DOCUMENT RESUME

ED 412 084 SE 060 596

AUTHOR	Williams, J. S.; Linchevski, L.
TITLE	Situated Intuitions, Concrete Manipulations and the Construction of the Integers: Comparing Two Experiments.
PUB DATE	1997-04-00
NOTE	25p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, April, 1997).
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Cognitive Processes; Comprehension; *Constructivism (Learning); Educational Games; Elementary Education; Foreign Countries; Integers; *Intuition; Learning Strategies; Mathematical Concepts; Mathematics Instruction; Problem Solving; Teaching Methods
IDENTIFIERS	Abacuses; Israel; United Kingdom

ABSTRACT

This paper further develops an instructional method called here "process-object linking and embedding". The idea is to link the familiar mathematical processes to objects in a familiar situation, then re-embed the new link through mathematical symbols into their mathematical construction. It makes use of children's extra-mathematical, ethnomathematical, or everyday knowledge to link situations to and "unpack" processes in the mathematics with which they are already confident. It is designed as a means of overcoming the problem of intuitive gaps recurring in children's mathematical development. This process is illustrated here in the case of two attempts to teach integers. It is based on the principle that intuitions arising outside mathematical experience can be imported into mathematics: strategies and concepts arising in the extra-mathematical situation can be modeled with the aid of tools and representational devices. The design of appropriate activity does need to address the whole social situation in which the child's intuitions arise. Here we find a method in which the intuition for "fairness" is carried into a classroom game in which integers are modeled particularly effectively. Contains 41 references. (PVD)

Situated Intuitions, Concrete Manipulations and the Construction of the Integers: Comparing Two Experiments

by

J.S. Williams
L. Linchevski

Situated Intuitions, concrete manipulations and the construction of the integers: comparing two experiments.

Williams,J.S. (presenting),University of Manchester,
Linchevski,L. Hebrew University of Jerusalem.

Abstract

This paper develops further our instructional method called here "process-object linking and embedding". It is designed as a means of overcoming the problem of intuitive gaps recurring in children's mathematical development, and is illustrated here in the case of two attempts to teach integers. It is based on the principle that intuitions arising outside mathematical experience can be imported into mathematics: strategies and concepts arising in the extra-mathematical situation can be modelled with the aid of tools and representational devices. The design of appropriate activity does need to pay attention to the whole social situation in which the child's intuitions arise. Here we find a method in which the intuition for 'fairness' is carried into a classroom game in which integers are modelled particularly effectively.

Introduction and theoretical background

Recently, work on situated cognition (Rogoff and Lave, 1988, Lave and Wenger, 1991, Lave, 1996) has been developed and related to teaching and learning in classrooms. Various attempts to adapt processes of apprenticeship and peripheral participation to classrooms have been developed or discussed (e.g. Brown et al, 1989, Collins, 1994 and The Cognition and Technology Group at Vanderbilt, 1994). We agree broadly with the critique of Heckman and Weissglass (1994). They say that many examples of curriculum innovation which appeal to a situated learning approach have only a little 'authenticity'. They see authenticity in the sense one would expect from Lave and Wenger's (1991) case studies, i.e. authentic learning is acquired by the individual engaging in a community of practice. A feature of the cases studied, especially daily life and vocational activity, is that social goals and purposes tend to subsume learning goals: most learning is picked up by the way, incidentally, from old hands and through experience, with the occasional conscious guiding (see Billett, 1994, Wood, 1995 and Young, 1993).

The classroom situation is quite different to vocational and everyday learning contexts in which the learning goals are implicit within the process, even emergent. To be sure, all learning is situated, and so the process of legitimate peripheral participation is relevant in classrooms: to the induction of children, and new teachers, into classroom practices. Such a perspective should reveal a lot about the hidden curriculum, peer group influences, the incommensurability of school and everyday maths and the 'acquisition of children' by disabilities and disaffection (see a number of papers in Chaiklin and Lave, 1993). But this implicit learning is not the overt purpose of education; indeed it is often quite opposed to the learning goals the educational institution believe it is supposed to promote.

In our view such analyses lead logically to a critical approach to education in school, to a 'critical mathematics education' (Engestrom, 1994), in which the whole social system which contains the classroom is brought into the analysis and into question. This suggests the critical examination of the classroom within the wider system, its institutions and culture. Indeed elsewhere we propose work of this kind, especially in pre-vocational education. The notion of turning the classroom into a community of inquiry, as for instance proposed by the Children's Philosophy Movement is an apparently coherent and even viable one (see Splitter and Sharp, 1995, Lipman et al, 1980, and Williams, 1997).

But in this paper we argue that it is possible to achieve **some** progress through a partial approach, which attempts to make use of 'authentic' situations familiar in daily life activity, whose sense gives rise to transferable intuitions with which children can build mathematics. And we compare the authenticity of two situations used in our instructional method.

We identify some key points in children's construction of mathematical knowledge in school in which they must extend their conceptual structures, such as the operations on the integers and the flexible partitioning of fractional quantities (Semadeni, 1984). Sfard (1991) includes these in a collection of such gaps which cause problems in mathematical development, where operational conceptions must give rise to structural conceptions, and processes must be mentally reified as objects. She describes the vicious circle which frustrates this development: one must handle the processes as if they were objects first, possibly instrumentally in Skemp's (1978) sense, in order to mentally reify them.

We propose an instructional method which we will call "process-object linking and embedding" (POLE). The idea is to link the familiar mathematical processes to objects in a familiar situation, then re-embed the new link through mathematical symbols into their mathematical construction.

It makes use of the children's extra-mathematical, ethnomathematical or every-day knowledge to link situations to and 'unpack' processes in the mathematics with which they are already confident. The intuitive development of strategies in the situation then involves representation of these processes literally as "objects"in the situation. Through the activities the children are brought to mathematise, and especially encouraged to use the mathematical signs which facilitate transfer into the mathematical 'voice'. Finally the children's activity becomes mathematical, and we speak of the children reifying the processes through their learning to use the new symbols in flexible ways, proceptually, (Gray and Tall, 1994).

