinking in the problem-posing task, the interpretation of a solution in the DWR problem, and the generalization and the simultaneous coordination of two dimensions in the pattern problem. The division involving in one of the computational exercises ($11.28 \div 3.6 = ?$) is the same as that involving in the solution of the DWR problem, although the former involves the decimals and the latter involves the whole numbers. Such design allows for the comparisons of students' division skills when it is decontextualized and it is embedded in the solution of the DWR problem. The problem-posing task was designed in such a way that it has the same mathematical structure as the pattern problem, thus, such design of the problem-posing task allows for examination of the relatedness of student solution processes in solving the pattern problem and the problems they posed in the problem-posing task.

Students had 30 minutes to complete these tasks, 15 minutes for the four computation exercises and the problem-posing task and another 15 minutes for the DWR problem and the figural pattern problem. Students were asked to stop even if they had not finished the four computation exercises and the problem-posing task after 15 minutes. They were not allowed to change anything about their responses to the four computation exercises and the problem-posing task while they were working on the DWR problem and the figural pattern problem even if they finished them early. The tasks were administrated in such a way that students were unable to change their problem-posing responses after they had seen the figural pattern problem.

All data was collected via students' written responses. For computational exercises, only a final solution was required; for the DWR problem, students were asked both to provide a numerical answer and to write down their solution processes. For the problem-posing task, students were asked to write down problems they posed; for the pattern problem, students were asked to extend the pattern by drawing the fifth and seventh figures and describe how they knew what the seventh figure would look like. Although written protocols have some limitations when compared with verbal protocols collected from individual subjects, there are also some practical advantages of written protocols when conducting cross-national studies. Moreover, researchers (e.g., Becker, 1992; Cai, 1997) has been established the validity of using written responses of open-ended tasks as a means of assessing U.S. and Asian students' mathematical problem-solving and reasoning processes.

Data Coding

Each response for the computation exercises was coded as correct or incorrect. Student responses to the problem-posing task were coded along two dimensions. The coding scheme for analyzing student responses to the problem-posing task was based on prior research in solving pattern problems in general (e.g., Simon, 1979) and solving the figural pattern problem in specific (Cai, Magone, Wang, & Lane, 1996). In solving pattern problems, one needs to induce a rule based on given elements of a pattern, then to extend the pattern using the rule (Simon, 1979). Thus, the problems students posed in this study were first classified into extension problems, non-extension problems, or others. An extension problem refers to a problem questioning the pattern beyond the four given figures. A non-extension problem refers to a problem questioning the given figures in the pattern. Figure 2 shows examples of each type of problems.

Insert Figure 2 about here

In order to induce a rule and extend a pattern, one needs to *compare* the given elements in the pattern and find what is the same in the changes of the pattern (Simon, 1979). Moreover, in solving the figural pattern problems, students are not only asked to extend the pattern to find the fifth and seventh figures, but also asked to describe the rules in the figural pattern. Therefore, each posed problem was also classified into a factual problem, a comparative problem, or a rule-problem (See Figure 2 for examples).

Student responses for the DWR problem were coded using a classification scheme adapted from the one used by Silver et al. (1993) to code U.S. students' responses to a similar division problem. Each response was examined with respect to four distinct aspects: (1) solution processes, (2) execution of procedures, (3) numerical answers, and (4) explanations of the solutions. This classification scheme has been shown appropriate to code Chinese student responses to similar DWR problems (Cai & Silver, 1995). Student responses for the figural pattern problem were coded using a classification scheme adapted from the one used by Cai et al. (1996) to code U.S. students' responses to a similar pattern problem. Each response was examined with respect to three distinct aspects: (1) correctness of the figures and evidence of a description, (2) solution strategies, and (3) drawing errors.

RESULTS

The results are reported in five sections. The first section reports results for the computational exercises. The second section reports results for the DWR problem and the relatedness of students' performance on decontextualized and contextualized divisions. The third section provides an analysis of U.S. and Chinese students' problem-posing responses with respect to the types of problems that students generated. The fourth section presents a summary of U.S. and Chinese students' solutions to the figural pattern problem. Last section provides an examination of the relatedness of students' problem posing and their solving of the figural pattern problem.

