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ABSTRACT

This teaching module with its 80-minute videotape of lessons from U.S., Japanese, and German classrooms is designed to help moderators facilitate discussion among educators. This guide is intended to help moderators make appropriate decisions about sessions and suggest items to consider when discussing the Third International Mathematics and Science Study (TIMSS) videotape, "Eighth-Grade Mathematics Lessons: United States, Japan, and Germany". The guide is organized into sections on: (1) preparation for sessions to discuss the TIMSS videotape; (2) descriptions of six algebra and geometry lessons from the U.S., Japan, and Germany that appear on the videotape; (3) research methods and findings from the TIMSS videotape classroom study; (4) background information on education in the U.S., Japan, and Germany; (5) frequently asked questions and answers; (6) handout/transparency masters; (7) lesson tables; and (8) lesson transcripts. (AIM)

ATTAINING
EXCELLENCE

TIMSS AS A STARTING POINT TO EXAMINE TEACHING

MODERATOR'S GUIDE TO EIGHTH-GRADE MATHEMATICS LESSONS:

UNITED STATES, JAPAN, AND GERMANY

U.S. DEPARTMENT OF EDUCATION
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U.S. DEPARTMENT OF EDUCATION
OFFICE OF EDUCATIONAL RESEARCH AND IMPROVEMENT

U.S. Department of Education
Richard W. Riley
Secretary

Office of Educational Research and Improvement
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Acting Assistant Secretary

National Center for Education Statistics
Pascal D. Forgione, Jr.
Commissioner

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Ronald W. Cartwright
Acting Director

National Institute on Student Achievement, Curriculum, and Assessment
Joseph Conaty
Director

Media and Information Services
Cynthia Hearn Dorfman
Director

September 1997

PARTNERS IN DEVELOPING

Attaining Excellence: TIMSS as a Starting Point to Examine Teaching

U.S. Department of Education
National Center for Education Statistics (NCES)
555 New Jersey Avenue, NW
Washington, DC 20208
Telephone: (202) 219-1333
Fax: (202) 219-1736

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
555 New Jersey Avenue, NW
Washington, DC 20208-5726
Telephone: (202) 219-2116
Fax: (202) 219-2053

National Council of Teachers of Mathematics (NCTM)
1906 Association Drive
Reston, VA 20191-1593
Telephone: (703) 620-9840
Fax: (703) 476-2970

American Federation of Teachers (AFT)
555 New Jersey Avenue, NW
Washington, DC 20001-2079
Telephone: (202) 879-4400
Fax: (202) 879-4545

National Council of Supervisors of Mathematics (NCSM)
Post Office Box 10667
Golden, CO 80401-0600
Telephone: (303) 274-5932
Fax: (303) 274-5932

Eisenhower National Clearinghouse (ENC)
1929 Kenny Road
Columbus, OH 43210-1079
Telephone: (614) 292-7784
Fax: (614) 292-2066

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Deborah Ball, University of Michigan
Cathy Carroll and colleagues, Mathematics Renaissance
Alfred Manaster, University of California, San Diego
Lew Romagnano, National Council of Supervisors of Mathematics
James Stigler, University of California, Los Angeles
Gail Burrill, National Council of Teachers of Mathematics
Alice Gill, American Federation of Teachers
Bonnie Walker, National Council of Supervisors of Mathematics
Len Simutis, Eisenhower National Clearinghouse
Lois Peak and colleagues, National Center for Education Statistics
Rima Azzam and colleagues, Pelavin Research Institute
Keith Tidman, American Institutes for Research
Pat O'Connell Ross and colleagues,
the Office Reform Assistance and Dissemination
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Ruth Chacon and colleagues, The Widmeyer-Baker Group

Contents

INTRODUCTION TO THE *MODERATOR'S GUIDE*

OVERVIEW OF THE TIMSS PROJECT

The 1995-1996 Third International Mathematics and Science Study (TIMSS) is the largest, most comprehensive international study of schools ever conducted. The study tested a half-million students from 41 nations in 30 languages at three different education levels (fourth, eighth, and twelfth grades) to compare their achievement in mathematics and science. The study also involved analyses of students, teachers, schools, curricula, instruction, and policy in order to understand the educational context in which teaching and learning take place.

Additionally, TIMSS included an extensive videotape survey of eighth-grade mathematics lessons in the United States, Japan, and Germany. The TIMSS Videotape Classroom Study is the first attempt to collect videotaped observations of classroom instruction from nationally representative samples of schools and classes. The purpose of gathering this information was to understand better the processes of classroom instruction in different cultures in order to improve student learning in our schools. The primary aim was to show teachers (and others) some of the different possibilities for structuring and presenting lessons so they can look at their own teaching with the fresh perspective provided by an international lens. This module was designed to help teachers think about why they teach as they do, what thinking lies behind the choices they make, what goals they emphasize through a lesson, and how they instruct and interact with students.

The purpose of the TIMSS Videotape Classroom Study is not to prescribe any single way to teach or to learn. Nor does TIMSS suggest that U.S. teachers should duplicate either Japanese or German methods. Teaching in the United States, Japan, and Germany has evolved from different goals and cultures, yet educators in one country can be stimulated to reflect on their own behavior by observing their counterparts in other countries.

USE OF THE *MODERATOR'S GUIDE*

This teaching module, with its videotape of lessons from U.S., Japanese, and German classrooms, is designed to help you, the moderator, facilitate discussions among educators. The TIMSS Videotape Classroom Study offers resources that can be useful in improving U.S. education for all students, but the effective use of these resources requires systematic planning. Hence, as the moderator, you should read the *Moderator's Guide* and view the videotape prior to facilitating a discussion. The guide will help you to make appropriate decisions about your own sessions and suggest things to consider as you plan for and facilitate the discussion of the TIMSS videotape, *Eighth-Grade Mathematics Lessons: United States, Japan, and Germany.* The guide is intended to be flexible in that it can be modified and used repeatedly. **You are permitted to reproduce any of the pages in this guide for use in your discussions.**

GOALS FOR SESSIONS BASED ON THE *MODERATOR'S GUIDE*

Viewing and discussing the videotaped lessons offers us an opportunity to:

- talk together about teaching in the United States, Japan, and Germany.
- talk together about a shared example of teaching that is not our own and not our colleagues'.
- develop a common language for discussing teaching.
- develop new norms for teaching.
- discuss the elements of teaching that make mathematically strong lessons.
- learn about teaching, learning, and children from watching and talking about instances of practice.
- learn about ourselves and others while learning other countries' ways of thinking about mathematics teaching and learning.

DEVELOPMENT OF THE VIDEOTAPE

The teachers in the accompanying videotape volunteered to participate and agreed to public distribution of the videotape. The six lessons in this production were not among the lessons actually taped as part of the TIMSS study, because those teachers were promised that their videotapes would be kept confidential. Nevertheless, the teachers and lessons in the accompanying videotape are representative of those in the actual study. The teachers shown are ordinary teachers, working within the constraints of their various systems. The purpose of the videotape is not to present extraordinary teaching for U.S. teachers to imitate. Instead, it is intended to help viewers discuss how teaching may relate to student learning.

ORGANIZATION OF THE *MODERATOR'S GUIDE*

- **Preparation for Sessions to Discuss the TIMSS Videotape—*Eighth-Grade Mathematics Lessons: United States, Japan, and Germany.*** This section provides guidelines for using the videotape to facilitate group discussions and suggests how to prepare for those discussions, including possible focus areas and agendas.
- **The Six Lessons: A Discussion Guide.** This section provides a series of descriptions, with pertinent sidebars, of the U.S., Japanese, and German geometry and algebra lessons that appear on the videotape. The descriptions include tips to help you facilitate the discussion sessions.
- **Research Methods and Findings of the TIMSS Videotape Classroom Study.** This section provides further information on the TIMSS Videotape Classroom Study, as well as detailed explanations of the methodology and the results.
- **Background on Education in the United States, Japan, and Germany.** This section briefly describes education, schooling, and curricula in each of the three countries.
- **Frequently Asked Questions and Answers.** This section provides information about the TIMSS study that may be useful in dealing with discussion participants' questions.

- **Handout/Transparency Masters**. This section provides master copies for reproduction of pre- and postdiscussion participant questions and general questions for participants to keep in mind while viewing the lessons. They can be duplicated for use by participants or photocopied onto transparencies.
- **Lesson Tables.** This section contains tables that provide a graphic overview of the complete subject matter and the instructional format of each of the six lessons.
- **Lesson Transcripts.** This section contains transcripts of the videotaped portions of the six lessons.

Additional discussion resources included as part of the teaching module: The Japanese standards *Mathematics Program in Japan (Kindergarten to Upper Secondary School)* and *Fostering Algebraic and Geometric Thinking: Selected Standards from the NCTM Standards Documents.*

PREPARATION FOR SESSIONS TO DISCUSS THE TIMSS VIDEOTAPE—

EIGHTH-GRADE MATHEMATICS LESSONS: UNITED STATES, JAPAN, AND GERMANY

In preparing to facilitate the discussion of the TIMSS videotape *Eighth-Grade Mathematics Lessons: United States, Japan, and Germany*, you will want to study this *Moderator's Guide* carefully and view the videotape once or twice. Then, think carefully about how to conduct your viewing and discussion session. For help in preparing your session, this guide includes some suggestions based on field tests in which a variety of audiences viewed the videotape.

In addition, you also will find it helpful to have the overview module to this Resource Kit, *Attaining Excellence: TIMSS as a Starting Point to Examine U.S. Education.* Two other modules you may want to refer to are *TIMSS as a Starting Point to Examine Student Achievement* and *TIMSS as a Starting Point to Examine Curricula.*

SESSION SIZE

The most successful sessions occur with small groups of up to 20 people, although the materials can also be successfully used with groups of as many as 100. The larger the group, the more time needed. For example, larger groups may need time to divide into small groups of four to six people to discuss key issues around teaching and learning and then reconvene and share their thoughts with the larger group. Also, large groups may need multiple TV monitors or a screen large enough for all participants to view it.

PARTICIPANT MIX

When preparing for a viewing of the videotape, think about the audience's knowledge of TIMSS in general. For example: Do participants have knowledge of what TIMSS is? What do they know about the findings other than the comparison of scores they may have heard about through the media?

Even if the participants already have some information about TIMSS, the study is so large and complex that you should discuss it before showing the videotape. The overheads provided in the booklet, *Introduction to TIMSS:*

The Third International Mathematics and Science Study, provided in this Resource Kit, will help you with this introduction. Another option is to use two sessions, one to focus on TIMSS as a whole and another to focus on the teaching module.

Think about who your participants are and where they come from. This will help you tailor the discussion. Are they elementary teachers, secondary math teachers, principals, or some other group? An understanding of each group, and the lens through which they are likely to view and discuss the videotape, will help you anticipate the issues that can emerge during the discussion.

For example, principals have a tendency to focus on general issues around teaching and learning and the system structures that support or inhibit teachers' efforts to improve practices. Most principals are not mathematicians, but they may support improvements in teaching practices that help students experience mathematics differently from the way they themselves did as students. Principals may feel that they are learning some mathematics from viewing the videotape. This can also be the case with non-math teachers. Tapping into non-math teachers' knowledge about teaching and learning in other subject areas enables them to connect their experiences to the discussion of mathematics teaching and learning. Secondary mathematics teachers are more inclined to focus on a lesson's mathematical content and on the cultural reasons for differences in teaching practices. Be prepared for the possibility that some participants may become defensive when viewing lessons from other countries.

ORDER OF THE LESSONS

The order in which the lessons appear on the tape is the result of careful field testing. Geometry lessons are shown first, from the United States, Japan, then Germany, followed by algebra lessons from these countries. However, as the moderator, you may choose to change the order, as other orders may be more effective in your situation.

The videotape is designed so that discussion leaders will show the U.S. lesson first. This reduces the defensiveness that sometimes occurs if participants have seen the Japanese lessons first and feel the need to defend U.S. teachers.

An alternative, however, is to show the Japanese lesson first with the idea that, in order to look at U.S. teaching and learning from a fresh perspective, participants need first to experience looking at a different style of teaching and learning. Viewing the Japanese lesson first can help audiences begin to draw comparisons immediately when the U.S. lesson is shown.

FOCUS AREAS FOR THE SESSION

General Focus

First, offer guidance to participants about how to view the lessons overall. The general guidelines and questions in the box below will help you set the initial focus for the participants.

GENERAL SUGGESTIONS FOR VIEWING THE LESSON

Stay focused on the lesson itself: What do you notice? What do you hear? What inferences do you find yourself making and why? Look for patterns that provide clues to how and what the student/teacher was thinking.

Draw on your experience with teachers and students, and with teaching and learning, but also try to look beyond your assumptions and experiences to see with fresh eyes.

- What do you think is the teacher's goal? What does he/she seem to want students to learn? What do you think they are learning?
- What does the teacher do? Are there key moves or moments in the lesson? Are there crucial missed opportunities?
- Why do you see this lesson in this way? What does this tell you about what is important to you? Look for patterns in your thinking.
- What questions about teaching and learning did viewing the videotape raise for you?
- Are there things you would like to try in your classroom as a result of viewing the lessons? How would you need to prepare yourself and your students to try these things?

Specific Focus

Following presentation of the general guidelines, choose a specific focus area from the initial session. Focus areas to choose from include the following:

1. Mathematics instruction;
2. Communication between teacher and students; and
3. Teacher beliefs about mathematics, teaching, and learning.

The focus may dictate the lessons you need to show, the stage to set for viewing the tape, the order in which to show the lessons, and the questions to raise for discussion. Each focus area is discussed below, along with suggestions for specific questions to ask participants.

1. Mathematics Instruction

The first, and perhaps most important, focus area for the session may be the teaching methods used in the taped lessons and the learning that students experience. When leading a discussion on the teaching and learning of mathematics, try to help participants focus on the analysis of teaching and learning, rather than on what they like or dislike about a particular teacher. As the discussion unfolds, focus on key questions that can be raised again and again.

QUESTIONS ABOUT MATHEMATICS INSTRUCTION

- What is the mathematics of the lesson?
- What seems to be the teacher's mathematical goal?
- How does the lesson flow?
- Are there logical connections between the parts of the lesson?

If responses focus on topics or ideas, probe for mathematical process goals, and vice versa, for example:

You've mentioned a topic. Are there others? Would you say there are also goals that have more to do with mathematical reasoning or thinking? **or...**

You've mentioned mathematical reasoning. Would you say that there are also goals that focus on particular mathematical topics? What do the teacher's goals seem to be? What do you think this teacher wants students to learn? What kinds of things do they seem to be learning? What evidence do you have for this?

If the discussion moves in other directions, such as cultural or organizational issues, the key questions shown in the box below can pull the discussion back into focus. It is helpful to display the questions before playing the videotape so participants can use them in viewing and discussing the lessons. An overhead of the questions appears in the handout section. Consider making copies for participants for reference while viewing and discussing the lessons.

2. Communication Between Teacher and Students

Another key focus area for the session's participants is the communication between the teacher and the students during the taped lessons. In analyzing the communication, you may find it helpful to look at the roles of the teacher and students with regard to the mathematics discussed in each lesson. Again, the point is not to focus on what participants might judge as good or bad, but on what can be inferred about student learning from specific evidence in the videotaped lessons.

QUESTIONS ABOUT COMMUNICATIONS BETWEEN TEACHERS AND STUDENTS

- What does the teacher do to orchestrate discussion in the lesson? What are the questions posed to students? When and how are they posed? How do the questions elicit mathematical thinking among the students?
- What does the teacher do to use students' ideas in the discussion? Are most students involved? How are students' ideas used?
- What decisions does the teacher appear to make in regard to students' ideas or discussion? Here, you can probe for more detail by asking:
  Do there appear to be ideas that the teacher is pursuing? Are there times when the teacher decides to provide more information, clarify an issue, model a strategy, or let students struggle? What do you think that says about the teacher's goal?
- What do the students do in the lesson discussion? What do their verbal and nonverbal communication suggest about their mathematical understanding?

3. Teacher Beliefs About Mathematics, Teaching, and Learning

A third focus area for discussion is what the teacher seems to believe about mathematics, teaching, and learning. The goal is to understand what the teacher values and believes and how this seems to influence his or her teaching of mathematics and, in turn, student learning. Such a focus can help participants consider their own perspectives more explicitly.

QUESTIONS ABOUT TEACHERS' BELIEFS ABOUT MATHEMATICS, TEACHING, AND LEARNING

- What does this teacher seem to believe about mathematics? About the way students learn? About the role of the teacher?
- What specific evidence can you find that indicates the teacher's patterns of thinking? His or her apparent theories of teaching and learning?
- What role do you think culture plays in teacher beliefs about teaching and learning?

LENGTH OF SESSION AND SAMPLE AGENDAS

It is strongly recommended that you plan at least a three-hour initial session. Experience shows that anything under three hours provides insufficient time for discussion, and even with three hours, you will likely have time to show only two lessons. Because the lessons provoke a lot of thought and discussion—particularly, as demonstrated by field tests, the U.S. and Japanese lessons on geometry—you may want to add three more hours, for a total of six, to show all the lessons. In field tests, for example, it took up to four hours to view and discuss just the geometry and algebra lessons of the United States and Japan. Alternatively, you may want to schedule a second session rather than trying to achieve everything in one lengthy session. For example, you might choose to view the German lessons in the second session, after participants have had a chance to digest and learn from the experience of the first session. Each of the six lessons runs approximately 15 minutes.

To help you in planning the sessions, two sample agendas follow, one for a six-hour session and one for a three-hour session.

SAMPLE SIX-HOUR AGENDA

(Include two 15-minute breaks and a 45-minute lunch)

(20 minutes) *Introduction:* Have participants answer the prediscussion questions as they enter (see the Handout/Transparency Masters section). Explain the TIMSS study and the TIMSS Classroom Videotape Study. Use overheads summarizing TIMSS (see handouts in the accompanying booklet, *Introduction to TIMSS: The Third International Mathematics and Science Study*). Consider providing additional information on the study's findings contained in "Research Methods and Findings of the TIMSS Videotape Classroom Study" in this *Moderator's Guide.*

(15 minutes) *Set the stage:* Before viewing the videotape, set the stage by saying, "As you watch each lesson, try not to evaluate or make judgments about whether the instruction is good or bad. Instead, try to focus on what is happening in the lesson and the teaching and learning that take place." This will keep the discussion grounded in the events on the videotape and will provide a place to return to should the discussion become evaluative. Raise the key questions and let participants choose which ones interest them as a framework for viewing the lessons.

(10 minutes) *Distribute materials:* Hand out lesson scripts, lesson tables, and focus questions to each participant for use during the viewing and discussion of the videotape.

(15 minutes) *View the videotape introduction:* Use the section "Research Methods and Findings of the TIMSS Videotape Classroom Study" in this document to clarify any questions or issues raised in relation to the study and its methodology.

(90 minutes) *View geometry lessons*: View the U.S. lesson and organize participants in small discussion groups using the focus questions. Then, have the small groups share the main points of their discussions with the larger group. View the Japanese lesson, then discuss it, again using the small-group format with a large-group follow-up discussion. If time permits, show the German lesson.

(90 minutes) *View algebra lessons*: Follow the same procedure as with the geometry lessons.

(45 minutes) *Across all lessons:* Go back to the original focus areas. Ask small groups to look for similarities between the U.S. geometry and algebra lessons. Then ask them to think about similarities between the Japanese geometry and algebra lessons. Have the small groups share key points with the whole group. Relate the groups' thinking to the findings of the TIMSS Classroom Videotape Study wherever possible. Finally, ask the participants to answer the postdiscussion questions (see the Handout/Transparency Masters section).

SAMPLE THREE-HOUR AGENDA

(Include a 10-minute break)

(20 minutes) *Introduction:* Have participants answer the prediscussion questions as they enter (see the Handout/Transparency Masters section). Explain the TIMSS study and the TIMSS Classroom Videotape Study. Use overheads summarizing TIMSS (see handouts in the accompanying booklet, *Introduction to TIMSS: The Third International Mathematics and Science Study*). Consider providing additional information on the study's findings contained in "Research Methods and Findings of the TIMSS Videotape Classroom Study" in this *Moderator's Guide.*

(15 minutes) *Set the stage:* Before viewing the videotape, set the stage by saying, "As you watch each lesson, try not to evaluate or make judgments about whether the instruction is good or bad. Instead, try to focus on what is happening in the lesson and the teaching and learning that take place." This will keep the discussion grounded in the events on the videotape and will provide a place to return to should the discussion become evaluative. Raise the key questions and let participants choose which ones interest them as a framework for viewing the lessons.

(10 minutes) *Distribute materials:* Hand out lesson scripts, lesson tables, and focus questions to each participant for use during the viewing and discussion of the videotape.

(15 minutes) *View the videotape introduction:* Use the section "Research Methods and Findings of the TIMSS Classroom Videotape Study" in this document to clarify any questions or issues raised in relation to the study and its methodology.

(80 minutes) *View geometry lessons*: View the U.S. lesson and have small groups discuss the lessons with the help of the focus questions. Then, have the small groups share the main points of their discussion with the larger group. View the Japanese lesson then discuss it using the same procedures. If time permits, show the German lesson.

(30 minutes) *Across both lessons:* Go back to the original focus area. Ask small groups to look for similarities and differences between the two lessons and then to share the main points of their discussion with the whole group. Relate the groups' thinking to the findings of the TIMSS Classroom Videotape Study wherever possible. Ask the participants to answer the postdiscussion questions (see the Handout/Transparency Masters section).

RECOMMENDATIONS TO THE MODERATOR

- Don't show the algebra lessons by themselves. They don't stimulate as much discussion as the geometry lessons and can result in defensiveness, depending on the audience.
- Make sure to preserve discussion time; don't cut this down!
- Allow an extra hour for discussion for each pair of lessons that you show.

FOLLOW-UP SESSIONS

After the first viewing and discussion of the taped lessons, some participants might incorrectly feel that they have completed their work with these materials. You should encourage them to regard the first viewing as only the beginning of an extended journey into examining teaching practices and offer them additional opportunities to view and discuss the videotaped lessons or related issues. Follow-up sessions will allow participants to explore certain topics in greater depth, to reflect on their own practices, and to discuss their considered reflections and observations with colleagues. In addition, these sessions are opportunities to introduce new discussion themes related to the initial session. You may wish to consider the following themes:

1. Examining other focus areas

Use another one of the specific focus areas suggested above that was not used in your first session. For example, "teachers' beliefs about mathematics teaching and learning" is an especially useful focus area in the second or third session for those whose first session focused on mathematics instruction.

2. Connecting the six lessons to the findings of the TIMSS videotape study

Familiarize participants with the findings of the TIMSS Classroom Videotape Study contained in this *Moderator's Guide.* Encourage discussion of whether and how these particular lessons reflect specific findings of the larger study.

3. Examining the mathematical content

Exploring the mathematics of certain lessons, such as the Japanese geometry lesson, is an option for a two-hour session. Concentrating on mathematical content and goals can spur examination of local standards and curricula.

For example, you may want to look at the phrasing and emphasis of the local standards. By examining the content sequence of the local curriculum, you can see whether or not the repeated topics would add a new perspective or help deepen understanding.

Questions to ask might also include the following: Is this important mathematics? How does this mathematical concept develop in later grades? What is the value of understanding this particular mathematical concept?

4. Examining the lessons through the *NCTM Standards*

You might choose to examine the lessons using the analytic framework from the National Council of Teachers of Mathematics' (NCTM) *Professional Standards for Teaching Mathematics,* exploring specific evidence of NCTM's recommendations. (See *Fostering Algebraic and Geometric Thinking: Selected Standards from the NCTM Standards* provided in this module.)

5. Examining the Japanese standards

You might arrange a session to study where the Japanese mathematics lesson was situated in the curriculum. You can examine the primary grade standards to obtain an idea of the experience students have coming into the eighth grade. Another option is to compare the Japanese standards to the NCTM standards, looking for similarities and differences. [See *Mathematics Program in Japan (Kindergarten to Upper Secondary School)* provided in this module.]

6. Viewing the same videotaped lessons again to focus in greater detail

You might plan a session so that participants focus in detail on the types of questions the teachers raise or critical moves the teachers make in each lesson. Participants may look for critical opportunities missed by the teacher during the lesson and discuss these in detail, brainstorming about what the teacher could do differently and why.

7. Exploring the same lesson taught differently

Teachers might agree to try out a lesson similar to one on the videotape in their own classrooms and then return to share the results in detail, using the videotape for reference and comparison.

8. Exploring interesting practices

As a result of the first viewing of the videotape and the first discussion session, participants may try different practices that intrigue them. These might include the following: using a variety of student solutions; posing only one or two problems; posing a thought-provoking problem at the beginning of a lesson; working to develop concepts; and discussing and sharing ideas with the whole class, with teacher-facilitated summaries at the end of the lesson. Participants may then share with others what they did and how it worked. Remember that new practices might not work the first time without proper preparation by teachers and students.

Such experimentation might lead to the formation of an ongoing study group for examining teaching practices. If a dialogue is sustained beyond the first session, it will likely yield greater results in further refining teaching practices.

THE SIX LESSONS: A DISCUSSION GUIDE

The accompanying videotape, *Eighth-Grade Mathematics Lessons: United States, Japan, and Germany*, contains six lessons, one in geometry and one in algebra from each of the three countries. The lessons on the videotape are different from, but representative of, those in the TIMSS Videotape Classroom Study. The lessons for the study were filmed with the understanding that they would be kept confidential, while the ones on the videotape in this Resource Kit were filmed separately with the understanding that they would be distributed publicly. However, researchers used the same procedures to analyze these six lessons as they did for those in the TIMSS Videotape Classroom Study and chose lessons that were similar to those they observed in the actual study. Hence, the accompanying videotape accurately illustrates the findings of the TIMSS Videotape Classroom Study.

This section of the module describes the sections of each lesson shown on the videotape. Sidebars provide mathematicians' and students' perspectives on the lessons, the responses of the videotaped teachers to the TIMSS questionnaire concerning their lesson goals, tips from experienced moderators, and common questions and comments you may need to deal with after participants view the videotape.

Remember, these lessons are intended to represent typical teaching in these countries. They are not ideal lessons, nor are they intended to prescribe what teachers should or should not do.

Mathematician's Perspective

This lesson is dominated by about 65 straightforward applications of the definitions and most elementary properties of vertical, supplementary, complementary, right, and straight angles and congruent triangles. The formula giving the sum of the angles in an n-gon is stated and applied for small values of n. The lesson concludes with a review of the definitions and properties mentioned along with the definitions of equilateral, isosceles, and scalene triangles (not shown in this videotape).

U.S. GEOMETRY LESSON: ANGLES

In this lesson, students practice using what they know about vertical, complementary, and supplementary angles to calculate the sizes of various angles. The teacher concludes by presenting the formula for finding the sum of the interior angles of any polygon.

Part 1: Presenting and Checking Warm-Up Problems

The teacher begins by presenting four diagrams, all drawn on the chalkboard. Each contains intersecting lines and rays that create vertical and supplementary angles. Students are asked to find the measures for ten angles. The teacher helps find four of the angles by asking questions and providing information. Then he asks students to find the rest. After about 40 seconds, the teacher works through the

Moderator Questions

- What aspects of angles is this lesson about?
- What's the place of vocabulary? How do we teach mathematical terms? How do students learn them?
- Is this a geometry lesson? What makes it a geometry lesson?
- What do students appear to be learning?
- How do you deal with arithmetic in the eighth grade?
- What seems to be this teacher's view of teaching mathematics?
- What's the role of practice?
- How would you teach this differently?
- What does the students' perspective tell us about what we in the United States think mathematics, teaching, and learning are about?
- Do you agree/disagree with the mathematician's perspective? Why, or why not?

remaining problems in a similar way, by eliciting responses from students using questions such as "If this angle is a right angle and this is 30 degrees, what does F have to be? And what's left for angle E? They all have to add up to...?" The warm-up activity lasts about five minutes.

Part 2: Checking Homework

The teacher asks students to take out the worksheet that was assigned earlier in the week for homework. The worksheet, "Types of Angles," includes definitions of terms (such as "supplementary"), sample problems with solutions, and about 40 problems for students to solve. The teacher checks students' answers through a question-and-answer format: "The complement of an angle of 84, Lindsay, would be.... [16] Are you sure about your arithmetic on that one? [Six?] Six. Six degrees. Albert, number four." Moving through the homework in this way continues for about ten minutes.

