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ABSTRACT

This teacher support packet is based on Oregon's Academic Content Standards and Performance Standards which require higher skills of Oregon students at all grade levels. Under these standards, in order to receive a Certificate of Initial Mastery and at benchmark grade levels, students need to achieve a specific score on the statewide assessment and produce classroom work samples which exemplify this high standard. This teacher support package is intended to provide a clear description of: (1) performance standards in mathematics for each benchmark and the Certificate of Initial Mastery level; (2) examples of problems from the state multiple choice mathematics test which are at the level required by the performance standards; (3) examples of problems from each strand of the content standards which have typically been difficult for students; and (4) examples of open-ended mathematics tasks, student solutions, and the scoring guide used for these and for classroom work samples. The specific number, type, and the minimum scores required on classroom assignments are also included. (DDR)

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# MATHEMATICS

## Teacher Support Package

**Helping Students Work Towards  
The Certificate of Initial Mastery  
and Benchmark Expectations**

October 1996

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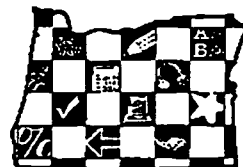
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## Introduction

Recently, the Oregon State Board of Education adopted new academic Content Standards and Performance Standards which will require higher skills of Oregon students at all grade levels. In order to receive a Certificate of Initial Mastery, and at benchmark grade levels along the way, students will need to achieve a specific score on the statewide assessment and produce classroom work samples which also exemplify this high standard.

This teacher support packet is intended to provide a clear description of the following:

- performance standards for mathematics for each benchmark and the Certificate of Initial Mastery level;
- examples of problems from the state multiple choice mathematics test which are at the level required by the performance standards;
- examples of problems from each strand of the content standards which have typically been difficult for students;
- examples of open-ended mathematics tasks, student solutions, and the scoring guide used for these and for classroom work samples.

After reviewing this packet, teachers and students should have a better understanding of the type of mathematics problems and tasks they are likely to encounter on state multiple choice and open-ended assessments and of the level of mathematics proficiency required to meet the performance standards for each benchmark and the Certificate of Initial Mastery.

The specific number, type and minimum scores required on classroom assignments are also included in this packet which will assist teachers in planning for classroom activities and using scoring guides.

Additional teacher assistance materials in mathematics will be developed and distributed during the 1996-97 school year

# Mathematics Performance Standards

## 3 Components of the Mathematics Performance Standards:

Grades 3, 5, 8, 10

- State Multiple Choice Tests  
Tied to the Content Standards
- State Open-ended Mathematics Assessment  
Tied to the Content Standards
- Work Sample Collection
  - Calculations and Estimations
  - Measurement
  - Statistics and Probability
  - Algebraic Relationships
  - Geometry

To meet or exceed the performance standards students must achieve the following scores on state open-ended tests and classroom assignments scored using the official state scoring guide.

Adopted Performance Standards	Grades 3 & 5	Grades 8 & 10
<p>State mathematics multiple choice tests take about one hour for most students to complete. Grade 3 tests contain 40 multiple choice questions. Tests at grades 5, 8 and 10 contain 60 multiple choice questions. Students must achieve the following scores to meet or exceed the performance standards on state mathematics multiple choice tests.</p>	<p>Within five mathematical tasks, students must demonstrate the ability to solve accurately and demonstrate understanding of calculations and estimations, measurement, statistics and probability, algebraic relationships and geometry. On a scale of 1 to 6, students must:</p>	<p>Within five mathematical tasks, students must demonstrate the ability to solve accurately and demonstrate understanding of measurement (8th grade only), statistics and probability, algebraic relationships and geometry. On a scale of 1 to 6, students must:</p>
<p>MEET STANDARD      EXCEED STANDARD</p>	<p>MEET EXCEED STANDARDS</p>	<p>MEET EXCEED STANDARDS</p>
Grade 3	<p>202</p>	<p>215</p>
Grade 5	<p>215</p>	<p>231</p>
Grade 8	<p>231</p>	<p>239</p>
Grade 10	<p>239</p>	<p>249</p>
	<ul style="list-style-type: none"> <li>• Show an understanding of the mathematical concepts related to the task</li> <li>• Choose strategies that can work and carry out the strategies chosen</li> <li>• Explain the reasoning at each step, using diagrams, symbols and/or vocabulary</li> <li>• After solving the task, review the work and show why the solution is reasonable in relation to the task</li> </ul>	<ul style="list-style-type: none"> <li>• Show an understanding of the mathematical concepts related to the task</li> <li>• Choose strategies that can work and carry out the strategies chosen</li> <li>• Explain the reasoning at each step, using diagrams, symbols and/or vocabulary</li> <li>• After solving the task, review the work and show why the solution is reasonable in relation to the task</li> </ul>



# Oregon's Statewide Assessment

## Reporting Categories for Multiple Choice Mathematics Assessment

### **#1: CALCULATIONS AND ESTIMATIONS**

Select and apply mathematical operations in a variety of contexts.

- Estimation
- Operations with Whole Numbers
- Operations with Fractions
- Operations with Decimals
- Operations with Integers

### **#2: NUMBER SENSE**

Apply number theories, mathematical rules and algorithms. (beginning in 1997, this will be reported as part of #1)

- Place value
- Rounding
- Numerals and words
- Meaning of Fractions
- Primes, Factors, Multiples
- Ratios
- Matrices
- Relationships among whole numbers, decimals, fractions, percentage, integers, exponents

### **#3: MEASUREMENT**

Select and use units (standard and nonstandard) and tools (metric and U.S. Customary) of measurement.

- Length
- Perimeter
- Weight
- Area
- Volume
- Time
- Temperature
- Money
- Angle
- Rate
- Estimate measurements
- Use formulas
- Use indirect measures (e.g. trigonometry, scale drawings)

## **#4: STATISTICS AND PROBABILITY**

Collect, organize, display, interpret and analyze facts, figures and other data.

- Interpretation of number lines
  - bar graphs
  - line graphs
  - circle graphs
  - histograms,
  - stem and leaf plots
  - charts
  - tables
- Probability - experimental and theoretical
- Odds
- Mean
- Median
- Mode
- Scatter plots
- Draw inferences and make predictions from data

## **#5: ALGEBRAIC RELATIONSHIPS**

Describe and determine generalizations through patterns and functions and represent them in multiple ways.

- Numeric and geometric patterns
- Arithmetic and geometric sequences
- Variables and open sentences
- Linear and nonlinear equations
- Inequalities
- Matrices
- Functions (e.g. linear, exponential, polynomial, direct, inverse, step, trigonometric)

## **#6: GEOMETRY**

Reason about geometric figures and properties and use models, coordinates and transformational geometry to solve problems.

