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ABSTRACT

Research has often found little or no significant differences in overall performance for math students using graphing calculators compared to traditional (non-calculator) classes. However, when performance is divided into procedural and conceptual levels, significant differences appear at the conceptual level. This study investigates the relation of the factors of gender, spatial visualization, mathematical confidence, basic algebra ability, and classroom graphing calculator utilization to conceptual mathematical performance with graphing calculators in college algebra. The subjects were undergraduate students enrolled in a college algebra class in which graphing calculators are required. Graphing calculator utilization was observed and rated on a scale ranging from negative to intensive. The study revealed little variation over the utilization rating scale, with scores only ranging from 6-9 inclusive on a scale of possible values from 3-15. This indicates that the integration of this technology was not yet a major part of the instructional process. The data collected in this study may serve as a baseline for data collected in the future as the reform process changes the teaching of college algebra. The lack of gender correlations supports the equity of graphing calculator-enhanced instructional practices in college algebra. Results show that the variables of spatial visualization and mathematical confidence are related to conceptual mathematical performance when graphing calculators are utilized as a tool. An appendix provides the guidelines and procedures for classroom graphing calculator utilization observations. (Contains 14 references.) (Author/SWC)

Learning with Technology: Research on Graphing Calculators

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Research has often found little or no significant differences in over-all performance for math students using graphing calculators compared to traditional (non-calculator) classes (Scott, 1994; Tolias, 1993). However, when performance is divided into procedural and conceptual levels, significant differences show up at the conceptual level (Tolias, 1993). What factors relate to mathematical performance with graphing calculators at the conceptual cognitive level? Factors strongly suggested for consideration are gender, spatial visualization, mathematical confidence, basic mathematical ability, and classroom graphing calculator utilization.

Background

Two levels of mathematical understanding are defined by researchers in the field of mathematical learning. Hiebert and LeFevre (1986) divide mathematical knowledge into procedural and conceptual. Procedural knowledge is "familiarity with the symbol representation system and rules, algorithms, and procedures"(pg. 9), while conceptual knowledge is "a connected web of knowledge, a network in which the linking relationships are prominent as the discrete pieces of information" (pg. 3-4). For Hiebert and LeFevre, instruction should foster conceptual knowledge construction. Richard Skemp's (1987) cognitive learning theory delineates two types of understanding -- instrumental and relational. For mathematics, instrumental understanding is the application of a rule or procedure, while relational understanding involves relating a task to an appropriate schema (knowing both how and why in problem solving). For Skemp, relational understanding and conceptual understanding are the same and should be the goal of instruction.

The National Council of Teachers of Mathematics (NCTM) in it's 1989 publication Curriculum and Evaluation Standards for School Mathematics (commonly referred to as the Standards) fosters conceptual mathematical learning and relates it to multiple representations of the graphing calculator:

> The 9-12 standards call for a shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understanding, multiple representations and connections, mathematical modeling, and mathematical problem solving. The integration of ideas from algebra and geometry is particularly strong, with graphical representation playing a connecting role. Thus, frequent reference to graphing utilities will be found throughout these standards. (1989, p. 125).

The graphing calculator which is often referred to as a hand-held computer shows promise in recent research as a tool to assist the learner construct conceptual knowledge in mathematics in the areas of algebra and functions (Estes, 1990; Shoaf-Grubbs, 1992; Tolias, 1993). Graphing calculators have an advantage over computers in mathematics classrooms in their lower cost and smaller size for portability. Because of these advantages, the graphing calculator has gained widespread acceptance as a powerful tool for mathematics classrooms (Dick, 1992; Wilson & Krapfl, 1994). Although the use of graphing calculators has become extensive in high school, community college, and university mathematics classrooms in the last few years, little is known about how and why graphing calculators make a difference in mathematical understanding. Much of the initial research has been in the form of comparisons of achievement and/or attitudes for groups using calculators and traditional, non-calculator groups (Wilson & Krapfl, 1994). These studies of <u>educational evaluation</u> of the impact of the graphing calculator on the teaching and learning of mathematics are not <u>educational research</u> which add knowledge about how or why graphing calculators facilitate student construction of mathematical knowledge (Bright & Williams, 1994). "For research effectively to guide curriculum development and instruction, we need to find out <u>why</u>" (Dunham & Dick, 1994, p. 440).