U.S. Eighth-Graders' Perspective

After viewing U.S. and Japanese lessons, U.S. students who participated in the field testing of this module's materials were asked which class they would choose to learn mathematics in, and why? They said that they would like to be in the U.S. class because "the teacher explained well and students were learning." When asked to give specific evidence of "explaining well," some students replied that "the teacher started sentences for the students to help them, and he went step by step by step."

Teacher Questionnaire

On the questionnaire, the teacher checked the following as the subject matter content of this lesson: geometric congruence and similarity (from a list of mathematical topics). He reported that the material was mostly new. He wanted students to learn angle relation (vertical, supplementary, and complementary) from this lesson. He indicated that this class contains students of mixed abilities. He reported that this lesson was "very similar to the way I always teach."

Part 3: Assigning Seatwork

The teacher distributes a worksheet, "Types of Angles (Continued)," that contains two sample problems with solutions and 15 problems that ask for measures of angles shown in drawings. The teacher introduces the worksheet by working through several problems with the students, asking questions such as, "If angle 3 is 120 degrees and angle 3 and angle 1 are vertical, what must angle 1 be equal to?" While the students work individually on the rest of the problems, the teacher assists individual students.

Part 4: Providing Extra Help on Challenging Problems

While assisting students on the worksheet problems, the teacher receives many questions on problem 37 (Find the measure of two angles that are equal and supplementary) and problem 38 (Write an equation that represents the sentence: The product of 12 and a number is 192). He decides to work these problems with the class. For problem 37, he says, "Two angles are supplementary. Therefore, they must add up to 180 degrees. But they are equal, so let's call one QRS and the other SRT (he draws a figure on the chalkboard). Each one of them has to be...?" After both problems have been answered and discussed, the students return to their worksheets.

Part 5: Checking More Homework and Introducing a New Formula

After the students finish the worksheet, the teacher asks them to get "the worksheet we did on Friday after the quiz." This worksheet has two problems. The first contains a map of two streets intersecting at a 45-degree angle with a triangular-shaped piece of land between the streets. A boundary line divides the piece into two parking lots. The task includes finding the measure of the angle formed by the property line and First Street and suggesting a more equal way to divide the lots. The teacher elicits the answers to these questions from the students, helping when the students are stuck. The second problem involves finding the sum of the interior angles for a six-sided figure. The teacher asks students for the answers they found using their protractors. The teacher then presents a formula and asks students to try it.

Moderator's Tips

- Sometimes it is hard to get the conversation to go beyond the surface level and into the specifics of analyzing practice. Use questions to help push the conversation into deeper levels.
- Sometimes the conversation can become focused on good or bad teaching. Use questions, like those on the next page, to help get the discussion back into focus.
- You may want to wait to share the teacher's goal for the lesson (as shown in the teacher's response to the questionnaire), until the participants have shared their own interpretations of the teacher's mathematical goal(s).

Part 6: Previewing the Upcoming Schedule

The teacher concludes the lesson by announcing the topic for the next day and informing students of the dates for the next quiz and the next exam.

Common Questions/Comments

- This is a vocabulary and subtraction lesson.
- There appears to be little more than recall required of students.
- The students were spoon-fed the information.
- This isn't how I teach; this can't be typical.
- Some things need to be taught this way; I don't think he always teaches this way.
- He didn't help students with the arithmetic errors.
- The teacher is working so hard!
- Q: Is this one period?
  A: Yes.
- I wonder if he considers this "teaching the basics."

Mathematician's Perspective

There are three applications (of increasing complexity) of the general principle that any two triangles with the same base and a vertex on the same line parallel to that base have the same area. The first application is to replace two straight-line segments forming the boundary between two land regions with one line segment so that the areas of the two regions are unchanged. The second is to find a triangle whose area is the same as that of a given quadrilateral. The final application is to find a triangle whose area is the same as that of a given pentagon. (Note the progression in abstraction and complexity, even though all three applications rely on the same fundamental geometric principle.)

JAPANESE GEOMETRY LESSON: AREAS OF TRIANGLES

In this lesson, students apply a concept to solve a problem involving the areas of irregularly shaped quadrilaterals. After working the problem on their own, students share their solution methods with the class.

Part 1: Linking Yesterday's Lesson to Today's Topic

The class begins with a ritual bowing by the students, as in almost all Japanese classrooms. The teacher begins the lesson by asking, "Do you remember what we did last period?" Then he walks to the front of the classroom where a TV monitor is connected to a computer, and he uses it to show a triangle between two parallel lines. A student replies that they studied how to obtain the area of a triangle constructed between parallel lines. As the teacher shows various triangles that can be formed by moving the top vertex along a line parallel to the base, he reminds the students that the areas of these triangles are the same because the base and the height are always the same. The teacher says they will use this result as "the foundation today."

Moderator Questions

- What aspects of triangles is this lesson about? Is this important mathematics?
- How many methods are there for solving the original problem? What are they?
- What are students doing? What mathematical reasoning is taking place?
- What seem to be the teacher's views of teaching, learning, and mathematics?
- What are the similarities to the U.S. lesson? The differences?

Part 2: Posing the Problem

The teacher draws a figure on the board representing two pieces of land, each piece owned by a student in the class. The boundary is a line bent in the middle. The owners would like to make the boundary straight without changing the areas of the two pieces of land. The teacher asks where he should draw the boundary. After a brief question-and-answer session to clarify the problem, and several predictions by the students, the teacher asks the students to work on the problem, "First of all, please think about it individually for three minutes."

Part 3: Working on the Problem

The students work individually on the problem while the teacher observes and assists them. The students' task is to develop a method to solve the problem, so the teacher gives hints to the students instead of showing them what to do. For example, the teacher asks one student, "Is there a method that uses the area of triangles?" and says to another student, "The question is... that there are parallel lines somewhere." After three minutes, the teacher suggests that students may want to work together. He adds, "And for now I have placed some hint cards up here so people who want to can refer to them." He tells the students they can think about the problem themselves, with a friend, or discuss it with the assistant teacher.

U.S. Eighth-Graders' Perspective

U.S. students who participated in the field testing said that they were surprised that the teacher laughed and the students laughed too. They thought the Japanese class would be very strict, with no talking allowed. They were intrigued by the teacher's relation with the students. "He seemed to like them!" But, students thought he did not explain as well as the teacher who presented the U.S. lesson on angles.

Teacher Questionnaire

On the questionnaire, the teacher checked the following as the content of this lesson: perimeter, area, and volume, basics of one- and two-dimensional geometry, geometric transformations and symmetry, problem-solving strategies, and "other" mathematics content. He reported that the material was half review/half new. This is a class of mixed abilities. He reported that this lesson was "similar to the way I always teach." He described this lesson as very typical of the lessons he normally teaches.

Part 4: Students Presenting Solutions

For about ten minutes, the students discuss the problem with each other, the teacher, or the assistant teacher. The teacher asks two students to draw their solutions on the chalkboard while the other students finish their discussions. The teacher then asks all the students to return to their seats and attend to the students' presentations. While a series of students explain their solutions, the rest of the students and the teacher ask questions and request clarifications. The solutions involve drawing parallel line segments, one connecting the two endpoints of the boundary and the other parallel to the first. By moving the vertex along the parallel line segment, a new straight boundary can be formed that retains the same areas.

Part 5: Reviewing Students' Methods and Posing Another Problem

The teacher reviews and clarifies the students' methods and asks how many students used each method. Then the teacher presents a follow-up problem, which is to change a quadrilateral into a triangle without changing the area. The teacher puts a figure on the board and says, "Without changing the area, please take the quadrilateral and make it into a triangle. Please think for three minutes and try doing it your own way." The students work on the problem at their desks, and the teacher assists students as necessary. After about three minutes, the teacher again tells them to discuss their solutions with one another.

Part 6: Summarizing the Results

During the students' seatwork, the teacher encourages students to find as many solutions as possible. He draws ten quadrilaterals on the chalkboard and asks students to show their solutions on the figures. After about 20 minutes, he briefly reviews the solutions and asks which students found each solution. All of the methods involve drawing a diagonal that divides the quadrilateral into two triangles, then drawing a line parallel to the first through the opposite vertex of one of the triangles, and then changing the shape of that triangle by moving the vertex along the parallel line until the entire figure is a triangle. The teacher ends the lesson by suggesting that, for homework, the students try to change other polygons, such as pentagons, into triangles with equal areas.

Moderator's Tips

- Before having the participants view this lesson, tell them that in the previous lesson the students explored the areas of triangles constructed between two parallel lines.
- Refer to the lesson script before the group views the lesson. The script gives more information about this lesson than does the videotape.
- The geometry problem posed in part two often intrigues participants. You may want to stop the videotape at that point and explore the problem. Alternatively, you may elect to defer the discussion about the actual mathematics to later.
- You may want to wait to share the teacher's goal for the lesson, as shown in his response to the teacher questionnaire, until the participants have shared their own interpretations of the teacher's mathematical goal(s).
- Sometimes it is hard to get the conversation to go beyond the surface level and into the specifics of analyzing practice. Use questions to help push the conversation into deeper levels.
- Sometimes defensiveness and cultural stereotypes emerge after viewing this lesson. Use focus questions to help get the discussion back on track. You may have to respond by re-posing the original focus question.
- Point out that while walking around the classroom, Japanese teachers typically note student methods and choose which students will be asked to share their methods on the blackboard. The teacher is looking for the strategies he expects will emerge from this problem and is developing a plan as to how to use them in his summary of the lesson.

Common Questions/Comments

- Q: Is this a high-level class?
  A: There is no tracking of Japanese students through the ninth grade. All students receive the same standard curriculum.
- Q: Isn't the student population homogeneous in Japan? Japan doesn't have the same diversity we do.
  A: There is variability of mathematical knowledge and there is social diversity. In fact, Japanese teachers cite differences in students' ability as a challenge that limits their teaching more than any other factor.
- Q: Don't the Japanese have higher IQ's than other cultures?
  A: There has been a great deal of research proving that this is not the case.
- Q: Who is the second teacher in the room?
  A: During the videotaping, many Japanese principals stayed in the room. Also, because of the extensive internship involved in becoming teachers, it is not uncommon to have an intern in the room.
- Q: Do most Japanese students wear uniforms?
  A: Almost all middle schools require them.
- Q: How are students assessed in Japan?
  A: The teachers at each school get together by grade level and develop exams to assess student understanding and to make instructional decisions. Students who want to enter high schools or universities must pass exams.
- Q: How long is this class period? Do Japanese teachers have more time than do U.S. teachers to teach lessons?
  A: Class periods average between 45-50 minutes, the same as in the United States.
- Q: When does the teacher take roll?
  A: A student monitor takes roll before the teacher arrives.
- Q: Don't Japanese classrooms experience interruptions?
  A: No, classes in Japan are rarely interrupted by outside agents such as loudspeakers or visitors.
- Q: Are computers common in Japanese schools?
  A: Virtually all junior high schools have computers, but the level of computer use varies from school to school.
- Q: What does "Onegaishimasu" mean?
  A: It means, "Please (teach us.)" Virtually all Japanese lessons and classes begin with this standard greeting and bow.

Mathematician's Perspective

Students share answers to homework problems using the formulas for the volume and mass of a rectangular solid to find the mass of three solids. A related, but still significantly different, problem is given to find the height when the length, width, specific gravity, and mass are known. Students confusing units while solving the problem leads the class to a discussion of dimension analysis. Ultimately, they draw a conclusion that the formula for mass can be used to find any one variable if all the others are known. Several applications of this conclusion are presented as additional problems.

Moderator Questions

- What seems to be the mathematical focus of the lesson?
- What are students doing? What is happening to enable the students to reason mathematically?
- How is this lesson similar to the U.S. angles lesson and the Japanese lesson on areas of triangles? How is it different?
- How does this teacher seem to view mathematics, teaching, and learning? What specifics can you give?

GERMAN GEOMETRY LESSON: VOLUME AND DENSITY

In this lesson, after reviewing homework, the students and teacher work through several problems involving the relation between volume and density.

Part 1: Sharing Homework

The lesson begins with the teacher asking a student to present her homework results. The student uses an overhead projector to explain what she did. There are three problems. One of them is, "A rectangular bowl of glass with a width of 14.6 cm and a length of 8.4 cm is filled with 17 mm of quicksilver (density 13.6 g/cm^3). What is the mass of the quicksilver?" After the student explains her solutions, the teacher leads the class in a discussion of the results. The teacher says, "Who confirms this result?" and "Does anybody else have any other suggestions for an alternative?" When mistakes are noted, the presenter makes corrections on the transparency. Reviewing the homework in this way continues for about 10 minutes.

Part 2: Revisiting Previous Materials

The teacher asks, "Yesterday, you put together what you know about calculation. Who can remember what we said?" The students say, when nominated, that they can calculate the surface, volume, and mass of a rectangular solid. The teacher asks for the formulas, and the students provide them. The teacher says that they will develop a fourth calculation during today's lesson.

Part 3: Posing a Problem

The teacher presents a problem using the overhead projector. The problem reads, "An iron sheet with a length of 0.5 m and a width of 20 cm weighs 3.9 kg. Calculate the height (thickness) of the sheet ($7.8\ g/cm^3$)." The teacher leads a brief discussion of the problem, reminding students to think about "our three-step [method]: given, wanted, and calculation path."

Part 4: Working on the Problem Together

The teacher asks for a volunteer to work the problem on the chalkboard. The volunteer begins working the problem by recording the given information. The teacher monitors the ensuing 20-minute discussion on how to solve the problem. The student at the board tries to solve the problem while taking suggestions and corrections from the other students. One student says, "I would convert that into centimeters." The volunteer responds, "I wouldn't." The teacher says, "Would you give him a reason," and the student says, "Well, then the numbers are a little bigger and the density would be easier to calculate." During this activity, several other students take their turn at the board. The activity concludes when the students agree on the answer.

Moderator's Tips

You may want to wait to share the teacher's goal for the lesson, as shown in her response to the teacher questionnaire, until the participants have shared their own interpretations of the teacher's mathematical goal(s).

Teacher Questionnaire

On the questionnaire, the teacher checked the following as the content of this lesson: common and decimal fractions, estimation and number sense, measurement units and processes, perimeter, area, volume, equations, inequalities, and formulas. She reported that the material was half review/half new. This is a class of average abilities. On the questionnaire, she reported that this lesson was "similar to the way I always teach." She described this lesson as mostly typical of the lessons she normally teaches.

Common Questions/Comments

- The problems are real but the focus is on the procedures, not the concepts.
- The students didn't practice many problems.
- Q: What is rho?
  A: Density.
- Q: Do German classes have tracking or ability groups?
  A: Yes. In Germany, students are usually separated into three tracks after the fourth grade, and each track attends a different school. All students receive a similar curriculum, but it is taught at a different level of complexity.

Part 5: Summarizing the Result

Following the completion of the problem-solving activity, the teacher says, "Is anyone able to say what we just did, what you learned?" Students volunteer that they learned to calculate the length, width, and height of a rectangular solid. The teacher fills in the statement of this calculation into the table she reviewed at the beginning of the lesson.

Part 6: Assigning Seatwork

The teacher offers three types of problems that differ in their level of difficulty and asks students to choose those they would like to do. She reminds them of an important point about units that they had discussed previously and then lets them choose the problems and begin working individually. During part of this work time, the teacher meets with four students at the board who are having difficulties with the earlier problem. The class ends while students are working, and the teacher suggests that they save their unfinished problems until class tomorrow and work at home on their home exercise book.

Mathematician's Perspective

Four warm-up activities begin the lesson. Two require finding integer solutions to equalities involving exponential expressions. One requires finding a volume of a box whose sides turn out to have integer length. The last is to evaluate the quotient of two monomials. The focus of the lesson is announced to be least common denominators. Two examples of adding two rational expressions are discussed. In the first, the denominator of the first fraction is a linear factor of the quadratic denominator of the second fraction. In the second example, the two fractions have the same denominator, which is linear. The lesson concludes with students working on homework exercises, all of which seek least common denominators for two or three given denominators—the fractions are not given—that may be integers, monomials, or linear or quadratic polynomials.

U.S. ALGEBRA LESSON: COMPLEX ALGEBRAIC EXPRESSIONS

In this lesson, after some warm-up problems, the teacher presents the problem $1/(x - 7) + 1/(x^2 - 49)$ and asks students to find the least common denominator. After explaining the correct way to solve the problem, the teacher assigns multiple tasks for seatwork, and students work on their own for the rest of the lesson.

Part 1: Presenting and Checking Warm-Up Problems

The teacher asks students to solve "warm-up" problems displayed on the overhead projector. The problems include finding the largest integer n for which $2 > n!$ and finding the number of cubic inches in the volume of a rectangular solid if the side, front, and bottom faces have areas of 12, eight, and six square inches, respectively. Students work on their own, during which time the teacher moves around the classroom helping students. After 13 minutes, the teacher reconvenes the class to share the solutions. The teacher asks students for the answers, which she records on the transparency. For the last problem, she asks, "How did you get it?" and the student describes the process.

Moderator Questions

- What appears to be the mathematical focus of this lesson?
- What do you think students are learning? Give examples of apparent mathematical learning.
- What is the purpose of review? How does it help student learning? What is review?
- How might this lesson be taught differently? How does this teacher use the overhead projector?
- When the teacher was walking around the room, how many students appeared to understand? How was student understanding used to make instructional decisions?
- How is this lesson like the U.S. angles lesson? What patterns emerge in both lessons? What can that tell us about U.S. views on teaching, learning, and mathematics?
- Do you think this teacher is teaching according to the recommendations of the NCTM standards? What did you see that makes you think that?

Part 2: Presenting and Discussing Problems

The teacher presents the problem $1/(x - 7) + 1/(x^2 - 49)$ on the overhead projector and says to the students, "Yesterday we worked on least common denominators. Try this problem." While the teacher passes out the homework worksheets, the students work on this problem. After about one minute, the teacher asks for the solution. Some students have difficulty, so the teacher explains each step. She then continues the lesson by presenting a second problem, $[5/(x + 6)] - [(2 - x)/(x + 6)]$, and warns students that "this one looks easier but there is a trick to it." Students work on the problem for about one and a half minutes. During this time, the teacher moves from desk to desk, checking students' work.

U.S. Eighth-Graders' Perspective

After viewing both U.S. and Japanese lessons, U.S. algebra students said that they would rather be in this classroom because it felt more like their classroom. They said that the way things were in this classroom was what they were used to seeing. Non-algebra students said that this was obviously a "smart" class. When asked if they understood the lesson, they answered, "No." However, they pointed out that they had not had this math, and said they thought the students in the class did understand. They thought this was the "hardest math" because of all the Xs.

Moderator's Tips

You may want to wait to share the teacher's goal for the lesson, as shown in the teacher questionnaire, until the participants have shared their own interpretations of the teacher's mathematical goal(s).

Teacher Questionnaire

On the questionnaire, the teacher checked the following as the content of this lesson: problem-solving strategies (draw a picture, make a simpler problem, and guess and check), and adding and multiplying algebraic fractions with like and unlike denominators. She reported that the material was mostly new. This is a class of high-ability students. On the questionnaire, she reported that this lesson was "very similar to the way I always teach." She described this lesson as very typical of the lessons she usually teaches.

When the teacher announces the answer, some students ask for an explanation. The teacher provides a brief explanation by asking students to complete several steps leading to the answer.

Part 3: Assigning Multiple Tasks for Seatwork

The teacher says, "For the remainder of the period there are about five things that I would like you to work on in the following order." These include finishing a test, correcting the previous day's homework, and finishing a worksheet for which a graphing calculator is needed. When these tasks are completed, students are to work on the next day's homework. The homework requires students to find the least common denominator (LCD) of rational expressions. Exercises include finding the LCD of 4x and 8x; 3x-6 and 12x-24; and 12, 18, and 30. Students work on these assignments individually as the teacher circulates to assist them. The seatwork activity lasts about 12 minutes. The lesson ends with this activity.

Common Questions/Comments

- This must be an algebra class.
- People might think this is "reform" because students are seated in groups and using calculators.
- I can't figure out the mathematical focus of this lesson.
- This must be a review day.
- The lesson seemed disconnected, disjointed, and focused on procedural learning. I couldn't find the concept.
- The students are obviously learning because this is an advanced class.
- Why did the teacher tell a student to look at the overhead when she was writing? What does that accomplish?
- The teacher did what I have often done: Find one student who gets the answer I'm looking for, and then move on in the lesson in an effort to cover the material.

Mathematician's Perspective

The teacher presents algebraic solutions to six linear inequalities. Three similar word problems that can be answered by solving linear inequalities are presented. In all the applications, an added feature is that the solution sought is the largest integer satisfying the inequality. They originally solve the first in two other ways, but then they discuss how setting up an inequality facilitates finding the answer. The solutions for the other two begin by carefully using tables to set up the inequalities and conclude by using the tables to develop careful algebraic solutions.

JAPANESE ALGEBRA LESSON: ALGEBRAIC INEQUALITIES

In this lesson, after briefly checking homework, the teacher poses a problem that can be solved by creating and solving an algebraic inequality. Students work on the problem, then share their solutions with the class. The teacher then poses some follow-up problems.

Part 1: Sharing Homework

The teacher asks six students to write the solutions for six of the homework problems on the chalkboard. The problems involve solving inequalities, such as $6x - 4 < 4x + 10$. While they are working, the teacher checks whether the students have completed their homework. The class spends about seven minutes answering the problems, explaining methods, and checking correctness of solutions.

Part 2: Posing the Problem

The teacher introduces the main problem for the day by saying, "I will have everyone use their heads and think a little, okay? Until now,

Moderator Questions

- What seems to be the mathematical focus of this lesson?
- Do you agree or disagree with the mathematician's perspective? Why or why not?
- What issues are raised for you with the students' perspectives? What does this say about our educational system?
- How was this lesson similar/different from the Japanese geometry lesson? What patterns emerge as clues for how Japanese teachers view teaching, learning, and mathematics?
- How is this lesson similar/different from the U.S. lesson? What are the differences in the way we think about mathematics, teaching, and learning and the way the Japanese teacher thinks about teaching and learning?
- Would you teach this topic differently?

we've just done calculation practice, but today you will have to use your heads a little." The teacher then displays a poster with the problem: There are two different types of cakes, one costing 230 yen and the other, 200 yen. You want to buy 10 cakes, but you don't want to pay more than 2,100 yen. How many of each type should you buy? The cakes costing 230 yen "look more delicious." The teacher clarifies the problem by restating it in several ways and then asks the students to work individually on the problem for about three minutes, using whatever methods they would like.

Part 3: Students Presenting Solution Methods

After about six minutes during which the teacher observes and assists students as they work individually, the teacher asks a student to share her solution method. The student reports that she computed the cost for ten 230-yen cakes and that the total was too much. She reduced the number of 230-yen cakes by one and computed again. She says she had planned to continue this process but ran out of time. Other students, who used the same method, build on her explanation and report that the solution is three 230-yen cakes and seven 200-yen cakes.

Part 4: Teacher and Students Presenting Alternative Solution Methods

The teacher introduces another method by saying, "I've thought about it too, so... what do you think about this way

U.S. Eighth-Graders' Perspective

When U.S. students viewed this Japanese lesson, they often said that the mathematics seemed simple to them. When pushed for specifics, they said they had done "cake" problems in elementary school. They were bothered by the fact that the teacher "made" a student get up and share when she didn't understand; they thought that was "mean." When asked if the student seemed uncomfortable, they answered, "No." Many also thought that the teacher talked too much and too fast and didn't explain well. The U.S. algebra students viewing this lesson said that this appeared to be a lower level class because the mathematics was easy, the classroom was messy, and there was no computer in the room.

of thinking?" He then describes a method of subtracting the savings of a cheaper cake (30 yen) from the amount needed to buy all delicious cakes (2,300 yen). One would need to subtract seven times to get below the 2,100 yen, so one could buy only three delicious cakes. Not all students understand his explanation so he asks another student, whom he knows has used an algebraic inequality, to explain. The student verbally describes the inequality $230x + 200(10-x) < 2,100$. After the student's presentation, the teacher challenges students to provide explanations that others can understand.

Part 5: Teacher Elaborating on a Student's Method

The teacher indicates that the last student's presentation on using algebraic inequalities captured his goal for the lesson. He says he would like to review the method carefully so others will understand: "To tell you the truth, I was going to set up today what Rika set up, but I wanted you all to come up with a number of ways to [solve] it. So we are going to try to do the problem using an inequality equation." He then spends about seven minutes leading the students step by step through the method. At the end of this discussion, he notes the advantages of using this method over the trial-and-error methods presented earlier in the lesson: "The answer will come out quickly... you don't need to figure out each number one by one."

Moderator's Tips

Point out that teachers in Japan often view student mistakes as valuable learning opportunities, which can further understanding for the individual student and the class as a whole. Allow participants to share their own interpretations of the teacher's mathematical goals before discussing them with the whole group.

Teacher Questionnaire

On the questionnaire, the teacher checked the following as the subject matter content of this lesson: equations, inequalities and formulas, and problem-solving strategies. He reported that the material was half review/half new. This is a class of mixed abilities. On the questionnaire, he reported that this lesson was "similar to the way I always teach." He described this lesson as mostly typical of the lessons he normally teaches.

Part 6: Posing and Solving Follow-Up Problems

The teacher presents two similar problems and asks the students to use the method just discussed to solve these problems. Here is one problem: "You want to buy 20 apples and tangerines all together for less than 2,000 yen. Apples are 120 yen each and tangerines are 70 yen each. Up to how many apples can you buy?" He reminds the students about the advantages of using this method and then gives the students about 12 minutes to work on the problems. As he assists students, he asks two of them to write their methods on the chalkboard.

The teacher completes this part of the lesson by talking through the students' work displayed on the board.

Part 7: Summarizing the Lesson Objective

The teacher summarizes the major point of the lesson: "What we talked about today was solutions using inequality equations. That is, when you work out problems, instead of counting things one by one and finding the number, it's usually easier if you set up an inequality equation and find the answer." Then the teacher distributes another worksheet for homework.

Common Questions/Comments

- Q: What is a 200-yen cake?
  A: Yen is the unit of currency in Japan.
- I was shocked to see paint chipping off the wall! It made me feel better about where I teach.
- There was a window washer outside washing windows, and none of the students looked at him!
- It seems this might be a school in a lower socioeconomic area. The students are wearing different uniforms, and the classroom looks less kept up.
- The students weren't actively involved but seemed comfortable sharing their strategies when they didn't understand.
- The teacher shared "his" method of solving inequalities, and "there were only two problems the *whole* period."
- Q: Is this an algebra class? Is this a class of lower abilities than the geometry class?
  A: No, students are not separated by ability in Japan until the tenth grade. All eighth graders have this curriculum.

Mathematician's Perspective

Five quick warm-up activities begin the lesson. Three involve numeric or symbolic exponentiation, another is a percentage calculation, and the last is the computation of the difference of two rational numbers. A quick review of three methods of solving systems of two linear equations follows; all examples given in the review have integer coefficients. The central problem of the lesson is to solve a system of two equations with two unknowns; rational forms are in both equations, but each is reducible to a linear equation with integer coefficients. After the class solves this system, the teacher describes the general method of solving such equations. The lesson concludes with three problems similar to its central problem.

GERMAN ALGEBRA LESSON: SYSTEMS OF EQUATIONS

In this lesson, after some brief warm-up problems, the students and teacher work collaboratively on solving a complex system of equations: $(2y\text{-}5)/9=5(x\text{-}1)/6\text{-}5y$ and $(3x+1)/12=(8/3)(y\text{-}2)+33x/2$.

Part 1: Presenting Warm-Up Problems

The lesson begins with two minutes of quickly paced warm-up exercises. The teacher asks six questions, including "Eight to the third power?" "Twelve percent of 120?" and "Five factorial?" Students answer orally, and the teacher confirms the response or asks if others agree.

Part 2: Reviewing Previous Material

After the warm-up activity, the teacher asks, "What have we done lately?" After a student replies that they have studied "equations with two variables," the teacher encourages students to describe the solution methods they have learned. Students respond by identifying the methods of "equating," "substituting," and "adding." The teacher asks them to give examples of how such

Moderator Questions

- What seems to be the mathematical focus of this lesson?
- What did students seem to be learning? Give specific examples of learning.
- What are the similarities/differences compared to the U.S. and Japanese algebra lessons?
- Given the similar student performances of German and U.S. students, what are the similarities in the apparent views of mathematics teaching and learning?

methods work. With some prodding, students generate a system of equations and illustrate the method while the teacher records their verbal descriptions on the chalkboard. They work on three examples of systems of equations during this review activity, which lasts about seven minutes.

