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ABSTRACT

This booklet is designed to be used with a video of the classroom of second-grade teacher Victoria Bill. The 40-minute video shows the teacher using a variety of manipulations, facilitating both large- and small-group discussions, and applying a problem-solving approach to math. The text in the booklet is based on spoken comments made by observers as they watched the videotape, and provides added insight into the instructional methods used in the class. The video and the accompanying booklet are divided into 16 events, each representing a change in the activities in the classroom. The discussion of each event begins with an assessment and overall description of the event, followed by discussions of classroom management, problem solving, and teaching strategies used in each example. (ND)

# School Development Library

## *A Second-Grade Math Lesson With Victoria L. Bill*

Victoria Bill is a second-grade teacher. The 40-minute video of her classroom shows her using a variety of manipulatives, facilitating both large and small group discussions, and applying a problem-solving approach to math.



NCREL

# A Second-Grade Math Lesson With Victoria L. Bill

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NCREL's School Development Library consists of video and print resources designed to support educators in their efforts to improve classroom instruction. These materials include audiotapes, case studies, the *Pathways to School Improvement* Internet server, print guidebooks containing teacher and expert commentary, video programs and videotapes of actual classrooms, and CD-ROMs, which include both a classroom video and the guidebook in electronic format.

The classroom videos are not scripted; they provide an example of real elementary or high school instruction to be used as models or cases for educators to study. They are examples of good instruction that is consistent with established and developing content standards. These videos are designed to be used as part of an ongoing professional development program that includes the use of other classroom videos, information, and resources.

Some of this material was adapted from Strategic Teaching Framework (STF), an NCREL/IU hypermedia project, under the direction of Thomas Duffy, Professor of Instructional Systems Technology at Indiana University; Beau Fly Jones, Senior Researcher and Director of the Teaching and Learning Center at NCREL; and Randy Knuth, Director of the Center for Scaling Up at NCREL.

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# Victoria Bill

## Suggestions for Using the Perspectives With the Video

Victoria Bill is a second-grade teacher. The 40-minute video of her classroom shows her using a variety of manipulatives, facilitating both large- and small-group discussions, and applying a problem-solving approach to math.

The video is actual footage of a second-grade class filmed during January. It is divided into 16 events, each division representing a change in the activities or flow of the classroom.

The Victoria Bill perspectives booklet is designed to be used with the Victoria Bill video. The text in this booklet is based on spoken comments made by various people as they watched the accompanying videotape. NCREL tape-recorded, transcribed, categorized, and, in some cases, condensed these comments. This booklet's intent is not to be a verbatim transcript but rather to capture the flavor of the viewer's reactions to the classroom. These perspectives were shared by Victoria Bill, the classroom teacher; Dr. Pete Kloosterman, a mathematics researcher; and the NCREL/ IU project team.

We suggest that you use the perspectives booklet as you view the video to add insight into the instructional methods used in the class.

# Event 1

# Introduction – Beginning of class procedure with small group activities

**Classroom Management**

Victoria Bill

The groups are working on labs. The groups are situated around the room and labeled one to five. I make up six labs, and those six labs last over six weeks. A new lab is introduced to the whole class on one day. Then the next day one group gets that lab. The group usually stays with that lab for a set number of days, usually three. The groups rotate from lab to another lab until they have completed all six.

What I usually do is either call one team at a time or call teams at opposite ends of the room. One person from the group, the group leader, gets the manipulatives and is in charge of returning them. I established that routine. The next week another group leader takes over.

NCREL/IU

These kids really know what is expected of them in the groups. This kind of cooperation takes time to build. It's going to take the first month or six weeks of school to get them as comfortable as they are right now with that routine.

Victoria systematically has the students get their materials so there is no wasted time, and they can get straight to work on their math lesson for the day. This is a routine they probably do every day, and it is a specific procedure they have been taught. Victoria calls on the groups who are at opposite ends of the room so that they can go get their materials to start their lab. By doing this, she establishes a routine that the students follow to get their activity and bring it back to their groups. By calling group numbers, she directs students from different locations to get their materials and avoids traffic pattern problems.

In order for the students to get their materials as smoothly as they do here, Victoria obviously has had to teach this procedure. In doing so, most likely she told the students what she wanted them to do and explained why they need to do this. She has modeled for them and has had them practice going to get their materials and bring them back to the table.

In the beginning, to make sure that the students are performing the routine effectively, Victoria reinforces correct behaviors by providing rewards on her sticker charts or with verbal praise.

| Event 1 cont. | Introduction – Beginning of class procedure with small group activities |
|---|---|
| **Problem Solving** | |
| NCREL/IU | It is very important to note that Victoria previously modeled the procedure for doing the lab, not the answers that they're supposed to get.<br><br>They don't know what the outcome for the lab is supposed to be. Victoria models the process and allows them to determine their own outcomes and solutions, and then she questions them to help them make inferences about relationships, concepts, and principles.<br><br>Victoria uses a problem-solving approach by giving the students a complex problem to solve. She gives them a particular task and a specific process, and they're supposed to determine some relationship or some concept through the procedure they're doing. |
| **Teaching Strategies** | |
| Victoria Bill | This is the second-grade class. We're having a third grader come and help with the computer, to assist in giving directions to the students, getting them to think, and helping them with their questioning techniques. |
| NCREL/IU | Each lab is modeled for the whole group by Victoria. As she introduces new labs, she introduces one on a particular day so that all the students know how to work through the lab and what they're responsible for doing. Most of her labs last a week, although some only last three days. Each group works through a lab; some for three days, some for five days.<br><br>At this time there are six labs. Two of them involve using computers. One of the groups on the computer has a third-grade student who comes and helps them with their activity. Other lab activities involve determining probability, manipulating money, and measuring water. It looks like there is a group doing some addition problems as well. There are also, on the board around the room, multiplication problems and addition problems with the names of individual students on them. The students are responsible for solving these problems at some point during the lab activity time. |

# Event 2 — Probability – Small group using probability to predict outcomes

**Assessment**

NCREL/IU

Victoria's questioning strategies have multiple purposes. One is to determine where the students are in their thinking processes. By questioning the students about their predictions, she is performing an informal assessment to see how their thinking is progressing and whether or not there are any flaws to be addressed. She's also assessing their ability to process what they are doing and whether the procedure they're using is the correct one. If the group was on the wrong track, she would have questioned them in order to guide them toward the correct process. Rarely is there a time when Victoria finds it necessary to directly challenge a child's answer.

Victoria's trying to help the students construct some problem-solving abilities here. If they seem to be way off with their understanding, she questions them to get them to explain their thinking. If there are major errors in a child's problem-solving schema, questioning can help that child understand and correct those errors. This is much more effective in building problem-solving abilities than simply telling the child that he or she is wrong.

**Classroom Management**

Pete Kloosterman

When you are working with groups, you want to situate yourself with your back against the wall of the classroom. Then when you're not looking down at the group, you can look up and see what all of the other groups are doing. There may be too much equipment in the room for her to do that. But it's just a good management technique to always be in a position where you don't have to turn around to see what the rest of the class is doing.

NCREL/IU

There's a critical aspect to classroom management here. While some people may be concerned about Victoria turning her back to the rest of the classroom while working with this group, it's obvious that the kids know the procedures and what's expected of them to such a degree that she doesn't have to worry about monitoring the rest of the class every second. It may be that if the cameras weren't there, she would have been on the other side of this group so she could see more of the class. However, if you notice the group in the background, they seem to be on task.

**Classroom Management, cont.**

NCREL/IU, cont.

The procedures Victoria has developed are so well implanted within the students' minds that she doesn't have to monitor them every second. In fact, by not "hawking" the class, she's conveying an expectation of responsibility from each of the groups and, therefore, each of the students.

**Problem Solving**

Victoria Bill

When we're working with probability it's good to say "about the same." They need to hear that language ("about an equal chance") because it builds up an understanding of probability.

Pete Kloosterman

They're doing a probability activity here where they have two colors of chips in the bag. Probability is one of those notions that rarely gets taught in elementary schools: The idea that if you have more yellows than reds, you ought to have a better chance of getting a yellow than red. It's a fairly simple idea that you can work with in the second and third grade. Yet teachers don't tend to do that because they don't realize how simple that idea really is. But all that she is really doing is just trying to help the kids to think, "Well if there is more of one color, that's the color we're likely to get."

