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ABSTRACT

This report describes the results of a three-year study, the Informed Instruction Project, that investigated different approaches to mathematics instruction for students with learning disabilities and at-risk for special education services. The project focused on two strands of inquiry: (1) the effects of a computer-based diagnostic assessment system in aiding special education teachers to remedy students' arithmetic misconceptions and (2) the effects of specific curricular interventions on students in a mainstreamed setting. The three sections of the report discuss the findings. The first section presents an overview of project dissemination and products. The second section is an overview of the findings, "The Informed Instruction Project: Preliminary Findings and Implications for Future Research in Mathematics for Students with Learning Disabilities" (John Woodward and Juliet Baxter), written for the National Center to Improve Instruction in November 1995. This paper describes the implementation of a diagnostic system (TORUS) designed to detect student misconceptions in addition and subtraction that was used in a year-long, qualitative study by a special education teacher, who used direct instruction and TORUS to supplement resource room math classes. The third section contains the technical overview of the project, "Action-based Research on Innovative Mathematics Instruction and Students with Special Needs" (Juliet A. Baxter and Deborah K. Olson). (Contains references.) (Author/LC)

Informed Instruction in Mathematics

Final Project Report

H180G20032

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May 23, 1996

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Overview of the Report

Over the last three years, the Informed Instruction Project has investigated different approaches to mathematics instruction for students with learning disabilities and those who are at-risk for special education services. The project has focused on two strands of inquiry: 1) the effects of sophisticated, computer-based diagnostic assessment system in aiding special education teachers remedy arithmetic misconceptions and 2) the effects of specific curricular interventions on these students in mainstreamed setting.

Early pilot studies and discussions with general and special educators conducted prior to this research suggested that the major emphasis of this project should be on the second strand -- innovations in mathematics curriculum at the elementary level. The success of new approaches in mathematics for many regular education students, a finding reported here as well, makes this focus even more imperative. The attached paper, "It's What You Take For Granted When You Take Nothing for Granted: The Problems with General Principles of Instructional Design," presents the educational implications of different methods of teaching mathematics. In the end, researchers on the Informed Instruction Project articulated why special educators need to reconsider traditional, basic skills approaches to the subject as well as developed intervention strategies best suited to the needs of students with learning disabilities and those at risk for special education services.

This report comprises three sections:

1. *Project Dissemination: Articles, Papers, and Professional Development Activities.* Dissemination activities are listed along with research and professional development activities which will extend beyond the project.

2. *Overview of Findings.* This overview article was written in November, 1995 for the National Center to Improve Instruction (NCIP). It describes work and findings to date and provides the reader with a sufficient overview of our efforts to date.

3. *Publications and Technical Reports.* Two articles have been accepted for publication. A technical report is also included. This report describes action research methods which were used to generate intervention techniques which were part of our final research efforts. Three other articles, which are in preparation, are not included in this section because of their preliminary state.

Section 1: Project Dissemination

This section provides a brief description of the overall project activities and dissemination efforts conducted over the last three years as well as activities which will continue in the near future after termination of funding. The journal article, book chapter, and technical report appear in Section 3 of this report.

Journal Articles and Book Chapters

Woodward, J., & Baxter, J. (in press). The effects of an innovative approach to mathematics on academically low achieving students in mainstreamed settings. Exceptional Children.

Woodward, J., Baxter, J., & Scheel, C. (in press). It's what you take for granted when you take nothing for granted: The problems with general principles of instructional design. In T. Scruggs & M. Mastropieri (Eds.), Advances in Learning and Behavioral Disorders. New York: JAI Press.

Journal Articles in Preparation and Technical Reports

Baxter, J., & Woodward, J. (in preparation). Problems of academic diversity in innovative mathematics classrooms: Can the teacher teach everyone?

intended audience: practitioners and researchers.

Woodward, J., & Baxter, J. (in preparation). Meeting the needs of academically low achieving students in innovative mathematics classrooms.

intended audience: practitioners and researchers.

Baxter, J., Woodward, J., Olson, D., & Kline, C. (in preparation). Action-based research on innovative mathematics instruction and students with special needs.

intended audience: practitioners.

Presentations at Major National Conferences

An Observational Study of Innovative Mathematics Practices. Pacific Coast Research Conference, Laguna Beach, California, 1995.

Presenters: Woodward and Baxter

The Informed Instruction Project. National School Board Association Technology + Learning Conference, Atlanta, GA, 1995.

Presenter: Woodward

Conceptual Approaches to Mathematics Through Technology. Association for Supervision and Curriculum Development. New Orleans, LA, 1996.

Presenter: Woodward

Inservice and Professional Development Presentations

- Nine inservices at Edison Elementary School, Eugene 4J School District, Eugene, Oregon 1993-4. (20 elementary school teachers)

  Presenters: Baxter and Woeste

- Informal Assessments Techniques for Everyday Mathematics: How to Assess the Progress of Academically Low Achieving Students. Central Kitsap School District, Silverdale, WA March, 1995. (50 elementary school teachers)

  Presenter: Woodward

- Instructional Strategies for Working with Academically Low Achieving Students. University of Chicago Mathematics Project (UCSMP) Inservice Training on Everyday Mathematics, Chicago, IL August, 1995. (50 elementary school teachers)

  Presenter: Woodward

Future Research, Dissemination and Professional Development

Even though the Informed Instruction Project (H180G20032) ended on March 30, 1996, a variety of activities stemming from the project will continue into the near future. First, one more study at Silver Ridge Elementary School will be conducted during the 1996-97 school year. The district will provide a support person to work with two third grade teachers in the attempt to study the deployment strategy developed during the project. This will be a naturalistic study which will examine how an individual acting as an instructional aide (or for that matter, a special educator teaming with two grade level teachers) can help maximize the instructional opportunities for mainstreamed students with learning disabilities and those at risk for special education in mathematics.

Second, research data collected over the three years will be analyzed and papers in preparation will be submitted for publication. Likely journals will be:

Elementary School Journal, American Educational Research Journal, Remedial and Special Education, and Arithmetic Teacher. The qualitative and cumulative nature of the work on the Informed Instruction Project has resulted in a longer time than usual in respect to distilling conducted research into publications for a professional research community.

Finally, we will continue to work with the Everyday Learning Corporation and UCSMP's National Science Foundation Project to disseminate instructional strategies for students in special education and other academically low achieving students. The Everyday Learning Corporation, publisher of Everyday Mathematics, conducts nine major training sessions throughout the country each summer. UCSMP has a four year grant to develop inservice trainers and materials for the Everyday Mathematics program. Our research findings will complement these efforts.

Section 2: Overview of Findings

The Informed Instruction Project: Preliminary Findings and Implications for Future Research in Mathematics for Students with Learning Disabilities

John Woodward

University of Puget Sound

Juliet Baxter

Educational Inquiries

November, 1995

Research for this project was funded by the US Department of Education, Office of Special Education Programs (H180G20032).

Overview of the Project

Over the last three years, the Informed Instruction Project has investigated different approaches to mathematics instruction for students with learning disabilities and those who are at-risk for special education services. The project, which is in its final year, has been funded by the US Department of Education, Office of Special Education Programs. Research over the three years has focused on two strands of inquiry: 1) the effects of sophisticated, computer-based diagnostic assessment system in aiding special education teachers remedy arithmetic misconceptions and 2) the effects of specific curricular interventions on these students in mainstreamed setting.

Early pilot studies and discussions with general and special educators conducted prior to this research suggested that the major emphasis of this project should be on the second strand -- innovations in mathematics curriculum at the elementary level. Detailed reasons for this are apparent in the summary of findings from our computer diagnosis research described immediately below.

Computer-Based Diagnosis of Misconceptions

The first strand of the Informed Instruction Project looked at the use of an expert system (TORUS) designed to detect student misconceptions in addition and subtraction. It culminated in a year-long, qualitative study of a highly skilled, special education teacher who used direct instruction curricula and TORUS to augment her for resource room math classes.

The TORUS Program

TORUS (Woodward, Freeman, & Howard, 1992) is a diagnostic system that was originally developed under an innovative assessment in technology grant from the US Department of Education, Office of Special Education Programs. Further support came from Apple Computers and Neuron Data, Inc., of Palo Alto, California.

The TORUS system was built upon previous technology-based assessment systems in arithmetic, specifically the BUGGY Project (Brown & Burton, 1978). However, TORUS was tailored to the characteristics of students with learning disabilities. Data on over 300 intermediate and middle school students with learning disabilities from five sites were used as an empirical basis for designing the program.

For both addition and subtraction, the TORUS diagnoses provide: 1) total percent correct on a sheet of 25 problems, 2) the percent correct on specific types or "subtypes" of problems (e.g., subtraction problems with multiple carries, borrows from zeros), and 3) evidence of chronic errors or "bugs." For example, certain answers to problems involving borrowing and zeros are highly predictable (e.g., middle zeros a minuend or top number of a subtraction problem like 2005). Our analyses of middle school students' worksheets yielded 15 common addition bugs and 40 common subtraction bugs. TORUS can detect these chronic errors and as a technology assessment system, it reflects the move toward a deeper understanding of the learner and the categories or patterns of errors she or he is making (see Woodward & Howard, 1994).

<u>TORUS Research</u>

In the preliminary phase of the project, interviews with general and special educators as well as math specialists suggested that TORUS alone may do little to improve conceptual understanding. In fact, Richard Burton, one of BUGGY's original programmers, described an intervention study (Friend & Burton, 1981) where he and his colleagues showed teachers how to interpret and use BUGGY diagnoses. Burton and his colleagues felt this information would provide a better foundation for teaching subtraction more conceptually.

The results of Burton's study, unfortunately, were discouraging. He and his colleagues concluded that more significant curricular changes were required if teachers were to alter their mathematics instruction in any substantive way.

Ironically, the increased access to technology (specifically, calculators) made the extensive practice required to "learn" the common subtraction algorithm less necessary, thus undercutting the utility of a program like BUGGY. That is, with less of a need for computing multi-digit subtraction problems by pencil and paper -- procedures which were as susceptible to casual errors as they were to systematic bugs -- BUGGY's utility decreased in a corresponding manner.

In contrast, many special education researchers have long felt that generic, research-based practices such as direct instruction provide an optimal method of instruction. It has been argued that instructional techniques along with specific methods for designing curriculum exist irrespective of a particular content area. To be sure, the content area (e.g., math, science, writing) ultimately influences which principles are used. Nonetheless, these researchers argue that curriculum design principles provide an independent framework for how best to teach concepts and problem solving as well as sequence material in the most efficient manner.

In math, direct instruction tends to emphasize: 1) mastery of facts and computational algorithms 2) extensive practice on one or two step traditional word problems and 3) key or clue word methods for solving these problems. It is hoped through highly systematic, teacher-directed instruction, a student's general math abilities (e.g., conceptual understanding of topics and operations, problem solving) will be enhanced. As a behavioral approach, the ultimate intent is to teach for generalization. This orientation is significantly at odds with contemporary research in elementary school mathematics as well as the 1989 NCTM Standards. Furthermore, contemporary work in cognitive psychology questions the basic notion of teaching skills in a decontextualized manner with the hope for transfer within or outside of a domain (see Lave, 1988; Prawat, 1991).

The first research study in the Informed Instruction Project provided the opportunity to determine the effectiveness of direct instruction in teaching concepts and operations. It also enabled researchers to examine the impact of TORUS on teacher behavior and student learning. We chose an exemplary special education teacher, one with considerable experience and skill in direct instruction, for this study. The research was conducted over the course of an entire school year. Over this time we documented the effects of TORUS (and the Direct Instruction program Corrective Subtraction) in more detail than what would have been available through a brief experimental study with intervention and comparison groups.

The research focused on two variables: 1) the extent to which a program like TORUS would aid the teacher in identifying and remedying misconceptions at a conceptual as well as procedural level (i.e., would her remediation stress basic concepts like place value and regrouping over massed practice on the type of problems with which students were having difficulties) and 2) how keyword methods such as the ones used in Corrective Subtraction generalize to other types of word problems.

For example, direct instruction programs like Corrective Subtraction contain many problems of the form: *Jane had 45 tennis balls. She gave away 13 balls to her friend. How many balls did Jane have left?* Students are taught to look for words like *gave away* and determine that it implies subtraction. Key words act as synonyms for operations. We designed the study to examine the extent to which this method would enable students to solve problems that: 1) contained conflicting language, 2) were written without keywords, or 3) required students to represent or explain the problem using manipulatives such as unifix cubes. A conflicting language problem, for example, uses keywords, but their meaning is other than what the keyword typically connotes. For example, "Jane had 45 more tennis balls than Sue. Sue had 13 balls. How many balls did Jane have?" In this problem, subtraction is required even though *more* typically connotes addition.

Through the study, a research assistant gave the participating special education teacher bi-monthly TORUS assessments of student performance. Each report, which indicated levels of mastery, subtype performance, and lists of bugs (along with examples), was explained to the teacher for each student who participated in the study. As new students were added to her math class throughout the year, new TORUS reports were generated. When students reached the end of the Corrective Subtraction program, they were given the Individual Mathematics Assessment (IMA), which contained an array of subtraction word problems. The examiner read each problem to the student and prodded or prompted the student when necessary. All sessions were tape recorded and transcribed for qualitative analysis.

Findings from the bi-monthly TORUS reports and the IMA essentially confirmed the main hypotheses of the study. TORUS reports did assist the teacher in addressing specific problem-type errors and bugs. Students were eventually able to reach a mastery level of performance on the TORUS assessments. However, a more conceptual understanding of subtraction was missing.

Analyses of student IMA protocols suggests extremely limited problem solving skills. Students were generally able to solve subtraction problems where the key words matched the appropriate operation. However, they generally erred in conflict language problems and had little or no capacity to solve more complex, ill-defined problems. When asked how they solve word problems, they were prone to talk about the subtraction algorithm rather than a problem solving strategy.

In the end, researchers concluded that when used in these circumstances, TORUS would likely become a vehicle for streamlining rote computational practice. Rather than enhance a greater conceptual understanding of subtraction, it would lead to a "putting out brush fires" approach where teachers would merely instruct a student to a point of mastery on those problem subtypes which were causing him or her difficulty. Past data on middle school students with learning disabilities indicates that students rarely reach or sustain this level of mastery.

As Richard Burton suggested at the onset of this study, achieving a greater emphasis on concepts and meaningful problem solving requires an entirely different curricular and pedagogical orientation. Findings from this study and prior research on middle school students in special education (Woodward, 1992) -- students being taught with traditional remedial or direct instruction programs -- suggested that the Informed Instruction research would be more productive if it concentrated on the innovate mathematics curricula activities described in the original proposal. Consequently, research conducted over the remainder of the three years focused on research-based, innovative curricula.

Curricular Intervention Research

While the term "innovative approaches" may connote a range of curriculum and teaching techniques, its use in this project is more specific. Researchers selected a curriculum that not only represented the intent of the 1989 NCTM Standards, but reflected significant research and development efforts. For this reason, we chose the Everyday Mathematics from the University of Chicago Mathematics Project (UCSMP). The background of this program, as well as the characteristics that differentiate it from typical special education curricula, are described in the next section.

The UCSMP Everyday Mathematics Program

Everyday Mathematics reflects over six years of work by mathematics educators at UCSMP. Research and development efforts, which now exceed $20 million, are funded by grants from the National Science Foundation as well as several major corporations.

Initially, developers translated mathematics textbooks from over 40 countries. Comparative analysis of elementary school texts indicated what many in this country had long suspected: the United States had one of the weakest mathematics curricula in the world. Important mathematical concepts were taught too slowly, tasks surrounding concepts (e.g., measurement, geometry) were too simplistic, and there was too much repetition.

To remedy these problems, developers at UCSMP created a curriculum that de-emphasized computations and changed the way concepts were reintroduced. Major concepts have been sequenced across grade levels so that when they reappear they are presented with greater depth. This structure has been a commonly cited feature of Japanese mathematics curricula.

Everyday Mathematics teaches problem solving in a notably different fashion. Unlike traditional math word problems, which are often conducive to a key word approach, problems or "number stories" are taken from the child's everyday world or from life science, geography, and other curriculum areas. Developers are in strong agreement with other mathematics educators in their view that students come to school with informal and intuitive problem solving abilities. Consequently, the curriculum designers draw on this knowledge as a basis for math student-centered problem solving exercises. Students are encouraged to use or develop a variety of number models which display relevant quantities (e.g., total and parts; start, change, end; quantity, quantity, difference) to be manipulated in solving these problems. While the third grade level of

Everyday Mathematics is rich in problem solving, very few of the exercises are of the one and two step problems which commonly appear in traditional commercial curricula for general and special education students.

Mathematical activities such as temperature or sunrise and sunset are extended over a significant portion of the school year. Data are taken from thermometers or daily newspapers and analyzed at regular intervals to determine averages, discuss negative numbers, or to graph data. Math "games" are offered as a means of providing automaticity fact practice or for reinforcing major concepts. For example, two students draw from a deck of cards and place each card in one of eight slots on a board. The goal is to create the largest number. Developers suggest that this activity reinforces an understanding of place value in a game-like context.

Finally, the program stresses conceptual development, as indicated by the vocabulary taught as a central part of each instructional unit. Throughout the year, students learn about arrays and factors as they study multiplication and mean, median, mode, and randomly sampling as they measure and collect data.

The Everyday Mathematics program emphasizes a series of important NCTM Standards. Students spend a considerable amount of time identifying patterns, estimating, and developing number sense. Multiple solutions for problems are encouraged and discussed. Finally, an array of math tools and manipulatives -- calculators, scales, measuring devices, unifix cubes -- are considered an important part of the daily lessons.

This program, like other emerging university research-based approaches, is fundamentally different from typical special education curricula. Recent literature characterizes the common special education model as a combination of behavioral analysis, mastery learning, and some limited form of direct instruction. In many instances, teachers merely provide additional practice worksheets when students are having problems with a particular concept or operation. Most of all, the dominant concern in special education mathematics instruction is a mastery of facts and computational algorithms. As in the case of the direct instruction program described in the TORUS study, traditional word problems are often taught using a keyword approach. Finally, calculators are rarely emphasized as a day-to-day instructional tool. All of these characteristics sharply contrast with the Everyday Mathematics approach.

Intervention Research

In order to develop an effective intervention model for mainstreamed students with learning disabilities and those who are at-risk for special education, it was essential to conduct lengthy, systematic observations of classroom practice. Observations helped identify the consistent problems general education teachers faced in meeting the needs of these students. Researchers also documented seemingly effective or "promising" practices.

This initial phase of research into innovative practices was followed by a phase of research where the problems identified in our observations were addressed and promising practices were gradually incorporated into classroom instruction. All of these efforts are best described as action research. The final model, which was piloted in three different classrooms, formed the basis of an experimental study to be conducted this fall.

Each phase of research will be summarized below. Research reports and dissemination papers are now being written for professional journals and other forums. Obviously, we have no data to report on the last study, which was being conducted as this dissemination paper was written.

Observational Research

Quantitative and qualitative research was conducted during the first year in three elementary schools at the third grade level. Nine classrooms and 205 students (n = 104 in the intervention schools and 101 in the comparison schools) participated in the study. All schools were selected on the basis of comparable demographic characteristics as well as similar attitudes the teaching staff had toward mathematics instruction. Two schools were selected as intervention sites by the fact that they were using the Everyday Mathematics Program. The third school in this quasi-experimental design was using Heath Mathematics, a traditional approach to the subject.

All third graders took the Iowa Test of Basic Skills (ITBS) and a special measure designed for this study, the Individual Mathematics Assessment (IMA), was administered to a stratified sample of 20 students in each condition. Total test and problem solving subtest scores on the ITBS were used as a basis for determining selection of students for the IMA. Seven high students (i.e., above the 67th percentile), six average students (between the 34th and 66th percentile) and seven low students (below the 34th percentile) were randomly selected. Items on the IMA covered a range of grade level topics, from pattern analysis to multistep addition and subtraction problems. As with the TORUS IMA, tests were individually administered, tape recorded, transcribed, and scored quantitatively with a rubric as well as analyzed qualitatively. The ITBS and the IMA were administered both in October and April.

Systematic qualitative observations and teacher interviews were also conducted throughout the year. Researchers focused on the instructional techniques that the five teachers in the intervention schools used to teach the Everyday Mathematics program as well as the overall impact of the approach on the lowest academic third of the students. These students either had a learning disability in math and/or reading or they were at-risk for special education services.

Analysis of the ITBS for the year favored the Everyday Mathematics program. When compared to the intervention school, total test results for this measure indicated no decline in performance, and significant differences in the area of concepts. The lack of a decline on the ITBS was important insofar as Everyday Mathematics placed relatively little emphasis on computational practice, and, in fact, stressed student-derived algorithms. Total group IMA results also significantly favored the Everyday Mathematics schools.

A more fine-grained, qualitative analysis of IMA results indicated, however, that the greatest benefits of the program were for the average ability students. Their answers to the spring IMA as well as the problem solving strategies they used most closely approximated the high achieving students.

Unfortunately, there were only modest changes in the low ability students. These students tended to move from an "I don't know" state to an unstructured, if not random, approach to the multistep word problems. The greatest deficiencies were not in computational ability (as indicated on the easier IMA problems), but in the logical - linguistic abilities needed to decompose complex problems into an appropriate schema. To be sure, weakness in computational skills, along with a reluctance to use a calculator as part of the problem solving process, became apparent on the more cognitively demanding tasks.

A traditional, task analytic interpretation of these students' poor performance (e.g., a firm grasp of the algorithm would have enabled these students to be better problem solvers) is an inadequate explanation. The difficulties these students face is best indicated through a sample problem from the IMA. This problem was part of the spring IMA. An alternative version of the problem involving witches, ghosts, and goblins was administered in the fall.

<u>IMA Problem 6 (spring version)</u>

Your school is collecting box tops for play equipment. Each fifth grader must bring in 35 box tops, each fourth grader must bring in 28 box tops, and each third grader must bring in 19 box tops.

How many box tops would be brought in by 97 third graders and 165 fifth graders?

Shifts in reasoning by students in the lowest academic third of the class are exemplified in the transcript below. Strategic behavior in the spring tends to rely on the numbers "just as they are presented" in the problem without a systematic effort to find appropriate categorical relationships. This pattern is in direct contrast to the average and high ability students at the intervention schools.

Problem 6 - Witches and Ghosts Fall	Problem 6 - Boxtops Spring
Student: About 304 Interviewer: About 304, How did you get that? Student: Guessed Interviewer: Can you tell me a little more how you got the answer? Is there anything you can use here (pointing to tool kit) to show me? Student: No, I just guessed.	Student: 1,562. Interviewer: How did you get that? Student: I plussed 35 plus 19 plus 28 plus 97 equals 1,562. Interviewer: How did you figure out to get those numbers? Student: (shows on pencil and paper) 35 19 28 + 97 1,562 <u>Notes</u>: Student thinks about it. Asks to use pencil and paper. On paper (See attached):

Observational research helps explain the marginal progress of the low ability students over the course of the year. These students required much more direct assistance than other students in the class. At the same time, they engaged in an array of avoidance behaviors during whole class discussions. For example, the lowest students frequently sat quietly and did not volunteer when teachers asked questions. They relied extensively on those around them to re-explain directions, to answer paired or small group tasks, and to complete workbook tasks.

