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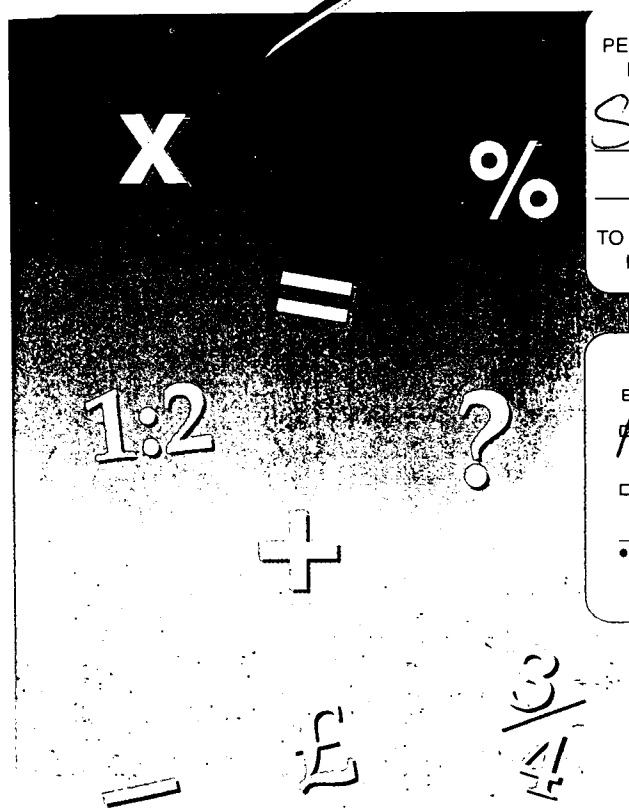
ABSTRACT

This document is an information resource for teachers based on the findings of the Assessment of Achievement Programme (AAP), a major research program funded by The Scottish Office Education and Industry Department. This book presents information about the 1994 AAP mathematics survey and contains sections with sample survey items on student achievement; problem solving and inquiry; information handling; number, money, and measurement; and shape, position, and movement. Teachers can use the information presented here for personal study, staff meetings, and staff development. (Author/DDR)

5-14 links

MATHEMATICS

Feedback



Information for teachers – from the Assessment of Achievement Programme

WHAT IS *FEEDBACK*?

Feedback is an information resource for teachers, based on the findings of the Assessment of Achievement Programme (AAP), a major research programme funded by The Scottish Office Education and Industry Department. This *Feedback* presents information about the 1994 AAP mathematics survey.

Read *Feedback* to discover:

- ❒ the **aspects** of the 5-14 guidelines which were assessed.
- ❒ the **key findings** of the survey, against which you can compare the attainment of your own pupils.
- ❒ **teaching issues** based on the AAP survey findings.

Use *Feedback* for:

- ❒ **personal study** – reflecting on the achievement of your own pupils, reviewing assessment of class work and assessment tasks.
- ❒ **staff meetings** – reviewing, collectively or in groups, teaching programmes in relation to the 5-14 attainment targets.
- ❒ **staff development** – considering effective teaching strategies, and helping student teachers as part of their professional training.

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Please note: you are free to photocopy or adapt *Feedback* for use in your institution.

Introduction

The Assessment of Achievement Programme (AAP)

The Assessment of Achievement Programme (AAP) was established by The Scottish Office Education and Industry Department (SOEID) in 1981 to monitor the performance of pupils in Scottish schools in particular areas of the curriculum. Since 1983, there have been regular surveys in three core curricular areas – English language, mathematics and science. The main objectives of the AAP are to describe national levels of attainment and to provide evidence about changes in these levels over time. The surveys are intended to inform the SOEID, education authorities, teachers and other interested parties about the achievement of pupils, and to indicate ways of improving teaching and learning.

The 1994 mathematics survey

The fourth AAP survey of pupils' attainment in mathematics was carried out in 1994 by a research team at the University of Strathclyde. This team also carried out the third survey in 1991.

Samples of pupils at P4, P7 and S2 stages were selected to be representative of pupils in all mainstream schools. Over 9,000 pupils completed the assessments.

The survey involved both written and practical mathematics tasks, including practical problem solving.

Assessment was based on the curriculum defined in *National Guidelines Mathematics 5-14*. The main reporting categories adopted by the survey were *Information handling*; *Number, money and measurement* and *Shape, position and movement*. Reporting sub-categories were closely related to strands of the 5-14 guidelines.