The teaching method involves drawing attention to intuitions and strategies which arise outside mathematics to fill the gap, and so extend their conceptions or to build the necessary mathematical knowledge. This method draws on the phenomenological approach of Freudenthal (1983) and Treffers (1987). But we expect the situation to provide richer access to common sense strategies if it recalls and mentally replays an experienced situation. The recall and simulation of the situation is 'imported' from the child's everyday culture into school.

Literature on integers

Negative numbers usually demand an algebraic frame of reference for the first time. While counting numbers are constructed by abstraction from real objects and quantities, and operations performed on them are related to concrete manipulations, operations on negative numbers and the properties of these numbers are usually given meaning through formal mathematical reasoning. Moreover, some of these properties contradict intuitions that have been developed in constructing the counting numbers, (for example, you can't get something from nothing!). Over the years this situation has led people in the mathematical community to one of two positions.

One alternative has been to completely avoid any attempt to give practical meaning to the negative numbers, and to recommend treating them formally from the outset (Fischbein, 1987; Freudenthal, 1973). The other alternative is to look for an embodiment, a 'model' that will satisfy the need for providing a practical intuitive meaning to negative numbers, arithmetical operations on them, and the relations between them (e.g. Thompson and Dreyfus, 1988; Peled, Munkhopadhyay and Resnick, 1989; Munkhopadhyay, Resnick and Schauble, 1990; Liebeck, 1990; Janvier, 1985).

Fischbein (1987) argues against the use of the existing models for negative numbers. They do not satisfy the criteria of 'comprehensiveness', 'obviousness' and 'correctness'. Moreover, the very definition of the negative numbers makes it impossible for there to be such a model, because these objects cannot be described directly and realistically. Their existence and the relations among them can only be deduced formally. Fischbein therefore concludes that the topic of negative numbers should be taught only when the students are ready to cope with intra-mathematical considerations and justifications, using 'at least' the inductive-extrapolation method (Freudenthal, 1973, p. 281).

Let us now consider the three requirements suggested by Fischbein: comprehensiveness, correctness and obviousness. The requirement that a single model should satisfy the need for comprehensiveness in teaching a mathematical concept is practically impossible to fulfil. Rejecting models because they are only partial would lead to rejection of all the existing models in mathematics education, since by definition every model has aspects that are not in the concept and vice versa (Ost, 1987).

The requirement of correctness in models is especially interesting. Resnick and Ford (1981) also claims that the main purpose of a model is to create a mental image of 'goodness' and 'correctness' for the system of concepts being learned. According to these views the purpose of a model is not merely to provide a well-defined interpretation for a mathematical theory but also to give the theory or concept a 'correct representation'. This cannot possibly be fulfilled. Every mathematical theory has or can be given alternative models that provide the user with different images of the concepts in the theory and the relations among these concepts. Fischbein's requirement of correctness stems from the fact that the new concepts being acquired are often extensions of existing concepts (Semadeni, 1984). Therefore the proposed model must preserve the intuitions and schemes that were constructed in the narrower frame and transfer them to the extension. When this condition is satisfied, the person using the model has a feeling of 'correctness'; if it is not satisfied, the person has a feeling of 'fabrication' or 'obscurity'.

Inherent in the 'obviousness' criterion is the requirement to avoid artificial conventions that would make a model seem detached from reality. Moreover, in order for the model to fulfil its cognitive function it must describe a reality that is meaningful to the student, in which the extended world (for example, the world which contains negative numbers) already exists and our mathematical activities allow us to discover it (e.g. Vinner, 1975). In the specific case of negative numbers this world must include the practical need for two sorts of numbers. It is also necessary to present situations in this world in which the relevant laws can be deduced without 'mental acrobatics' (Janvier, 1985), and without inducing a feeling of contradiction with known truths.

Experiment 1: review

In Linchevski and Williams (1996), we described an experiment in teaching the negative integers to sixth-grade students, with an attempt to fulfil the third of Fischbein's (1987) criteria, that of 'obviousness' for addition and subtraction of integers. The construction of the integers essentially involves the construction of an equivalence class of pairs of natural numbers, involving a recognition of the 'sameness' of a class of pairs such as {(5,0), (6,1), (7,2)...} and the attachment of some label or sign, eventually this will of course be +5.

We wanted this to be constructed intuitively. Thus, the 'procept' (Gray and Tall, 1994), for the integer will attach itself to an action-in-situation (which holds some meaning and can evoke intuition), a representation on an abacus (which can be manipulated independently) and some label, initially just a verbalisation "5 more in", but which in a later episode becomes the formal mathematical symbol, "plus 5".

Our teaching followed the approach of Diriks (1984) and others using the double abacus. It was based on a model in which the neutralisation of equal amounts of opposites allows every integer to have many physical representations (Lytle, 1994).

We presented the children with a dancers-game , a simulation-game in which the children represent the processes of 'entering and leaving' on an abacus, and so represent these processes as objects (beads on the double abacus) in their activity before they must do so mentally through the symbols. The game is played with cards (initially blue and yellow, later these become plus and minus: + and -) which represent dancers coming and going through the disco gates. Each child records the traffic at their gate, and is periodically required to report the status at their gate, (such as "4 more out", or just "4 out") and combine all the results to see if too many dancers have entered. Strategies for dealing with the abacus when it fills up include those described below as 'cancellation' and 'compensation'. These form the intuitive basis for the operations on the abacus needed later. A notable result is that cancellation (and un-cancellation) arises more often and apparently more naturally than compensation, and forms the basis for abacus manipulations which "go through zero", such as +3 take away +6. The compensation strategy, in which -6 is added, instead of taking away +6, rarely arises.

The children are later asked to check occasionally if the tallying has developed correctly by "taking away the cards"from their abacus. This is the intuitive root of subtraction, and so we can say that subtraction is introduced in the situation as an inverse, but when carried out on the abacus it is a concrete extension of the 'take-away' schema. They take away yellow beads from the yellow pile and blue from the blue, where necessary 'uncancelling', i.e. adding the same number of beads to both wires of the abacus.