- Congruence
- Similarity
- Two- and Three-dimensional figures
- Pythagorean Theorem
- Geometric Shapes
- Transformations including reflections, rotations
- Coordinate Geometry

# Multiple Choice Mathematics Examples

These multiple choice examples represent the level of difficulty students would have to demonstrate on the state test to meet the standard. Students need to answer 60 - 70 percent of questions of similar difficulty correctly to achieve a score at the standard level.

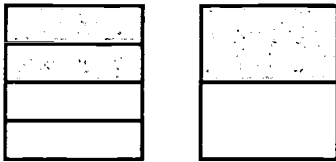
# Examples Representing the Standard

## GRADE 3

### Mathematics

#### Calculations & Estimations

These diagrams show that



- A.  $\frac{1}{2} = \frac{2}{4}$
- B.  $\frac{2}{6} = \frac{1}{2}$
- C.  $\frac{7}{8} = \frac{1}{2}$
- D.  $1 + 1 = 2$

#### Measurement

A board is 24 inches long. How long is it in feet?

- A.  $1\frac{1}{2}$
- B. 2
- C. 36
- D. 72

#### Algebraic Relationships

Mari is showing her brother how she can count by threes. If she starts at zero and goes past 100, which one of these numbers will she say?

- A. 10
- B. 36
- C. 50
- D. 100

#### Statistics and Probability

A class of students kept this chart of the birds they saw over a period of days. It shows how many birds they saw and when. The chart indicates that

	Robins	Sparrows	Pigeons
Morning			
Afternoon			

- A. you're likely to see fewer birds in the morning.
- B. you're more likely to see pigeons in the afternoon.
- C. you're more likely to see sparrows than robins or pigeons.
- D. you should see as many robins in the morning as pigeons.

#### Geometry

What name can you use for each shape?



- A. Parallelogram
- B. Square
- C. Rectangle
- D. Triangle

# Examples Representing the Standard

## Grade 5 Mathematics

### Calculations & Estimations

About what percentage of this triangle is shaded?



- A. 20%    B. 33%    C. 50%    D. 67%





### Measurement

How far would you walk if you walked all the way around this park?

- A. 800 feet  
B. 1200 feet  
C. 1600 feet  
D. 12,000 feet

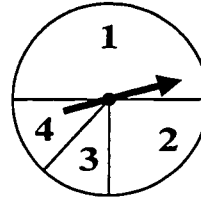
### Geometry

Which figure is NOT a parallelogram?

- A.     C.   
B.     D. 

## Statistics and Probability

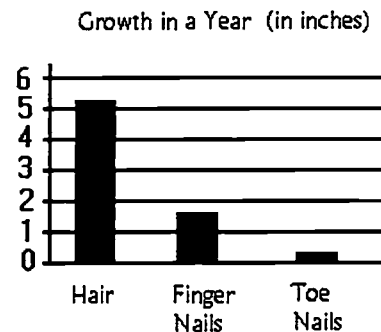
If you were to spin the spinner below, which of the following is most likely to be true?



- A. You are just as likely to spin a 1 as a 3.  
B. You are twice as likely to spin a 1 as a 3.  
C. You are just as likely to spin an odd number as an even one.  
D. You are twice as likely to spin a 1 as a 2.

## Algebraic Relationships

This chart shows how much various parts of your body grow in a year:



If Nadine's hair was 3 inches long at the beginning of the year and she didn't cut it for a year, about how long (in inches) would it be at the end of the year?

- A. 5    B. 7    C. 8    D. 15

# Examples Representing the Standard

## Grade 8 Mathematics

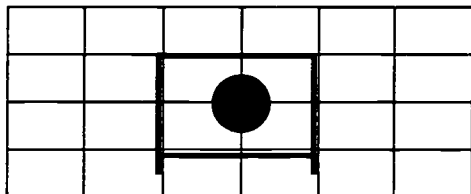
### Calculations & Estimations

Joshua drove 301 miles in 6 hours, 41 minutes.  
At what speed did he drive?

- A. Less than 40 miles per hour
- B. About 45 miles per hour
- C. Over 50 miles per hour
- D. Exactly 55 miles per hour

### Measurement

Use the scale drawing below to estimate the diameter of the hole in the center of the drawing.



1 foot

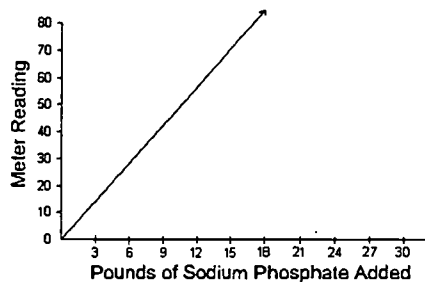
- A. 8 inches
- B. 4 inches
- C.  $\frac{1}{2}$  inch
- D.  $\frac{1}{8}$  inch

### Statistics and Probability

There are 2,400 students at a community college. A random sample of 100 students is taken and 16 are found to be business majors. How many students can be predicted to be business majors in the entire college?

- A. 16
- B. 160
- C. 240
- D. 384

## Algebraic Relationships



Look at this graph:

If the reading on the meter is 74, how many pounds of sodium phosphate should be added?

- A. About 7.5
- B. About 15
- C. 74
- D. There's no way to tell because 74 isn't indicated on the graph.

## Geometry

Of the line segments shown here, only segments A and B are the same length:

- A. \_\_\_\_\_
- B. \_\_\_\_\_
- C. \_\_\_\_\_
- D. \_\_\_\_\_

The four segments, when joined at the endpoints, could form a

- A. trapezoid.
- B. parallelogram.
- C. rectangle.
- D. pentagon.

# Examples Representing the Standard

## Grade 10 Mathematics

### Calculations & Estimations

Which series is ordered from LARGEST (at the top of the column) to SMALLEST (at the bottom)?

A	B	C	D
10	$6.8 \times 10^4$	$6.8 \times 10^4$	$9.6 \times 10^3$
$9.6 \times 10^3$	$9.6 \times 10^3$	$9.6 \times 10^3$	$6.8 \times 10^4$
$6.8 \times 10^4$	10	10	10
0.451	0.451	$\frac{3}{7}$	0.451
$\frac{3}{7}$	$\frac{3}{7}$	0.451	$\frac{3}{7}$

### Measurement

What is the radius of the circle you'd draw with a compass set at 1 inch?