Some in special education would likely attribute their performance to the "discovery-oriented" nature of innovative mathematics. Contrary to these expectations, though, all five teachers in the intervention schools would best be described as utilizing active or effective teaching principles (Brophy & Good, 1986). Daily lessons consistently followed the pattern of 1) reviewing previous material 2) framing the day's lesson (e.g., eliciting background knowledge, providing advanced organizers) 3) modeling, when appropriate, how to complete specific tasks 4) guided practice and 4) small group, pairs, or independent seatwork.

The only significant deviations from effective teaching techniques were on those occasions when teachers probed for more than one method for solving a problem (usually during the guided practice phase) as well as the general tendency only to call on volunteers. Otherwise, teachers tended to ask a mix of higher and lower order questions and maintain sufficient pace throughout the hour-long lesson. Perhaps the only lag time in the lesson occurred when students used manipulatives such as coins or unifix cubes.

Teachers were well aware, if not anxious, about the difficulties the "lower half" of their class was having with the lessons. This was evident in the informal conversations with these teachers throughout the year as well as end of the year interviews. Innovative curricula such as Everyday Mathematics de-emphasize computational practice. Consequently, the teachers had no ready anchor or measure of independent performance like the worksheets of traditional, basic skills curricula.

With the greater emphasis on concepts and non-traditional problem solving, difficulties and challenges emerged. This was most evident in the balance between an introduction or exposure to a concept (e.g., using rectangles to introduce arrays and multiplication) and those occasions when students were expected to have mastered a concept. Teachers frequently felt uncertain of what students were "supposed to know" and what they were just "exploring." Also, they didn't have taxonomies or rubrics to scale performance in any systematic fashion. Thus, they didn't know how to expand student discourse (e.g., move from one word answers to longer descriptions or explanations) or help students use a variety of representations to integrating their understanding of a concept.

Overall, dependent measures and weekly observations indicated that the Everyday Mathematics was generally successful for average and above average students. Its increased cognitive demands -- both in terms of conceptual understanding and problem solving -- required more time and attention than what one teacher in a classroom could provide on a day-to-day basis.

Action Research

Throughout the second year, researchers followed an action research model to remedy problems identified in the previous year's observation. Perhaps the most significant change was the addition of an instructional aide who could perform a variety of teaching duties. This allowed the teacher to create different forms of classroom organization. Most notably, the lowest third of the class was able to participate in concentrated, small group instruction for 20 minutes each day. This enabled the teacher to modify the type of problems used during the active teaching portion of the lesson as well as provide greater opportunities for participation and feedback.

The aide assisted throughout the period by actively monitoring and assisting low achieving students either during whole class instruction or small group/paired activities and independent seatwork. Monitoring entailed a variety of behavior management strategies from prompting students so that they would attend to the lesson or making sure that they had appropriate materials (e.g., workbook, ruler). Aides also reminded the low achieving students of the teacher's instructions or what was done during the previous day's lesson.

The instructional aide worked in conjunction with the teacher to increase participation and discourse. The aide reminded the low students of strategies or answers that they had practiced during small group work or in a previous lesson. The teacher, in turn, made sure to call on these students by modifying questions to ensure successful participation.

The teacher and aide also used small group or independent work time tutoring the lowest students on an individual basis. This frequently involved further, alternative explanations of a concept, use of manipulatives for demonstration purposes, and helping students articulate their understanding by rephrasing their statements.

A final technique for increasing discourse occurred at the end of the lesson each day. The teacher spent approximately 10 minutes discussing what students had learned from the lesson and their various activities (e.g., paired or small group work). She made sure to call on the lowest third of the students. Moreover, she worked at rephrasing student answers and extending their descriptions of what they had done during the independent or small group work. At first, the teacher tried to expand on one or two word answers to longer answers). Gradually, she worked toward more sophisticated forms of student discourse (e.g., explanation).

Data from the action research phase of the Informed Instruction Project suggests positive effects for the model. Day-to-day levels of participation increased dramatically. There were also positive changes in attitude and ability as measured by the "How I Feel About Math" inventory and the IMA. These findings, along with systematic observations throughout the action research phase of the project, suggested that the intervention model merited further investigation in the form of an experimental study.

Experimental Research

An experimental study will be conducted from September 1995, to March 1996. Students in two third grade classrooms will be randomly assigned to one of two conditions: an instructional assistance model described above and a comparison condition where an instructional aide performs more typical duties (e.g., working with individual students, grading seatwork). An array of dependent measures -- attitudinal as well as cognitive -- have been developed to assess the impact of the intervention. This study should provide some evidence for how best to serve students with learning disabilities and those at risk for special education in mainstreamed classrooms.

Tentative Conclusions

The intervention model which will be tested this fall is guided by the realization that complex forms of literacy such as those expressed in new approaches to mathematics require significant changes for the classroom teacher. At the very least, the teacher needs to provide academically low achieving students and those with learning disabilities greater opportunities to: 1) actively and continuously participate in classroom discussions, 2) succeed in non-traditional problem solving exercises, and 3) work in highly supervised settings where there are many opportunities for dialogue with and feedback from the teacher. Our year-long observational research suggests that this is exceedingly difficult for even highly skilled, veteran teachers when working alone with a classroom of 25 or more students with diverse academic abilities.

Our intervention model is one way of re-configuring classroom organization to address the needs of low students. It draws on the work of Robert Slavin (Slavin, Madden, Karweit, Livermon, & Donlan, 1990) and others who advocate adding instructional personnel during key academic periods of the day and re-deploying students across classrooms by grade (or ability) level. The resulting instruction involves small homogeneous groups and large or whole class heterogeneous groups. Small group instruction in our model is relatively fluid; that is, low achieving students may receive modified instruction depending on the task or, in the case of concepts which are relatively new to the entire class (e.g., geometry), average and high ability students are added to the small group. This is one way of reducing the potential stigma of the "same low group" of students.

The added personnel in our model -- an instructional aide -- is a proxy for many individuals who might be available for instructional purposes. Special education resource room teachers working with grade level teachers could also fulfill this role.

Differences with Traditional Instructional Methods

The proposed intervention model differs from traditional behavioral methods for teaching mathematics in a number of ways. First, the approach differs from hierarchical or task analytic models that stress mastery of facts and computational procedures and limited forms of problem solving. To be sure, a mastery of facts as well as some facility with computational algorithms is necessary. These goals can be accomplished through distributed practice (particularly in the case of facts) and conceptually-guided instruction. In other words, concepts should precede and accompany practice on algorithms. Furthermore, such practice should be limited and when possible, performed as homework.

Algorithms and concepts. The wide availability of calculators raise the question of whether or not algorithms should be taught in the traditional, most efficient manner. Alternative algorithms may promote greater conceptual understanding even though they are less efficient. In double digit multiplication, for example, the common algorithm inadvertently teaches students to mislabel the carries and confuse what kind of place value operations are being performed in the products below the line. This is evident in the following two digit by two digit operation:

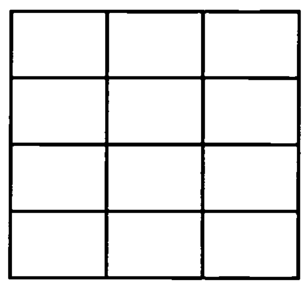
$$\begin{array}{r} 47 \\ \underline{\times\, 35} \\ 235 \\ \underline{141} \\ 1645 \end{array}$$

A more conceptually based algorithm would stress place value in the following manner.

$$\begin{array}{r} 47 \\ \underline{\times\, 35} \\ 35 \\ 200 \\ 210 \\ \underline{1200} \\ 1645 \end{array}$$

In all likelihood, students will actually compute problems of this kind in contexts where calculators are available. Thus, the more inefficient algorithm helps make explicit the role of place value.

In a related manner, multiple representations for a concept or topic are encouraged. Again, with multiplication students can be shown how multiplication applies in geometry and measurement. Everyday Mathematics uses the concept of arrays to teach multiplication, geometry, and measurement. A 4 x 3 array such as the one shown below is a geometric object (i.e., rectangle) that has been systematically subdivided. Each unit length on the perimeter serves as a basis for measurement and, by extension, for calculating area through multiplication. In this way, students can see the way common partitions (e.g., a row or column of squares) forms the basis for the operation and that multiplication is a more efficient form of repeated division.



4 x 3 = 12

Problem solving. A more fundamental difference involves the nature of mathematical problem solving. Traditional word problems are typically artificial exercises which contain key words. Students often solve these problems by quickly skimming for key words or performing the operation contained in the most recent set of worksheet problems (i.e., "If this unit and the worksheets were division, these word problems must be division"). Unfortunately, a key word approach teaches students to solve problems superficially. At best, they must make only a few discriminations which ultimately trigger the appropriate algorithm.

Two or three sentence word problems promote the illusion that students are extending their understanding of mathematical concepts when, in fact, the problems are conveyed in a relatively artificial manner. Most traditional word problems are unrelated to a child's world. Finally, this approach to problem solving also imparts the notion that problem solving is done quickly and with little conceptual effort. Highly contextualized problems which draw on number sense, alternative solution methods, and mathematical concepts create great difficulties for students who would otherwise be highly proficient at traditional word problems. This has been a long-standing criticism of commercial textbooks (and by extension, special education methods) by the NCTM and math researchers for almost two decades (Schoenfeld, 1992).

In those instances where Everyday Mathematics provides two or three sentence problems, students are encouraged to create number models and alternative representations of the problem (e.g., drawings, diagrams). Furthermore, key words are controlled. However, most problems in a program like Everyday Mathematics, are non-traditional. Students work from maps, collect data, measure and calculate, or estimate. Grocery store scenarios are created so students can discuss different ways of making change and hence, develop number sense. These contexts create a basis for further conceptual development.

Problem solving is much more closely related to the student's every day world, and problems often take more than five minutes to complete. Unlike the traditional model of problem solving, which presents limited exercises of one kind with the hope of generalization, this approach immerses students in mathematical problem solving with the intent of building a rich, flexible schema.

Discourse. Perhaps the most striking finding from the first year of research involved the nature of classroom discourse. Average and high ability students tended to engage in many more classroom discussions than the lowest third of the students. The students in the top two-thirds of the class tended to answer more questions, pose more mathematical problems and solutions, and were generally more willing to discuss their hunches or insights. Differences between students were also evident on the IMA described earlier.

With the reduced need for computational practice (along with an extended time for mathematics), more time should be devoted to structured student discussions. Academically low achieving students, in particular, need assistance in a continuum of linguistic skills. Many are reluctant to talk in class and need many opportunities to extend their descriptions of strategies, answers to problems, etc. Over time, these students need to move from describing to explaining and defending their ideas. For elementary students, this continuum of logical - linguistic development will span several grades. Discourse strategies for developing these abilities in students are a core element or our future research efforts.

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Section 3: Publications and Technical Reports

The Effects of an Innovative Approach to Mathematics on Academically Low Achieving Students in Mainstreamed Settings

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Abstract

This article presents the results of a year-long study of an innovative approach to mathematics and its impact on students with learning disabilities as well as those at-risk for special education. There is a considerable interest in the field regarding current mathematics reform, particularly as it reflects the simultaneous and conflicting movements toward national standards and inclusion. However, most commentary on mathematics reform tends to be analytical in nature, and criticisms largely have been directed at the NCTM *Curriculum and Evaluation Standards* alone. Empirical findings reported here suggest that innovative methods in mathematics are viable for students with average and above average academic abilities. Results also indicate that students with learning disabilities or those at-risk for special education need much greater assistance if they are to be successful in mainstreamed settings. Critics of the NCTM *Standards* may regard these findings as confirming their suspicions about the reform. However, it is the success of the majority of students in this study that raises significant questions about commonly advocated methods in special education for teaching mathematics.

Introduction

The National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* (NCTM, 1989) reflect a high level of consensus within the mathematics education community about current and future directions of the discipline. The *Standards* are intended as a policy document for professionals in mathematics education as well as a vision of excellence, one which attempts to move the field well beyond the minimal competencies of the back-to-basic movement of the 1980s (Bishop, 1990).

While the*Standards* are the most visible component of math reform for many, particularly special education researchers, it should be noted that they reflect almost two decades of research, curriculum development, and related policy documents by the NCTM and other professional organizations. The research, which draws extensively on cognitive psychology and child development (e.g., Gelman & Gallistel, 1978; Grouws, 1992; Hiebert, 1986; Putnam, Lampert, & Peterson, 1990), is a considerable enhancement of the knowledge base which led to the "New Math" movement of the early 1960s.

Recent mathematics education research has also yielded detailed analyses of elementary and secondary math concepts (Carpenter, Fennema, & Romberg, 1993; Hiebert & Behr, 1988; Leinhardt, Putnam, & Hattrup, 1992). More recently, a series of research-based curricula have emerged (e.g., *Everyday Mathematics*, Bell, Bell, & Hartfield, 1993). Finally, policy documents such as *An Agenda for Action* (NCTM, 1980) and Everybody *Counts* (National Research Council, 1989) consistently argued for significant changes in the role of computational practice and the type of problem solving found in most commercial textbooks, as well as an increased role for technology.

Despite the depth of the reform in mathematics, special educators have most of their concern over the potential impact of the NCTM *Standards*, which they feel reflect a wider, national standards movement. There is little mention in the *Standards*, or for that matter, *Goals 2000*, regarding the role of students with disabilities or how their unique needs will be addressed. For example, the *Standards* press for higher student performance through more challenging curriculum: specifically, a greater emphasis on conceptual understanding and having students solve longer, less well-defined problems. Pushing *all* students to achieve higher academic goals would seem to directly clash with the move to include more and more special education students in general education classrooms where little if any additional support is provided (Carnine, Jones, & Dixon, 1994; Fuchs & Fuchs, 1994). After all, problems accommodating students with learning disabilities in *traditional*, general education classrooms are well documented in recent case study research (Baker & Zigmond, 1990; Schumm et al., 1995).

Special educators also question the curricula and pedagogy advocated in the *Standards*. Newly proposed methods and materials are often at odds with the effective teaching model which was articulated by Good and his colleagues (Good & Grouws, 1979; Good, Grouws, & Ebmeier, 1983) and later embraced by mathematics researchers in special education (Darch, Carnine, & Gersten, 1984; Kelly, Gersten, & Carnine, 1990; Gleason, Carnine, & Boriero, 1990). Some special educators suggest that the instructional methods and materials proposed in the *Standards* are particularly ill-suited to the needs of academically low achieving students and those with learning disabilities because they are "too discovery-oriented" (e.g., Carnine, et al., 1994; Hofmeister, 1993). They also suggest that the *Standards* are nothing more than a recycling of old reforms (i.e., the New Math movement of the early 1960s). Finally, Hofmeister (1993) argues at length that the *Standards* are elitist, that what is generally proposed has little or no empirical validation.

Even those special educators who appear more sympathetic to the *Standards* exhibit difficulty and confusion when attempting to translate the mathematics research of the 1980s into a special education framework. Gersten, Keating, and Irvin (1995), for example, misconstrue constructivist discourse as teacher-directed example selection. Also, traditional cognitive interpretations of student misconceptions in arithmetic are uncritically equated with constructivist theory.

Without systematic evaluation, the ways in which current mathematics reform might "play out" for students with learning disabilities or those at risk for special education is likely to remain speculative or only at the level of policy debate. At the very least, such evaluation would help determine whether any problems with innovations in mathematics rest in the nature of the curriculum and pedagogy or the more traditional problem of educating students with learning disabilities in mainstreamed environments.

Purpose of the Study

The purpose of this study was to examine the effects of an innovative approach to mathematics instruction on academic performance of mainstreamed students with learning disabilities and academically low achieving students who are at risk for special education. This research was part of an extensive study of teachers in three elementary schools, two of which were in the third year of using a new, university-based math reform curriculum. Nine third grade classrooms were the focus of systematic observations, teacher and student interviews, and academic assessment. Quantitative as well as qualitative data were collected in the attempt to triangulate on the effects of innovative curriculum and teaching techniques on target students (see Patton, 1980). Because of the extent of the data, this report will concentrate on the academic growth of students over the course of the year. Observation and interview data are described elsewhere (see Baxter & Woodward, 1995).

Method

Participants

Teachers and schools. The participants in this study were nine third grade teachers and their students in three schools located in the Pacific Northwest. The two intervention schools were selected because they were using the *Everyday Mathematics* program (Bell et al., 1993), which is closely aligned with the 1989 NCTM *Standards*. A third school, which acted as a comparison, was using *Heath Mathematics* (Rucker, 1988), a more traditional approach to mathematics. Five third grade teachers taught in the two intervention schools and four in the comparison school.

The schools were comparable along many variables. All were middle class, suburban elementary schools with similar socio-economic status (determined by the very low number of students on free or reduced lunch), as well as other demographic information provided by the districts.

Schools were also comparable in the general beliefs held by the staff regarding mathematics instruction. First through fifth grade teachers at each school completed the Mathematics Beliefs Scale (Fennema, Carpenter, & Loef, 1990), an updated version of the Teacher Belief Scale (Peterson, Fennema, Carpenter, & Loef, 1989). This measure has been used in a number of studies investigating the effects of innovative mathematics instruction. Differences between the staffs at the intervention and comparison schools were non-significant ($\underline{t}_{(1,41)} = .94$; $\underline{p} = .36$) on this scale.

Students. A total of 104 third grade students at the two intervention schools participated in this year long study. At the comparison school, 101 third graders participated. Forty-four students from the intervention and comparison schools were excluded from the data analysis because they were not present for either the pretesting or posttesting. Twelve students were classified as learning disabled on their IEPs, and they were receiving special education services for mathematics in mainstreamed settings. Seven students with learning disabilities were in the intervention schools and five were in the comparison school.

It should be noted that interviews with teachers in all three schools indicated that more students could have been referred for special education services in mathematics but were not for a variety of reasons. Some teachers mentioned that the special education teacher primarily served low incidence students (e.g., autistic, students with physical disabilities) or students who had reading problems. There was "little room left" to serve students for math.

Three teachers in the intervention schools chose not to refer students, and in two cases, they retained students in the general education classroom for mathematics instruction -- because they did not want to contend with the logistical problems of sending students out for mathematics at important or inconvenient times in the day. These teachers were also skeptical of the quality of mathematics instruction in the special education classroom. They felt that the traditional direct instruction approach to the subject did little to teach students the mathematics they needed for success in future grades.

Consequently, a wider pool of students was selected as a focus for this study. The mathematics subtest of the ITBS, administered in October, was used as a basis for further identifying students who were at-risk for special education services in mathematics. The 34th percentile was used as a criterion for selecting these students. In addition to the seven students with learning disabilities at two intervention schools, nine other students were identified based on total subtest performance on the ITBS. At the comparison school, another 17 students were identified. This resulted in a total of 16 students at the intervention schools and 22 at the comparison school who were considered academically low achieving in mathematics or were identified as having a learning disability in mathematics.

Materials

Intervention schools curriculum. As mentioned earlier, the two intervention schools in this study were using the *Everyday Mathematics* program. This program reflects over six years of development efforts by mathematics educators at the University of Chicago School Mathematics Project (UCSMP). The project has been funded by grants from the National Science Foundation as well as several major corporations. Initially, program developers translated mathematics textbooks from over 40 countries. Comparative analysis of elementary school texts indicated that the United States had one of the weakest mathematics curricula in the world (Usiskin, 1993). Among the many shortcomings, important mathematical concepts were taught too slowly, tasks surrounding concepts (e.g., measurement, geometry) were too simplistic, and there was too much repetition (Flanders, 1987).

To remedy these problems, developers at UCSMP created a curriculum that de-emphasized computations and changed the way concepts were reintroduced. For example, when major concepts reappear later in the year or in the next grade level, they are presented in greater depth. This structure is common to Japanese mathematics curricula (Stevenson & Stigler, 1992; Stigler & Baranes, 1988).

The UCSMP materials also emphasize innovative forms of problem solving. Unlike traditional math word problems, which are often conducive to a key word approach, problems or "number stories" are taken from the child's everyday world or from life science, geography, and other curriculum areas. The program developers are in strong agreement with other mathematics educators (e.g., Carpenter, 1985) in their view that students come to school with informal and intuitive problem solving abilities. The developers drew on this knowledge as a basis for math student-centered problem solving exercises. In these exercises, students are encouraged to use or develop a variety of number models which display relevant quantities (e.g., total and parts; start, change, end; quantity,

quantity, difference) to be manipulated in solving these problems. While the third grade level of *Everyday Mathematics* is rich in problem solving, very few of the exercises consist of the one- and two-step problems that commonly appear in traditional commercial curricula for general and special education students.

Automaticity practice is achieved through the use of math "games." Students roll dice and add or subtract the numbers as a way of practicing math facts. Concepts are also developed through games. For example, two students alternate drawing cards from a deck and place each card in one of eight slots on a board. The goal of the game is to create the largest number eight-digit number. Developers suggest that this activity reinforces an understanding of place value in a game-like context.

The *Everyday Mathematics* program emphasizes a series of important NCTM *Standards*. Students spend considerable time identifying patterns, estimating, and developing number sense. The are encouraged to come up with multiple solutions for problems. Finally, the students are taught to use an array of math tools and manipulatives (e.g., calculators, scales, measuring devices, unifix cubes), and these materials play an important role in daily lessons.

Comparison school curriculum. The comparison school used the *Heath Mathematics Program.*, a traditional approach to mathematics. Lessons are structured around a systematic progression from facts to algorithms with separate sections on problem solving. Facts and algorithms are taught through massed practice, and students can be assigned as many as 50 facts and 20 to 30 computational problems at a time. Story problems involve one or two sentences and are generally of one type (i.e., they are directly related to the computational problems studied in the lesson or unit). Unlike the *Everyday Mathematics* program, there is far less emphasis on mathematical concepts and a much greater focus on computational problems. Teachers in the comparison school often supplemented the *Heath* program with worksheets containing more facts, computational problems, and occasional math exploration activities.

Procedures

Observational, interview, and academic performance data were collected over the 1993-94 school year. All third grade students in the three participating schools were administered the mathematics subtest of the Iowa Test of Basic Skills during the third week in September and again in the last week of April. In addition to this traditional measure of mathematics achievement, a stratified sample of third graders was given an innovative test of problem solving ability. ITBS problem solving subtest and total test scores were used as a basis for randomly selecting students in the intervention and comparison schools. ITBS scores were matched and t-tests were performed to determine comparability of the samples. This process continued until there were non-significant differences between the intervention and comparison groups ($t_{(1,38)} = .80$; $p = .38$ for problem solving; $t_{(1,38)} = .11$; $p = .75$ for total test score). The Informal Mathematics Assessment, which is described below, was administered to a total of 20 students in the two intervention schools and 20 comparable students in the comparison school during mid-October and again during the first week of May.

The nine participating teachers were systematically observed two to three times per week throughout the course of the year. Researchers interviewed the teachers informally during the year and formally in June at the end of school. Details of the observational instruments and findings as well as the interviews can be found elsewhere (see Baxter & Woodward, 1995).