They then play with cards which have signs on them +3, -4 etc., instead of colours. They model the recorded value on an equivalent abacus as an integer, represent them in symbols and record the action as a series of sums. The mathematical extension is then more or less complete. But the essential point is that they develop some intuitive sense of the processes and objects as well, and can translate to some extent back to the abacus and situation from the symbols.

Assessment of the success of this teaching (see Linchevski and Williams, 1996) will be clearer in the conclusion of this paper when we compare the two experiments. In general the children who completed the sequence of instruction were able to perform symbolic calculations of addition and subtraction with few errors. In one case out of six the child made 4 errors out of 20, which she was able to correct with the aid of the abacus. Their calculations in some cases used the abacus and in others not, but all their explanations in response to questioning they appealed to the abacus rather than the disco situation.

The degree of obviousness consequently depended on the actual calculation: -8 take away -3 is 'obvious' because you take the three minuses (yellows) away from the eight minuses (yellows) and are left with 5 minuses. But when the calculation "goes through zero"the explanation is indirect, so +3 take away +8 is less obvious. You have to see that one abacus representing +3 could be 8 pluses (blues) and 5 minuses (yellows), say, so you can then take the 8 blues away and you are left with the 5 yellows, minus 5. This is less obvious: they see it as a calculation to perform rather than an instantaneously obvious result.

When asked to justify calculations "in the disco"the children were able to do this for additions, but not for subtractions. So adding +3 and -5 is intuitive in the situation. But the inverse involved in the subtraction makes this indirectly formal, the children at this stage did not cope with this. On the other hand the situation has made the integers themselves acceptable as both processes and relations or ordered pairs (-2, the process of 2 going out, and the comparison of 2 fewer after than before), and it has justified for the children the abacus manipulations they will require to "go through zero", and the equivalence class of abacuses which might allow them to select a convenient representation for an integer.

Methodology

We described the first study in Linchevski and Williams, (1996) as a series of teaching episodes with small groups of year 6 children who are new to integer operations as such (in contrast to those studied in Shiu, 1981 and Shiu and Bell, (1981). We acted as their teachers, but presented ourselves to the children as researchers hoping to improve the teaching of others. The early sessions were developmental, and we refined the design of the games and of the lessons in the light of responses from children. We were particularly interested to identify opportunities and the obstacles the children meet, the role of the abacus and the situation in their discussions and arguments, and their constructions and suggestions.

All the lessons were videotaped for later analysis, with interesting and illustrative moments transcribed for study.

When the series of games/lessons was more or less defined, two groups of children were taken through the whole process and were assessed at the end as to their understanding of integer addition and subtraction, through a written test and an interview where they were, in groups, asked to explain their calculations in rotation.

The second experiment detailed here involved a similar procedure. The teaching series was developed and two groups in the UK were studied, and a further two groups were later studied in Israel.

The method is designed to study the teaching method itself through the reactions of children engaged in classroom interaction. For technical reasons of data collection we use small groups, though we have checked that the games can be played in large classes. It is therefore of interest to us to record how children respond, discuss, explain and solve the problems presented in the game. We identify the obstacles and opportunities.

It is not our intention to test a specific method of instruction in the classic 'teaching experiment' sense of treatment versus control groups, and the interpretation of the children's behaviour. Nor do we generalise about how other children will respond to the teaching approach: we expect major variations and indeed observed major variations in our few groups.

It is therefore a method which has more in common with the constructivist teaching experiment (Cobb and Steffe, 1983) than the classic teaching experiment. But with this difference, we are content to deal with the group, record its interaction and mostly assess the individual through the group interaction. This is because we are more concerned here with the relation between the instruction and the groups dynamic than we are with determining specific learning paths of the individual.

Analysis of experiment 2: the dice games.

We describe in some detail the findings of our next teaching experiment with a double abacus but a new situation involving children recording team points scored on the throw of dice, and in which children spontaneously develop the 'compensation strategy' in which points are added to one team rather than subtracted from the other. This is represented and formalised as an intuitive basis for subtracting integers. The abacus facilitates the transfer of the compensation strategy from the situation of point-scoring to the mathematics.

Game 1: A pair of dice (say yellow and blue) is thrown alternately by two teams (the blue and the yellow team, corresponding to the two colours for the abacus beads, and the plus and minus numbers respectively, later on). The scores for the teams are decided by the scores on the dice, and recorded by each team on a double abacus (containing blue and yellow beads). The winning team is the first to get 8 (or more) ahead of the other. The way they record on the abacus is up to the children to discuss: anything goes as long as it is fair to each team. The children may be ready for the next game when they are cancelling the two dice.

Game 2: This time on each turn the yellow and blue dice are thrown as before (and they are expected to cancel the two to a single score for yellows or blues) but there is a third die (labelled add or subtract) which is used to decide whether to add or subtract the given amount. Thus (3b, 2y, sub) means subtract one blue, and 2b, 5y, add means add 3 yellows. Otherwise the scoring takes place as before. In case of confusion the children can be encouraged to throw the two coloured dice first, calculate the result as say, 2 blues, or five yellows and so on. Then, after a moments delay, they can throw the die which is used to decide to add or subtract them. As before we expect to see cancellation and compensation strategies, and we expect these to be discussed and mastered.

Game 3. We return to the first game and ask how this could be played with only one die. Suggestions may include many interesting games. The one we want to follow up uses a single die with +3,+2,+1, -1,-2 and -3 on the faces. These are interpreted from the point of view of the blues. +3 means 3 for the blues, -3 means 3 for the yellows. The blues win if they get to +8, and the yellows if they get to -8. This game involves consolidating the strategies developed with the blue and yellow dice, but using them with the signed integers.

Game 4. The abacus begins with an equal number of beads on each wire (not empty, usually 9). Now we return to the second game, but with two dice: the add-subtract die and the signed integer die, (faces: -3, -2, -1, +1, +2, +3). The children are, after a time, asked to record games and check them, with the right hand integer column referring to the state of the abacus:

E.g.	add (+3)	+3
	sub (-2)	+5
	add (-1)	+4
	sub(-3)	+7
	add(+1)	+8 game ends.

