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ABSTRACT

This paper examines the mathematical conversations of middle school students as they use computer-mediated graphic representations to construct a shared understanding of the basic concepts of probability under two conditions. The first condition used graphical representations to depict important abstract relationships visually, and the second condition used animation to augment the graphic representations to see if animation helped students grasp abstract relationships. Thirteen pairs of students (11 boys and 15 girls) divided themselves into the two condition groups. The analysis of student think-aloud protocols focused on how the visual and dynamic elements of the computer-based graphical representation influenced students' interpretations of the mathematical concepts. Data suggest that students with access to animated representation had qualitatively different mathematical discussions than those who had only the static representation. The conversations of students who used animated representations used a narrative structure and were richer than those of the students who used static representations. (Contains 3 figures and 37 references.) (SLD)

# CONSTRUCTING UNDERSTANDING: THE ROLE OF ANIMATION IN INTERPRETING REPRESENTATIONS

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*"Mediating technologies do not stand between the user and the task. Rather, they stand with the user as resources used in the regulation of behavior in such a way that the propagation of representational state that implements the computation can take place...These tools permit the people using them to do the tasks that need to be done while doing the kinds of things people are good at: recognizing patterns, modeling simple dynamics of the world, and manipulating objects in the environment."*

*Edwin Hutchins*
*Cognition in the Wild (pages 154-155)*

## I. PROBLEM AND SIGNIFICANCE

Mathematics is a domain in which cognitive technologies traditionally mediate both learning and practice. These cognitive technologies are often either physical or symbolic artifacts, and computer-mediated cognitive technologies offer new ways to combine the manipulablity of physical artifacts with the computational power and abstraction of symbols. Dynamic graphical representations are one such resource for mathematics education that allow students to access complex concepts in mathematics while doing the types of mathematical activities they are already good at. This paper examines the mathematical conversations of middle school students while they use computer-mediated graphic representations to construct a shared understanding of the concepts of basic probability. The analysis illustrates how mathematical learning and problem solving is "stretched over" the complex cognitive system of individuals, the physical environment, and social interaction.

The study compares student conversations under two conditions. In both conditions, students are given access to a computer tool which they use to investigate the concepts of mathematical probability. However, each condition received slightly different versions of the computer software. The first version employs graphical representations to visually depict important abstract relationships that are an important aspects of normative probabilistic reasoning. In the second condition, this important graphic representation is augmented with animation to determine what role dynamic animation can play in helping students grasp abstract relationships. The analysis of the students protocols focuses on how the visual and dynamic elements of a computer based graphical representation influence students' interpretations of the mathematical concepts.

The data suggests that the students with access to an animated representation had qualitatively different mathematical discussions than those who had the static representation. The conversations of the animation group used a narrative structure, perhaps suggested by the animation, to organize their talk about the relationship of two complex ideas of probability. The non-animation group described the same relationship in a different and perhaps less rich manner.

This paper will begin by discussion the theoretical assumption that undergird both the design of the computer learning environment and the analysis of the data. Second, the paper will describe the two conditions of the study and the methodology of the data collection and analysis. Next, the results of the qualitative analysis of the student transcripts will be reported. The paper ends with a discussion of the findings and their instructional implications.

## II. THEORETICAL FRAMEWORK

The students involved in this study did not come *tableau rasa* to the problem solving task. They brought to the situation a collection of intuitions that they had derived from their experiences with uncertainty. Much of this experience comes from playing games. Games and game playing are an essential part of childhood in our culture, and in many of the games children play, the outcome of the game is based on chance—on the outcome of random devices such as dice, cards or spinners.

The collection of intuitions people develop from their everyday experience with uncertainty and the way they use these intuitions in reasoning in probabilistic situations are, for the most part, "different in kind" then normative mathematical reasoning. Mathematicians, when they reason probabilistically, employ a tightly inter-connected mental model that integrates all the relevant mathematical factors into a coherent theory. By "mental model," I mean the ability to generate predictions about the behavior of a complex system by mentally simulating changes to the system's representational state over time according to causal relationships between the parts of the model (Johnson-Laird, 1983; Genter & Stevens, 1983; Roschelle, 1991; White, 1993). In probability some of these aspects are: an understanding of random events; knowledge of all possible outcomes and how the outcomes are partitioned (i.e. the event space); a causal understanding of the relationship between the random events and the event space that would lead to a predicted frequency of outcomes (i.e. a probability distribution); and an understanding of the between sample size and predictability (i.e. the law of large numbers).

However, research has documented that most adults are not very good at probabilistic reasoning. The most established explanation of this poor performance is that adults use heuristic reasoning strategies in situations of uncertainty and that this leads to systemic errors in their conclusions (see Shaughnessy, 1992 for an overview of research on probabilistic reasoning).

In our work with middle school students we have found that students typically reason in a number of non-normative ways as well—leading to probabilistic statements that are mathematically unsound. In general, students reason from a set of poorly connected qualitative cases(Vahey & Enyedy, 1996). A qualitative case is a group of observed regularities that students apply to a limited set of situations on an ad hoc basis (c.f. Rochelle, 1991). While Individual cases may have some internal consistency, as a set they have no coherence. Many of the qualitative cases students use over the course of a single half hour interview are contradictory (Vahey & Enyedy, 1996).

First, students often either do not realize that there is an event space or they do realize that the event space is relevant to their problem. Students often predict that any game based on a random event or a series of random events is a fair game. For instance one student stated, "Yeah, so they can have the same probability to get whatever the heads and tails is, so it is gonna be fair because its like half—its half the chance you can get a tail or a head." Alternatively, some students state that because a game is based on a random event and thus inherently unpredictable. The reasoning continues, that if one cannot make any predictions about the outcome of the game, the game must be fair. For example, a different student said, "The coins aren't that important cause its just luck." She elaborated her answer in writing stating, "The coins is just data and the data in this game is not that important."

For the students that recognize that relevance of the event space, the difficulty lies in enumerating and partitioning the event space. Preliminary analysis of written pre-test questions that asked 7th grade students to reason about probability, showed that students' ability to enumerate the entire event space of a game based on two events varied with the exact context of the game (Enyedy and Vahey, in preparation). The best performance was on a question that asked the students to name all the possible outcomes of a game based on two coin flips. In this situation about half of the seventh graders were able to name all four possible outcomes. Most of the students that did not name all four outcomes did not differentiate the order in which events took place and thus under counted the total number of outcomes in the event space. For instance, Heads-Tails was often not differentiated from Tails-Heads. It is only when the event space is attended to and enumerated that it can the be partitioned into favorable and unfavorable events and a probabilistic judgment can be made.

Because reasoning from a coherent mental model is different in kind from reasoning from qualitative cases, innovative pedagogical solutions must be employed to change students world view. Our pedagogical approach is derived from our commitment to two theoretical perspectives—constructivism and distributed cognition.

<u>Constructivism</u>

Constructivism is based on the premise that ultimately students construct their own knowledge from their interpretations of their experience. Further, it is assumed that this process is more effective when students are actively making sense of and organizing their experiences. Many traditional pedagogues for teaching mathematics do not provide students with rich experiences with mathematical phenomena or support students in their sense making or organizational efforts.