NCREL/IU

The problem that they're working on here has to do with probability, which is an NCTM standard. What happens here is that each student has his own bag of two different colored markers or chips. Each bag has a different proportion of one colored marker to the other. In fact, it's interesting to note that the bag Jeremiah is using has an almost equal number of yellow and red chips.

The procedure in this lab is for each student to randomly draw the chips out of the bag and then record what is selected. As you can see, Jeremiah is marking in his notebook the total number of times he chooses a particular color. In the end, after each child has used this procedure on each bag, the members of the group will compare their results to determine similarities and whether or not there are any general conclusions that can be drawn about their results.

**Problem Solving, cont.**

NCREL/IU, cont.

The questioning that Victoria is doing here helps the students by activating their thought processes so they can begin a problem-solving scheme. The process they're going through in the lab could be very rote–just "pull it out of the bag and write it down on the paper." But, instead, by having them think about their prediction, Victoria is trying to actively involve them in the thinking process. She's helping them develop thinking that will transfer to other problems the students will encounter in the future.

**Teaching Strategies**

Victoria Bill

All of the students are responsible for recording their lab. They label it by its lab number, or sometimes they put down probability. I have some coding that I put in their notebook. But they're responsible for having some kind of written record from that lab.

I usually check them either daily or by day three when it's completed. For example, the probability won't be completed for three days because it usually entails working with three different bags. At the end I want a generalization. So by day three I question them about what they observe. I usually go to their group at that time and ask for their thoughts.

NCREL/IU

It's crucial to note that Victoria is very careful not to say, "You're getting the right answer" or "You're getting the wrong answer."

At this point she's asking them basically to do two things–to make a prediction and then follow the given procedure. So they're formulating some hypothesis about what it is they're going to end up with, and then they actually perform the process in this lab and determine whether their hypothesis is correct or not. She doesn't interfere with nor explicitly direct their building of understanding of what's happening. She makes sure she only moves in when there seems to be a problem with the process the students are using.

Victoria also uses words such as *prediction,* which is an implicit way of building math vocabulary. Building this vocabulary is crucial for student understanding and is an important part of the NCTM standards for teaching mathematics.

# Event 3 — Money group – Small group exploring alternative representations of money problems

**Assessment**

NCREL/IU

With this group, we see Victoria performing an informal assessment to determine where the students are in the problem-solving process as a group as well as individually. She is also assessing whether the students are making the connection between the concrete and abstract representations.

At the same time, Victoria is assessing the process the students are using in this lab. By making sure that the process is correct, she can be assured that any problems encountered in the learning are not due to errors in that process.

Victoria's questioning strategies are used here not only to help the children build problem-solving strategies, but as an assessment tool. The answers that the children provide give her immediate information on what the students are thinking and how they have interpreted the process they are going through. The questions used here continually emphasize process. There is very little or any emphasis on the "right" answer.

**Classroom Management**

NCREL/IU

She's got a procedure for how they're supposed to work in their groups, and that keeps them all on task and going through the process. When she says "I need you," it's a way to draw all the students in and keep them actively involved in the process because she has stopped them and she's calling on individuals at this point. So in order to make sure that everyone is following her and staying up with what's going on, she has them all count.

Notice that when Victoria leaves this group she leaves them with something to do. By having them show another way to represent the solution, she sustains the group process. She never leaves a group just sitting without something to do.

**Problem Solving**

Victoria Bill

The students get a cup with an amount written on the side. One person in the group starts. The others are responsible for that child's thinking. They're also responsible for helping contribute to solving that problem. So it's the whole team's problem. It has to be one problem with everybody giving their thoughts. You can't just let them have individual cups and each work on his or her own.

**Problem Solving, cont.**

Victoria Bill, cont.

Normally they're responsible for two ways of solving the problem, then they can move on to the next cup. It moves much quicker when I'm not there. But they must have an equation to map the money. So if they use fourteen nickels, they would put either fourteen times five or they would write five fourteen times. In other words, I have taught them to map with the money. That occurs through whole class instruction when I do it on the overhead. You have to build that or they can't go into these labs and work. The purpose of getting these equations down is to establish a language so that they can talk about their solution when they come together as a whole class. It's also massive practice.

Pete Kloosterman

She's asked for a multiplication problem after an addition problem was given. This is something that you certainly want to do in mathematics. Addition is a simple way of expressing the answer. Multiplication is more advanced, and you want to lead them from the way that they know on to multiplication.

NCREL/IU

Problem Solving is a multiple-step procedure in this class. In all of the labs, Victoria helps the children really understand the problem-solving process by allowing them to move from very concrete solution representations to abstract ones. In this case she is moving them along the continuum from concrete to abstract by having them first represent the solution with the coins. Second, each student is expected to verbalize his or her solution to the other members of the group. Third, the students provide a symbolic form of the solution by writing the appropriate equation in their notebooks. In this way, there is always a link between what the children are doing, thinking, saying, and writing. There is always a reason for writing the equation, and the equation is always tied back to some physical manifestation of the problem and its solution.

The approach that Victoria takes here and throughout this math class is to encourage multiple ways of getting to an answer as well as multiple representations of that answer. This lab is really built around these two ideas. The students are required in this lab to determine multiple ways to represent 76 cents. These representations are physical, verbal, and symbolic. In some cases, even the physical representation may stay the same, but each child develops an alternative symbolic representation. Here we see Victoria using questioning and discussion techniques to develop these multiple representations. In fact, the

**Problem Solving, cont.**

NCREL/IU, cont.

multiple representations are used to lead into the discussion and the understanding of the different mathematical or arithmetic operations of addition and multiplication.

It is important that Victoria is also having them compare two numbers all the way through. She states, "Thirty-five—do you need more or do you need to take some away to get to 76?" So they're always comparing to see if 35 is less than or greater than or equal to 76. Then when a dime is added, she still wants to know if it is less than or greater than 76. All the way through, they're making that comparison. This builds their ability to understand the relationship among numbers and quantities. It is another way to continue the link between the abstract and the concrete.

Victoria's questioning also builds another part of the larger problem-solving task for the students. She always wants the students to check their solutions and their thinking to determine "reasonableness." Victoria is helping build a self-check mechanism into any Problem Solving that the students may do in the future. Its also helps the students to evaluate where they are in the solution process at that time and then decide what they need to do in order to proceed.

**Teaching Strategies**

Victoria Bill

By saying "Let's count," I was signaling her [the student] that she was not successful. I know she didn't feel sure she was right. They know that I ask frequently whether it's right or wrong. I also have one student check another student. So they know my pattern; I'll check anyone.

If a teacher were watching this, he or she might say, "Oh she only did that because that child was wrong." Whether the student is right or wrong, you need to say, "Let's check."

Often teachers will say, "Oh, do you need more?" You have to be careful not to always ask that question because they catch on to that very quickly. So often I'll say, "Do you have to take some away or add more? Why?" and then get them to tell me.

They were having difficulty with 35, 45, and 55, so I repeated it–35, 45–so they were getting it all connected and they were hearing a pattern.

**Teaching Strategies, cont.**

Victoria Bill, cont.

What I did was shoot Aaron's method down. My goal was to get him to his multiplication. So I took him where I wanted to go. Meanwhile, he doesn't believe "10 and 10 and 10" and soon is wrong. But the other children proved that was their method. I should have acknowledged that and said, "You're right, let's write that down." Then I should have gone on and taken care of Aaron's method and then linked the two. It makes the others feel not confident. Aaron is the only one really following the multiplication. You can see the other ones are not connecting with it. However, I believe they would have connected had I done the repeated addition.

I should have gotten Jameeca to say, "Oh, I'm done," and let the others say, "No, you're not." Normally I would do nothing. I think I guarded them quite a bit because I'm on tape here. You should let them regulate; let them check themselves.

Pete Kloosterman

She asks questions that have very short, quick answers. Those don't require a whole lot of thinking but they're very good for keeping everybody on task. If you're always going around getting answers quickly, then everybody's got to be on their toes. One of the disadvantages of thinking-type questions is that if a student takes two-three minutes to answer, you tend to lose the rest of the group.