- A.  $\frac{1}{2}$  inch
- B. 1 inch
- C. 2 inches
- D. About 3.14 inches

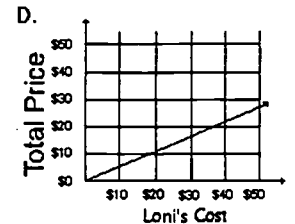
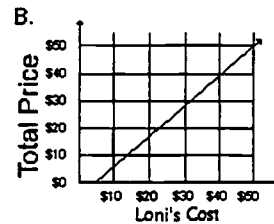
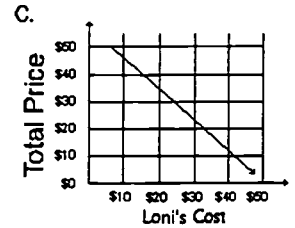
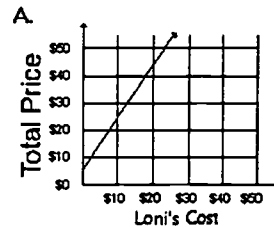
## Geometry

The teacher drew a triangle on the board. She asked a student to draw a line anywhere inside the triangle, just so the endpoints of the line were on different sides of the triangle. Would the line the student drew be parallel to a side of the triangle?

- A. Yes, it would have to be parallel.
- B. It could be, but wouldn't have to be.
- C. No, it couldn't be parallel.
- D. No, but it would be perpendicular to at least one side.

## Algebraic Relationships

Loni is opening a mail order house. For the clothes she sells, she wants to charge twice what they cost her plus a \$5.00 shipping and handling charge. Which chart shows her pricing plan?



# Examples Representing the Standard

## Grade 10 Continued

### Statistics and Probability

What is the MEDIAN number of pages in one chapter of this eleven-chapter book?

CHAPTER	PAGES	CHAPTER	PAGES
Chapter 1	15	Chapter 7	16
Chapter 2	13	Chapter 8	18
Chapter 3	16	Chapter 9	18
Chapter 4	13	Chapter 10	15
Chapter 5	17	Chapter 11	13
Chapter 6	13		

- A. 13   B. 14   C. 15   D. 16



## Multiple Choice Answer Key

Problems Solving	GRADE 3	GRADE 5	GRADE 8	GRADE 10
Calculation and Estimation	A	C	B	B
Measurement	B	C	B	B
Algebraic Relationships	B	C	B	A
Statistics and Probability	C	D	D	C
Geometry	D	D	A	B

# Samples of Difficult Multiple Choice Items

From an item analysis of statewide results, we know which problems the students have the most difficulty solving. Sample items from each of the reporting categories follow, with the related benchmark statement from the content standards.

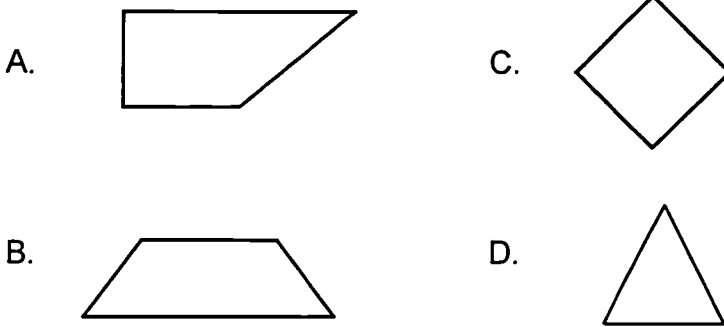
Looking at the results for a classroom of students, there will be some strengths to celebrate and some areas to focus attention on. These samples are provided to assist with that process. Not all schools will need to focus extra attention on all content strands.

**Mathematics problems that 3rd grade students  
had difficulty solving**

**Geometry**

**Benchmark Standard** - Identify properties of geometric figures and combinations of shapes.

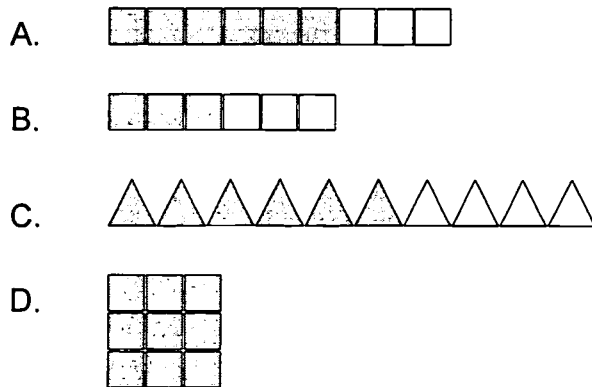
**Example** - Three of the following figures could be folded in the middle, making two equal halves. One figure could not have two equal sides. Which one?



**Calculation and Estimation**

**Benchmark Standard** - Apply concepts of place value and grouping in whole number operations.

**Example** - Which of these represents six-ninths?



## Measurement

**Benchmark Standard** - Develop understanding of measurement and concepts related to length, perimeter, weight, area, volume, time, temperature, money and angle.

**Example** - Which of these could be measured in meters?

- A. How much a whale weighs
- B. The length of a soccer field
- C. How long it takes to run a mile
- D. How much soda pop to buy for a school party

## Statistics and Probability

**Benchmark Standard** - Collect, organize, display and describe simple data using number lines, bar graphs and line graphs.

**Example** - With a table showing the games played by the White Sox, identify which game the White Sox lost by the most points?

Day	White Sox	Red Sox
Tuesday	6	3
Wednesday	4	6
Thursday	2	3

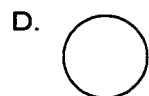
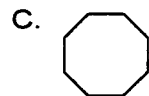
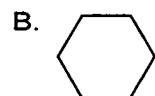
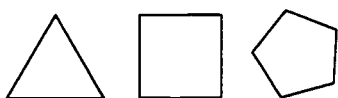
- A. Tuesday's game
- B. Wednesday's game
- C. Thursday's game
- D. There's no way to know.

**Grade 3 Continued**

**Algebraic Relationships**

**Benchmark Standard** - Recognize, create, describe and extend numeric and geometric patterns.

**Example** - What is the next shape in this set of figures?

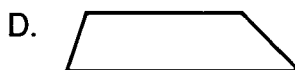
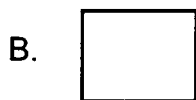


## Mathematics problems that 5th grade students had difficulty solving

### Geometry

**Benchmark Standard** - Construct, draw, measure and compare shapes.

**Example** - Which figure is NOT a parallelogram?



### Calculation and Estimation

**Benchmark Standard** - Model, explain and perform calculations on whole numbers, fractions and decimals, by hand and with a calculator

**Example** - Rod is having a picnic and wants to make 36 cheeseburgers. He knows the following information:

1 pound of hamburger	= 4 patties
1 pound of cheese	= 12 slices
1 cheeseburger	= 1 hamburger patty, 1 cheese slice

How many pounds of hamburger and cheese should Rod buy to make 36 cheeseburgers?

- A. 9 pounds of hamburger and 3 pounds of cheese
- B. 9 pounds of hamburger and 12 pounds of cheese
- C. 6 pounds of hamburger and 3 pounds of cheese
- D. 3 pounds of hamburger and 6 pounds of cheese

## Measurement

**Benchmark Standard** - Develop understanding of measurement related to length, perimeter, weight, area, volume, time, temperature, money and angle.