Part 3: Posing and Working on the Problem

The teacher writes the following system of equations on the chalkboard: $(2y-5)/9=5(x-1)/6-5y$ and $(3x+1)/12=(8/3)(y-2)+33x/2$. After giving students a minute to think about the problem, he asks for students to volunteer suggestions on how to proceed. Students take turns coming to the board to work on the problem, taking questions and advice from their peers and the teacher. After about ten minutes working together in this way, the teacher asks students to record the partial result in their notebooks and continue solving the problem. He gives them about five minutes to find the solution.

Moderator's Tips

Wait to share the teacher's goal for the lesson (as shown in the teacher's response to the questionnaire), until the participants have shared their own interpretations of the teacher's mathematical goal(s).

Teacher Questionnaire

On the questionnaire, the teacher checked the following as the subjects of this lesson: equations, inequalities and formulas, and problem-solving strategies. He reported that the material was half review/half new. This is a class of average ability. On the questionnaire, he reported that this lesson was "similar to the way I always teach" and that the lesson was mostly typical of the lessons he normally teaches.

Part 4: Sharing the Result

The teacher asks students to describe the methods they used to finish the problem. One student suggests the method of addition, and the teacher asks her to show her work on the chalkboard. She works at the board to complete the problem with help from the teacher and the other students. She occasionally asks questions of the teacher, and debates points with her peers. She finishes the problem in about six minutes.

Part 5: Summarizing the Objective and Assigning Seatwork

When the student completes the problem and returns to her seat, the teacher asks the students to summarize what they learned about solving "complicated problems" similar to this one. The teacher says that the main thing is to think about what method will be best to use for different types of systems. He then assigns a problem from the exercise book. For about seven minutes the students work independently. The teacher monitors their work and occasionally assists students until the lesson ends.

Common Questions/Comments

- I was amazed at the mental mathematics!
- There weren't any books.
- Q: It's interesting how the desks are set up; is that typical?
  A: Yes, it is common to find desks arranged in this order in German eighth-grade classrooms.
- There were no practice problems during class time.
- Like the U.S. lessons, this German lesson focused on using procedures to solve problems as opposed to conceptually understanding the origin of the methods.
- I was shocked to see the teacher stick his tongue out at the students!

RESEARCH METHODS AND FINDINGS OF THE TIMSS VIDEOTAPE CLASSROOM STUDY

This section describes the methodology used by the researchers in conducting the TIMSS Videotape Classroom Study, as well as the goals and findings. The information here will help you be more informed about the study so that you can better facilitate the discussion sessions, both anticipating and reacting to participants' questions and comments.

INTRODUCTION TO TIMSS VIDEOTAPE CLASSROOM STUDY

The Third International Mathematics and Science Study (TIMSS) included a videotape survey of eighth-grade mathematics lessons in the United States, Japan, and Germany. Funded by the U.S. Department of Education's National Center for Education Statistics and the National Science Foundation, the TIMSS Videotape Classroom Study is the first attempt to collect videotaped records of classroom instruction from nationally representative samples of teachers. The purpose of gathering this information was to build a better understanding of the processes of classroom instruction in different cultures in order to contribute to efforts to improve student learning in schools.

The TIMSS Videotape Classroom Study was conducted in 231 classrooms in the United States, Japan, and Germany. Innovative multimedia database technology helped to facilitate the management and analysis of the videotapes.

The videotape study had four goals:

1. Provide a rich source of information regarding what takes place in eighth-grade mathematics classes in the three countries.
2. Develop objective observational measures of classroom instruction to serve as quantitative indicators, at the national level, of teaching practices in the three countries.

3. Compare actual mathematics teaching methods in the United States and the other countries with those recommended in current reform documents and with teachers' perceptions of those recommendations.
4. Assess the feasibility of applying videotape methodology in future wider scale national and international surveys of instructional practices.

RESEARCH METHODS USED

The study sample included 231 eighth-grade mathematics classrooms: 81 in the United States, 50 in Japan, and 100 in Germany. The sample was designed to be representative of eighth-grade classrooms in the three countries. The findings are representative of the instruction received by eighth-grade students in each country.

Researchers videotaped one lesson in each classroom during the school year. Tapes were encoded, stored digitally on CD-ROM, and accessed and analyzed using multimedia database software developed specifically for this project. Teams of coders who are native speakers of the three languages transcribed and then analyzed all lessons. Analyses focused on the content and organization of the lessons as well as on the instructional practices that teachers used during the lessons.

TIMSS was based on randomly chosen, nationally representative samples of eighth-grade students in each country. The TIMSS Videotape Classroom Study sample was designed to be a random subsample of approximately half of the TIMSS classrooms in the United States, Japan, and Germany.

Sampling was conducted in two stages. Researchers first sampled schools, then classrooms within schools. The exact procedures used varied by country, but in each case procedures were expected to yield a sample representative of the instruction received by eighth-grade mathematics students in each nation.

A project coordinator initially contacted the teachers in each country to explain the study's goals and schedule the date and time for videotaping. Because teachers knew when the taping would occur, there was a possibility that they might attempt to prepare in some special way. To mitigate any bias caused by the teachers' preparations, researchers gave the teachers in each country a common set of instructions and told them that the goal was to see what typically happens in the mathematics classrooms of their country. The researchers indicated that they wanted to see what the teachers would do were they not being videotaped.

Researchers collected two kinds of data in the TIMSS Videotape Classroom Study: videotaped lessons and questionnaire responses. They also collected supplementary materials (e.g., copies of textbook pages or worksheets) deemed helpful for understanding the lesson. Each classroom was videotaped once, on a date convenient for the teacher. One complete lesson—as defined by the teacher—was videotaped in each classroom. A primary purpose of the questionnaire was to obtain teachers' judgments of how typical the videotaped lesson was compared to what observers would normally see in their classroom.

Teachers and students who appeared in the TIMSS Videotape Classroom Study were guaranteed confidentiality; videotapes of their classrooms can be studied only by researchers who have applied for and received a license from NCES. In addition, TIMSS collected five "public use" tapes in each country as examples to help communicate the results of the study. Teachers and students who appear in the public-use tapes were not part of the main study but agreed to be taped and to have their lessons made available for public viewing. From these five tapes, two lessons from each country were chosen for inclusion in the videotape accompanying this module.

The success of any videotape study hinges on the quality, informativeness, and comparability of the tapes collected. What is captured on a videotape is determined by not only what transpires in the classroom but also the way the camera is used. Therefore, researchers constructed standardized procedures for camera use and trained videographers in the application of these procedures. Only one camera was used to tape each lesson, and it was usually focused on the teacher.

CODING AND EVALUATING

Researchers conducted a field test with nine classrooms in each country before data collection. They used these field-test tapes, in part, as a basis for developing event codes.

Once the tapes were collected, they were sent to the project headquarters at The University of California, Los Angeles (UCLA), for transcription, coding, and analysis. The first step in this process was to digitize the videotape and store it in a multimedia database, together with scanned images of supplementary materials. Digital videotape offers several advantages over standard videotape for use in studies. The resulting files are far more durable and lasting, and they will not degrade with the repeated playing or replaying of segments that is required for thorough analysis. Digital video also enables random, instantaneous access to any location on the videotape, a feature that makes possible far more sophisticated analyses.

The videotapes were then transcribed, and the transcripts were linked by time codes to the videotape. German and Japanese transcripts were translated into English. Transcription of videotapes is essential for coding and analysis. Without a transcript, coders have difficulty hearing, much less interpreting, the complex stream of events that flow past in a classroom lesson.

Using the software developed for this project, coders had instant access to the videotape as they worked with the linked transcript, making it easy to retrieve the context needed for interpreting the transcript. All event codes were marked, stored, and linked to a time code on the videotape, all within the same database.

Objectives of coding

In deciding what to code, researchers had to keep two goals in mind: They wanted to code aspects of instruction that relate to current definitions of instructional quality, and they wanted the codes to provide a valid picture of instruction in three cultures.

To achieve the first goal, they sought ideas about what to code from the research literature on the teaching and learning of mathematics, and from reform documents—such as the National Council of Teachers of Mathematics' *Professional Standards for Teaching Mathematics*—that make recommendations about how mathematics ought to be taught. Researchers wanted to code both the structural aspects of instruction, that is, those things that the teacher most likely planned ahead of time, and the real-time aspects of instruction, that is, the processes that unfold as the lesson progresses.

The second goal was an accurate portrayal of instruction in the United States, Japan, and Germany. Toward this end, researchers took care to make sure that their descriptions of classrooms in each country made sense from within those cultures and not just from the U.S. point of view. They wanted to be sure that, if different cultural scripts underlie instruction in each country, this study would have a way to discover these scripts. For this reason, they sought coding ideas from the videotapes themselves.

Process of coding

In a field test in May 1994, researchers collected nine tapes from each country. A team of six code developers—two from the United States, two from Japan, and two from Germany—spent the summer watching and discussing the contents of the tapes in order to develop an understanding of how teachers construct and implement lessons in each country.

The process was a straightforward one: They watched a tape, discussed it, and then watched another. As they worked their way through the 27 tapes, they generated hypotheses about what the key cross-cultural differences might be. These hypotheses formed the basis of the codes, i.e., objective procedures that could be used to describe the videotapes quantitatively. The code developers also outlined hypotheses about general scripts that describe the overall process of a lesson, and they devised ways to validate these scripts against the videotape data.

In this way, they developed codes describing dimensions along which the lessons varied, including the type of mathematics studied, the ways in which lessons were organized, and the kind of thinking students were engaged in during the lesson. Great emphasis was placed on the development of codes that could be applied with high levels of inter-rater reliability.

Once the list of "what to code" was completed, a group of four code developers, all of whom had participated in the initial viewing and discussion of the 27 field-test tapes, developed the specific coding procedures. Two code developers were from the United States, one from Japan, and the other from Germany. Three developers—one from each country—were doctoral students in either psychology or education, all with classroom teaching experience. The fourth code developer, a doctoral student in applied linguistics from the United States, helped work through the technical issues involved in coding classroom discourse.

Some codes were used to indicate the frequency of events, others to indicate the duration of various kinds of activities. In all cases, great emphasis was placed on establishing the inter-rater reliability of the codes before putting them to use. Once codes were defined and coding procedures specified, the developers checked the reliability of the codes. Only after they reached at least 80 percent agreement among themselves on independent judgments of the same code did coding begin. Actual coding was completed by a separate group of coders trained by the developers in implementing the codes. Coding of the main study tapes did not begin until coders proved reliable with each other and with the code developers on at least 80 percent of their judgments.

PRELIMINARY FINDINGS OF THE TIMSS VIDEOTAPE CLASSROOM STUDY

In the analyses completed thus far, a number of cross-cultural differences have emerged. These findings can be grouped into five categories, each of which will be explored in some detail:

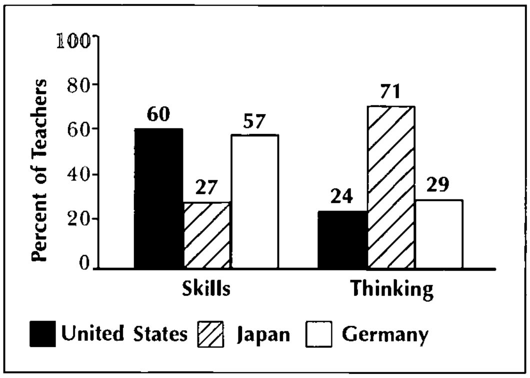
1. **Structure and Delivery of the Lessons**—U.S. and German teachers stress skill acquisition as the goal of instruction; Japanese teachers stress understanding.
2. **Type and Level of Mathematics Taught**—In both the level and richness of content, Japanese and German classrooms appear more advanced than U.S. classrooms.
3. **Student Thinking During the Lessons**—Japanese students appear to engage in different kinds of mathematical thinking during the lesson than U.S. or German students.
4. **Teachers' Views of Reform**—Most U.S. teachers report familiarity with reform recommendations; only a few apply the key points in their classrooms.
5. **Achievement in the Three Countries**—Japanese students scored significantly higher than U.S. students in both mathematics and science. U.S. and German students had similar scores in both mathematics and science.

1. Structure and Delivery of the Lessons

Exhibit 1 shows the percentage of teachers who gave responses in each of these two categories. Japanese teachers focused on thinking and understanding, while U.S. and German teachers focused on skills.

EXHIBIT 1:

PERCENTAGE OF TEACHERS WHO DESCRIBE THE GOAL OF THE VIDEOTAPED LESSON AS "SKILLS" VS. "THINKING"



Evaluating a classroom mathematics lesson is difficult unless we first know what the teacher is trying to accomplish in the lesson. On a questionnaire, researchers asked teachers whose lessons were videotaped to state what they wanted students to learn in that lesson. Most of the answers fell into one of two categories:

Skills—answers that focused on students being able to *do* something: perform a procedure or solve a specific type of problem.

Thinking—answers that focused on students being able to *understand* something about mathematical concepts or ideas.

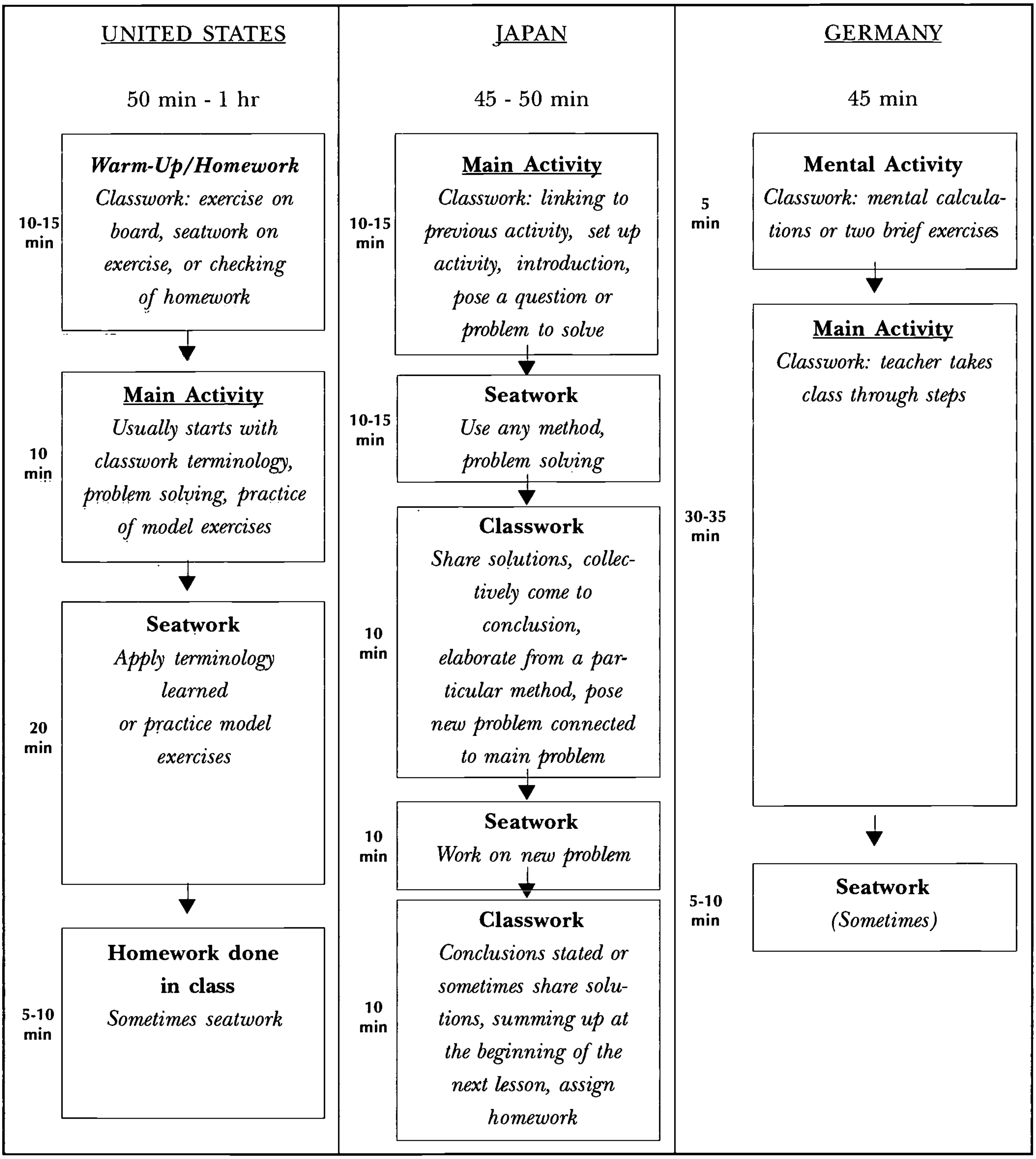
These different goals lead Japanese teachers to construct lessons different from those of U.S. and German teachers.

A clear distinction exists between the way the lessons are put together by Japanese teachers, and the way they are put together by U.S. and German teachers. The different ways follow from different instructional goals, and they are probably based on different assumptions about the role of problem solving in the lesson, the way students learn, and the proper role of the teacher.

U.S. and German lessons tend to have two phases: an initial acquisition phase and a subsequent application phase. In the acquisition phase, the teacher demonstrates and/or explains how to solve an example problem. The explanation might be purely procedural (as most often happens in the United States), or it can include development of concepts (more often the case in Germany). Yet, the goal in both countries is to teach students a particular method for solving the example problem(s). In the application phase, students practice solving examples on their own while the teacher helps students who are experiencing difficulty.

Japanese lessons generally follow a different script. Problem solving comes first, followed by a time in which students first share the solution methods they have generated and then work jointly to develop explicit understandings of the underlying mathematical concepts. Whereas students in U.S. and German classrooms must follow their teachers through the solution of example problems, the Japanese students have a different job: to invent their own solutions and then reflect on those solutions in an attempt to increase understanding.

HOW TIME IS USED IN A TYPICAL LESSON



UNITED STATES

50 min - 1 hr

10-15 min — ***Warm-Up/Homework*** *Classwork: exercise on board, seatwork on exercise, or checking of homework*

10 min — **Main Activity** *Usually starts with classwork terminology, problem solving, practice of model exercises*

20 min — **Seatwork** *Apply terminology learned or practice model exercises*

5-10 min — **Homework done in class** *Sometimes seatwork*

JAPAN

45 - 50 min

10-15 min — **Main Activity** *Classwork: linking to previous activity, set up activity, introduction, pose a question or problem to solve*

10-15 min — **Seatwork** *Use any method, problem solving*

10 min — **Classwork** *Share solutions, collectively come to conclusion, elaborate from a particular method, pose new problem connected to main problem*

10 min — **Seatwork** *Work on new problem*

10 min — **Classwork** *Conclusions stated or sometimes share solutions, summing up at the beginning of the next lesson, assign homework*

GERMANY

45 min

5 min — **Mental Activity** *Classwork: mental calculations or two brief exercises*

30-35 min — **Main Activity** *Classwork: teacher takes class through steps*

5-10 min — **Seatwork** *(Sometimes)*

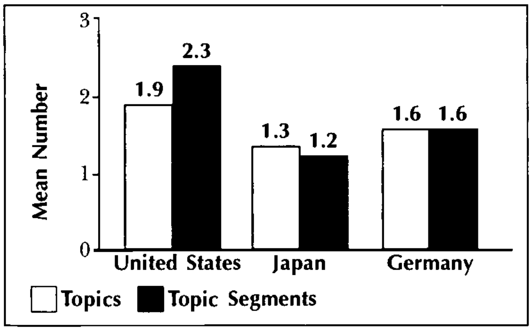
59

Coherence of the Lesson

In addition to these differences in goals and scripts, researchers also found differences in the coherence of lessons in the three countries. Students will be more successful in making sense of instruction that is more coherent. The greatest differences in coherence were apparent between U.S. lessons and Japanese lessons; researchers using several criteria found U.S. lessons to be less coherent than Japanese lessons.

EXHIBIT 2:

AVERAGE NUMBER OF TOPICS AND TOPIC SEGMENTS PER LESSON IN EACH COUNTRY



First, U.S. teachers switched from one topic to another within lessons more than Japanese teachers did. As shown in Exhibit 2, U.S. lessons contained significantly more topics and topic segments than did Japanese lessons.

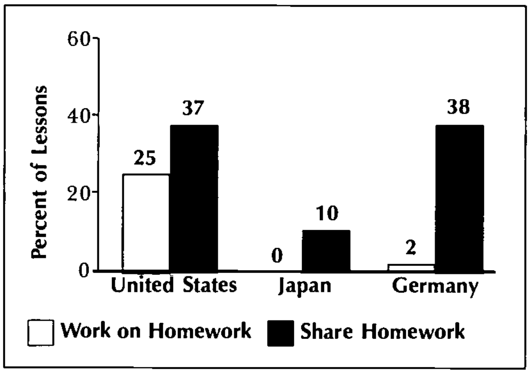
Second, the changes from topic to topic or from one segment to another in U.S. lessons often were not linked together by the teacher. Compared to U.S. and German teachers, Japanese teachers were significantly more likely to provide explicit links or connections between different parts of the same lesson.

Third, U.S. teachers devoted significantly more time during the lesson to irrelevant diversions than did German or Japanese teachers. U.S. lessons were more frequently interrupted by outside events, such as announcements or visitors. This was true for 28 percent of U.S. lessons, 13 percent of German lessons, and zero percent of Japanese lessons.

Homework During the Lesson

EXHIBIT 3:

PERCENTAGE OF LESSONS IN WHICH CLASS WORKED ON AND SHARED HOMEWORK



Another cross-national difference was in the role homework played in the lessons. If homework was attended to during the lesson, it could happen in two ways: The class might review and share the results of homework assigned during the previous lesson, or the students might be given time to work on their homework for the next day. Exhibit 3 shows the percentage of lessons in which students actually worked on or shared homework.

Japanese students never worked on the next day's homework during class and rarely shared homework results. Both U.S. and German students shared homework frequently, but only U.S. students spent time in class working on the next day's homework. When we calculate the total percentage of class time devoted to assigning, working on, or sharing homework, we get a similar result: Only 2 percent of lesson time in Japan involved homework in any way, compared with 8 percent in Germany and 11 percent in the United States.

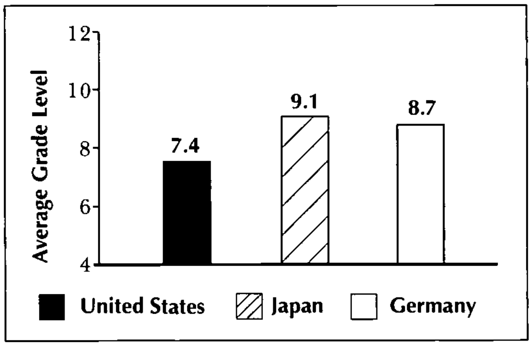
On the questionnaire, researchers asked teachers whether they had previously assigned homework that was due for that day. Whereas 55 percent of both U.S. and German teachers said that they had assigned such homework, only 14 percent of Japanese teachers reported assigning homework.

2. Type and Level of Mathematics Taught

It is not possible, a priori, to say that one mathematical topic is more complex than another. However, the TIMSS researchers could judge how advanced a topic is based on its placement in mathematics curricula around the world. The videotape study made use of the TIMSS curriculum analyses to estimate the average level of mathematical content in the videotaped lessons in each country.

EXHIBIT 4:

AVERAGE GRADE LEVEL OF CONTENT IN THE VIDEOTAPED LESSONS BY INTERNATIONAL STANDARDS

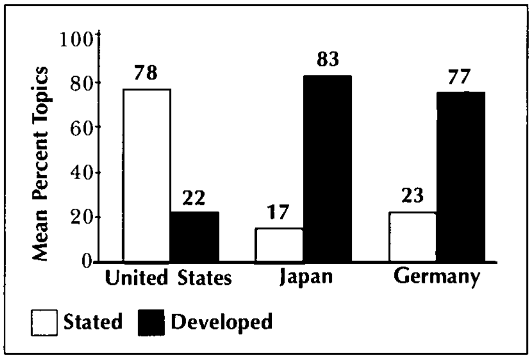


For the curriculum analysis, experts in the 41 TIMSS countries identified the grade levels at which various topics in the TIMSS framework were most emphasized in their country, so that researchers could determine the average grade level for each particular topic. Researchers coded the content of each lesson in the videotape study according to the same framework and compared the topics taught with the international average.

Exhibit 4 shows the average grade level of topics covered in the videotape sample. By international standards, the mathematical content of U.S. eighth-grade lessons was at a seventh-grade level on average, whereas German and Japanese lessons placed at the high eighth- or ninth-grade levels.

EXHIBIT 5

AVERAGE PERCENTAGE OF TOPICS IN EACH LESSON THAT CONTAINED CONCEPTS THAT WERE DEVELOPED VS. ONLY STATED

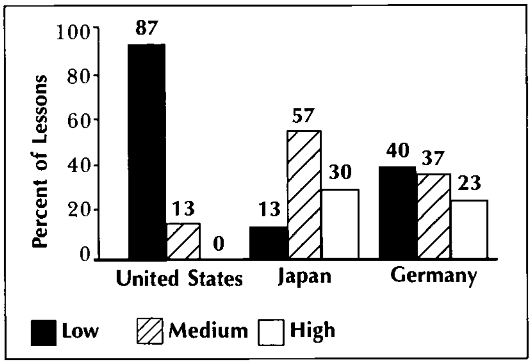


The nature of the content also differed across countries. For example, most mathematics lessons include some mixture of concepts and the application of those concepts to solving problems. How concepts are presented, however, varies across countries. Concepts might simply be stated, as in "the Pythagorean Theorem states that $a^2 + b^2 = c^2$," or they might be developed and derived over the course of the lesson. Exhibit 5 shows the percentage of topics in each lesson that contained concepts that were developed as opposed to only stated. More than three-fourths of German and Japanese teachers developed concepts when they included them in their lessons, compared with approximately one-fifth of the U.S. teachers.

As part of the videotape study, U.S. college mathematics professors evaluated the quality of mathematical content in a representative subsample of the videotaped lessons. They examined 30 lessons in each country, basing their judgments on a detailed written description of each lesson's content. Descriptions were altered to disguise the country of origin (deleting, for example, references to currency that might identify the country).

EXHIBIT 6:

PERCENTAGE OF LESSONS WITH CONTENT JUDGED TO BE OF LOW, MEDIUM, OR HIGH QUALITY

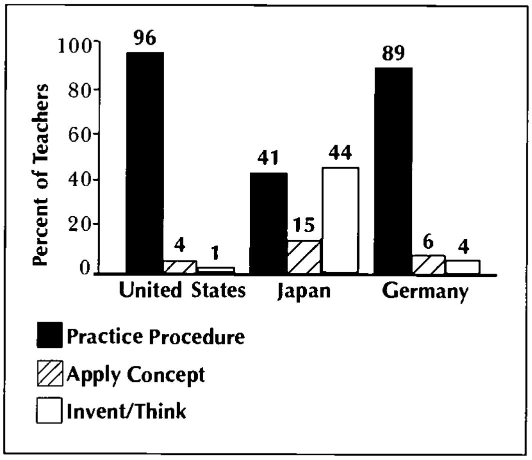


The professors completed several in-depth analyses, the simplest of which involved global judgments of the content quality of each lesson on a three-point scale (low, medium, high). The judgments are summarized in Exhibit 6. Whereas 30 percent of the Japanese lessons and 23 percent of the German lessons received the highest rating, none of the U.S. lessons did so. Eighty-seven percent of U.S. lessons received the lowest rating, compared with 13 percent of Japanese lessons and 40 percent of the German lessons.

3. Student Thinking During the Lessons

EXHIBIT 7:

AVERAGE PERCENTAGE OF SEATWORK TIME IN EACH COUNTRY
SPENT WORKING ON THREE KINDS OF TASKS



When researchers examined the kind of work students engaged in during the lesson, they found a strong resemblance between Germany and the United States, with the situation in Japan distinctly different. Three types of work were coded in the videotape study: Practicing Routine Procedures, Applying Concepts to Novel Situations, and Inventing New Solution Methods/Thinking.

Almost all students' seatwork time in the United States and Germany was spent in practicing routine procedures, compared with 41 percent in Japan. Japanese students spent nearly half their time inventing new solutions and engaging in conceptual thinking about mathematics.