Conceptual Framework and Hypotheses

Several factors that have been associated with performance in traditional mathematics have also been mentioned by researchers as possible links to achievement by students using graphing calculators. The relations of gender, spatial visualization, and mathematical confidence to traditional mathematical performance have been heavily investigated. Such factors show promise for exploratory correlational research in graphing calculator utilization. Some studies indicate that women are not disadvantaged by the integration of graphing calculator technology and in some instances outperform males (Boers and Jones, 1992; Dunham, 1990; Ruthven, 1990). Since this achievement by females is contrary to previous gender studies in non-technology mathematics, gender is a variable that needs further investigation. Because of the visual representations produced by the graphing calculator, spatial visualization has been suggested as a factor for research. Whether the calculator increases spatial ability as Shoaf-Grubs (1993) reported or offers "an alternative <u>source</u> of visual images for those who cannot create their own" (Dunham, 1995), spatial visualization as a factor related to mathematical performance with graphing calculator utilization merits further study. While Dunham (1990) reports gains in confidence for students in graphing calculator classes, Shoaf-Grubbs (1993) describes comments of increased confidence from female students with the graphing calculator as a tool for checking solutions. Is this confidence with using technology related to performance with the technology? Thus gender, spatial visualization, and confidence are prominent factors for further study in relation to conceptual mathematical performance in calculator-enhanced algebra instruction.

Other factors that may be related to performance in graphing calculator-enhanced classrooms are basic algebra ability and classroom graphing calculator utilization. Understandably, a student's basic algebra ability will influence her performance in studying algebra with or without the enhancement of technology. How strong is that relationship when graphing calculators are utilized? Because of classroom variation in the utilization of the calculator, when examining multiple classrooms, this information should be collected as a possible factor for further study.

Research Question

The research question guiding this study is:

What is the relation of the factors of gender, spatial visualization, mathematical confidence, basic algebra ability, and classroom graphing calculator utilization to conceptual mathematical performance with graphing calculators in college algebra?

Five proposed hypotheses associated with this question are:

#1. There is a positive relation between Basic Algebra Ability and Conceptual Mathematical Performance.

#2. There is no relation between Gender and Conceptual Mathematical Performance.

#3. There is a positive relation between Classroom Graphing Calculator Utilization and Conceptual Mathematical Performance.

#4. There is a positive relation between Spatial Visualization and Conceptual Mathematical Performance.

#5. There is a positive relation between Mathematical Confidence and Conceptual Mathematical Performance.

Methods

Sample

The study consisted of the quantitative analysis of the five factors of gender, spatial visualization, mathematical confidence, basic algebra ability, and classroom graphing calculator utilization through correlational statistical techniques.

This study was conducted at a major Rocky Mountain university during the Spring semester of 1996. The subjects were undergraduate students enrolled in one of the nine sections of Math 1400 -- College Algebra who completed all four of the measurements administered for this study. This was the second year that graphing calculators were required for this course. Students must meet prerequisites for admission to this course through one of the following:

1. Grade of C or better in a prerequisite course (a non-credit elementary algebra)
2. Sufficiently high ACT/SAT score in math
3. Performance on the university Math Placement Exam passing arithmetic, elementary algebra, and intermediate algebra

Students in College Algebra are generally classified as freshmen, sophomores, or juniors. The majority of the students need to take additional math coursework in calculus.

Students in all sections used the same textbook, had the same assignment of topics and suggested homework, and took the same exam at the same time. The textbook used for the course was College Algebra: A Graphics Approach by M. G. Settle (1995). It was selected by the previous year's instructors who wanted a textbook that included specific calculator activities. Purchase of a graphing calculator such as the TI-81 was recommended for all students, and this model was depicted in the textbook with instructions for operation. Instructors also used this model for demonstrations in class with an overhead projection device and large screen.