In response to the perceived lack of conceptual understanding of mathematical concepts, the demonstrated inability to perform complex problem solving, and the difficulty in communicating mathematically, most major organizations of professional mathematicians and many mathematics educators have adopted a constructivist philosophy and advocate more active and guided discovery approach to learning including more communications-intensive activities (Cf. NRC, 1989, 1990a, 1990b, 1990c & 1991; and the National Council of Teachers of Mathematics (NCTM), 1989, 1991 & 1995a). One such recommendation published by the NCTM (Curriculum Evaluation Standards for School Mathematics, 1989) proposes that mathematics curricula should develop students mathematical language abilities so that students are able to:

> ". . . reflect upon and clarify their thinking about mathematical ideas and relationships; formulate mathematical definitions and express generalizations discovered through investigations; express mathematical ideas orally and in writing; read written presentations of mathematics with understanding; ask clarifying and extending questions related to mathematics they have read or heard about; (and) appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas."

These goals are a radical departure from earlier conceptualizations of the mathematics teaching and learning enterprise. They redefine both mathematical competence and the pedagogy by which it is achieved. Mathematical competence measured not by the ability of a student to instantly recall mathematical procedures, but by a student's ability to make use of and communicate her mathematical understanding in complex problem solving. These goals criticize the traditional pedagogy of mathematics classrooms for portraying the teacher as a disseminator of information and the students as intellectually and linguistically passive. A new pedagogy is endorsed in which students actively engage in mathematics through sustained investigations of mathematical concepts and where the teacher's role is to create opportunities for learning and to guide students in their construction of mathematical understanding.

We have adopted a pedagogy of learning through inquiry an embedded it into the computer-meditated Probability Inquiry Environment (PIE) we are designing. We have adopted this approach because it provides a method of learning mathematics that is based on making sense of one's own experience through a cycle of conjecture, observation, and analysis. This sense making activity requires that the students organize their experience into conceptual structures that can later serve as the foundation for traditional computational skills. In the case of probability this means we want the students to understand randomness, frequency, and compound events prior to learning the procedures, algorithms, and symbolic notation for computing the theoretical probability of an arbitrary event.

Distributed Cognition—the role of social interaction

The inquiry process is driven by encountering problematic experiences (Dewy, 1938). Our second theoretical assumption is that people try to resolve these experiences through interaction with the environment and with other people. We start from the position that through interaction knowledge is socially constructed and thus mediated though language (Lave, 1987, Vygotsky, 1986). That is, we want our students to understand that certain quantitative patterns exist in our world which mathematicians have developed a precise way of describing. The process by which students come to understand these patterns and relationships is through social interaction. Students produce and refine mathematical descriptions in an iterative fashion by resolving ambiguous propositions and negotiating any differences in interpretations. It is through social interaction that they create an inter-subjective understanding of the domain which is eventually internalized.

Distributed cognition—the role of external representations

In order for inquiry learning environments based on these ideals to be successful, they require that we provide students with tools and resources that will aid and guide them in their construction of understanding and will encourage that understanding to be closer to the normative mathematical model of probabilistic reasoning. Our goal is create graphic representations that support the social process of inquiry and that maintain a high fidelity with the mental model of probability used by mathematicians.

Graphical representations provide an accessible perspective on mathematics which does not sacrifice mathematical rigor or computational power. They are more accessible than other mathematical resources because of the meaningful analogical mappings between the structure of the representation and its referent (Barwise & Etchemendy, 1995; Myers, & Konolige, 1992). The structural similarities between the graphic image and the source domain can be used to visually highlight the salient features of a concept. Additionally, in many instances graphic images are less abstract than the symbolic notation that is usually used to teach students mathematics. Thus graphic images can provide a more context dependent, experiential way in which to investigate a mathematical concept. This implies that the accessibility that graphic representations provide may be due to the different set of cognitive resources and background information that they draw upon (Stenning & Inder, 1995)

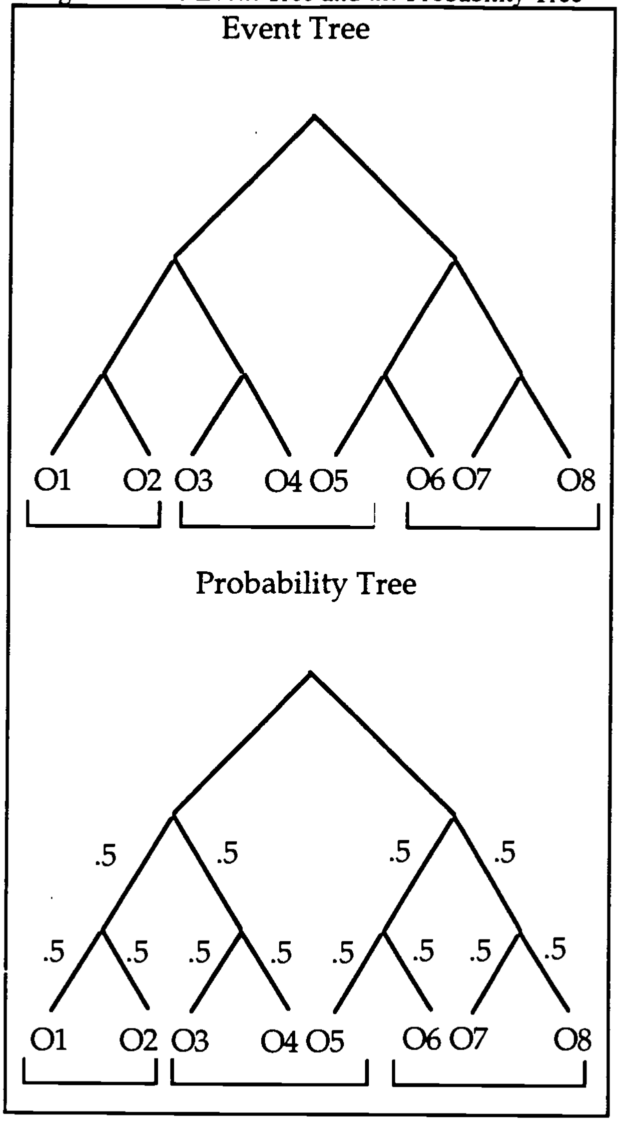
The inferences that can be drawn from graphic representations, however, are not merely helpful during the learning process. Mathematical proofs have been presented showing that the diagrams such as Euler's circles and Venn diagrams are as complete and rigorous as the formal symbolic notations (Hammer, 1995; Shin, 1991). Similar arguments about the effectiveness and thoroughness of the tree structure have been made in the domain of probability (Shafer, 1995).

The point here is not that graphical images are a universal representational system for cognition. Graphical representations are not independent of language, rather graphical representations are non-linguistic resources for mathematical discourse and problem solving. Graphical representations can be used as a public reference object that can be talked about, gestured to, and manipulated in students efforts to create a shared understanding. From this perspective graphical representations serve as "comprehensible input" and "output" that clarifies the ambiguity of verbal discourse (Krashen, 1982) as students communicate about a complex subject matter. The physical presence of graphical representations make them ideal public resource for communication and a convenient entry point for private understanding.