4. Teachers' Views of Reform

Considerable effort has been invested in the reform of mathematics teaching in the United States in recent years. Agreement exists among experts about what good instruction should include. The main goal of much of mathematics reform is to create classrooms in which students are challenged to think deeply about mathematics by discovering, understanding, and applying concepts in new situations. Numerous documents—including the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards* and the *Professional Standards for Teaching Mathematics*—encourage teachers to adopt new practices and point to some features of preferred instruction.

Although many of the current ideas stated in such documents are not defined in such a way that they could be directly coded, it is possible to view some of the indicators developed in the videotape study in conjunction with these current ideas. When the videotape data are viewed in this way, in some respects Japanese lessons come closer to implementing the spirit of current ideas advanced by U.S. reformers than do the U.S. lessons. Japanese students were asked to solve problems, generate alternative solution methods, and explain their thinking more often than were U.S. students. In contrast, there were other ways in which Japanese lessons departed from current U.S. reform recommendations. For example, eighth-grade Japanese lessons emphasized abstract, symbolic problems more than real-world hands-on problems and almost never required calculators. Thus, Japanese lessons follow a distinct pattern that cannot be labeled as either traditional or reform-minded in the U.S. sense.

U.S. teachers believe that they are implementing current reform ideas in their classrooms. When asked to evaluate their videotaped lesson, almost three-fourths of the U.S. teachers rated it as being reasonably in accord with current ideas about the teaching and learning of mathematics. They were more than twice as likely to respond this way than either Japanese or German teachers. Teachers who said that their videotaped lesson was in accord with current ideas about the teaching and learning of mathematics were asked to justify their responses. Although the range of responses to this question was great, the vast majority of U.S. teachers referred to surface features, such as the use of manipulatives or cooperative groups, rather than to the deeper characteristics of instruction, such as the depth of understanding developed by their students.

The study suggests that written reports on reform disseminated to teachers have little impact on practices in the classroom. One reason for this may be that teachers may not have widely shared understandings of what such terms as "problem solving" really mean, leading to idiosyncratic interpretations in the classroom. Videotaped examples of high-quality instruction, tied to descriptions of what quality instruction should look like, can help address this issue.

5. Achievement in the Three Countries

Japanese students were among the highest-scoring in the 41 TIMSS countries in eighth-grade mathematics. U.S. and German students scored below the international average. There was no statistically significant difference between the U.S. and German students' average scores.

BACKGROUND ON EDUCATION IN THE UNITED STATES, JAPAN, AND GERMANY

SCHOOL ORGANIZATION

United States

The U.S. school system is difficult to define because practices differ among the thousands of school districts in the country. Within-class grouping or individualization of instruction is common in elementary schools in reading and mathematics. In middle and high schools, students are frequently grouped by ability in mathematics classes. In the United States, 80 percent of principals of eighth graders reported that they provide different ability-based classes in mathematics, but only 17 percent reported this in science.

In the United States, educational expectations and teaching standards can also differ substantially among communities, based on a neighborhood's economic status and parents' expectations for their children's futures. Minority students are overrepresented in lower level classes and in schools in poorer areas.

There are various procedures for dealing with students whom teachers judge as having not learned the course material. Such students can be promoted anyway, retained in grade, moved to a lower tracked class, or given remedial assistance. Principals reported that 4 percent of the students in their schools were required to repeat grade eight.

The U.S. system does not use high-stakes gateway examinations to regulate entrance into further schooling before the end of twelfth grade. Seventeen states currently conduct an exit examination as a requirement for high school graduation, but, in most cases, this is a minimum-competency test. Scores on college entrance examinations such as the SAT and ACT are given considerable weight by most selective universities, although non-selective schools may not require them at all.

Japan

Students in Japanese elementary and junior high schools are rarely tracked or grouped by academic ability. During the nine years of compulsory education, all students study the same nationally determined curriculum, regardless of differences in motivation or ability. Until the end of ninth grade, most students are promoted automatically, whether or not they understand the material. Students who are overly or insufficiently challenged by classroom assignments can receive extra help after school from a teacher, or their parents can pay to enroll them in a *juku*, which is a private after-school class. In Japan, a substantial amount of remedial and enrichment instruction is provided by the private sector.

Before Japanese students enter high school, they are required to take a high school entrance exam covering five core subjects, including mathematics and science. Examination scores and previous academic performance are used to divide students into high-, medium-, and low-level high schools. Those with the best scores are accepted into the best academic high schools, which prepare students for the best universities. The least able students are accepted only by the lesser ranked commercial or vocational high schools, which prepare graduates to enter the labor force after high school. Students and parents clearly understand the consequences of academic performance and the examination at the end of ninth grade with respect to students' future careers and life choices. Japanese students say that the examination motivates them to study harder during junior high school.

After graduating from high school, most Japanese students enter the labor force or begin vocational training. Approximately one-third of high-school graduates apply to a university or two-year college, most of which require an entrance examination. Competition on the entrance examinations for prestigious universities is intense, although some lower ranked colleges will accept most who apply.

Germany

Schools in Germany are controlled by the 16 federal states, so there are differences in the requirements and rigor of schools. One large difference is that those states that were part of the German Democratic Republic (East Germany) require students to attend the Gymnasium for only eight years (graduating at grade 12), as opposed to nine in the former West Germany.

The German school system starts the sorting process much earlier than do Japan and the United States—usually at the end of the fourth grade. This is accomplished through a system of gateway examinations and ability grouping, which differs considerably from the Japanese. Most German students attend one of four types of schools:

- Gymnasium (academic), which provides a demanding, academic curriculum through grade 13 and leads to the Abitur exit examination (necessary to attend the university) and university study. About one-third of secondary-school students attend the Gymnasium.
- Realschule (commercial), which provides a moderately paced curriculum ending at grade 10 and leads to a school-leaving certificate and vocational training or further study at a Gymnasium. One-fifth of all secondary-school students attend Realschule.
- Hauptschule (general/vocational), which provides practically oriented instruction ending at grade nine and leads to a school-leaving certificate and vocational training or employment. Children of immigrants, foreign-workers, and other non-Germans are overrepresented in the Hauptschule. One-fourth of secondary-school students attend Hauptschule.
- Gesamtschulen (mixed), which provides academic, commercial, and vocational programs. This is the newest form of school, and about 15 percent of students attend these.

At the end of the fourth grade, children's teachers recommend which of these schools the children will attend. Parents can, and frequently do, override teacher recommendations if they believe that their child deserves to be placed in a higher track. If the student is unable to keep pace, however, he or she will have to repeat a grade, and, after repeated failure, will be returned to the next lower level of schooling. Principals reported that 5 percent of all students were required to repeat grade eight.

Most German students finishing the Hauptschule at the end of grade nine or Realschule at the end of grade 10 receive a diploma, and most states do not require an exit exam. About 10 percent of the students receive only a school-leaving certificate instead of a diploma. Approximately one-third of German students are enrolled in a Gymnasium, and about one-fourth of these end their studies before taking the Abitur examination at the end of grade 13. Few students who sit for the Abitur fail it, although those with a lower score may not be able to enter their preferred university or field of study.

CURRICULUM

United States

U.S. students attend school approximately 180 days per year, five days per week. Each day, school usually runs from about 8:00 a.m. until mid-afternoon, with a lunch break and five- to seven-minute breaks between classes. Schools vary in the ways they organize students. Middle schools commonly include either grades seven to nine or six to eight, although variations exist. In some schools, the student body is subdivided into "houses" or "blocks," which include several classes of students and a single group of teachers, to strengthen continuity in student-teacher and student-student relationships. In other schools, students change teachers and class-mates at the end of each period.

Course content and textbooks usually differ in the higher and lower level classes. In the eighth grade, lower level classes typically focus on a review of arithmetic and other basic skills, with a small amount of algebra. Higher level classes focus more heavily on algebra, with a small amount of geometry.

U.S. eighth graders spend considerably more hours per year in mathematics classes than their Japanese and German counterparts. U.S. students average 143 hours of mathematics, while German students receive 114 hours and Japanese students 117 hours. U.S. students' instructional time is both longer and more compressed, because it takes place within a school year of approximately 180 days compared with 188 in Germany and 220 in Japan. However, TIMSS found that, by international standards, the topics taught in U.S. eighth-grade mathematics classrooms were at a seventh-grade level, while topics observed in the German and Japanese classrooms were at a high eighth-grade or even ninth-grade level.

Japan

Japanese schools are in session 220 days per year, five days per week, and two Saturday mornings per month. School usually starts at 8:00 a.m., ends by mid-afternoon, and includes a lunch break, 5- to 15-minute breaks between various periods, and a homeroom meeting at the beginning and end of each day. The number of classes per day is frequently reduced for special seasonal events, school-wide meetings, and other activities. Junior high schools include grades seven to nine. Students in a given class remain together throughout the day, and a different teacher comes to the students' classroom for each subject.

Japanese public schools offer a single curriculum for all students through the end of ninth grade. In mathematics, all eighth-grade students study a curriculum focused heavily on algebra and geometry. Review of arithmetic is not included in the official curriculum goals and textbooks. TIMSS' observers noted that there are differences in students' ability to keep up with the curriculum both within each classroom and also between schools whose students come from families with predominantly high or low economic backgrounds. However, the Japanese system is designed so that teachers throughout the country strive to meet similar standards for presentation of content, while allowing almost unlimited variation in students' standards of performance.

The Japanese curriculum is more tightly controlled and centralized than in the United States. While a typical U.S. high school may have hundreds of courses, the typical Japanese school offers under 50. Even the electives are controlled by the government. Japan places a greater emphasis on having all students master the same content rather than providing students with many options. Japanese schools emphasize the importance of trying and struggling to master the material, rather than just memorizing the right answer.

Germany

German students attend school approximately 188 days per year. School usually starts around 7:45 a.m. and ends around 1:15 p.m., with 10- to 25-minute breaks between classes. There is no lunch period, and most students return home for lunch at the end of the school day. Gymnasium usually includes students from grades five to 13, Realschule grades five to 10, and Hauptschule grades five to nine. Eighth-grade students remain together throughout the day, with teachers changing classrooms. Classes are usually kept together for several years, and students develop a strong sense of unity.

Classes in grades five to nine typically cover the same content in all three types of German schools, although there is considerable difference in the depth and rigor of instruction among the three types. Gymnasium students usually learn through a theoretical approach, while Hauptschule students learn through a practical approach to the same content. In eighth-grade mathematics, the German curriculum is focused mostly on geometry and algebra for all three types of schools, with some mixture of other topics.

Within most schools, all eighth graders follow the same course of study in mathematics and science, regardless of their ability level. Seventy-five percent of the schools reported that they provide only one course of study in mathematics, and 90 percent provide only one course in science. The German system tends to divide students of different ability levels into separate schools rather than separate classes within schools.

TEACHER PREPARATION AND TRAINING

U.S. teachers reported more years spent in college than teachers in all but a few of the 40 other TIMSS countries. Nearly half of the teachers of U.S. eighth-graders had a master's degree, a proportion that was exceeded by teachers in only four other TIMSS countries. In Japan, few teachers had more than a bachelor's degree with teacher training. In Germany, teachers complete 13 years of primary and secondary school, followed by about six years of study at a university, after which they write a thesis and pass an examination to receive a degree considered equivalent to a U.S. master's degree.

U.S. teachers lack the long and carefully mentored instruction in teaching that Japanese and German teachers receive. In all three countries, prospective teachers first take a mixture of courses in education and in academic subject areas leading to graduation from college. In Germany, prospective teachers must pass a state examination at the end of college and spend two years in a teaching apprenticeship that progresses from classroom observation, to assisted teaching, and finally to unassisted teaching under the close supervision of a mentor teacher. They also attend seminars on their subject areas once or twice a year. In Japan, prospective teachers must pass certification and employment selection exams. In their first year, they have a reduced teaching load and must spend at least 60 days in closely mentored teaching and 30 days of further training at resource centers. Senior teachers mentor and assist junior teachers throughout their careers. They have many casual opportunities to share advice, ideas, and teaching materials. Over three-fourths of Japanese teachers say they meet to discuss curriculum at least once a month.

By contrast, prospective teachers in the United States typically spend 12 weeks or fewer in student teaching near the end of their undergraduate training. Some teachers may be hired without any formal training. Licensing requirements vary by state, and most induction programs depend on the district's desire and ability to provide it.

The teacher of U.S. eighth-grade mathematics and science students is typically a woman in her 40s—the average age of teachers in most of the TIMSS countries—with about 15 years of teaching experience. The teacher of German students is typically a man near the age of 50 who has been teaching for about 19 years, and the teacher of Japanese students is typically a man in his late 30s who has been teaching for 14 years.

U.S. teachers teach more hours per year than their Japanese and German counterparts. U.S. mathematics teachers teach 26 periods per week, compared to 24 for German and 16 for Japanese teachers. U.S. teachers are typically at school about eight hours each day and are expected to be in the building during school hours, although many come earlier or stay later. Japanese teachers are usually at school about nine hours each day, staying until student club activities end. Japanese schools also are in session for a half-day two Saturdays per month. German teachers' schedules are similar to college professors; they are not required to be in the building when they are not teaching. When school is over around 1:30 p.m. most return home, where they do their planning and grading of papers.

FREQUENTLY ASKED QUESTIONS AND ANSWERS

What is TIMSS?

The Third International Mathematics and Science Study (TIMSS) is the largest, most comprehensive international study of mathematics and science achievement ever conducted. More than 500,000 students from as many as 41 countries participated in the assessments, which were administered in 30 languages to pupils in five grade levels in 1995. TIMSS analyses include student outcomes, instructional curricula, and cultural context.

Why is TIMSS important?

In addition to being the largest and most comprehensive international study of mathematics and science achievement, TIMSS goes beyond comparison of students' scores. It examines student achievement, teaching, curricula, and the lives of students and teachers. The study's innovative research techniques include analyses of textbooks and curricula, videotapes of teaching, and ethnographic studies. Until TIMSS, no large nationally representative study ever observed U.S. classrooms to assess how teachers teach.

The result is a complete and accurate portrait of how U.S. mathematics and science education differs from that of other nations. TIMSS is a treasure trove of data that combines qualitative and quantitative information. The study can help us to define what we mean by "world-class" performance and to set standards for what our children need to know and be able to do in order to compete with their international peers.

TIMSS also is vital to our strategies to improve schools. If mathematics and science education is to improve in the United States, we must carefully examine how other countries' policies and practices help students achieve. Not everything will be applicable to the United States, but this comparison opens new lines of investigation and reveals several aspects of education in the United States that can be improved.

TIMSS helps us measure progress toward our national goal of improving children's academic performance in mathematics and science. TIMSS is more than a scorecard for the mathematics and science events in the "education Olympics." It is a diagnostic tool for helping us to examine our nation's progress toward improvement of mathematics and science education. Students, teachers, and principals shared information about their backgrounds and their attitudes, experiences, and practices in the teaching and learning of mathematics and science. Ultimately, TIMSS is important because it can illuminate how our education policies and practices compare to those of the world community.

Why should the United States be concerned about what students in other countries learn?

One of the driving forces behind TIMSS is the recognition by policymakers that mathematical and scientific literacy affect economic productivity. World-class competence in mathematics and science is essential to compete successfully in today's interdependent global marketplace. TIMSS provides a comparative international assessment of educational achievement in these two subjects and the factors that contribute to it.

By studying what children in other countries learn, we can see what our own children are capable of achieving. We also can better understand the effects of our own curricula and materials by viewing them through the prism of other countries' practices.

Who conducted TIMSS?

TIMSS is being coordinated by the International Association for the Evaluation of Educational Achievement (IEA), an independent international cooperative of research centers and departments of education in more than 50 countries. TIMSS has the largest complement of participants of any of IEA's international studies.

TIMSS was designed by task forces including members from the many participating countries. These groups were involved in developing the tests and reviewing instruments, questionnaires, and sampling plans. IEA monitored the sampling process, quality control, scaling of tests, and training. In addition, an International Steering Committee continues to monitor the activities and progress of the study, and a U.S. Steering Committee has been established to give advice regarding the implementation of the study in the United States.

The international TIMSS is funded by the National Center for Education Statistics (NCES) of the U.S. Department of Education, the National Science Foundation (NSF), and the Canadian Government. Dr. Albert E. Beaton directs the study's international activities with his staff at the International Study Center at Boston College.

TIMSS in the United States is funded by NCES and the NSF. NCES is overseeing the collection, analysis, and reporting of the U.S. data through a contract with Westat, Inc., and Dr. William Schmidt of Michigan State University, the national research coordinator for TIMSS.

How was the study conducted?

TIMSS used data collection methodologies that go beyond those used in two previous international studies of mathematics and science. Student achievement was measured through written tests that included multiple-choice and open-ended questions. In many countries, samples of students also were selected to engage in performance assessments (design experiments, test hypotheses, and record findings through hands-on activities).

In addition, students, teachers, and administrators completed questionnaires that solicited information on 1,500 issues, such as student background, teacher instructional methods, and a country's commitment of staff and materials to science and mathematics instruction. TIMSS also analyzed the curricula in participating countries though an ambitious review of textbooks, curriculum guides, and other materials.

Beyond these approaches, NCES designed two new analysis methodologies that were carried out in the United States, Japan, and Germany. In the TIMSS Videotape Classroom Study, teachers in eighth-grade mathematics classes were videotaped for the purpose of studying classroom interactions. The videotapes offer insights into the organization of lessons and instructional methods. In the ethnographic case study, researchers conducted in-depth interviews with and observations of teachers, students, administrators, and parents in these countries. The case studies focus on the implementation of national standards, teachers' lives, the role of school in adolescents' lives, and methods of dealing with ability differences.

Why were different methods used?

TIMSS includes five parts: assessments, questionnaires, curriculum analyses, videotapes of classroom instruction, and case studies of policy topics. The study was designed to bring a variety of different and complementary research methods to bear on important policy questions. The use of multiple methods has three major benefits. First, it strengthens the conclusions of the study, because researchers are able to verify key findings by comparing results based on different research methods. Second, it provides broader information, because different types of data can be gathered and acquired than through any single method or instrument. Third, the use of multiple methodologies enriches the public's understanding of the contextual meaning of key findings. Each of the five parts represents an important advance in its field. Taken together, they provide an unprecedented opportunity to understand U.S. mathematics and science education from a new and richer perspective.

If the teachers in the videotaped lessons are just average teachers, then why are they being held up as models?

These teachers are not presented as models but rather as primary sources that can help us better understand teaching and learning in the three countries. The lessons and teaching on these videotapes are typical of the lessons and teaching observed in the TIMSS Videotape Classroom Study. Because these lessons are typical, they show us what ordinary day-to-day teaching is like in these countries, and they provide a reasonable point of comparison.

The purpose of these videotapes is not to present extraordinary teaching for U.S. teachers to imitate, but to help viewers discuss how typical teaching relates to student learning.

Isn't it unfair to compare an inadequate U.S. teacher with better Japanese and German teachers?

The teachers who are on this videotape are similar to those in the TIMSS Videotape Classroom Study and therefore are typical of those observed in most eighth-grade classrooms in each country.

Also, it must be noted that the teacher is not the only one who influences what takes place in the classroom. A country's examination system, curriculum, books, materials, structure, schedule, and training and support of new teachers also have a significant impact on teaching and learning.

How comparable are the student populations? Doesn't this videotape compare average U.S. students with the best in Japan and Germany?

The two U.S. lessons shown on the videotape are from mixed- and high-ability classes. The two Japanese lessons are from mixed-ability classes, and the two German classes are from average-ability classes. Japanese public schools offer a single curriculum for all students through the end of the ninth grade. Unlike those in the United States, students in Japanese elementary and junior high schools are rarely tracked or grouped by academic ability. There is a widespread belief in Japan that the nine years of compulsory education must offer the same nationally determined curriculum to all students. Until the end of ninth grade, there are no gateway exams, and all students are automatically promoted.

By contrast, the German school system usually sorts students into one of three types of schools at the end of the fourth grade. Classes in grades five to nine cover basically the same content in all of the three types of German schools, although there is a considerable difference in the depth and rigor of instruction among the three school types. However, in eighth-grade mathematics, the German curriculum focuses mostly on geometry and algebra for all three types of schools, with some mixture of other topics.

In the United States, the content students study depends on the track in which they have been grouped. Four out of five principals of schools with an eighth grade reported that they provide different ability-based classes in mathematics. In the eighth grade, lower level classes typically focus on a review of arithmetic and other basic skills, with a small amount of algebra.

Higher level classes focus more heavily on algebra, with a small amount of geometry. In order to compare classes with similar content, the two U.S. lessons on the videotape had to be chosen from mixed- or high-ability classes that study geometry and algebra.

Why do students in other countries do better?

It is beyond the bounds of a study such as this to suggest reasons for variations in student performance. We can, however, offer observations that give an audience the facts needed to make an informed judgment.

It can be observed from the TIMSS study that:

- The content of U.S. mathematics classes requires less high-level thought than that of classes in Japan and Germany.
- U.S. mathematics teachers' typical goal is to teach students how to do something, while Japanese teachers' goal is to help them understand mathematical concepts.
- Japanese teachers widely practice many of the U.S. mathematics reform recommendations, while U.S. teachers do so less frequently, even though most U.S. teachers report familiarity with the reforms.

Is it fair to compare less diverse countries to the United States?

While the United States is more ethnically diverse than Japan, Japanese schools are not tracked by ability until high school. Thus, the typical Japanese eighth-grade classroom is likely to have students with a greater range of academic abilities than its U.S. counterpart. Also, during the period of this study, Germany was in the early stages of reunifying the East and West, adding a considerable diversity of experiences, cultures, and incomes.

During interviews in this study, teachers in all three countries frequently described student diversity as a challenge. U.S. teachers referred primarily to differences in U.S. students' social, economic, or ethnic background. German teachers referred to differences between children of German citizens and those of their country's foreign workers. Japanese teachers referred to the wide differences in academic ability within each classroom.

Moreover, because the United States, Japan, and Germany are three of the world's strongest economic powers, our children will work together in the global marketplace with children from Japan and Germany. All of our children, whatever their race or background, can and must be educated to the same high levels as their Japanese and German peers.

Don't other countries appear to have better results because they don't have the problems that we have?

No education system is perfect; each has its own advantages and disadvantages. Many of the countries whose students outperformed those in the United States have lower incomes, employment levels, and standards of living.

How does teaching differ in the three countries?

Experts recommend that well-taught lessons should be focused on having students think about and come to understand mathematical concepts. In contrast, U.S. and German eighth-grade mathematics teachers usually explained that the goal of their lesson was to have students acquire skills. The U.S. and German emphasis on skills above understanding also carries over into the types of mathematical work that students are assigned to do at their desks during class.

U.S. teachers rarely develop concepts, in contrast to German and Japanese teachers who usually do so. In Germany, the teacher often does the mental work in developing the concept, while the students listen or answer short questions designed to add to the flow of the teacher's explanation. Japanese teachers, however, design a lesson in such a way that the students themselves derive the concept from their own struggle with the problem. When a lesson included a mathematical concept, it was usually simply stated in U.S. classrooms. U.S. lessons were interrupted more frequently by announcements, visitors, and so forth, than those in Germany and Japan.

A panel of experts evaluated the mathematics used in these lessons. None of the U.S. lessons were considered to contain a high-quality sequence of mathematical ideas, compared to 30 percent of the Japanese and 23 percent of the German lessons. Instead, the lowest rating was assigned to the mathematical reasoning used in 87 percent of the U.S. lessons, but to only 40 percent of the German and 13 percent of the Japanese lessons.

Should we imitate other countries?

This study is not intended to suggest that the adoption of another country's education system is desirable or even feasible. Rather, the goal is to promote a better understanding of our current system from a global perspective. By seeing alternative ways of teaching and learning, our educators, policymakers, and the general public can have a discussion that moves beyond preconceptions about our schools and teaching. TIMSS' findings can serve as an objective basis for thinking about how to improve, not replace, the U.S. education system.

Many people think Japanese education is teacher-focused, authoritarian, and centered on memorization of facts. The videotapes offer a different picture of Japanese education. Which picture is true?

These videotapes present typical Japanese classrooms and show how their teachers teach and their students learn. The goal in a typical Japanese eighth-grade mathematics classroom is understanding, not memorization. Students are required to explain their reasoning and encouraged to explore alternate solutions and explanations. A student in a U.S. mathematics classroom is more likely to be given a formula to memorize; a Japanese student would be required to derive the formula.

How are the goals of teaching different in the three countries?

German and Japanese teachers put a greater emphasis on understanding and developing concepts, while U.S. teachers emphasize learning how to do things. In Japanese classrooms, a concept can be an interesting puzzle to analyze and comprehend, while in the U.S. classroom, it is a tool to be used to do something else.

How is the life of a teacher different in these countries?

U.S. teachers spend more years in college than do teachers in all but a few of the 40 other TIMSS countries. Teachers of U.S. and German eighth-grade students teach more classes per week than do Japanese teachers. German teachers spend the shortest amount of time at school and come and go during the day depending on their schedules, much like U.S. college professors. U.S. and German teachers do not have the rich informal opportunities to learn from each other and to share questions about teaching-related issues that are enjoyed by their Japanese colleagues. In addition, U.S. teachers lack the long and carefully mentored introduction to teaching that Japanese and German teachers receive.

What does TIMSS show about mathematics reform?

Most U.S. mathematics teachers have heard of the National Council of Teachers of Mathematics (NCTM) standards, and many believe that they are putting them into practice. Ironically, in many respects, everyday Japanese teaching appears closer to the goals of the U.S. mathematics standards than the teaching in the United States.

HANDOUT/TRANSPARENCY MASTERS

PREDISCUSSION QUESTIONS

1. When you think of a typical U.S. eighth-grade mathematics classroom, what are the characteristics of teaching and learning that come to mind?

2. When you think about a typical Japanese eighth-grade mathematics classroom, what are the characteristics of teaching and learning that come to mind?

POSTDISCUSSION QUESTIONS

1. In viewing and discussing the videotapes, what was interesting or surprising to you? What did you learn about how different countries view teaching, learning, and mathematics? What about the process of teaching mathematics?

2. What questions about teaching and learning did viewing the videotaped lessons raise for you? How can you pursue these questions? Are there things you would like to try in your classroom as a result of viewing these lessons?

GENERAL SUGGESTIONS FOR VIEWING THE LESSONS

STAY FOCUSED ON THE LESSON ITSELF.

- What do you notice?
- What do you hear?
- What inferences do you find yourself making and why?
- What patterns provide clues to how and what the student/teacher was thinking?

DRAW ON YOUR EXPERIENCE WITH TEACHERS AND STUDENTS AND WITH TEACHING AND LEARNING, BUT ALSO LOOK PAST YOUR ASSUMPTIONS AND EXPERIENCES TO SEE WITH FRESH EYES.

- What do you think is the teacher's goal? What does he/she seem to want students to learn? What do you think they are learning?

- What does the teacher do? Are there key moves or moments in the lesson? Are there crucial missed opportunities?

- Why do you see this lesson in this way? What does this tell you about what is important to you? Look for patterns in your thinking.

- What questions about teaching and learning did viewing the videotape raise for you?

- Are there things you would like to try in your classroom as a result of viewing the lessons?

QUESTIONS ABOUT MATHEMATICS INSTRUCTION

- What is the mathematics of the lesson?
- What seems to be the teacher's mathematical goals?
- How does the lesson flow?
- Are there logical connections between the parts of the lesson?

QUESTIONS ABOUT COMMUNICATION BETWEEN TEACHER AND STUDENTS

- What does the teacher do to orchestrate the discussion in the lesson? What are the questions posed to students? When are they posed? How do the questions elicit mathematical thinking in the students?

- What does the teacher do to use students' ideas in the discussion? Are most students involved? How are students' ideas used, and what seems to be the purpose for student ideas?

- What decisions does the teacher appear to make in regard to students' ideas or discussion?

- What do the students do in the lesson discussion? What does their communication suggest about their mathematical understandings?

QUESTIONS ABOUT TEACHERS' BELIEFS

- What does this teacher seem to believe about mathematics? About the way students learn? About the role of the teacher?

- What do the clues in the specific evidence tell you about patterns of thinking? About apparent theories of teaching and learning?