NCREL/IU

There seems to be a big difference with this group from the group we just saw. In the probability group, the students are working individually, whereas this group is working more in a cooperative manner. You'll notice that they're responsible for each other's work, and each person in the group has to understand the other's thinking.

**Teaching Strategies, cont.**

NCREL/IU, cont.

In this lab the group is given a cup that has "76 cents" written on the side. They are also given a tray of money and are responsible for two or three ways of representing 76 cents with the money. Victoria is having them work cooperatively. They are all responsible for going from that concrete representation to an abstract representation, which is the written equation in their notebook. Each student is allowed his or her own way of thinking about and constructing the representation. Incorrect mathematics is addressed. Conditions are set for students to discover that the equation 7 x 10 = 70 + 5 = 75 + 1 = 76 conveys an incorrect message because 7 x 10 does not equal 70 + 5. Victoria helps students learn the importance of recording problems that are true mathematically correct and offers students an alternative way of recording. For example: 7 x 10 = 70

70 + 5 = 75

75 + 1 = 76

Although Victoria offers them an alternative way of recording the problem, she does not change their work, the way they thought about the problem.

At the same time, each member of the group is responsible for understanding the other members' thinking. In fact, when Victoria was questioning the students, she included everyone in the group, and each one was responsible for being able to pick the problem up where it was and contributing to it. Victoria checks students when they are right or wrong. When Jameeca said that "45 plus 10 more was 65" they counted it, and they also counted it when they said "7 x 10." So in this instance, it looks like she's checking just because Jameeca was wrong, but that's not always the case.

| Event 4 | Probability revisited – Checking understanding of probability group. |
|---|---|
| **Assessment** | |
| NCREL/IU | Victoria left Jeremiah and this group to go to the money group, and now she has returned to see how things are going. Apparently, Jeremiah may have been having trouble. When she came back though she assesses Jeremiah's progress by asking him which color he is getting most often. When he states that he has selected more red than yellow, she glances at his written work and realizes that there must be a problem with the process he is using, because the number of reds and yellows in the bag is virtually equal.<br><br>Victoria didn't think that Jeremiah was just putting one in and pulling the same one out. She was concerned that he just wasn't mixing the markers around well. But one thing that she might have done is let him continue on the way he was going, because later on the three students in that group would have compared their answers for each bag. At that point, Jeremiah would have seen that his results were different from the other two.<br><br>With each of these groups, she's done a very quick, informal assessment. That's one of the things she does very well. She can almost immediately determine where they are, what they're doing, and where any problems might lie in their thinking process or in the process they're using to do the lab. She immediately works on that by giving a series of questions. Instead of saying, "Jeremiah, you're doing that wrong," she begins by asking, "Jeremiah, how are things going?" With Brent she is pursuing a more complex line of questioning. In general these questions include asking about the results of the procedure or process and then asking, "Why did you get these results? Does this bear out your hypothesis?" In this class why and how you get an answer is more important than the answer itself. |
| **Classroom Management** | |
| Pete Kloosterman | I noticed she started with this group of three, went over to the other group, and came back to this group. We don't know what the rest of the groups are doing. I am assuming that they are doing fine because she is only working with these two groups. Good teachers have one eye on the group they are working with and another eye on the rest of the class. In fact many times the teacher will go from group to group depending on which group seems to be struggling the most. |

# Probability revisited – Checking understanding of probability group.

**Classroom Management, cont.**

NCREL/IU

Just before Victoria leaves this group, she says to Brent, "You're so smart" because he had figured out that 30 was two times more than 10, and that was 20 more. She reinforced the last part. The use of videotapes can help teachers hear incorrect statements such as this which they might not notice normally during classroom time. Viewing the videotape helped Victoria notice that this statement was not correct and that it could cause students to develop misconceptions. By praising him and patting him on the arm, she's providing a reward for his having stuck with the problem and for giving a good explanation. This is basic to Victoria's classroom management style and also to her teaching style, which really encourages the children to be open and work through complex material without giving up.

**Problem Solving**

NCREL/IU

When Victoria was with this group the first time, she talked about predicting what would happen. When she comes back, she notices immediately that Jeremiah is having problems with the process, so she deals with that and lets him continue his work, whereas she talks to Brent on a different level, asking him about his results. He states that he is getting more red than yellow. Victoria isn't satisfied with just a correct result, however. She immediately asks him why he got the results he did. In this class it's not just good enough to get a correct answer, you must understand how and why you got the answer and be able to explain it to others. This questioning provides for the building of math vocabulary, conceptualization, and mathematical communications. Victoria is helping Brent to make mathematical connections.

**Teaching Strategies**

Victoria Bill

When I asked Jeremiah why he was getting all of one color, I should have asked him how he's been reaching in. It might have been that he was just reaching in and not shuffling it around. That would have been a perfect opportunity to let him do it on his own and then later on, someone else would use that bag. I could just let him compare after day three with the other students.

Brent said, "Thirty is 20 more than 10." I know what he's saying: It's two times more. Actually it's three times more and that could be a danger later on. I knew what he was thinking though, but I should have repeated back to him, "Oh, you mean 20 more."

**Teaching Strategies, cont.**

Pete Kloosterman

Throughout the tape she asks lots of questions, which is really good. It's one thing to ask questions such as "What do you think the answer is going to be?" But it's also good to ask questions such as, "Why do you think that's going to come out that way?" I think one of the real important things is to ask questions that allow the kids to express how they've come up with the answer they've gotten. Why does red keep coming up more?" is a good example of trying to get the kids to express their thinking.

NCREL/IU

It is interesting that Victoria makes a quick informal assessment of what Jeremiah is doing and determines that there might be a problem with the process that he is using because of the answer he gave. But instead of focusing on the fact that he seems to be getting an incorrect answer, she assesses his process. She immediately tries to evaluate what error he might be injecting into the process. So she focuses on the process and works with him on that and helps him to understand, in a little more detail, what the process is so that he can get a correct result and gain a deeper conceptual understanding.

**Assessment**

NCREL/IU

By beginning a new task, Victoria could watch the entire process they were using in this lab. It is possible then to assess how well the students are performing their various individual tasks and how the group process is working. Victoria also uses this opportunity to make comparisons in order to assess student understanding of the concepts involved in the lab. These concepts include fractions, measuring, estimation, comparisons and conservation.

**Classroom Management**

Victoria Bill

We've talked about how they will measure the water in the cup, what they are going to be comparing, and that they will respond on the recording sheet. I also go through all the "do not's." For example, "You will not throw water at each other." It is important to do that. I go through what they will and will not do, how they will question each other, who will pour. Then they have to check with someone to see if they did it correctly. So one child holds the cup, the other child checks it. They each have a role to play, and then it rotates so they all get a turn at each role.

NCREL/IU

There seems to be some confusion about what these students are supposed to be doing when Victoria first walks up. It appears that in order to keep the students working, Victoria has to reinforce their roles for them as well as the process for the lab. But notice how quickly she gets them actively involved in the work by redefining roles and the task at hand.

Because Larry is looking off at something else, Victoria pulls him in a couple of times by calling his name and asking him questions, and that pulls him back into the discussion. And so then she has Larry fill the cup if it needs more water to keep him involved in the process.

As with the money group Victoria leaves this group with another problem to do. With the roles now defined, she can feel pretty confident that they will stay on task during the next few minutes.

**Problem Solving**

Victoria Bill

I find it very helpful to let them measure the amounts out and then pour them into the same size of cup to compare, as opposed to just letting them use the measuring cups. It's a better way for them to compare the amounts.

**Problem Solving, cont.**

Victoria Bill, cont.

I should have maybe said, "One cup of water," or "One-eighth of a cup of water" Some of the kids may have been focusing on the cup. These little things make a difference. Introducing this to the whole class also makes a difference. I know that they're probably safe there.