Instruments

For correlational analysis, the following tests were administered to students:

1. Spatial visualization -- Paper Folding Test, VZ-2 of the <u>Kit of Factor-Referenced Cognitive Tests</u> published by Educational Testing Services.
2. The Mathematical Self-Concept Scale by Annette Gourgey.

Student performance on the first class exam, which was a review of basic algebra covered in the first three weeks of class with only very minimal calculator utilization, served as the basic mathematical ability variable measure.

To quantify the factor of classroom utilization of the graphing calculator, the researcher and two assistants observed classrooms three times during the period of intensive study of functions utilizing graphing calculators. Observers rated the technology utilization in the classrooms according to researcher-developed guidelines (see Appendix A).

Students' scores on the third class exam, after nine weeks of intensive study of functions, were divided into subscales for the mathematical cognitive levels of procedural and conceptual. The subscale for conceptual knowledge was the Conceptual Mathematical Performance measure. The correlation coefficients for the relations of gender, spatial visualization, mathematical confidence, basic algebra ability, and classroom graphing calculator utilization to Conceptual Mathematical Performance were determined.

Results

Descriptive Statistics

Complete data on all of the measures was collected for 144 students in College Algebra with 65 male (45%) and 78 female (54%) participants.

The cumulative ratings by the three observers appear in Figure 1 to depict the Classroom Graphing Calculator Utilization measure for the nine sections of Math 1400. Each section was observed on the same three class meeting days with one observation per rater whenever possible. Individual observations were scored on a scale from one to five using the guidelines in Appendix A. Rating guidelines included both instructional and student utilization of the technology. The lowest possible cumulative rating of 3 would occur when the three individual ratings were each at the Negative Utilization level, with a score of 6 representing consistent Minimal Utilization ratings, 9 for consistent Moderate Utilization ratings, 12 for consistent Frequent Utilization ratings, and the highest cumulative rating of 15 for consistent Intensive Utilization ratings. All nine sections received cumulative ratings that were between 6 and 9, inclusive, indicating little variation in utilization of the graphing calculators in the Minimal to Moderate range across the nine sections.

Table 1 presents the mean, standard deviation, minimum, and maximum scores for the measurements of the variables. Basic Algebra Ability (BSCALG) scores are the results of the first exam in College Algebra administered early in the course after three weeks of review before the introduction of functions and the utilization of the graphing calculator for graphing purposes. The measure for Classroom Graphing Calculator Utilization (CLSUTL) was the ratings reported for each section in Figure 1.