This can lead to formal sums such as +3 sub (-2) = +5, and then +3 + (-2) = +5.

Analysis 1: the children develop and negotiate strategies: cancellation, compensation and equivalence.

First we make some general comments and explain some of the strategies identified in the first game, then we analyse an illustrative section of transcript, then we discuss its generalisation and the comparison with the first experiment.

We see the use of 'fairness' by the two teams as the embodiment of 'equivalence' in the situation, which will in time become abstracted as a mathematical concept or principle. There are a number of elements of this concept of equivalence, and it is used to justify intuitively, (and then, when challenged by the group, explicitly) a number of action-schemata. They implicitly use 'comparison', 'cancellation' and 'compensation' in the game, with varying degrees of obviousness and social negotiation.

Comparison means considering only the difference between the blue and yellow values, e.g. the columns of beads on the abacus. This usually happens first on the abacuses, and then on the dice, where it becomes cancellation.

Cancellation happens most naturally on the dice: (5,1) is 'equivalent' to 4 for the blues. But in some cases they first notice when they transfer (3,3) to the abacus that "its the same"and only then do they look at cancellation of the dice. Sometimes the cancellation on the dice only arises, or sticks, when the team are pushed because they are short of beads on the abacus to score with, then one strategy is to use fewer beads by cancelling the dice.

(Note: Cancellation of the dice is a pre-condition in our sequence for moving to game two, and one group of children was stuck at this stage for so long we never completed the sequence with them.)

Compensation, e.g. taking from the blues instead of giving to the yellows comes easily in the game to most groups when they need it, and in the group below straight away, because one boy sees the need to avoid running out of beads from the start.

Un-cancellation, or adding to both sides of the abacus generally developed later in this second experiment, whereas in the first we had explicitly encouraged this part of the concept of equivalence through our questioning. We saw it in the first experiment as a necessary part of the development of conservation of the cardinality of the integer, and as an important pre-requisite for the management of subtraction later. But in this case the use of compensation usually obviated the need for this. Here is a possibly general notion. The situation allows certain schema which may be conceptually justified to drop because they are not needed: a kind of local version of a famous historical-cultural principle.

(Finally we observe some schema for combining two abacuses. It seems however that they can manage the combination of two abacuses simply using natural numbers.)

The script: the game has been going on for five rounds. The four children, Tom, Keren, Lior and Adi, compare the two columns of beads on an abacus, counting the difference and co-ordinating the two abacuses to check their team's scores. This is absolutely natural 'in the situation'. The column comparison, and combination of the two abacus-scores is not even discussed. They have just thrown 6 blue, 5 yellow, we code this as (6,5):

Tom: Put one blue over (mumbling), so it won't finish.. (he is motivated by the coming problem of running out of beads, which never occurs)

Teacher: Tom has a suggestion to put only one blue

Keren: 5,6 its exactly the same

Lior pushes one blue bead to the front (cancellation of dice)

Teacher: Do you all agree?

The teacher here is sharing the strategy already agreed by three of the children: they all agree. Then Lior takes some blues and yellows and pushes them to the back of the abacus, thereby cancelling the abacus but maintaining the difference. The group are puzzled, it is not so obvious to them.

Tom : You do it so you won't run out of beads.

Tom: 1,5.

Adi moves 5 blues and one yellow, she has enough beads and does not cancel.

Lior: You are 2 ahead of us.

Teacher: (to Adi): You moved all the yellows and all the blues ?

Adi: I prefer it...

The teacher is checking to see if she understood cancellation. She knows but doesn't use, it seems more playful perhaps to move more beads over if you can. Soon Keren throws the dice and:

Lior: We can have 5 yellows, or we take away from the blues instead of adding to the yellows.

Here there is an explicit statement of compensation, allowing the intuitive idea to be verbalised, questioned: i.e. the subject of reflection. They decide to take away the blues.

Lior then has a suggestion: since the number of points for the teams are exactly even, he says why not push the beads back on the two abacuses so that they are both level. The group rejects this idea, though it is not clear why.

Adi: OK, you can say there is a difference of 4.

Lior: So take away.

Adi: There is a difference of 4.

Lior: So take it away it will be even, one point and one point.

Teacher: Let her do it the way she wants to do it.

Adi: Now I added one, (5, 3) , we are one point ahead.

Lior: Why one?

Adi: Because here I have 5 and here I have 4. (Comparison of the columns on the abacus.)

Tom's turn: before he throws he pushes one blue and one yellow to the front of the abacus, demonstrating equivalence, not just cancellation! Adi disagrees, probably she understands comparison, maybe also cancellation, but does not yet accept equivalence as such.

Adi: I don't agree, what are you doing?

Tom: So what, it doesn't make any difference.

Teacher: Why did you move the beads?

Tom: To make it higher.

On Keren's turn, she throws the dice and pushes back one yellow (compensation). Then Adi throws 5 blues and one yellow. Lior wants her to take away 4 yellows (cancellation and compensation), though there are enough beads so compensation is not strictly necessary.

Adi: But why, its 1, 5, .. Oh yes ...you are one ahead of us...

Adi appears to change her mind about accepting the compensation, goes back to the original position (there are enough blues so she doesn't have to compensate) but finally everyone is confused, and she goes back to what Lior wanted. Tom's turn, now Lior again instructs:

Lior: Take away one from them...

Tom: OK I'll take one

Lior takes away one,..

Adi throws the dice, she gets (5,3). Here she thinks she has the idea we call 'compensation'.

Adi: So I am taking 2 from the yellows (she nods to herself) so now I've got it.

Lior uses the two wires: he takes blues away and pushes yellows to the front (mixture of compensation and straight-forward adding).
Tom's turn; he is compensating on his abacus, but suggests "Can we add to their abacus instead of to ours?"Teacher says Yes. Then Adi has her turn, points to the other abacus and asks Keren (who is across the table with the other abacus) to "add one"to hers.

Lior tries to explain to Tom (his team mate) how they can benefit from using both abacuses to create a situation in which one abacus records only blues while the other records only yellows. "Each time I'll take from those (points to the yellows) and you take from those (the blues)"This idea gets dropped and, in fact, the group later abandons the second abacus as unnecessary.