**Example** - Monica wants to put tile on her kitchen floor. Her kitchen is perfectly square. One side of the kitchen is 9 feet. Each tile is 1 square foot. How many tiles does she need to cover the entire floor?

## Statistics and Probability

**Benchmark Standard** - Formulate and solve problems that involve collecting and analyzing data.

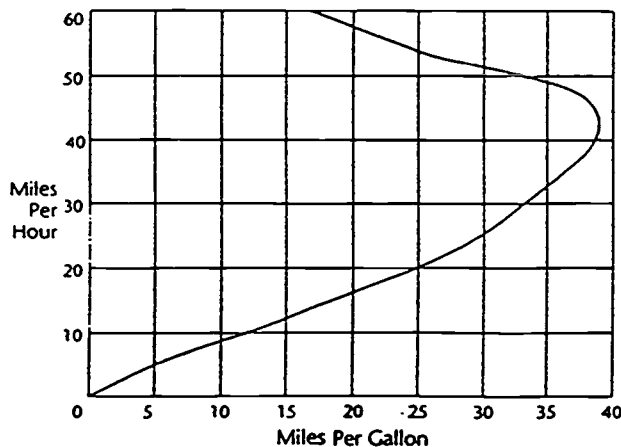
**Example** - There are 5 basketball teams in a tournament. Each team plays each of the other teams once. How many games would be played all together in this tournament?

- A. 30   B. 15   C. 12   D. 10

## Algebraic Relationships

**Benchmark Standard** - Create and use tables, graphs and rules to represent and describe mathematical relationships.

**Example** - Given a line graph showing miles per hour and miles per gallon, determine the approximate miles per hour when the car gets the greatest miles per gallon.



- A. 60   B. 45   C. 30   D. 15

## Mathematics problems that 8th grade students had difficulty solving

### Geometry

**Benchmark Standard** - Visualize and represent geometric properties of two and three dimensional figures.

**Example** - Which of the following objects is most like a *geometric plane*?

- A. A table top
- B. A glass of water
- C. An ice cream cone
- D. shoe box

### Calculation and Estimation

**Benchmark Standard** - Perform calculations on whole numbers, fractions, decimals and integers using paper and pencil, calculators and/or computers

**Example** - To get a particular color of paint, you're supposed to mix 3 parts teal blue paint with 1 part white paint and 2 parts hunter green paint. You need to make 8 gallons of the special paint. If this paint comes only in gallon cans, how many gallon cans of hunter green paint do you need to buy?

- A. 2 gallons
- B. 3 gallons
- C. 4 gallons
- D. 6 gallons

### Statistics and Probability

**Benchmark Standard** - Create, interpret and analyze charts, tables, frequency and central tendency and graphs to make conclusions.

**Example** - Sue played a card game several times last month. Her scores for five games were 135, 142, 187, 210, and 156. What was Sue's average score?



## Measurement

**Benchmark Standard** - Estimate and use tools, scale drawing, models and formulas to calculate length, weight, angle, volume, distance, area,, perimeter, and speed.

**Example** - Each of the three blocks in the figure below weighs the same. The weight of each block is closest to:

- A. 3 units
- B. 6 units
- C. 9 units
- D. 20 units

## Algebraic Relationships

**Benchmark Standard** - Represent mathematical relationships using tables, graphs, patterns, generalizations and equations.

**Example** - Given a table of collected data, identify the graph representing the same information.

The students in a class kept track of the money raised for charity during one week. This information is shown in the table below.

DAY	AMOUNT
Monday (M)	\$8.00
Tuesday (T)	\$12.00
Wednesday (W)	\$22.00
Thursday (Th)	\$39.00
Friday (F)	\$18.00

Four of the students started to construct a bar graph showing this information. Which of their four approaches shown below is the best way to start the graph?

# Mathematics problems that 10th grade students had difficulty solving

## Geometry

**Benchmark Standard** - Explore, deduce or prove characteristics of figures, using transformations, coordinates and/or other geometric properties.

**Example** - Beginning with a triangle, if you draw a line segment with endpoints on different sides of the triangle, would this line be parallel to a side of the triangle?

## Calculation and Estimation

**Benchmark Standard** - Use the relationships among whole number, decimal, integer, percent, exponent, and integer operations.

**Example** - Order this series from largest to smallest

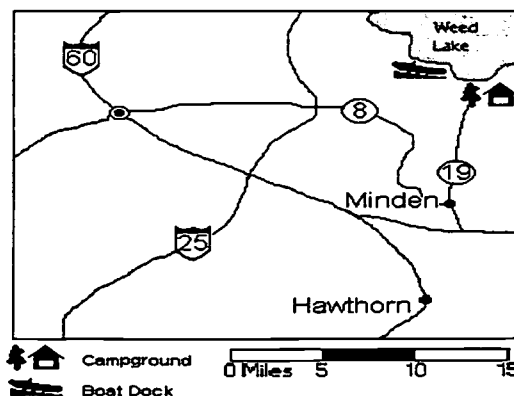
$6.8 \times 10^4$ , 10, 0.451,  $\frac{3}{7}$ ,  $9.6 \times 10^3$

## Measurement

**Benchmark Standard** - Measure perimeter, area, volume, weight, angle, temperature, and distance of regular and irregular shapes using standard and/or nonstandard units of measurement.

**Example** - According to this map, about how many miles is it by road from Hawthorn to the campground at Weed Lake?

- A. 15 to 20
- B. 25 to 30
- C. 35 to 40
- D. more than 40



## Statistics and Probability

**Benchmark Standard** - Create, analyze, draw inferences and make predictions from charts, tables and graphs summarizing data from real-world situations.

**Example** - Given the number of pages in each chapter of an eleven chapter book, determine the mean, median, and mode.

CHAPTER	PAGES	CHAPTER	PAGES
Chapter 1	15	Chapter 7	16
Chapter 2	13	Chapter 8	18
Chapter 3	16	Chapter 9	18
Chapter 4	13	Chapter 10	15
Chapter 5	17	Chapter 11	13
Chapter 6	13		

## Algebraic Relationships

**Benchmark Standard** - Represent and analyze discrete structures and continuous functions using tables, graphs, matrices, generalizations and equations.

**Example** - The chart below shows the total cost for making various units of product, (after an initial \$100 startup expense). What is the per unit charge that will allow for breaking even after selling the first 50 units?