Results for the Computation Exercises

Chinese students were quite successful in solving the computation exercises. In three of the exercises, more than 90% of the Chinese students had correct answers. For each of the four computation exercises, a significantly larger proportion of the Chinese than U.S. students had correct answer ($p < .01$). A majority of the Chinese students (70%) had correct answers for all four computational exercises, but only about 30% of the U.S. students had correct answers for all four computational exercises. The results for each exercise are shown in Table 1. Among the four exercises, the last one was the most difficult one for Chinese students. The common error for those Chinese students who had incorrect answers for the last exercise was due to the difficulty dealing with the decimal point. In fact, nearly 60% of these Chinese students who had an incorrect answer for this exercise provided either 31.33 or .3133 as their answers. However, the first computation exercise was the most difficult task for the U.S. students. In fact, only about one-third of the U.S. students obtained a correct answer for the first computation exercise. The common error for these U.S. students was their incorrectly adding numerators and denominators to find the sum of two fractions with unlike denominators.

Insert Table 1 about here

Results for the DWR Problem

Solution processes and execution of procedures. The correct solution of DWR problems requires not only correct execution of a division computation (computation phase) but also a correct interpretation of computational results with respect to a given story situation (sense-making phase). The majority of U.S. and Chinese students recognized the DWR problem as a problem which required a division computation. In particular, 89% of the U.S. students and 94% of the Chinese students selected division procedures. In addition to the division procedure, a few U.S. students used other appropriate procedures, such as repeated addition and repeated subtraction to solve the problem. No Chinese student used appropriate procedures other than division. A small proportion of U.S. (7%) and Chinese (2%) students used non-division procedures, such as multiplication, and these students used alternative procedures in inappropriate ways in attempting to solve the problem.

A larger percentage of Chinese students (87%) than U.S. students (66%) executed the procedures correctly ($z = 4.99$, $p < .01$). Students tended to give various numerical answers. The proportions of Chinese and U.S. students who gave an answer of 31 or 32 were similar. In fact, about 36% of the Chinese students provided an answer of 32; while 42% of the U.S. students gave an answer of 32. About 35% of the Chinese students gave an answer of 31 and 30% of the U.S. students gave an answer of 31. The rest of the U.S. and Chinese students provided 31 and the whole number remainder, 12 and the fractional remainder, or 12 and the decimal remainder as their answer. U.S. students appeared to be more comfortable expressing the remainder as a whole number; while Chinese students felt more comfortable expressing the remainder as a fraction or decimal. In fact, in approximately 20% of the Chinese student responses, the remainder actually appeared as a fraction or decimal. In contrast, in approximately 22% of the U.S. student responses, the remainder was a whole number.

Interpretations and sense-making. Although more than two-thirds of the U.S. and Chinese students performed an appropriate computational procedure correctly, only about 38% of the U.S. and 31% of the Chinese students provided appropriate and complete explanations for their answers. Examples of appropriate explanations accompanying a final answer of 32 included: "If you need 31 buses. These 12 people need one bus. 31 + 1 = 32." A few students provided appropriate explanations to support a final answer of 31: "If you have 31 buses, there are 12 people left over. You choose 12 buses to hold the 12 people, each bus holds extra 1 person, therefore, you just need 31 buses."

Over 25% of the Chinese students computed the result of the division computation and then directly rounded up or rounded down the computational result (31.3, $31\frac{1}{3}$, or 31 R12) to obtain an answer of 32 or 31 respectively; these students did not provide a justification for their decision to round up or down; only 3% of the U.S. students did this. Over 45% of the U.S. students did not provide any explicit explanation of their answers; while 27% of the Chinese students omitted an explanation. A small proportion of the U.S. and Chinese students provided explanations that were considered inappropriate. Rather than reasoning about the situation, these students explicitly applied a computational rounding rule to deal with the remainders obtained in their calculations. Typically, a student performed the division calculation to get a computational result like 31.33, and in their response they wrote "3 is less than 5, so I just take the remainder away."

In solving the DWR Problem, a student response was considered as correct if the student provided 32 with or without an appropriate interpretation or an answer other than 32 with an appropriate interpretation. Based on this criterion of correctness, it was found that about 46% of the U.S. and 41% of the Chinese students provided correct solutions for the DWR problem.

Decontextualized and contextualized division. Over two-thirds of the Chinese students obtained the correct answer for both the decontextualized and contextualized division problems, which is significantly higher than that for U.S. students, 52% ($z = 4.81$, $p < .01$). About 15% of the Chinese students performed the decontextualized division computation correctly, but failed to perform the contextualized division computation correctly; another 15% of the Chinese students performed the contextualized division computation correctly, but failed to perform the decontextualized division computation correctly. About 10% of the U.S. students performed the decontextualized division computation correctly, but failed to perform the contextualized division computation correctly; another 20% of the U.S. students performed the contextualized division computation correctly, but failed to perform the decontextualized division computation correctly. Only about 5% of the Chinese students failed to perform both correctly, but about 20% of the U.S. students failed to perform both correctly.