Event trees

In our environment we have made an effort to link together a number of representations at different levels of abstraction to enable students to coordinate different informative perspectives on the domain of probability. The analysis presented here revolves around a graphical representation called an *Event Tree* which represents of the abstract event space (all possible outcomes and their inter-relations) spatially by a vertical hierarchy of branching nodes. Each branch represents an atomic event, while each node (except for the terminal nodes—henceforth called outcomes) represent decision points. The event tree is identical to a probability tree except that the probabilities at each node are not provided. The two types of tree structures are shown in Figure 1.

Figure 1. The Event Tree and the Probability Tree




We incorporated a graphical representation of the event space, the event tree, into our environment to help accomplish the following instructional objectives:[1]

- to highlight and make relevant the event space. The structure of the tree, combined with the simulation of many trials, are used to demonstrate the relationships and relevance of the chain of random events leading to an outcome.
- to enumerate the event space. The structure of the event tree concretely shows all the possible events and outcomes within the event space.
- to partition the event space. The bottom row of the event tree is labeled with the various outcomes and how these outcomes are partitioned with respect to the teams competing for points.
- to afford computation of probabilities. The bottom row of the event tree provides a centralized space to compute probabilities using simple arithmetic and quantitative comparisons.

[1] It should be noted here that this analysis rests on the assumption that all outcomes are equally likely. Although this assumption holds for both games, it was not always the case that the students held this assumption. Some students believe that some of the outcomes were more likely than other. For example, some students said that three heads hardly ever comes up. Interestingly, the same pair also said that fairness depended on both teams possessing the same amount of outcomes. This global incoherence did not seem to bother them even when it was pointed out to them. While this warrants further study, it is beyond the scope of the present paper.

## II. METHODOLOGY

Thirteen pairs of students were divided into two conditions and given the task of determining if a game of chance was fair to all participants. The students were recruited from the seventh and eighth grades of an urban middle school. The study was relatively balanced in gender (11 boys and 15 girls). Except for one pair, the students chose their own partners and grouped themselves by sex.

The format of the study was an adaptation of an open clinical interview. The students worked together in pairs at a desk with a computer (pencil and paper were also available but were not used by any of the students). An interviewer was in the room seated behind the students and would sometimes engage the students in discussion. For the most part, these conversations were to clarify something the students did not understand or to pursue a topic raised by the students in their conversations. The majority of the discussions, however, were between the two students.

The difference between the two conditions revolved around the context and features of the event tree representation. Five pairs of students were given a version of PIE where the rules of the game were contextualized in a story of a competition and the event tree was animated. The remaining eight pairs of students were given a version of PIE where the rules of the game were contextualized as a coin game and the event tree was not animated (these differences are discussed in greater detail below.) In most other aspects the two versions of the computer software were isomorphic.[2]

[2] There were two minor differences between the two versions of the software that did not seem to effect the students conversations. First, in the animated condition the game was based on a race to ten points at which time a team would win a match. The non-animated condition did not have this secondary scoring structure. This did not seem to affect the study because students did not focus on this secondary scoring structure, but instead looked the two team point totals (which both displays supported in the same manner). The second difference was that the two games had slightly different partitioning of the event space. Again, this did not seem to affect this analysis because

Description of the Probability Inquiry Environment

The games the students were asked to judge the fairness of were games where two teams were competing for points. Which team scored a point was determined by the outcome of three coin flips. In both conditions, the rules of the games were established so that one team won a point if any one of five of the eight possible outcomes occurred and the other team won a point if any of the reaming three outcomes occurred.

PIE uses a context that is genuinely interesting to middle school students—games of chance—to engage them in doing mathematics (Enyedy and Vahey, 1996; Mannon, 1996). Games and game playing are an essential part of childhood in our culture. In many of the games children play, the outcome of the game is based on chance—on the outcome of random devices such as dice, cards or spinners.

However, in adopting the rich activity structure of games for mathematics, we wanted to attempt to avoid some of the problems of selective attention and bias that can occur in when students engage in game playing. For example, when students play a game favorable experiences are remembered more vividly and are more accessible then non-favorable occurrences. As a result students miss-assign probabilities for certain events based on their subjective experience (Falk, 1989; Kahneman & Tversky, 1973; Mannon, 1996). We also wanted to focus their attention on the frequency of events over the course of a game and not just on the end result of who won. For these reasons, we chose to make the goal of the activity judging the fairness of a game rather than playing the game. Judging the fairness of the game is intended to frame the activity as an objective investigation where the students are impartial judges. Theories of cognitive development (Piaget and Indler, 1975), as well as our pilot studies (Enyedy and Vahey, 1996; Vahey, 1996), gave us reasons to expect that judging the fairness of a game, while perhaps not as appealing as playing the game, is still an interesting activity for middle school students.

Shifting the activity from game playing to judging the fairness of a game had the additional benefit of making the activity structure ideally suited for facilitating mathematical inquiry. PIE's version of inquiry is based on a cycle of conjecture, observation, and analysis rather than the more constrained paradigm of hypothesis testing taught in scientific method. We decided to adopt an inquiry approach to mathematics because it provides a method of learning mathematics that is based on making sense of one's own experience. This sense making activity requires that the students organize their experience into conceptual structures that can later serve as the foundation for traditional computational skills. In the case of probability this means we want the students to understand randomness, frequency, and compound events prior to learning the procedures, algorithms, and symbolic notation for computing the theoretical probability of an arbitrary event. In addition, during the process of learning specific mathematical concepts via inquiry the students are also learning an array of meta-skills, such as data-collection, data-analysis, modeling, inference, deduction and presentation. These meta-skills enable students to move beyond memorizing mathematical facts and procedures and enable them to begin to participate in the practice of mathematics.

The steps that are involved in the process of systematically investigating a mathematical concept are physically represented in the buttons along the top of the computer screen (see Figure 2). These buttons, which are always present on the screen, change the text, graphics and tools to match the types of activities associated with the steps in the "inquiry cycle". These steps are; understanding the rules of the games, exploration of intuitions (unfortunately this feature was not fully developed at the time of this study), developing predictions and hypotheses, collecting data and data analysis and finally drawing and presenting conclusions (cf. White and Frederiksen, 1995).

Once the students were satisfied that they understood the rules they were asked to answer some questions and make predictions about the outcome of the game. The predictive questions serve three functions; 1) to highlight salient features of target conceptions, 2) to create a frame of reference against which the data analysis will be performed, and 3) to provide an opportunity to hear alternative conceptions of one's peers and refine one's own thinking through social interaction.

The students are asked three sets of predictive questions. Each question relates to an aspect of probability that is needed to mathematically judge whether or not the game is fair. The first set of questions revolve around the main goal of the activity: Is this a fair game? Why or why not? And what do you mean by fair? The second set of questions ask for the students to make specific predictions about the points each team will have scored after ten turns and to provide a rationale for their prediction. The third set of questions ask whether or not any one event will happen more often than any other.

After completing the predictive questions, the students run a simulation of the competition. The simulation shows each coin flip in turn and "grays out" the outcomes that are no longer possible. When the three coin flips are completed one outcome remains blue. A histogram, located directly below the game board, automatically keeps track of who scored the point. The histogram can me made to show either the distribution by outcomes or by teams, and can display the data from the current game or for all the games played in the session. The students control the pace and duration of the game. The students also control how many games they play and at what point they wish to make their conclusions. When they reached this point, they were asked questions that followed up their predictions.