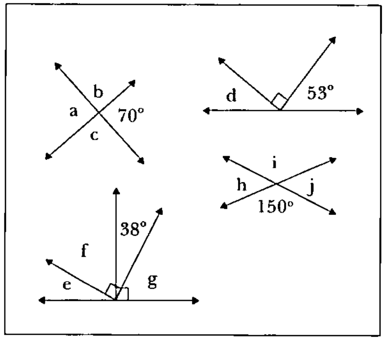
- Are there common cultural theories of teaching and learning that seem to underlie this teacher's beliefs?

LESSON TABLES

While the videotapes and their transcripts provide highlights of each of the six lessons, the following tables provide overviews of the entire lessons. Although the videotape shows roughly 12 minutes of each lesson, the full lessons were significantly longer, ranging anywhere from 37 minutes to a little over 51 minutes. The tables break each lesson down into components and briefly describe what takes place during the time specified. The tables use abbreviations such as CW (whole-class work) and SW (seatwork). In addition to providing information on lesson components not included in the videotape, the tables also include problems, diagrams, formulas, and other items discussed during the lesson but not shown on the videotape. Thus, the tables serve as valuable supplements to the videotape and should be referred to for a complete understanding of the lessons. The tables are organized to reflect the order of the lessons in the videotape, starting with the geometry lessons for the United States, Japan, and Germany, followed by the algebra lessons for these countries in the same order.

U.S. LESSON ONE: GEOMETRY—ANGLES

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
00:01	CW: Working on Tasks/Situations (2 min 15 sec)	(Chalkboard)  m<a = m<b = m<c = m<d = m<e = m<f = m<g = m<h = m<i = m<j = What is the angle that is vertical to the 70-degree angle? (No answer) Which angle is vertical to angle A? Students: Angle 70. Teacher: Therefore, angle A is 70 degrees. What is the supplementary angle to angle A? Student 1: Angle B. Teacher: Angle B and angle C. What do the supplementary angles add up to? Students: 180 degrees. What is the other angle that I indicated in the diagram besides the 53-degree angle? Student 2: Right angle. What is the size of a right angle? Student 2: 90 degrees. Teacher: Right angle is 90 degrees. What other angle is left in the diagram? Student 3: 37 degrees. Why is it 37 degrees? Student 4: Because 53 degrees and 37 degrees add up to 90 degrees, and with the other 90 degrees, it adds up to 180 degrees.

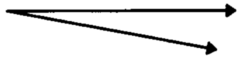
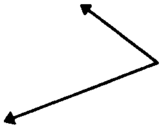
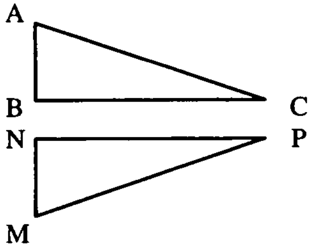
U.S. LESSON ONE: GEOMETRY—ANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
		Why do they have to add up to 180 degrees? Student 5: Because it is a straight angle. Teacher: A straight angle is 180 degrees.
02:16 CW:	Setting Up Tasks/Situations (2 sec)	Figure out the rest of the angles.
02:18 SW:	Working on Tasks/Situations Individually (40 sec)	
02:58 CW:	Sharing Tasks/Situations (2 min 15 sec)	Students: Angle G = 60 degrees Angle F = 60 degrees Angle E = 30 degrees Angle H = 30 degrees Angle J = 30 degrees Angle I = 150 degrees
05:13 CW:	Sharing Homework (9 min 46 sec)	Let's get out the worksheets that I gave you earlier this week to make sure that you understand complementary, supplementary, and angle measurements. (Worksheet pg.103) Find the measure of a complement of an angle of the given measure. 1) 38° 2) 7° 3) 84° 4) 11° 5) 67° 6) 29° 7) 53° 8) 46° 9) 1° 10) 52° 11) 45° 12) 73° Find the measure of a supplement of an angle of the given measure. 13) 2° 14) 101° 15) 92° 16) 82° 17) 15° 18) 135° 19) 44° 20) 149° 21) 168° 22) 174° 23) 59° 24) 179° Answers: 1) 52° 2) 83° 3) 6° 4) 79° 5) 23° 6) 61° 7) 37° 8) 44° 9) 79° 10) 38° 11) 45° 12) 17° 13) 178° 14) 79° 15) 88° 16) 98° 17) 165° 18) 45° 19) 136° 20) 31° 21) 12° 22) 6° 23) 121° 24) 1°

U.S. LESSON ONE: GEOMETRY—ANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
		(Worksheet p.103 part b) Find the measure of each angle. 1  2  3  Find the complement of each angle. 7. 40 8. 83 9. 16 Find the supplement of each angle. 10. 75 11. 130 12. 5 △ABC = △MNP  13. Angle A 14. BC 15. Angle C A 16. AC What kind of an angle is it? Acute. 14 degrees. Acute. 41 degrees. Obtuse. 155 degrees.

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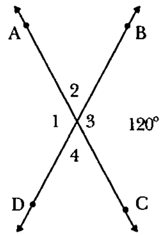
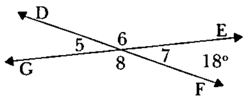
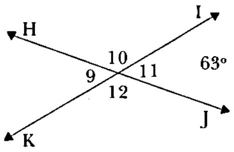
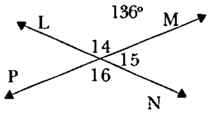
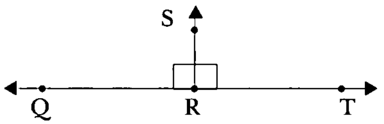
U.S. LESSON ONE: GEOMETRY—ANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

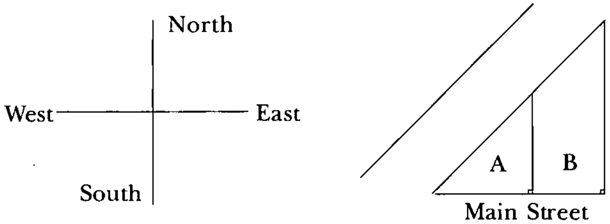
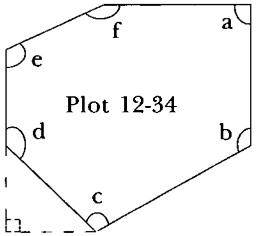
Time	Description of Activity	Description of Content
		7. 50 8. 7 9. 74 10. 105 11. 50 12. 175 ≅ This symbol means congruent, which means identically equal to. 13. Angle M 14. NP 15. Angle P 16. MP
14:59	CW: Working on Tasks/Situations (9 min 46 sec)	I'm going to give you now a worksheet based on these kinds of angles. (Worksheet p.104) ***Example 2:*** Use the figure at the right. Find the measure of each angle. a. <1 b. <2  ***Solution*** a. <3 and <1 are vertical angles, so *m*<3 = *m*<1 = 120° b. <2 and <3 are supplementary angles, so *m*<2 = 180° - *m*<3 = 180° - 120° = 60° What angle must 1 be equal to? 120 degrees. What can you say about 2 and 3. They're supplementary. If 3 equals 120 degrees what is 2? 60. If 2 is equal to 60 what must 4 be equal to? 60.

U.S. LESSON ONE: GEOMETRY—ANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

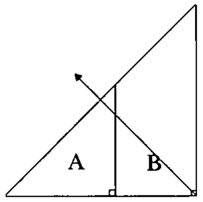
Time	Description of Activity	Description of Content
16:53	SW: Working on Tasks/Situations Individually (9 min 26 sec)	(Worksheet p.104 continued) Use the figure at the right. Find the measure of each angle  25. <5 26. <6 27. <7 28. <8 Use the figure at the right. Find the measure of each angle.  29. <9 30. <10 31. <11 32. <12 Use the figure at the right. Find the measure of each angle.  33. <13 34. <14 35. <15 36. <16 **Spiral Review** 37. Angle *QRS* has the same measure as its supplement. Find *m<QRS* 38. Write an equation that represents the sentence: The product of 12 and a number *k* is 192. *(Lesson 4-7)* 39. Find the sum: - 26 + |-28| *(Lesson 3-2)* 40. Find the measure of an angle that is complementary to an 83-degree angle. *(Lesson 5-3)*
26:19	CW: Sharing Tasks/Situations (41 sec)	Teacher: Problem 37  Two angles are supplementary and they add up to 180 degrees, but they are equal. Each one has to be a 90-degree angle.
27:00	SW: Working on Tasks/Situations Individually (51 sec)	(continuous)
27:51	CW: Sharing Tasks/Situations (1 min 23 sec)	Teacher: Problem 38 12k = 192
29:14	SW: Working on Tasks/Situations Individually (3 min 26 sec)	(continuous) The worksheet on Friday we can go over that today.

U.S. LESSON ONE: GEOMETRY—ANGLES (CONTINUED)

Time	Description of Activity	Description of Content
32:40	CW: Sharing: Homework (5 min 15 sec)	(Worksheet number 3) Exercise 15 Two lots are positioned on a downtown city block, as shown below.  a. What is the angle that First Street makes with Main Street? b. What is the angle that the property line between the two lots makes with First Street? c. What is the angle that the property line between the two lots makes with Main Street? d. Could you suggest a more "equal" way to divide the two lots? (Make a sketch) Exercise 35 A survey is made of a piece of property as shown below.  a. Measure each of the labeled angles and summarize them in a table. b. The surveyor knows that the sum of the angles for a plot of land that has 6 sides should be 720 degrees. What was the total of your angle measurements? c. Suppose point D is moved down to the same level as the point C, so that the angle D is a right angle. What happens to the other angles? Will they still add up to 720 degrees? Check this out.

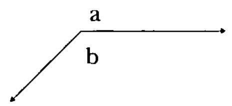
U.S. LESSON ONE: GEOMETRY—ANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
		Exercise 15. a. 45 degrees. b. 135 degrees. c. That one was given to you. It's 90 degrees.  d. Middle of First Street down to the corner
35:44		Exercise 35 b. Teacher asks students the measurements they found. c. Yes, the angles still add up to 720 degrees because there are still 6 angles.
37:55	CW: Working on Tasks/Situations (3 min 13 sec)	There is a formula, but we'll go over it during spring break, and I'm going to give you a hint right now. (N - 2) x 180° tells me how much the angles will add up to. How many degrees in the figure? 6 sided. 6 subtract 2 is 4 times 180 is 720 degrees. 5-sided figure. 540 degrees. All 5-sided figures contain 540 degrees. Triangle. 180 degrees. Square. 4 subtract 2 is 2 times 180 is 360 degrees. What is an equilateral triangle? Equal on all sides. How many degrees in each angle of the equilateral triangle, if each side is equal? 60 degrees. What is an isosceles triangle? Two angles and sides are the same. What is a scalene triangle? None of the sides are equal.

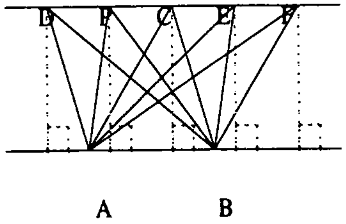
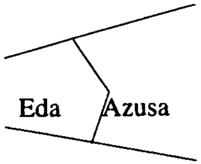
U.S. LESSON ONE: GEOMETRY—ANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
41:08	Other (1 min 22 sec)	(Going over future activity)
42:30	CW: Working on Tasks/Situations (5 min 2 sec)	What do two supplementary angles add up to? 180. How can you remember supp and comp and what they add up to? C comes before s in the alphabet and 90 comes before 180. What can you tell me about vertical angles? Vertical angles must be equal. What is a straight angle? 180. A right angle? 90. An acute angle? Less than 90. Greater than 0. Straight angle? 180. Reflex angle?  a is reflex angle. b is obtuse angle.
47:32~ 48:28	Postlesson Activity	

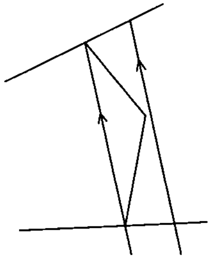
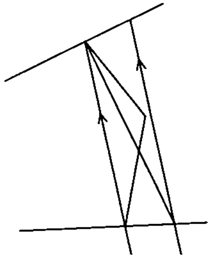
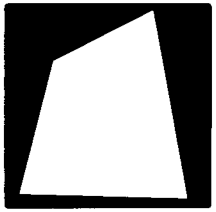
JAPANESE LESSON ONE: GEOMETRY—AREA OF TRIANGLES

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
00:01	Prelesson Activity	
00:27	CW: Working on Tasks/Situations (59 sec)	(Teacher shows a figure on computer screen.) The triangles between two parallel lines have the same areas. 
01:26	CW: Setting Up Tasks/Situations (2 min 39 sec)	(Teacher draws a diagram on chalkboard.) There is Eda's land. There is Azusa's land. And these two people's border line is bent, but we want to make it straight. 
03:34		Try thinking about the methods of changing this shape without changing the area.
04:05	SW: Working on Tasks/Situations Individually (2 min 59 sec)	
07:04	SW: Working on Tasks/Situations in Small Groups (12 min 16 sec)	People who have come up with an idea for now work with Mr. Ishikawa, and people who want to discuss it with your friends, you can do so. And for now I have placed some hint cards up here so people who want to refer to those, please go ahead.

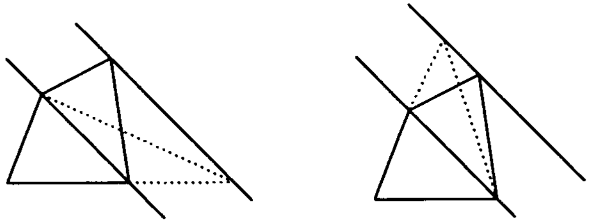
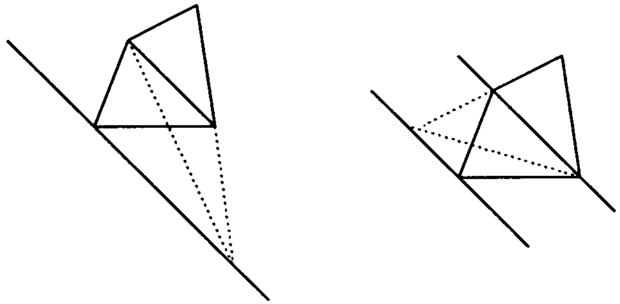
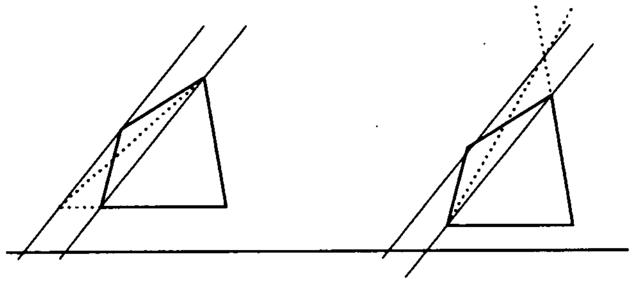
JAPANESE LESSON ONE: GEOMETRY—AREA OF TRIANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
19:20	CW: Sharing Tasks/Situations (3 min 37 sec)	 First you make a triangle. Then you draw a line parallel to the base of triangle. Since the areas of triangles between two parallel lines are the same we can draw a line here. (See the diagram)  We make a triangle and draw a line parallel to the base of the triangle by fitting it with the apex. Since the length of the base doesn't change and the height in between the parallel lines doesn't change. So wherever you draw it the area doesn't change with the triangle that we got first.
22:57	CW: Setting Up Tasks/Situations (42 sec)	(Chalkboard) 
23:25		Without changing the area please try making it into a triangle.
23:39	SW: Working on Tasks/Situations Individually (3 min 8 sec)	
26:47	SW: Working on Tasks/Situations in Small Groups (19 min 24 sec)	Then people who are done please go to Mr. Ishikawa again. And people who want hints I will leave hint cards here, so please look at them and try doing it. It's also fine to do it with your friends. (Hint cards unidentified.)

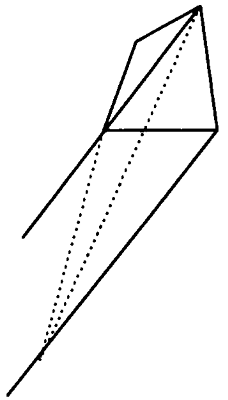
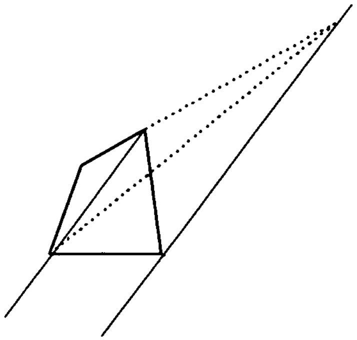
JAPANESE LESSON ONE: GEOMETRY—AREA OF TRIANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
46:11	CW: Sharing Tasks/Situations (2 min 47 sec)	We will make them ABCD. (Draw a diagonal line AC and make a triangle by drawing a parallel line going through D.)  (Draw a diagonal line AC and make a triangle by drawing a parallel line going through B.)  (Draw a diagonal line BD and make a triangle by drawing a parallel line going through A.) 

JAPANESE LESSON ONE: GEOMETRY—AREA OF TRIANGLES (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
		(Draw a diagonal line BD and make a triangle by drawing a parallel line going through C.)  
48:58 49:47	CW: Assigning Homework (49 sec)	Pentagon ABCDE. Let's try making the pentagon into a triangle...I'll make that then a homework.
50:25 ~50:45 Postlesson	Activity	

GERMAN LESSON ONE: GEOMETRY—VOLUME AND DENSITY

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
00:25	Prelesson Activity	
00:48	CW: Sharing Homework (10 min 56 sec)	1. A rectangular bowl of glass with a width of 14.6 cm and a length of 8.4 cm is filled with 17 mm quicksilver (density 13.6 g/cm^3). What is the mass of the quicksilver?* a=8.4cm; b=14.6 cm; c=1.7 cm m $m = a \cdot b \cdot c \cdot S$ $m = 8.4 \cdot 14.6 \cdot 1.7 \cdot [13.6 g/cm^2]$ $m = 2835.4368 = 2835.43(6)$ (student erases 6) *European style is to use commas instead of decimals. We have substituted decimal points for the commas here for clarity.
07:43		2. What is the mass of a shop window (density 2.6 g/cm^3) with a height of 2.30 m, a width of 2.65 m, and a thickness of 8.5 mm? a= 265 cm; b=230 cm; c=85 cm m $m = a \cdot b \cdot c \cdot S$ $m = 265\ \ 230 \cdot 85 \cdot 2.6$ $m = 13469950$
08:37		3. An open balcony has to be covered with lead sheet (length 4.30 m, width 2.35 m). The thickness of the lead is 2 mm, and its density is 11.34 g/cm^3. What is the mass of the surface?
11:44	CW: Working on Tasks/Situations (21 min 11 sec)	Who can remember what we can calculate since yesterday? - Surface of a rectangular solid - Volume of a rectangular solid - Mass of a rectangular solid
12:41		How did we calculate the surface of a rectangular solid? $O = (a \cdot b + a \cdot c + b \cdot c) \cdot 2$
13:12		What is the formula for the volume? $V = a \cdot b \cdot c$
13:43		What is the formula for the mass? $M = V \cdot \rho$
14:19		Goal: You see an empty space on the transparency. And at the end of the lesson you will write in a fourth point—I hope—which you will be able to calculate as well. (Look at solutions to task 1)
14:26		Read the exercise! An iron sheet (rho equals 7.8 grams per centimeters cubed) with a length of 0.5 meter and a width of 20 centimeters weighs 3.90 kilograms. Calculate the height (thickness) of the sheet.

GERMAN LESSON ONE: GEOMETRY—VOLUME AND DENSITY (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
15:04		How could we solve this problem? Student 1: Maybe the formula H equals...M divided by rho divided by A divided by B. Student 2: H equals...(Teacher interrupts student's formulation.)
15:36 17:02		Write on the board: what is given, what are we looking for, calculation path. given: $a = 50$ cm $b = 20$ cm $m = 3900$ g $\rho = 7.8 \frac{g}{cm^3}$ to determine: thickness h
21:08		Student: $V = a \cdot b \cdot c$ $c =$ (Try rejected, it's impossible.) $m = a \cdot b \cdot c \cdot \rho \| \div a \div b \div \rho$ $\frac{m}{a \cdot b \cdot \rho} = c$ $\left[\frac{kg}{dm \cdot dm \cdot \frac{g}{cm^3}}\right]$ Student: $c = \frac{3.9}{5 \cdot 2 \cdot 7.8}$ $c \doteq 0.05$ Correction of the unit: $\left[\frac{kg}{dm \cdot dm \cdot \frac{kg}{dm^3}}\right]$
28:13		Student: Why is it still 7.8? Repeat the three relations we have stated earlier. $7.8 \frac{g}{cm^3}$ $7.8 \frac{kg}{dm^3}$ $7.8 \frac{t}{m^3}$ Can it always be 7.8 or does the number have to change? Student 1: Yes it can stay the same, but it's always different numbers...bigger ones. Student 2: The number can stay like this because the measurement always get bigger as well. Can you explain that?

Time	Description of Activity	Description of Content
		Student 2: Decimeter cubed is bigger than centimeters cubed, thousand times. Student 3: A kilogram is thousand times as big as one gram. If you have a volume that is 1,000 times bigger, what can we say about the weight if the material is the same? Student 1: It has to be thousand time bigger as well.
32:32		Back to our problem. Which number did we get, and what is the unit? $c = 0.05 \quad \left[\frac{kg \cdot dm^3}{dm \cdot dm \cdot kg}\right]$ $c = 0.05dm$ $c = 0.5cm$ The height of the sheet of iron is 0.5 cm.
35:10		I will come back to the top (of our list of possible calculations). Is anybody of you able to say what we just did and what you learned? Student 1: We can calculate the length, height, or width of a rectangular solid. Student 2: Two of the length, width, or height, the mass, and rho. Student 3: We can calculate the length, width, or the height of a rectangular solid if we have the mass and rho. Teacher: We can change the formula for the mass.
36:55	CW: Setting Up Tasks/Situations (2 min 20 sec)	(Teacher hands out three stacks of worksheets for the power group, the middle group, and the basic group. The students decide to which group they want to belong and choose their worksheet.) Two rectangular rods are to compare. The iron rod (spec. weight = 7.8) and the aluminum rod have a width of 5 cm and a height of 3 cm. The iron rod weighs 15.500 kg, the aluminum rod 4.860 kg. Calculate: a) the length of the iron rod b) the specific weight of the aluminum (The aluminum rod has the length of the iron rod.) A 3-cm-thick copper plate (specific weight = 8.9) is 50 cm long and 30 cm wide. How thick must be a a) Aluminum plate (specific weight = 2.7) b) Lead plate (specific weight = 11.4) of the same size, when they are supposed to have the same weight like the copper plate?

GERMAN LESSON ONE: GEOMETRY—VOLUME AND DENSITY (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
		A receptacle has a rectangular shape, and the base area is 40 cm times 30 cm. It is filled with oil (specific weight = 0.93) and has a weight of 55.800 kg. Calculate the clear height of the receptacle. A rectangular sandstone block (specific weight = 2.3) is 30 cm wide, 20 cm high, and weighs 303.600 kg. Calculate the length. A crystal column (specific weight = 2.7) has a quadratic base area and a height of 30 cm. The weight is 5.184 kg. Calculate: a) the length of the edges of the base area. b) the surface area of the column. A step of a staircase made of sandstone (specific weight = 2.3) has a weight of 0.345 t. The step has a height of 20 cm. Calculate the base area.
39:15	SW: Working on Tasks/Situations Individually (3 min 17 sec)	(Teacher explains to a group of students the calculation of the unit one more time while the other students are working individually.)
42:32	CW: Teacher Talk/Demonstration (20 sec)	We discussed two difficult things today. One appeared in your homework. Then we clarified why the number stays the same.
42:53 ~43:32	CW: Working on Homework (39 sec)	Homework: Do number 7a and 7c page 83 from your textbook. Whoever wants to work ahead with the exercise that you got can do that.

U.S. LESSON TWO: ALGEBRA—COMPLEX ALGEBRAIC EXPRESSIONS

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
01:07	Prelesson Activity (1 min 5 sec)	
02:12	CW: Setting Up Tasks/Situations (4 sec)	Start [the] warm-up and I will be with you in a minute. 1) What is the largest integer n for which $2^n > n!$ 2) Find the number of cubic inches in the volume of a rectangular solid if the side, front, and bottom faces have areas of 12 in², 8 in², and 6 in². (Hint: Draw a picture.) 3) Find an ordered pair of integers (a,b), a>b, such that $a^b + b^a = 100$ 4) What is the quotient when $6x^{2a+b-c}$ is divided by $2x^{a+2b+3c}$?
02:16	SW: Working on Tasks/Situations Individually (12 min 51 sec)	
15:07	CW: Sharing Tasks/Situations (1 min 31 sec)	(Teacher elicits the solutions from the students.) 1) 3 was the largest one 2) 24 3) Student 1: 2 minus 6 Teacher: Almost, but read the directions again it says a > b. Student 1: 6 and 2 4) $2x^{a-b-4c}$
16:38	Other (40 sec)	(Discuss future project)
17:18	CW: Setting Up Tasks/Situations (12 sec)	Review: Yesterday we worked on least common denominator. Try this problem. What is the least common denominator? $\frac{1}{x-7} + \frac{1}{x^2-49} =$
17:30	SW: Working on Tasks/Situations Individually (1 min 13 sec)	(Teacher passes out homework for two nights, pages 83 and 84.)

U.S. LESSON TWO: ALGEBRA—COMPLEX ALGEBRAIC EXPRESSIONS (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
18:43	CW: Sharing Tasks/Situations (1 min 30 sec)	(Teacher elicits the solutions from the students.) Student 1: $\frac{7x}{x^2\text{-}49} =$ Teacher: How did you get 7x? Student 1: To get x - 7 to x^2 - 49 you have to square 7 and x. (Teacher: nuh.) You have to multiply 7 · 7 and x · x. Teacher: Not quite...you have to do some factoring. Student 2: x - 7 is a factor of x^2 - 49 so we find the other factor x+7 and add 1. $\frac{x+7+1}{x^2\text{-}49} =$
21:13	CW: Setting Up Tasks/Situations (14 sec)	(Teacher writes a situation on the board.) $\frac{5}{x+6} - \frac{2 - x}{x+6} =$
21:27	SW: Working on Tasks/Situations Individually (1 min 24 sec)	
22:51 22:58	CW: Sharing Tasks/Situations (46 sec)	Subtracting is the same as what? Adding the opposite. $\frac{5}{x+6} + \frac{-2+x}{x+6} = \frac{3+x}{x+6}$
23:37	CW: Setting Up Tasks/Situations (59 sec)	Finish test (Tasks/Situations unidentified) Correct homework (yesterday's ditto) (Tasks/Situations unidentified) Finish graphing calculator worksheet (Tasks/Situations unidentified) And if you finish all of these, then you may start on your homework ditto number 83 Homework: number 83 Least Common Denominator: Find the LCD of rational expressions having the given denominators. Example: $3x$ - 3; $6x$ -12 Solution: $3x - 3 = 3(x - 1)$; $6x - 12 = 6(x - 2) = 2 \cdot 3(x - 2)$ $\text{LCD} = 2 \cdot 3(x - 1)(x - 2) = 6(x - 1)(x - 2)$ 1) 12; 18 2) 9; 15 3) 5; 7 4) x^2y; x 5) $4x$; $8x$ 6) x^2y; xy^2 7) $18x^2$; $24x$ 8) 15; 18; 30 9) x^2y; xy; y^3 10) $3x$ - 6; $12x$ - 24

U.S. LESSON TWO: ALGEBRA—COMPLEX ALGEBRAIC EXPRESSIONS (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
		11) $8x + 8; 12x + 12$ 12) $15x + 45; 18x - 36$ 13) $9x - 9; 18x + 18$ 14) $x^2 - 1; x^2 + 2x + 1$ 15) $y^2 - 1; y^2 - 2y + 1$ 16) $n^2 - n - 6; n^2 + n - 2$ 17) $x^2 + x -20; x^2 -2x - 8$ 18) $x^2 - 9x + 14; x^2 - 3x - 28$ 19) $x^2 - 10x + 25; x^2 - 9x + 20$ 20) $x^2 - 16; x^2 + 2x - 24$ 21) $y^2 + 10y +21; y^2 -6y -27$
24:36	SW: Working on Multiple Assignments Individually (11 min 37 sec)	
36:13- 36:27	Postlesson Activity	

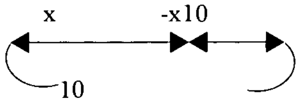
JAPANESE LESSON TWO: ALGEBRA—ALGEBRAIC INEQUALITIES

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
00:01	Prelesson Activity	
00:15	CW: Sharing Homework (6 min 45 sec)	Solve inequalities. (Teacher assigns six students to write the solutions on the board.) (1) $6x-4<4x+10$ $6x-4x<+10+4$ $2x<14$ $\frac{2x}{2}<\frac{14}{2}$ $x<7$ (2) $2x-6 \leq 7x+4$ $2x-7x \leq 4+6$ $-5x \leq 10$ $\frac{-5x}{-5} \geq \frac{10}{-5}$ $x \geq 2$ (3) $1.2x-4.2>0.4x+0.6$ $(1.2x-4.2>0.4x+0.6) \times 10$ $12x-4x>6+42$ $8x>48$ $\frac{8x}{8}>\frac{48}{8}$ $x>6$ (4) $3(x+4) > 5x+2$ $3x+12 > 5x+2$ $3x-5x>2-12$ * $-2x<-10$ $\frac{-2x}{-2}<\frac{-10}{-2}$ $x<5$ (5) $4(x-2) \leq 5(2x-3)$ $4x-8 \leq 10x-15$ $4x-10x \leq -15+8$ $-6x \leq -7$ $x \geq \frac{7}{6}$ (6) $1.8x+2>0.5x+0.7$ $18x+20>5x+7$ $18x-5x>7-20$ $13x>-13$ $x>-1$ *(Corrected) In the above solution, $-2x<-10$ should be $-2x>-10$
07:00 08:51 10:24	CW: Setting Up Tasks/Situations (3 min 24 sec)	(Teacher states lesson goal.) Today will be the final part of word problems. (Chalkboard) You would like to buy 10 cakes all together for less than 2,100 yen in which one cake is 230 yen each and the other cake is 200 yen each. If you want to buy as many 230-yen cakes as possible, what is the maximum number that you can buy? Think about what you need to do to find out how many you can buy and find the answer.
10:34	SW: Working on Tasks/Situations (5 min 24 sec)	

JAPANESE LESSON TWO: ALGEBRA—ALGEBRAIC INEQUALITIES (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
15:58	CW: Sharing Tasks/Situations (9 min 24 sec)	<Thinking method 1> 230-yen cakes x 3 ... x 8 x 9 x 10 690 2,070 2,300 yen 200-yen cakes x7 x 1 14.00 2.00 2,090 2,270 <Thinking method 2> You want to buy ten 230-yen cakes which would cost 2,300 yen You are short 200 yen. You want to substitute some with 200-yen cakes to make up the shortage. How many do you have to substitute? If you buy seven 200-yen cakes, you save 210 yen, which would take care of the shortage of money. Which means you buy three 230-yen cakes. <Thinking method 3> Make the number of 230-yen cakes x. Then the number of 200-yen cakes becomes 10-x. 230 yen 200 yen x 10-x 230x+200(10-x) ≤ 2,100
25:34	CW: Working On Tasks/Situations (9 min 4 sec)	(The teacher passes out worksheets.) We are going to try and do [the problem] using an inequality equation. Buy x amount of 230-yen cakes. 