Lior: I add 3 to us and take 1 from them.

Tom: You could have added 4 to us.

Lior: I know, but I don't want them to have anything on their wire...

There is a degree of play in their manipulation of the cancellation and compensation: there is more fun in the mathematics than in the game itself by now, and its time to go on to the next game, which poses more of a challenge.

The development of a number of strategies are seen here interweaving in the game. The children explain to each other "it doesn't make any difference", implicitly its fair to both sides, and this justifies the strategies the children develop. There is some explicit formulation of strategies, and some clear questioning by Adi, who then acquires a new strategy.

But it doesn't matter too much if she doesn't fully understand a strategy at a certain point, as long as she doesn't think it is unfair. The need to proceed with the game, and the 'right' of the player to select their own strategy when they have their own move, seems to encourage the children to be comfortable enough to operate instrumentally and to observe each other's moves, with perhaps the insight, "I've got it now", coming later.

Analysis 2: The introduction of symbols , the translations of the strategies and intuitions into mathematics: children speaking with two 'voices'.

In game 3 the pair of dice are replaced by a single dice with the signed integers, -1,-2,-3, +1,+2,+3 on it. The teacher establishes that the plus indicates points for the blue team and the minus for the yellow. This move to the use of + and - symbols on the dice introduces a 'mathematical voice' into the activity (Wertsch, 1991). In many cases the use of the signs is accepted as merely a formal sign for blue and yellow, the children are able to accept this for the sake of playing the game. The problem of connecting the sign with addition or subtraction of the scores then is delayed to a later point, after the game has been established and the playing has proceeded formally.

In other groups the meaning is discussed first and there is an immediate crisis, an apparent regression before the children can proceed. This involves the co-ordination of the use of the sign for the colour with the mathematical meaning of the sign as adding or subtracting (which in this context has to be to the blue score, with the opposite meaning for the yellows!)

Teacher: We will take Adi's idea, but because it is mathematics we are going to use the symbols + and - instead of blue and yellow colours on the dice.

Adi: Oh, we forgot it was mathematics completely

Teacher: So instead of colours we put these signs

Adi: How?

Teacher: We take the viewpoint of the blues .. we could take the view of the yellows, but from the blues, when we have +3 it means 3 points for the blues.. just like you suggested with the dice with a blue 3 on it ..if we had -1, who gets points?

all: The yellows

Blue Team Pupil: One for you

Teacher: So if you get this minus 3, so?

Blue Team

Pupil:	From the viewpoint of the blues, so I take 3 from me?
Teacher:	Yes, three from you or 3 points for him
Adi:	So yellows will always win!
Teacher:	Why?
Adi:	Because if its a minus, we lose..
Tom:	But if its plus, you will be winning.
Teacher:	If minus comes up.. it means?
Adi:	To take away.
Tom:	No, to take away from the blues
Lior:	Or to add to the yellows, its the same
Adi:	How do you know if its yellow or blue?
Tom:	Its from the viewpoint of the blues,..
Teacher:	Its not exactly, it means the plus is for the blue, but when you have -1 for example,..
Tom:	(yellow team) Its one for us,
Teacher:	Yes it adds to the yellow
Tom:	One yellow

Keren doesn't understand, she thinks "they are always going to win"whether its a plus or minus (we say in English 'heads you win, tails I lose')

Tom:	Don't you see, when you have 3 and 3, 2 and 2 (shows the die) there are pluses for everyone and minuses for everyone, and minus is like its plus for us (the yellow team) and for you its minus, its luck its not certain (for us)..
Teacher:	When the yellows see -2 on the die, what do they think?
Adi:	It means 2 is added to the yellows, or taken from the blues.
Teacher:	Exactly ..
Adi:	Aha. Got it: for us its exactly what is on the dice, and for them its the opposite.
Tom:	OK, lets start playing.

Although Keren did not speak throughout the above, she followed and demonstrated on the abacus and has no problem with the game. But Adi struggles for the first 2 rounds of the group, the group show her how it goes and explain why: it is an important feature of the game that she has to be brought to understand so the game can proceed.

For example she gets -1, she hesitates, Tom says "its for us", Adi: "OK, OK, but what am I to do?". She moves one yellow forward, hesitantly. Tom gets -3 and pushes 3 yellows forward,

Adi:	Why?
Teacher:	What does it mean?
Tom:	This game is very slow!
Adi:	Its less for him
Tom:	Its less for you
Keren:	Its less for us
Lior:	Its always from the point of view of the blue
Teacher:	Can you create -3 on these two dice.

Adi places 3 more blues on the pair of dice. Lior says "Adi, its the opposite". Adi changes the dice. Lior says: "correct".

Keren:	Did you understand, its exactly the opposite
Lior:	Its exactly the opposite, its minus 3, can you imagine it in yellows?

Keren agrees. The game continues, Adi gets -3 again. She moves 3 blues backwards, although there is no need to do so, she could have used the yellows, but she thinks of the sign and connects it with subtraction. Tom says "excellent Adi, wonderful".

We might think that Adi has little understanding but just follows the rules of the game instrumentally from this point. We are concerned that she (and others) understand the + and - as adding and subtracting "from the point-of-view of the blues", but that this can be represented as adding blues and yellows respectively to the teams points. Until this consolidation takes place, we cannot expect to smoothly proceed to the fourth game where the signs must be acted on as objects (i.e. where we have subtract plus 1 or subtract minus 1 as well as addition).

The use of the terms 'minus' and 'plus' introduce a mathematical 'voice' which ensures some association of the integer with their previous concept of addition and subtraction. From this game on, the mathematical voice becomes prevalent, but the traces of the game-situation are intertwined, giving meaning to the formal mathematics (in Vygotsky's terms the scientific conceptions) in the everyday terms and experiences of the children, (in Vygotskyan terms their everyday conceptions).