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the proportion between the two games remained constant at 5 to 3.

Figure 2 Play mode of the animated condition

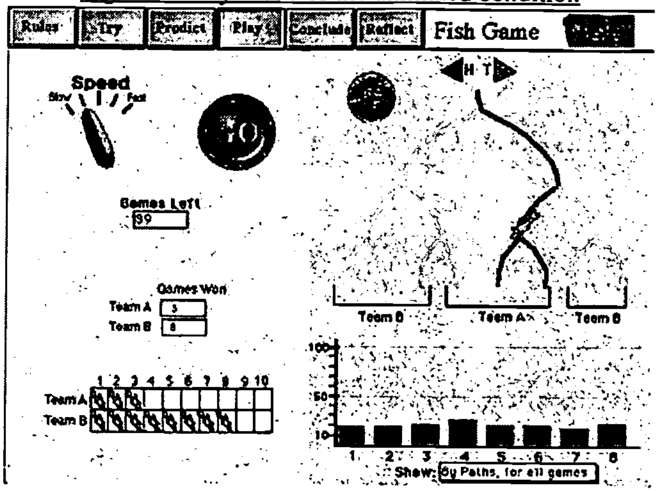
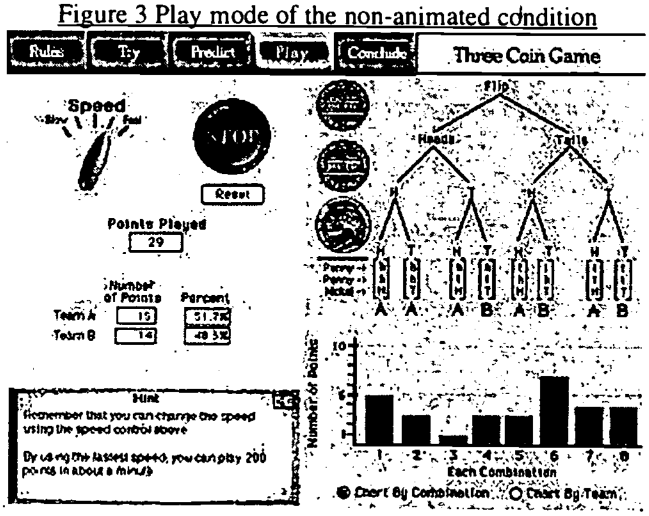



Figure 3 Play mode of the non-animated condition




## The Differences Between Games

The game that the students were asked to play in the animated condition was embedded in a story of a competition between two groups of fishermen in a race to catch ten fish. It is clearly a fictitious situation where the fishermen set up gill-nets in the various branches of a system of branching rivers and fishes are released one at a time to swim down the river until they are caught by one team or another. The students are told that the branch of the river the fish chooses depends on the outcome of a coin—heads the fish goes left and tails the fish goes right (see Figure 2.) In this version of PIE the event tree is contextualized as a river system within a watershed (see Figure 2). The river system can be directly mapped to a prototypical event tree for three coin flips (Figure 3).

In the non-animated condition students were told that teams were playing a game based on flipping three coins one a t a time. The rules were: Team A gets a point whenever both pennies land on Heads, or the nickel lands on Heads, or all three coins land on Heads; otherwise Team B gets the point. The event tree, in this version of PIE, was a prototypical event tree (see Figure 3).

## Framework for Data Analysis

All sessions were video taped, and those taped were transcribed. The names used to identify students below are all pseudonyms. Areas of the transcript in which the students discussed the event space were transcribed in closer detail and the relevant gestures were recorded and incorporated into the transcript[3]. Additionally, all the students interactions with the computer were captured and written out to a file.

For this study, I examined the only excerpts with respect to how the students talked about the tree and how they used the tree to reason about the probability of events and the fairness of the game. In the tradition of grounded theory (Glaser & Struass, 1967) I used the data to produce refine my initial conjectures and define new ones, eventually settling upon coding students behavior according to the two perspectives described below. I believed that the talk between students would reflect any contrasts in understanding of the event tree. The framework I chose uses students talk and gestures to code the perspective they are adopting for that conversational turn. Coding was based on contrast classes in the verbs the students' used when talking about the event tree. These codes do not make any strong commitments as to the knowledge that students do or do not possess at the time of the conversational turn. Instead they reference what the students are attending to at the moment.

This paper argues that students talked, gestured, manipulated and interacted with the event tree in two distinct and complementary ways. It is likely that these two sets of behaviors correlate with a local perspectives that focus a students' attention on certain aspects of the tree and away from others.

The first perspective, the "outcome perspective," is characterized by attending to the outcomes of the event space that are represented along the bottom row of the probability tree. The second perspective, the "process perspective," is characterized by attending to the structure of the event tree (especially the internal nodes) that represents the process by which outcomes occur.

Both perspectives fulfill different functions in the problem solving process. The outcome perspective allows students to quantitatively compare the partitioned event

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[3] The transcripts are arranged in three columns the first identifies the speaker, the second is the discourse and the third column is used for the gestures that were coded for that turn.

Boldface was used to indicated possessive verbs coded as the outcome perspective and underlining was used to indicate the action verbs coded as the process perspective.

The transcript conventions I used are derived from Gumprez (1982) and are as follows:
[]= comments by researcher
()= best guess at what was said
-- =false start
. . . =pause
wor:::d= lengthening the word
== = latching and overlaps
word[1]= where a gesture begins

space and use this proportion to judge the fairness of the game. However, this comparison rests on the assumption that all outcomes of the event space are equally likely. To understand why this is the case, students need a way to understand the process by which a series of coin flips produces an outcome. The process view allows students to understand the relationship between the event space and the expected probability distribution.

## IV. RESULTS

Example of the Outcome Perspective

The example below illustrates a typical stretch of student dialogue that I have coded as reasoning using the outcome perspective. It was prototypical of this type of talk in that it demonstrates both the use of possessive verbs used and horizontal gesturing that are focused on the bottom row of the event tree. In this episode Lilly spontaneously interrupts me while I am explaining the computer interface and states that the game is unfair. At first she is unable to verbally articulate her point. However, using the image as a resource for communications she reveals by gesturing that she is attending to the bottom row of the tree—first as she counts the outcomes and second as she points to a group of outcomes. She eventually changes tack and instead of trying to articulate why the game is unfair, she re-partitions the event space to make it a fair game. This shifting of the task from explaining their reasoning to fixing the game turns out to be a common strategy. After re-partitioning the event space she returns to explain why the games are unfair, by counting up the outcomes for each team and comparing the two quantities.

It is interesting that in this example the gestures are used in two different ways. In the first turn Lilly uses her gestures as a resource in the communicative process. Her verbal utterances, with five false starts, would be difficult or impossible to interpret without the accompanying gestures that clarify ambiguities of her speech. References to the event tree in this case give Lilly a public resource to aid her in conveying meaning where her mathematical register seems to fail her. In her second turn, however, Lilly uses her gestures as a resource for her cognitive activity. She counts up the number of outcomes for each team using her finger to point to each outcome in turn. This use of pointing changes the cognitive activity involved from counting small objects in her visual field to the easier activity of counting the number of time she moves her hand (Kirsh, 1996).