JAPANESE LESSON TWO: ALGEBRA—ALGEBRAIC INEQUALITIES (CONTINUED)

CW: Whole-class work
SW: Seatwork

<table>
<tr><th>Time</th><th>Description of Activity</th><th>Description of Content</th></tr>
<tr><td></td><td></td><td>(Equation)
$230x + 200 (10 - x) \le 2{,}100$
$230x + 2{,}000 - 200x \le 2{,}100$
$230x - 200x \le 2{,}100 - 2{,}000$
$30x \le 100$
$\le \frac{10}{3}$ x (3.3...)

<u>Answer: You can buy up to three of 230-yen cakes.</u>
(Chalkboard: Setting up an inequality equation facilitates finding the answer.)</td></tr>
<tr><td>34:39</td><td>CW: Setting Up Tasks/Situations
(1 min 5 sec)</td><td rowspan="3">There are two problems on the right side. Try to set up an inequality equation by yourself in the same way and try to solve the problem.
(Worksheet)
You would like to buy 20 apples and tangerines all together for less than 2,000 yen, in which one apple costs 120 yen each and one tangerine costs 70 yen each. Up to how many apples can you buy?
You would like to buy 15 pears and persimmons and a basket all together and for less than 1,000 yen, in which one pear costs 70 yen each, one persimmon costs 50 yen each, and a basket costs 80 yen. You want to buy more pears than persimmons. Up to how many pears can you buy?

Student 1:
<table>
<tr><th></th><th>120-yen apples</th><th>70-yen tangerines</th><th>Total</th></tr>
<tr><td>Amount</td><td>x</td><td>20-x</td><td>20</td></tr>
<tr><td>Sum</td><td>120x</td><td>70(20-x)</td><td>2,000</td></tr>
</table>
$120x + 70 (20 - x) \le 2{,}000$
$120x + 1{,}400 - 70x \le 2{,}000$
$120x - 70x \le 2{,}000 - 1{,}400$
$\le 50x \quad 600$
$\frac{50x}{50} \le \frac{600}{50}$
$x \le 12$
<u>Answer: 12 apples</u></td></tr>
<tr><td>35:44</td><td>SW: Working on Tasks/Situations
Individually
(11 min 29 sec)</td></tr>
<tr><td>47:15</td><td>CW: Sharing Tasks/Situations
(2 min 33 sec)</td></tr>
</table>

JAPANESE LESSON TWO: ALGEBRA—ALGEBRAIC INEQUALITIES (CONTINUED)

CW: Whole-class work
SW: Seatwork

<table>
<tr><th>Time</th><th>Description of Activity</th><th>Description of Content</th></tr>
<tr><td></td><td></td><td>Student 2:
<table>
<tr><th></th><th>70-yen pears</th><th>50-yen persimmons</th><th>Basket</th><th>Total</th></tr>
<tr><td>Amount</td><td>x</td><td>15-x</td><td></td><td>15</td></tr>
<tr><td>Sum</td><td>70x</td><td>50(15-x)</td><td>80</td><td>1000</td></tr>
</table>

$$\begin{aligned} 70x+50(15-x)+80 &\leq 1{,}000 \\ 70x+750-50x+80 &\leq 1{,}000 \\ 70x-50x &\leq 1{,}000-750-80 \\ 20x &\leq 170 \\ \frac{20x}{20} &\leq \frac{170}{20} \\ x &\leq \frac{17}{2}\ (8.5) \end{aligned}$$

<u>Answer: 8 pears</u></td></tr>
<tr><td>49:48</td><td>Teacher Talk/Demonstration
(39 sec)</td><td>(Teacher states lesson summary.) When you work out problems instead of counting things one by one and finding the number, it's usually easier if you set up an inequality and find the answer.</td></tr>
<tr><td>50:27~
51:38</td><td>Postlesson Activity</td><td>(The teacher passes out worksheets for the next lesson.)</td></tr>
</table>

GERMAN LESSON TWO: ALGEBRA—SYSTEMS OF EQUATIONS

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
00:54	Prelesson Activity	Greeting
01:01	CW: Working on Tasks/Situations (19 min 34 sec)	(Determine) 8 to the third power? 512
01:30		Second binomial theorem? $(a-b)^2 = a^2 - 2ab + b^2$
01:47		12 percent of 120? 14.40.
02:00		5 factorial? 120
02:26		A to the third power times A to the seventh power? A to the tenth power.
02:45		One-half minus one-third? One-sixth.
03:10		(Review) What have we done lately? Student: Equations with two variables Teacher: Systems of Equations (on Blackboard)
03:50		What various methods do you know to solve systems of equations? Setting them equal
04:04		Give an example. Student: 2y plus 3x equals 5. And 2y plus 5x equals 37. Teacher: I. $x = 3 - 4y$ Continue: Student: II. $x = 7 + 3y$
05:37		Which other method do we know? Substituting
05:50		Give an example for that. Student: I. $y = 3 - 4x$ II. $x - y = 3x$
07:13		How do we solve this? Substitute y in II.
07:44		What was the third method? Method of addition

GERMAN LESSON TWO: ALGEBRA—SYSTEMS OF EQUATIONS (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
08:14		Give an example. I. $3y + 17x = 1{,}438$ II. $-3y + 28x = 17$
09:25		How do we solve this? I. + II.
10:00		Solve this problem: $\frac{2y-5}{9} = \frac{5}{6}(x-1) - 5y \wedge \frac{3x+1}{12} = \frac{8}{3}(y-2) + \frac{33x}{2}$
11:50		What would you suggest? Eliminate the parentheses: $\Rightarrow \frac{2y-5}{9} = \frac{5x-5}{6} - 5y \wedge \frac{3x+1}{12} = \frac{8y-16}{3} + \frac{33x}{2}$
13:41		Find a common denominator: $\Rightarrow 4z-10=15x-15-90y \wedge 3x+1=32y-64+198x$
18:00		Combining terms: $\Rightarrow 94y + 5 - 15x = 0 \wedge -195x + 65 - 32y = 0$
20:35	CW: Setting Up Tasks/Situations (5 sec)	Copy this in your notebook. Think about the next step.
20:40	SW: Working on Tasks/Situations Individually (5 min 27 sec)	
26:13	CW: Working on Tasks/Situations (8 min 39 sec)	What method do you want to use? Method of addition: I. $94y+5-15x = 0 \quad \mid .(-13)$ II. $-32y+65-195x = 0$ I'. $-1{,}222y-65+195x = 0$ II.+I'. $1{,}254y = 0$ $\Rightarrow \underline{y = 0}$ $\Rightarrow$ yI. $5-15x = 0 \quad \mid -5$ $\Rightarrow -15x = -5 \quad \mid :(-15)$ $\Rightarrow x = \frac{1}{3}$

GERMAN LESSON TWO: ALGEBRA—SYSTEMS OF EQUATIONS (CONTINUED)

CW: Whole-class work
SW: Seatwork

Time	Description of Activity	Description of Content
33:34		Summarize how one has to proceed with this kind of problem: First eliminate the parentheses. Next multiply with the common denominator. Next find the approach to solve the system of equations.
34:52	CW: Setting Up Tasks/Situations (27 sec)	Solve number 12 page 195. 12a) $\frac{3x}{4}+\frac{7}{12}=2-\frac{2y}{9}$, $\frac{2y}{5}+\frac{3}{10}=1+\frac{x}{2}$ 12b) $\frac{x}{3}+2=\frac{y}{2}+\frac{5}{6}$, $\frac{y}{4}+1=\frac{3}{10}-\frac{3x}{5}$ 12c) $\frac{2y-5}{9}=\frac{5}{6}(x-1)-5y$, $\frac{3x+1}{12}=\frac{8}{3}(y-2)+\frac{33x}{2}$
35:19	Working on Tasks/Situations Individually	Who knows what "coefficient" means? It means to work together, to combine.
42:39~ 43:07	Postlesson Activity	Greeting and clean-up

LESSON TRANSCRIPTS

The following transcripts are based on the teacher/class dialogues from the lessons on the videotape, which provide highlights from six complete lessons. The transcripts represent both dialogue and utterances such as "Uhm" and other fillers. Wherever the dialogue was not clear, empty parentheses were inserted. To keep the transcripts to a reasonable length, only the portions of the lessons shown on the videotape were included, providing brief descriptions of what happened during the parts of the tapes that were cut out. Three vertical dots (⋮) are used to indicate where sections of the lessons have been omitted. Important actions of the students and teachers are shown in italics. The gaps in the time code on the left-hand column of the transcript correspond to the omitted sections. For a full understanding of the content of the complete lessons and for problems, diagrams, formulas, and other items mentioned in the transcripts, refer to the lesson tables.

THIRD INTERNATIONAL MATHEMATICS & SCIENCE STUDY
EIGHTH-GRADE MATHEMATICS LESSONS: UNITED STATES, JAPAN, AND GERMANY

00:00:00 VO: Third International Mathematics and Science Study.

00:00:07 VO: This video contains excerpts from six mathematics lessons taped in eighth-grade classrooms in the United States, Japan, and Germany. The first three are geometry lessons; the second three are algebra lessons.

00:00:21 VO: The lessons shown here are not those collected in the actual TIMSS Videotape Classroom Study, because the teachers who participated in that study were guaranteed anonymity.

00:00:32 VO: However, the lessons shown are similar to those taped for the TIMSS Videotape Study and are representative of teaching in the three countries.

00:00:41 VO: These lessons are shown to encourage discussion, but are not intended to prescribe how teachers should teach.

00:00:48 VO: Before viewing the videotaped math lessons, Dr. James Stigler from UCLA, who directed the video study, provides us with a brief background.

00:00:59 VO: Dr. Stigler, you are responsible for the video study. Tell us about it.

Dr. James Stigler, Professor, Department of Psychology, UCLA

00:01:03 JS: The TIMSS Video Study was conducted in three countries, Germany, Japan, and the United States. It was really a very simple study. We took national samples of eighth-grade mathematics teachers in each country, and we videotaped them teaching in their classrooms. We had two goals for this study. The first goal was to find out how we teach mathematics in the United States. Actually, up until this point, we've had very little detailed information at a national level about what teachers actually do in the classroom. The second goal, of course, was to find out how they teach mathematics in other countries, particularly in a country like Japan, where we know that student achievement levels are very, very high. Of course, teaching is not the only factor that would account for high student achievement in a country like Japan. But we believe that teaching is very, very important for achievement, and that's why we wanted to do this videotape study.

00:01:56 VO: How was the study conducted?

00:02:00 JS: The study was conducted over about a seven-month period. We had one camera person in each country traveling around from place to place, shooting in a different classroom every day or two. The participants in the study, the teachers, were selected at random. First, we selected schools at random across these three nations; then we selected eighth-grade teachers in that school at random; and, finally, we randomly selected the math period that that teacher taught that was going to be included in the study.

00:02:30 VO: Is there anything we should know about your findings that would help us understand the lessons we're about to see?

00:02:37 JS: Well, I think that some general findings would be very useful. First of all, the goals that teachers have for these lessons are very different across countries. German and

American teachers, on one hand, have a very different goal than Japanese teachers. We asked teachers, "What was the main thing you wanted students to learn from this lesson?" The majority of German and American teachers said they wanted students to learn how to solve a particular kind of problem, so they stressed skills. Japanese teachers, on the other hand, tended to stress understanding. They said, "We want students to understand some mathematical concept or principle." Now, because of these different goals, lessons appear to follow a very different script in these countries. In German and American lessons, we find that they generally are divided into two parts. The first part is where the teacher explains or demonstrates how to solve the kind of problem that the teacher wants the students to learn how to solve. So, in the first part of the lesson, the teacher might work an example at the board or work an example collaboratively with the class. Then, in the second part of the lesson, students will be given time to practice what the teacher had taught them. Japanese lessons proceed in the exact opposite direction. They tend to start with a very challenging or rich problem and the teacher will say, "Think about this problem and see if you can think of a method for solving this problem." Students will then work on their own or in groups for the first part of the class period. After that, the class gets back together and the teacher will ask various students to share the method they came up with for solving the problem. At the end of the lesson, the teacher will try to summarize or bring everything together in order to highlight the particular mathematical concept or principle that the teacher wanted the students to understand. I think it's very helpful to know how the lessons flow in order to begin watching these lessons.

00:04:29 VO: What do you think we can learn by viewing and discussing these examples?

00:04:34 JS: I don't think you can learn what's the right way to teach. That's not the goal here. The goal here is to use these examples to help you think about the way we teach in the United States. So, by studying teaching in Japan, by studying teaching in Germany, it helps us to more clearly see our own teaching, to see that it's really just one alternative form of teaching, and that there are others. So, really, the goal is to help us understand and discuss and to begin to be able to talk about teaching in ways that would be useful when we go about trying to improve it.

00:05:08 VO: Great, let's take a look at the videotape.

00:05:10 JS: OK.

00:05:11 Please note that the subtitles in this videotape are literal, rather than idiomatic, translations of the dialogue in each lesson. They provide the gist of the dialogue and are not intended to capture everything said.

U.S. Lesson One: Geometry—Angles

Classroom setup: There are approximately 23 students in the class. They are sitting in five rows. One student is sitting at the front behind a large desk, facing the other students. There is a large chalkboard on the front wall.

Part 1
Presenting and Checking Warm-Up Questions

00:00:00	T	(*To class*) What is the angle that is vertical to the seventy-degree angle?
00:00:04	T	Vertical angles are formed by what, Juan?
00:00:07	S	Umm... (I don't know. I was just stretching.)
00:00:09	Ss	Ha, ha, ha.
00:00:11	T	Don't get nervous (you do stretching.) When I intersect lines I get vertical angles. Right? Look at your definitions. I gave them to you. You have them. (You don't?) You can look them up.
00:00:24	T	Here we have vertical angles and supplementary angles. ...Angle A is vertical to which angle?
00:00:32	Ss	(Seventy)
00:00:33	T	Therefore angle A must be...
00:00:35	Ss	Seventy degrees.
00:00:36	T	Seventy degrees. Go from there. Now you have supplementary angles. Don't you?
00:00:42	T	What angle is supplementary to angle B?
00:00:46	S	A.
00:00:46	T	I mean...I am sorry. Angle A.
00:00:48	S	B.
00:00:48	T	B is and so is?
00:00:50	Ss	C.
00:00:50	T	C. Supplementary angles add up to what number?
00:00:56	S	One eighty.
00:00:56	T	One hundred eighty degrees. So if you know one is seventy the other one has to be?
00:01:01	S	Hundred ten.

00:01:01	T	A hundred ten. Go from there. ...Okay. You have all your information. So we already figured these out. (*Teacher begins writing on board.*) We have the measure of angle A is seventy degrees. B is a hundred ten and C is a hundred and ten. We know that.
00:01:19	T	What information are we given in the second problem for D?
00:01:23	Ss	Fifty-three degrees.
00:01:24	T	(*Walking away from board*) Okay. You—two things. You have fifty-three degrees.
00:01:29	T	What is the other angle I've indicated in there?
00:01:31	S	D.
00:01:32	T	Mike?
00:01:32	S	Right angle.
00:01:32	T	It's right angles, which add up to?
00:01:34	Ss	Ninety degrees.
00:01:35	T	Ninety degrees. Okay?
00:01:38	T	What is left?
00:01:39	S	Uh.
00:01:41	T	Somebody just gave me the answer. (*Teacher walks back to the board.*)
00:01:42	Ss	Thirty-seven.
00:01:43	T	(*Writing on board*) Thirty-seven degrees. Right?
00:01:47	T	Why thirty-seven degrees Jamie? (*Teacher turns to face class.*)
00:01:51	T	Carrie?
00:01:51	S	Because thirty-seven and fifty-three equals ninety.
00:01:56	T	Thirty-seven and fifty-three equals ninety. The middle angle is ninety. And why did they all have to add up to one eighty?
00:02:03	S	Because it's...it's a...
00:02:05	T	Because it's a...? Veronica.
00:02:08	T	What is this angle called here?
00:02:10	S	A straight...
00:02:10	T	Straight angle. And a straight angle adds up to?
00:02:13	S	One eighty.

00:02:13	T	One eighty. Okay. You think in a couple of minutes you can figure out the rest of them?

(*Students begin working on problems on their own. After realizing many students are having difficulty with two problems in particular, the teacher helps them as a class. Then the teacher turns to the homework. The students take out a worksheet that was assigned earlier in the week.*)

Part 2
Checking Homework

⋮

00:06:15	T	(*Standing in front of class, reading from paper*) Okay. What is the complement of an angle of thirty-eight degrees? Tracy?
00:06:24	T	If you didn't get a chance to do it, do it now. Complementary angles add up to what, Tracy?
00:06:31	T	(*Other students raise their hands.*) Relax, give...give... give Tracy a chance.
00:06:34	T	Look at...look up at the top definition right here. Complementary angles add up to?
00:06:39	S	Ninety degrees.
00:06:39	T	Ninety degrees. So if I have an angle of thirty-eight degrees...
00:06:44	S	(*Not Tracy*) Fifty-two.
00:06:45	T	Thank you, Tracy.
00:06:46	Ss	Ha, ha, ha.
00:06:48	T	If I have an angle of thirty-eight degrees, what is ninety minus thirty-eight?
00:06:53	S	Fifty-two.
00:06:54	T	Fifty-two. So the complement would be fifty-two degrees. Right?
00:06:59	T	What is the complement of an angle of seven degrees? Ho?
00:07:03	S	Eighty-three.

00:07:03	T	Eighty-three. The complement of an angle of eighty-four, Lindsey, would be...
00:07:09	S	Sixteen.
00:07:10	T	You sure about your arithmetic on that one?
00:07:14	S	Six?
00:07:15	T	Six. Six degrees. Albert, number four.
00:07:19	S	Umm, seventy-nine degrees.
00:07:22	T	Number five. Joey?
00:07:24	S	Thirty-three.
00:07:26	T	Sure about that? Claudia?
00:07:28	S	Twenty.
00:07:28	T	Twenty-three. Gotta be careful with the arithmetic. Number six. Jamie.

(*The teacher reviews the remaining problems with the class.*)

Part 3
Assigning Seatwork

⋮

00:14:59	T	(*Standing in front of class*) All right. I'm gonna give out your worksheet...based on these kind of angles and let you get started on it. (*Teacher walks to back of room to get worksheets.*)
00:15:12	S	Will we need a protractor?
00:15:14	T	You will not need a protractor. This is gonna be by observation. Just like the warm-up. (*Teacher begins passing out worksheets.*)
00:15:43	T	All right.
00:15:46	T	Okay. When you get the worksheet, let's look at the example on the top. These are very similar—you didn't get one?
00:15:52	S	We need two more.
00:15:53	T	We need two more. Okay. (*Teacher hands students worksheets.*)

00:16:01	T	All right. Look at the examples on the top.
00:16:04	T	(*Walking back and forth in front of room*) Similar to your warm-up. Look at the figure on the right...and find the measure of each angle. If angle three is one hundred twenty degrees...and angle three and angle one are vertical, what must angle one be equal to?
00:16:22	S	One hundred twenty.
00:16:22	T	One hundred twenty degrees.
00:16:25	T	What can you tell me about angles two and three?
00:16:31	S	They are vertical.
00:16:32	T	Two and three are not vertical.
00:16:35	T	One and three are vertical. Two and four are vertical. ...Two and three are supplementary. So if three is a hundred and twenty, what must two be equal to?
00:16:46	S	Umm, sixty?
00:16:47	T	Sixty. If two is sixty, what must four be equal to?
00:16:52	Ss	Sixty.
00:16:53	T	Okay. (*Teacher begins circulating among students; looks at worksheet.*)
00:16:58	T	All the rest is done the same way. Any questions? I'm curious to see when you get down to thirty-seven and thirty-eight, you're gonna have to think a little bit. Curious to see what you can come up with for those. You do not need a protractor. This is all by observation.

(*Students start working on the worksheet, and the teacher begins helping individual students.*)

Part 4
Providing Extra Help on Challenging Problems

⋮

00:25:48	T	I thought you had sixty-two.
00:25:50	T	No, you had—you had an eight at the end. You had—there was an eight there at some place. One seventy-eight or something like that.
00:26:01	T	Okay. (*Student walks into classroom.*) (*To student sitting at front of room*) You have the attendance? Did you (bubble) it? (*Student at desk hands attendance sheet to other student.*)
00:26:05	T	What do you mean? Which is angle QRS? (*Walks to board and begins drawing angle*)
00:26:13	S	(Zero)
00:26:15	T	(*To attendance checkers*) Oh those are—uh...yes. Somebody came in the first period and got them. Thanks.
00:26:28	T	(*Finishes drawing and speaks to class*) I don't wanna give it away.
00:26:30	S	It is ninety?
00:26:31	T	It's gotta be. Think about it.
00:26:34	S	It's ninety.
00:26:35	T	Look at problem thirty-seven. Two angles are supplementary. Therefore they must add up to one hundred eighty degrees (*Teacher begins drawing more on figure on board.*), but they are equal, so let's call one QRS and the other SRT. Each one of them has gotta be...
00:26:36	S	Ninety.
00:26:54	T	A ninety-degree angle.
00:26:54	S	Oh, okay. I get it now.
00:26:58	T	That's the only way.

(*Students begin working on problems individually. The teacher then calls attention to a particularly difficult problem and works it through with them. The teacher then proceeds to review a worksheet with the students.*)

Part 5

Checking More Homework and Introducing a New Formula

⋮

00:36:46	T	Oh. Seven eighteen. That's pretty darn close. Within five degrees. How many got seven twenty? Within five degrees. (*Students raise hands.*)
00:36:53	S	Yeah (I got).
00:36:53	T	Okay.
00:36:54	S	I got seven twenty exactly.
00:36:56	T	Seven twenty exactly. You were accurate with your protractor. If I move that bottom angle...started out like this (*Teacher begins to draw figure on board.*); let me see if I can re-create it here for you.
00:37:11	T	One (was) here, then the line came down here then went back here. One, two, three, four—this is about what it looked like. This was B, A, F, E, D, and C.
00:37:27	T	(*Working on board*) If I took...this angle...and moved it...down here...and made it across this way. Moved D down here, should that change the sum—the total of my...?
00:37:44	S	No.
00:37:45	S	No.
00:37:45	T	Angles?
00:37:46	Ss	No.
00:37:46	T	It should not. Why? I still have how many angles? Joey.
00:37:51	S	You still have six.
00:37:52	T	I still have six angles.
00:37:55	T	There is a formula, and we are gonna go through this in...after spring break, but I am gonna give you a hint right now (*points toward figure on board*). If I take the number of sides...and I subtract two...and I multiply that number times one hundred eighty

		degrees...that will tell me how many degrees these add up to. How many sides in this figure?
00:38:25	S	(Six)
00:38:27	S	(How many) sides?
00:38:28	T	How many sides in this figure? One, two, three, four, five, six. Right? Number of sides subtract two.
00:38:37	T	Gives me what?
00:38:38	Ss	Four.
00:38:38	T	Four. What is four times one hundred eighty degrees?
00:38:42	S	Uh.
00:38:45	S	Seven hundred twenty.
00:38:46	T	Should be seven hundred twenty. Right? How many...how many degrees should there be in a five-sided figure?
00:38:56	S	Uh.
00:38:56	T	A pentagon.
00:39:00	S	(Five seven)
00:39:02	T	(*Walking away from students*) Take the formula... number of sides is five. You don't have to do it in your head. You have pencil and paper.
00:39:11	T	Number of sides is five...subtract two and multiply it by one hundred eighty degrees.
00:39:19	S	Five hundred forty.
00:39:19	T	Five hundred and forty degrees. All five-sided figures contain five hundred forty degrees.
00:39:29	T	Triangle has how many sides?
00:39:31	Ss	Three.
00:39:31	T	Take away two is one. One times one eighty. A triangle contains eighty degrees.
00:39:37	S	One hundred eighty degrees.
00:39:38	T	One hundred eighty degrees. Thank you Liz. A square. Four sides or rectangle subtract two is two

times one eighty is three hundred sixty degrees. You can always figure out the total number of degrees in a figure by taking the number of sides, subtracting two, and multiplying by one eighty.

(*The teacher uses this rule to introduce a brief discussion of isosceles triangles, the topic of the next day's lesson.*)

Part 6
Previewing the Upcoming Schedule

:

00:41:08 T (*Standing in front of class*) Tomorrow we are gonna go over triangles...Friday you're—uh tomorrow I will introduce triangles. We will review for the quiz. Your quiz on Friday will contain complementary angles, supplementary angles, vertical angles, and that's about it. Next week...next week we're gonna finish this unit.

00:41:30 T I wanna finish the unit. (*Student raises her hand.*) Let me finish, and then you can ask questions or tell me who is going on vacation early or what. Next week I want to finish the unit because I don't wanna continue the unit past skin—uh spring break. The unit test next week will be on Thursday. ...Because I am afraid some of you may be leaving early for vacation and will not be here on Friday.

(*The class ends with students working individually. They get a quick review of rules and definitions regarding triangles, and discussion of an upcoming quiz and field trip.*)

153

Japanese Lesson One: Geometry—Areas of Triangles

Classroom setup: Approximately 36 students, seated in six rows of six. The teacher's desk is at the front of the room, with a large chalkboard on the front wall. A computer is set up at the front, with a large TV monitor for all of the students to see. There is a second teacher standing in the back of the classroom.