NCREL/IU

Victoria makes excellent use of the students' prior knowledge with this lab. Virtually all students by this age have had some experience with measuring, possibly even with a measuring cup, and with fractions. They just haven't yet linked their experience with mathematical terminology and symbolic representations. Nor have they systematically determined relationships between quantities. Here Victoria is having them use every-day items that they are familiar with to begin systematically determining quantitative relationships. It is important to always try to relate what is being learned in the math class to some prior experience the children already have. At this age that experience has mostly come in the form of manipulating real objects. When the children see the math as simply representing actual physical relationships, they are much more likely to understand it and have the ability to use it later in other situations.

The students are also learning the higher-order thinking skill of analysis. Victoria is having the students compare the different sizes of cups, and then she asks them to predict how many $^1/_3$ cup or $^1/_4$ cups it will take to fill the larger cup. The students must hypothesize which cup must be used more to fill the larger cup. Victoria even builds in math vocabulary by asking them to "prove it to me." She is having them prove their own hypothesis about the relationship between the two different-sized cups.

By providing the students with the opportunity to make comparisons, make hypotheses, and test results, Victoria is helping the children build their own understanding and ways to look at problems. These skills are exactly the skills they will need and will use as they progress through school and into adult life.

Finally, this approach to teaching fractions helps the students gain some insight into the "reasonableness" of their answers. Often when students first encounter fractions, it is in a symbolic format. The children are provided with algorithms or procedures for adding, subtracting, multiply-ing and dividing fractions. These procedures, however, seem very abstract to the students; and, therefore, they just perform them without truly understanding them. Students then have a difficult time determining

**Problem Solving, cont.**

NCREL/IU, cont.

whether an answer arrived at through a given procedure is "reasonable" or not. By providing the children here with the opportunity to gain skills and experience with fractions, they won't have nearly the difficulties later that most students experience with understanding fractions and their manipulation.

**Teaching Strategies**

Victoria Bill

I told them too much here. "Tell me why" is enough. Either that or let them do it a couple more times and then come back.

NCREL/IU

Victoria uses many formal cooperative group learning techniques in her classroom. Here, she is using specified group roles to help the students stay actively involved and working. At first, the students seem to have finished their work or are not on task. Victoria quickly reactivates them by reminding them of their roles and by giving them a new problem to begin. Each student is assigned a role in the process. One student holds the cup, one person puts the water in; and, later, one person is responsible for making sure the cup is full.

Introducing a new problem allows Victoria to watch the process this group is using, all the way through. Notice that she presents a problem and then asks them for their expectations. It is important to help the students begin work with some expectation—then the work is aimed at determining if the expectations are correct. Too often a child will simply calculate an answer and never assess whether that answer makes any sense at all. Having an expectation builds in that self-checking.

Victoria could have made estimating a part of the early lesson on thirds. The children did not fill the cup accurately, and she emphasized the precision. Then later she talked about errors in measuring and she took control to make sure they got it "right." She could have been more effective if she had talked about filling the cup partially as "estimation" and talked about the kinds of errors that can occur in estimating. Then they could have projected ahead as to whether there will be more than or less than a cup as a result of how the child was filling the "thirds." In essence both fractions and estimation could have been combined very nicely here.

**Classroom Management**

Victoria Bill

I needed to establish a way for everything to stop in the room. Either I say "Stop" or I clap. At that point everybody freezes for a second. They clean up and pass everything to the group leader. Then the group leader waits to be called on to return the items.

This took two to three days to establish. It's done step by step and I model how to do it . Then they practice. I say, "Let's watch group two do it," and we watch how they clean up; then I say, "Let's watch group three, four, and five do it, all at the same time." I remark how efficiently they do it. That's a practiced routine the first two days of school. Everybody carries it the same way, and there are no spills this way.

Each team's items are going to be put on their side of the room; not across the other side of the room. You get done faster, and there's not going to be a collision.

Pete Kloosterman

To shift to the large-group activity, she made an announcement. Obviously this is a cue. The kids know when this happens they're supposed to get ready to move and go on to something else. Kids pick up fairly readily on those types of cues.

She's got them trained so they don't spend a lot of time getting materials; that's really good. The first few times you use manipulatives, you have to give kids some time to sit and play with the materials. They need to explore those things. As you use them more often, you need a system set up to get the materials out and quickly distributed. You want to spend your time doing mathematics, not passing out materials.

Getting students to begin working quickly with the manipulatives is like getting them to work in groups. When you start out, you have to go through how the team leader will get the materials. Once the students are used to that routine, it doesn't take very long to get the materials ready to use. It's crucial as you are developing these management skills that you're very consistent in what you expect. If you say, "Group leaders come to the front and get your materials" and somebody doesn't do it, you've got to get him or her right away and make those expectations really clear. The worst thing that you can do is half the time admonish them for not coming, and the other half let it slide.

**Classroom Management, cont.**

NCREL/IU

First of all Victoria claps and says, "Let's stop" as a signal for the students to stop what they are doing. The signal is part of the procedure. The kids know they're supposed to start putting their stuff away so that they can take it back to where it belongs. Victoria tells them to put their things away a group at a time. Again, she uses groups that are on different sides of the room so there's no problem with congestion. Then she calls all group leaders to the front to get their manipulatives and hands them out one at a time. Apparently Larry hasn't responded to her call, so she very quickly says, "Larry, ready?" to get him to come up and get the manipulatives. It's a very positive way to keep Larry doing the task he's supposed to be doing.

This management style is very integrated into the rest of the class. In other words, Victoria doesn't stop the entire class for off-task or inappropriate behaviors. With just a quick comment or action she is able to pull students back into the flow of the class.

Victoria's management style is very directive in nature; but it is not always necessary or even advantageous to use such a style. However, it is important to realize that in the past Victoria has had some behavior problems. Many of the children come from a very depressed neighborhood and simply don't have a lot of structure in their home lives. In these cases it is important for the school to provide that structure. It is also more difficult for these children to know how to behave in a structured environment. Victoria, therefore, has chosen to provide a structured learning environment by incorporating many procedures into an overall directive management approach.

**Teaching Strategies**

NCREL/IU

From this event it is clear that Victoria's teaching style is a combination of behavioral and cognitive strategies. While working on lessons, she places great emphasis on the children's thinking about and discussing problems. They must be inventive. However, during transitions, Victoria emphasizes orderly and efficient behavior. Often too much time is spent in the classroom on things other than learning. Victoria's strategies are essential to maximizing the time she has to engage the children in mathematics.

# Event 7

# Story about Jeremiah – Introducing class problem and assigning roles

**Classroom Management**

NCREL/IU

The children have put their labs away and gotten manipulatives. Everyone is trying to get settled again. Victoria says, "The story is about Jeremiah... let's see who's ready?" and immediately praises Jeremiah for being ready. By praising Jeremiah, Victoria gets the other students focused in on her and she gives them time to get settled so she can start talking about the story. She also has Jeremiah's Rocklord, and she walks around showing it to the class.

At this particular time, the manipulatives have to do with things that the students collect. As a reward, the students get to bring in their collections and have the story problem be about them. In order to be able to do that, they have to be cooperating in their group and working well together. If they've been having trouble with someone or in class, they are rewarded as soon as they start to behave appropriately or to get along or to work well in the groups.

**Problem Solving**

Victoria Bill

I usually don't even like to read the problem off that paper. I would prefer to have big garbage bags and say, "Here they are," and have something in them and shake. I mean real life. I go on to say that Jeremiah's going to give Michael 84 Rocklords. Well, that involved Jeremiah and Michael. I could have made it a problem for the whole class maybe by saying, "Well, it's gonna take 84 for all of us to play with him"— all of us in the primary and they know that's first, second, and third. "Do you think he has enough? Will he have some left over?" Maybe we can invite more children to play with them. In other words, get more children involved than just Michael and Jeremiah. The children all know they have a part though. But now, I make much more of an effort to make it a true dilemma for all of us.

When I said, "Now go discuss it," I lost the whole feeling of my story. I've turned it into math class. My fear of turning it into math class is that they're going to pull out those numbers and just do a procedure. For example, one of the groups went to the three numbers and just added them all together.