Results for the Problem-posing Task

In total, 181 U.S. students posed 861 problems (mean = 4.76) and 223 Chinese students posed 1588 problems (mean=7.12). In average, Chinese students posed significantly more problems than U.S. students ($t = 4.90$, $p < .001$). This mean difference is partially due to the fact that a larger percentage of U.S. than Chinese students did not generate any problems and a larger percentage of Chinese students generated 10 or more problems. In fact, about 15% (28 of 181) of the U.S. students did not generate any problems; while only 3% (7 of 223) of the Chinese students did not. Over 28% (63 of 223) of the Chinese students generated 10 or more problems; while only 16% (29 of 181) of the U.S. students generated 10 or more problems. Interestingly, the proportions of U.S. and Chinese students who generated 1 to 9 problems are very close.

Although Chinese students posed a significantly higher mean number of problems than U.S. students, proportions of the extension or non-extension problems for both samples were very similar. Table 2 shows the percentages of U.S. and Chinese students who posed extension or non-extension problems. Chinese students generated a total of 206 extension problems and U.S. students generated 105 extension problems. In particular, over 80% of the problems posed by U.S. and Chinese students are non-extension problems. About 12% (105 of 861) of the problems posed by U.S. students and 13% (206 of 1588) of the problems posed by Chinese students are extension problems. About 32% (72 of 223) of the Chinese students and 28% (51 of 181) of the U.S. students generated at least one extension problem. In particular, 14 Chinese and 19 U.S. students generated all extension problems.

Insert Table 2 about here

About 76% of the problems generated by the U.S. students and 68% of the problems generated by the Chinese students were comparative problems. The majority of their comparative problems involved the comparison of the number of dots in figures. For example, "How many more dots are there in the fourth figure than in the third figure?" Nearly 85% (151 of 181) of the U.S. students and 90% (198 of 223) of the Chinese students had at least one comparative problem. Although a majority of the problems generated by the U.S. and Chinese students were comparative problems, only about a quarter of U.S. students and one third of Chinese students' extension problems were comparative problems. About 40% of U.S. and Chinese students' extension problems were factual problems. For example, "How many dots are there in the 10th figure?" Another 35% of U.S. students and 30% of Chinese students' extension problems were rule problems. For example, "What is the rule in which each figure changes from the previous one?"

Results for the Figural Pattern Problem

Correctness of figures and drawing errors. Recall that students were asked to draw the fifth and seventh figures. A larger percentage of Chinese (85%) than U.S. students (61%) drew both the fifth and seventh figures correctly ($z = 5.49$, $p < .001$). Overall, 71 U.S. and 33 Chinese students drew either the fifth or seventh figure incorrectly. For those who drew either the fifth or seventh figure incorrectly, about 55% (40 of 71) of U.S. students and 60% (20 of 33) of the Chinese students drew both figures incorrectly. Over one third of the U.S. and Chinese students drew the fifth figure correctly, but failed to draw the seventh figure correctly. Only a few U.S. and Chinese students drew the seventh figure correctly but drew the fifth figure incorrectly.

Those who drew either the fifth or seventh figure incorrectly were included in the analysis of drawing errors. The majority of the drawing errors seemed to result from student difficulties in coordinating the two dimensions of the problem: the numbers of dots and the shapes of the figures. About 65% of U.S. (46 of 71) and 55% of Chinese (18 of 33) students' incorrectly-drawn figures correctly showed one of these two dimensions. For example, they correctly drew all 18 or 24 dots for the fifth or seventh figure respectively, but did not maintain the shape of the figures. About 35% of U.S. students and 45% of Chinese students' incorrect drawings showed serious errors in both the shape and in the number of dots.

Evidence of description and solution strategies. Over 90% of the U.S. and Chinese students provided descriptions for their solutions. In some cases, students' solution strategies were readily apparent from their descriptions. In other cases, no strategy was apparent because the students' descriptions were either incomplete or unclear. There were seven different solution strategies used by at least one U.S. or Chinese student in solving this figural pattern problem. Table 3 describes each of these solution strategies and shows the percentage distributions of U.S. and Chinese students who used these strategies.