AAP and the 5-14 curriculum

The assessment tasks were matched to levels A - E and 'beyond level E' as these are defined in the attainment targets of *National Guidelines 5-14*. Members of the 1994 AAP Mathematics Project Committee advised on the match of all new tasks to 5-14 levels and an independent consultant re-examined the match of tasks to levels in the 1991 survey.

What Our Pupils Achieved

In general

- Attainment in many of the tasks was good. For detail about pupils' performance, readers should consult the fuller report *AAP Mathematics 1994* published by SOEID. Inevitably, since the purpose of this document is to raise issues relevant to teaching, the examples chosen often illustrate poor performance or interesting patterns of error.
- Performance in *Information handling* was good. There was evidence of progression in problem-solving skills from P4 to S2, but in extended problem-solving tasks pupils preferred a random, rather than systematic, approach and were reluctant to show working or to handle materials. There were weaknesses in *Number, money and measurement* and in *Shape, position and movement*, particularly at S2.
- Performance was higher if items were in a context which encouraged a method different from the common algorithm and lower when the context was unfamiliar or involved new terms or concepts.

At P4

- Pupils' performance was poorer than in 1991 in *whole number* in basic processes and applications.
- They found information from tables well but had more difficulty interpreting graphs and tables.
- Girls performed better than boys in *Information handling*, but there was little difference in the other attainment outcomes.

At P7

- Pupils' performance was poorer than in 1991 in *whole number* and *decimals, fractions and percentages* in both basic processes and applications.
- They found information from tables well but had more difficulty interpreting graphs and tables.

- The performance of boys was better in *Information handling* and in *Number, money and measurement.*

At S2

- Performance was poorer than in 1991 in *whole number* and *decimals, fractions and percentages* in basic processes and applications.
- S2 pupils did not perform significantly better in working with *whole numbers* than P7.
- Pupils found information from tables well and had less difficulty interpreting graphs and tables than P7 pupils.
- They performed well in line symmetry but had difficulty with rotational symmetry.
- The performance of girls was better in *Information handling* and the performance of boys was better in *Shape, position and movement.*

Problem Solving and Enquiry

What was assessed?

- ❒ interpretation of the problem
- ❒ selection of a strategy
- ❒ carrying out the task
- ❒ reporting back

The tasks were presented as short and extended response items in the written tests. There were also extended problem-solving tasks in the practical survey where pupils could use practical materials and report verbally to the field officer.

Key findings

- ❒ Where the same task was used at more than one stage, there was a noticeable increase in the percentages of pupils successfully solving the problem.
- ❒ If a simple diagram would solve the problem, pupils preferred that strategy, otherwise they resorted to a written description.
- ❒ Most pupils used a 'trial and improvement' strategy in preference to a more systematic approach.

Teaching issues

What systematic strategies should we introduce pupils to, and at what stage?

Should we teach a set of problem-solving strategies to pupils? If so, how do we teach these strategies?

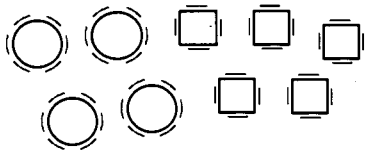
Do we give enough emphasis to insisting upon pupils setting down their working of a problem?

Example 1

A party of 32 people arrived to find seats at the tables.

The round tables seat 6. The square tables seat 4.

Each table has to be full.



What is the least number of tables they need?

They will need_____ round tables and _____ square tables.

Correct solutions were produced by 32% of P4 pupils, increasing to 66% at P7. The incorrect answer of '2 round tables and 5 square tables' was given by a third of P4 pupils and a tenth of P7 pupils.

How do you think they arrived at that conclusion?

Example 2

In an extended task, P7 and S2 pupils were given a kilt pin and, from a manufacturer's list of boxes, asked to select the smallest box which would hold the pin.

About 40% at P7 and 60% at S2 were able to select and implement an appropriate strategy without prompting. Having found an apparent solution, few considered whether there might or might not be a better solution.

Do these results simply reflect lack of experience with problem solving?

Example 3

A block of chocolate fudge is made in the shape of a cuboid.

It measures 10 cm by 7 cm by 3 cm.

1 Six of these blocks can be packed in a box measuring 30 cm by 14 cm by 3 cm without any wasted space.
Describe in words, or sketch, how this can be done.

2 Six of the blocks can also be packed into a box measuring 30 cm by 7 cm by 6 cm.
Describe in words, or sketch, how this can be done.