Excerpt 1.

Lilly: It is a little unfair that **team A won't get** B[1]-- and then team B won the--(it should be)[2] --is it--so that-- you should put team B[3] over here and team A[4] over here

Int: So you think its because the. . .nets are==

Lilly: ==uh huh more like team B is more than team A. It looks like its (1,2,3,. . 1,2,. . 1,2,3)[5] **they got five**. Its unfair don't you think so?

Gestures

1: quick "counting points" along the bottom row
2: points to the bottom row of the tree
3: flat hand held vertically sweeps from them middle of the tree left
4: hand sweeps from the middle of the tree to the right
5: counting off while pointing to spots along the bottom row of the tree

Examples of the Process Perspective

Students also successfully reason from the event tree using the process perspective. In the example below, Dee makes inferences about the fairness of the game based on the internal nodes of the tree's relationship to the outcomes. Dee noticed that certain nodes, once passed, pre-determine the outcome of the game. In effect Dee is inventing Alpha-Beta pruning to constrain the event space which must be considered to determine the fairness of the game. This type of reasoning is coded as the process perspective because Dee is focused on the internal nodes of the tree and reasoning about the process which leads to outcome rather than quantifying the outcomes.

Excerpt 2.

Dee: ...and I think B has a better chance...here [1]there's a 50-50, at both of these places2 there's 50-50 chances. but here and here[3], they always end up at B, so there's no 50-50 chance, I mean, there is up here[4], but no matter where they go they always end up at B. here they can go either way[5], that's even, and then this one goes with Team A[6]

Gestures

1: points to the top intersection of the event tree
2: points to both intersections on the first level of the event tree
3: points to both intersections on the first level of the event tree
4: points to the left intersection of the second level of the event tree
5: points to the intersection second from the left of the second level of the event tree

6: points to the intersection third from the left of the second level of the event tree

An additional example of Alpha-Beta pruning to limit the event space.

Excerpt 3.

Int so um do you think that any river happened more then any other river

Peggy yep

Lilly yeah[1], over here see it looks like they got more chance to go to here[2] then to down to team B's nets that all. The fish go down to river like they only have one chance to go to team A but they have more chances to go to[3] team B

Gestures

1: trace the left path down the tree
2: points to the left outcome along the bottom row
3: points to spots along the bottom row of the tree

In both examples students' talk make explicit use of the event tree as a public resource as they refer to its internal structure. The majority of Dee's references are marked verbally with the word "here" and gesturally with a point to a particular node of the tree. Dee's utterances if presented without her gestures would be ambiguous but coherent. The meaning of Lilly's speech seems to be more dependent on the gestures that refer to parts of the event tree.

More interestingly, Lilly does not identify specific nodes that are strategic to the outcome of the game. Rather, she first prunes of the entire right hand side of the tree structure with her calling attention to the left side of the tree with the words, "over here," and her vertical gesture. Presumably she has noticed the perfectly balanced symmetry of the right half of the event tree and moved on to analyze a more consequential feature of the event space. With this move accomplished, Lilly then compares the partitioning of the smaller event space and finds that there is only one opportunity for team A and three for team B, adding more evidence to her earlier assertion that the game was unfair.

As mentioned earlier, to produce quantities for the relative frequencies of outcomes students need to draw upon their understanding of the process of events and the causal relationships in the event structure. In the example below, the students use the process perspective to reason about the event space and conclude that certain events are slightly more likely than others. The discussion demonstrates the use of active verbs and vertical traces to talk about the causal events that lead to outcomes.

Excerpt 4.

Int But do you think one path is--do you think the fish is gonna go down one path more likely than the others?

Tom I don't know

Paul starts typing short answer

Paul How much is in there[1]? probably one sixteenth I guess 1: points to a prediction bar that is pulled most of the way down

Int what . . . one 16th of what?

Paul Like if they have ten fish right? If I go in each box[1] they got eight right, and I got two, two fish left. They probably will go[2] in each side

Int OK

Paul 'cause its outside

Int Do you agree with that?[to Tom] That it would go to the outside?

Paul Right. . .it maybe that it goes in each box here, but two fish left it will go to the outside or inside

Gestures

1: traces vertical paths down the event tree
2: Traces paths down each of the outside paths of the event tree

Examples of the Combination of Both Perspectives

Students demonstrated the ability to fluidly switch between the perspective to leverage both the outcome and process views for problem solving. Cases where the two students collaborate to coordinate these two perspectives led to successful problem solving. In the example below, Ming and Xu attend to different features of the environment, Ming takes the process perspective and Xu takes the outcome perspective. The discussion shows the students negotiating what are the salient aspects of the event space. Ming initially believes that because team A has the center paths that they will win the game. His partner however is focused on the bottom row and uses the outcome approach to argue for ignoring the spatial location and only focusing on the cardinality of the partitioned event space.

Excerpt 5.

Ming I think its not fair you now **team B kind of has routes in the middle[1].** And it usually ends in the middle. It doesn't it?

Xu shakes head

Ming how?. . .I don't know

Xu **OK 1,2,3,4,5,6,7,8. 1,2,3,4. .five for team B and team A is three out of eight**
Ming what?
Xu **its three out of eight. . .look team B gets five of them that's just not fair**
Ming yes its not fair

Gestures
1: traces a path down the middle of the tree

It is important to notice that in cases where one student coordinates both the outcome and process perspectives the student's gestures and talk are also coordinated. For example, in both Vicki's second and last turns of the excerpt below, when she talks about the process by which the event space is related to the expected probability distribution, she coordinates her gestures that highlight the internal nodes of the event tree (e.g. vertical traces) with verbs that describe the relationship in terms of spatial relationships and motion (e.g. "go down there" or "you flip a tail and you go here"). Likewise, when she compares the proportional quantities of outcomes she uses gestures that refer to the bottom row of the tree (e.g. horizontal counting) and verbs that describe teams as possessing outcomes (e.g. "they have only three").

Excerpt 6.
Int so now if you want to answer the questions
Vicki all right the game is not fair ==
Int ==why isn't it fair?
Vicki because[1] if you try all the probabilities go down there is like eight of them and like **they only have three**[2] and it goes to team A and[3] **five to team B** so its gonna be unfair to team A though
Juan but when its red here its my turn and when its red over her its her turn right?
Int Oh no,==
Vicki ==no you keep going till you flip
Int no there is nobody actually playing. Its the fisher men
Vicki yeah so look you[4] flip the coin heads or tails or you go here or over here and then you flip a tail and you go here[5]. So **there are all**[6] **(the nets of) the teams** so its going to be unfair.

Gestures
1: traces paths down the tree
2: points to the bottom row of the tree
3: points to the bottom row of the tree
4: point to the first intersection at the top of the tree
5: traces a path down the tree
6: horizontally traces the bottom row of the tree

An Extended Example

The above excerpts demonstrate what student discourse looks like when students are problem solving from the outcome perspective, the process perspective or both. However, the examples above sacrifice the context of the interaction to clearly illustrate my conceptual framework. The extended example below shows how two students reason about the Fish Game using the event tree representation over the course of the activity which lasted approximately thirty minutes. What is important about this extend example is that it highlights the social aspects of this task and the on-going negotiations that the students perform to construct a shared understanding of the game and of what aspects of the situation are salient.