The other groups added two of them together and then subtracted it.

| Event 7 cont. | Story about Jeremiah – Introducing class problem and assigning roles |
|---|---|
| **Problem Solving, cont.** | |
| Victoria Bill, cont. | I didn't do enough build up of that story problem in the beginning to make them feel comfortable, and I didn't even hear the language throughout. Usually they don't just talk about numbers. There's a lot of talk about Rocklords. It's more of a maintenance of the story. Those popsicle sticks really stand for those Rocklords. |
| Pete Kloosterman | The problem she's working on with the 84 Rocklords out of three bags is a fairly straight forward problem. It is a good way to get them to talk about it and discuss it<br><br>We just saw the buckets of sticks that were rubber banded together. There are teachers who worry that they can't use manipulatives because they don't have any money to buy them. You can see that you can buy all of the sticks that you are ever going to need for a few dollars. A lot of things can be made by hand or out of paper, so materials should not be an obstacle if you are creative in how you put activities together. |
| NCREL/IU | The roles of the group members really represent the components of the problem-solving process. When a student is assigned a role, it does not mean that he or she is the only one responsible for that activity. Indeed all students are responsible for all of the activities. The assigned student is responsible for making sure that the group does not forget about a particular activity. He or she is also responsible for asking the questions that are attached to the role. Thus the recorder will be vigilant for what should be recorded and when; however, it is something the group discusses and each individual does. In later events, notice that when Victoria visits a group she directs questions to particular students; and their roles are not relevant to that questioning.<br><br>The primary reason for the collaborative group is to provide a mechanism for the students to be able to talk to each other about the problem. This serves several purposes. It engages them in talking mathematically—creating a mathematical discourse community. This is one of the instructional objectives. It also provides a mechanism for the students to test their ideas, both by presenting them and by comparing them to the ideas of others. Finally, it provides an opportunity to evaluate the ideas of others. Since Victoria focuses on problem representation, there is no single correct answer. So the group discussion is not a competition between ideas but rather a sharing and evaluation as to whether each representation is adequate. |

| Event 7 cont. | Story about Jeremiah – Introducing class problem and assigning roles |
|---|---|
| **Problem Solving, cont.** | |
| NCREL/IU, cont. | The group structure—assigned roles but each member is responsible for the activity—is a mechanism for promoting these goals. |
| **Teaching Strategies** | |
| Victoria Bill | Ideally, I try to use items that they bring in. I believe that whole month was focusing on things that they collect. Very often I pick a focus. It could have been even more meaningful to have a couple of bags sitting in the middle. Kids like exaggerated numbers. How many kids have a collection of something that large? That made Jeremiah feel great. |
| Pete Kloosterman | The idea that this problem is about Jeremiah and his Rocklords is a nice little trick. Personalizing the problems keeps the kids more interested and involved in the things that you're doing.<br><br>She is obviously a very high-energy, very excitable person who is delighted to be in front of the classroom. There are people who would worry that they can't be that bright and cheerful all of the time. You have to be friendly and firm and warm with the kids, but you don't have to be bubbly the way that she is all of the time. There are a lot of good teachers who aren't nearly that bubbly. |
| NCREL/IU | Victoria spends a long time on a problem. She is able to do this because she addresses many skills in the context of the problem. In this problem, the children are involved in numerous instances of rounding, estimation, equations, addition, subtraction, and counting by tens. This contrasts to end-of-chapter problems that address only one skill at a time and consume minimal time. The extended problem is a very important teaching strategy. By using it as a vehicle for numerous objectives, the problem provides an anchor for the children's thinking; that is, it provides a context for thinking. This is more like the way we engage in learning every day. As teachers, we think about our classroom and think about mathematics for the students—our teaching needs provide the problem context for a wide variety of mathematical thinking. |

**Teaching Strategies, cont.**

NCREL/IU, cont.

Victoria's use of cooperative groups has evolved during the year. At first she used informal pairs—two students sharing ideas about their work and solutions. Probably what Victoria would do is read the problem and say that now she needs to retell the story. So she would just start talking about the story. The students now don't just re-read the story, they put it in their own words. So she had to show them how to do that and they've practiced that throughout the year to get to the point where they are now. When it came time to be a checker and to check each other's understanding, Victoria may have asked the kids questions, modeling the types of questions that they would be asked, and that they needed to ask each other.

For each one of the roles, she had to build each role in. She might start with retelling the story and then add the other role of checking, being more explicit that this is what a checker would do. Then she might add another role and then another. She might do that before the students got into their groups of two or three so they could see the process that they needed to go through. Of course it's one thing to know what the role is; it's another thing to do it—and so she might have the kids practice asking each other questions that a checker might ask. So she probably went through it that way, very systematically, taking one small piece at a time and building on each piece.

Finally, when she shifted toward using formal cooperative groups of three and four, she first discussed the roles the students would play in this situation. The students actually came up with the ideas for the roles that they would play and the questions they should ask. These roles, however, had developed naturally from the foundation that Victoria had already laid for their development.

**Assessment**

NCREL/IU

The reason she is reinforcing the fact that they discussed well in their group may be that in the past this group had difficulty working together or staying on task. By guiding them and reinforcing them in the steps they are using, she is making sure they get off to a good start. It may well be, in fact, that Victoria chose to go to this group first based on her assessment of their prior abilities and/or problems they may have encountered in the past.

**Classroom Management**

Victoria Bill

Now when you're building team roles, naturally, you would have said, "I like how team 1 discussed...," "I heard this...," or "I heard that...." So it's more than saying I heard you discuss; it includes what is discussed. It includes praising them and getting over to that sticker chart right away because they're watching.

They're building up for something, maybe a sundae or a pizza with me at lunch, or a bag of chips and playing the record they want to hear. Every term it's a new chart. Some teachers just give them a sticker, and that's okay, that's enough.

Pete Kloosterman

Stickers or other sorts of rewards in the short term are usually necessary to get kids to do the types of roles that you expect. Eventually you'd like to not have any sort of rewards; you'd like the kids to be working so well that you don't need to reinforce that.

It will depend on the students you have and your situation as to how quickly that can happen. I've seen situations where within a couple of weeks there were no external rewards, everything worked pretty well. In other situations, rewards were still needed after several months for some kids to stay involved in the groups.

NCREL/IU

Reinforcing the roles establishes that in the group each child has a particular role to play and that keeps them on task. Victoria reinforces group behavior by using the sticker chart or through verbal praise.

**Problem Solving**

NCREL/IU

Victoria is performing a mentoring role here. She is modeling the phases of the problem-solving process she designed, which involves the students retelling the story so Victoria can determine what they know about the problem.

**Problem Solving, cont.**

NCREL/IU, cont.

Victoria uses probing questions that go beyond the students' roles. These questions are the same questions she would mentally ask herself as she goes through the problem-solving process in mathematics. In this way, the students begin to pick up on the whole way in which they should approach any problem.

Further, these questions help the students internalize the problem-solving process to the point that they begin to ask these fundamental mathematical questions every time they approach a problem they wish to solve.

**Teaching Strategies**

Pete Kloosterman

Once groups are working, it's best to stay away from making comments to the whole class because that interrupts their train of thought and tends to get them away from what it is that you want them to be working on. From time to time, though, you have to do it.

NCREL/IU

Victoria seems rather surprised when the suggestion is made that all three bags be put together in order to have enough Rocklords to give 84 to Michael. Notice that she catches herself and is careful not to show disapproval or to tell the students they should do it her way. Instead, she realizes that although adding two bags will provide more than enough Rocklords to do as the problem asks, adding three is a perfectly acceptable way to solve this problem. It's just a different perspective.

By allowing the students to solve the problem their way, Victoria helps the students build confidence in their ability to solve mathematical problems. This confidence cannot be overemphasized as a necessary aspect of any child's development in mathematics.

**Assessment**

NCREL/IU

Victoria must quickly assess how the group is working together and where they are in the problem-solving process. This gives her some ideas about where to start asking questions. These questions are tailored to the group based on this initial assessment.

Victoria then continues to assess each individual member of the group by asking him or her questions. If a student is having difficulty, she then continues questioning until the difficulty or misconception is cleared up or the student has some strategy for dealing with his or her problem.

**Classroom Management**

Victoria Bill

Beginning teachers need to have a way of saying to the students, "I need you." Usually in my room it's a signal with my hand. I raise it, and they know to "give me your eyes." If my hand is coming forward, it means "settle down." I have different signals.