Measures

Two different measures were administered to assess the effects of the intervention. The third grade level (Form G) of the Iowa Test of Basic Skills was used as both a pretest and as a posttest. The norm referenced test has well documented reliability and validity. It is a highly traditional, multiple choice form of assessment which measures computations, concepts, and problem solving skills.

The second measure, the Informal Mathematics Assessment (IMA), was an individually administered test of problem solving abilities. The intent of this measure was to examine the problem solving processes or strategies a student used in deriving an answer, as well as the answer itself. In this respect, it is consistent with the call for assessment which is more closely aligned with math reform and the NCTM *Standards* (Romberg, 1995). Students were also given a range of mathematical tools and representations which they were encouraged to use as part of the problem solving. The IMA "tool kit" included a calculator, ruler, paper and pencil, poker chips, and number squares with ones, tens, and hundreds values.

The six items on the test were based on an analysis of third grade mathematics texts, innovative materials which subscribe to the 1989 NCTM *Standards* as well as more traditional texts. In order to prevent fatigue and possible frustration, particularly with academically low achieving students, the items on the IMA were relatively brief, and the examiner read each one to the student. While the IMA took approximately 15 minutes to administer, students were given as much time as they wanted to complete each item. Alternate form reliability for the pre- and posttest versions of this measure was .87.

Figure 1 presents a word problem from the IMA. As with other word problems on the test, it was written to exclude key words (e.g., *each* and *every* often are taught as key words which signify multiplication or division). After the examiner read the problem to the student, s/he carefully noted if the student reread the problem, what calculations were made, and what tools or manipulatives were used. Finally, s/he asked the student to, "Tell me how you got that answer." This form of inquiry has been shown to be a valid method of determining how young children solve mathematics problems (Siegler, 1995). All sessions were tape recorded and transcribed for later scoring and qualitative analysis.

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insert Figure 1 about here

Individual student protocols were scored with a rubric which was analytically derived from the NCTM *Standards* and related literature on innovative mathematics assessment (Lesh & Lamon, 1992). A five point scale was used for each item, with the highest score reflecting both the quality of the student's answer as well as the process used to derive the answer. Inter-rater reliability for scoring the student protocols was .93.

Finally, the IMA protocols were subjected to a categorical analysis. Researchers examined student answers in an effort to classify different kinds of problem solving behavior. The extent to which students used manipulatives provided in the tool kit (particularly paper, pencil, and calculators) and the strategies they used to solve problems (e.g., guessing, using numbers provided in the problem in random order, decomposing problems into subunits) were analyzed. Inter-rater reliability for the categorical analysis was .88.

Results

Data for this study were analyzed quantitatively and qualitatively. The quantitative data provided a broad framework for gauging the relative changes in academic performance for students at the intervention and comparison schools. This was particularly important as two different types of academic measures were used to assess growth in mathematics. The protocols from the IMA, along with classroom observations and teacher interviews enabled a qualitative analysis of the effects of the innovative curriculum on students with learning disabilities and academically low achieving students.

The ITBS

The ITBS functioned as a traditional measure of achievement. Pretest scores from the fall for the total test and all subtests were used as covariates in an Analysis of Covariance (ANCOVA). Results are presented for the total sample and the three ability groups.

Total sample. Results of the ANCOVA show a significant difference between groups ($F_{(1,202)} = 29.12$, $p < .001$) on the concepts subtest, favoring the intervention group. All other differences were statistically non-significant. Table 1 provides descriptive statistics for the two groups on the total test and all three subtests. Generally, students at the two intervention schools indicated mixed growth over the year as measured by the ITBS. Mean percentiles for fall and spring indicate that total test performance was stable, with noticeable increases in the area of concepts and slight to considerable decreases in computations and problem solving, respectively. The comparison students declined slightly over the course of the year in all areas.

insert Table 1 about here

Analysis by ability group. ANCOVAs were performed in a similar manner for students at the three different ability levels as determined by the total test score on the ITBS in the fall. Academically low achieving students, which included the 12 mainstreamed students with learning disabilities in mathematics, scored at or below the 34th percentile. Average ability students scored from the 35th to the 67th percentile, and high ability students scored above the 67th percentile.

Results of the ANCOVAs for the academically low achieving students indicated non-significant differences for the total test and all three subtests. Table 2 provides descriptive statistics for these two groups of students who scored below the 34th percentile in the fall on these measures. In general, students in both schools showed modest improvement. The most dramatic gains were in problem solving for the intervention students and in total score for the comparison students.

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insert Table 2 about here

ANCOVA results for average ability students were significant only in the area of concepts. Like the total sample comparisons, the results favored the intervention students ($F_{(1,66)} = 8.05$, $p < .01$). All other differences were non-significant.

For the high ability students, ANCOVA results indicated significant differences favoring the intervention students on concepts ($F_{(1,95)} = 12.75$, $p < .001$) and problem solving ($F_{(1,95)} = 5.12$, $p = .03$). Descriptive statistics for average and high ability students for the intervention and comparison groups on these measures are provided in Table 3.

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insert Table 3 about here

<u>Informal Assessment of Mathematics (IMA)</u>

An ANCOVA was performed on spring test results of the IMA for the total sample of students tested (i.e., 20 per condition). The fall IMA test scores were used as a covariate. Results strongly favor students in the intervention group ($F_{(1,37)} = 9.85$, $p < .01$).

Data were further analyzed by ability group. Due to the small sample sizes, further ANCOVAs were not conducted for high, average, and low ability groups. Instead, those data are presented descriptively in Table 4 along with the descriptive data for the total sample. Data for the three ability groups are also presented graphically in Figure 2. Data suggest that the greatest effects, at least by ability, were for the average students (i.e., those between the 34th and 67th percentile).

insert Table 4 and Figure 2 about here

Qualitative analysis of IMA protocols. The primary purpose of this year-long case study was to investigate the effects of an innovative curriculum like *Everyday Mathematics* on students with learning disabilities and those at-risk for special education. Therefore, protocols of all of the students in the intervention school who were given the IMA were carefully analyzed along a variety of dimensions.

Protocols were first examined categorically using constructs associated with the scoring rubric as well as the theoretical guidelines used to develop the IMA (e.g., those emanating from the 1989 NCTM *Standards*; recent research, particularly on innovative assessment in mathematics). Transcribed protocols and examiner notes taken during the individualized administration of the IMA enabled researchers to determine the extent to which students used manipulatives, calculators, paper and pencil, and the "reasoning" used to derive answers to specific problems.

Categorical analysis of protocols by ability groups across time indicated some similar behavior among all of the students. There were no discernible differences, for example, in the use of manipulatives as part of the problem solving process. By spring, all students tended to increase their use of paper and pencil for problem solving. The extent to which students in different ability groups used calculators remained constant, with high ability students using calculators over twice as frequently as academically low achieving students (71% versus 29%).

The most noticeable differences between students of different academic abilities were evident in the way students reasoned out problems, particularly the longer, more complex word problems shown in Figure 1. Three distinct categories of student reasoning emerged from the protocols which can be related to the problem solving literature in mathematics. These categories are shown in Figure 3 below.

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insert Figure 3 about here

The first category of Confusion and Uncertainty most directly pertains to students with learning disabilities and other academically low achieving students. As the data indicate, these students continued to guess, merely repeat numbers presented in the problem, or quickly respond, "I don't know," once the examiner finished reading the problem. Even with prompts or gentle attempts to get them to work a part of the problem, the students often appear to have little or no framework for simplifying a problem. Average ability students are far less likely to react this way by spring.

If there was any shift in this categorical behavior among the academically low achieving students, it was to move from giving up on the problem in the fall to an attempt to use numbers in the problem, albeit incorrectly in the spring. Figure 4 below presents a protocol for Problem 6 in the fall and its alternate version in the spring. The spring version of the problem, not shown in Figure 1, describes the collection of box tops for playground equipment. Like the fall version of the problem, students are given extraneous information and if they answer it correctly, they generally do so in three steps. The correct answer is 7618 box tops.

Figure 4 is a protocol of a mainstreamed student with learning disabilities in one of the intervention schools. The shift in the way he works the problem reflects a common pattern found among the lowest third of students: numbers presented in the problem are used, but with no association to the correct operations or categories. To solve the problem correctly, the student would need to 1) multiply the 19 box tops times the 97 third graders 2) multiply 35 box tops times the 165 fifth graders and 3) add the two products together. As the protocol indicates, the student with learning disabilities uses relevant as well as irrelevant information (e.g., 28 box tops for fourth graders) in the linear order presented in the problem.

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insert Figure 4 about here

In contrast to the academically low achieving students, average and high ability students spent more time conceptualizing the IMA problems before they worked them. For example, when working Problem 5 presented in Figure 1, many students used an "if-then" logic to talk through the problem prior to computing it on paper or using a calculator. This verbal restatement served as an important way to mediate what would have otherwise been an immediate and incorrect answer (usually in the form of adding or multiplying the distance from home to school twice, ignoring the intermediate 238 steps of walking back home to get the book). Moreover, Problem 6 was a clear occasion for high ability students (and many average students by spring) to carefully discern the relevant information from the problem and divide it into subproblems. Again, students restated the problem verbally in a simplified form as they worked it on paper or used a calculator. Both the conditional logic and the tendency to clearly decompose a problem into relevant subproblems was missing with the academically low achieving students.

Discussion

The results of this study suggest that the innovative curriculum benefited the majority of students in the intervention schools. Quasi-experimental comparisons indicated no overall decline in ITBS total test scores for the entire sample. In fact, most intervention students maintained or significantly improved performance levels on ITBS subtests directly related to the design of the intervention curricula (i.e., concepts and problem solving for average and higher ability students).

Improved performance was also evident on the IMA alternative assessment, a measure which is more closely aligned with recent reforms in mathematics. Quantitative and qualitative changes on the IMA were particularly evident for average achieving students at the intervention schools. They tended to more closely approximate the behavior of high achieving students in their ability to restate and decompose problems as well as use calculators as an integral part of problem solving.

Some mathematics reformers (e.g., Romberg, 1995) may view these findings as highly encouraging insofar as performance at the intervention school was not undercut by a lowering of scores on traditional measures. The findings from the IMA in this study tend to complement overall trends in the ITBS data. As Romberg and other would argue, an innovative form of assessment like the IMA is critical in documenting the varied and more subtle effects of mathematics reform.

As for students with learning disabilities and their academically low achieving peers, data from this study indicate only marginal improvement in their learning. Quasi-experimental results even suggest that students at or below the 34th percentile in the comparison school made more dramatic gains in total test performance on the ITBS total test (i.e., from the 20th to 30th percentile versus 24th to 26th percentile) and ITBS Computations subtest than similar students at the intervention schools. Surprisingly, low achieving students in both intervention and comparison schools made impressive gains on the problem solving subtest of the ITBS, at least in terms of percentile change.

Changes on the IMA for these students were much more modest, particularly for students in the comparison school where their mean performance over time remained at the same 40 percent correct level. Low ability students at the intervention school fared better, but their gains were not comparable to average ability students. Spring scores were still below 50 percent correct on this measure. Moreover, qualitative analyses of the data indicate that these students still exhibited high levels of confusion and uncertainty when answering many of the IMA problems, and tended to just repeat numbers rather than conceptualize and logically simplify complex problems. Unlike their average and above average peers, they struggled to incorporate calculators into their problem solving process.

Although some might interpret these data as supporting special educators' criticisms of the current mathematics reform, we hesitate to do so. In fact, the general success of students at the intervention schools raises a series of complex questions which go well beyond the polemics against the *Standards* in the recent special education literature.

An evaluation of the direct impact of the 1989 NCTM *Standards* would be a difficult, if not an impossible endeavor. Few in the mathematics education community would suggest that the *Standards*, which were designed as a framework for reform, provide a sufficient blueprint for daily instruction. For this reason, current study investigated an innovative curriculum, one which was closely aligned with the *Standards* but based on other sources (e.g., the translation of elementary and secondary textbooks from other countries which consistently score favorably in international comparisons). As a university, research-based effort, the curriculum also reflects field testing in a variety of settings and multiple revisions. Essentially, the *Everyday Mathematics* program represents the *Standards* and much more.

Viewed in this light, data from this study do not support the contention of critics from special education that reform efforts which represent the *Standards* are elitist. Rather, the data clearly suggest that the curriculum benefited the majority of students. Observations and interviews conducted as part of this study (Baxter & Woodward, 1995) indicated that a teacher's capacity to meet the needs of the lowest achieving students was complicated by many factors, only part of which may have been due to the structure and content of the curriculum. Equally problematic were the limited educational resources available to mainstream teachers (e.g., personnel, contact time, specific pedagogical techniques). If anything, these findings are consistent with recent mainstreaming research, which suggests that a variety of classroom organizational, instructional, and institutional variables inhibit the success of these students when they are taught in regular education settings (e.g., Baker & Zigmond, 1990; Schumm et al., 1995).

Therefore, the fact that the innovative curriculum met the needs of the majority of students in the intervention schools is cause for special educators to begin reconsidering the adequacy of many of their current instructional practices. It should be remembered that with the innovative curriculum, students discuss multiple solutions to problems, defend problem solving methods, and use an array of tools to work out solutions and demonstrate answers These practices differ substantially from current special education and past general education methods.

A careful analysis of even the most widely cited special education methods for teaching mathematics suggests a considerable difference in structure and content (Woodward, Baxter, & Scheel, in press). Special education curricula tend to place excessive emphasis on acquisition of facts, a rote mastery of the algorithms for basic operations, and key word solutions to traditional one- and two-step word problems (see Darch, et al., 1984; Silbert, Carnine, & Stein, 1989). Cognitive-based research not only questions whether teaching algorithms (VanLehn, 1990; Woodward & Howard, 1994) or problem solving (Hegarty & Mayer, 1993) can be successful over the long term, but more significantly, if these kinds of instructional experiences adequately prepare students for the kind of learning found in the intervention classrooms in this study. As Resnick (1989) has suggested, new forms of literacy do not follow a traditional hierarchy of preskills to a final point where students actually solve complex, ill-defined problems. Instead, skills need to be mixed with challenging activities.

It would appear, then, that a continued focus on (and condemnation of) the 1989 NCTM *Standards* is misplaced. Given the direction of research in mathematics education over the last two decades, the profound changes in technology which have devalued rote computational abilities, and findings such as the ones in this study, more attention should be placed on new instructional approaches for students with learning disabilities.

Implications for Practice

A lengthy discussion of instructional strategies which address the needs of students with learning disabilities and those at risk for special education in innovative mathematics classrooms would go well beyond the intent of this article. In the time following the research reported above, however, action research and empirical research has been conducted in the attempt to craft strategies for academically low achieving students (Baxter, Woodward, Olson, & Kline, 1996; Woodward & Baxter, 1996). These efforts to date suggest two levels of intervention.

First, new forms of literacy in mainstreamed settings, ones which promote classroom discourse and small group activities, argue for significant changes in classroom organization. Slavin, Madden, Karweit, Livermon, and Donlan's (1990) work in deploying students for small group, homogeneous instruction during portions of a lesson holds promise for innovative mathematics classrooms. This practice generally requires a cooperative working relationship for grade level teachers and additional instructional assistance. This latter role may be fulfilled by paraprofessionals or special educators working in mainstreamed environments.

Deployment strategies have been a significant feature of our recent research (e.g., Woodward & Baxter, 1996).

Specific pedagogical techniques comprise a second level of intervention. Work over the last decade in reading (Bos & Anders, 1990; Palinscar & Klenk, 1992) and writing (Englert, Raphael, & Anderson, 1992) provide important insights into the ways in which complex forms of literacy can be modified for students with learning disabilities through a balance of explicit strategies, a careful attention to cognitive process (e.g., the methods a student uses to derive an answer, the quality of a student's explanation) over product, and teacher-student dialogue. Yet techniques such as scaffolding or strategic feedback need to be understood in a content dependent fashion. Broad instructional principles such as those commonly associated with the effective teaching literature are likely to be insufficient. Instead, advances in our understanding of how students with learning disabilities might benefit from new approaches to mathematics depend upon innovative curricula and a teacher's subject matter knowledge.

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Figure 1

Sample Problems from the Informal Mathematics Assessment -- Fall Version

Problem 5

Your friend Kelly is in the other 3rd grade class. Kelly just got this problem for homework and has no idea how to do the problem. How would you solve the problem?

Homework problem: It is 534 footsteps from my house to school. I left for school, but after I walked 238 footsteps, I remembered that I had forgotten my overdue library book. I returned home, got the book, and went to school.

What distance did I walk?

Problem 6

The rules of Witches Guild are very strict. Black hats must be worn in public at all times. Broomsticks are to be replaced yearly and goblin gowns must not contain any patches. On Halloween, each witch must scare 13 people -- no more and no less. Each goblin must scare 33 people, and each ghost must scare 19 people.

How many people were scared by a group of 135 ghosts and 273 witches?

Figure 2

Informal Mathematics Assessment Results by Ability Groups

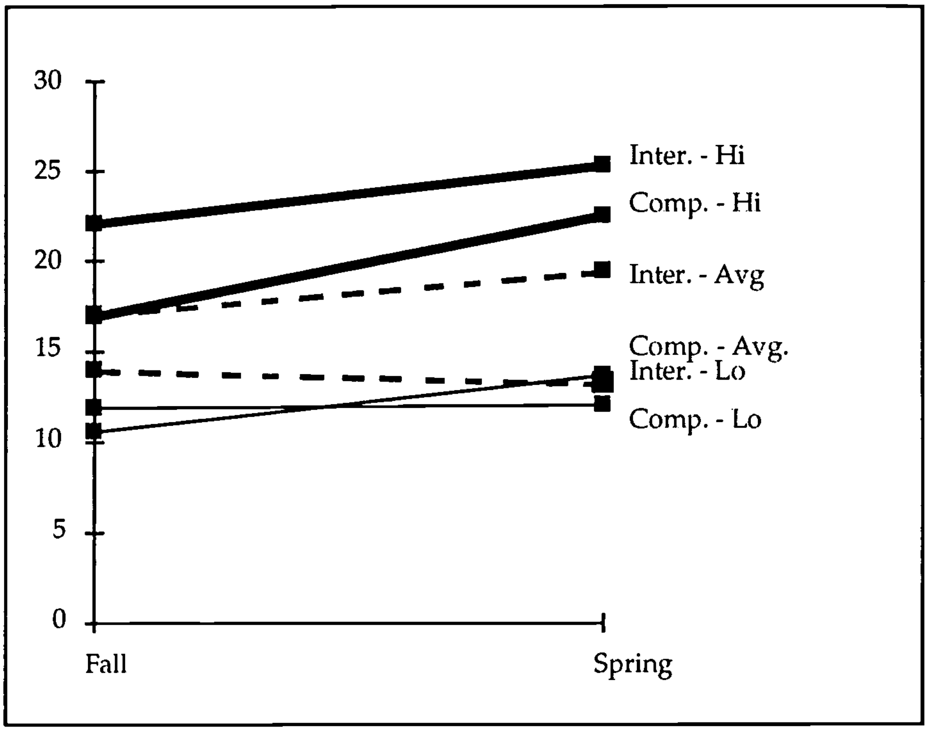

Figure 3

Reasoning Strategies Used by Different Ability Groups

Confusion and Uncertainty

	Fall	Spring
High (N= 7)	3*	1
Average (N = 6)	5	2
Low (N= 7)	6	5

*students

Conditional Reasoning

	Fall	Spring
High	5	5
Average	3	3
Low	0	1

Decomposition

	Fall	Spring
High	5	5
Average	3	6
Low	0	2

Figure 4

Shift in Reasoning for a Student with Learning Disabilities

Problem 6 - Witches and Ghosts Fall	Problem 6 - Boxtops Spring
Student: About 304 Interviewer: About 304, How did you get that? Student: Guessed Interviewer: Can you tell me a little more how you got the answer? Is there anything you can use here (pointing to tool kit) to show me? Student: No, I just guessed.	Student: 1,562. Interviewer: How did you get that? Student: I plussed 35 plus 19 plus 28 plus 97 equals 1,562. Interviewer: How did you figure out to get those numbers? Student: (shows on pencil and paper) 35 19 28 + 97 1,562 Notes: Student thinks about it. Asks to use pencil and paper. On paper (See attached):

Table 1

ITBS Scores 1993-94: Total Sample

Group	N	Fall Mean	Fall Sd	Fall Mean Percentile	Spring Mean	Spring Sd	Spring Mean Percentile
Total Test							
Intervention	104	56.2	13.2	71	68.08	11.85	71
Comparison	101	47.59	13.23	58	61.48	11.4	55
Computations							
Intervention	104	23.88	5.57	65	28.13	4.71	60
Comparison	101	19.85	6.36	52	27.02	4.8	51
Concepts							
Intervention	104	16.66	5.19	59	21.72	4.45	67
Comparison	101	15.53	4.64	51	18.9	4.37	47
Problem Solving							
Intervention	104	15.65	5.0	82	18.22	4.73	70
Comparison	101	12.21	4.38	61	15.55	4.22	60

Table 2

ITBS Scores 1993-94: Students at or below 34th Percentile

Group	Fall				Spring		
	N	Mean	Sd	Mean Percentile	Mean	Sd	Mean Percentile
Total Test							
Intervention	16	32.0	6.51	24	48.38	10.55	26
Comparison	22	32.0	7.86	20	50.86	9.9	30
Computations							
Intervention	16	15.5	4.97	30	23.44	5.35	32
Comparison	22	14.09	4.84	25	23.05	5.26	32
Concepts							
Intervention	16	9.31	3.46	12	13.69	3.2	18
Comparison	22	10.77	3.21	22	14.55	2.39	24
Problem Solving							
Intervention	16	7.19	2.43	25	11.25	5.72	37
Comparison	22	7.14	3.31	25	11.73	5.72	41

Table 3

ITBS Scores 1993-94: Students above the 34th Percentile

		Fall			Spring		
Group	N	Mean	Sd	Mean Percentile	Mean	Sd	Mean Percentile
High Ability Students							
Concepts							
Intervention	61	19.00	4.19	74	24.14	2.43	83
Comparison	37	19.78	3.28	80	22.60	2.70	74
Problem Solving							
Intervention	61	18.69	2.94	92	20.53	2.10	91
Comparison	37	16.00	2.88	82	18.84	2.41	77
Average Ability Students							
Concepts							
Intervention	27	14.74	3.47	47	19.82	2.96	54
Comparison	42	14.29	2.63	38	17.12	3.52	34

Table 4

Means and Standard Deviations for the Informal Mathematics Assessment

Group	N	Fall Mean	Fall Sd	Fall Mean % Correct	Spring Mean	Spring Sd	Spring Mean % Correct
Total Group							
Intervention	20	17.68	6.57	59	20.85	6.21	70
Comparison	20	14.3	3.57	48	14.95	5.06	50
High Ability							
Intervention	7	22.14	5.9	74	25.29	3.64	84
Comparison	7	17.0	1.83	57	19.43	4.76	65
Average Ability							
Intervention	6	16.86	4.78	56	22.57	4.72	75
Comparison	6	14.0	3.85	47	13.17	4.67	44
Low Ability							
Intervention	7	10.5	2.66	35	13.67	3.2	46
Comparison	7	11.86	3.02	40	12.0	1.91	40

<u>It's What You Take for Granted When You Take Nothing for Granted</u>

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Abstract

For decades, special educators have relied on broad, content-independent teaching strategies to tailor mainstreamed instruction to meet the needs of students with disabilities. Yet, how well these general strategies align with optimal practices suggested by researchers and educators within a particular discipline, such as mathematics, is an important and complicated issue. We begin our examination of this issue by briefly reviewing research in mathematics education, contrasting the differences between general and special education approaches to the teaching of mathematics. Next we delineate the basic assumptions that guide the direct instruction approach to curriculum design. We then analyze the impact of the direct instruction approach to teaching mathematics in light of the results from two studies of direct instruction in mathematics that focused on conceptual understanding and problem solving. We conclude with a discussion of an emerging, alternative approach to teaching academically low achieving students, one that is rooted within current research in mathematics education.