Insert Table 3 about here

A larger percentage of Chinese (83%) than U.S. (69%) students had clear indications of using one of the seven identified solution strategies ($\underline{z} = 3.31$, $\underline{p} < .01$). However, the percentage distributions of using these strategies between U.S. and Chinese students are quite similar. For example, Strategy 4 was the most frequently (about one third) used strategy for both U.S. and Chinese students. The second most frequently used strategy was strategy 5, which was employed by about one sixth of both the U.S. and Chinese students. The strategy 6 is the least frequently used strategy for both samples.

It should be indicated that almost all U.S. and Chinese students who had correct fifth and seventh figures provided descriptions of how they obtained their correct figures. The majority of the U.S. and all Chinese students used written words to describe how they knew what the seventh figure would be. Only a small proportion of the U.S. students used both written words and pictures to describe their solutions.

Relatedness between Problem Posing and Problem Solving

Since the problem-posing task was designed according to the figural pattern problem and the mathematical structure involved in the problem-posing task and the figural pattern problem were the same, it affords an opportunity to examine the relatedness of student performance in problem-solving and problem-posing for two samples. To examine the relatedness of students' problem posing and problem solving, students in each sample were, first, divided into two groups according to their problem-posing responses: the extension group and the non-extension group. The extension group consisted of those students who generated at least one extension problem for the problem-posing task; the non-extension group consisted of those students who did not generate any extension problems. Then, student performance on the figural pattern problem between extension and non-extension groups were examined.

For the U.S. sample, the extension group consists of 51 students and the non-extension group consists of 130 students. For the Chinese sample, the extension group consists of 72 Chinese students and the non-extension group consists of 151 students. Table 4 shows the percentages of students with correct fifth and seventh figures as well as percentages of students with clear indication of using strategies by group (extension v.s. non-extension groups) and by nation (U.S. and China). For both U.S. and Chinese samples, students in the extension groups performed significantly better than those in the non-extension groups. In particular, a larger percentage of students in the extension group than in the non-extension group had correct fifth and seventh figures. Similarly, for both samples, a larger percentage of students in the extension group than in the non-extension group had clear indications of using solution strategies in solving the figural pattern problem. These differences of percentages are all statistically significant ($z = 1.96 - 3.97$, $p < .05$). Superior performance of the students in the extension group is also evident by the fact that for those 14 Chinese and 19 U.S. students who generated only extension problems, they all, except one Chinese and two U.S. students, drew both fifth and seventh figures correctly and provided a clear indication of using solution strategies.

Insert Table 4 about here

Although a significantly larger percentage of Chinese (85%) than U.S. students (61%) drew both the fifth and seventh figures correctly, the difference is not significant if the U.S. students in the non-extension group are excluded in the analysis. In fact, 84% of the U.S. students in the extension group drew both fifth and seventh figures correctly, which is almost identical to that of Chinese students (85%). Similarly, although a significantly larger percentage of Chinese (83%) than U.S. students (69%) provided clear indications of using solution strategies, the difference is not significant if the U.S. students in the non-extension group are excluded in the analysis. Indeed, 89% of the U.S. students in the extension group provided clear indications of using appropriate strategies.

Second, to examine the relatedness of students' problem posing and problem solving, students' solution strategies for solving the figural pattern problem and the problems they posed were examined. Recall that students used seven different strategies to solve the figural pattern problem. Students' uses of solution strategies appeared to be related to kinds of problems they posed. For example, of those students who used Strategy 5 (see Table 3), nearly 85% of U.S. and Chinese students' posed problems explicitly questioned the numbers of dots in figures, such as "How many dots are there in the 10th figure?" or "How many more dots are there in the fourth figure than in the third figure?" Of those who used strategies 1, 2, 3, 4, 6, and 7, only 48% of Chinese students' and 42% of U.S. students' posed problems explicitly questioned the number of dots in figures. Using the solution strategy 5, students focused on the total number of dots in each figure to describe how to get the next figures. The first figure has 6 dots, the second has 9, the third has 12, the fourth has 15, ⋯, therefore the seventh figure has 24 dots. In this case, the student did not mention the shape of each figure explicitly, and may actually have ignored the shape of the figures. Using solution strategies 1, 2, 3, 4, 6, and 7, students appeared to realize both the shape and number of dots in each figure. Thus, for both samples, students who mainly focused on the number of dots to solve the figural pattern problem appeared to generate problems explicitly involving the number of dots for the problem-posing task.