Part 1
Linking Yesterday's Lesson Topic to Today's Topic

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(*The teacher is standing in front of the class.*)

00:00:01	S	Stand. (*Students stand.*)
00:00:14	S	Stand straight.
00:00:16	S	Bow.
00:00:17	Ss	(*Bowing*) Onegaishimasu.
00:00:18	T	Okay. (*Students sit.*)
00:00:27	T	(*Turns on computer monitor*) Umm, do you remember what we did last period?
00:00:30	S	(*Stands*) We did mathematics.
00:00:32	T	Sakurai, what kind of thing did we do?
00:00:35	S	Huh?
00:00:35	S	We did mathematics.
00:00:35	S	I don't know.
00:00:39	S	Huh? Hmm. The last period?
00:00:42	T	Umm. This study okay?
00:00:46	S	Is it that?
00:00:47	T	Yes.
00:00:47	S	Obtain the area of triangle which are in the places in the parallel lines.
00:00:52	T	That's right, huh? (*Teacher pointing to diagram on monitor*) Umm, we did a study that says on the parallel lines...the triangles on it, umm, of the same base or height are all the same...like this. For example, here. (*Monitor shows two parallel lines with two fixed points on the bottom line. Two lines are drawn from each of the base points*

		to meet on the top line, forming a triangle. The point on the top line moves back and forth, forming several different triangles.)
00:01:10	T	This.
00:01:18	T	We did a study, okay? That says since all of...umm these become the same height, the areas become equal, okay? Umm, having this as the foundation we will be going to study today.

(The teacher moves toward the chalkboard, reaches for chalk and a wooden triangle. Students get out their notebooks and get ready to take notes.)

Part 2

Posing the Problem

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00:02:13	T	(*Drawing diagram on board*) Umm. Right now, over here, okay? ...there is Eda's land. (*Teacher labels left portion. Eda is a student in the class.*)
00:02:21	T	It's okay, huh? Okay. There is Eda's land.
00:02:24	T	Okay? Over here is Azusa's land, okay? (*Teacher labels right portion. Azusa is another student in the class.*)
00:02:24	S	Ha, ha.
00:02:31	T	Is it okay? Let's say that there is a land like this.
00:02:35	T	And. Is it okay? Azusa.
00:02:37	S	(*Azusa*) Yes.
00:02:38	T	(*Pointing to line separating the two*) And these two people's... hmm...border line is bent like this, but we want to make it straight, okay?
00:02:46	T	Eda.
00:02:46	S	(*Eda*) Yes.
00:02:47	T	Is it okay here? (*Teacher indicates border giving Eda larger space.*)
00:02:50	S	(*Eda*) Yes.
00:02:50	T	Is that okay?

00:02:51	Ss	Ha, ha, ha.
00:02:51	T	Then we'll end today's class okay?
00:02:53	Ss	Ha, ha, ha.
00:02:53	T	Ha, ha.
00:02:55	T	Azusa, is it okay here? (*Teacher indicates border giving Azusa larger share.*)
00:02:57	S	(*Azusa*) Ahhh.
00:02:59	T	No? (*Teacher moves border to the right, gradually decreasing Azusa's share.*)
00:02:59	S	(*Azusa*) No.
00:03:00	T	Where would you like it?
00:03:02	S	It would be better if mine were wider.
00:03:04	S	A lot more (to the bottom)?
00:03:05	T	Huh? A little more over here?
00:03:06	S	More.
00:03:06	T	Around where would you like?
00:03:07	S	Continue on over. (*Teacher moves border to the left.*) More. More.
00:03:09	T	More over here?
00:03:09	S	More. More. More. There.
00:03:11	Ss	Ha, ha, ha.
00:03:11	T	Oh. Ha, ha.
00:03:12	T	Eda. Is it okay here?
00:03:14	S	No.
00:03:14	T	It's not okay, right?
00:03:14	Ss	Ha, ha, ha.
00:03:15	T	Then, where would it be good?
00:03:18	T	(*Points to student—Shimizu.*) Shimizu. Around where do you think would it be good?
00:03:19	S	Huh?
00:03:20	T	Approximately.
00:03:22	S	That line...well. (*Student points to board.*)
00:03:25	T	Well, try doing it.
00:03:26	S	Huh?
00:03:26	T	Approximately. Estimate. *(Student walks to board; teacher hands her the pointer.)*

00:03:29	S	Umm.
00:03:30	T	Yes.
00:03:31	S	Umm. [Take it] between the—this line and this line.
00:03:33	T	Yes. (*Student sits down.*)
00:03:34	T	We got an estimate that says isn't it okay if it's in the middle? I see. How about other people? Okay? Then, well in your notebook, okay? Draw a figure like this and...please try thinking about it a little using methods of changing this shape without changing the area. (*Teacher places sign on chalkboard, "Think about a method of changing the shape without changing the area."*)
00:03:53	T	Okay?
00:03:55	T	Okay, then everybody...let's try thinking about it. The [work] time is...would you think about it for three minutes? First of all, please think about it individually for three minutes. Okay, begin.

(*Students begin working on problems individually, and the teacher circulates among them.*)

Part 3
Working on the Problem

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00:06:20	T	(*To student*) First of all, draw a figure and...
00:06:21	S	Draw the figure and...
00:06:22	T	That of last time. Is there a method that uses the area of the triangles? (*Teacher moves on to other students.*)
00:06:35	T	[That's] sharp.
00:06:37	T	You were able to make this a triangle, right? Okay? Then if you do what...okay? Would you get triangles with the same area? Would you make this the base?
00:06:49	T	[The question is] that somewhere there are parallel lines, okay?
00:06:52	T	Hmm. We did it like this and like in the last class (like

		we did it like this)...we get a triangle.
00:07:04	T	(*Walks to front of class. To class*) Okay. Then since the three minutes are up so...umm...people who have come up with an idea for now go to Teacher Ishikawa (*standing at back of room*) and do it [with him], and people who want to discuss it with his/her friends, discuss it with your friends. And for now I have placed some hint cards up here so people who want to refer to this, refer to it. Now then, umm, in three minutes... umm...we'll think about it and please try doing it with your friend or by yourself. Okay, begin.

(*Students get up and start moving. Some discuss in groups. Others look at hint cards. Still others talk with the teacher. During this time, the teacher identifies two students to present solutions to the class. Those students start preparing their explanations on the board.*)

Part 4
Students Presenting Solutions

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00:19:48	S	(*Standing at front of room at chalkboard, holding pointer, and explaining his solution*) Hmm. First of all, we make a triangle, okay?
00:19:51	S	Ha, ha.
00:19:52	Ss	Ha, ha, ha.
00:19:52	S	What are you saying?
00:19:53	S	(*At the board*) You talk too much.
00:19:54	Ss	Ha, ha, ha, ha.
00:19:55	S (b)	You make a triangle, right? And then at here...
00:20:02	S (b)	Draw a line para...para...parallel over here also, and...we make over here as the base.
00:20:09	S (b)	As the base. Here.
00:20:11	S (b)	Yes.
00:20:12	S (b)	And then we make it the height and this triangle and...

00:20:16	S (b)	That's the height? ( ).
00:20:17	S (b)	Okay?
00:20:19	S (b)	Which is it?
00:20:20	S (b)	(I think it's not that.)
00:20:21	S (b)	This triangle and a tri...somewhere.
00:20:25	S (b)	Ha, ha.
00:20:25	T	(*Standing to the side of the class.*) Umm. The red triangle.
00:20:27	S (b)	Oh. It's this, right?
00:20:28	T	Yes.
00:20:29	S (b)	[The red triangle] is.
00:20:30	S (b)	The area is...
00:20:32	T	(*Teacher walks to board and outlines triangle for student.*) Over here. Over here. Over here. (*Teacher walks away.*)
00:20:33	S (b)	What is it?
00:20:34	S (b)	(Ya—)
00:20:34	S (b)	Well, they are the same, okay?
00:20:34	Ss	Ha, ha, ha.
00:20:36	T	The triangle over here.
00:20:37	S (b)	Actually, and the triangle over here.
00:20:39	T	Yes.
00:20:42	S (b)	The fact is that the areas are the same, okay?
00:20:43	T	Hmm.
00:20:44	S (b)	Since the base and the height are the same. So...first of all...the fact is we can draw a line here.
00:20:52	S (b)	Yes, yes. Well I don't know what I am saying but...
00:20:54	Ss	Ha, ha, ha.
00:20:56	T	No. We can understand enough, right?
00:20:57	S (b)	Oh. You understand?
00:20:58	T	Is there anybody who does not understand?
00:20:59	S	Ha, ha.
00:21:00	Ss	Ha, ha.
00:21:00	T	Oh. You don't understand?
00:21:02	S	I also don't understand.
00:21:03	T	You don't understand? Then one more time then with this side. This time, please explain it, Inuma. [You're

saying] that it was a good explanation. Okay, then, applause. Wonderful.

(Students applaud. Inuma then walks to the board to present her explanation. She does so and returns to her seat.)

Part 5
Reviewing Students' Methods and Posing Another Problem

00:22:31 T (*Pointing to diagram on board*) Okay. Umm, okay for now? Since it's hard to see, we will make it clearer. The areas of this triangle and the red triangle over here [and] this triangle and the yellow triangle over here are the same areas, so we want to do it so here becomes straight like this, okay? So that the corner here is gone. The angle okay? It's that then we were able to draw a straight line here. [That's what we can] say. People who were able to d—who did it like this? (*A few students raise their hands.*)

00:22:51 T [People who can say] I drew it like this over here? [People who say] I was able to draw it. [People who say] I was able to do it this way? [People who say] they were able to do both? (*Students raise their hands accordingly.*)

00:22:56 T Okay. That's good. Umm, okay then? Next, making this as the basis, okay? Oh, I don't know if it's making it as a basis or not. A quadrilateral. (*Places cutout of quadrilateral on board.*)

00:23:06 T Oh. Of course it can be crooked like this. Well for now (Naranai) quadrangle. [Taking] this quadrangle ...without changing the area...make it into a triangle. (*Teacher writes below the quadrilateral, "quadrilateral ==> triangle."*)

00:23:25 T Take this shape of the quadrangle without changing the area and please try making it into a triangle. Okay, then...please take three minutes and try doing it in your own way. Okay, begin.

(*Students begin working individually. After three minutes, the teacher asks that the students who are done go over their work with the other teacher. Those who have not finished should look at the cards, ask the teacher, or work in groups.*)

Part 6
Summarizing the Results

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00:46:37 T For the convenience of explanation we will put in symbols. (*Teacher begins to write on board.*)

00:46:41 T There were some who were doing it already labeled, okay? We will make them A, B, C, and D, okay? And right now in the beginning we draw a diagonal line [through] A to C and...draw a diagonal line from A to C, and we will make a triangle. We were able to make two, right? And if you ask which ones are the triangles we found that goes through D are...this and...this, right? This one is made like this on the bottom side like this. This one is on the top side. Since this triangle and the original triangle are the same, so it becomes the fact that we changed this quadrangle into a shape like this, right?

00:47:20 T Next we drew in the same way [through] AC, but this time we drew the parallel line on the B side. Okay? Those are this and...this, okay?

00:47:31 T It's this and this, okay. Is everything okay? Then I will ask.

00:47:35 T About how many people are there who say he/she found this I—were able to find it? (*Many students raise their hands.*)

00:47:40	T	Okay. People who say he/she were able to find this one. (*Other students raise their hands.*)
00:47:44	T	There are about the same [number of people] huh? Umm, how about this one?
00:47:47	T	The D one. The sharp triangle. How about this one?
00:47:50	T	Okay. That's good. Thank you. Okay. What is it? You don't understand? (*Teacher pointing to figures on board*) On this side this time we will draw it on the BD side. Then in the same way, which one is it?
00:47:50	S	I don't understand all of them.
00:48:01	T	Ones that we drew [through] BD and drew a parallel line [through] vertex A are this and...this, okay? They are the triangle that we make on this side and the triangle we make on the top, right? And also the ones we drew a line [through] BD and drew a parallel line on the C side are this and...which one is it?
00:48:22	T	Which one is it?
00:48:24	T	Is there none?
00:48:25	T	Is this it?
00:48:26	T	Huh?
00:48:28	T	It's wrong? Is this it?
00:48:30	T	This? Huh? Is it? Okay then...for the time being? We don't have much time but with the computer...I will explain it a little, okay? (*Teacher walks to the computer.*)
00:48:40	T	It's something we have done.
00:48:43	T	Well the screen is not showing up.
00:48:44	S	It's showing.
00:48:45	T	It's showing. Right, now, well the symbols are different from over there but on BD...for now...we'll draw [a line]. (*Computer shows line.*) And we will connect A.
00:48:45	S	It came out.
00:48:56	T	The parallel line is done. It's two parallel lines. Then there are a lot of them like this which are the same area with this triangle, okay? (*Computer changes figure to form different quadrilaterals.*)

00:49:05	T	There are many of them but...within these...since the fact is we're making the quadrangle into a triangle, [meaning] all we have to do is lose one of the angles. The angle here, okay? (*Teacher points to monitor.*) When it becomes a straight line, then that will become a triangle.
00:49:20	T	(We will do it) with the fact that in the same way...even if it's this side when here becomes straight...we get a triangle. Then we have done quadrangles. Next what do you want to do? ( ).
00:49:29	S	Five [Pentagon].
00:49:30	S	Six [Hexagon].
00:49:31	T	Ha, ha, ha.
00:49:32	S	We like five and six [sided].
00:49:33	T	Ishizaki. You (*Bell rings.*) want to do six by all means. Then five. Fix pentagons into triangles. (*Teacher points to board.*)
00:49:39	T	This. A pentagon's—try drawing a pentagon of your liking. Then...pentagon...
00:49:40	S	I don't have a clue.
00:49:44	S	That's impossible.
00:49:45	T	It's impossible?
00:49:46	S	It's impossible.
00:49:46	S	You don't know if it is impossible until you try it.
00:49:47	T	Okay? Then let's try making the pentagon.
00:49:49	S	Ha, ha.
00:49:50	T	Into a triangle.
00:49:52	S	From right now?
00:49:53	T	No. I'll make that homework.
00:49:56	T	Okay? People who are interested you can do ten-angled [or] twenty-angled [or] one hundred-angled [or] anything but...

(*After a brief exchange about different geometric figures, the class ends.*)

German Lesson One: Geometry—Volume and Density

Classroom setup: There are approximately 24 students in the classroom. They are sitting in pairs at two-student desks, arranged in four rows. There is an overhead projector and screen at the front of the classroom.

Part 1
Sharing Homework

(*The class begins with a review of the homework. One student, Wilma, is chosen to come to the front of the room to present her solutions to the class, using the overhead projector. She has just presented a model solution to a homework problem. The teacher is standing at the side of the classroom.*)

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00:03:08	T	(*To class*) Who confirms this result? (*Some students raise their hands.*)
00:03:10	S	(I don't.)
00:03:14	S	That is the same.
00:03:15	T	Hmm?
00:03:17	S	That is the same.
00:03:18	T	You say you don't.
00:03:19	S	Nn nh.
00:03:20	T	Did you discover your mistake?
00:03:21	S	No. Not yet. I was just going to check it.
00:03:23	T	Then—maybe it is just a calculator mistake. You have to go through it. Umm, does anybody want to alter Wilma's result a little bit?
00:03:39	T	Dan—Mirco?
00:03:40	S	To three...point—well to three digits after the point. Four point—well two thousand eight hundred thirty-five point four three seven grams.
00:03:50	T	You rounded up correctly. And that was an agreement between us that we would round up grams and kilograms after the third digit. (*Wilma begins correcting her work on the transparency.*)

00:04:01	S	I see.
00:04:10	T	Yes. And now we have...
00:04:12	S	Grams or kilograms?
00:04:14	S	Three seven.
00:04:14	T	Yes. Look. There is your justified question. Lutz?
00:04:19	S	Grams.
00:04:23	T	Can you give her an explanation?
00:04:27	S	Yes, because, uh...because to the power of three always gets rounded up three points after the point.
00:04:36	T	That is correct. That is the reason for the seven. Right?
00:04:40	S	Yes.
00:04:40	T	Four three seven. And now Wilma's question was grams or kilograms or what?
00:04:47	S	(*Wilma*) Yes. Grams or kilograms?
00:04:50	T	You already had written grams.
00:04:52	S	(*Wilma*) Yes.
00:04:54	T	Were you sure about that?
00:04:56	S	(*Wilma*) No.
00:04:56	T	She wasn't sure about that. Who can help her...then? Katrin.
00:05:02	S	Umm, it is grams because it is centimeters cubed...and then the unit gets stated in grams.
00:05:09	T	Yes. Do you have something else in mind?
00:05:15	S	No.

(*The class goes over a total of three problems. Wilma presents the solutions for all three, as her classmates comment on her solutions.*)

Part 2
Revisiting Previous Material

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00:11:44	T	(*Standing in front of class*) Yesterday...you guys put together what you know how to calculate already and... (*Teacher turns on overhead projector—paper is covering transparency—teacher moves it down to reveal top of transparency.*) I wrote that down on the transparency. We claim that. We hope—I hope that this is actually true...what you guys can calculate...Who can remember what it said? (*Students raise their hands.*)
00:12:24	T	Matthias.
00:12:25	S	We can calculate the surface of a rectangular solid.
00:12:30	T	Yes.
00:12:31	S	(Maybe.)
00:12:32	Ss	Ha, ha.
00:12:33	T	Natasha.
00:12:34	S	We can calculate the volume of a rectangular solid.
00:12:36	T	Yes right. Timo.
00:12:38	S	We can calculate the...the mass of a rectangular solid.
00:12:41	T	Mm hmm. (*Teacher moves paper down to reveal more of transparency.*) And again you see that you guys were right on target. Let's go back to the first one. You remember how you can calculate the surface of a rectangular solid?
00:12:59	T	Hauke.
00:13:00	S	Well, with the formula O [Oberflaeche = surface] equals A times open parenthesis A times B plus A times C plus B times C close parenthesis times two.
00:13:10	T	All right.

(*The teacher then reviews the formulas for volume and mass. Following this, the teacher indicates they will learn a fourth formula and introduces an exercise related to it.*)

Part 3
Posing a Problem

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00:14:19	T	(*At overhead*) I have an exercise for that.
00:14:26	T	Michaela. Would you please read aloud?
00:14:28	S	Yes. (*Student reads from transparency.*) An iron sheet...umm RHO equals seven point eight grams per centimeters cubed with a length of zero point five meters and a width of twenty centimeters weighs three point nine zero kilograms. Calculate the height in parentheses thickness of the sheet.
00:14:54	T	Mm hmm.
00:15:04	T	Sven?
00:15:07	S	Maybe the formula H equals...umm M divided by (RHO)...divided by A divided by B.
00:15:14	S	(Divided by O?)
00:15:17	T	You are already very far with your thoughts. Yes. What would you have said Lothar?
00:15:24	S	Well. H equals...
00:15:29	T	Yes. You want the same thing again. You want to convert the formula right away in your heads. Maybe we can first do that slowly so everybody can understand this, uh, thought. In our three steps. Given, wanted, and calculation path.
00:15:49	T	Who dares to do that here in the front? Here in the front?

(*One student, Lutz, volunteers. He walks up to the board. As Lutz is working on a problem at the chalkboard in front of the class, some students have their hands raised.*)

Part 4
Working on the Problem Together

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00:18:27	T	Lutz, would you please turn around?
00:18:31	Ss	Ha, ha.
00:18:38	T	There are a few people who would like to tell you something.
00:18:39	S	(*Lutz*) Yes. I know.
00:18:40	Ss	Ha, ha.
00:18:42	T	So then.
00:18:47	S	(*To Lutz*) I would convert that into centimeters.
00:18:49	S	I wouldn't. Nn nh.
00:18:51	T	Would you give him the reason?
00:18:53	S	Well, then the numbers are a little bigger, and rho would be better to calculate as well.
00:19:01	T	Can you repeat that one more time so everyone can understand you?
00:19:04	S	Yes, I would convert that into centimeters because one can calculate that better with the density.
00:19:11	S	Umm.
00:19:11	T	Because the density is stated in grams per centimeters cubed. Mm hmm. Bjoern?
00:19:18	S	I would calculate it in decimeters because it is stated in kilograms.
00:19:23	T	That was important. Would you please listen?
00:19:26	S	I would convert it into decimeters because it is stated in kilograms.
00:19:31	T	Sonye says but...
00:19:32	S	Well, one can also convert A and B into centimeters, and then one converts M into grams, too. Well then...
00:19:39	T	Both are possible. (*To Lutz at chalkboard*) You are supposed to decide.
00:19:45	S	Or take decimeters.
00:19:46	S	Math.
00:19:48	T	And...we will follow your calculation path along.

00:19:49	S	Poor Lutz.
00:19:52	S	Lutz you (don't need to stand like this).
00:19:53	S	Take decimeters.

(*Lutz continues to work on the problem. The teacher then asks another student to take over for Lutz. Different students take turns working out the problems on the board, with the rest of the class watching and providing comment. Then the class arrives at a solution.*)

Part 5
Summarizing the Results

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00:35:10	T	(*Standing at overhead in front of class*) I will come back to the top. To the fourth one. Are any of you able to say what we just...did and what you learned? (*Teacher points to transparency.*)
00:35:32	T	Michaela?
00:35:33	S	(Well) we can calculate the length, width, or height of a rectangular solid.
00:35:40	T	Well, yes. But there must be something given as a premise. We can calculate the length, the width, or height.
00:35:49	S	(Yes). Two of the...of the length, width, or height. The mass and the ( ) and rho.
00:36:00	T	You mean the right thing. We hear it one more time. Coming from the back. From (Sonye).
00:36:05	Ss	Ha, ha.
00:36:07	T	What?
00:36:07	S	I see. We can.
00:36:12	T	Mm hmm.
00:36:12	S	Well, we can calculate the length, width, or the height of a rectangular solid if we have the mass...and...and rho.

00:36:25	T	Correct. I will sum it up a little more briefly (*Teacher writes on transparency.*) and write down we can...change the formula...for the mass. Everything you just said is contained in it.
00:36:55	T	We will leave that on the board for orientation... Because you are now supposed to find out if you are able to do it yourselves.

(*The teacher picks up pieces of paper with problems from her desk and arranges them for the students to take at the side of the classroom. The teacher then returns to the board.*)

Part 6
Assigning Seatwork

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00:37:49	T	Look over here one more time. It is very important that you don't forget this step. (*Teacher points to step written on chalkboard.*) That you write down the units in parentheses...Reduce them, and then, at the end, find the real unit of the solution.
00:38:14	T	Okay...Here are the problems.

(*The students begin working on the problems individually. The class ends with the teacher assigning homework.*)

U.S. Lesson Two: Algebra—Complex Algebraic Expressions

Classroom setup: There are approximately 27 students in the class. They are sitting with their desks together in groups of four. There is an overhead projector and screen at the front of the room.

Part 1
Presenting and Checking Warm-Up Problems

(*The class begins with the students working on warm-up problems individually. The teacher is circulating among them.*)

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00:13:54 T (*To student*) All right?

00:13:55 S Is this it?

00:13:59 S Did I get number two right?

00:14:04 S For one I got three, and for two I got twenty-four, I mean for two I got twenty-four.

00:13:56 T I'd have to look it up, but I think you are right. Minus...B minus four C? No, there—the last part is minus...(*Teacher moves to another student.*)

00:14:08 T But two asks you for a pair of ordered—three asks you for…a pair of ordered integers.

00:14:13 T No. I got…

00:14:15 S I haven't done three yet. I got…got twenty-four.

00:14:20 S Mrs. Maddock? (*Teacher walks toward student.*)

00:14:23 S Umm, are these all right?

00:14:26 T Very, very good. Now this one…looks good. Now…here, if I had said to you, simplify that, what would you have written?

00:14:38 S X…to the third.

00:14:40 T To…very good. How did you get the third? What did you do to the five and the two?

00:14:45 S I…I subtracted them.

00:14:48 T All right. What do you want to do to those exponents?

00:14:53 T Subtract them.

00:14:53	S	Okay.
00:14:54	T	Same method. (*Teacher walks toward front of room and puts transparency with problems on overhead projector.*)
00:15:05	T	(*To class*) Okay, may I have your attention. Let's...look up. I've seen some n—nice work. Umm...and uh...listen carefully. First one, I think almost everybody had. Jenny you had...
00:15:05	S	For which one?
00:15:22	T	First one.
00:15:22	S	Umm...three.
00:15:23	T	Three. Three was the largest one and that was pretty much a guess and check. Number two. I think most people had it. Molly, what did you get?
00:15:34	S	Umm...I got twenty-four.
00:15:36	T	You got twenty-four. All right. And I—as I walked around I saw that most of you had it. Anybody want to ask about that?
00:15:41	T	All right. Uh...three was a little trickier.
00:15:46	T	Uh...Carrie?
00:15:48	S	Two and six?
00:15:49	T	Almost, but read the directions. It says A is greater than B.
00:15:53	S	Oh six...and two.
00:15:55	T	Six and two. Yeah. (*Teacher writes answer on transparency.*)
00:15:56	T	Lot of...uh, math is just reading the directions carefully and understanding it. All right, last one. Uh, a number of you got...eventually with a little bit of a hint. Diana?
00:16:09	S	Uh...two X mi...to A minus B minus four C. (*Teacher writes answer on transparency.*)
00:16:13	T	All right. How did you get it?
00:16:15	S	Uh...well when you divide the—you subtract the exponents.
00:16:20	T	All right.

00:16:21 S And two A minus A is A. B minus two B is negative B, and...negative C...minus three C is minus four C.

00:16:28 T All right. Question on that...anybody? Anybody get four done and four right?

00:16:34 T Quite a few of you. Okay. Nice job.

Part 2
Presenting and Discussing Problems

(*The teacher writes a least common denominator problem on the transparency, similar to ones they worked on the previous day. She also passes out homework for two nights.*)

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00:18:41 T Okay, uh...let's have least common denominator. Molly?

00:18:46 S Uh...isn't it X squared minus forty-nine?

00:18:48 T (*Writing student's answer on transparency*) X squared minus forty-nine. And...what's your numerator, Molly?

00:18:53 S Uh...the answer?

00:18:55 T Yeah.

00:18:55 S Seven X...

00:18:57 T How did you get seven X?

00:19:00 S Uh...because to get...from X minus seven to X squared minus forty-seven

00:19:09 S Forty-nine.

00:19:10 S Forty—oh. You have...you have to square...seven and X.

00:19:21 T No.

00:19:22 S You have to mult— you have to multiply, umm, seven times seven and X times X.

00:19:27 T Not quite Molly. You have to do some factoring.

00:19:31 T Uh, Serti.

00:19:31 S You need to times...x m—you need to— you...ugh...umm...X...minus seven is obviously a

178

		factor of x squared minus forty-nine so you get the other fact—so you find the other factor.
00:19:47	T	Which is?
00:19:48	S	Which is X plus seven.
00:19:50	T	Right.
00:19:50	S	And then you add one. So it's X plus ( ).
00:19:53	T	Okay...so you—first you multiply one times that, which gives you the X plus seven. And then the one comes from here.
00:20:01	T	Why do you add the one?
00:20:03	T	Because we were adding these two fractions. And this one already has the denominator we want.
00:20:08	T	All right?
00:20:09	S	Right.
00:20:09	T	So X plus eight. How many got it? (*Some students raise their hands.*) Okay, Megan, look at the overhead when I'm writing please. Uh...okay. Question about it, Allison. Does it make sense now?
00:20:21	S	Wait.
00:20:21	T	Question?
00:20:22	S	Why can't you do what I did?
00:20:24	T	You have to find...a least common denominator...a least common multiple that includes both of these as factors. And X minus seven is not a f...
00:20:35	S	(*Interrupting*) No, I found, I...I...I got the least ( )—the bottom right. The top one I thought was...seven X don't you use to multiply?
00:20:44	T	I'm mult—I want to turn this denominator into X squared minus forty-nine, right?
00:20:50	S	Yeah. So...
00:20:50	T	So I— in order to do that...I have to multiply it by the identity element...X plus seven over X plus seven.
00:20:59	T	And so I do one times X plus seven, which gives me X plus seven.
00:21:06	T	All right. And then I have a one here. Umm...this one

		looks easier but there is a trick to it. (*Teacher writes new problem on transparency.*)
00:21:37	T	Raise your hands when you think you have it. It's just a small something that people tend to forget. (*Some students raise their hands.*)
00:21:51	T	Jot it down when you think you have it. (*Teacher walks over to check work of student who has raised his hand.*) No, I don't think so.
00:21:59	T	Show me again.
00:22:11	T	(*Circulating among students, checking their work*) I haven't seen what I think is right yet.
	S	I got (E minus X over)
00:22:30	T	Oh...anybody? Justin. Justin, I think that's...what's your answer?
00:22:37	T	No.
00:22:41	T	Anybody think he has it or she has it? I haven't seen it yet.
00:22:45	T	I can't read that.
00:22:49	T	All right. X plus three over X plus six. (*Teacher walks back to front of room.*)
00:22:52	Ss	Why?
00:22:54	T	Why. All right.
00:22:57	T	Subtracting is the same thing as what?
00:23:00	Ss	Adding the opposite.
00:23:01	T	Adding the opposite. (*Teacher writing on transparency*) So this is the same as...that. All right? Now our denominator is the same, and your denominator is just like a label so your denominator remains X plus six.
00:23:14	T	And from there, where do we go, Rog?
00:23:17	S	You do...five plus negative two, which is three...plus X.
00:23:23	T	Plus X. Oh—Alexa?
00:23:25	S	Why wouldn't you have to make the X negative two if you were...
00:23:29	T	Cause it was negative and a...the...subtraction makes it the opposite. Anybody else?