3 **Find the dimensions of another box which will hold exactly 6 blocks of this fudge.**

Most P7 pupils used a diagrammatic approach to the first two parts; 59% opted for a written description to part 3, but two thirds of these answers were unsatisfactory. The same problem produced similar approaches with S2 pupils but with somewhat better results.

Why do you think pupils encountered difficulties with a written description?

Information Handling

What was assessed?

The written tests assessed the strands

- display
- interpret

The practical tests assessed the strands

- collect
- organise
- display

Key findings

- **Performance was high overall compared to the other attainment outcomes.**
- **At P7, performance fell significantly when interpretation went beyond reading graphs and tables, with a smaller fall at S2.**
- **20% of P4 pupils succeeding in drawing, unaided, a bar graph from data they had collected.**
- **About half of P7 and S2 pupils successfully drew a bar graph with a non-unitary scale from data they had collected.**

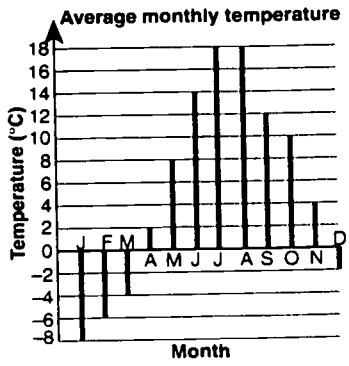
Teaching issues

Do we provide sufficient progression in the amount and complexity of interpretation of data? What contexts might be introduced to achieve this?

Do our programmes of study give sufficient emphasis to the use of computer spreadsheets and databases to increase experience in handling and interpreting data? Can you identify the progression in these which is needed for each school stage?

Example 4

This graph shows the average monthly temperature throughout last year.



Ski-ing was possible when the temperature was below 5°C.

Give the names of all the months when ski-ing was possible.

Answer_____

Level E

Only 32% of P7 pupils managed to give the correct answer and only 54% of S2, despite, at both stages, the example being set within the theme of *The Ski Trip*.

Can you identify any factors which may have caused pupils so much difficulty?

Number, Money and Measurement (1)

What was assessed?

The written tests assessed the strands

- range and type of numbers
- measure and estimate
- round numbers

The practical tests assessed aspects of these strands, particularly estimation and rounding.

Key findings

- P4 pupils handled place value with two-digit numbers fairly well but few succeeded when there were three digits.
- Many P7 pupils had difficulty with place value in decimals.
- Many P7 and S2 pupils were insecure with other than the simplest fractions and unfamiliar with other than the simplest whole number percentages.
- At P7 and S2, converting from one metric unit to another gave the greatest difficulty when the conversion factor was 1000.
- In the practical tests, on average less than 10% at P7 and 20% at S2 demonstrated adequate skill in estimating the answers to calculations.

Teaching issues

Is sufficient emphasis given in teaching programmes to understanding place value?

Would introducing percentages through decimal, rather than common, fractions make the concepts accessible to more pupils?

What strategies could be adopted to improve pupils' abilities in estimating answers?

Example 5

Complete this table.

The first row has already been filled in for you.

Decimals	Fractions
0.50	$\frac{1}{2}$
0.10	
0.15	

Level D

Most P7 pupils correctly identified 0.10 as a tenth, but the success rate fell from 72% to only 16% with 0.15; the answers one-fifteenth and one quarter were the most common incorrect responses for the latter question.

How might pupils have arrived at these incorrect answers ?

Example 6

In a test a boy scored 32 out of 50.

Write his result as a percentage.

Answer _____%

Level D

Only 22% of P7 pupils and 38% of S2 gave the correct answer; this compares to 31% at P7 and 43% at S2 in 1991. The most common incorrect answer at both stages was 32.

Why does such a simple example give so much difficulty?

Number, Money and Measurement (2)

What was assessed?

The written tests assessed the strands

- ❒ patterns and sequences
- ❒ functions and equations

Key findings

- ❒ **Most P4 and P7 pupils were able to complete number sequences with simple rules.**
- ❒ **Many S2 pupils had difficulty in constructing formulae and in substitution into algebraic expressions.**
- ❒ **Solution of equations by S2 pupils was well done, but performance fell sharply as equations became more complex.**

Teaching issues

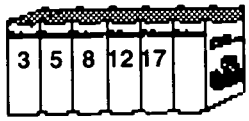
Are algebraic ideas introduced early enough to pupils? If not, should we review the stage at which algebra is begun?