Pete Kloosterman

Notice, the student who didn't have a pencil. She quickly dealt with it and got it out of the way rather than dwelling on it. Some teachers will get frustrated or upset because students should be able to keep track of their own pencils. Victoria did a nice job of just getting on with the task at hand and not worrying about the superfluous things.

NCREL/IU

One of the students, Rafael, in this group has become distracted by either the cameras or something else. Victoria quickly draws his attention back to the group by saying "I want your eyes here." This strategy doesn't interrupt the group work but gets the student back on task. It is an excellent classroom management technique.

In another instance, one of the children didn't have a pencil. She quickly provides a pencil for the boy. In both cases she dealt with the problem very quickly without interrupting the group.

**Problem Solving**

Victoria Bill

The child's definitely going on in a very rote way. And knowing that he usually does that in his group, I'm purposely disrupting his method and throwing in questions that will throw him off track and force him to think.

**Problem Solving, cont.**

Victoria Bill, cont.

In our room, when we estimate, we put the number down and circle it at the top. Usually I don't like to do estimation when they solve a problem because the kids learn, "Well why bother, I'm going to solve it anyway." So if I do estimation, I just estimate. I let them check with a calculator.

NCREL/IU

The students here have used the manipulatives to represent the problem statement. They have placed the appropriate number of popsicle sticks in each of three cups to represent the number of Rocklords in each bag. This allows the students to concretely represent what they know about the problem. In fact, they are answering the first question through the use of manipulatives.

**Teaching Strategies**

Victoria Bill

The kids learn very quickly that we have children on different levels in the room. He needed to practice counting tens and ones; that's where he is. So it was an opportunity to assess him at that point—to give him practice doing that.

NCREL/IU

Victoria asks the students to estimate here first before they actually solve the problem. She emphasizes the use of estimation as a part of the problem-solving process. Estimation in this context is used as a constant self-check mechanism. It allows the individual to determine if a proposed solution seems feasible and if the end result of that solution—the answer to the problem-solving process—is reasonable.

Another reason for doing estimation here was that one of the students didn't think there would be enough if they added the two bags of 71 and 23. Once again, Victoria helps the child determine for himself through estimation that adding these two bags together is plenty.

It's interesting to see Victoria become a member of each group when she walks up and begins talking to them. Here, for instance, she doesn't ask the same questions she did with the first group. This group has already answered the first questions, so Victoria adapts her questions to where this group is within the process. This keeps the children on task and doesn't interrupt their thought processes or the group process as a whole.

**Problem Solving**

Victoria Bill

This group is definitely at a different level. They're still working on assigning jobs to everybody and that's okay.

Pete Kloosterman

Obviously she's spent time going over group roles before. With groups of three or four, roles become very important. They keep the groups doing what they are supposed to be doing. You're going to have to go over those roles and what they mean, or kids are not going to be involved in them.

**Teaching Strategies**

NCREL/IU

Group roles and shared interdependence are being reinforced here when Victoria praises the children when each of them takes the responsibility for placing the appropriate number of manipulatives in one of the cups. This group is not as far along as the previous group in the problem-solving process, but they worked out a way for each person in the group to be involved, and so Victoria rewards that work. Victoria is willing to take the students where they are; and, in this case, they must work out their roles before they can proceed with the problem-solving process.

## Event 11

## Begin whole-group debrief–Jeremiah modeling the reader's job of retelling the story

**Classroom Management**

NCREL/IU

When Victoria says "I want to discuss," that's a signal for the students to stop working in their groups and to focus on the whole group discussion.

When Victoria says, "Record," that's a signal for the students to write down what she is putting on the board or to write another group's solution in their notebooks. This gives the children specific responsibilities in the group discussion and keeps them on task during that discussion.

**Problem Solving**

Pete Kloosterman

This is a fairly typical second-grade problem. We can have 71 + 23 + 35 as one problem, find the sum of that, and subtract 84. She is getting them to discuss the problem and think about different ways of doing it. Each group has gone about it in a little bit different way, which is nice. In mathematics we tend to think there's a right way and wrong way, and if you don't do it my way, you're doing it wrong. That is not the case. Kids need to be flexible and realize that there are lots of different ways to get an answer.

NCREL/IU

Beginning with collaborative groups and then moving to whole-group discussion gets all of the children involved in thinking about the problem. It also permits Victoria to see what sort of alternative representations the children are generating. She can then use them in the whole class discussion to show how to achieve the same goal in several different ways. Having the alternatives come from the children rather than Victoria gives them ownership.

The movement from collaborative groups to whole class also permits the children to see Victoria modeling the thinking strategies that they were just engaged in. It is valuable to let a child engage in a problem before modeling it since then the child will have a better sense of what is important and not important to the process. Finally, this approach permits Victoria to focus the children on particular issues or problems.

We would expect that when beginning this overall teaching method, Victoria used guided, whole-group discussions most of the time; and as the children gained proficiency, she let them work longer and longer on their own. It is like learning to ride a bicycle—we begin with training wheels and then give less and less support. Victoria's directing of the class is a form of support.

# Begin whole-group debrief–Jeremiah modeling the reader's job of retelling the story

**Teaching Strategies**

NCREL/IU

Victoria brings the children back into a whole class discussion. Notice that she begins by saying that she wants to discuss. Then she indicates the need for a reader to retell the story—one of the roles in the collaborative groups and one of the first Problem Solving tasks. In essence, the children have worked at Problem Solving and now Victoria is going to guide them in the process. She is modeling the process but at the same time engaging them in the process.

Notice that even when Victoria directs the class, she never gives children answers or identifies something as "wrong." The goal is to promote student thinking about the problem. There are two ways that two bags can be combined (35 + 71 or 23 + 71), and she allows the students to determine how the combining occurs—later the third alternative will be considered.

Since this class does not use a textbook, each student is responsible for keeping track of the discussion by writing everything in his or her notebook. Throughout the lesson, as Victoria writes on the board, she cues them to write in their notebooks. This gives Victoria a record of what the students have done during the discussion and holds them accountable for staying with the discussion so that they can quickly answer any questions that she asks. Her questions are short and quick and the students must understand the discussion in order to answer her questions when called upon. This all leads to active participation by the students. It keeps them on task, accountable, and in the flow of the discussion.

# Event 12 — Whole-group estimation – Estimating to determine if there are enough Rocklords

**Assessment**

NCREL/IU

When Victoria says, "It comes between 70 and 80. Which is it closer to? You record," she is using the recording procedure as a quick assessment of the students' ability to estimate. The procedure may also be used to determine if the students are on task and keeping up with the discussion.

Later, Victoria can use the notebooks as an overall assessment tool for the lesson to determine if the students are performing as expected and if there are any systematic errors in the mathematics. In this way, Victoria always has some idea about how the students are doing with particular objectives and whether or not they need extra work in a particular area.

**Problem Solving**

Victoria Bill

I said, "How could I check?" They have to learn how to check themselves and how to regulate themselves.

I use specific language, usually like "Round it off to the nearest ten." The students hear that language now, so when you're getting up into hundreds and you start saying, "Round it to the nearest ten, round it to the nearest hundred" you've got that built in.

Pete Kloosterman

She is talking about estimating. When she says "estimate," she also says "round off," which is what we often think of as estimating. You want kids to be flexible in how they estimate. Another way is to say, "Seventy-one, how many tens is that near? Twenty-three, how many tens is that near? How many tens do we have altogether?"

By focusing on place value—how many tens we have and combining tens and not worrying about the ones—you can really set them up for more advanced strategies at a later time.

Here we're talking about the tens, from 60 to 70. They started counting at 60 and ended up with 70, which is actually 11 numbers. They only used 10 fingers. They've obviously done this before. They know that when they hit 61, that's when they bring up the first finger rather than on 60, the first number said. Counting on fingers is great. The kids need some way to remember things; fingers are as good a manipulative as popsicle sticks, Cuisenaire rods, or base 10 blocks.

# Whole-group estimation – Estimating to determine if there are enough Rocklords

**Teaching Strategies**

Victoria Bill

Having the children put their hands on their heads was a quick way of assessing everybody. You can tell whose hands are going up confidently and who is just following along.