One of the many educational trends with direct implications for students with learning disabilities and others at-risk for special education is the national standards movement. As the National Council of Teachers of Mathematics, the National Council for History Standards, the National Committee on Science Education Standards and Assessment, and other disciplinary committees move to establish higher standards for student performance, special educators fear that there has been little consideration for the lowest academically achieving students in the conceptualization of these standards. Raising standards seems even more problematic at this time, when students who are likely to fail in general education classrooms increasingly remain in mainstreamed settings because of inclusionary policies (Carnine, Jones, & Dixon, 1994b; Fuchs & Fuchs, 1994). Furthermore, special educators argue that some standards (e.g., math, science) endorse pedagogical methods that are ill-suited to the needs of academically low achieving students (Grossen, Romance, & Vitale, 1994; Hofmeister, 1993).

If the legacy of innovation and school change efforts is any guide, it is unlikely that any of the standards, even those that are successfully written and accepted at a national level, will be implemented in *their entirety* in public school classrooms. The recent political debate over the History Standards gives credence to this observation. Translating policy to practice, assuming clear policies even exist, is exceedingly complex. In this case, the focus on standards by some special educators belies a more fundamental problem for the field: the tension between the content independent strategies commonly used in special education and what current research *within* specific disciplines suggests is effective practice.

The role of broad-based, content-independent strategies has been central to special education for a long time. Metacognitive strategy instruction, curriculum based measurement, and direct instruction exemplify generally recommended methods for tailoring mainstreamed instruction to meet the needs of students with disabilities. Whether included as components of general academic instruction (e.g., the use of a mnemonics metacognitive strategy for peer editing or draft revision in writing) or used as comprehensive alternatives to traditional instruction, these strategies can be and are applied *across* academic content areas. Thus, for example, direct instruction programs for mathematics and social studies share many common pedagogical and curriculum design features. Yet how well these content independent strategies align with optimal practices suggested by various disciplines remains an important and unresolved issue.

This chapter will explore this alignment issue in-depth, through the specific context of direct instruction and recent developments in the discipline of mathematics education. This particular context has been chosen for several reasons. First, direct instruction has a considerable presence in the special education literature. Direct instruction researchers and curriculum developers argue that its methods -- pedagogical and curricular -- are particularly well suited to students with learning disabilities and those at-risk for special education (both groups will hence be referred to in this chapter as academically low achieving students).

Second, direct instruction represents a relatively unique intervention strategy in special education because of the extent of its commercially available curricula. The direct instruction programs in mathematics range from beginning level materials for first graders to remedial programs in fractions, ratios, and algebra for secondary level students.

Finally, direct instruction researchers have been particularly critical of recent developments in the field of mathematics education. Many prominent direct instruction researchers (Carnine, 1992; Engelmann, Carnine, & Steeley, 1992; Chard & Kameenui, 1995) claim that the 1989 NCTM Standards are both ill-defined and generally anathema to the needs of academically low achieving students.

This chapter is divided into four sections. The first section briefly reviews some of the main developments in the field of mathematics education during the last 15 years, as well as trends in special education math research over that same time period. This discussion, although far from exhaustive, is included as a means of contrasting the differences between general and special education approaches to mathematics.

The second section delineates several basic assumptions that guide the direct instruction approach to curriculum design. Rather than simply restating the instructional design principles, which are articulated more comprehensively elsewhere (e.g., Engelmann & Carnine, 1982; Kameenui & Simmons, 1990), the principles are discussed more broadly. These assumptions ostensibly enable curriculum developers to meet the needs of all academically low achieving students because they "take nothing for granted." Thus, developers assume that students know little, if anything, of a subject before they learn it, that print or electronic curriculum is the dominant factor in instruction, that lesson efficiency is paramount -- meaning that minimal time is spent on dialogue or ancillary activities, and that knowledge is rule-based or algorithmic. Understanding these assumptions is an important backdrop to the remainder of the chapter.

The third section of this chapter summarizes two recently conducted studies involving highly traditional and direct instruction approaches to mathematics. The results are disconcerting, at least in regard to students' conceptual understanding and their problem solving skills. The fourth and final section, then, discusses these findings through the direct instruction assumptions stated earlier as well as current mathematics research. This section also includes a brief discussion of an emerging, alternative approach for teaching academically low achieving students, one that is rooted within current research in mathematics education.

Mathematics Research in General and Special Education

Trends in Math Education for General Education Students

At the beginning of the 1980s, *An Agenda for Action* (NCTM, 1980) presented a broad outline for changing mathematics instruction at the elementary and secondary grades. The document proposed that problem solving "should not be limited to the conventional 'word problem' mode (p. 3)," but that it should include problem formulation, the investigation of patterns, and the use of imagery, visualization, and spatial concepts. It also stressed the importance of redefining the nature of basic skills to be more than routine practice on computations. In fact, the report recommended that computational practice not be isolated from conceptual development and that fluency in computational algorithms *not* be a prerequisite to calculator use.

Empirical research, as well as analyses of US math programs, conducted throughout the 1980s corroborated many of the issues and problems presented in *An Agenda for Action*. Studies of student misconceptions in subtraction, for example, strongly suggested that without a conceptual foundation, students tend to produce creative, but highly erroneous solutions to simple computational problems (Van Lehn, 1983, 1987). Examinations of curricula used in public schools revealed a protracted emphasis on arithmetic, with considerable redundancy from year-to-year in the elementary and early middle school grades (Flanders, 1987; McKnight, et al., 1987).

The research also exposed weak instruction in problem solving. For example, researchers found that when students routinely encounter only two or three sentence word problems that contain key words, problem solving quickly becomes little more than the rapid application of basic operations (e.g., "*gave away* means to subtract"). Consequently, students tend to develop the view that all math problems can be solved "in less than five minutes" (Doyle, 1988). When presented with more complex, ill-defined, or longer problems, they predictably flounder. Or, when problems appear relatively simple, they apply operations with little thought to the completeness or appropriateness of the answer (Schoenfeld, 1988).

By the late 1980s, documents such as *Everybody Counts* (National Research Council, 1989) and the *NCTM Standards* (1989) repeated many of these same themes. They, too, suggested that mathematics education needed to focus more on conceptual understanding, genuine problem solving, and an increased use of computers and calculators as natural tools for learning mathematics.

In addition, the research from this time yielded detailed descriptions of how students come to understand mathematical concepts and how instructional techniques influence students' thinking. Among other things, researchers found that when computational algorithms are linked to an array of representational systems, student understanding is more robust (Fuson, 1990; Hiebert, 1986; Janvier, 1987; Lampert, 1986). It should be noted that the notion of representational systems for these researchers went well beyond the traditional connotation of manipulatives. Graphic representations, such as pictures or sketches, and verbal explanations were included with physical objects as tools for teachers to use to promote conceptual understanding (Hiebert & Carpenter, 1992; Hiebert, Wearne, & Taber, 1991; Lesh, Post, & Behr, 1988).

One of the central, albeit most difficult instructional features to emerge from this research was the role of discourse. A number of qualitative and action research studies offer lengthy accounts of the complex nature of dialogue and scaffolding associated with the development of student thinking in the elementary grades (Ball, 1993; Lampert, 1986, 1990; Resnick, 1989; Wood, Cobb, & Yackel, 1991). To conduct successful, substantive discussions, teachers need a considerable understanding of mathematics, as well as rich models of the learners' knowledge states at a given time (Ball, 1993; Leinhardt & Smith, 1985). In fact, these discourse-oriented classrooms, the curriculum emanates from the teacher (*rather than from printed or electronic materials as will be seen shortly*) (Williams & Baxter, in press). This need for considerable subject matter knowledge and more appropriate pedagogical skills, also led researchers in the field to the unavoidable conclusion that major changes in teacher preparation and inservice are needed (Cobb, 1988; Cohen, McLaughlin, & Talbert, 1993; Schifter & Fosnot, 1993).

Finally, one of the most important areas in recent mathematics education research involves curriculum analysis. There are now a number of books that offer extensive treatments of key concepts in elementary and middle school mathematics (Carpenter, Fennema, & Romberg, 1993; Grouws, 1992; Hiebert, 1986; Hiebert & Behr, 1988; Leinhardt, Putnam, & Hattrup, 1992). These analyses of curriculum have been accompanied by recommendations for new forms of assessment that math educators believe must be implemented if true reform is to occur (Kulm, 1990; Lesh & Lamon, 1992; National Council of Teachers of Mathematics, 1995; Romberg, 1995).

<u>Trends in Math Education in Special Education</u>

In contrast to the extensive efforts within the field of mathematics education, there has been far less research on mathematics in special education.

Of the research conducted, however, at least two main trends can be seen. The first involves researchers' attempts to articulate the nature of learning disabilities in math. Studies on how students acquire and become fluent in math facts suggest that students with learning disabilities experience developmental delays when compared to their non-disabled peers (Bahr & Rieth, 1989; Goldman, Pellegrino, & Mertz, 1988; Kirby & Becker, 1988). In fact, students with learning disabilities seem to retain counting strategies much longer than non-disabled students, and tend to require much more intense practice in order to use direct retrieval methods (Hasselbring, Goin, & Bransford, 1988). These findings are consistent with the more general information processing deficits exhibited by students with learning disabilities (Kolligian & Sternberg, 1987; Swanson & Cooney, 1991).

Similarly, the math problem solving research in special education supports the broader contention that students with learning disabilities act impulsively and use an array of suboptimal strategies when trying to complete traditional story problems (Montague, Bos, & Doucette, 1991; Montague, 1992). These studies underscore the notion that these students require limited, but highly generalizable metacognitive strategies for identifying key words, drawing pictures as a way of simplifying the problem, and completing problems (Case, Harris, & Graham, 1992; Goldman, 1989). It is important to note that researchers tended to use highly conventional content (e.g., simple story problems) in these examinations of the different characteristics associated with a learning disability in mathematics.

A second research trend, one only generally aligned with diagnostic research, is the use of content independent instructional strategies. Traditional content is modified by applying behavioral principles like task analysis and mastery learning (e.g., Fuchs, Fuchs, & Fernstrom, 1993; Stevens & Schuster, 1988; Sugai & Smith, 1986; Wilson & Sindelar, 1991). For example, special educators are encouraged to teach subtraction by starting with simple one and two digit problems that do not require regrouping (i.e., borrowing) and progressively work toward more difficult problems as students master each problem type (Howell & Morehead, 1987; Silbert, Carnine, & Stein, 1990). In fact, this task analytic, hierarchical orientation reflects the way elementary mathematics in general education was taught prior to the current wave of research.

Although many of the instructional strategies used in special education research follow a behavioral framework, few appear in the form of a complete curriculum. In this respect, the direct instruction programs in mathematics (e.g., *Corrective Mathematics: Subtraction, Connecting Math Concepts, Mastering Ratios*) constitute a distinctive contribution. These comprehensive print and technology-based instructional programs take a troubled or "naive" learner, step-by-step, from virtually no understanding of the domain toward a mastery of complex material (Carnine, 1989). In *Connecting Math Concepts* (Engelmann & Carnine, 1991), the most recent of the direct instruction mathematics programs, students progress across five instructional levels from counting and numeral identification to ratios and decimal multiplication. This represents a considerable development in mathematical ability, particularly when one considers that students who eventually solve complex computations and difficult word problems purportedly begin with little or no knowledge of mathematics.

The methods used to create direct instruction programs (Carnine et al., 1994b; Engelmann, et al., 1992) have evolved measurably since the early 1970s (see Woodward, 1993). The framework remains largely behavioral, however the principles for designing instruction across a range of content areas have become much clearer. This can be seen in the books (Carnine & Kameenui, 1992;

Engelmann & Carnine, 1982; Kameenui & Carnine, in press; Silbert, et al., 1990), special issues of professional journals (*Journal of Learning Disabilities*, 1991; *School Psychology Review*, 1994), and federally-funded centers for dissemination (i.e., Center for the Study of Improving Math Instruction and the National Center to Improve Instructional Tools for Educators, University of Oregon) on direct instruction approaches to curriculum design.

Two important themes pervade this body of literature. First, is the assertion that instruction design principles can be applied independently of content and that these principles, rather than the content, are foremost in developing curricula (i.e., the content of any discipline is secondary to the design principles). And second, the content of commercial programs is generally substandard and poorly structured. Commercially available programs -- whether in mathematics, science, spelling, or reading -- suffer from inadequate design. They fail to provide sufficient practice and review, the strategies are not explicit enough, and they require students to engage in too many repetitive or peripheral activities. These criticisms appear repeatedly in the direct instruction mathematics literature (Darch, Carnine, & Gersten, 1984; Kameenui & Griffin, 1989; Kelly, Gersten, & Carnine, 1990).

Direct instruction curriculum developers and researchers have also focused their criticisms on the NCTM Standards (National Council of Teachers of Mathematics, 1989). As previously noted, some have argued that the Standards are vague, and that they are nothing more than recycled reform efforts from the New Math of the 1960s. These critics consider the Standards as particularly troublesome for academically low achieving, both because they contain exceedingly high expectations and because constructivism, which is purported to be the sole pedagogical foundation for the reform, is elitist (Carnine, 1992; Chard & Kameenui, 1995; Hofmeister, 1993). All of this assumes, of course, that the

Standards are intended to be instructional blueprints rather than a framework for conceptualizing the research, curriculum analyses, and assessment efforts mentioned earlier. Given these criticisms, as well as the extensive presence of direction instruction in mathematics for academically low achieving students, it is important to take a closer look at the key assumptions which underlie the direct instruction programs.

The Direct Instruction Approach: Assumptions About Learners and Teaching

Arguments for the utility of direct instruction for academically low achieving students are grounded in broad claims about general intellectual characteristics of students who fail in school. In turn, these characteristics are used to support a number of highly specific curriculum design principles. While the direct instruction literature offers empirical support for the effectiveness of these principles, one study or even a set of studies rarely make explicit the basic assumptions which lie behind the direct instruction curricula that are used in everyday classrooms. Several of the key assumptions about learners and teaching are presented below. They are, in fact, what direct instruction curriculum developers "take for granted" when they design programs such as *Connecting Math Concepts.* These assumptions provide a foundation for the last two sections of this chapter.

Assumptions About Academically Low Achieving Students

Since its inception, direct instruction has focused on students who are at-risk for failure. At times, these students are portrayed as capable, but poorly taught. In other writings, academically low achieving students are described as prone to confusion and misconceptions (Engelmann & Carnine, 1982). As might be expected, the greatest array of deficits is associated with students who have learning disabilities. Chard and Kameenui (1995), for example, criticize the 1989 NCTM Standards because they fail to take into account the cognitive and academic characteristics of students with learning disabilities. They suggest that the poor or inconsistent memory skills, substandard strategies (particularly in the area of metacognition), irregular attention, learning rates which are slower than non-learning disabled peers, linguistic difficulties, and poor motivation of these students are likely to result in extremely unsuccessful experiences if instructional methods aligned with the Standards are used. However, Chard and Kameenui do not specify how many of these deficits any one student with a learning disability may possess or the extent to which these characteristics are representative of other "diverse learners" or academically low achieving students. This broad, multiple deficits perspective is apparent in other writings as well (see Kameenui & Carnine, in press).

Carnine (1991) implies that while all academically low achieving students may not have all these traits, direct instruction methods nonetheless offers a common remedy for their learning problems. That is, irrespective of content area, and regardless of a student's particular deficit or difficulty, it is critical to "begin at the beginning," and make few, if any, assumptions as to what the student might know about a subject. In this respect, academically low achieving students -- from those whose achievement is below average to those with learning disabilities -- have undifferentiated needs. More important, the instructional remedy for all is well-designed curricula (i.e., print or electronic materials).

Assumptions About Teaching and Curriculum

Curriculum is instruction. There are several reasons for direct instruction's emphasis on curriculum in mathematics instruction. First, much of the early process-product research indicated that the materials or curriculum development component for a math lesson is difficult for elementary school teachers (Good, Grouws, & Ebmeier, 1983). A second and more general reason is that educational change is best achieved through curriculum. As Doyle (1992) sardonically notes, it is easier to replace curricula than teachers. Direct instruction researchers argue that substantive changes in the design of print or electronic materials can help teachers overcome what research otherwise points worked to as negligible differences between general and remedial or special education instructional practices (Ross, Smith, Lohr, & McNelis, 1994; Ysseldyke, O'Sullivan, Thurlow, & Christenson, 1989).

Another clearly important reason for the emphasis on curriculum is that instructional design principles cannot be realistically implemented without it. Direct instruction curricula follow a "research-development-dissemination" paradigm based on the belief that content is best structured by outsiders (e.g., professional curriculum developers) and then disseminated to practitioners (Woodward, 1993). The result is that teachers are presented with a carefully developed sequence for teaching math facts as well as an "easy to hard" sequence for teaching computation algorithms (e.g., in subtraction, students begin with non-regrouping of one and two digit numbers and increase toward three and four digit numbers with multiple instances of regrouping). It is also evident in the practice provided on math facts and algorithms in these programs, which always follows a consistent pattern of massed and distributed practice, along with precise discriminations (e.g., when to and when not to regroup).

Finally, detailed curricular materials allow the scripting of teacher-student interactions. Scripting ostensibly ensures a high fidelity of program implementation and enables developers to control the pace of the lesson, the nature of explanations, and the way errors are corrected. To be sure, much of the scripted formats conform to the general process-product findings from a decade ago, with teachers asking predominantly low order questions, giving concise feedback, conducting lessons at a brisk pace, and so forth (Brophy & Good, 1986; Rosenshine & Stevens, 1986).

Efficiency is the key. Direct instruction researchers and curriculum developers assume well-designed materials are "efficient." That is, the materials must be designed so that students can progress at a faster rate than normal and obtain performance levels that are comparable to their non-learning disabled or non-disadvantaged peers (Carnine, 1991). The goal, then, is to have academically low achieving students work through carefully developed strands of information, with a minimum of distractions or time consuming activities, so they can focus on the kind of "practice to mastery" needed to achieve high levels of performance on a skill.

This need for efficiency requires curricula to be stripped of unnecessary activities, and explains why the direct instruction mathematics programs contain few, if any, manipulative activities. In addition to empirical questions about the efficacy of manipulatives in helping learn concepts such as counting or place value (see Evans, 1990), it is believed that the time and organization needed to conduct manipulative-based activities is too great. The goal of direct instruction mathematics curricula, from the beginning, is to move students as quickly as possible to highly structured forms of symbol manipulation (e.g., practice on computational algorithms such as long division). This is clearly illustrated in an early lesson from *Connecting Math Concepts*.

Initially, students use stick lines as a way of linking number to numerals, a common practice in many traditional programs. To represent the numeral 6, for example, the student draws six lines (i.e., IIIIII). However, once students reach the ten units, drawing stick lines becomes time consuming and inefficient.

Quantities for ten are expressed through T's, which stand for the number ten. The numeral 34, for example, would be represented as TTT IIII.

This seemingly minor example nonetheless epitomizes the abstract and symbolic emphasis of even the initial phases of the program. In fact, the majority of lessons in the first level of *Connecting Math Concepts* rarely go beyond paper and pencil activities in which students practice tracing numerals, counting, and adding simple one and two digit numbers. The emphasis on symbol manipulation and the almost exclusive use of paper and pencil activities persists across all five levels of the program. Workbook exercises constitute the main form of instruction, and pencils are the dominant mathematical tool. Pattern analysis, a variety of manipulatives, measurement tools, calculators, and games, which are central to innovative mathematics (Carpenter, 1985; Fuson, 1992; Resnick, Bill, & Lesgold, 1992), are foreign to *Connecting Math Concepts*.

Part of what direct instruction enables developers to achieve such high levels of efficiency is a reliance on "sameness" and "big ideas." These curriculum developers begin by surveying the traditional content of a discipline for any underlying qualities which are (or can be made) similar. In a related manner, the developers search for "big ideas" or key concepts, principles, or heuristics that can be used to link together what otherwise would be fragmentary information (Carnine, Dixon, & Kameenui, 1994a; Chard & Kameenui, 1995). In *Connecting Math Concepts*, for example, an arrow is used as a heuristic across math facts, computational problems, and traditional story problems. As Figure 1 indicates, students learn to associate different numbers in a problem by writing them in one of three places. In the case of math facts, the three numbers compose a "family," which generate two addition and two subtraction facts (e.g., 7, 1, and 8 generates 7+ 1 = 8, 1 + 7 = 8, 8 - 1 = 7, 8 - 7 = 1). Simple addition problems such as 43 + 75 can be cast in this format.

[Insert Figure 1 here]

Again, the novelty of the arrow heuristic in this program is the way it is extended across topics and different levels of the curricula. Through extensive discrimination and practice exercises, students learn to identify "big" and "small" numbers in traditional word problems. Explicit strategies such as the one conveyed in Figure 2 teach students to search through a problem for these numbers and place them on the arrow in the appropriate places. One of the presumed virtues of sameness is that, if used consistently across a range of problems and "types of knowledge" (e.g., math facts, story problems), it is an efficient means of teaching students *more* in *less* time. Unfortunately, exactly how students learn big and small numbers in any linguistically meaningful sense (i.e., other than being *shown* examples of each of the problems) is not clear. As the studies discussed later in this chapter suggests, this approach is highly problematic.

[insert Figure 2 here]

Knowledge is algorithmic. Once big ideas have been identified, they are taught to mastery across the curriculum using a "multiple strands" approach. This is a complex scheme in which, for each big idea or strand, new skills are gradually introduced, massed practice is provided, and further work on the skill is distributed across many subsequent lessons. The goal is to develop algorithmic knowledge; that is, rule-based and/or declarative knowledge which can be applied quickly to problems with distinct features. Once the student recognizes that regrouping is required, the student borrows. If the problem contains words like *each* or *every* it is a multiplication or division problem.

An example of this graduated approach toward mastery can be found in *Mastering Ratios*. Initially, students just identify numbers and associated units in simple one or two sentence word problems. Thus, for several lessons, problems like, "A train travels 75 miles in 2 hours, how far would it travel in 3 1/2 hours?" students merely identify miles and hours as the key units in the problem. Gradually, they associate numbers with units in an equation, then construct ratios and solve them through equivalence, and finally work two and three sentence ratio problems to mastery (see Moore & Carnine, 1989).