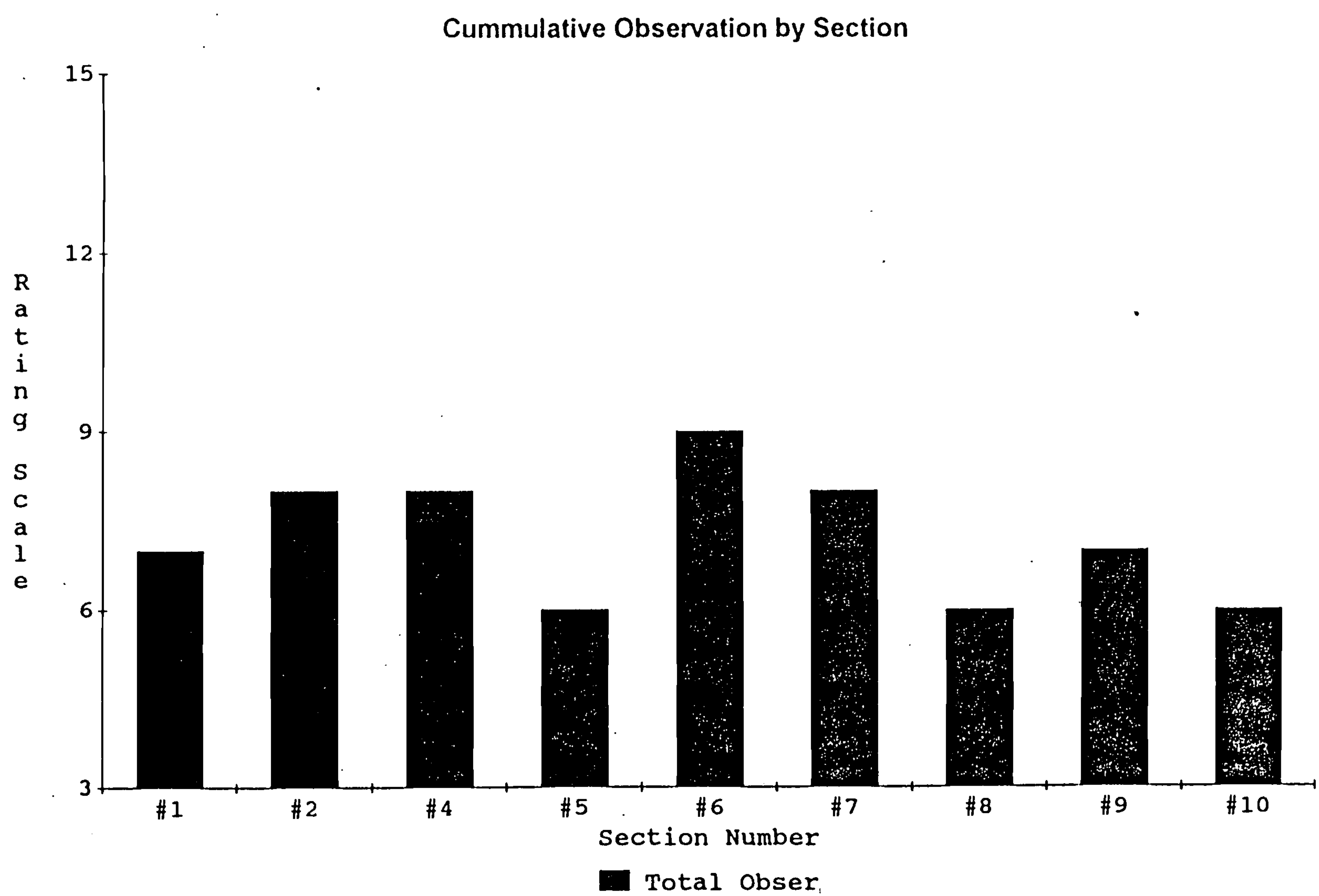

Figure 1. Cumulative Observation Ratings for Classroom Graphing Calculator Utilization by Section Number.

Table 1

Descriptive Statistics for Variables. (n=144)

Variable	Mean	Std Dev	Scored Min	Scored Max	Possible Min	Possible Max
BSCALG	76.62	11.70	48	100	0	100
CLSUTL	7.09	1.12	6	9	3	15
SPAVIS	12.82	3.28	3	20	0	20
MCONF	87.94	20.87	34	132	27	135
PRCDMPRF	48.97	12.71	5	70	0	70
CNCMPRF	20.65	7.50	0	30	0	30

NOTE: The labels with the factor represented in parentheses are as follows: BSCALG (Basic Algebra Ability), CLSUTL (Classroom Graphing Calculator Utilization), SPAVIS (Spatial Visualization), MCONF (Mathematical Confidence), PRCDMPRF (Procedural Mathematical Performance), and CNCMPRF (Conceptual Mathematical Performance).

The instrument used to measure Spatial Visualization (SPAVIS) was the Paper Folding Test: VZ-2 of the Kit of Factor-Referenced Cognitive Tests (1976 [Reprinted 1995]). Mathematical Confidence was measured using Gourgey's Mathematical Self-Concept Scale (1982). Procedural Mathematical Performance and Conceptual Mathematical Performance are the two subscales of the third exam in College Algebra after eight weeks of study of functions enhanced with the utilization of the graphing calculator. These subscales were obtained by dividing the questions of the third exam as to whether they were procedural knowledge items or conceptual knowledge items as determined by an expert panel.