DISCUSSION

Similarities and Differences between U.S. and Chinese Students

Similar to what was reported in Cai (1995), results of this study revealed both similarities and differences between U.S. and Chinese students' mathematical problem solving and problem posing. Results of this study showed that Chinese students performed significantly better than U.S. students on four computation exercises. This was not a surprising finding since prior comparative studies involving U.S. and Chinese students (e.g., Cai, 1995; Lapointe et al., 1992; Stevenson et al., 1990) have consistently reported that Chinese students outperformed their U.S. counterparts, especially on computation tasks.

In solving the DWR problem, similar percentages of U.S. and Chinese students chose the correct procedures, but more Chinese students correctly executed these procedures. A similar percentage of U.S. and Chinese students provided appropriate interpretations of their solutions. Thus, the results suggest that Chinese students outperformed U.S. students on the computation phase of solving the DWR problem, but not on the sense-making phase. However, for both samples, students were more successful on the computation phase than on the sense-making phase. Thus, the results of this study suggest the cognitive complexities of the DWR problem for both U.S. and Chinese students. The difficulty of this type of problem derives not from computational requirements but rather from the sense-making requirement included in interpreting the computational result, as suggested in other studies (Cai & Silver, 1995; Silver et al., 1993).

Overall, Chinese students appeared to perform better than U.S. students in solving the figural pattern problem. In particular, a larger proportion of Chinese than U.S. students correctly extended the pattern to the fifth and seventh figures. In addition, a larger percentage of Chinese than U.S. students apparently used appropriate strategies. However, the kinds of drawing errors made by U.S. and Chinese students and the kinds of solution strategies used by U.S. and Chinese students were similar. For example, for both samples, the majority of the drawing errors seemed to result from student difficulties coordinating the two dimensions of the problem: the numbers of dots and the shapes of the figures.

This study extended previous cross-national studies by including the examination of U.S. and Chinese students' problem posing in mathematics. The results of this study showed that both U.S. and Chinese students were able to formulate mathematical problems based on a given situation. Chinese students generated more problems than U.S. students, but the proportions of each type of problem generated by U.S. and Chinese students were almost the same. For example, the proportions of the extension problems generated by both samples were very close. The proportions of the U.S. and Chinese students who generated at least one extension problem were also very close.

In particular, the results of this study showed that about one third of the U.S. and Chinese students were able to see mathematical structures and formulate problems questioning beyond the four given figures (i.e., extension problems). However, for both samples, the extension problems that students posed were only a relatively small proportion of the total problems they posed. In fact, only about one eighth of the problems posed by students in both samples were extension problems. As was reported before, the majority of the U.S. and Chinese students were able to correctly extend the pattern into the fifth and seventh figures. In particular, over 20% of the U.S. sample and 40% of the Chinese sample who correctly extended the pattern into the fifth and seventh figures did not generate any extension problems. Why were so many responses non-extension problems? Why did so many students generate only non-extension problems? In the problem-posing task, the first four figures of the pattern were given. It seems to be natural to pose a problem like: "What is the fifth figure?" or "What would the fifth figure look like?" However, only a few U.S. and Chinese students posed this problem.

One of the plausible interpretations of this finding is the novelty of the problem-posing activities for both U.S. and Chinese students. In the U.S., although the mathematics education community (e.g., Brown & Walter, 1988; NCTM, 1989, 1991; Silver, 1994) calls for integration of problem-posing activities into the school curriculum, problem-posing activities have rarely been implemented into classrooms. Chinese schools use national-unified mathematics textbooks which rarely include problem-posing activities. Problem-posing activities are also not often included in classroom instruction in China.

Links between Posing and Solving

This study also examined the relatedness of problem solving and problem posing from an international perspective. Similar to what was reported in other studies (e.g., Ellerton, 1986; Silver & Cai, 1996), the results of this study showed the direct link of problem posing and problem solving. In particular, this study appears to support the direct link between problem posing and problem solving from two aspects. First, the results of this study suggest that problems posed by students seemed to be related to their solution strategies for solving the figural pattern problem. For example, students who mainly focused on the number of dots to solve the figural pattern problem appeared to generate problems explicitly involving the number of dots for the problem-posing task.

Second, the results of this study suggest that students who generated extension problems tended to perform better on the figural pattern problem than those who did not generate extension problems. Not only have a larger percentage of the students who generated extension problems (extension group) than those who did not generate extension problems (non-extension group) correctly extended the pattern into the fifth and seventh figures, but also a larger percentage of the students in the extension group than those in the non-extension group had clear indications of using appropriate solution strategies. Those who generated all the extension problems performed almost perfectly in solving the figural pattern problem. The problem-posing task and the figural pattern problem contain the same figural pattern. Those who generated the extension problems for the problem-posing task might have mentally extended the figural pattern. The main requirement for solving the pattern problem is to extend the figural pattern, so students who generated extension problems for the problem-posing task had an advantage for solving the figural pattern problem since they have already mentally extended the figural pattern. Hence, it seems natural that students in the extension group performed better than those in the non-extension group in solving the pattern problem.