Like the big and small number strategy mentioned earlier, carefully sequenced practice in conjunction with specific strategies for quickly translating word problems into their computational form enables students to work through problems efficiently, with an emphasis on the proper algorithm (e.g., making fractions equivalent in the case of ratio problems). A systematic analysis of lessons for *Connecting Math Concepts* reveals that students work an average of eight sets of skills per lesson (with a range of seven to ten). Some of these skill sets are new and accompanied by introductory, massed practice. Others are included as part of the distributed practice scheme. As Chard and Kameenui (1995) suggest, this is one of the hallmarks of instructional design for students who have retention problems or need a graduated, step-by-step approach to instruction.

Coupled with detailed teacher scripts (as specified in the teacher's lesson book), the intent is to enable students to practice skills to high levels of mastery. The desired outcome is for students to be able to recognize the critical features of a problem (i.e., make the appropriate discriminations) and respond with the correct answer. In subtraction, for example, different forms of borrowing are described in the teacher's manual as a series of subtypes. Through a carefully sequenced set of steps, students are taught to identify a critical feature of the problem (e.g., that the number on the bottom of the ones column is the big number, which means that one must borrow). Then, highly practical language is used for the regrouping. In the example shown in Figure 3, students "borrow from the tens," by re-writing the 43. This is done by crossing out the 4, writing a number one less than 4 and writing the 1 that is borrowed in front of the 3.

[insert Figure 3 about here]

The purpose of the teacher-directed language, it seems, is to communicate the functional steps of regrouping efficiently. Thus, the student writes 1 less than 4 rather than 10 less than 40 and puts a 1 next to the 3 rather than a 10. Throughout lessons on regrouping in subtraction students work through various subtypes, with practice sets that include a mix of addition problems to enhance discriminations. Manipulatives, which are infrequently recommended at the end of lessons if time permits, are not integrated into core of the lesson. Furthermore, calculators rarely play a role in direct instruction math curricula. In *Connecting Math Concepts*, the most recent program, teachers are instructed not to use calculators as a substitute for paper and pencil mastery of the algorithm.

This focus on algorithmic knowledge is likely based on several factors. Concern about the learner characteristics mentioned earlier, particularly those related to memory skills, lead to a curriculum with extensive daily practice activities. In addition, the notion that academically low achieving students are easily confused results in explicit, step-by-step strategies where children learn "one way" to solve problems. Finally, a considerable portion of the curricula -- from facts to the algorithms for common operations to word problems -- is highly traditional, which enables the consistent application of rules to highly similar problems.

Naturalistic Research on the Direct Instruction Approach to Mathematics

The following section summarizes two recently completed research studies into the use of highly traditional and direct instruction mathematics programs in naturalistic settings. In both studies, students spent a considerable portion of their time mastering computational algorithms and working story problems using a key word approach. A range of measures were used to document the effects of the instructional programs. In particular, TORUS (Woodward & Howard, 1994), an artificial intelligence program for detecting misconceptions in subtraction, was a key dependent measure in both studies.

Study 1: Developmental Trends in Subtraction Misconceptions

The purpose of the first study was to examine the impact of traditional approaches to subtraction on academically low achieving, general education, elementary-aged students and a small cohort of middle school students receiving special education in mathematics. Researchers assessed student competence in subtraction at three intervals over the course of the year. By measuring performance across the year, researchers were able to document the subtle effects of traditional methods for teaching subtraction (i.e., methods which focus on a rote mastery of algorithms) by examining the relationships among fact knowledge, criterion performance on subtraction computations, and misconceptions for different ability groups. Van Lehn (1983), in earlier work on the Buggy Project (see Brown & Burton, 1978; Van Lehn, 1990), originally proposed an analysis of student competence based on multiple criterion measures.

Participants included 143, third grade, fourth grade, and middle school students from two rural schools in the Pacific Northwest. Only those students for whom there were test data for the entire year were included in the analysis. The final cohort, therefore, consisted of 72 third graders, 61 fourth graders, and 10 middle school students, with approximately equal numbers of girls and boys at the elementary level. Eight of the ten students in the middle school cohort were boys, and all students were receiving 45 minutes of resource room instruction per day in mathematics.

All students were given the Computations Subtest of the Metropolitan Achievement Test in early October and again in May. Fall scores were used to classify students by ability level (i.e., academically low, average, and high achieving in math computations). Students at or below the 34th percentile comprised the academically low achieving group, those between the 35th and 66th percentile were identified as academically average achieving, and those above the 66th percentile were classified as academically high achieving.

The study focused on was on the academically low achieving students in the third and fourth grade and the 10 middle school students with learning disabilities who had IEPs in mathematics. Interviews with both general and special education teachers in the elementary school suggested that many students who scored at or below the 34th percentile could have qualified for special education in mathematics but were not served because of limited resources. In fact, only six of the third graders and nine of the fourth graders had IEPs, but in each case they were being served for a reading disability. Twenty-eight of the third graders received Chapter I services in reading and math. At the fourth grade level, 27 students received Chapter I services for reading and math.

Elementary school teachers used the *Scott Foresman Mathematics* program (Scott Foresman, 1985), a traditional basal program. The third grade program presents subtraction through daily computational exercises, and lessons frequently include traditional two and three sentence subtraction word problems, which can be answered by looking for key words. The program progresses from easy forms of subtraction (i.e., subtraction without regrouping) to multidigit problems requiring regrouping. The fourth grade program follows a similar pattern, but contains fewer lessons devoted to subtraction. At both levels, the most concentrated practice on subtraction occurs from November to February.

The single middle school resource room teacher used the direct instruction program *Corrective Mathematics: Subtraction* (SRA, 1981), which fully reflects the direct instruction approach to mathematics and, more generally, curriculum development. Massed and distributed practice is used to teach increasingly difficult computation and word problems. Unlike traditional basal programs, *Corrective Mathematics: Subtraction* contains a much higher level of cumulative review and a greater emphasis on discriminating between types or subtypes of problems (e.g., addition and subtraction word problems, programs which require regrouping and those which do not). Also, strategies for translating word problems using key words such as *per, each, gave away, bought* are taught explicitly. Thus, in a word problem like, "*Sally gave away 15 ball . . .*" in a word problem, students are taught that *gave away* means to subtract.

Student performance was assessed in October, February, and early May during the academic year. At each interval, they were given a 50 item, timed, subtraction facts test and a 25 item TORUS Level 1 subtraction test. The TORUS Level 1 subtraction test is part of an artificial intelligence program (TORUS) that systematically analyses student responses to each problem by accessing a library of misconceptions or bugs. After that analysis, TORUS then generates a report describing criterion performance (e.g., raw score, percent correct by categories), problem type difficulties (i.e., the kinds of problems on which students exhibit difficulties but their answers are not predictable), and specific misconceptions, or "bugs," where students err in highly predictable ways as suggested by answers which match those predicted by the TORUS program. For further details on TORUS, see Woodward and Howard (1994).

Immediately following each assessment interval, half of the middle school students with learning disabilities were randomly selected for individual interviews using think-aloud techniques (Ginsburg, Kossan, Schwartz, & Swanson, 1983). Each student was shown five problems from their recent TORUS test, most of which he or she had answered incorrectly and were asked to "talk through" his or her solution methods for each problem. They were also asked specific questions about place value, the role of zeros, and how they might check their work for errors or how they could determine whether their answers were correct. The purpose of these questions was to gauge the students' conceptual understanding, as well as the ways in which they checked or evaluated the correctness of their work.

A complete analysis of the third and fourth grade data can be found elsewhere (Woodward, 1992; Woodward & Battle, 1995), but Table 1 below presents descriptive statistics for all of the cohorts in the study. Generally, students in the different ability levels across the two grades followed common patterns, particularly those in the lowest achieving groups. The lowest third of the students tended to begin the year with little understanding of subtraction. In essence, they demonstrated little knowledge of regrouping. Over time, particularly as subtraction was either taught in the third grade or reviewed in the fourth grade, they improved significantly. By May, their performance plateaued, resulting in means of 68 and 57 percent correct on Level 1 of the TORUS test for academically low achieving third and fourth graders respectively.

Distinct shifts in the nature of errors complemented this pattern in the raw score performance data. In October, the academically low achieving students had few identifiable misconceptions or bugs besides inverting when regrouping was called for (i.e., always subtracting the smaller number from the larger number irrespective of position in the minuend and subtrahend or the S - L bug).

As raw score performance improved, however, an increased number of misconceptions appeared. By May, the low academically achieving students exhibited a preponderance of bugs associated with zeros and multiple borrows. This was true for both the third and fourth grade subsample. Previously conducted research also found that academically low achieving students tended to plateau at approximately 70 percent correct and performance included a similar array of bugs (Woodward & Howard, 1994). It should be noted, as well, that subtraction fact knowledge was generally uncorrelated with the students' raw score performance.

Interestingly, the middle school students in the *Corrective Mathematics: Subtraction* program tended to plateau *at the same level* as the academically low achieving third and fourth graders. While their mean performance in October on the TORUS test was substantially higher (46 percent correct versus 20 percent correct), their May performance was comparable to that of the academically low achieving third and fourth graders in May (i.e., 68 percent correct). Again, the middle school students' performance on the TORUS tests was also uncorrelated to their subtraction fact scores.

The pattern of misconceptions over time for these students, however, differed considerably from the low achieving third and fourth graders, presumably because these middle school students began the year with some knowledge of regrouping. Nine of the ten middle school students exhibited at least one significant misconception other than inverting when regrouping or the S-L bug in October and February. Eight of ten still had at least one misconception in May. Again, details of their misconceptions can be found elsewhere (Woodward, 1992; Woodward & Battle, 1995).

In May, these middle school special education students showed the same type of difficulties with zeros and multiple borrows as did the third and fourth grade subsample of academically low achieving students. Their think aloud protocols provide greater insight into the nature of their misconceptions. Figure 4 presents computational solutions to selected TORUS tests which were used as part of the student interviews. Problems, and their solutions, come from four different middle school students. One of the most intriguing aspects of these examples is that while all four students used erroneous algorithms, the first two students produced answers that are incorrect while the remaining two produce correct answers.

[insert Figure 4 here]

John's method represents a typical "shortcut" strategy, one which is often taught directly in math class. That is , when the configuration of two adjacent middle zeros appears in the minuend(e.g., 5009), teachers often instruct students to "cross out the 500" when regrouping is required. Unfortunately, students tend to generalize this strategy and apply it whether or not regrouping is required in the ones column. This was a typical multiple borrow, zero misconception that we observed throughout the year.

Paul's error represents another common, but incorrect strategy. Rather than regrouping systematically one column at a time from right to left, he moved freely across columns. Paul began by regrouping from the ten's column, then moved abruptly to the thousand's column for further regrouping. Thus, he wrote 10 one hundreds when nine would have been the correct amount. Misconception data from the elementary students indicated that these interior zero borrow errors with zeros were also a common problem for academically low achieving third and fourth graders by the end of the year.

George and Ringo's solutions reveal the kind of errors that TORUS cannot detect because of the highly unusual nature of their repair strategies, ones which happenstantially produce correct answers. In George's case, once he "recognized" the need to regroup, he "moved all the way to the left," and regrouped all of the minuend columns at once except the ones column. Then, as George systematically worked each column from right to left, he realized upon finishing the subtraction in the hundred's column that he didn't need to regroup in the thousand's column. To repair the problem, he "added one back to the 2" and finished subtracting.

Ringo's correct answer was an artifact of the problem type (i.e., four digit by four digit, alternate regrouping). When Ringo was asked how he solved the problem, he explained, "It's easy. I just divide it into two problems." This enabled him to transform $\begin{array}{r}2493\\ \underline{-1556}\end{array}$ into two easier problems: $\begin{array}{r}24\\ \underline{-15}\end{array}$ and $\begin{array}{r}93\\ \underline{-56}\end{array}$. Alternate regrouping in this instance produced fortuitous success.

Overall, the findings from this study are consonant with many of the findings from the Buggy research a decade ago (Van Lehn, 1983, 1988). The think aloud protocols suggest that students are prone to repair impasses or troublesome steps in a subtraction problem spontaneously, and as Van Lehn (1988) noted, they are frequently influenced by other problems on the page. During the think aloud sessions, for example, many students abruptly changed an answer to a problem completed earlier based on some feature of the problem they were currently working. In addition, criterion performance on the TORUS test across time was clearly related to the evolution of misconceptions. As students moved from 40 percent correct to 65 percent correct, they exhibited different kinds of errors. They "graduated" from relatively simple S-L bugs to diverse zero and multiple borrow errors.

Undoubtedly, the most distressing finding was the general lack of conceptual knowledge. Think aloud protocols consistently revealed that the students had little way of discussing the meaning of place value, the role of zero, or even how they might check their answer other than "doing it over again." Implications of these findings will be discussed later.

Study 2: Direct Instruction in Subtraction Computations and Word Problems

Baxter and Fabry (1995) conducted a follow-up study of the *Corrective Mathematics: Subtraction* program at the elementary school level. Data were collected at in November on nine fourth and fifth grade students with learning disabilities in mathematics (eight boys and one girl). All students had IEPs in mathematics.

Students were initially screen for their computational abilities in subtraction. Mean performance on TORUS Tests was 32.6 percent correct. Students were taught in two different groups for approximately eight weeks each. This was due to the fact that additional students entered the resource room program in late January, approximately at the time when the first cohort of students were finishing the *Corrective Mathematics: Subtraction* program. Thus, students were taught in groups of four and five respectively.

The participating teacher in this study was carefully selected because researchers wanted to ensure high fidelity of program implementation. A special education teacher with 13 years of experience and extensive training in direct instruction teaching methods was chosen for the study. In fact, she had been using direct instruction programs like *Corrective Mathematics: Subtraction* for almost ten years. In addition, researchers also conducted two lengthy observations in her classroom to confirm that she was implementing the program as intended.

TORUS assessments were administered at two week intervals to track computational performance over the course of the *Corrective Mathematics: Subtraction* intervention. Word problem solving ability was also evaluated, once the students completed the program, using an innovative instrument (the

Informal Mathematics Assessment or IMA) employed in other research (Woodward & Baxter, 1996). The IMA was modified to reflect an adequate range of addition and subtraction problems. Addition problems were included because they had already completed instruction in this area.

The eight items on the IMA were based on an analysis of third grade mathematics texts, as well as the *Corrective Mathematics: Subtraction* and Level C of *Connecting Math Concepts* in which subtraction is taught to mastery. To prevent fatigue and possible frustration, the items on the IMA were relatively brief, and were read to students by the examiner. In most cases, the IMA took about 20 minutes to administer, however all students were given as much time as they wanted to complete each item. Alternate form reliability for the original version of the IMA was .87.

A trained interviewer administered the IMA to students individually. After reading each word problem, she recorded *how* the student derived his or her answer, along with the answer itself. For example, she carefully noted whether the student reread the problem, what calculations the student made, and what tools or manipulatives the student used. When a student finished a problem, the interviewer asked, "Tell me how you got that answer." This form of inquiry has been shown to be a valid method of determining how children solve mathematics problems (Siegler, 1995). Each interview was tape recorded then transcribed for later scoring and qualitative analysis. During the sessions, students were given a range of mathematical tools and representations which they were encouraged to use as part of the problem solving. The IMA "tool kit" included a calculator, ruler, paper and pencil, poker chips, and number squares with ones, tens, and hundreds values. Figure 5 presents some of the word problems from the IMA.

[insert Figure 5 here]

The TORUS results for the nine students showed a distinct pattern of improvement over time. For two of the students, a simple review of subtraction proved sufficient. Almost immediately their mean performance exceeded 90 percent for the eight week intervention. For the other students, the *Corrective Mathematics: Subtraction* program was far less successful. While they gradually improved, they never scored above 70 percent correct on any of the bimonthly TORUS probes. At the end of the program implementation, TORUS analyses revealed that these seven students continued to have at least one significant misconception -- a pattern similar to trends in the data for academically low achieving students in Study 1.

Of greater interest were the quantitative and qualitative analyses of the IMA data. Results overwhelmingly indicated exceedingly poor problem solving skills. These students worked an average of 2.4 problems correctly (35 percent correct) with a standard deviation of 1.24. The easiest problems were the ones that had "consistent language" (i.e., the key word was consistent with the operation). Thus, two thirds of the students were successful on Problem 5 (see Figure 5). However, on a similar problem (see Problem 7, Figure 5),which had conflict language, not one student answered the problem correctly. On a second, but linguistically easier, conflict problem in the IMA, only two of the nine students produced the correct answer.

A qualitative analysis of the interview protocols revealed a range of student difficulties in problem solving. Typically, students answered the problems *immediately* after the interviewer finished reading the problem. In many cases, they merely "echoed" the last number read in the problem. Moreover, students tenaciously clung to their first answer despite the interviewer's prompting and evidence which suggested that the answer might have been wrong. For example, when students answered Problem 7 incorrectly, the interviewer restated the problem, pointing out that Jason had *fewer* comic books than Erin. In spite of this explicit prompt and the students' knowledge that 61 was greater than 46, most still claimed that Jason had 61 comic books.

Perhaps the most consistent and disturbing characteristic that students exhibited during the problem solving was a failure to mediate problem solving in any way. As stated earlier, most students responded to problems instantaneously. With the exception of the calculator, they showed little facility in using the tools (e.g., ruler, poker chips) provided as part of the IMA. Most important, when asked how they solved the problem, students would either reply, "I did it in my head," or describe the algorithm used (e.g., addition) rather than how they might have conceptualized the relations between quantities in the problem. These tendencies are clearly reflected in the sample protocol shown in Figure 6. Once the interviewer finishes reading Problem 6, the student answers immediately in this way.

[insert Figure 6 about here]

The student appeared to be jumping from one number to the next. He initially offered 538 as the answer, misreading one of the two numbers that appeared in the problem. He then tried 238, the last number in the problem. Finally, he used the calculator to add the only two numbers explicitly stated in the problem. Each time the interviewer asked for an explanation, the student replied with a new answer. The student's drawing was also revealing. Rather than constructing a pictoral representation of the path walked to fetch the forgotten library book, the student tried to find the correct algorithm. For this student problem solving equals basic computations.

Discussion

Admittedly, the findings from these two studies are not definitive, however they are consistent with major findings from current mathematics education research (Bottge & Hasselbring, 1993; Hiebert & Carpenter, 1992; Lave, 1988; Van Lehn, 1990). Furthermore, we argue that the assumptions behind the direct instruction programs also help explain the results. To be sure, a direct instruction interpretation of the elementary school data from Study 1 would likely concentrate on the inadequacies of the commercial curricula -- its poor range of examples, its excessively massed (but meagerly distributed) practice, and strategies which are far from explicit. This is obviously a more difficult case to make with the middle school data, as students were taught with *Corrective Mathematics: Subtraction*. The following discussion of the studies' findings will be framed around the role of concepts in teaching basic operations and algorithms, and recent conceptualizations of problem solving.

Concepts and Operations

The TORUS analyses from both studies suggest, generally, that low achieving students have difficulty achieving consistently high levels of competence on computational problems. In both traditional and direct programs, academically low achieving students have a hard time breaking through the "70 percent ceiling." As students move from little or no knowledge of regrouping to larger, more complex problems, they tend to move from one type of bug to the next. At the 70 percent level, they exhibit recurrent and varied problems with multiple borrows and zeros.

Current math education researchers would argue that these patterns reflect fundamental problems with the mastery learning model, which plays a central role in direct instruction and other special education strategies, such as curriculum based measurement. A careful analysis of curricula like *Corrective*

Mathematics: Subtraction and *Connecting Math Concepts*, however, reveals that the issue is not one of disfluency per se, but rather the disassociation of computations from concepts. While current math education research indicates that some level of algorithm fluency is desirable for elementary students, difficulties invariably arise when instruction on operations moves quickly from a brief conceptual orientation to having students do little more than apply algorithms to a range of computational problems in a rote fashion.

The direct instruction approach, with its major emphasis on the explicit strategies for *how to do* algorithms, epitomizes this orientation. As Figure 3 indicates, lesson scripts for teaching subtraction direct the teacher toward functional rather than conceptual explanations. Students are shown how to "borrow the 3" rather than explicitly identify 3 tens when they regroup. The problem with this kind of mechanical regrouping is evident in John's error (see Figure 4). Not only does he regroup immediately when he sees adjacent interior zeros (e.g., 4006, 3001, 5009), but he "borrows from 500." And as the Study 1 middle school protocols show, students tend to have little if any conceptual framework for describing their actions.

In addition to problems with the language used to teach operations and algorithms, the intricate massed and distributed practice schemes of the curriculum also diminish opportunities for conceptual development. Lessons are filled with textbook and workbook practice problems, punctuated only by occasional teacher-directed modeling, simple checks for understanding, and directions for the next activity. The teacher's guide strongly recommends that students move quickly from one set of practice activities to the next, with minimal time spent on discussion or additional teacher development activities. This is particularly odd given recent direct instruction issues papers (Carnine, 1991; Carnine, et al., 1994b), which readily cite Porter's (1989) critique of traditional mathematics programs as being too computationally intensive. *Connecting Math Concepts*, *Corrective Mathematics: Subtraction*, and other direct instruction approaches to mathematics are best characterized as computationally intensive.

The conceptual portions of these programs, when they occur, are entirely teacher directed and somewhat one dimensional. As mentioned in section two of this chapter, students in the first level of *Connecting Math Concepts* move quickly from stick figure associations of numbers for numerals to more abstract symbols. Rather than draw ten lines, students write "T" for ten (e.g., 34 = TTT IIII). Students are rarely given the opportunity to represent concepts through multiple representations such as manipulatives or pictures. Alternative methods for solving computational problems -- methods which draw attention to the conceptual foundations of an operation -- are antithetical to the direct instruction method.

In contrast, innovative mathematics approaches recommend that computational practice be frequently integrated with conceptual activities and explanations, and that a range of representational systems be used to provide the foundation for this instruction. Pattern blocks, unifix cubes, diagrams and drawings, along with verbal explanations mutually reinforce an understanding of the concepts that underlie operations and their algorithms. The goal of a multiple representational approach is to build a rich, flexible schema, one that operates as a makeweight to the kinds of misconceptions that were so apparent in Study 1 (Hiebert & Carpenter, 1992).

Finally, innovative math classrooms are filled with mathematical tools. Rulers, calculators, scales, and geoboards are readily available instruments for calculating, measuring, or visualizing mathematical concepts. This notion of appropriate and available mathematical tools stems from the situated cognition literature (Collins, Brown, & Newman, 1989; Lave, 1988), as well as the need to link mathematical understanding *from the beginning* to a child's world (Carpenter, 1985; Usiskin, 1993).

The appropriate balance between conceptual understanding and algorithmic practice for academically low achieving students is a particularly acute issue given what can be accomplished in a fixed amount of instructional time. In as much as these students generally take longer than their average and above average achieving peers to achieve competence on a task, it is questionable whether academically low achieving students really need to become fluent in three and four digit subtraction problems. Even when these students are sufficiently grounded in the relevant concepts, they may be prone to make far too many random errors or what Van Lehn (1983, 1990) calls "slips." With limited instructional time, it seems more prudent to balance computational skills with conceptual understanding and proficiency in the use of calculators.