00:23:31	S	Oh right. Right. Okay.
00:23:34	T	Okay. Uh, for the remainder of the period there are about...five things that, uh, I would like you to work on in the following order.

(*The teacher goes over assignments, which include correcting a worksheet from the day before and a graphing calculator worksheet. The students then begin to work individually, and the teacher circulates among them.*)

Part 3
Assigning Multiple Tasks for Seatwork

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00:34:40	T	Is this making sense, Jess?
00:34:42	S	Yeah.
00:34:43	T	Okay.
00:34:43	S	That's not exactly what we got.
00:34:45	S	What did you get?
00:34:48	S	Umm, I have a question.
00:34:46	S	Mrs. Maddock..
00:34:50	S	(*Showing his calculator to the teacher*) When I graph...number umm...when I graph this one, the parabola it goes down so I can't see what the Y coordinate...
00:34:57	T	Parabola? Okay, there's a way to fix it. Push...press window.
00:35:01	T	Window, window. It's...this one.
00:35:04	T	Now if—you want to make your Y minimum lower so that that...vertex will show. So, arrow down to Y MIN...and go over one. Arrow over one. No, the other direction. To the right.
00:35:18	T	Uh, two. Let's go over once more cause we want to change that Y minimum to something lower like sixteen. So type a six in...enter and graph. Now graph.

00:35:31	S	(*Separate conversation*) I'm not there yet.
00:35:33	S	I'm still figuring out the...I'm way off.
00:35:34	S	Oh. Okay.
00:35:38	T	Yeah. And if that isn't right keep playing with the window.
00:35:41	T	Yeah, you have to do trace.
00:35:42	S	I did do a trace. Okay...okay so I did a bad trace.
00:35:47	Ss	Ha, ha, ha. (*Teacher begins circulating among students again.*)
00:36:01	T	It's so quiet in here I can't believe it.
00:36:10	T	If you can just stay a minute or two and finish, that will be very nice, or get your lunch and come back.

(Then, the class ends.)

Japanese Lesson Two: Algebra—Algebraic Inequalities

Classroom setup: There are approximately 36 students in the room. They are sitting in six rows of six. The teacher is standing behind his desk at the front of the room. On the front wall is a large chalkboard.

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00:00:00	S	All stand. (*Students stand.*)
00:00:10	S	Onegaishimasu.
00:00:11	Ss	Onegaishimasu.
00:00:12	T	Okay. Onegaishimasu. (*Students sit.*)
00:00:15	T	Okay, we're going to start our homework answer comparisons, so, uh, please take out handout number nine.
00:00:22	T	Now I'll have you write it. Please write ( ), right? Okay, it's the one here.
00:00:27	S	What?
00:00:27	S	Oh.
00:00:28	T	(*Selecting students to write on board*) One two three four five six. Okay, then please write it. Okay.

(*The teacher checks to make sure everyone has completed the assignment. The six students write solutions to the problems on the board. The teacher and class discuss.*)

Part 2
Posing the Problem

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00:07:00	T	(*Erasing board*) Okay, then, uh, today will be the final part of the sentence problems so...then uh...I will have everyone use their heads and think a little, okay? Until now we've just done calculation practice, but today we will have your heads a little so...asking you to use thinking methods; how to think and how to, uh, look for it and think about it may be a little difficult, you

		know. More difficult than just simply calculating, that is. Right?
00:07:25	T	Well, then, we'll go ahead. [*Teacher places poster-sized paper on the board with problem three: "Problem 3. You would like to buy 10 cakes all together for less than 2,100 yen in which one (type of) cake is 230 yen each and the other (type of) cake is 200 yen each."*]
00:07:27	T	Okay. Well, then...please look at...the problem.
00:07:33	T	Hachino, can you see it?
00:07:34	S	Yes.
00:07:35	T	Can you see?
00:07:35	S	I can see.
00:07:36	T	You can see. Okay. Then [let's do it].
00:07:43	T	Please read the problem...in English.
00:07:45	S	Ha, ha.
00:07:48	T	Okay. Makoto, please read the problem.
00:07:51	S	(*Reading from board*) You would like to buy ten cakes all together for less than two thousand one hundred yen, in which one cake is two hundred thirty yen each and the other cake is two hundred yen each.
00:08:02	T	Yes.
00:08:03	T	Do you understand the meaning of the problem?
00:08:07	T	Abe, do you understand what this problem means?
00:08:09	T	You have two-hundred-thirty-yen cakes and two-hundred-yen cakes, right? The two-hundred-thirty-yen cakes are a wee bit more expensive.
00:08:14	T	And...you have ten people in your family, so you want to buy cakes so that each person gets one cake. However, I have only two thousand one hundred yen.
00:08:24	T	Which cake...seems more delicious?
00:08:29	S	The two-hundred-thirty-yen one.
00:08:31	T	The more expensive one is somehow more desirable, right?
00:08:33	T	And so...you want to buy as many expensive cakes...as you can, but what's the maximum that you can buy?
00:08:44	T	That's the problem. (*Teacher unfolds second part of*

		problem from problem 3 poster on board: "If you want to buy as many two-hundred-and-thirty-yen cakes as possible, what is the maximum number that you can buy?")
00:08:46	T	Understand?
00:08:48	T	There are cakes that are two hundred and thirty yen and two hundred yen and...you only have two hundred ten yen. But you need to buy ten.
00:08:56	T	But the two-hundred-thirty-yen one looks more delicious, so you want to buy as many as possible.
00:09:01	S	Nine.
00:09:02	T	But you only have two thousand one hundred yen so...so in fact, how many two-hundred-thirty-yen cakes can you buy?
00:09:09	S	Nine.
00:09:10	T	So then today I am going to have you all think about how to find the answer. I will pass out paper, so, umm, please try and think about how to solve it.

(*Students begin working on the problem individually. After a while, the teacher stops the class and asks a student to present her solution.*)

Part 3

Students Presenting Solution Methods

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00:17:12	T	Okay then, Yokogake.
00:17:13	T	How did you think about it?
00:17:15	S	(*Yokogake*) Heh?
00:17:17	S	What?
00:17:18	T	Okay, go ahead.
00:17:20	S (Y)	(*Standing*) I
00:17:22	T	Mm.
00:17:22	S (Y)	Didn't understand it at all, but...
00:17:25	T	Mm hmm.

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00:17:28	S (Y)	First of all...
00:17:29	T	Mm hmm.
00:17:30	S (Y)	I thought that I should calculate...
00:17:36	T	Mm hmm.
00:17:36	S (Y)	How many of the two-hundred-thirty-yen one that I could buy and...
00:17:36	T	Mm hmm. (*Teacher begins writing on board.*)
00:17:40	S (Y)	In the beginning...when I did it with ten...
00:17:44	T	Mm hmm.
00:17:45	S (Y)	It ended up being two thousand and thirty yen, so...
00:17:48	T	Mm hmm.
00:17:50	S (Y)	It's not good because it's over the amount, so...
00:17:51	T	Mm hmm.
00:17:53	S (Y)	And then next when I did it with nine...
00:17:54	T	Mm hmm.
00:17:57	S (Y)	When I did it with nine...it was two thousand seventy yen, and...it was okay, but...
00:18:03	T	Mmm.
00:18:04	S (Y)	You need to buy ten, so when I calculated it...
00:18:09	T	Mmm.
00:18:10	S (Y)	To buy one two-hundred-yen-cake...
00:18:15	S (Y)	Two...two thousand seventy plus...
00:18:18	T	Mm hmm.
00:18:20	T	Two hundred yen is...
00:18:21	T	Mm hmm.
00:18:23	S (Y)	With two thousand...two hundred seventy yen and...
00:18:26	T	Mm hmm.
00:18:26	S (Y)	You go over.
00:18:27	T	You go over don't you? Okay.
00:18:28	S (Y)	So then...you keep reducing the numbers and...
00:18:28	T	Keep reducing and...when you do it with eight on this side and two this side then?
00:18:32	S (Y)	When I did it...
00:18:33	T	When you did it?
00:18:34	S (Y)	Time ran out and...

00:18:35	T	Time ran out and...
00:18:36	S (Y)	(I couldn't do it) to the end.
00:18:38	T	You couldn't do it to the end.
00:18:39	T	Okay.
00:18:40	T	Raise your hand...if you say that when this way of thinking came up, it was really similar to yours.
00:18:45	T	Yokogake started counting from ten but...

(*The teacher asks other students how they solved the problem.*)

Part 4

Teacher and Students Presenting Alternative Solution Methods

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00:21:40	T	(*Referring to the same problem*) Then...I've thought about it too, so...what do you think about this way of thinking? Do you all understand it?
00:21:46	T	(*Writing on board*) You bought...ten...two-hundred-thirty-yen cakes.
00:21:52	T	You're told to buy a lot, and so, in reality, you want to buy all two-hundred-thirty-yen cakes, right? (*Written on board: "Method of thinking. Ten 230-yen cakes."*)
00:21:56	T	Then how much money is needed?
00:21:58	S	Two thousand three ( ).
00:21:59	T	(*Continues to write on board, illustrating verbal explanation*) In reality, two thousand three hundred yen is required, right?
00:22:02	T	But you're short two hundred yen.
00:22:05	T	You are short...two hundred yen.
00:22:09	T	He's short, so to tell you what that person thought...that he would buy a cake that is...thirty yen cheaper than...the two-hundred-thirty-yen cake. Buy a cake that is thirty yen cheaper and...you buy a cheap cake and...replace this

needed two hundred yen in a cake that is thirty yen cheaper, okay? [*On board.* (*"Short 200 yen"*) (*"30-yen cake"*)]

00:22:26 T You're short two hundred yen, you know.

00:22:27 T But let's buy them a cake—not two hundred thirty yen, but a cake...that is thirty yen cheaper.

00:22:32 T Then thirty yen is going to float. With each...thirty yen is going to float.

00:22:35 T How many cakes that are thirty yen cheaper do you need to buy inorder to save the two hundred ten yen?

00:22:40 T This needed part. (*Pauses*)

00:22:46 T How many cakes that are thirty yen cheaper do you need to buy to save two hundred ten yen? Can you save two hundred ten yen?

00:22:51 S Seven.

00:22:52 T Yeah. If you buy six, six times three is one hundred eighty yen, so you are still twenty yen in the red, right?

00:22:58 T However, if you buy seven of these...if you buy seven...you will have two hundred ten yen left over, right? The money right? [*On board* (*"Seven"*)]

00:23:05 T That two hundred ten yen is applied to this two hundred yen, you know.

00:23:09 T Then if you buy seven of the cakes that are thirty yen cheaper, if you do that, then what about this side?

00:23:13 S Three.

00:23:14 T It's three...did anyone do it like that? Someone who did it like this?

00:23:19 T There probably isn't anyone right?

00:23:20 T Start off by buying ten.

00:23:21 T You're short two hundred yen, so...let's bury the missing cost with a cake that is thirty yen cheaper.

00:23:26 T You can save it if you buy seven, so this is three. In that way.

00:23:30 T It was ( ) right? Then I'll ask you. Okay...Rika.

00:23:33 T Then, how did you think...about this one?

00:23:36 S (*Rika*) (*Standing*) Umm...the total two hundred thirty

		yen, oh...It's for some amount, and make that amount X and...umm...you need to buy ten of the two-hundred-yen one, so ten...make it ten minus X and...and the to...total has some amount of two—two-hundred-thirty-yen ones...and the two-hundred-thirty-yen ones are two hundred thirty X and...the two-hundred-yen one is two hundred...bracket ten minus X.. and then...then...umm...two hundred—two hundred thirty X plus two hundred bracket ten minus X...is less—less than minus, uh, less than or equal to two thousand one hundred yen, and you form the inequality equation.
00:24:42	T	Okay.
00:24:42	T	You said that this was the inequality equation, right?
00:24:45	T	Okay. (*Student sits.*)
00:24:45	T	(*To class*) Did you understand the meaning?
00:24:49	T	Perfect.
00:24:50	T	You'll get it better with Rika's explanation than with mine.
00:24:51	T	Okay. Try and raise your hands.
00:24:53	T	People who say that they got it with Rika's explanation. (*Some students raise their hands.*)
00:24:55	T	One person?
00:24:57	T	Only Kanzaki? Two people? Three people?
00:24:59	T	Four people? Just four people is it? Five people? Okay.
00:25:02	T	Then...please explain it next, Ryo.
00:25:04	T	Please explain it in a way that is a little more understandable.
00:25:05	T	Try and explain it in a way in which...a few more people will say...that they understood.
00:25:12	T	The method of explanation is okay with this. (*Teacher begins to draw chart around Rika's solution method on board.*)
00:25:15	T	Okay.
00:25:17	T	Go ahead.

(*The teacher passes out a worksheet and works through it with the class. The discussion of the worksheet continues.*)

Part 5
Teacher Elaborating on a Student's Method

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00:33:11	T	Which is easier, doing it one by one or using an in equality equation?
00:33:16	S	An inequality equation.
00:33:16	T	It's easier with the inequality equation, isn't it? And so today what I would like you to do from now is we did this in the method of thinking but...inequality equation, right?
00:33:26	T	(*Begins writing on small chalkboard.*) I would like you...umm to know...the good qualities of...finding the answer by...the answer by...setting up an...inequality equation right...inequality equation...inequality equation so...we thought about it...with a problem like this. [*On Board* (*"Having you know the good qualities of finding the answer by setting up an inequality equation."*)]
00:33:52	T	If you were to solve it without using an inequality equation you need to check it out quite a lot, one by one a lot.
00:33:57	T	Yokogake could solve it because it was ten (*points to Yokogake's solution on board*), but what if you were to buy one hundred of these two cakes together...ninety figure out one hundred and figure out ninety-nine and figure out ninety-eight and figure out ninety-seven and you need to figure out all of the numbers between one and a hundred, don't you?
00:34:16	T	However, if you used a method like this that Rika used...(*teacher pointing to Rika's solution on board*) the

		answer will come out quickly.
00:34:22	T	Therefore you don't need to figure out each number one by one (*pointing to Yokogake's solution*), so working it out by making...an inequality equation has a lot more...good qualities...than counting it one by one. That's what it's about, all right?
00:34:35	T	So then, and so...if there are good qualities like that then...we're saying that so, umm, there are two problems on the right side.

Part 6
Posing and Solving Follow-Up Problems

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00:34:43	T	This time, please buy twenty apples and oranges all together.
00:34:46	T	If you count it one by one, you will be in an incredibly terrible situation.
00:34:50	T	In the same way that we just did the cake situation, set up an inequality equation by yourself and find out up to how many apples you can buy.
00:34:56	T	Either that, or if it were the problem at the bottom, try to solve a problem about how many pears can you buy by setting up an inequality equation, work it out, and find an answer.
00:35:04	T	Because finding the answers one by one is hard, I wonder if you see the numerous good points of setting up inequality equations and, well, that you'll set up inequality equations yourself and try to find the solutions. That's what it's all about, okay?
00:35:16	T	Is it okay?
00:35:19	T	Okay. Then and so...eh...people who haven't written this here write it and then problem one. Try to set up

		an inequality equation by yourself in the same way and try to solve the problem.
00:35:32	T	Okay. Go ahead.

(*Students begin working on problems individually. After about 11 minutes, the teacher goes over the problems with the class.*)

Part 7
Summarizing the Lesson Objective

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00:49:48	T	What we talked about today was...the answer from inequality equations...that is...when you work out problems instead of counting things one by one and finding the number, it's usually easier if you set up an inequality equation and...find the answer.
00:50:03	T	That's why although it may be tedious...uh...about the applied problems of inequality equations, okay? Rather than looking for the answers one by one, you can get them by translating the parts written in Japanese into mathematical terms and solving [them].
00:50:17	T	Because an inequality equation has a good quality like this. This is what we talked about.
00:50:22	T	Is it okay?
00:50:24	T	Is it okay?

(*The teacher ends the class by passing out a homework assignment.*)

German Lesson Two: Algebra—Systems of Equations

Classroom setup: There are approximately 15 students in the classroom. Nine students are seated along the right-hand and back walls. From the left-hand wall, two groups of students are seated in short rows. There is a large open space on the right-hand side of the room. There is a large chalkboard on the front wall.

Part 1

Presenting Warm-Up Problems

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00:00:54	T	(*Standing in front of class*) Well, good morning.
00:00:55	Ss	Good morning.
00:00:57	T	Arne, did you want to change places? I mean, we can also put you up here with a mike on you. Eight to the third power. (*Teacher begins walking back and forth in front of class.*)
00:01:14	T	Gabi.
00:01:15	S	Hundred twenty-eight.
00:01:18	T	I don't know at what point you miscalculated there. We'll have to go over that again right away. Sebastian?
00:01:21	S	Five hundred twelve.
00:01:22	T	Please calculate it for us.
00:01:24	S	Uh well. Eight squared is sixty-four. That times eight is five hundred twelve.
00:01:29	T	Yes, Gabi.
00:01:30	S	Yes, that's right.
00:01:30	T	Yes. You agree with that? Second binomial formula. We have aspecialist for that. Right, Rieke?
00:01:36	S	A minus B in parentheses squared.
00:01:37	T	Rieke speak (a little [louder])—look I got this thing around me. Man, why don't you at least speak up?
00:01:39	S	Yes.
00:01:41	S	A minus B in parentheses squared equals A squared minus two A B plus B squared.
00:01:47	T	And twelve percent of hundred twenty.

00:01:57 T Claudia.
00:01:58 S (Fourteen point forty)?
00:01:59 T Right on. And five factorial?

(*The teacher asks a few more review questions.*)

Part 2
Reviewing Previous Material

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00:03:10 T (*Walking back and forth in front of class*) Gesa, your turn. Very briefly. What have we done lately?
00:03:14 S Umm...umm.
00:03:17 Ss Ha, ha, ha.
00:03:18 S Umm.
00:03:19 T Well, guys. Take note of this. Finally, Gesa got put in her place, right? (*Teacher sits on empty desk at front of room.*) Usually she...she is gabbing away all over the place and now she's sitting all small and mhm... Gesa, come on. Go on.
00:03:29 S Well, I don't know anymore. Umm, this thing with two variables.
00:03:31 T Well? (*Teacher walks to chalkboard.*)
00:03:35 T Name?
00:03:38 S Huh? X and Y.
00:03:39 T No. The heading.
00:03:41 S Equation with two variables.
00:03:43 T (*Begins writing*) All right. Okay. [*On board,* (*"Systems of equations"*)]
00:03:50 T Okay, Hannah? What methods do you know? What various methods do you know to solve systems of equations?
00:03:58 S Umm, equating?
00:04:00 T Well, then.
00:04:04 T (*Begins writing on board again*) Christian, will you give

		an example for that?
00:04:16	T	Christian. Hello?
00:04:21	T	(*Turns around to face class*) Christian. Christian. Come on. Let's forget about it. Patrick? (*Teacher walks to back of room among students.*) Any example where you apply the method of equating?
00:04:27	S	Mhm.
00:04:30	S	In a problem or what?
00:04:32	T	Yes. (*Teacher walks back toward front of room.*)
00:04:33	S	Mhm...two Y...
00:04:38	T	Go on.
00:04:39	S	Plus three X...equals five. And the other one, two Y plus five X equals thirty-seven.
00:04:51	T	And you want to use the method of equating on this?
00:04:55	S	Yes, mhm.
00:04:56	Ss	Ha, ha, ha.
00:04:57	T	Ha, ha.
00:04:58	S	Maybe. Yes.
00:04:59	T	Well? Hannah?
00:05:01	S	Umm...two X plus three...
00:05:03	T	Yes.
00:05:06	S	Uh...
00:05:07	T	Mm hmm. You're right. Three minus four Y. And now? Yes? Well, go on.
00:05:12	Ss	Ha, ha, ha.
00:05:13	S	Yes, but I didn't think of that.
00:05:15	T	You didn't say that?
00:05:16	S	No ( ).
00:05:17	T	But Sven, continue then.
00:05:19	S	Okay. And well, Y equals...seven plus seven plus three (Y).
00:05:30	T	(*Writing student's solution on board*) Okay, and what can you do now? (Fokko)?
00:05:32	S	Equating it.
00:05:33	T	Okay. (*Teacher writes on board.*) Method. Right?
00:05:37	T	And so on. Okay which other method (Ina)?

00:05:41	S	Substituting.
00:05:43	T	Okay.

(*The teacher continues reviewing the material with the students.*)

Part 3
Posing and Working on the Problem

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00:09:57	T	(*To Class*). Oh, boy. But I'm granting that you are shy because you actually should have known this better. (*Teacher reading from book*) Let's see.
00:10:10	T	Mhm. (*Teacher begins writing problem from book on board. Finishes writing, closes book, sits on empty desk at the side of the room. Students look at problem on board and discuss among themselves.*)
00:11:42	T	(*Student raises his hand.*) Are you raising your hand, Patrick?
00:11:43	S	Yes.
00:11:44	T	Okay. Can you wait a little? Maybe some more people will raise their hands. Let's wait some more. (*Other students raise their hands.*) Yep. Yes, four. Well, that's pretty nice already. Okay, Patrick, what are you suggesting?
00:11:57	S	Well, first I would get rid of the parentheses.
00:12:00	T	Do you also want to do it yourself?
00:12:02	S	Yes ( ).
00:12:03	T	Come on then.
00:12:13	T	And nice and loud please. Right? So everyone understands what you're doing.
00:12:18	Ss	Ha, ha, ha.
00:12:19	T	Uh huh.
00:12:20	S	Well.
00:12:26	Ss	Ha, ha, ha.
00:12:28	T	Hey. That is...well let's go. Mm?

00:12:28	S	Well, ha, ha.
00:12:31	S	(*Begins writing on board*) This stays the same for now.
00:12:36	S	And then I'm doing the distributive law well.
00:12:41	T	Yes.

(*The teacher and students continue working on the problem. Students then copy what has been done so far in their notebooks and work on problems individually.*)

Part 4
Sharing the Result

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00:26:13	T	(*Sitting on empty desk at front of room. To class*) Yes...then let's see. We should all be done with the copying down by now. Some of you already had different ideas. If you would perhaps briefly explain them. Rieke?
00:26:22	S	(*Rieke*) Umm...times minus thirteen.
00:26:24	T	Well—can you tell me what you want to do? What method will you want to do?
00:26:29	S (R)	The method of equating. Oh no—that is the method of adding.
00:26:30	S (R)	No. Addition.
00:26:33	T	Okay, you want to use the method of addition. And then you want to—you just said something with thirteen.
00:26:34	S (R)	Yes.
00:26:38	S (R)	Yes. Umm, that—we take the equation times minus thirteen.
00:26:39	T	Why? Why?
00:26:42	S (R)	Then the—then that's exactly one hundred ninety-five X and then ( )...
00:26:47	T	Hold on. Hold on. I think—complete it first?
00:26:52	S	(*Another student*) The left, well, the left—left equation times thirteen.

00:26:55 T I think that's what she wanted. She wanted to go into that direction, Sven. And if she's making a little mis take just doing it mentally, that's no big deal. I think Rieke just may show us what she's thinking of. Right?

00:27:06 S Now?

00:27:07 T Umm, should we wait till tomorrow? (*Rieke walks to chalkboard.*)

00:27:15 T And Jochen. Hello. The board is here and not there.

00:27:19 T Okay.

00:27:20 S (*Rieke*) Should I write it down again?

00:27:21 T Yes. I think that would be good for an overview, to write it down your way.

00:27:25 S Maybe first like this. (*Student writes on board.*)

00:27:26 T Yes.

00:27:36 T (*While Rieke writes*) If she writes it down like that. Gesa, think of that hint again. If you want to use the method of addition, how are you supposed to write down the second equation?

00:27:43 S Further down.

00:27:44 T No (because you)—no, no. That's not what it's all about.

00:27:48 T No.

00:27:50 T Sorted. That she also has Y now—on top she wrote Y some number X. And that is exactly the same in the—mm hmm.

00:27:52 S I see.

00:28:02 T Arne?

00:28:04 S Yes?

00:28:05 T Behave yourself for once.

00:28:08 T Okay, Rieke. Loudly.

00:28:10 S (*Rieke*) And this one times minus thirteen.

00:28:11 S (*Another student, to Rieke*) No, thirteen.

00:28:12 S (*Rieke*) Yes, times minus thirteen.

00:28:14 S No, thirteen.

00:28:15 S (*Rieke*) If you add this up, then this must be plus in order for this to be omitted.

00:28:19	S	Oh, I see. Uh. Excuse me.
00:28:20	Ss	Ha, ha, ha.
00:28:22	T	Gotcha.
00:28:23	S	Yes.
00:28:27	T	Oh, now we're getting serious. Sven, get busy. Ninety-four times minus thirteen.
00:28:32	S	Ninety-four times?
00:28:33	S	Minus thirteen.
00:28:35	S	Umm (should I roughly estimate it?)
00:28:38	Ss	Ha, ha, ha.

(*The teacher continues going over the problems with the students.*)

Part 5
Summarizing the Objective and Assigning Seatwork

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00:33:27	T	(*Sitting on empty desk at front of classroom*) Well, Gesa. This problem. What would you say? It wasn't that easy was it?
00:33:32	S	No.
00:33:34	T	Well, how—how is it then with complicated problems like that? How do you have to proceed? Maybe you should summarize before you start writing it down.
00:33:41	S	First get rid of the parentheses, then multiply with the common denominator…
00:33:47	T	Yes.
00:33:48	S	Then…uh…umm make it so there is an equal number of Ys ( )…
00:33:57	T	Yes, that actually is the most difficult part. Right? To find the approach. How should I solve the problem now? Right? (*Teacher walks to chalkboard*) So that this—let's say this times minus thirteen. (*Teacher points to Rieke's solution.*) I think that was—that means to write it

		down like this with minus three so you can see that a multiple of fifteen is hundred ninety-five. Because it wouldn't have worked so nicely with the Ys. Right, Holger?
00:34:17	S	Huh?
00:34:18	S	No (that would have been)...
00:34:19	T	Something with ninety-four written there and back there it's thirty-two. (*Teacher walking toward class*) With multiplying we would have gotten into really high numbers. Right? (*To student*) You would have seen it?
00:34:26	S	What? No.
00:34:28	T	What?
00:34:28	S	If I wouldn't have seen that that works, I would have equated them or something. I think I would have used the method of equating.
00:34:35	T	Yes. Yes. (*Teacher walking toward board*) But, where would you have wanted to equate something? You would have—take a look you would have had to divide by thirty-two. Here. (*Teacher points to board.*)
00:34:41	S	Yes, of course, that would have been very difficult.
00:34:42	T	And then you would have had a major fraction. Right?
00:34:43	S	Yes.
00:34:44	T	(*Walks back toward class*) Well, if possible always remember not to make fractions. It's better to multiply bigger numbers. You got that so far?
00:34:51	S	Mm hmm.
00:34:52	T	(*Picks up book, opens it, looks at problems.*) Then I'd like to ask you to try something on your own after you're done copying this. On page hundred ninety-five we've got nice problems like that. To let you know right away this one was the most difficult of all.
00:35:05	T	Well, I think you should do the problems twelve and thirteen.
00:35:14	T	Twelve is enough for now.

(*The teacher closes the book and puts it down. Students begin working on the problems. The teacher circulates among them. After about eight minutes, the class ends.*)

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