The example begins with both students spontaneously talking about their interpretations of the game prior to watching a demonstration or answering any of the predictive questions. Both Vicki and Juan begin with non-normative views of the event space's relevance. Juan thinks that the turn taking aspects of games is the key feature that makes a game fair and keeps participants from fighting. Vicki has a more common naive conception. She believes that the partitioning of the event space is irrelevant because the game is based on coin flipping, and coin flipping is a "fifty-fifty" event.

Excerpt 7
Vicki Yeah, so they can have the same probability to get what ever the heads and tails is. So it is gonna be fair. 'Cause its like half. . . its half the chance you can get a tail or a head.
Int is that what you think it is?[to Juan]
Juan so then it won't (get in a fight)[1]. it would have to be fair
Int so now that's for the whole game its fair. . . .cause you said [to Vicki] that one—for one flips its fair
Vicki yeah one flip is fair. . . but how bout this when you have two team B's right here[2]

because you set up the team like this how come you have two team B's rivers instead of team A Int right so the each get there are 8 spots==

Vicki ==oh==

Int ==where the fish can end up and they each get a number of the spots so they each get to set up there nets so that they catch the fish that comes down one river or another

Vicki hmhum. . . .( long pause) OK the next one what made you think the game was fair or unfair how do you answer that?

Juan because we explained it

Int so you said above that it was a fair game and that fair means that each team has the same probability?—or each flip has the same probability?

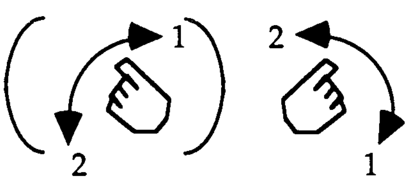
Vicki each flip

Int you can . . . So its fair? Why did you say the game is fair? You can Just==

Juan ==cause they each[3] get[4] there own[3] turn[4]

Gestures

1: wave hands back and forth keeping them a constant distance from each other
2: [gesture not clear]
3: both hands pointing diagonally up and out.
4: Alternates raising and lowering hands




*NOTE the left hand is obscured by Juan' body

By the time I prompt them to begin to record their reasoning, Vicki has re-evaluated the relevance of the partitioned event space. However, Juan is still struggling with understanding the game itself, and so Vicki attempts to point out the relevant features of the task. In her explanation she uses both the process perspective and the outcome perspective.

Excerpt 8

Int so now if you want to answer the questions

Vicki all right the game is not fair ==

Int ==why isn't it fair?

Vicki because[1] if you try all the probabilities go down there is like eight of them and like they only have three[2] and it goes to team A and[3] five to team B so its gonna be unfair to team A though

Juan but when its red here its my turn and when its red over her its her turn right?

Int Oh no,==

Vicki ==no you keep going till you flip

Int no there is nobody actually playing. Its the fisher men

Vicki yeah so look you[4] flip the coin heads or tails or you go here or over here and then you flip a tail and you go here[5]. So there are all[6] (the nets of) the teams so its going to be unfair.

Gestures

1: traces paths down the tree
2: points to the bottom row of the tree
3: points to the bottom row of the tree
4: point to the first intersection at the top of the tree
5: traces a path down the tree
6: horizontally traces the bottom row of the tree

In predictive question two, the two students continue to debate over what is salient about the game as they attempt to make quantitative predictions about the outcome of a number of turns. Vicki believes that Team B will catch more fish because of the asymmetry in the event space. Juan, however, does not agree that this will make a difference. Juan, who at this point is controlling the mouse, pulls the prediction bars to show that Team A will catch more fish, perhaps because of the central location of team A's nets. Vicki does not agree, and Juan in the last turn of the excerpt changes the bar to reflect her opinion.

Excerpt 9

Int Ok so they are going to release ten fish one at a time and swim down. Do you think one team is going to catch more fish than the other? There will only be ten fish to catch.

Juan	one team is gonna catch more than the other
Int	and which team? this team a-- the red bar and team B. You can just slide this bar up and down and how many==
Juan	==team' a
Vicki	==team B team B
Juan	No team A
Vicki	team B
Juan	A
Vicki	B, B
Juan	there[1]Vicki No:::o team B
Juan	Why? just because they got two[2] doesn't mean==
Vicki	==no I mean you only have three that go to team A though but we have five ways to team B, so (if you get the chance)
Juan	Ok
Vicki	Yeah yeah yeah[3]

Gestures
1: pulls bar to show that Team A score more points
2: [refers to two grouping of nets]
3: [Juan is pulling bar to show that B will win 7 of the ten]

However, even though Juan has agreed to use Vicki's answer for the formal prediction that the computer records, he is not convinced. There are at least two reasons why Juan may have entered Vicki's answer instead of his own. First, it could be a social move to avoid further argument and to allow them to continue to make progress on the activity. However, it is also possible that Juan agreed to input her answer because her explanation was more complete and articulate than his own. Vicki had presented evidence from the outcome perspective that gave specific quantities of outcomes to each team. Juan's rationale, stated in the example below, casts doubt on a key aspect of Vicki's argument but does not present any positive argument or causal reason why team A would score more.

Excerpt 10
Int	OK so do you guys both agree with that?
Vicki	Yeah
Juan	Nope
Int	you don't agree?
Juan	Team A , I don't know what they mean
Int	just tell me what you think
Juan	I think just because team b has more[1] ways to get it , that doesn't mean team A[2] cant get more than team B[3]
Int	so do you think that team A is going to get the same amount or more
Juan	probably the same amount or more

Gestures
1: traces the bottom row of the tree with his pinkie and index finger extended
2: points to the left
3: points to the right

In the third predictive question, which asks about the relative frequency of the outcomes, Vicki manipulates the histogram to show a normal distribution. She pulls the bars to show that the frequency of the outcomes in the center will be higher than those on the outside. Juan then takes the mouse and creates a random pattern with the bars of the histogram. When I ask them for their rationale Juan states that because the events are random you will not be able to tell what the outcomes will be. Vicki, however, believes that you will be able to make some predictions about the frequency of outcomes, reasoning that three heads is an unlikely outcome and so the corresponding part of the histogram will be low. In some senses both students are correct. Juan is right in that you cannot accurately predict at any given time what the outcome will be. What he ignores is that over a large number of trials you are able to predict relative frequencies based on the structure of the event space. Vicki is right when she notices that three heads is an unlikely outcome. In fact, it will only happen about one eighth of the time. However, she does not yet realize that any particular event is equally unlikely.

Excerpt 11
Int	and why did you choose this pattern Juan?
Juan	cause you never know where they are gonna go so its you are not gonna be sure
Vicki	But I think there is more a chance that your gonna go down the middle
Int	why do you think it is more in the middle?
Vicki	because if you separate these two[1] right, if you go to the head then they are going to be separate too then you wont likely go to a head again. You—you are going to go to the tail or something like that. So it wont be on the outside so if you give them less. . . .so the middle part is gonna be higher

Gestures
1: spreads index and middle fingers apart at the first intersection

After the predictive questions, the students run the simulation. When they are finished I ask them whether or not they still disagree about the game. They do not. The empirical evidence in this case was not enough by itself to convince Juan that the game was unfair.