Furthermore, the findings of this study provided empirical support to the theoretical argument offered by Kilpatrick (1987) that the quality of the problems students pose might serve as an index of how well they can solve problems. The findings of this study provided empirical evidence and support the hypothesis that subjects pose only problems that they can solve, or they think they can solve. This hypothesis was particularly supported because students who generated extension problems tended to perform better than those who did not generate extension problems in solving the figural pattern problem.

Future Studies

The findings from this study and others (e.g., Becker, 1992; Cai, 1995) suggest both the complexity of examining mathematical performance differences and the complexity of interpreting the cross-national performance differences. Researchers have explored what cultural and educational factors might influence students' learning of mathematics. In cross-national studies, however, researchers have less frequently examined potential influences of classroom instruction on students' learning of mathematics. Future studies are needed to document and analyze U.S. and Chinese mathematics teachers' complex cognitive processes of their classroom instruction, then to examine how U.S. and Chinese teachers' cognitive behaviors are correlated with their students' thinking and reasoning. Fortunately, some researchers (e.g., Leinhardt, 1993; Shulman, 1986) have developed effective research paradigms and methods for the study of teachers' cognition and instruction. These research paradigms and methods could be integrated into such comparative studies of teachers' cognition and classroom instruction. Furthermore, the TIMSS videotape study of classroom instruction directed by Stigler (U.S. Department of Education and National Center for Education Statistics, 1996; Stigler, 1997) has provided value in examining classroom instruction to understand cross-national similarities and differences in the teaching and learning of mathematics.

REFERENCES

Becker, J. P. (Ed.) (1992). Report of U.S.-Japan cross-national research on students' problem solving behaviors. Columbus, OH: ERIC/SMEAC Clearing House (ED 351/204).

Bradburn, M. B. & Gilford, D. M. (1990). A framework and principles for international comparative studies in education. Washington, D. C.: National Academic Press.

Brown, S. I., & Walter, M. (1983). The art of problem posing. Philadelphia, PA: Franklin Institute Press.

Cai, J. (1995). A cognitive analysis of U.S. and Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. (Journal for Research in Mathematics Education monograph series 7), Reston, VA: National Council of Teachers of Mathematics.

Cai, J. (1997). Beyond computation and correctness: Contributions of open-ended tasks in examining U.S. and Chinese students' mathematical performance. Educational Measurement: Issues and Practice, 16(1), 5-11.

Cai, J. & Silver, E. A. (1994). A cognitive analysis of Chinese students' mathematical problem solving: An exploratory study. In D. Kirshner (Ed.), Proceedings of the Sixteenth Annual

Meeting of North American Chapter of the International Group for the Psychology of Mathematics Education (Vol. 2, pp. 3-9). , Baton Rouge, LA: Louisiana State University

Cai, J. & Silver, E. A. (1995). Solution processes and interpretations of solutions in solving a division-with-remainder story problem: Do Chinese and U.S. students have similar difficulties? Journal for Research in Mathematics Education, 26(5), 491-497.

Cai, J., Magone, M., Wang, N., & Lane, S. (1996). Assessment: Describing student performance qualitatively. Mathematics Teaching in the Middle School, 1 (10), 828-835.

Ellerton, N. F. (1986). Children's made-up mathematics problems: A new perspective on talented mathematicians. Educational Studies in Mathematics, 17, 267-271.

Husen, T. (1967). International study of achievement in mathematics: A comparison of twelve countries, Volume I & II. New York: John Wiley & Sons.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), Cognitive science and mathematics education (pp. 123-147). Hillsdale, NJ: Erlbaum.

Lapointe, A. E., Mead, N. A., & Askew, J. M. (1992). Learning mathematics. Princeton, NJ: Educational Testing Service.

Leinhardt, G. (1993), On teaching. In R. Glaser (Ed.), Advances in instructional psychology (Vol. 4, pp. 1-54). Hillsdale, NJ: Lawrence Erlbaum Associates.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics (1991). Professional standards for teaching mathematics. Reston, VA: Author.

Robitaille, D. F. & Garden, R. A. (1989). The IEA study of mathematics II: Contexts and outcomes of school mathematics. New York: Pergamon Press.