Problem Solving

It is ironic that direct instruction researchers are so critical of traditional basal math programs, because the nature of problem solving activities in both approaches is virtually identical. Most certainly, direct instruction programs differ in the sequence of problems and how cumulative review techniques enhance discrimination between different types of problems. And as previously mentioned, students are taught to use key words to find "big" and "small" numbers and to associate numbers with the words or units that follow (e.g., 23 miles, 16 yards, 2 hours) and construct ratio equations. Arrows and tables are complementary heuristics for organizing different problem solving exercises. No doubt limited strategies such as key words and numbers/ units have evolved from a "sameness analysis" of traditional math texts. In fact, the notion of sameness analysis as a content-independent instructional design technique contrasts sharply with the cognitive theories which have influenced the nature of problem solving in mathematics today.

The problem with sameness. Direct instruction researchers write about sameness analysis with considerable enthusiasm. At times, the search for sameness appears to be almost phenomenological, with the eventual outcome that information is entirely restructured and connections which were otherwise ignored or unseen are made explicit (Carnine, 1991; Kameenui, 1991). This enthusiasm may also stem, in part, from the claim that the search for sameness is an important characterization of the moment-to-moment activities of the brain (Carnine, 1992).

A more modest description of sameness is that, as an instructional design principle, it is a way of systematically organizing what appears on the surface to be disparate examples (e.g., Engelmann, et al., 1992). Sameness, along with big ideas, makes instruction more coherent.

To the credit of direct instruction curriculum developers, their examinations of traditional basal materials tend to be meticulous and highly empirical. As a working principle, sameness is applied carefully to a restricted class of information. Yet as much as sameness analysis may yield well organized instruction and related activities, its results can also be misleading and highly limiting. This problem with sameness can be seen in an historical analogy from another domain.

Attempts to classify species through comprehensive taxonomic systems, which began with Aristotle, reached a watershed in the eighteenth century. The systems for identifying sameness across plant and animal life developed by Linnaeus and Buffon, for example, reflected a long search for rigorous, logical classification systems (Mayr, 1982). Yet as an avalanche of new life forms began to be discovered in the tropics, these taxonomic systems collapsed. They could not include the array of new species either collected or described by New World explorers. Further reconsideration of established taxonomies was required by advances in technology, particularly the microscope. As Gould (1995) has suggested, the rigorous systems of Linnaeus and others were far from objective. Instead, they reflected deeply held subjective assumptions.

By comparison, if mathematical problem solving consisted solely of two and three sentence word problems which were based on an identification and translation of key words (or problems which directly availed themselves to ratios and tables), the sameness analysis found in direct instruction programs might be adequate. This is clearly not the case.

There is little reason to believe that students who solve problems of the kind found in direct instruction math programs are at all prepared for the kind of problem shown in Figure 7. This problem exemplifies current directions in mathematics and can be found in innovative mathematics curricula (Bell, Bell, & Hartfield, 1993) as well as the current assessment literature (Lesh & Lamon, 1992). It might be added that the push for this kind of problem solving is nothing new, predating the 1989 NCTM Standards by almost a decade (see *Agenda for Action*, NCTM, 1980).

[insert Figure 7 here]

There is also one other significant problem with the direct instruction notion of sameness. Many of the "instant answers" that students provided in the think aloud protocols and IMA problems could be interpreted as typical of the impulsive answers that come from academically low achieving students. While this may be partially true, the behavior also reflects an unfortunate uniformity in the way students in direct instruction math programs answer problems, regardless of whether they are math facts or word problems. This, in turn, stems from the assumption that knowledge is algorithmic, and that students should respond to teacher-directed questions or written problems with convergent answers as quickly as possible.

It is in this sense that there is a curious asymmetry to the entire notion of sameness analysis. That is, for all of the excitement associated with uncovering relationships between concepts (e.g., using "big numbers" and "small numbers" as fact families and a means for representing word problems) the process used by curriculum developers in determining these relations is entirely different from that teachers are supposed to use to teach the concepts. The search for sameness by curriculum developers is distinctly painstaking, analytical, and arguably creative. It is by nature non-algorithmic. Yet once the relations are detected and transformed into a careful series of examples guided by an explicit strategy or rule, one might suspect that the deep linkages between concepts will inadvertently appear to learners as simply more rules and facts.

Again, current research in mathematics suggests that by actively representing concepts in different ways, as well as solving problems like the one presented in Figure 7, students will develop a broader sense of the connections between concepts and comprehend underlying relationships. Moreover, systematic discourse techniques can and should be used to prod student thinking (Ball, 1993; Lampert, 1990). For students with learning disabilities and those at-risk for special education, one could argue that this kind of classroom interaction is critical and must be done with a considerable level of teacher assistance (Reid & Stone, 1991; Tharp & Gallimore, 1988).

<u>An Emerging Alternative to Direct Instruction in Mathematics</u>

The direct instruction literature suggests there are only two instructional alternatives in light of the current directions in mathematics education. The choice is either constructivism, defined as discovery learning, or direct instruction (Carnine, et al., 1994b; Engelmann, et al., 1992). Not only is this a false dilemma, but many leading mathematics and cognitive educators would justifiably dispute this oversimplification of constructivism (e.g., Ball, 1993; Lampert, 1990; Prawat, 1992), we agree that in its extreme form (e.g., von Glaserfeld, 1991) there are many reasons to believe that this form of pedagogy would not be effective with most academically low achieving students.

Fortunately, cognitively oriented efforts in special education (Bos & Anders, 1990; Englert, Raphael, & Anderson, 1992; Palincsar, Anderson, & David, 1993) offer thoughtful frameworks for addressing the needs of academically low achieving students. What follows is a description of our current attempts to apply these frameworks to what mathematics research and curriculum analyses suggest is effective instruction. What we propose differs significantly from direct instruction assumptions about learners and pedagogical practice.

Reconsidering learners. As mentioned in section two of this chapter, an important rationale for direct instruction evolves from a somewhat unified view of the academically low achieving student. Chard and Kameenui (1995) tend to view students with learning disabilities through the lens of multiple deficits (e.g., low memory skills, substandard strategies, irregular attention). Kameenui and Carnine's (in press) while using the term "diverse learner," essentially argue for instructional approaches which "begin at the beginning" and assume the learner knows little or nothing about the subject matter.

It is our view that academically low achieving students are extremely heterogeneous in learner characteristics as well as instructional needs. Many of these students can benefit greatly from innovative approaches to mathematics that parallel the way Englert and her colleagues (Englert & Mariage, 1991; Englert, Raphael, & Anderson, 1992) have adapted innovations in writing instruction to this population. In essence, no one fixed curriculum can provide a satisfactory solution. Moreover, solutions are more likely to come as much from improved pedagogy as they are from revised print or electronic materials.

Reconsidering instruction. Our current research efforts (Woodward & Baxter, 1995) reflect an attempt to distill the central findings from the mathematics education research over the last 15 years, findings which have been discussed throughout this chapter. Our work is grounded in innovative curricula with an emphasis on adapting the materials and pedagogy for academically low achieving students. We believe that mathematics instruction should, *from the beginning*, emphasize conceptual understanding, multiple representational systems, and problem solving in authentic, child-oriented contexts. As for the latter, many children, even academically low achieving students, have considerable but informal mathematical knowledge which can be used as a basis for instruction. These shifts in curricular and pedagogical practice require fundamental changes in classroom practice. In this regard, we strongly disagree with direct instruction advocates who suggest than what happens in the classroom is essentially a-theoretical (see Gersten, Keating, & Irvin, 1995).

For academically low achieving students, our research suggests that teachers should provide more practice on central concepts at the expense of other supplemental or extension exercises which may be a suggested part of the day's lesson. These students need increased opportunities for practice, feedback, and success on a more limited set of exercises than their more capable peers. However, a focus on central concepts is just that: conceptual development, not practice on a fragmented series of skills which are designed to achieve procedural competence alone. Ongoing conceptual development, as Nesher (1986) suggests, is the underpinning of procedural practice. However, we also recommend that all students, irrespective of their learning needs, should get massed and distributed practice on math facts and limited computational problems. For students with considerable learning problems, fluency practice for subtraction may not exceed two digit by two digit problems. Rather, students should become facile in using calculators for these problems as well, and use of tools like calculators should be developed concurrently with paper and pencil skills.

Most of all, problem solving should involve problems which are reflected in the students' world. For young children, this might mean grocery store scenarios, counting or measuring objects in the classroom, and using manipulatives. Problem solving should also incorporate a range of mathematical tools and representational systems (e.g., calculators, scales, pictures). Students should gradually work on problems that have multiple solutions and take more than five minutes to complete. Teachers and capable peers in the classroom can be used to scaffold and assist in the problem solving process.

Achieving this kind of instruction, particularly in mainstreamed classrooms, requires a shift in classroom resources and organization. Students need to be grouped flexibly, both over time and, if necessary, within the lesson. For this, we borrow the deployment strategy from Slavin and his colleagues (Slavin, Madden, Karweit, Livermon, Donlan, 1990). By providing more resources for small group instruction (e.g., a skilled classroom aide, a Chapter I or special education teacher who works jointly with two or more grade level teachers during the math period), there are increased opportunities for teacher led instruction, feedback, and most important, discourse development for academically low achieving students.

For example, with careful teacher assistance and scaffolding, students can be encouraged to discuss how they will plan to solve the problem, how drawings or manipulatives might be used to simplify the problem or represent a concept, and eventually, how one student's solution to a problem may differ from another student's methods and answer. For students who are reluctant to talk -- a common phenomenon in discourse-oriented classrooms -- the teacher must gradually move them from simple, extended speech (i.e., one or two word answers to complete sentences and longer descriptions) to more detailed explanations, and ultimately, to argumentation. Such discourse development can occur in small homogeneous group settings, as well as large group or whole class instruction. These scaffolding techniques are in line with reciprocal teaching and interactive instructional methods discussed elsewhere in the special education literature (see Bos & Anders, 1990; Palincsar & Brown, 1986; Reid & Stone, 1991).

Closing Remarks

For decades, special educators have adapted instruction for academically low achieving students by drawing on broad, content independent strategies. Task analysis has been a particularly appealing technique which has been applied to mathematics because the subject matter has been viewed primarily as basic arithmetic with a clear, step-by-step hierarchy. Research within mathematics education, however, presents a fundamentally different vision of the subject. In this view, mathematics has moved far beyond basic skills --- not just because of advances in research -- but because of the increasing role of common and increasingly inexpensive technologies as day-to-day mathematical tools.

Advances in mathematics research and the growing presence of calculators and computers present a significant challenge to common practices typically found in remedial and special education. It would be awkward, if not arrogant, to suggest that the innovations in mathematics which have been described throughout this chapter, ones which have a surprisingly high level of consensus within the mathematics education community (see Putnam, Lampert, & Peterson, 1990), are fundamentally wrong. As we have argued throughout this chapter, there is much more to the current mathematics reform than the 1989 NCTM Standards.

It is also be difficult to see how, irrespective of approach -- one predicated on basic skills or one based on the type of innovative methods described above -- students would somehow "reach the same place" over time. In other words, it does not seem reasonable to assume that after a thorough mastery of computations and traditional problem solving, academically low achieving students will somehow acquire the competence to work on the kind of ill-defined problems represented in Figure 7. Findings from cognitive research on problem solving and transfer (e.g., Gick & Holyoak, 1983; Lave, 1988), as well as the results from the two studies described in this chapter, suggest that a common outcome is doubtful. Instead, it appears that the current state of mathematics research and innovative curriculum development, along with the increasing presence of technology argues strongly for a serious reconsideration of content independent curricular approaches to math instruction.

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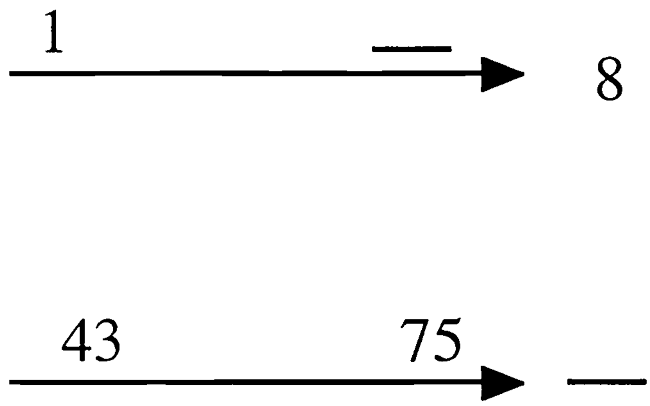


Figure 1. Direct instruction heuristic for associating numbers in a problem.

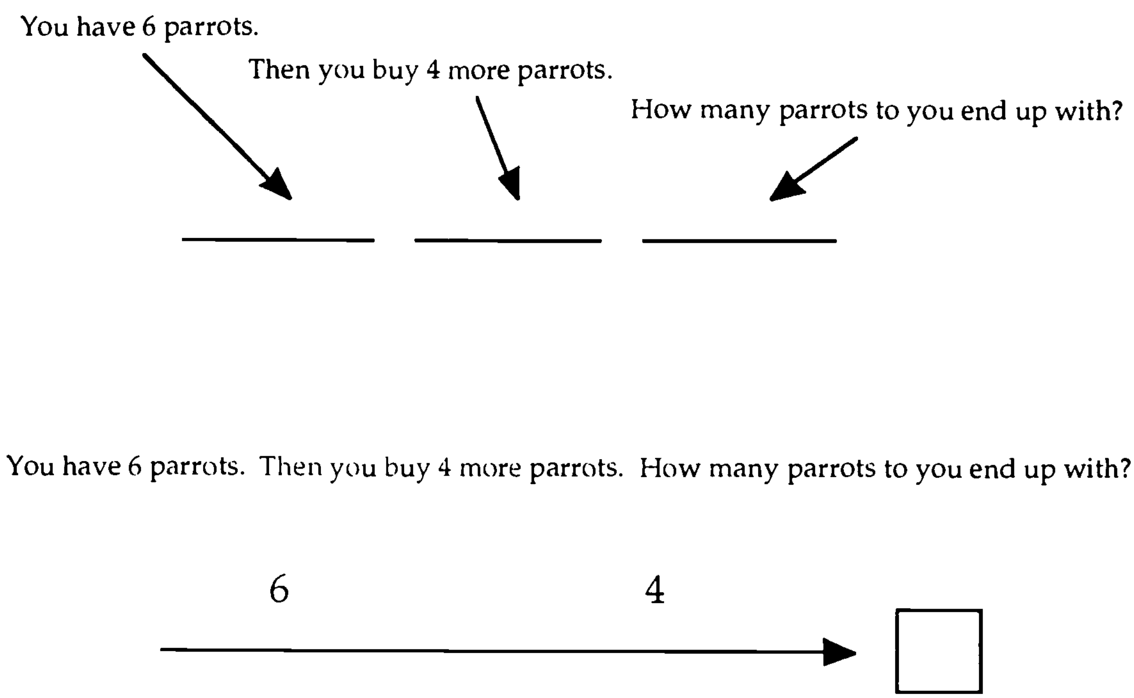


Figure 2. Explicit strategy for solving simple word problems.

$$\begin{array}{r} 5\,{}^{3}\cancel{4}\,{}^{1}3 \\ \underline{-\,1\,2\,9} \end{array}$$

Figure 3. Regrouping in a direct instruction program.

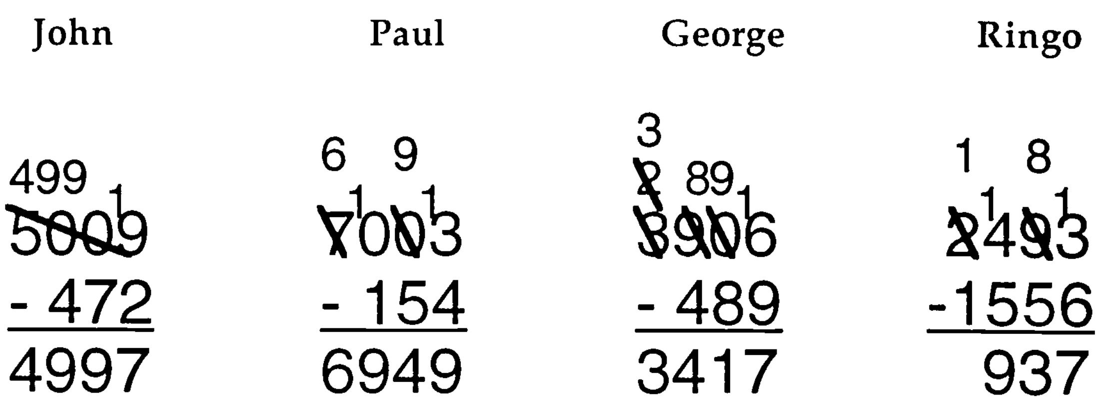


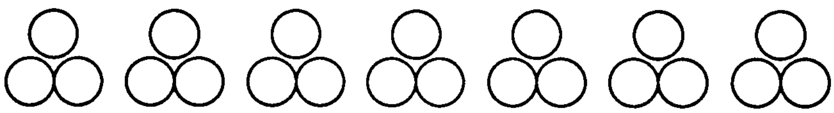
Figure 4. Four different types of bugs.

Problem 1

Here's a pattern. What comes next?

Problem 2

How many balls are in this picture?



Problem 5

Lindsey has 57 tennis balls. If Lindsey gets 16 more, she will have as many as Morgan. How many tennis balls does Morgan have?

Problem 6

Your friend Kelly is in the other 3rd grade class. Kelly just got this problem for homework and has no idea how to do the problem. How would you solve the problem?

Homework problem: It is 534 footsteps from my house to school. I left for school, but after I walked 238 footsteps, I remembered that I had forgotten my overdue library book. I returned home, got the book, and went to school.

What distance did I walk?

Problem 7

Erin has 46 comic books. He has 15 more comic books than Jason has. How many comic books does Jason have?

Figure 5. Sample Problems from the Modified IMA

(Interviewer reads the problem to the student.)

Student: "538."

Interviewer: "538 footsteps I walked?"

Student: "Yeah."

Interviewer: "How did you figure that out?"

Student: "Because it's right there -- 238." (Points to the number in the problem.)

Interviewer: "238?"

Student: "Yeah." (Picks up the calculator, shows the answer, puts it down.) "So then 772 is the answer."

Interviewer: "772. Can you show me this problem using a drawing?"

Student: "How did you want me to draw?"

Interviewer: "Can you draw the problem out on paper? However you think it should look."

Student: (Writes on yellow paper 534 + 238 = 772 horizontally.)

Figure 6, Sample Student Response to Problem 6 of the Modified IMA

Students are given the following information:

Two gunmen hold up the a bank 30 minutes after it opens in the morning. One was a middle-aged man, who pointed a large gun at a frightened teller and demanded cash. Another man held his hand tightly in his pocket holding what also appeared to be a large gun. The tellers were told to fill a laundry sack which the robbers gave them. The tellers then filled the sack with money, most of it in small bills. The two men escaped just minutes before police arrived.

While the bank manager complained that the police didn't respond fast enough, the police claimed that the bank's silent alarm did not go off during the robbery. They learned of the robbery from a customer who was about to enter the bank when the robbery was in progress. Instead, the customer ran next door and phoned the police.

The bank manager stated that, according to his accounting, the amount stolen was close to a million dollars. Fortunately, the amount will be covered by insurance. The manager is asking for any assistance in solving the crime and is posting a $3,000 reward for information leading to the arrest of the two gun men.

Could the events really have happened as they have been reported? Could the two robbers escape with a million dollars in small bills in a laundry sack? Analyze the situation. What suggestions would you offer for solving this crime? Write a brief report to the police assigned to the case explaining your reasoning. Give enough details so they will understand.

Figure 7. The bank robbery problem.

Table 1

Means and Standard Deviations for the Torus Test

		October			Feburary			May		
	N	M	SD	Mean % Correct	M	SD	Mean % Correct	M	SD	Mean % Correct
Instructional Group										
Third Grade	72	8.0	5.48	20	27.3	11.53	68	30.8	8.79	77
Low	28	5.9	2.60	15	24.1	13.06	60	27.0	10.79	68
Average	31	6.9	3.47	17	28.1	10.28	70	33.0	7.11	83
High	13	15.1	8.11	38	32.2	9.29	81	33.5	3.8	84
Fourth Grade	61	16.4	12.32	41	28.9	10.04	72	28.9	10.93	72
Low	20	8.0	6.74	20	23.8	10.39	60	22.9	11.47	57
Average	29	16.9	10.48	42	31.1	9.62	78	30.0	10.43	75
High	11	29.3	13.01	73	32.2	7.63	81	36.6	4.20	92
Middle School	10	18.2	13.67	46	26.6	6.00	67	27.1	8.56	68

It's What You Take for Granted When You Take Nothing for Granted:

The Problems with General Principles of Instructional Design

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DRAFT

Technical Report:
Action-based Research on Innovative Mathematics Instruction and Students with Special Needs

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May 13, 1996

TECHNICAL REPORT

Purpose of the research

The need to understand mathematics at a conceptual level is one of the defining characteristics of an information society. Rapid advances in technology have significantly decreased the value of efficient computational skills. Reform efforts such as the National Council of Teachers of Mathematics (NCTM) and the America 2000 initiative reflect the shift toward a conceptual approach to teaching math. Many special educators are now realizing that the changing world of work -- and more immediately, the main streamed classroom -- make mathematics a crucial instructional issue for students with learning disabilities.

In an earlier study we found that both high ability and average students performed well when taught with an innovative mathematics curriculum; however, students with special needs did not perform well when main streamed in these classrooms (Woodward & Baxter). Our conclusion was that additional resources were critical for these students to learn mathematics. For the present study we investigated the impact of a second teacher in the main streamed classroom. Although two teachers in a classroom is hardly realistic in these days of declining school budgets, we wanted to see the techniques that teachers used when they did have the time to work with special needs students.

The purpose of this study was to examine the effects of an innovative approach to mathematics coupled with additional resources (i.e., a second teacher) on academic performance of main streamed students with learning disabilities and academically low achieving students who are at-risk for special education. This research was part of a case study of teachers in an elementary school that was working to implement the mathematics reform. One fourth grade classroom was the focus of systematic observations, teacher and student interviews, and academic assessment. Quantitative as well as qualitative data were collected in the research design to triangulate on the effects of innovative curriculum and teaching techniques on target students
(see [Patton, 1980 #138]).

METHOD

Setting

School. The study was conducted in an elementary school located in the Pacific Northwest. The school served 200 to 220 students in grades K through 5. Students from a range of socio-economic backgrounds attended the school.

Participants

Teachers. The participants in this study were two fourth grade teachers and their students. Both teachers were experienced elementary teachers, each having taught for more than 20 years. Both teachers had completed extensive in-service programs in an innovative mathematics curriculum. One of the teachers conducted in-service workshops. The two teachers had taught as a team for a number of years at the elementary school. During that time they had developed what they called "an eclectic" approach to mathematics instruction. Neither teacher had found a mathematics curriculum at the fourth grade level that addressed the topics they valued in a way that they agreed with. Thus, they used a number of mathematics curricula to teach the topics that they felt were critical for fourth graders. They emphasized problem solving and de-emphasized computation. They also encouraged small group work and the sharing of alternative solutions to problems. The topics that they identified as critical for the year were the following:

problem solving heuristics,
Geoboards (spatial reasoning and geometry)
fractions
decimals
multiplication
game theory

One teacher, Sarah, had primary responsibility for the class, while the other teacher, Kathryn, team-taught mathematics (40 minutes three times a week). The school in which these two teachers taught served primarily middle class (determined by the very low number of students on free or reduced lunch) students.