Excerpt 12

```
Int    OK but you both disagree still
       on whether it is fair or not,
       still
Juan   Fair cause, we don't know
       which the fish is going to go.
       So its not gonna be our fault.
       Or the fisherman's fault.  The
       fish is gonna decide which way
       he is gonna go.
Int    so the fish is gonna act
       pretty randomly? cause we're
       flipping a coin, and you don't
       agree with that[to Vicki], why
       don't you
Vicki  Because you know how many ways
       its gonna be the--end up.  The
       fish like they have eight ways
       right? and you always have the
       two teams.  They are not gonna
       be 4 & 4, if it is 5 & 3 so
       its not an even number so if
       you go to 4&4 it'll be (fai)r.
```

However, Juan finally adopts Vicki's rationale when I change the task. I switch from asking why the game is fair or unfair to asking them to fix the game. In re-partitioning the event space Juan seems to coordinate both perspectives. First he focuses on the quantitative structure and assigns an equal amount of outcomes to each team. But at the end of the conversational turn he seems to check his reasoning using gestures associated with the process perspective.

```
Int    so since you both said it
       wasn't a fair game, or well
       you [to Juan] were more
       undecided, how can you make it
       a fair game?
Vicki  like you separate evenly
Int    what do you mean?
Vicki  like if you[1] end up eight ways
       you separate them like those[2]
       two team A
Juan   it would be even if put two[3]
       team B's and two team A's[4] or
       just one[5] team B and one team
       A[6] with the same amount[7]
       of......
Int    rivers?
Juan   ==yeah==
Vicki  ==Yeah==
```

Gestures

```
1:  points to left halve of screen
2:  rotates hand
3:  with fingers in a "V" points to the
      left of the bottom row
4:  with fingers in a "V" points to the
      right of the bottom row
5:  with index finger extended points to
      the left-side of the bottom row
6:  with index finger extended Juan
      points to the right-hand side of
      the bottom row
7:  with finger in a "V" he traces up
      and down the middle of the tree
```

Quantitative results

There were no significant quantitative difference between frequency of the outcome and process perspectives in the two conditions. In the animated condition 22 propositions about the event space were coded as outcome and 8 propositions were coded as process. In the non-animated condition 21 propositions were coded as outcome and 3 propositions about the event space were coded as process (Pearnson Chi-Square= 1.65, DF=1, p= 0.199).

There were no qualititive differences between the propositions coded outcome perspective in the two conditions. However, there were qualitative differences between the two conditions in the propositions that were coded as the process perspective. In the animated condition, the students statements about the process by which the probability distribution came to be were organized around the metaphor of motion which was directly mapped onto the event tree (see Excerpts 2,3,4 & 6).

In the non-animated condition the students did not use the metaphor of motion across space to discuss the process by which a probability distribution might occur. Instead, as seen in the examples below, they talked about the process in terms of some patterns of coin flips being more representative of what would happen because they were more random. This metaphor has a low fidelity with the event tree because it there is no close mapping between the "patterns" and the spatial representation of the event space. As a result, none of the students in the non-animated condition talked about any causal mechanism that could be used to determine the expected distribution of coin flips across all possible outcomes.

Excerpt 13.

```
Debby um::m, I ch-chose those--I
      guess cause it's like too much
      of a pattern...um..like Tails
      tails tail uh==___
Jill  tails tails[1]==
Debby yeah, seems like less of  a
      chacne for it to go on that
      same one all three times,and
      um like tails heads tails,
```

like, in a pattern like that, and like heads tails heads

INT uh, huh

Debby some of them seem like less of a chance, and also I think this one right here[2]. It's like all three like um it's all three in a row, all the times its gonna land on the same one

Gestures

1: points to a non-specific spot on the tree

2: use the mouse to circle the left-most path of the tree (Heads-heads-heads)

In the example below, taken from the non-animated condition, when the students try to reason about the expected probability distribution they use the metaphor of taking possession of an object to organize their explanation. Again, this metaphor does not have a high fidelity with the spatial representation of the event tree.

Excerpt 14.

Jen Um I don't think this game is fair, because....its like--there is like no way that team b to get their own--it's like an advantage for team a because all you have to do is just get 2 heads or 1 head with the nickel..and then um____

Liz and its easy to just get two heads

I know..its easy to just get a head for um for the nickle..so I don't think it is fair.

## V. DISCUSSION

The transcripts suggest that both versions of the event tree were successful in the limited instructional objectives laid out at the beginning of this paper. However, the short activity of this study was not enough to provide students with a lasting, coherent concept of probability. While all the student pairs interviewed concluded that the game was unfair (the mathematically correct conclusion) and successfully incorporated references to the partitioned event space in their explanation, preliminary analysis suggests there were no significant gain scores on the pre/post-test.

I claim, however, that the transcripts show that the students had two distinct ways of talking about and referring to the trees, which I have labeled the outcome perspective and the process perspective. The pairs of students, taken as a cognitive system, demonstrate the ability to collaboratively coordinate these two perspectives and successfully reason about the game. Further, the presence of animation facilitated talking about the event tree in a different manner. I will now discuss the origins of both perspectives, their complimentary functions in the process of learning and reasoning and provide an account for why the qualitative difference occurred between the two conditions.

It is important to clarify that I am not making strong claims in this section about students conceptual understanding of the game and of probability. The reader should not conclude that because a student has focused on the outcomes, and used that perspective to successfully reason about the game's fairness that she does not understand the process that produced the outcomes, especially in light of the fluency in which some students switched between the perspectives in their discussions. What I am arguing is that the students' discourse reveals what they are attending to at the moment and that what they are attending to may correlate with some local conceptual structure that emphasizes some aspects of the problem and de-emphasize others.

For the students of this study, evidence suggests that the coordination of the process and outcome perspectives via references to the event tree contributed to successful problem solving and learning. The event tree served as a public resource for communication in two ways. First, in both conditions the event tree helps students when they get into linguistic trouble with the mathematical register of probability. At points in the activity where the students seemed to be struggling for the appropriate word or phrase to express their ideas students made gestures that referenced the event tree and helped clarify the ambiguity of their verbal speech. Second, the event tree provided the students with a way to visualize and talk about the abstract event space. However, the students' interpretations of the animated event tree differed in that the students were better able to connect and coordinate aspects of the event space's quantitative and narrative structure and come closer to a causal model of probability.

### Quantitative Structure

It is my hypothesis that the outcome view of the event tree is in part facilitated by the visual structure of the representation. Static images often are used successfully to convey conceptual information. They are easily interpreted as objects or concepts, but are usually not interpreted as actions or processes (Trversky, 1995). In particular, the hierarchical structure of the tree is often associated with taxonomies of objects (Kress & van Leeuwen, 1996) where the lower leaves are subordinate and more specific exemplars of the category of objects represented in the leaves above them. The absence of labels for the internal nodes further focuses attention on the bottom row of the trees and as result focus students attention on the outcomes of the event space.