It seems this discussion centres on the problem of whether the +/- sign means add/subtract or blue/yellow. They are confronting the need to consolidate both concepts flexibly into a new procept: the integer +2 has to mean on the one hand 2 points for the blue team and on the other hand adding 2 for the blue (and taking 2 from the yellow) and vice versa for -2. The resolution seems here to be aided by the social orientation of the fairness of the game: Tom points out that for every plus for the blues there is a minus, and for every minus (which he says is a plus for the yellows: his own team) there is a plus (which is a minus to his team). In the notion of fairness and compensation here one sees the genesis of a new mathematical conception, minus-minus is plus.

In other introductions to the game this discussion was less involved, the children accepted minus is yellow and plus is blue, and re-discovered the compensation rule in the course of the game, probably this was facilitated by the use of the abacus: since the same manipulations can be remembered and hence transferred from one game to the next rather as language transfers when the same sign signifies two different 'signifieds'.

In another group, Sara, Dror, Limor and Ella are playing game 3.

Early on there is a discussion of the connection of the symbols with the team-colours:

Sara gets +3 and she gives 3 to the yellows

Dror and Limor: No!

Sara: Aha, its from the viewpoint of the blue.

She corrects and gives 3 to the blues.

This continues smoothly, they talk to themselves and then 'compensate' very early: in Dror's first go gets -1 and takes one from the blues

Teacher. What is the other possibility?

Dror shows by sliding the yellow half way...

Sara: Minus 2.

Ella: Minus 3.

Dror: Zero.

Ella: Minus 3.

Dror: Zero.

Sara: Minus 1.

Now the language is all plus and minus, superficially integers, and the situated language is lost, and we maybe have a pseudo-concept (Wertsch, 1991). They are still aware of the situation (because they have checked repeatedly to see if they are 8 points ahead. E.g. the yellows are winning and say "We have 8 , we won....".) But they are using the terminology of plus and minus, and they connect this with adding points for the blues and the yellows.

Analysis 3: Towards the completion of the mathematical voice in game 4.

Game 4 begins with the abacus showing six of each colour bead, and they are now introduced to the new dice marked "add"and "sub" (to distinguish from + and -) with the integer dice marked +3,+2,+1,-1,-2,-3. The teacher asks the group some questions before playing.

The teacher shows the dice 'sub -2' and asks Dror: "what would you do?"Dror takes forward two blues, Ella waves her hand nervously disagreeing. The teacher asks: "What would you do?"She pushes up (to take away) two yellow beads to the middle to show her preference. Clearly she doesn't see equivalence of the two.

Later Sara is given sub -3 by the teacher: she takes three yellows away. Dror gets add +3: he takes away three yellows. The teacher asks: "what else can you do?"He picks three blues up to show you can add 3 blues. Limor gets add +3: she takes three yellows away and adds "I could have added plus 3".

Later they play a game, starting with an abacus of 6 beads each. At an early point Ella makes a mistake and is challenged.

Ella throws sub -1: she takes away a blue. Dror stops her: she picks up the minus 1 die.

Ella: If I had only this I should've added one to us.

Sara: But you have a subtract.

Ella: Look minus one its one for you or take away one from us. Now I have to subtract minus one, so I have to subtract one.

Dror: OK but from whom do you subtract?

Ella: From the yellows.

Shortly, Ella gets 'sub +1' and comments "it means to subtract 1 from the blues, subtract -1 means to subtract from the yellows". She obviously got the point.

Limor gets sub -2 hesitates,

Limor: I can take two from us or add two to them.

Dror gets sub -3; he takes 3 yellows. Sara gets sub -1 and takes one yellow. Ella gets add +1 and takes a yellow. The end of first game. All the children have some confidence already. The next game is fluent.

All the discussion seems to be in a mathematical voice, sub +2, add -1 etc. but it is still understood in the situation of the game in concrete and intuitive terms, and when errors are made they draw attention to the game... thus the equivalence of subtracting the minus and adding the plus were already intuitive, but are here expressed in mathematical voice, in formal terms.

But Ella has zero on the abacus and ends up with 2 blues on the abacus (after subtracting -2). "It doesn't make sense.."she is persuaded it does make sense by Limor. She agrees "it's OK in the game, but not in mathematics."There is still some accommodation to be done!

Analysis 4: the role of the abacus and the situation finally in their explanations after the assessments:

A test was set:

Integers test

1.	+3 add -2	6.	+2 sub +4
2.	-5 add -2	7.	-5 sub -2
3.	-4 sub +2	8.	-4 sub -5
4.	+4 sub +2	9.	0 add -2
5.	+2 add -5	10.	0 sub -2

All but one of the children got 100 % correct on the assessment. About half made use of the abacus as a manipulative, the child with 4 mistakes (out of 10) did not use the abacus.

When asked to explain some of their calculations, they appeal to the situation to justify their methods:

Lior: (+9 - +13) "We have 9 on the abacus, 9 to the blues and then you are taking away thirteen from them, so you take away 9 that you have and you give 4 to the yellows: so its -4.

Keren: (-4 - -5 = +1) "There are 4 to the yellows and we have to take 5 from them, so we take the 4 that they have and we add one blue one."

Ella: (-7 + -8 is -15) "I had 7 yellows and I had to add 8 more yellows so all together I had 15 yellows."

Sara: (-13 + +7 = -6) "I had 13 yellows but I had to add 7 to the blues, so instead I take away 7 from the yellows and I ended up with 6 yellows."

Dror: (+5 - +7 is -2): "I am explaining from the viewpoint of the blues. Plus 5 is like 5 blues and we have take 7 blues, so we take 5 blues and we add -2 which means 2 yellows."

Limor: (+28 - -9) "is +37..."Dror interferes "28 plus 9: its like minus minus is like plus, that's the reason why the result is not 19 but 37"

The contrast here with the disco game is remarkable. In no case in the first experiment were explanations appealing to the disco-situation volunteered, the appeal was always to the abacus. (When pressed, an addition sum could be *applied* to the disco situation.) Yet here the children are keen to make sense of the sums with reference to the game!

Comparing the two experiments

Here I want to compare the success and the differences of the two experiments. They both involve establishing concreteness of natural numbers in a situation in such a way as to extend the concrete meaning to the integers later.

They both involve justifying strategies with the abacus by reference to the situation, and the representation of processes in the situation by objects (beads) on the abacus, which themselves are then manipulated on.