I've set a routine that when my hands go up, your hands had better go up with me and you had better finger count. Many of them don't need finger counting, but we're doing it all together and it's my way of assessing. I can tell who's with me and who's not.

You get a signal "Record, let's go," and when it's during the whole class lesson it's independent. Usually they put a shield up. Usually the hands go up right away. They shield, and that means it's independent work and that's my means of assessing their understanding. I usually get around that room and it tells me where I want to go. I can take the response that I want, whether its an incorrect one or if someone came up with a different method.

Pete Kloosterman

A lot of teachers would have been uncomfortable leaving the student with that much time to come up with the word estimation. The average wait time in some studies is about a half a second before the teacher begins to help the student with the answer. Let them stop and think about it for a number of seconds before you try and give them a hint or let another student help them to get an answer.

One of the nice things about the demeanor in this classroom is that mistakes are not looked at as bad. Certainly, students make mistakes. They treat each other as though it is part of what learning is all about. It's all right to make a mistake. They discuss it until they get it right, which creates a very positive atmosphere. Many teachers think that a kid's self-concept or self-esteem will be ruined if he or she makes a mistake and anybody else knows it. In fact, I think that self-esteem comes from being able to deal with making mistakes, knowing that it's all right and that everybody makes them. Mistakes can be corrected and that's what learning is all about.

**Teaching Strategies, cont.**

NCREL/IU

The recording mechanism or procedure that has been established in this class performs four important functions. One, the children must keep track of the discussion by recording in their notebooks the work on the board and other information that Victoria deems important or relevant to that discussion. Two, the children are often asked to transform the verbal discussion of the mathematics into a symbolic form; that is, they are asked to take what they have heard in the discussion and translate that into arithmetic sentences or equations. Third, the idea of multiple perspectives is reinforced by having a record of all the different approaches or solutions to a problem. By making the children accountable for understanding all of these solutions, Victoria is helping to broaden the children's understanding of mathematical Problem Solving. Finally, the children are asked to make predictions or answer questions that Victoria poses. This not only helps keep them on task and thinking about the problem, it also gives Victoria a quick assessment tool she can use to determine if the children are following and understanding the discussion.

Note that when Victoria uses the mathematical term estimate in this lesson, she also uses words or phrases like "it's about how many?" "round it off," or "it's close to...." These phrases help pair the words of the students with mathematical terminology so that she can then help build their math vocabulary.

When Victoria puts the circle over the bag, it's a cue that she always uses to indicate estimation. This is just a convention that Victoria has developed for her own class. These conventions are nice as teaching aids; however, it is extremely important to remember that once they are adopted, the teacher must be consistent in their use.

# Event 13 — Whole-group debrief – Jeremiah and Michael represent the problem correctly

**Problem Solving**

Victoria Bill

When kids are first doing these problems, they want to take that first set. We as teachers say, "You have to take the ones." Some teachers are resigned to the fact that kids like the tens, and they're going to start with the 70 and the 20, so they force them to say 70 and 20. Well, that can become just as rote. Kids usually want to take the whole, that first part, and bring the whole thing down and start with the 71. You have to say, "Can you give me some of it to start? I can't work with all of it. I can't think about that. Give me part of it first. What part do you want?" Then you can break it down to the tens and the ones.

They had the fingers; they had it linked to the formal, but then went back to the concrete. You can take reversals here. When it's something I want automatized I go to the formal. I always give the kids who still are going need a tool an opportunity to use their fingers and then finally go on to the concrete. When it's something I'm building, I go to the concrete, maybe with some fingers or to the actual item, and then to the formal. So, it depends on where I am in the development stage with them.

Pete Kloosterman

The student has just done the problem using what I'd call the "front-end strategy," where you look at the highest place values first. You add the 70 and the 20 and then add the 1 on later. Many second-grade teachers would cringe at the thought of doing that because we are starting at the left. In standard addition, you always start at the right according to the rules of the algorithm. But, in fact, what we want kids to do is to be flexible in the way that they think of numbers and to recognize that there's not always a rule to follow. If you can think of it in terms of 70, 80, 90 and then add on the 1, that's perfectly acceptable. There's no reason why you can't do that.

There is a lot of research indicating that when you ask kids to do a problem in their heads, it's much easier to start with the largest place value first. In fact, those kids who are good at doing mental computations have figured it out on their own. Say we had 79 and 78. If when you add those two together you add the 8 and the 9, you have to think about carrying or regrouping that ten. If you think about 78 and 79, and add 70 and 70 to get 140 first, then think of 8 and 9 as 17, then your answer is 140 and 17 equals 157. That's an easier way to remember it. It doesn't require as much memory space as the paper algorithm does.

**Problem Solving, cont.**

NCREL/IU

The incorporation of multiple objectives into the lesson is key to Victoria Bill's approach to teaching mathematics. At this particular spot in the lesson, she incorporates place value, estimation, addition, subtraction, multiplication, and counting by ones and tens. She has purposely designed this lesson in this way to accommodate all the children in the class. Many of these children are on different mathematical ability levels, and this problem gives each of them an opportunity to practice their skills in the areas in which they may be having particular difficulties.

Notice, then, how carefully Victoria chooses who answers a question. In this case, she calls on a student who has been having difficulty with place value to answer a question that deals with place value. By constantly assessing and understanding how the students are progressing, Victoria is able to allow the children to practice skills when they most need it.

**Teaching Strategies**

Victoria Bill

Usually what you're doing is building; you're working on place value. You have many subgoals in a lesson, too. But my goal in this lesson was to perform this problem mentally. So, if they're going to perform this mentally, these kids are usually not working on place value. I believe at this time I only had two children working on place value and understanding. Do I abandon it? No. I keep it up for those two kids—maybe not in this particular lesson, but later on

Nakeesha was right, but she was not sure of how to say it. I let her talk and didn't let her feel she was wrong. But she is definitely a child who needs to work on place value.

Steven did exactly what my goal was for that lesson, and I didn't listen to him well. I said, "Well, we're working on this right now." Ideally, I should have gone right over beside him. We have two sides of our notebook—A and B—so there can be two methods working at the same time. I could have gone and put his problem there. He went to the 35, I believe, and then took 10 and then later on, I think, he went over and took the 3 from the 23. So he really worked with all three amounts and just pulled out his and then added what was left. I mean, he had the strategy that I was looking for.

**Teaching Strategies, cont.**

Pete Kloosterman

By asking them to record the next step, she obviously thinks that they're able to do that on their own. They should be because they've gone through the problem different ways. What most teachers would have done was to quit before they had discussed alternative solutions to the problem. By getting kids to verbalize and talk about their different solutions, it helps them to see that there are several ways to do the problem.

NCREL/IU

Throughout this lesson, Victoria has the students count along with her. Here, she has them count the 3 in 23. This reinforces the fact that there are three ones. Earlier she has them count by tens to reinforce the notion of a "tens" place value. By having the students use their fingers, Victoria is letting them know that it's okay in this class to use fingers as manipulatives. They are easy to use and always with you. Aside from that, the counting is a way to keep the students actively involved in the lesson. It's just too long for the children to sit while she asks one or two students to hand her manipulatives or perform some operation.

The overhead is used to show the manipulatives that are being used to the entire class. Notice that she has her transparency divided into three parts. The most important division is from top to bottom. She is using this to illustrate the part/whole relationship between the numbers of manipulatives. The top part represents the whole, and then she pulls down the parts that are being used. In this case she pulls down the 71 and the 20. She leaves the three ones at the top since they are not being used at this time.