Given the nature of the task (a quantitative comparison of expected point totals of the two teams) there is a strong functional reason for the dominance of the outcome view in the student conversations. The bottom row of the event tree reveals the quantitative structure of the event space. The terminal leaves highlight the cardinality of the outcomes and how they are partitioned. The event tree, viewed from the outcome perspective, is an efficient external representation for quantitative reasoning. First, it frees up the cognitive resources that a student would need to produce, organize and partition the event space. Second, it avoids some of the limitations of short term memory by providing an external record of all the possible outcomes of the event space. Students who are reasoning from the outcome perspective can count off different combinations of outcomes to produce quantitative proportions. Student can then use these proportional quantities to reason about the situation and solve problems by comparing quantities and leveraging their skills in arithmetic and algebra.

The outcome perspective, which highlights some aspects of the quantitative structure of the event space, is a natural point from which to build towards mathematical symbolic notation. The outcome perspective provides an experiential base for proportional quantities, fractions and percents in a manner that highlights their relevance in reasoning. This experience and focus on quantities in the abstract leads students to begin to leverage arithmetic and algebra to reason probabilistically, setting the stage for the introduction of powerful and elegant abstract symbolic notation that when understood make computation more efficient. Computation and quantitative comparisons are essential components of probabilistic reasoning. The manner in which the outcome perspective emphasizes these aspects of probability make it an essential piece of a student's conceptual repertoire and a perspective that must be facilitated by the structural features of our learning environment.

Narrative Structure

It may be that other aspects of the visual structure of the event tree in our environment encourage students to adopt a model driven process perspective. Semiotic analysis of visual images postulates that images can be interpreted as a narrative or process if the image has an actor, a goal and a vector that connects the actor to the goal (Kress & van Leeuwen, 1996). The contextualized animated event tree meets these conditions. The fish of the animated condition is the actor, the bottom row of the rivers is the fish's goal, and the image of the river system provides a clear vector that connects the actor to its goal. As a result, the visual grammar of the event tree supports a process interpretation that narrates the events over time and space.

The process view contributes a different way to think about the game, fairness and probability. The process view highlights the narrative structure of the event space, leveraging both the structure of narrative and our spatial reasoning abilities to reason about probability. It brings to the forefront the active agent which is part of a system of activity and facilitates causal interpretations of the agent's activity over time and space. The perspective is inherently contextualized and provides a way of engaging the problem in a subjective manner that is situationally specific.

Further, it is likely that in more complex probabilistic setting the process perspective will become more important to computation. In cases where all events are not equally likely, one must begin to assign weights to the various branches of the event tree. In these cases, computation involves more than quantitative comparisons of the proportion of outcomes, it must account for the unequal probability of each atomic event in the compound event. In these types of situations the probability of each atomic event within the compound event is multiplied to produce the probability of a given outcome, making the relationship between the structure of event space and computation more salient.

The process perspective may contribute to students' understanding of the event space in at least three ways.

First (but not tested in this study), the process view is generative. Understanding the process that the tree represents in terms of nodes and branches, creates a structural framework which students can use to generate an event space for new situations. For example, once a student understands that the nodes represent events and that the branches represent outcomes for that event and can translate the events of a game into this framework, she can use that understanding to create an event tree for this new context. With the creation of the tree the student has enumerated the entire event space, and has positioned herself in a place to use the outcome perspective and the quantitative structure to solve specific problems. Creating the event tree requires information not highlighted by the outcome approach. It requires attending to the narrative structure of the events and representing them graphically. Further research needs to test the validity of this hypothesis.

Second, the process perspective allows students to personify probability. The narrative structure provides students with a means to immerse themselves in the process, identifying with an agent within the story. By adopting the viewpoint of an agent with a goal, or with someone who benefits by that agent's actions, students can reason about the process drawling upon situational knowledge of similar contexts. That is they can create a mental simulation of the process and "run" that simulation attending to and reasoning about the intermediate events. Subjective reasoning from the viewpoint of an agent within a process has been shown to be a productive strategy in scientific practice (Ochs, in Press). In the animated condition it may be that reasoning from "inside" the narrative allows students to recognize the salient aspects of the problem and begin to discover patterns that contribute to a better understanding of the quantitative structure of the event space. That is it may be that it is through the process perspective that the outcome perspective is discovered to be relevant. It is a telling sign that the subjective perspective was only adopted in the animated condition.

Third, Hall has proposed that certain representations allows student to consider both the situation and quantity simultaneously providing redundant constraints on the problem space (1990). It may be that the event tree provides a means for students to coordinate the situational structure of the event space (process view) with the quantitative structure of the event space (outcome view). The combined information constrains the students' model of the situation to a model where what is explicit in the quantitative structure and what is explicit in the narrative structure are both true. Students can thus check and evaluate their inferences against the union of both sets of constraints, giving them a much smaller problem space to explore. In more difficult probability tasks, with events that had unequal probabilities at each branch, the two views would have to be coordinated for the student to perform quantitative computations. In these situations the computing the probability of an outcome requires multiplying the probability of each of the individual events that lead to an outcome. This in turn requires an understanding of the internal structure of the event tree.

One of the arguments of this paper is that the process perspective that the students adopt when talking about and reasoning with the event tree is not a primitive strategy. Nor is it merely the structural origins of the outcome perspective, useful in learning the more abstract outcome approach and then abandoned when that perspective is mastered. I propose that the two perspectives provide complimentary functions that are entwined together in the actual problem solving of students. However, the non-animated version of PIE strips away the context of the Fishermen's competition, replacing it with the weaker narrative structure of a game of tossing three coins. In addition, when the fishermen context was removed so was the agent of the narrative—the fish. With the loss of the fish as an animated agent that highlighted the intermediate events on the event tree, one of the critical three elements that facilitates a narrative interpretation was lost (although the "graying-out" of branches of the tree still highlighted the internal structure of the event tree). Thus salient features of the narrative structure that supports the process perspective were suppressed and the resulting understanding of probability was less developed. Students, in this new environment did attend to intermediate events, did not consider adopting a subjective viewpoint, and took longer to notice relevant aspects of the quantitative structure.

We had unintentionally[4] created an environment where the abstract quantitative structure and partitioning of the event space remained exactly the same, but the situational and narrative context that highlights the process was absent. It appears that features of the environment, such as an animated agent, might be needed to highlight the narrative structure and provide a causal model to organize ones understanding between the partitioning of the event space and the probability distribution

## VI. CONCLUSION

In this paper I have examined the role that graphical representations can play in mathematics education. I have shown that graphic representations provide a valuable resource for mathematical discussions with no loss of mathematical power for some of the problem solving tasks within probability. In inquiry based learning environments, such as PIE, these discussions are critical to the development of students' construction of a shared understanding of the domain.

The PIE environment demonstrates one way in which graphical representations can be leveraged to achieve instructional objectives. Further, the analysis of the students talk revealed how features of the environment, such as rich contexts and animation, can facilitate important ways of interpreting the representations and ultimately the mathematical concepts. This analysis has important implications for designers of instructional activities, but also touches on issues that may be important to the more general audience of those engaged in the design of systems where interaction and interpretation play critical roles.

## Acknowledgments

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[4] I say unintentionally because this was not a controlled experiment. Rather the differences in PIE were the result of our iterative design cycle where each new version is user tested. The non-animated version of the environment was a letter version of the software in which some additional features were added or changed. Some of these changes appear to have been mistakes.

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