But the first led to an intuitive gap at the point where subtraction was introduced (it was a secondary concept, defined by inverse-addition), while the second introduced a gap earlier, when the signs are introduced to refer to teams in an arbitrary way. In this sense both have strengths and weaknesses, and a matching of the two at appropriate stages may be thought sensible.

The situation in the team game allowed the integers to be readily thought of as objects: 'points scored', whereas the integer in the disco situation is most readily seen as a process: dancers going in/out. Thus in the game-situation children seem to readily refer to the integers as objects, which can then be concretely added or subtracted.

Finally an important difference in authenticity appears relevant: a real game is socially valid and carries with it intuitions of fairness which proved important in generating rules and strategies. The disco-simulation was, as such, artificial and introduced some inauthenticity, but in any case did not carry with it such a productive range of intuitions.

General conclusions and discussion

This is intended now to generalise the instructional strategy; we call POLE an instructional method.

1. Building the link in the situation

   Solving problems posed in the situation should justify intuitively relevant strategies and operations, using only the number concepts readily understood. Processes in the situation are represented by objects on the manipulative, and are subject to manipulation through their representations 'by proxy'.

2. Attaching the link to the known numbers

   The activity should establish the modelling of the situation and the use of the representation with the new numbers , i.e. before new mathematical symbols are reintroduced.

3. Attaching the link to the new numbers

   The intuitive strategies are extended to the new numbers.

4. Embedding the link

   The formalization of these strategies and intuitions provide the new mathematical understanding sought: the gap has been 'filled'.

Two points in particular about the method. First, it is clear that the second situation carried more experiential reality (in Steffe's, 1996, terms) than the first. The notion of fairness was intuitive because the game was, though simple, a real one for the children. In contrast, the first situation was a simulation, and intuitive ideas about simulated situations do seem to be 'second hand'. This is a fundamental weakness of much of the work of those who call themselves 'situated learning' innovators. The use of even good quality simulation does not carry with it much of the intuitive richness of the social reality.

Second, an important feature of the design is the multiple representations involved, so that processes in one domain are mapped to objects in another. This allows activity with the objects to develop before the children have mentally reified the concept, thus breaking the vicious circle of which Sfard spoke. We think this approach may generalise to work with different representation of fractions, functions etc. in which one form appeals to process while another appeals to object. One implication might be the ordering of two of the 'big three' representations in children's representation of the function concept, for example from table (as representative of process) to graph (as representative of object).

The perception of the children's work as 'activity' within an activity system in Leont'ev's sense (Leont'ev, 1981, Wertsch, 1991, and also in Engestrom, 1996), helps to understand the proposed instructional method. The activity involves a sense of the social situation, use of the tool of the abacus (which is both a record of the situation and a manipulable representation of the integer), and ordinary language which is used to negotiate within the group. The integer is constructed in the social activity in a number of ways, but especially it begins as a process on the numbers already understood by the children. Its deeper meaning is formed through the activity, and through the discussion between children, on the social plane, before it is internalised intramentally.

The duality of the concept is visible in the situation presented to the children in the instructional sequence. Then it is visible in the activity of the children, and especially in the language of the children (for instance the process of going in and out is reified in the language when the children speak of the beads representing 'ins' and 'outs'.) Later, we encourage the children to symbolise, to mentally reify the integer, and they begin to manipulate the integers as objects which are added and subtracted. At this point we see the concept has become a mental entity for the individual: reification proper is complete.

In this sense we see activity theory as giving us a perspective for the design of instructional activity, in particular ones in which it is intended to 'fill' the cognitive gaps in children's extension of the number concepts through appeals to intuitions arising outside mathematics. We design social activity in which children engage with situations and tools, language and symbols which allow successive transformations from process to object.

References:

Billet, S. (1994). Searching for authenticity: a sociocultural perspective of vocational skill development. *The Vocational Aspect of Education*, 46(1), 3-16.

Brown, J.S., Collins,A. and Duguid,P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, 18(1), 32-42.

Chaiklin, S. and Lave, J. (1993). *Understanding Practice.* Cambridge, UK: Cambridge University Press.

Cobb, P and Steffe, L.P. (1983). The constructivist researcher ans teacher and model builder. *Journal for Research in Mathematics Education*, 14(2), 83-94.

Cognition and Technology Group at Vanderbilt (1994). The Relationship Between Situated Cognition and Anchored Instruction. *Educational Technology*, 34(8), 28-32.

Collins, A. (1994). Goal-Based Scenarios and the Problem of Situated Learning: A Commentary on Andersen Consulting's design of goal-Based Scenarios. *Educational Technology*, 43(9), 30-32.

Diriks, M. K. (1984). The integer abacus. *Arithmetic Teacher*, 31(7), 50-54.

Engestrom, Y. (1994). Non scolae sed vitae discimus. *Learning and Instruction*,1, 243-259.

Engestrom,Y. (1996). Interobjectivity, ideality and dialectics. *Mind, Culture, and Activity*, 3(3), 149-164.

Fischbein, E. (1987). *Intuition in Science and Mathematics*. Holland: Reidel.

Freudenthal, H., (1973). *Mathematics as an Educational Task.* Holland:Reidel.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht: Kluwer.

Gray, E.M. and Tall, D.O. (1994). Duality, ambiguity and flexibility: a 'proceptual' view of simple arithmetic. *Journal for Research in Mathematics Education*. 25(2), 116-140.

Heckman, P.E. and Weissglass, J.(1994). Contextualising mathematics instruction: moving beyond recent proposals. *For The Learning Of Mathematics*, 14 (1), 29-33.

Janvier, C. (1985). *Comparison of Models Aimed At Teaching Signed Integers.* Proceedings of the Ninth International Group for the Psychology of Mathematics Education, (pp 135-140). Utrecht, NL: University of Utrecht.

Lave, J. (1988). *Cognition in Practice*. Cambridge, UK: Cambridge University Press.

Lave, J. and Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge, UK: Cambridge University Press.