- A. \$3.50      B. \$2.00      C. \$175      D. \$1.60

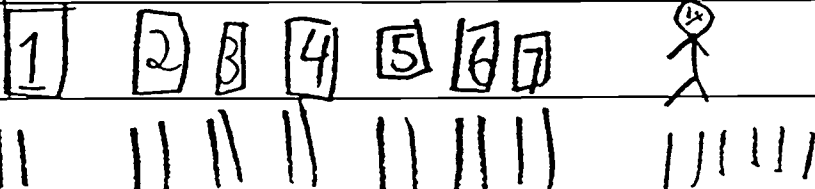
# Open-ended Mathematics Examples

These examples are a collection of elementary and middle level papers taken from the 1994-95 statewide assessment. Following the student papers are commentaries, based on the state scoring guide, which describe both the correctness (precisely, essentially, or not) and the scores (on the scale from 1 to 6) for each of the four dimensions. The quality of student samples are expected to improve (especially in “interpret reasonableness”) as the scoring guide stabilizes and teachers and students around the state become more familiar with the expectations of tasks such as these.

Garth went to the store to buy eggs. He thought, "I need enough eggs to last all week. I eat two eggs for breakfast each day and I need to give six eggs back to my neighbor because I borrowed some." When Garth asked the store clerk for eggs, he was told that eggs are only sold by the dozen. How many dozen will Garth have to buy?


ET11

Explain your thinking at each step and your answer(s).



I think the answer is two dozen because he needs 20 eggs and two dozen are 24 eggs.

3 then I drew a neighbor,

4. Then I drew six  I drew days eggs to represent the two represent ones he gets that the whole week garth borrowed from him.

2, I added two eggs to each

5. I added all the day because eggs together and got he eats two.

20 eggs which is greater than a dozen  
but less than two dozen and that's  
how I got my answer.

I checked my I think my answer is  
work with a right because it matches  
calculator my estimate.

$$\begin{array}{r} 7 \\ \times 2 \\ \hline 14 \\ + 6 \\ \hline 20 \text{ eggs} \end{array}$$

31

Page 4

PLEASE DO NOT WRITE IN THIS AREA



421511

Garth went to the store to buy eggs. He thought, "I need enough eggs to last all week. I eat two eggs for breakfast each day and I need to give six eggs back to my neighbor because I borrowed some." When Garth asked the store clerk for eggs, he was told that eggs are only sold by the dozen. How many dozen will Garth have to buy?

Explain your thinking at each step and your answer(s).

ER11

First I counted out how many days are in a week. He eats 2 eggs a day. So I do  $7 \times 2 = 14$ . He owes his neighbor 6 eggs because he borrowed some. Then I add  $14 + 6$ . Fourteen plus six equals twenty. The clerk at the store said that he could only get them by the dozen. So I punch in on the calculator  $12 + 12$ . Then I see that it says that  $12 + 12$  equals 24. So 12 is one dozen, so I know that  $12 + 12 = 24$ , he has to buy two dozen eggs. So that he could have two eggs for breakfast, he could give his neighbor six eggs and he will have two dozen eggs left from two dozen eggs.

Garth has to buy two dozen eggs to be able to have two eggs per day, per breakfast. And give his neighbors six eggs, with two extra eggs. Another way to solve the problem is by counting up the days in the week that equals seven. Add seven for it equals fourteen. Add six for his neighbor. Then count how much a dozen is. It is 12. So twelve isn't enough for twenty so I add another dozen so that equals 24 so he has to buy 24 dozen. The thing how I know it is

right is because I checked it with a calculator.

$$\begin{array}{r}
 14 \\
 + 6 \\
 \hline
 20
 \end{array}$$

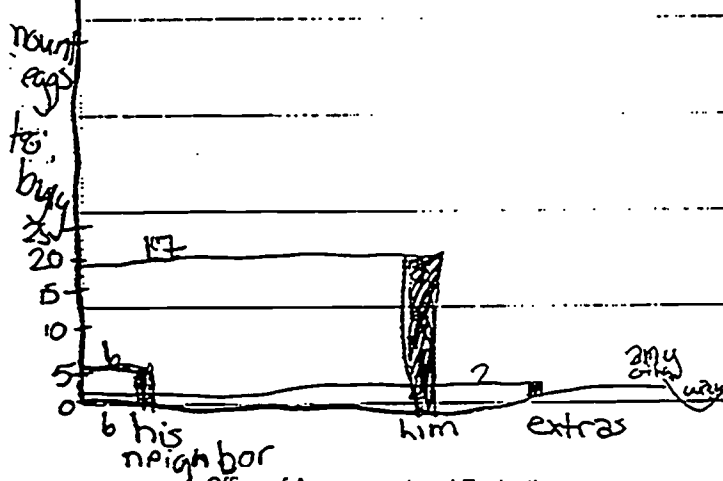
How many eggs to buy

The reason I know my solution makes

sense is because

The two ways I put it are the only ones that I think

makes sense, and I don't know





Your Name \_\_\_\_\_

Your Teacher's Name \_\_\_\_\_

CDS Code \_\_\_\_\_


4b.1

EB12

A recipe for cooking a turkey says to allow 20 minutes for each pound of turkey and to add 5 minutes per pound if the turkey is stuffed. Our turkey weighs 16 pounds and will be stuffed. To have dinner at 2 o'clock in the afternoon, what time does the turkey need to go into the oven? Allow 1/2 hour for cooling and carving. Explain your answer(s) and your thinking.

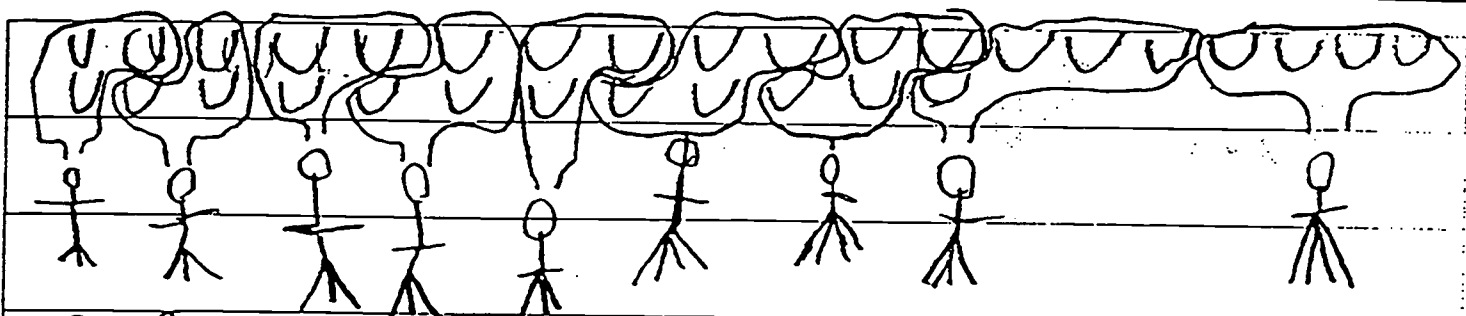
first we put the clock at 2:00 then we took  
30 minutes away now it is 1:30 next  
we took 16 20's away now it is 8:10 next I took  
16 55 away now it is 6:50  
they had to put it in at six fifty AM.  
We used the clock and move the  
hands. We checked it it was  
right

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Oakridge, Oregon was invaded by space creatures: the Fritzes, with four feet, and the Frogs, with three feet. Jill was hidden from them; but counted thirty-one of the creatures' feet as they marched by. How many Frogs and how many Fritzes were marching by Jill?