The results reported in Table 1 for Spatial Visualization and Mathematical Confidence for the sample population of College Algebra students tested in this study is consistent with data from previous studies. The mean of 12.82 for the sample population for spatial visualization using the Educational Testing Service's Paper Folding Test: VZ-2 is within the range of mean scores reported for the norming populations in the Manual for Kit of Factor-Referenced Cognitive Tests (1976). When Gourgey developed the Scale for the Measurement of Self-Concept in Mathematics, her sample population scored a mean of 94.53 with a standard deviation of 21.88 (range of 34 to 133) which are similar to the measurements for the college algebra students in this study where the mean was 87.94 with a standard deviation of 20.87.

Correlational Analysis

Using the statistical package of SPSS, the Pearson's Product-Moment Correlation Coefficient, r, was calculated for each of the independent variables in relation to the dependent variable Conceptual Mathematical Performance. In addition, the intercorrelations between the independent variables and with the Procedural Mathematical Performance measure were computed and appear in Table 2. The independent measures of Basic Algebra Ability, Spatial Visualization, and Mathematical Confidence exhibit significant statistical correlation to the dependent measure of Conceptual Mathematical Performance. Two other measures of mathematics abilities, the Basic Algebra Ability measure and the Procedural Mathematical Performance, are also correlated to Spatial Visualization and Mathematical Confidence.

Research Questions

The results of the correlational analysis were used to determine whether to retain or reject the five research hypotheses stated as null hypotheses as follows:

#1. There is no relation between Basic Algebra Ability and Conceptual Mathematical Performance. Rejected ($p < .01$). The original hypothesis that there would be a positive relation holds.

Question #2. There is no relation between Gender and Conceptual Mathematical Performance. Retained ($p > .05$). The original hypothesis that there would be no correlation holds.

Question #3. There is no relation between Classroom Graphing Calculator Utilization and Conceptual Mathematical Performance. Retained ($p > .05$). The original hypothesis thatthey would be positively correlated did not hold.

Question #4. There is no relation between Spatial Visualization and Conceptual Mathematical Performance. Rejected they would be positively correlated did not hold.($p < .05$). The original hypothesis that they would be positively correlated holds.

Question #5. There is no relation between Mathematical Confidence and Conceptual Mathematical Performance. Rejected ($p < .01$). The original hypothesis that they would be positively correlated holds.

Discussion and Educational Implications

The question guiding this study was: What is the relation of the factors of gender, spatial visualization, mathematical confidence, basic algebra ability, and classroom graphing calculator utilization to conceptual mathematical performance with graphing calculators in college algebra?

Looking at the lack of correlation of classroom graphing calculator utilization one possible explanations stands out. The results of the ratings of the observations for the classroom graphing calculator utilization showed little variation over the rating scale with scores only ranging from 6 to 9 inclusive on a scale of possible values from 3 to 15. This closeness in ratings leaves little opportunity to test the correlation of this variable to the dependent variable, conceptual mathematical performance.

The levels of the ratings for classroom graphing calculator utilization also explain an important aspect of this study in relation to a reform process in the teaching of college algebra that was just getting under way at this particular institution. All ratings are in the Minimum range up to the baseline of the Moderate range on the scale. This indicates that the integration of this technology was not yet a major part of the instructional process. The data collected in this study may serve as a baseline for data collected in the future as the reform process changes the teaching of college algebra.

The variable Gender did not correlate with Conceptual

Table 2

Correlations of the individual variables.

Variable	BSCALG	GENDER	CLSUTL	SPAVIS	MCONF	PRCDMPRF	CNCMPRF
BSCALG	1.000	.153	.068	.213*	.243*	.507**	.399**
GENDER		1.000	-.001	-.136	.065	-.014	-.077
CLSUTL			1.000	.125	.026	.053	-.061
SPAVIS				1.000	.112	.261*	.207*
MCONF					1.000	.325**	.433**
PRCDMPRF						1.000	.699**
CNCMPRF							1.000

N = 144 2-tailed Signif: * $p<.05$, ** $p<.01$

NOTE: The meaning for the abbreviations of the variables are in parentheses as follows: BSCALG (Basic Algebra Ability), GENDER, CLSUTL (Classroom Graphing Calculator Utilization), SPAVIS (Spatial Visualization), MCONF (Mathematical Confidence), PRCDMPRF (Procedural Mathematical Performance), and CNCMPRF (Conceptual Mathematical Performance).