Leont'ev, A.N. (1981). The problem of activity in soviet psychology. In J.M. Wertsch, (Ed). *The Concept of Activity in Soviet Psychology*, (pp 37-71).. Armonk, NY: M.E.Sharpe.

Liebeck, P. (1990). *Scores and Forfeits, an intuitive Model for Integers.* Educational Studies in Mathematics, 21(3), 221239.

Linchevski, L. and Williams, J. S. (1996). Situated Intuition, Concrete Manipulations and Mathematical Concepts: the case of integers. *Proceedings of the Twentieth International Group for the Psychology of*

Mathematics Education (vol. 3, pp 265-272). Valencia: University of Valencia.

Lipman, M., Sharp, A.M. and Oscanyan, F.S. (1980). *Philosophy in the Classroom..* Philadelphia: Temple University Press.

Lytle, P. A. (1994). Investigation of a model based on neutralization opposites to teach integers. *Proceedings of the Nineteenth International Group for the Psychology of Mathematics Education* (pp 192-199). Reclife, Brazil: Universidade Federal de Pernambuco.

Munkhopadhyay, S., Resnick, L. and Schauble, L. (1990). Social Sense-making in Mathematics: Children's ideas of Negative Numbers. *Proceedings of the Fourteenth International Group for the Psychology of Mathematics Education* (pp. 281-228). Mexico: Published by the Program Committee.

Ost, D. (1987). Models, modeling and the teaching of science and mathematics, *School Science and Mathematics, 87(5), 363-371.*

Peled, I., Munkhopadhyay, S. and Resnick, L. (1989). Formal and informal sources of mental models for Negative Numbers. *Proceedings of the Thirteenth International Group for the Psychology of Mathematics Education*, (vol. 3, pp 106-111), France.

Resnick, L. and Ford, W. (1981). *The Psychology of Mathematics for Instruction,.* Lawrence Erlbaum, New Jersey.

Rogoff, B. and Lave, J., (Eds.) (1988). *Everyday Cognition: its development in social context.* Cambridge, MA: Harvard University Press.

Semadeni, Z. (1984). A Principle Of Concretisation Permanence for the Formation of Arithmetical Concepts. *Educational Studies in Mathematics* 15, 379-395.

Sfard, A. (1991). The dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematicss* , 22, 1-36.

Shiu, C. M. (1981). *Teaching the addition and subtraction of negative whole numbers*, Nottingham, UK: Shell Centre.

Shui, C.M. and Bell, A.W. (1981). *Directed number.* Nottingham, UK: Shell Centre.

Skemp, R.R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher*, 26(3), 9-15.

Steffe, L.P. (1996). Sociocultural processes, a discussion, in H.M. Mansfield, N.A. Pateman and N. Bednardz (Eds). *Mathematics for tomorrow's young children*, (pp 79-99). Dordrecht: Kluwer.

Splitter, L.P. and Sharp, A.M. (1995). *Teaching for better thinking: the classroom community of inquiry*. Melbourne,Australia: Australian Council for Educational Research.

Thompson, P.W. and Dreyfus, T. (1988). Integers as Transformations, *Journal for Research in Mathematics Education* 19(2), 115-133.

Treffers, A. (1987). Three Dimensions: A model of goal and theory description. In *Mathematics Instruction*. Dordrecht: Kluwer.

Vinner, S. (1975). The naive Platonic approach as a teaching strategy in arithmetic. *Educational Studies in Mathematics, 6, 121-132.*

Wertsch, J.V. (1991). *Voices of the mind: a sociocultural approach to mediated action..* Cambridge, MA: Harvard University Press.

Wood, D. (1995). Theory, training and technology, part 1. *Education and Training, 37(1), 12-16.*

Williams,J.S. (1997). Scientific dialogue as evidence of Learning, *Paper to AERA-1997*, Chicago.

Young, M.F. (1993). Instructional design for situated learning. *Educational Technology for Research and development, 43(2), 53-69.*

email: JWilliams@fs1.ed.man.ac.uk

3/97

LIORA, 972-2-643-5830 RSVP TO +44-161-275-3484 JULIAN WILLIAMS
LINCHEVSKI
SE060596

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
Educational Resources Information Center (ERIC)

ERIC

REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: SITUATED INTUITIONS, CONCRETE MANIPULATIONS & THE CONSTRUCTION OF THE INTEGERS

Author(s): LIORA LINCHEVSKI, JULIAN WILLIAMS

Corporate Source: A.E.R.A. DIV C PAPER PRESENTATION, CHICAGO.

Publication Date: 4/1997

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic/optical media, and sold through the ERIC Document Reproduction Service (EDRS) or other ERIC vendors. Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following two options and sign at the bottom of the page.

	The sample sticker shown below will be affixed to all Level 1 documents	The sample sticker shown below will be affixed to all Level 2 documents	
☑ Check here **For Level 1 Release:** Permitting reproduction in microfiche (4" x 6" film) or other ERIC archival media (e.g., electronic or optical) *and* paper copy.	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN OTHER THAN PAPER COPY HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)	☐ Check here **For Level 2 Release:** Permitting reproduction in microfiche (4" x 6" film) or other ERIC archival media (e.g., electronic or optical), but *not* in paper copy.
	Level 1	Level 2	

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but neither box is checked, documents will be processed at Level 1.

LIORA ADD SIGN

"I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic/optical media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries."

Sign here→ please

Signature: [signature]	Printed Name/Position/Title: Dr JULIAN SCOTT WILLIAMS	
Organization/Address: DEPT of EDUCATION, UNIVERSITY of MANCHESTER, OXFORD RD M13 9PL, UNITED KINGDOM	Telephone: UK +44+161-275-3409	FAX: +44-161-275-3484
	E-Mail Address: JWILLIAMS@FS1.ED.MAN.AC.UK	Date: 28/5/97

(over)

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, or, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:

However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
1100 West Street, 2d Floor
Laurel, Maryland 20707-3598

Telephone: 301-497-4080
Toll Free: 800-799-3742
FAX: 301-953-0263
e-mail: ericfac@inet.ed.gov
WWW: http://ericfac.piccard.csc.com

(Rev. 6/96)