Students. A total of 28 fourth grade students participated in this year long study. Three students were classified as learning disabled on their IEPs, and they were receiving special education services for mathematics in main streamed settings. It should be noted that interviews with teachers indicated that more students could have been referred for special education services in mathematics but students were not for a variety of reasons. Some teachers mentioned that the special education teacher primarily served low incidence students (e.g., autistic, students with physical disabilities) or students who had reading problems. There was "little room left" to serve students for math. Consequently, a wider pool of students was selected as a focus for this study.

The mathematics subtest of the ITBS, which was administered in October, was used as a basis for identifying additional students who were at-risk for special education services in mathematics. The 34th percentile was used as a criterion for selecting these students. In addition to the three students with learning disabilities, six other students were identified based on total subtest performance on the ITBS. This resulted in a total of nine students who were considered academically low achieving in mathematics or identified as having a learning disability in mathematics.

Materials

The mathematics program emphasizes a series of important NCTM Standards. Students spend a considerable amount of time identifying patterns, estimating, and developing number sense. Multiple solutions for problems are encouraged and discussed. Finally, an array of math tools and manipulatives -- calculators, scales, measuring devices, unifix cubes -- are considered an important part of the daily lessons.

Procedures

Observational, interview, and academic performance data were collected over the 1994-95 school year. All fourth grade students were administered the mathematics subtest of the Iowa Test of Basic Skills during the third week in September and again in the last week of April. ITBS problem solving subtest and total test scores were used as a basis for determining a stratified sample of fourth graders. A stratified random sample of students was selected for individualized testing on a measure described below. This test, the Informal Mathematics Assessment, was administered to a total of 25 students during mid-October and again during the first week of May.

The teachers were systematically observed two to three times per week throughout the course of the year. Researchers interviewed the teachers informally during the year and formally in June at the end of school.

Measures. Two different measures were administered to assess students' understanding of mathematics. The fourth grade level (Form G) of the Iowa Test of Basic Skills, was used as both a pretest and as a post test. The norm referenced test has well documented reliability and validity. It is a highly traditional, multiple choice form of assessment that measures computations, concepts, and problem solving skills.

The second measure, the Informal Mathematics Assessment (IMA), was an individually administered test of problem solving abilities. The intent of this measure was to examine the problem solving processes or strategies a student used in deriving an answer as well as the answer itself. In this respect, it is consistent with the call for assessment that is more closely aligned with math reform and the NCTM standards (Romberg, 1990). Students were also given a range of mathematical tools and representations that they were encouraged to use as part of the problem solving. The IMA "tool kit" included a calculator, ruler, paper and pencil, poker chips, and number squares with ones, tens, and hundreds values.

The six items on the test were based on an analysis of fourth grade mathematics texts -- innovative materials that subscribe to the 1989 NCTM Standards as well as more traditional texts. In order to prevent fatigue and possible frustration, particularly with academically low achieving students, each item on the IMA was relatively brief and the examiner read each item to the student. While the IMA took approximately 15 minutes to administer, students were given as much time as they wanted to complete each item. Alternate form reliability for the pre- and post test versions of this measure was .87. Figure 1 presents a word problem from the IMA. As with other word problems on the test, it was written to exclude key words (e.g., each and every often are taught as key words that signify multiplication or division). After the examiner read the problem to the student, s/he carefully noted if the student reread the problem, what calculations were made, and what tools or manipulatives were used. Finally, s/he probed the student, asking, "Tell me how you got that answer." All sessions were tape recorded and transcribed for later scoring and qualitative analysis. Individual student protocols were scored with a rubric that was analytically derived from the NCTM standards and related literature on innovative mathematics assessment ([Lesh, 1992 #142]). A five point scale was used for each item, with the highest score reflecting both the student's answer as well as the process used to derive the answer. Inter-rater reliability for scoring the student protocols was .93.

Finally, the IMA protocols were subjected to a categorical analysis. Researchers examined student answers in an effort to classify different kinds of problem solving behavior. The extent to which students used manipulatives provided in the tool kit (particularly paper, pencil, and calculators) or what strategies they used to solve problems (guessing, using numbers provided in the problem in random order, decomposing problems into subunits) were analyzed. Inter-rater reliability for the categorical analysis was .88.

Classroom Observations. Twenty-five observations of the math class were made by the two authors between November 15, 1994 and April 30, 1995. They ranged in length from 45 minutes to 1 hour. The observer sat in back of the classroom and took notes, occasionally moving about the room to observe student work or student teacher interactions when it was appropriate, e.g. when students worked on their own or in groups and the teacher(s) also moved about the classroom. The observer did not interact with students in a teacher role. Descriptive field notes were constructed from the classroom notes, yielding 145 pages of data.

Interviews. A total of 8 open-ended interviews were conducted with the teacher (4 times) and 2nd teacher (4 times). The interviews were conducted by the first author and ranged in length from 30 minutes to 2 hours. The interviews were semi-structured in that a general interview guide was generated for each interview based on themes emerging from the analysis of field notes and questions arising from the observations. They were audio taped and transcribed verbatim, yielding 172 pages of data.

Analysis of Interview and observational data. Field notes and interview transcripts were shared between the researchers as they were written. The data analysis followed a constant comparative method(Glaser & Strauss, 1967). As data were collected, they were read and coded with descriptive labels. Some labels were suggested by the focus of the study (e.g. "teacher assistance"). Other labels emerged from the data (e.g. "homework"). As data collection continued, data were grouped into coding categories centered around these labels. This process included developing definitions of the coding categories, then comparing subsequent pieces of data to existing definitions to determine suitability for inclusion or exclusion in an existing category. Categories were internally refined as data collection continued. The coding process yielded analytic questions about emerging issues and relationships among the coding categories (Shelly & Siebert, 1992). These questions led to further exploration and examination of the data, as well as to formulating questions for the teacher interviews. This cyclical process was aided by the use of a data management program, The Ethnograph (Seidel, Kjolseth, & Seymour, 1985)

RESULTS

Data for this study were analyzed quantitatively and qualitatively. The quantitative data provided a broad framework for gauging the relative changes in academic performance for students. This was particularly important as two different types of academic measures were used to assess growth in mathematics. The protocols from the IMA, along with classroom observations and teacher interviews enabled a qualitative analysis of the effects of the innovative curriculum on students with learning disabilities and academically low achieving students.

Student outcomes on the ITBS and the IMA are currently being analyzed.

Teacher Strategies

The observational data revealed that the classroom teacher and the "assistant teacher" used multiple strategies to teach math in ways that would benefit all the students, including the slow learners. These strategies can be divided into two broad categories labeled 1) classroom organization and 2) math pedagogy content. Each of these broad categories is further subdivided into several areas.

Classroom Organization

Pair work. The teachers often gave the students exercises or problems to be worked out in pairs or small groups of 3 to 4 students. Since the seating structure of the class was organized into groups of 4 to begin with, working in two's or four's was not disruptive. Students are often given the direction to, "check your work with your partner before continuing" to encourage pair collaboration. Pair work allowed the teacher and second teacher to move throughout the classroom while the students worked. They checked on the progress of some students, repeated instructions to the low achievers and often got them started on the task at hand. Generally, they provided assistance where ever needed. When only one teacher was in the class it was an especially useful way of giving more attention to low achievers. When both teachers were available, one teacher stayed close to the low achievers while the other moved throughout the room.

Example: Fillmore #11, lines 260-333

Individual tutorial. The teachers often divided their teaching tasks: one teacher working with the whole group while the second teacher circulated among the students providing individual attention. The teacher working with individual students typically worked with a student for 5 to 10 minutes, asking questions and explaining as needed. For example, one day Sarah stopped to work with Daniel who was having trouble with a lesson on fractions.

> Sarah stopped to check on Daniel. Daniel had plotted 1/5 very close to the zero on the number line.
>
> Sarah asked Daniel, "If this bar was divided into 5 equal pieces would 1/5 be way over here?" Daniel looked at the number line. Sarah touched his paper using two fingers to show about how long a 1/5 subsection of the number line would be. She then jumped her finger along the number line 5 times and said, "Think about how big the pieces are." A light bulb went on for Daniel. He smiled and erased the 1/5. He then moved it closer to the 1-about one-fifth the distance from zero to one. (1/19/95)

In this exchange Sarah drew Daniel's attention to an important point in the lesson: students were to have some sense of the relative value of various fractions by placing them on a number line. Sarah provided a focus and offered a new way to think about fractions, that is dividing the bar into five equal pieces. Sarah left Daniel but returned in a few minutes, when the following exchange occurred.

> At 11:05 Sarah stopped to help Daniel. She asked him to pick a fraction. He picked 8 and 1/2. Sarah then asked, "where do you put the dot for 8 and 1/2?" (Here Sarah was asking Daniel where he would place 8 and 1/2 on the number line.) Daniel correctly plotted the point. Sarah smiled and told him, "Very good." Then she asked him to do 2 and 1/3, repeating her question, "Where do you put the dot?" He correctly placed the dot. Then Sarah asked him where to place the dot for 4 and 9/10. Again, Daniel correctly placed the dot. Sarah congratulated him and started to move on, when Kathryn walked up and glanced at Daniel's paper. Kathryn asked him to label the dots, as Daniel had only drawn dots rather than writing the name of the fraction as well. Daniel looked overwhelmed by Kathryn's request, "I don't remember them all." Sarah moved back to Daniel and said, "look at the dots and see if you can remember, figure them out." Sarah then moved on. Daniel looked at the 4 and 9/10 dot long and hard and then wrote 4 and 9/10. Kathryn pointed to his paper and said, "Good, you got it right up close." Daniel still appeared to be overwhelmed by the memory task, telling Kathryn, "I have to remember these." Kathryn told him to make some up. She pointed to another dot and asked him,

"What could that be?" Daniel was silent. Kathryn prompted, "Is it greater than 1/2 or less than 1/2?" Daniel looked at his paper and said, "Less." He then wrote 2 and 1/3, which was correct.

This episode is quite revealing. Sarah clearly returned to "test" Daniel and see how well he was understanding the relative position of the fractions. She smiled and praised his work. When the second teacher reminded Daniel to label the points that he had drawn for Sarah, Sarah returned to Daniel. She apparently realized that the other teacher had significantly increased the difficulty of the task for Daniel. Initially, Daniel was frustrated (he translated the labeling request as a memory task) and quit working, but Sarah returned to offer support. She made a general suggestion, "look at the dots and see if you can remember, figure them out," and then moved on. Sarah moved from specific questions such as, "If this bar was divided into 5 equal pieces would 1/5 be way over here?" to general prompts, tailoring her support to Daniel's needs. Sarah appeared to be quite skillful at offering only enough support to get a "stalled" student moving again.

In another episode Sarah again, helped a small group of students, but in this situation she began with a general question and then moved to more specific questions. The two episodes offer an interesting contrast in Sarah's tutorials. In this episode the students were working in small groups to solve a problem (see attached sheet p. 43 from Getting It Together). Four girls, all identified as low ability in math, were working together. They read their clues to each other and moved fava beans (substitutes for the M&M's in the problem). They all leaned forward and were eagerly reading their clues and touching the beans, when two of the girls, Joni and Rianna, were called from the room. The two remaining girls, Jennifer and Kisha, stopped work in frustration:

> Jennifer and Kisha groaned as half of their group left. Sarah quickly walked over and asked them, "Where are you on this one? "What do you know? What else do you know?" Jennifer was clearly frustrated and said that she didn't know anything. Sarah turned to Kisha and asked her to read one of her clues. Kisha did and Jennifer was quickly drawn into the problem again. (3/15/95:3)
>
> Even though the two girls were actively involved in the problem, Sarah stayed by their desks watching silently. At one point Jennifer noted, "But if we take one of these, these three won't equal what we want them to equal." Kisha looked puzzled by Jennifer's comment. Both girls appeared confused about what to do next. After a few moments Jennifer started to read clues again. Sarah did not intervene at this point, but she continued to watch the girls from a short distance. Later Jennifer concluded that eight beans were needed. Sarah asked her, "What's twice as many as 6?" Jennifer said, "8." Sarah then

asked, "What's 6 plus 6?" Jennifer smiled and thumped her head with her hand seeing her error, "12." (3/15/95:3)

In this exchange, Sarah offered very specific guidance. Jennifer, as a fourth grader, was not yet accurate with her single digit math facts. Sarah was aware of Jennifer's difficulty with math facts, as Jennifer had scored poorly on a series of timed math facts tests. Sarah stopped Jennifer, so that a computational error would not thwart her problem solving efforts. Sarah's decision not to intervene when Jennifer and Kisha became stalled is also noteworthy. By remaining silent she allowed the girls to decide on their next step (i.e., reread their clues), rather than having the teacher step in to show them the way to a solution.

In summary, Sarah's individual tutorials are strikingly flexible. She did not have one set of questions or procedures for tutoring students, rather she varied her questions and comments given each situation. When Sarah had a good idea of what was causing a student difficulty, she asked specific questions. In the exchange with Daniel, she looked at Daniel's number line and assumed that he did not understand the part whole relationship when thinking about one fifth. She then asked him a pointed question, "If this bar was divided into 5 equal pieces would 1/5 be way over here?" Again, when Jennifer made a computational error, Sarah quickly asked questions to help Jennifer find the correct sum. But Sarah's skill is not only with specific questions, she effectively remains silent as well. Sarah was able to identify situations where the students could struggle without becoming so frustrated that they gave up. In the exchange between Jennifer and Kisha, Jennifer sensed that removing three beans would cause problems, but she was not certain what to do next. Kisha was confused by Jennifer's concern. Sarah watched the exchange and allowed the girls to find their own next step, which they were able to do.

Classroom seating arrangement. At the beginning of the year, the seating arrangement appeared to be random??. The fourth graders from another homeroom who came into the class for math sat at the empty desks belonging to the fifth graders who left for their math class. By mid-January, however, the teachers instituted a seating chart based on ability grouping. The new arrangement was based on groups of 4, each group consisting of 4 desks that faced each other in a square. The low achievers were together in two groups of 4. One of these consisted of 4 girls and the other consisted of 3 girls and one boy. These two groups were side-by side, allowing a teacher to move easily between them. At the back of the class were two groups of the brightest students, each group consisting of 2 girls and 2 boys. The middle and front of the class contained the remaining students. In devising this seating arrangement, the teachers took into account ability, but also tried to balance gender among the groups. By the end of the year, the teachers thought that the ability grouping worked well:

...the top kids were able to take their lessons further and challenge each other in ways they wouldn't have been able to if

> they didn't have other top thinkers with them. The little slower kids, they were able to not just buy out of it and sit back and do nothing. They did have to do it, and by having the aide in here working with those slow kids, they couldn't just stop and do nothing which they are really inclined to do...

By working in small groups of students of about the same ability, the students couldn't sit back and wait for the brighter students to figure out problems. One teacher's observed that one of the slower students, Daniel, for example, grew in self-confidence:

> Had he been with his friends, they would have figured it out for him and he would have let them. He was at a team where he turned out to be one of the real power thinkers. It was nice he became a leader in his team. I think he has real confidence in has ability...He doesn't buy out like he use to.

Instructional groups. Towards the end of the school year (April) the teachers implemented a new strategy for dealing with the wide range of diverse abilities in the classroom. They divided the class into two instructional groups. One group of 18 students contained the average learners and low achievers. The other group consisted of 8 of the brightest students. The teacher worked with the latter group on the same instructional material, but at a faster pace, using more challenging questions and materials. This group first met at a table in the back of the classroom, but soon moved to the school's library. The larger group was taught by the second teacher. One teacher's explanation for the group had to do with meeting the needs of the brightest students:

> Mainly, it had to do with the top kids loosing interest in the pacing. It was hard for both of us to try to keep them with the other kids. They couldn't stand to take direction that slowly and their motivation and enthusiasm in math was starting to wane because of it. When we decided that was an okay thing to do, we tried to pull off the kids that we thought didn't need to go through step by step. There are some kids that are good in math, but they do benefit by hearing that step by step. Otherwise, they'll jump and end up making big mistakes.

The classroom observations confirmed that several of the bright students at the back two tables were showing signs of boredom indicated by making sarcastic remarks about the lesson, working on other projects, and talking more frequently among themselves.

Use of Instructional Aide. Two of the students, Joni and Rianna had an instructional aide, Diana, assigned to them for mathematics. Diana provided assistance to the students in several ways. She kept them on track during lessons, for example, noticing when Rianna is not paying attention and redirecting her back to the discussion going on in the classroom. She provided a lot of encouragement to the students and would occasionally drill them in their math facts. For example, when a work sheet was sent home with the students for multiplication math facts, Diana looked at Joni and Rianna's results and made up some flash cards for them to use. Lastly, she helped them to focus on homework assignments. Sometimes at the end of class, she took the two students into the instructional materials center across the hallway and reviewed with them the assignment for homework. After the students were grouped into ability groups, Diana extended her assistance to Joni and Rianna's group mates, Jennifer and Kisha.

Math Pedagogy

Through our observations we identified a number of features of the teachers' instruction that are in the spirit of the mathematics reform. These features include:

1. frequent use of manipulatives to explain topics and help students solve problems,
2. an emphasis on problem solving and de-emphasis on computation
3. inclusion of nonstandard topics such as geometry and game theory
4. frequent opportunities to talk about their mathematical thinking.

What is noteworthy are the ways that the teachers used these techniques to support the efforts of low ability students.

Frequent use of Manipulatives

The teachers used manipulatives in the majority of lessons. The problem solving lessons typically included beans or some other counting object, the fraction lessons were based on colored Fraction Bars, the decimal lessons relied on colored Decimal Squares, and the spatial reasoning lessons centered on Geoboards. Only the lessons on the function machine did not include manipulatives, instead, the function machine lessons relied on pictoral representations: graphs, charts, and a function machine diagram with arrows. In addition, the class acted out a function machine using large labels. Clearly, the students had the opportunity to use manipulatives when thinking about mathematical ideas.

But even when manipulatives are present and used by the teacher, they may not be used by students, especially low ability students who tend to watch as more skilled students lead the way (Woodward & Baxter). In the present study, we found that the teachers used a variety of strategies to encourage students to use manipulatives to support their thinking:

- modeling,
- homogeneous pairs and small groups, and
- monitoring of engagement by a classroom aide.
- games

Modeling. Both teachers used manipulatives to demonstrate mathematical ideas. Working at the overhead projector, they repeatedly used objects to help clarify their mathematical reasoning. In addition, the teachers frequently called on students to share their strategies at the overhead projector. Then a student or small group of students would draw or use objects to explain their solution to a problem. The students at the overhead received feedback from their peers as well as the teacher and the students listening again, saw manipulatives used to help think about mathematics.

Both teachers also used manipulatives when they worked with individuals or small groups of students. When trying to explain a concept both of the teachers would reach for the manipulatives to clarify a point or they would ask a student to answer a specific question using manipulatives. The teachers asked students to "show me" as often as they asked students to "tell me."

> At 10:40 Kathryn told the students that she was going to give them the directions needed to play a game called Match. Students take fraction bars of different colors and divide them into two piles. They then place bars in the center. If they find two bars that match (e.g., 1/2 and 2/4) then they can keep those two bars. Kathryn stressed that some of the tables would not use bars, they would only use cards that had the fractions written on them with no pictorial representation. Kathryn repeated that the tables in the back would not use bars, just cards. The other tables in the class would use bars.

In this episode, the teacher, Kathryn, is using the manipulatives in a game format to address the different ability levels in the class. She encouraged students who were comfortable with the relationships among the different fractions (e.g., one fourth is greater than one fifth) to use cards with only symbolic notations, no pictures. On the other hand, students who did not yet understand the relationships among the different fractions were directed to use the colored Fraction Bars. All of the children in the class played the same game, but by varying the representations used (symbolic versus manipulative) the teacher tried to challenge the different ability levels in her class.

Homogeneous Pairs and Small Groups. Part way through the school year the teachers decided to rearrange the students and place students of similar ability at the same tables. One concern that the teachers expressed was that the low ability students were letting the high ability students do all of the thinking. They intentionally wanted to group the students by ability. The teachers formed two groups of high ability students, two groups of low ability students and four groups of average ability. Whenever possible the teachers balanced the groups by gender.

Initially, the low ability groups required a good deal of adult (one of the teachers or the aide) encouragement. If an adult was not present then the students tended to sit quietly and make little progress on their group work. But after a while the low ability students began to form productive groups. Joni emerged as the initiator in her group. She would read problems out loud and urge the three other girls in her group to help out. She appeared to be highly motivated and by the end of the year, she and her group were able to work for five to ten minutes without adult encouragement. Daniel emerged as the leader of his group of three. He listened to the two girls in his group and asked questions when he didn't understand what someone else was doing. By the end of the year all of the students in these two groups touched and handled the manipulatives to solve problems or explain their thinking. This represents a significant change from earlier studies (Baxter & Woodward), in which low ability students sat back and watched other students handle manipulatives or the low ability students managed the manipulatives, handing them to higher ability students as they solved a problem. We saw clear evidence of "hands-on" as well as "minds on" work by the low ability students.

Monitoring of Engagement. Both teachers used manipulatives to monitor how involved students were in a particular problem. The vast majority of problems that the teachers gave the students required them to use manipulatives. The teachers could then walk around the room and watch the progress (or frustration) of groups of students by noting how they were using their manipulatives. For example, in one lesson four girls were diligently trying to solve a problem using Fava beans. Sarah stopped to observe their work. The girls kept counting and recounting their piles of beans, not always agreeing on how many beans were in a particular pile. Sarah interrupted and pointed out to the girls that some of their beans had split in half, thus causing confusion as to half beans versus whole beans. She suggested that they trade their broken beans for whole beans, so they could keep their totals straight.

During the lessons on fractions and decimals, the classroom aide was able to stay after class had ended and work with two of the students. Typically, the aide took the two girls into a separate room for the last 10 to 15 minutes of class. There she and the students worked through problems that the class was solving in small groups. The room was quiet and free of distractions, so the students settled down to work quickly and worked many more repetitions of a procedure than they would have been able to complete in class. The aide stopped them frequently and asked them questions to check their understanding.

> Diane, the aide, took Joni and Rianna into the IMC to work. The girls spread out at table in the back of the IMC and began to play Match. Joni was very excited about the game. She diligently checked her bars to see if she could find a match. She never seemed to find any patterns that would speed up her search (e.g., the orange tenths bars never match with the white twelfths bars). Rianna was involved with the game, but she simply scanned the bars quickly for a match, she didn't place her bar next to each of the other bars.

At times Diane would pull back two bars that one of the girls said was a match and ask them to check again. Diane was clearly doing the symbolic changes in her head (i.e., converting 4/5 into tenths to see if it matched the 8/10 bar), while the girls were simply lining the bars up next to each other to see if they matched. They never really discussed the close calls or looked for patterns that would make their comparisons easier.