Mathematical Performance, Basic Algebra Ability, Spatial Visualization, and Mathematical Confidence. Because students must meet rigorous prerequisites to enter the class, they enter college algebra with similar abilities which may have eliminated some of the gender differences seen in early mathematical ability studies. This lack of gender correlations supports the equity of graphing calculator-enhanced instructional practices in college algebra.

The results reveal that the variables of spatial visualization and mathematical confidence are related to conceptual mathematical performance when graphing calculators are utilized as a tool. This information is important in directing further investigation for the purpose of curriculum and instructional development.

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Appendix A

Guidelines and Procedures for Classroom Calculator Utilization Observations

The purpose of observations of the classroom is to determine the level of utilization of the graphing calculators by the instructor and students. The rating system in no way tries to judge the teaching of the instructor, only the quantity and quality of graphing calculator utilization occurring by both the students and instructor in a classroom at a particular moment in time. The highest and lowest ratings are extremes that will probably not exist in the sections of college algebra but are included to realistically identify all possible levels that exist. A panel of experts provided comments on initial drafts of the rating scale. Three raters simultaneously scored two sessions and then compared scoring to develop additional guidelines to insure reliability in ratings.

Each section will be observed and categorized according to the five-level rating system of graphing calculator utilization on three different class meetings. The dates for classroom observations were specifically chosen to coincide with instruction on topics concerning functions that would present opportunities for possible intensive use of the graphing calculators.

A point system will used so that individual ratings can then be cumulated for comparison. The scale for points is:

Level	Points
Intensive	5
Frequent	4
Moderate	3
Minimum	2
Negative	1

In an **Intensive** graphing calculator utilization classroom, the instructor would thoroughly integrate the use of the calculator to enhance learning as a natural part of the curriculum. When discussing problem solving and new concepts, the instructor would emphasize multiple solution methods which would include both algebraic and calculator solutions and the recall of graphical images as an integral part of instruction. The instructor might demonstrate new techniques with the calculator and encourage students to use their own calculators to follow along. The instructor would encourage students to help each other in utilizing the calculators. Throughout the class, students frequently would be engaged in working with their own calculators. Calculators would be utilized in numerous way including numerical calculations, algebraic substitutions, graphing, and exploration for conceptual development.

In a **Frequent** graphing calculator utilization classroom, the instructor would find frequent opportunities to utilize the graphing calculator in several ways for concept development and problem solving. Students would utilize their calculators on numerous occasions and be encouraged to do so by the instructor.

In a **Moderate** graphing calculator utilization classroom, the instructor would utilize the calculator for some concept development and problem solving while only using algebraic or other methods at other times even though the calculator could be utilized (thus missing opportunities for multiple methods). The instructor might work with calculator problem solutions mostly as a tool for checking or simple solutions, but not for multiple types of uses nor for exploration. The students would use their calculators moderately, simply listening or watching displays rather than working along on some occasions.

In a **Minimal** graphing calculator utilization) classroom the instructor would utilize the graphing calculator minimally by missing numerous opportunities for multiple methods of solutions that included calculators (as though the instructor just didn't understand how to utilize this tool for mathematical learning). The students would seldom use their calculators except for simple calculations.

In a **Negative** graphing calculator utilization classroom, the instructor would discourage calculator use in instruction. The instructor might refuse to help students who ask questions about how to manipulate the calculator and become upset with students who try to help each other with calculator utilization in class. When discussing problem solving, the instructor would only discuss non-calculator solutions whenever possible and convey a negative attitude when needing to use the calculator. Students would seldom bother to use their own calculators in class.