Explain your thinking at each step and your answer(s).

EX12



5 frogs + 4 Fritzes

1. I put 31 feet on the paper.

2. I put 5 frog on the paper.

3. I counted to see if there would

be enufe for all Fritzes to

go there and there were

4. I put 4 Fritzes on the paper.

5. I counted them and came up  
with 5 frops + 4 fritzles.


EB.11

A recipe for cooking a turkey says to allow 20 minutes for each pound of turkey and to add 5 minutes per pound if the turkey is stuffed. Our turkey weighs 16 pounds and will be stuffed. To have dinner at 2 o'clock in the afternoon, what time does the turkey need to go into the oven? Allow 1/2 hour for cooling and carving. Explain your answer(s) and your thinking.

$$20 + 5 + 16 + 2 + 1.2 = 44.2$$

you put + 20 and add 5 minutes and add 16 and add 2 and add 1.2 hour

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How many different double-topping pizzas can you make with four different toppings?

Explain your thinking at each step and your answer(s).

ETA1

SP

peperonie ✓ peperonie cheese

cheese ✓ peperonie onions

onions ✓ peperonie olives

olives ✓ cheese onions

cheese olives

onions olive

I used a sytematic list to solve the problem. I put peperonie with cheese, onions and olives, then I checked peperonie off after that. I put cheese with onions and olives then I checked cheese off, then I put onions and olives together and checked off onions. I couldn't use olives with anything because the rest were check off so I checked off olives. Then I counted the double-topping pizzas and the were 6

38

combinations

Garth went to the store to buy eggs. He thought, "I need enough eggs to last all week. I eat two eggs for breakfast each day and I need to give six eggs back to my neighbor because I borrowed some." When Garth asked the store clerk for eggs, he was told that eggs are only sold by the dozen. How many dozen will Garth have to buy?

Explain your thinking at each step and your answer(s).

E X14

MP

2. I think that is the answer because there's 12 eggs in a dozen. There are 7 days in a week.  $7 \times 2 = 14$ . So he needs 14 eggs for breakfast. He eats 2 eggs for breakfast. He needs to give six eggs back to his neighbor.  $14 + 6 = 20$ . That's how many eggs he needs. So he needs to buy 2 dozens of eggs.

Garth went to the store to buy eggs. He thought, "I need enough eggs to last all week. I eat two eggs for breakfast each day and I need to give six eggs back to my neighbor because I borrowed some." When Garth asked the store clerk for eggs, he was told that eggs are only sold by the dozen. How many dozen will Garth have to buy?

Explain your thinking at each step and your answer(s).

EC11

MP

- ① I know that there are 7 days in a week. So far he needs 7 eggs.
- ② Then the store told me that he eats 2 eggs for breakfast every morning, so there are 7 days a week he eats 2 every morning so  $7 \times 2 = 14$ .
- ③ Then I remembered that he also needed to give back 6 eggs to a neighbor that he borrowed. So I added 6 eggs to the 14  $6 + 14 = 20$  eggs. If the store only sells eggs by the dozen so I figured he can't buy just a half of a box so he would have to buy 2 boxes to get enough. Garth will have to buy 2 dozen.

Garth went to the store to buy eggs. He thought, "I need enough eggs to last all week. I eat two eggs for breakfast each day and I need to give six eggs back to my neighbor because I borrowed some." When Garth asked the store clerk for eggs, he was told that eggs are only sold by the dozen. How many dozen will Garth have to buy?

Explain your thinking at each step and your answer(s).

ET12

MP

Garth should get 2 dozen egg. Garth eats 2 egg for breakfast each week. so do this promlem  $2 \times 7 = 14$  or  $2+2+2+2+2+2+2=14$  and than add  $14+6=20$  but the eggs come in dozens but dozen + dozen = 24 then get the 20 up there and add 4 to the 20 then it is 24 then eat four more eggs for breakfast.



## Elementary Level Scores and Commentaries

### PAPER ID CU PS C V CORRECTNESS

**ET11**      **5**      **4**      **5**      **4**

**PRECISELY CORRECT - 2 dozen eggs are needed**

- 5      The work shows understanding of 2 eggs/day; days in a week; returning 6 to neighbor; dozen
- 4      The work uses number of days in a week; tallying; returning eggs to the neighbor
- 5      The pictures and symbols paired with written descriptions of the steps and why the student did each of the steps
- 4      There is evidence the student reviewed the work; the student used pictures and symbols as well as written descriptions (either one could have occurred first)

**ER11**      **4**      **3**      **3**      **3**

**ESSENTIALLY CORRECT -  $14+6=20$ ; 24 is 2 dozen, but  $24-20$  is not 2**

- 4      The work shows understanding of dozen; 2 eggs/day; return 6 to neighbor; and needing 20 eggs
- 3      The work begins with a reasonable strategy to find what most believe to be the solution, but the graph used at the end of the solution detracts from solving the task.
- 3      The reasoning is clear in the beginning, but gaps have to be inferred in the second half of the solution - 24 dozen;  $14+6=20$
- 3      A new strategy is used to arrive at the original solution, but throughout the work errors are made and not detected or corrected. 2 eggs left over (twice); 24 dozen; the graph

**EB12**      **4**      **4**      **0**      **0**

**PRECISELY CORRECT - The turkey goes in at 6:50 am**

- 4      The work shows understanding of cooling time; cooking and stuffing time for each pound; going back in time
- 4      The work describes using an actual clock with moveable hands as an efficient strategy for finding the solution
- 0      No information is provided as to why the numbers or strategies were used.
- 0      No evidence of looking back or checking the original solution is provided.

## Elementary Level Scores and Commentaries

PAPER ID CU PS C V  
CORRECTNESS

**EX72**      4      3      3      0

**PRECISELY CORRECT - 5 Frops and 4 Fritzles**

- 4      The work shows understanding of 31 feet; 4 footed fritzles and 3 footed frops
- 3      Pictures and symbols are clearly used to confirm the solution of the task, but the process to solve the task is not recorded
- 3      The reasoning is partially displayed with gaps that have to be inferred. (why start with 5 frops and then end up with 4 fritzles?)
- 0      No evidence of looking back or checking the original solution is provided.