Is there enough direct teaching of efficient methods of solving equations? Are pupils given enough practice in solving equations?

Example 7

Brian and Karen arrange their books on a shelf. Numbers on books are linked in a simple way.

Write the missing number on the book.



Level D

Despite the difficulty of the pattern of this sequence, there was a high performance by P4 pupils with 42% successful in completing it.

Example 8

Solve for x.

$3x + 7 = 16$

Answer _____

Level E

Performance by S2 pupils was slightly better than in 1991 with 63% able to find the correct solution.

Would you consider this a good level of performance? What strategies might pupils have used to find the solution?

Example 9

A parcel weighing N kg costs

$(10N + 30)$ pence to deliver.

What is the cost of delivering a 7 kg parcel?

Answer _____

Level E

Only around one third of S2 pupils correctly solved this problem and around one third made no attempt.

Why does this problem give so much difficulty?

Number, Money and Measurement (3)

What was assessed?

The written and practical tests assessed the strands

- add and subtract
- multiply and divide
- money

Key findings

- At P4, performance on basic processes was significantly lower than in 1991.
- P7 pupils' performance in working with whole numbers was much the same as in 1991, but that of S2 pupils was significantly lower than in 1991.
- Skill in using calculators, at all stages, was very high.
- When pupils had to decide which operation to select for a calculation, performance was significantly lower than where the operation was specified.

Teaching issue

Should more emphasis be given to performing basic calculations without the aid of a calculator?

Example 10

Subtract

82 - 36

Answer _____

Level B

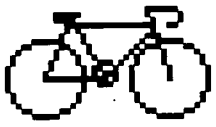
P4 performance fell from 59% to 43% between the 1991 and 1994 surveys. The answer of 54 was given by 17% of pupils.

Can you explain the likely reasoning behind the incorrect answer? Why is performance so low on this example?

Example 11

John is saving up for a new bicycle which costs £120.

He has already saved £72 and has a part-time job from which he can save £7 each week.



How many complete weeks will it be before he can buy the bike?

_____ weeks

Level D

42% of P7 and 54% of S2 gave the correct answer. The most common mistake at both stages was to round down to 6 weeks.

Why is there so little improvement in performance from P7 to S2?

Number, Money and Measurement (4)

What was assessed?

The written and practical tests assessed the strand

- decimals, fractions and percentages

Key findings

- Many P7 and S2 pupils had difficulty multiplying and dividing decimals by 10 and 100.
- Examples involving other than the simplest common fractions gave pupils considerable difficulty.
- Performance on basic processes was significantly lower at P7 and S2 than in 1991.
- In the practical survey, P7 and S2 pupils were very skilled at using calculators for calculations involving decimals. However performance was much lower if rounding was involved.

Teaching issues

Should more use be made of concrete materials to give understanding of the effect on place value of multiplying and dividing by multiples of 10?

Should the relationship between percentages and decimal rather than vulgar fractions be given more emphasis?

Example 12

Multiply

35·2 x 100

Answer _____

Level D

Only 36% of P7 pupils could answer this question correctly, a proportion which has remained low over successive surveys. A quarter gave 352 as the answer.

Example 13

Calculate

⅗ x 25

Answer _____

Level D

Only 39% of S2 pupils were correct and around one fifth made no attempt. The success rate was lower than in 1991 when 47% were correct.

Why does such a straightforward example give difficulty?

Number, Money and Measurement (5)

What was assessed?

The written and practical tests assessed the strands

- perimeter, formulae, scales
- measure and estimate
- time
- money

Key findings

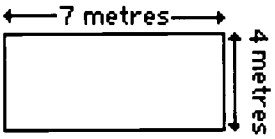
- At all stages, performance on applications of time was high.
- S2 performance was higher than P7 on many tasks involving applications of ratio, formulae and scale.
- Performance in aspects of perimeter and area was disappointing and some P4 pupils seemed unfamiliar with the term perimeter.
- Many P4 and P7 pupils had difficulty finding the actual weight of an object but more could put objects in order of weight using a two-pan balance.
- Few pupils could calculate correctly a time interval which crossed a change in hour.
- Most S2 pupils could convert a 24-hour clock time into am/pm notation.

Teaching issue

How might performance in perimeter and area be improved? With more use of practical activity? By introducing aspects of these at an earlier stage?

Example 14

Calculate the perimeter of the rectangle.