Shulman, L. S. (1986). Paradigms and research programs in the study of teaching: A contemporary perspective. In M. C. Wittrock (Ed.), Handbook of research on teaching (3rd ed., pp. 3-36). New York: Macmillan.

Silver, E. A. (1994). On mathematical problem posing. For the Learning of Mathematics, 14(1), 19-28.

Silver, E. A. & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. Journal for Research in Mathematics Education, 27 (5), 521-539.

Silver, E. A., Leung, S. S., & Cai, J. (1995). Generating multiple solutions for a problem: A comparison of the responses of U.S. and Japanese students. Educational Studies in Mathematics, 28(1), 35-54.

Simon, H. A. (1979). Models of Thought (Vol. I). New Haven: Yale University Press.

Stevenson, H. W. Lee, S., Chen, C., Lummis, M., Stigler, J. W., Liu, F., & Fang, G. (1990). Mathematics achievement of children in China and the United States. Child Development, 61, 1053-1066.

Stevenson, H. W., & Stigler, J. W. (1992). The learning gap. New York: Summit.

Stigler, J. W. (1997). Classroom mathematics instruction in Germany, Japan, and the United States: An introduction to the TIMSS videotape classroom study. Paper presented at the annual meeting of the American Educational Research Association; Chicago, IL; March 24-28, 1997.

Stigler J. W., Lee, S., & Stevenson, H. W. (1990). Mathematical knowledge of Japanese, Chinese, and American elementary school children. Reston, VA: NCTM.

U.S. Department of Education and National Center for Education Statistics (1996). Pursuing excellence, NCES 97-198, by Lois Peak. Washington DC: U.S. Government Printing Office.

Table 1

Percentages of Students with Correct Answer in Each of the Computation Exercises

Problems	$\frac{1}{5}+\frac{2}{3}$	$3480 \div 60$	$15.3 - 8.8$	$11.28 \div 3.6$
Chinese Students	95	93	94	83
U.S. Students	36	78	64	58
P-value	$p < .01$	$p < .01$	$p < .01$	$p < .01$

Table 2

Percentages of Students-Generated Problems by Category and by Nations

	U.S. (n=861)	China (n=1588)
Extension	12%	13%
Non-extension	83%	81%
Others	5%	6%

Table 3. Percentages of U.S. and Chinese Students' Solution Strategies and Their Descriptions

Strategy	Description	Percent of Students U.S. (n=181)	Percent of Students China (n=223)
Strategy 1:	Students focused on the number of dots in the three rows of each figure as a triplet and induced the rule of the triplets (Fig. 1: [1, 2, 3]; Fig. 2: [2, 3, 4]; Fig. 3: [3, 4, 5]; etc.)	5%	10%
Strategy 2:	Students looked at the dots in each row across the figures as a sequence and found the rule of each sequence (Row 1: {1,2,3,4, ⋯}; Row 2: {2,3,4,5, ⋯} Row 3: {3,4,5,6, ⋯}).	6%	12%
Strategy 3:	Students looked at the figures diagonally and realized each successive figure has one more diagonal of 3 dots. By adding one more diagonal column to the previous figure, then students would get the next figure.	6%	13%
Strategy 4:	Students realized that, from figure to figure, each row has one more dot than the corresponding row in the previous figure. From the fourth figure, the student added one dot to each row individually to get the fifth figure.	31%	26%
Strategy 5:	Students focused on the total number of dots in each figure to describe how to get the next figure. The first figure has 6 dots, the second has 9, the third has 12, the fourth has 15, ⋯, therefore the seventh figure has 24 dots. In this case, the student did not mention the shape of each figure explicitly, and may actually have ignored the shape of the figures.	17%	14%
Strategy 6:	Students removed the first row of the previous figure, then added a new bottom row that has one more dot than the previous bottom row, this would be the next figure.	1%	2%
Strategy 7:	Students found that the number of dots in the first row of a figure was equal to the number of each figure. The number of dots on the second row was one more than the first row, and the number of dots on the third row was one more than the second row.	3%	6%
No Clear Indication of Using a Strategy		31%	17%

Table 4

Percentages of Students with Correct Fifth and Seventh Figures as well as Percentages of Students with Clear Indications of Using Strategies by Group and by Nation

		Correct 5th and 7th Figures	Clear Indication of Using Strategies
U.S. (n=181)	Extension (n=51)	84	89
	Non-extension (n=130)	52	61
China (n=223)	Extension (n=72)	92	91
	Non-extension (n=151)	82	79