**EB11**      1      2      0      0

**NOT CORRECT -  $44.2 \neq 6:50$  am**

- 1      Only the numbers not the concepts from the task are used.
- 2      Although the processes and strategies are recorded, they are ineffective.
- 0      The reasoning is not provided.
- 0      No evidence of looking back or checking the original solution is provided.

**ETQ1**      4      4      3      4

**PRECISELY CORRECT - 6 different double topping pizzas**

- 4      The work shows understanding of 4 toppings; pairings of toppings; combinations
- 4      A systematic list of pairings with a check-off of toppings was completed
- 3      The work is presented in a clear and organized manner (the chart and the description) while the reasoning is partially displayed with gaps that have to be inferred. Why was there no olive, olive? Is cheese, olive the same as olive, cheese?
- 4      The student describes the strategy used and answer found, but doesn't present a different perspective.

## Elementary Level Scores and Commentaries

PAPER ID CU PS C V

### CORRECTNESS

**EX14**      **4**      **4**      **3**      **2**

#### **PRECISELY CORRECT - 2 dozen eggs are needed**

- 4 The work shows understanding of dozen; 2 eggs/day of the week; returning 6 to the neighbor; needing 20 eggs
- 4  $7 \times 2 = 14$ ;  $6 + 14 = 20$ ; 1 dozen is not enough
- 3 The reasoning behind the process of solving the task is clear until the leap from needing 20 eggs to knowing that means 2 dozen
- 2 The student starts with the answer of 2, then s/he supports that decision

**EC11**      **5**      **4**      **4**      **0**

#### **PRECISELY CORRECT - 2 dozen eggs are needed**

- 5 The work shows understanding of number of days / week; returning 6 to the neighbor; needing 20 eggs
- 4  $7 \times 2 = 14$ ;  $6 + 14 = 20$ ; 1 dozen is not enough
- 4 The student explains “why” at each step in solving (except “half a box”)
- 0 No evidence of a second look at the task or checking the original solution is provided.

**ET12**      **4**      **4**      **3**      **2**

#### **PRECISELY CORRECT - 2 dozen eggs are needed**

- 4 The work shows understanding of 2 eggs per day in a week; returning 6 to the neighbor; dozen; 4 extra eggs
- 4  $2 \times 7 = 14$ ;  $14 + 6 = 20$ ; 2 dozen = 24; 4 eggs are left
- 3 The reasoning is partially displayed with gaps that have to be inferred. (why add  $14 + 6$ ?)
- 2 The second look at the task may support the original approach (which is not recorded - the student begins with the solution), but may not present a different perspective

44

Laura wanted to buy a great birthday gift for her best friend. Unfortunately, when she opened her piggy bank, she found only 100 coins. They were only pennies, dimes, and half dollars. When Laura finished counting up all these coins, she found that she had exactly \$5.00. How many coins of each kind were in Laura's bank?

Explain your thinking at each step and your answer(s).

MT5!

I first thought to make a chart to see how many different possibilities there could be.

1¢	10¢	50¢	Coins Used
50	20	5	75
60	14	6	80
70	13	6	89
80	12	6	98
40	22	6	90
50	25	5	80
50	30	3	83
50	35	2	87
60	24	4	88
70	23	4	97
70	28	3	101
60	29	3	92
60	34	2	96
60	39	1	100

Then I found answer to the problem, 60 pennies, 39 dimes, 1 half dollar = 100 coins. =  $60¢ + 390¢ + 50¢ = \$5.00$ . The answer is 60 pennies, 39 dimes, 1 half dollar.

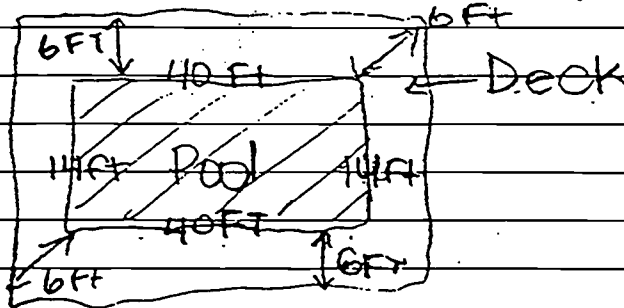
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Joe dug his own pool with a shovel. He also planned a rectangular concrete deck around the pool that would be 6 feet wide at all points. The pool is rectangular and measures 14 feet by 40 feet. What is the area of the deck?

Explain your thinking at each step and your answer(s).

MT9!

First I pictured the plot in my head:



I thought that I needed to know the area of the pool so I multiplied 14 and 40. 14 I found the area of the pool was

$$\begin{array}{r} \times 40 \\ 560 \end{array}$$

560 ft. To get the area of the pool deck I thought I should add the 6 ft on each the 14 ft and the 40 ft. 14 40

$$\begin{array}{r} 6 \\ + 6 \\ \hline 12 \text{ ft} \end{array} \qquad \begin{array}{r} + 12 \\ 12 \\ \hline 26 \text{ ft} \end{array} \qquad \begin{array}{r} 40 \\ + 12 \\ \hline 52 \text{ ft} \end{array}$$

Now the total measurements of the pool was 26 ft by 52 ft. I found the area to be 1352 ft by multiplying 26 and 52.

$$\begin{array}{r} 26 \\ \times 52 \\ \hline 1352 \text{ ft} \end{array}$$

1352 ft is the area of that pool deck with the pool inside. But the area of the pool deck alone is 792 ft because you have to take away the area of the pool which I found out earlier was 560 ft so it was  $1352 - 560 = 792 \text{ ft}$ . After I got this answer I tried to

think of other ways to get the right answer

$$40 \quad 14 \quad 46$$

$+6 +6 \times 20$  This was wrong because the 6 was  
 $46 \quad 20 \quad 920$

supposed to be doubled to 12 then added  
to 40, 14, and multiplied to  $52 \times 26 = 1352$ .  
I could not come up with another  
solution.

I checked my work by multiplying and  
adding on my paper than checking  
it on a calculator

On paper:

$$\begin{array}{r} 40 \quad 14 \\ +12 +12 \\ \hline 52 \times 26 = 1352 \text{ ft} \end{array}$$
$$\begin{array}{r} 52 \\ \times 26 \\ \hline 312 \\ +104 \\ \hline 1352 \text{ ft} \end{array}$$
$$\begin{array}{r} 720 \\ 560 \overline{)1352} \end{array}$$

By Calculator:

$$40 + 12 = 52$$

$$14 + 12 = 26$$

$$52 \times 26 = 1352 \quad 560 \overline{)1352} = 2.414$$

The basic rule to this problem is double  
the six and add it to both sides  
of height and width.

Frank jogged an average of 2.5 miles a day for 3 days. He jogged 4 miles on the first day and 1.25 miles on the second day. How many miles did he jog on the third day?