Answer ____ metres

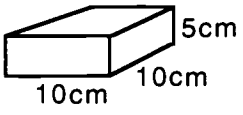
Level D

Only 54% of P7 pupils calculated the perimeter correctly, although this was an improvement on the 42% correct in 1991. Performance at S2 was the same. Nearly 30% at P7 and at S2 gave the answer as 28 metres.

Can you identify the main confusion in pupils' minds?

Example 15

What is the volume of this cuboid in cubic centimetres?



Answer _____ cm^3

Level E

Less than a quarter of P7 pupils could apply the correct formula to find the volume successfully.

Speculate on the kinds of incorrect answers which would have been given.

Shape, Position and Movement (1)

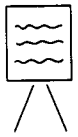
What was assessed?

The written and practical tests assessed the strands

- range of shapes
- symmetry

Key findings

- **Most P4 pupils could recognise and name common 2D and 3D shapes.**
- **At S2, over 60% of pupils could work with the relationship between radius and diameter.**
- **Most P7 pupils could complete symmetrical shapes and knew about lines of symmetry.**

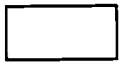


Teaching issue

Given good performance by pupils in this aspect of mathematics, should we review the pace at which the concepts are introduced?

Example 16

Draw the lines of symmetry in the following rectangle.



Level D

52% of S2 pupils showed both lines of symmetry but 29% included the diagonals as lines of symmetry. It appears that if the rectangle is much longer and thinner pupils are less likely to make this mistake.

Shape, Position and Movement (2)

What was assessed?

The written and practical tests assessed the strands

- ❒ position and movement
- ❒ angle

Key findings

- ❒ **Most P4 pupils could use simple map-type co-ordinates but had difficulty with compass bearings.**
- ❒ **Most P7 and S2 pupils could use map-type and Cartesian co-ordinates well.**
- ❒ **Most P7 and S2 pupils successfully read a grid reference from a map but were less successful at giving directions.**

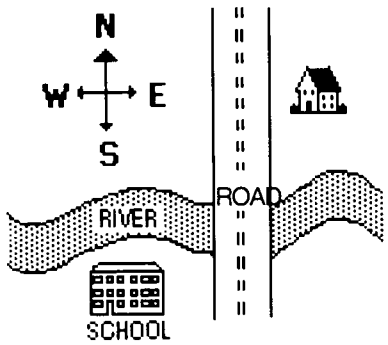
Teaching issues

What practical activities using a compass might be introduced to teaching programmes?

Would more use of computer packages involving movement assist understanding of this area of mathematics?

Example 17

The house is north of the river and east of the road.



Complete the sentence:

The school is _____ of the river and _____ of the road.

Level B

Fewer than a third of P4 pupils have answered this correctly in each of the last three surveys. 'South and east' and 'north and east' were common answers.

Example 18

A boat changes direction from East through South to South West.

Through what angle has it turned?

Answer _____°

Level D

Only one third of S2 pupils could give the correct answer, similar to the levels of performance in the previous two surveys.

Related publications

The full report on the 1994 AAP Mathematics survey is available from the research team. Copies can be obtained by sending a cheque for £19.50 (payable to the University of Strathclyde) to Department of MSTE. Faculty of Education (Jordanhill), 76 Southbrae Drive, Glasgow G13 1PP.

A limited number of additional copies of the summary report of the survey, *AAP Mathematics 1994*, can be obtained by writing to the SOEID Dissemination Officer, c/o SCRE, 15 St John Street, Edinburgh EH8 8JR. A copy for electronic downloading is available on the Educational Research in Scotland World Wide Web Server, accessible through the Internet and JANET (http://www.ed.ac.uk/~riu)

As a result of the concerns raised in the AAP mathematics report, HM Inspectors decided to investigate good practice in the teaching of mathematics, particularly number, and to publish a report giving recommendations designed to improve performance in both primary and secondary schools. Shortly after this inspection task was begun, the Third International Mathematics and Science Study (TIMSS) report was published. The findings of this report reinforced concerns about Scottish standards in mathematics in the early stages of secondary school. TIMSS found that Scottish 13 year olds performed poorly in mathematics compared to many other countries – Scotland was 26th out of 41 countries, an uncomfortably low position. HM Inspectors have taken steps to identify features leading to the strong performance of pupils in some of the highest scoring countries. Their conclusions, along with recommendations deriving from inspection of good practice in Scottish schools, will shortly be published in an HMI report on the teaching of mathematics.

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