In order to build interest and keep the children motivated to work on the problem, Victoria picks up the Rocklord and the two of them look at the work that she has asked the students to do. This not only is motivational, but ties back to the original problem—reminding the students that the actual problem to be solved involves having and then giving away Rocklords.

| Event 14 | Whole-group debrief – Jeremiah and Michael modeling the problem concretely |
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| **Classroom Management** | |
| Victoria Bill | The students are praised for these problem-solving methods of working backwards. So had I been able to get to my cooperative learning chart, I would have automatically said, "I like what Iris just did. What did she do? She worked backwards, that's neat." I would have praised her to the rest of her group. |
| NCREL/IU | The problem was written using Michael and Jeremiah's names. The students must have earned this privilege. This is now a really big deal for these two to have the opportunity to go to the front of the class and act out the problem. This is their reward for behaving well in class or getting along, sharing with each other, or something of that nature. At the same time they are linking back to the whole class. By having Jeremiah physically give the 84 manipulatives to Michael, Victoria is linking the concrete physical process with the abstract answer that the class has been working toward.<br><br>Getting a sticker in this class is usually a reward for either performing well in a group context or for expressing one's own thinking clearly in the context of the whole class. Here the emphasis is on being willing to volunteer a solution approach to the class and expressing it well. Iris has thought through her ideas and expressed them without reservation to the class. |
| **Problem Solving** | |
| Victoria Bill | Brent used a check to mark the number from the problem. They have really come up with these self-regulating devices. The checks were their creation. I saw some circled and I asked them what they were doing. Some were underlining things, some were crossing them out completely. Now I objected to that, I want to still see it in the end, so we can go back and look at our work. But they really came up with their own devices. Everybody does their own thing. As far as working up at the board, every day I choose different devices just to show everybody that they're all okay. When we're doing these inventive methods, they do need some way of regulating getting every quantity in. |

| Event 14 cont. | Whole-group debrief – Jeremiah and Michael modeling the problem concretely |
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| **Problem Solving, cont.** | |
| Victoria Bill, cont. | Ideally, the goal within another week or two would be to make three columns on that board. Then I would tell them that as soon as their group has solved it and has an equation, and everybody understands it and has the same problem recorded, the group leader or the checker should come up and record the problem on the board. So then, ideally you end up with three different methods on the board and you take your colored chalk and you start mapping the three problems at the same time, but you do it very tightly with the overhead. More so at that point than at this level. |
| Pete Kloosterman | We know that students learn better by talking with each other. In the real world, you don't do mathematics by yourself. You do it with other people and you have to be able to communicate what you've found out about mathematics to somebody else. And that is one of the reasons that there has been a big push for more discussion as part of mathematics classes. |
| NCREL/IU | Victoria asks for another way to solve the subproblem of addition. The first child adds 71 and 20 by adding 70 and then 20 and then the 1. This child has taken 71 and added 10 more, which is 81, and 10 more which is 91. This is an alternative solution to the addition problem they are presently working on. This is simply a part of the solution to the overall problem, but it is interesting that even here with simple addition Victoria is exploring solution alternatives.<br><br>When Victoria says, "Now do you have enough to give 84 away?" she focuses the students' attention back on the overall problem they have been trying to solve. They have until this point been preoccupied with the problem of adding together the number of Rocklords from the two bags. Now they must continue with the next step to finish solving the problem. At the same time, Victoria is using words that the students understand to help them think about representing subtraction in the next portion of the problem. She then calls on a group that has used subtraction to illustrate the solution to this part of the problem. |

# Whole-group debrief – Jeremiah and Michael modeling the problem concretely

**Teaching Strategies**

Victoria Bill

When I asked if it was my whole set or part of my set, they were right. While you're teaching, you have to learn to think, "What could they possibly be thinking?" I mean, they're right—it wasn't the whole set. But then that becomes a difficult issue—how to work with the whole within the parts. I thought it would be difficult, but it's not. I think one of the keys is moving to the individual pieces of paper as opposed to the part-whole device you see on the overhead.

Usually, you let a child start the counting and by the time he does one or two tens you can tell he understands it. Then I get everybody else actively involved by usually just nodding or raising my hand. On a normal day, it would be enough to get them all involved.

I saw that Iris was getting confused. A better technique would have been going back to the overhead then, or drawing in somebody else at that point. I really don't stay with one child. Someone starts it and another child usually has to pick it up because they have to know my thinking and I have to know their thinking. That's why they have to listen so intently to each other's method. But, that's the only way we're going to get children to move to different levels.

It's okay to disagree in my room as long as you say why you disagree. They learn to go through a problem and say, "What have we used, what haven't we used." It's a process of regulating—of checking.

NCREL/IU

When Iris says, "I want to use 80, not 84," Victoria immediately changes to do it Iris' way. In this class using your own approach to the solution of a problem has been so encouraged that Iris doesn't hesitate to voice her ideas. This is quite a change from a traditional classroom where the student would be expected to go along with the teacher's solution. Instead Iris is responsible for her own work, and, therefore, the teacher must respect and encourage that.

| Event 15 | Group 5's solution – Providing an alternative strategy |
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| **Assessment** | |
| NCREL/IU | Victoria performs a final assessment of the children's work as she goes around and checks the notebooks. She is also assessing the problem at hand, which involves adding four numbers together: 16 + 23 + 1 + 5. This gives her some idea about how well the children are able to handle this slightly more complex addition problem so she knows what to concentrate on during the next class period. |
| **Classroom Management** | |
| Pete Kloosterman | That little comment, "I need you, Jeremiah," took about one second to pull Jeremiah back into the discussion. She doesn't dwell on discipline; she does it real quick and pulls in the kids she is losing from the group.<br><br>By going around and having the kids whisper in her ear, it's implicit that she expects every student to come up with an answer and that she believes that every student can learn mathematics. |
| NCREL/IU | Allowing the students to whisper this final answer in her ear is Victoria's final way of giving the students a reward for their work in this class. This has been a very lengthy math class for second graders and they have been asked to concentrate during virtually all of this time. Through the use of a variety of teaching strategies, Victoria has been able to keep them actively engaged in the process of learning. |
| **Problem Solving** | |
| NCREL/IU | One group worked the problem using the bags of 71 and 23 while another group used 70 and 30 from the bags of 71 and 35. Once they had 70 plus 30 Rocklords, this group immediately determined they had enough to give 84 away. Victoria promotes going through the process this second group used in order to see an alternative to the first approach. This is another example of the use of multiple solutions to a problem in order to enhance the students' understanding of the mathematics involved. |

## Group 5's solution – Providing an alternative strategy

**Teaching Strategies**

Pete Kloosterman

She's engaging them in an activity to check and think of the problem again. This is very useful if it gets the kids thinking about the problem in a new way. As a teacher you have to decide if the kids need to move on to something else. The more ways you can look at a problem and get something out of it, the better chance the kids have of seeing connections among different mathematical ideas. But when you run dry and there is nothing else to say, it is time to quit and move on.

NCREL/IU

There is one girl here at the end who is having difficulty in adding the four numbers together. Victoria suggests a strategy of breaking up the problem into several steps as a way, which the girl proceeds to do. This has a strong feeling tone quality in that Victoria is patting the student on the back and touching her on the cheek. This has a motivational effect on the student. It also shows that Victoria has high expectations for the students. In fact, the touching indicates to all the students that she believes they are capable of doing this problem. Finally, the feeling tone and touching becomes a reward as the child figures out how to implement Victoria's suggested strategy. Note that Victoria is not interested so much in the answer this girl gets, but simply in the fact that she has been able to implement a strategy that will lead her to the answer.

# Event 16 — End – Following the procedure for leaving the class

**Classroom Management**

Victoria Bill

You give each student a turn to line everyone up. In other words, they're taking charge of this room. It's their room and they're lining the students up, and they monitor the behavior.

For the year, you set down your routines. Any teacher has his or her room the way it's going to be in the first five days. Usually day one and day two I work on routines of behavior, such as routines for the classroom lining up, hanging up their belongings, and sharpening pencils.

Pete Kloosterman

She's got them trained so they don't spend a lot of time putting the manipulatives away and that's really good. You need a system to quickly put away materials.

When she says, "Take the Rocklord and line people up," you don't have a clue what it means. But the students obviously know what she means and are ready to follow that routine. You don't want to make math dull by doing the same thing day after day. You want to spend your time varying the mathematics and not worrying about the management. And that's what she does very quickly and easily. She has them used to her routines of what to do next. They are ready for it, and it doesn't take long to make a transition from one activity to another.

NCREL/IU

"Okay, let's stop" is a signal for the ending procedure in the class. Notice that Victoria also uses a nonverbal hand gesture to indicate closure as well. At this point the students know to put away their manipulatives and notebooks and then wait until called on in order to line up and leave. Victoria gives the Rocklord to a student to use to line up the children. This is a final opportunity to reward a student in this particular class.

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