Figure 1

<u>Tasks</u>

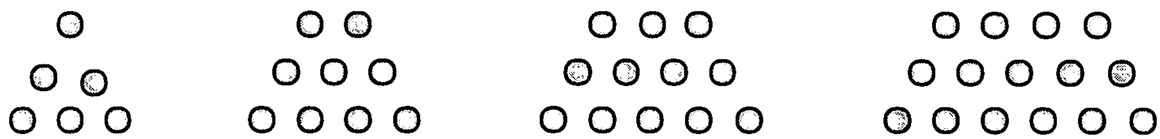
Computation exercise:

$\frac{1}{5} + \frac{2}{3} = ?$ $3480 \div 60 = ?$

$15.3 - 8.8 = ?$ $11.28 \div 3.6 = ?$

Problem-posing Task:

Mr. Miller drew four of the figures in a pattern, as shown below.



(1) (2) (3) (4)

For his students' homework, he wanted to make up some problems according to this pattern. Help Mr. Miller by writing as many problems as you can in the space below.

DWR Problem:

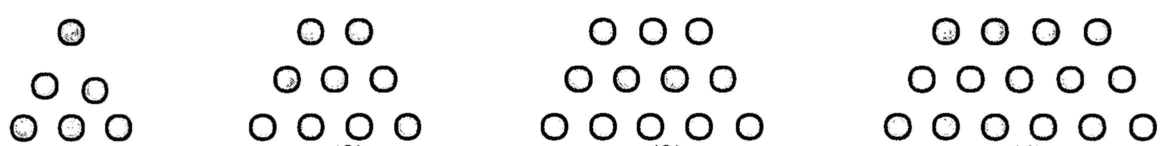
Students and teachers at Marquette Middle school will go by bus to have Spring sightseeing. There is a total of 1128 students and teachers. Each bus holds 36 people. How many buses are needed?

Answer: ____________________

Show all your solution processes below.

Pattern Problem:

Look at the pattern below.



(1) (2) (3) (4)

A. Draw the fifth figure.
B. Draw the seventh figure.
C. Describe how you knew what the 7th figure would look like.

Figure 2

A List of Sample Student Responses

Extension Problems

What is the 10th figure? [factual]
How many dots are there in the 100th figure? [factual]
What is the rule in which each figure changes from the previous one? [rule-problem]
How many more dots are there in the 100th figure than in the 99th figure? [comparative]

Non-extension Problems

How many dots are there in the second figure? [factual]
How many more dots are there in the 2nd row than in the 1st row in the 3rd figure? [comparative]
What is the shape of the fourth figure? [factual]
What is the total number of dots in all four figures? [factual]
How many more dots are there in the fourth figure than in the first figure? [comparative]

Other Responses
Which Are Not Mathematical or Irrelevant to the Given Pattern Situation

Why did Mr. Miller want to assign homework?
How many new students are there in Mr. Miller's class?
What materials were made up the each circle?
What are the least common multiples?

SE060598

ERIC

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
Educational Resources Information Center (ERIC)

REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: An Investigation of U.S. & Chinese students' Mathematical Problem Posing & Problem Solving

Author(s): Jinfa Cai

Corporate Source: Presented at 1997 AERA

Publication Date: April, 1997

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic/optical media, and sold through the ERIC Document Reproduction Service (EDRS) or other ERIC vendors. Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following two options and sign at the bottom of the page.

☒ Check here
For Level 1 Release:
Permitting reproduction in microfiche (4" x 6" film) or other ERIC archival media (e.g., electronic or optical) *and* paper copy.

The sample sticker shown below will be affixed to all Level 1 documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

Sample

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

Level 1

The sample sticker shown below will be affixed to all Level 2 documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN OTHER THAN PAPER COPY HAS BEEN GRANTED BY

Sample

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

Level 2

☐ Check here
For Level 2 Release:
Permitting reproduction in microfiche (4" x 6" film) or other ERIC archival media (e.g., electronic or optical), but *not* in paper copy.

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but neither box is checked, documents will be processed at Level 1.

"I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic/optical media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries."

Sign here→ please

Signature: [signature]	Printed Name/Position/Title: JINFA CAI / Assistant professor	
Organization/Address: Dept. of Math Sciences University of Delaware Newark, DE 19716	Telephone: 302-831-1879	FAX: 302-831-4511
	E-Mail Address: JCAI@math.udel.edu	Date: July 1, 1997