> The IMC was quiet and Joni and Rianna worked the entire time. They were relaxed and able to concentrate. They did not benefit (or feel pressured) by the discoveries of their peers (e.g., after class Ann told me that one of the students in the class had figured out that the orange and white bars never match).
> [OBSERVATION 1/17/95: p. 3]

In this episode, the instructional aide used the manipulatives to help the girls "see" the relationship between different fractions. She physically held two colored fraction bars next to each other and then had the girls compare shaded regions.

Games. Games were an integral part of the mathematics program. Sarah began the school year with Math Quest, a board game played by groups of students. The purpose of Math Quest is to move along a path by solving open-ended problems. Both skill and luck are involved, so all of the students were quite motivated. Each day a problem was given to each small group of students. The students had to solve the problem and then explain their solution to the class. The group then drew a card to advance on the board. At this point luck entered as the card might tell them to skip a turn, or go backwards on the board.

During the lessons on fractions and decimals, the teachers allowed time for games that helped the students practice ordering, adding, subtracting and understanding these concepts. (See example above)

Curricula.

Math Quest
Get It Together
Lane County Problem Solving
Geoboards
Real Math (functions and graphing)
Fraction Bars
Decimal Squares

Emphasis on Problem Solving and de-emphasis on computation

Problem solving integrated throughout the year. Problem solving was a major theme of the teachers' mathematics program. They began the year with 3 weeks of group problem solving and specific instruction of various problem solving heuristics. Throughout the year the teachers incorporated the problem solving heuristics into each mathematical topic, reminding the students to try different problem solving strategies whenever they were stumped by a problem. For example, when working on place value, Sarah directed the students to make a chart and then record successive rolls of three dice. She told the students make a three digit number from each roll of the dice. After five rolls of the three dice each child had five three-digit numbers. Sarah then told the students to add up their numbers and see who is closest to the number 2005.

> Voices ring out, "Oh, no. I went over." Sarah: "What did you learn from this, what are you going to do next time?" She asked some of the students for their number. One responded with 1912 and another with 1993. Sarah asked, "how far off is James from 2005? Is anyone closer that 12 away? How many went over?" Sarah asked, "Eric, what is your strategy for next time?" Eric replied that he did all the biggest numbers and went over. Sarah said, "I'm going to challenge you. Tomorrow we'll play this game and think what you can do to get close to 2005." Kathryn added, "What strategy will you use?" Sarah: "I'll let you stew on that for awhile." (observation #3)

In this episode, Sarah and Kathryn worked together to create an open-ended problem that the students not be able to solve quickly. The best strategy for winning the game was not immediately apparent, in fact, more than one strategy might lead to winning the game. In addition, the students were also encouraged to think about the problem for longer than one or two minutes. Too often students assume, based on their experience in math class, that problems can be solved in a few minutes. Spending a day or two working on a problem is a novel experience and one that many students rarely encounter.

Homework. The teachers assigned homework on a regular basis, but for a variety of reasons. At the beginning of the school year the homework did not directly relate to the topics addressed in class. Problem solving heuristics were the main focus of classroom lessons for the first part of the year, occasionally the teachers asked students to work on a problem that emphasized one of the heuristics studied in class, but more often the homework was multi-digit addition, subtraction, multiplication or division. The teachers used the homework to help students review their computational skills without taking lots of class time.

During the lessons on fractions and decimals the homework assignments were closely related to class lessons. Lessons typically began with a group correction of homework. It was not clear how diligent the students were in completing homework assignments. One student, Rianna, surreptitiously completed her homework during the group check. Kathryn commented on this a few times, but Rianna continued to fill in unfinished answers.

Timed Fact tests. The teachers devoted a three week period to timed tests of single digit math facts. During that time, the class had 5 minute timed tests. The students were encouraged to practice facts that they didn't know at home. Very little class time was devoted to these tests, but the teachers repeatedly stressed the importance of improving and getting their facts "down cold."

Nonstandard topics

The teachers devoted a major portion of the school year to "non standard" topics, such as geometry and game theory. Sarah felt strongly that the students benefited from working with Geoboards early in the school year. each student hammered in 100 nails to build their own board. Then they worked through a number of activities covering symmetry and basic vocabulary. Sarah wanted students to understand that mathematics was not just addition and subtraction. She also wanted students who didn't typically succeed in math to experience success. Every student was able to build a board and construct symmetrical designs. As Sarah explained in an interview:

> Kathryn and I have done geoboards almost every year, and it has been kind of an activity that was a real hands on constructivist thing they could work on. Outside, they'll pound their boards together, and then actually work on geometry, ideas of perimeter and area. I though it would be a real useful thing you can apply a lot of what you learn from the geoboards to fraction bars and decimals...I think it's a worthwhile way to approach the shapes. And it also is a diagnostic tool for us to use. Did you see how you could pick out how really primitive some of the kids were in figuring out how to copy a shape? It reminds me of that little kindergarten Draw A Man Test when you see how complex some kids see this geoboard by making really intricate figures out of it. Or as Tommy was talking about rotational symmetry and all of this kind of thing. And some of the kids can't make a hexagon on it. You know, so it really is a good diagnostic tool in that way if you need more information from Tommy about what level he's on.
>
> (interview 1/19/95)

Game theory was reserved for the end of year, but similar to the Geoboards it was a topic that appealed to many children who did not typically succeed in mathematics. By the end of the unit each student designed and constructed a game. The charm of this unit was that every game was enjoyed and appreciated by the students regardless of its sophistication or complexity. The game built by Joni was quite simple in design, but because she carefully built the game and checked for inconsistencies, the other children enjoyed playing the game.

> Our final project was to create a math game, themselves, and

> Joni did a really successful one. It wasn't as complicated as some were, but she definitely got it. They had to do a board game that used math in some fashion. Then they did a rough draft, then they tried it out with a friend. then they were supposed to refine it and fix up anything that was boring or that ruined the game. Joni's was not one of the most complicated ones, but she definitely got the idea that is had to have this quest and there was math involved. She did well with it and I think she felt really successful and happy with her final project.
>
> (interview 5/11/95)

Writing/Discourse

The teachers encouraged students to talk about mathematics in a variety of ways. During large group discussions, the teachers frequently asked students to come to the overhead projector and explain their solution to a problem. The teachers encouraged students to question each other and ask for clarification. The teachers also used small groups on a regular basis. The teachers' expectations were clear during small group work, all of the students were to explain their thinking to each other and ask each other questions. Many of the problems that the teachers asked the small groups to solve consisted of a set of clues that were passed out to the different group members. Each group member then had to read her or his clue to the other students in the group.

Writing about mathematics was not a large part of the teachers' mathematics program.

Techniques to address the needs of low ability students

In addition to the broader strategies effecting classroom organization and math pedagogy, the teachers used a variety of "smaller" techniques to facilitate the learning process of the low achievers. These techniques are grouped into two main categories: main teacher strategies and second teacher/aide strategies.

Main Teacher Strategies

When one teacher was conducting the lesson without the assistance of the second teacher, there were numerous strategies used to assist the low ability students. One of the most common strategies was repetition. Definitions, instructions and principles were repeated several times during the large group session until the teacher was sure that most of the class were paying attention. A second common strategy was to provide individual assistance, including repeating instructions, while the students worked on their own. Positive reinforcement and encouragement was also commonly used.

One very effective strategy was to call on the low ability students during the large class session for a piece or an answer, for low-level information, or, if necessary, ensure a successful answer by walking them through the problem. Here are two brief examples of this technique:

Fillmore #10, lines 280-285

> Kathryn says, "we're changing numbers to improper fractions. Here's 1 and 1/3. How do we turn that into an imperfect fraction?" She calls on Kisha and walks her through the answer by saying, "3/3 plus what? Right, 3/3 plus 1/3 = what? Right. 4/3".

Fillmore #15, lines 277-288

> She writes: 1) 7 10 80. She walks the students through this equation. They have to fill in +3 and x8. She calls on Kisha for the x8 answer. She can't get it at first, but after being walked through and asked what times 10 equals 80, she gets it right.

Second Teacher/Assistant Strategies

When the second teacher, Kathryn, or the instructional aide, Diana, were in the classroom, the techniques available for helping the low ability students were multiplied. As mentioned above, the second person can stand near the low ability students to help them remain focused on the task at hand:

Fillmore #3 lines 197-204

> Observer comments: Throughout this lesson Kathryn roams around the room, looking at the students' work. Most often, however, she hangs out near Caitlin and often leans over to her and asks her questions.

Fillmore #8 lines 163- 168

> I observe that Rianna is making faces at Amy and talking to the boy, Billy, who sits between them. Diana is standing behind her, watching Kathryn (conduct the lesson). She touches Rianna on the arm and bends over to talk to her.

Fillmore #13 lines 137 - 14

> Sarah is continuing the discussion of the function machine...Diana is working with Joni, Kathryn is standing behind Kisha. Occasionally she bends over and points out something in Kisha's book or in Rianna's book.

The second teacher often moved about the room, providing individual tutorials during individual work time and the instructional aide could often attend to the needs of several of the low ability students at one time. The next example shows Diana providing assistance to both Joni and Kisha. It also shows her breading problems down into parts or steps.

Fillmore #8, lines 225-236

> Diana has turned to help Kisha. She's counting each bar, walking through it with her: "Two times 1 is? Right. Two times 3 is? Excellent. Here's that 2/12 again. 2/5ths are?" ... Diana has turned back to helping Joni, "You're getting them wrong, slow down. You need to understand what you're doing." She makes her count each section out loud.

Constraints to effective teaching

While the teachers clearly used strategies involving classroom organization and pedagogical strategies to address the needs of the low achievers, they taught mathematics in a larger context that often provided constraints to effective teaching. Some of these constraints are described below.

Length of class. Typically the teachers allotted 45 minutes for a mathematics lesson, but due to special classes, assemblies, and interruptions instructional time was often reduced to 35 minutes. Both teachers expressed concern about the too brief mathematics lessons, but were unable to lengthen the period due to scheduling agreements with other teachers in he building. The textbook authors recommend one hour to one hour and 15 minutes for each mathematics lesson. The teachers were clearly frustrated about the lack of time and often ended lessons abruptly, with no closure or clear connection to homework. In the last few minutes of class students were typically concerned with putting away materials that were in use during class, such as number lines or fraction cards, turning in assignments, or picking up their homework for the night. Often times, the homework required some instruction from the teacher. This added another layer of complexity for the students as they then had several tasks to do at once. The impact on the low achievers and students with learning disabilities was that they often appeared confused or did not make the transition from the main lesson of the class to the instructions for their homework. At the end of class half of the fourth graders leave to return to their homeroom, while the fifth graders who are in this 4/5 split class return from their math class. This arrangement precluded any students getting help after class or extending the class to finish a topic. The following excerpt from the field notes describes one such class ending, in which the students were required to quickly switch from making designs on geoboards to doing a multiplication grid:

Fillmore#6 lines 365 - 411:

> Teacher: I'm going to count to 4 and I want to have eyes up here, just freeze what you're doing. On your homework for tomorrow, there's a multiplication grid. I know you have had

> some multiplication the third grade, we haven't done much of it in this class yet." She draws a grid and writes some numbers in some of the boxes...Some of the students in the back of the room have moved forward to see what she's doing. [I notice that Joni hasn't made the quick transition to this exercise, she's still drawing a picture on a grid in her packet.] Greg (substitute teacher) is walking through the class picking up the geoboards. Kathryn is trying to explain the grid and is filling in the boxes. She asks, "How come I can't fill in this row? If I had a 40, now can I fill in the rest?" She tells the students that there's a grid like this in their homework, and this one [on the board] is more difficult. Kathryn tells everyone to get a homework sheet on their way out. Greg has all the boards picked up. The fifth graders are coming into the classroom.

In this example, it is not clear that Joni understood of the directions concerning the homework assignment.

Split Class Arrangement. Mathematics was the only subject that was not taught to the 4/5 mix of students in Sarah's homeroom. The teachers had made the decision before the start of school that Sarah would teach mathematics to the 4th graders and the other 4/5 class (?) Teacher would teach math to the 5th graders. This decision had consequences for the students with special needs. Sarah was not as familiar with the 4th graders from the other class. These students included Joni, Rianna, and Andrea, all students identified as needing special education services. Thus Sarah could not assess their behavior or progress in the context of other subject areas or compare their progress with the previous year. Also, because she only saw them during that one period of the day, she did not have a close relationship to them as with "her" students. As mentioned earlier, at the end of the math class, the "outsiders" returned to their own 4th grade class and did not have the benefit of getting additional help from Sarah.

"Pull out" policy. Several of the student receiving special education services were pulled out of math class during the final 15 minutes for individualized related special education services, e.g. speech therapy (when math was moved to an earlier time to accommodate another school-wide program?). This meant that they were not part of the class during any "closure" or summary of the day's lesson and they did not receive any special instructions regarding their homework. They often left class in a hurry, grabbing their homework assignment as they went out the door. Quite often, they were in the middle of a small group project when they had to leave, as the following observation note describes. The two groups of 4 students each under observation each contain students who are pulled out. The groups are playing a game that involves clues that are given to each student, then read to the group. The group's task is to put a geometric design together based on the clues:

(Edison19, lines 213-232)

I move to stand between Daniel's group and Kisha's group, but still can't hear the reading of the clues very well. It's clear that they do not understand Kisha's clue. It's 9:15 and Caleb and Caitlin leave. This appears to leave Kelly and Ava frustrated and clearly stuck. A few minutes later Rianna and Joni also leave their group. Kisha and Andrea discontinue the reading of the clues and play with the blocks, building shapes. Andrea is trying to make an elaborate shape. In the other group, Kelly decides they don't have enough of the pattern blocks and goes for more. Andrea has returned to looking at the clues and is trying to solve the problem on her own. Kisha is not engaged with the process. Andrea tries reading her clue to Kisha, but after a few minutes they both give up. Kelly and Ava keep trying.

Effective use of special education aide. Due to scheduling problems the instructional aide, Diana, assigned to assist Joni and Rianna entered the classroom 15 minutes into the period (and was not present at all during the weeks the class was moved to an earlier time period). This meant that she had to assess what was going on in the class, decide what demands or expectations were made of the students and provide assistance - all without any consultation from the teacher. To make matters more difficult, the aide and the teacher did not have time during the school day to "check in" with each other, thus no information was shared about the lesson plan, the assignments or particular issues for each student. Sarah recognized the limitations in this arrangement:

> I think it would have made a huge difference if she (Diana, the instructional aide) could have been here at the beginning of the class every day instead of halfway through. Her schedule didn't allow any time to talk over the lesson so she couldn't anticipate what she was going to do with them. If she were a classroom assistant in the way that you would like to see, you would have some prep time together....Then you could talk about what happened yesterday, get feedback. Ideally, if you could have ten minutes before and then at some point in the day, have another 5 or 10 minutes so you can say, 'How did it go?' I think you could really improve your instruction if you had that pre and post conference with the aide each time....Memos and things like that, notes don't really capture it because you can't point to the problem. If you don't have a time set aside where the two of you can meet, it's not going to happen. (Interview, May 11, 1995)

Clearly, more effective use of an instructional aide could be made if preparation and planning time were made available to the teacher and aide.

Conclusion

Building in success - giving the low achievers opportunities to be successful often captured their attention and increased their enthusiasm for math.

Writing tasks and expecting discourse were usually beyond the ability of the low achievers. They were passive observers of the process, not participants.

Student Descriptions

Of the 9 students identified as in need of special education services, three will be more fully described here. All three were present at the beginning of the school year.

Joni. Joni was a slightly chubby, roundfaced girl of medium height. Her light brown hair hung to her shoulders or was sometimes worn in a perky ponytail. She was not in Sarah's homeroom, but came to her math class. Joni almost always arrived in class clutching a large, 3-ring notebook to her chest, the frightened, wide-eyed look on her face relaxing only slightly by the end of the school year.

something about her academic tests, scores, etc.

At the beginning of the year, observations showed that Joni often did not have her homework done. This situation was addressed by the special education instructional aide as described elsewhere in this paper. Sara also reported that Joni's mother was very supportive in making sure she had her homework done. Joni's first successful experiences in math appeared to occur in the geoboard segment of the curriculum, as observed in the following fieldnotes:

> When Joni's figure matches the figure in the packet, Kathryn uses a stamp pad that she is carrying around and stamps a butterfly on Joni's figure. Kathryn moves off and Joni does the next figure correctly by herself. .... While I am standing there, Joni does the rest of the page by herself and does them correctly. (Fillmore #5, Dec.1)

Both teachers attributed this early success to providing Joni with motivation and enthusiasm toward math class. Indeed, as the year wore on, Joni volunteered answers more frequently than any of the other low-ability students, sometimes, even when she did not have an answer. She demonstrated a remarkable enthusiasm for learning that is often lacking in low ability students. As the class progressed to fraction bars, Joni again struggled, as this observation of the introduction to fraction bars demonstrates:

> Kathryn asks Joni what she notices about the fraction bars. Joni doesn't respond and Kathryn asks another girl if she can make some observations about them. She also doesn't respond. Kathryn places the colored fraction bars on the overhead and again asks Joni what observations she can make. Joni responds that they all have different sizes. Kathryn rephrases this to having different parts and the parts are different sizes.....Kathryn asks what 1 unit of the bar should be called. This discussion goes on with different students and different fraction bars. Joni is rearranging the white bars on her desk. Kathryn is saying, "all these are the same size, the more pieces it is cut into the smaller the size. See if it's true. Let's name these fractions." She writes the fraction "1/3" and says, "one third." Then she writes "1/2" and

says, "one half." Joni is still playing with the white bars and not paying attention. Diana goes over, redirects her attention and whispers quietly to her. (Fillmore #7, January 10, 1995)

Several times throughout the year, it was obvious that Joni did not want to be singled out for attention from the instructional aide, Diana. She would sometimes turn away from her, looking very intently at the teacher in front of the class. Several times she appeared to forget to leave class early for the special education services she and several other students were receiving, and had to be paged to the special education classroom.

By the end of the year, Sarah saw Joni as a student with low retention ability, without a "number sense well established" and one who struggles "more than I think she even realizes." But her tenacity for trying to learn was well recognized and respected by the teachers.

Daniel . Daniel was small for his age, with a cherubic face that betrayed the fact that he was cleaver and witty. He was in Sarah's home room and at the beginning of the year he sat next to Tommy, an extremely gifted student in math. The teachers observed that Tommy often did Jonathan's thinking for him and that Daniel would sit back and let Tommy do the work. The switch to ability grouping came in part because of this type of situation. In Daniel's new table grouping he sat with three girls and, not surprisingly, he became the leader and problem solver of the group.

Test scores, etc......

Daniel was especially adept at hiding his low ability in math. His strategies for this included not volunteering answers, leaving the room to go to the bathroom during class, reading voice cues from others that his answer was not correct, and sitting in the front of the class to the very right of the teacher so that he was often not in her visual scan of the class. Because he was so quiet, he often managed to escape notice in the researcher's observations. One of his strategies for not exposing himself is evident in the following observation of a game that the students played with a partner. In this game the students are to pick two fraction cards and subtract them, the winner has the larger number. Daniel's partner is Anna:

> After Daniel returns from the bathroom, he and Anna each pick two cards. Daniel immediately says, "You win." Sara arrives and Daniel says that his partner is killing him. Sara asks to look at their cards. Daniel reads his as 6/12 minus 10/12 equals 4/12. Anna's answer is 2/5. Sarah asks which one is greater. Daniel uses the trick that Max (another student) demonstrated in class earlier to arrive at the conclusion that his partner won. Sarah stays with them as they each pick two more cards. Daniel's is 9/12 minus 1/12 equals 8/12 and his partner's is 9/10 minus 3/10 equals 6/10. Daniel wants to use the trick again but he can't multiply 12 x 6 until Sarah asks him what's 10 x 6 and then what's 2 x 6, then what is 90 plus 10. He uses this method to figure out

that Anna won again. Sarah says that she wants to see them do one ore so that she knows they understand it. Daniel draws 4/4 minus 1/4 equals 3/4. His partner has 10/12 has her answer. He says, "She won again." Sara asks, do you think 10/12 is greater than 3/4? He picks up on her tone and says, "Oh, what am I doing?" Anna responds with, "he won." Sarah writes on a piece of paper: 10 x 4 = 40 and 3 x 12 equals 36. When she does this Anna says, "Oh, I won." Then Sarah says, "What if we convert them? 9/12 is the same as 3/4 and is it bigger than 10/12?" Daniel says, "I have no idea what you're talking about." Sarah responds with, "Well, you have a strategy that's working for you, you should stick with it." She makes them do one more example. This time the answers are 1/6 and 2/10. After doing the cross multiplication trick, the two conclude that 2/10 is larger. Sarah talks about reducing the fractions by half. Daniel again, doesn't understand the concept, but he can figure out from Sarah's tone that their answer is wrong. Sarah gets interrupted by another student during their next example. Then she tells the class to figure out which partner won. Daniel says loudly, "Who won? Who cares?" He counts the number of cards he'd taken throughout the game and asked his partner how many she had. From this he concludes that she won because she had one more card. Joe, another student hears Daniel announce that Anna had won by one card and he says, "That's impossible!" He comes over to Daniel and Anna's table, but Daniel tells him, "Go away, go away!" Sarah restores order to class.

In this example, he demonstrates that he gives up in a game rather than do the computation to figure out who really wins. He also is skilled in reading the teacher's tone to determine whether his answer is right or wrong. Lastly, he deflects scrutiny from the other students.

Because Daniel was very adept at staying out the teachers' "spotlight," it wasn't recognized until mid-year that he had very low ability math skills. Example from Jill's observation.... By the end of the year he had been identified as needing special education services and was being pulled out of the end of math class, along with Joni and Rianna, for special services. He did not, however, have the instructional aide assigned to him.

Sara saw Daniel as a low-ability student but one who made other contributions to class. Since he was in her homeroom, she was able to report that he had especially well-developed artistic skills that were much admired and respected by the other students.

<u>Rianna</u> Rianna was a tall, thin, dark-skinned girl with straight black hair. She was very social and enjoyed giggling and talking with one or two other girls in math class. Rianna often did not have her homework done and quite frequently used the time in the beginning of the class when homework was being corrected to fill in the answers. She never volunteered and would have preferred never to be called on or noticed in any way.

test scores, etc.

Unlike Joni, Rianna was not enthusiastic about math. At times she appeared to be "getting it" but a closer check of her work would reveal many errors. The following excerpt from the observations reveals some of these characteristics:

> Everyone is working on their sums. I notice that Rianna and Kisha are still trying to figure out whether they have the smallest number, not realizing that the class has moved on to doing the actual computations. Kathryn goes over to their table and asks Joni how she's doing. The student responds that she has three done already. Kathryn asks to see Rianna's computations. Rianna responds that she's doing them in her head, but Kathryn says that she wants her to write it down.

As the year progressed Rianna seemed to be more focused on her relationship with her girlfriends in class than on math and her failures or low grades did not seem to bother her. Sara saw her as a student with definite learning problems and performing below-average in math, although since Rianna was only in her math class she did not have information about other academic areas. She saw that the assistance of the instructional aide was instrumental in Rianna having her homework done and feeling good about that aspect of class. She perceived Rianna as still lacking self-esteem and self-confidence to volunteer in class.

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