Explain your thinking at each step and your answer(s).

MTKI

4 miles  
+ 1.25 miles  
5.25 miles

2.5  
x 3  
7.5  
- 5.25  
2.25

1 mile

+ 1.25  
3 | 7.5 = 2.5

2.25 miles that third day

2.5  
2 miles x 3  
7.5 miles

3 miles 7.5 miles in all

4 miles  
1st day (4 miles)

5 miles

2nd day (1.25 miles)

6 miles 4  
1.25

7 miles

1 mile

7.5 miles

2.25 miles

each = 1 mile (turn page for explanation)

After reading the problem I decided to find the sum of the miles he ran for day 1 and day two. It was 5.25. Then I took 2.5 and multiplied it by 3, since you divided it by three to get the average, and I got 7.5. So if 7.5 is the total of miles he ran in the three days I just subtracted 5.25 (the sum of the first two days) to get the number of miles he ran the third day. My answer was 2.25 miles! I checked it by adding the number of miles he ran each day together. The sum was 7.5 which averaged to 2.5 when I divided it by 3.

Please turn the page if you need more space.

## 2nd explanation

I decided to draw a sort-of map showing the total number of miles & the individual number of miles for each day! Since the total number of miles was 7.5 ( $2.5 \times 3$ ) my map was 7.5 inches long, each inch representing 1 mile. I marked in the 1<sup>st</sup> days run (4 miles or 4 inches), then the second (1.25 miles or 1.25 inches). The amount of space left was 2.25 miles (or inches) so I checked by adding the days runs together getting a sum of 7.5 and averaging that out getting 2.5!

My answer is 2.25



Frank jogged an average of 2.5 miles a day for 3 days. He jogged 4 miles on the first day and 1.25 miles on the second day. How many miles did he jog on the third day?

Explain your thinking at each step and your answer(s).

MTK3

$$7.5 - 5.25 = 2.25$$

1st day 2nd day  
4 1.25

3rd day  
2.25

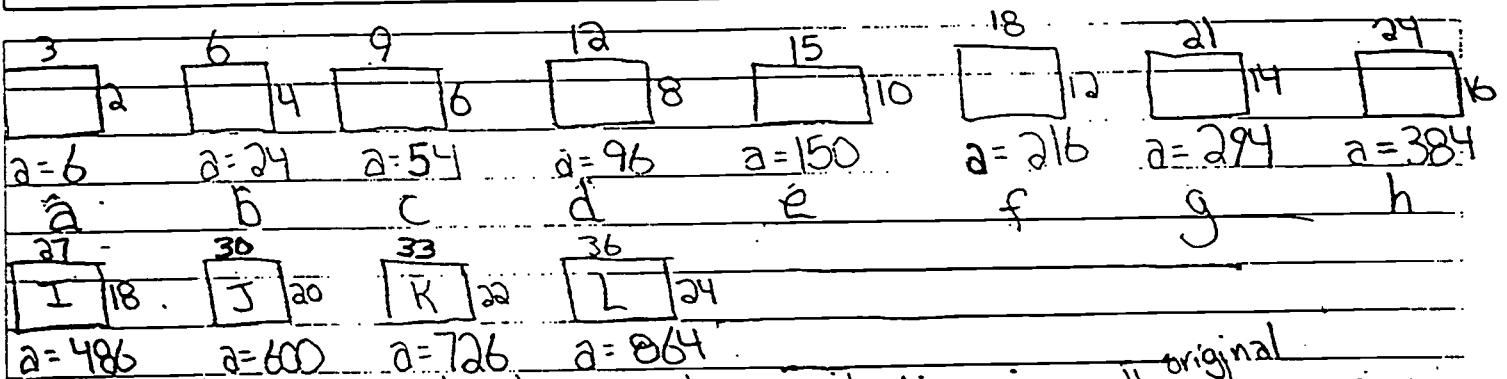
To solve this problem  
You had to times <sup>2.5</sup> by 3 so you can figure  
out the total miles <sup>Frank</sup> ran, then you take away  
the total miles of the first two days and  
you got the answer, Frank jogged 2.25  
miles on his third day of running.

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The length and width of a rectangle are each doubled, then the original length and width are tripled. What is the pattern for the changing areas?

Explain your thinking at each step and your answer(s).

MT31



First, I drew a rectangle, and gave it dimensions, the original dimensions are  $3 \times 2$ . Then I doubled that and got  $6 \times 4$ . Then I tripled the original dimensions, and got  $9 \times 6$ . Then I quadrupled the original dimensions, and got  $12 \times 8$ . And I kept on going. Then, I figured out the areas for each rectangle.

Then I started with rectangle C, and subtracted b, which is  $54 - 24 = 30$ . Then, I realized that B plus A = 30 which is C-B. So I decided to try it on another problem:  $E - D = 150 - 96 = 54$ .  $D + C = 96 + 54 = 150$ . But that didn't work out, but I thought I saw some thing else. In that problem,  $E - D = 54$ , and  $C = 54$ , so maybe that's the pattern. I'm going to try it on another set, to check it!  $F - E = D$ .  $216 - 150 = 66$ , but that didn't work either.

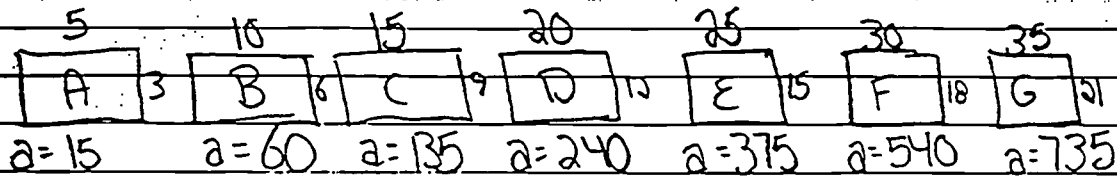
Now I think I'll try something new,  $a \times 4 = b$ ;  $b \times 4 = d$ ;  
 $6 \times 4 = 24$ ;  $24 \times 4 = 96$ ;  
 $C \times 4 = f$ ;  $d \times 4 = 384$ ;  $e \times 4 = 600$ ;  $f \times 4 = 864$ ;  
 $54 \times 4 = 216$ ; 384 is rectangle H!; rectangle J; rectangle L!

As far as I can tell right now, this looks like it might be the pattern, but to be sure, I'm going to figure out what the rectangles would be in the way I did it first.

I think that I got the pattern, you take the area of any rectangle, times it by 4, and you will get a rectangle that's coming up. The only problem with this pattern is that you will

only get every other rectangle.

I am now going to see if this pattern works on rectangles with different dimensions.



$a \times 4 = b$	$b \times 4 = d$	$c \times 4 = F$	$d \times 4 = ?$
$15 \times 4 = 60$	$60 \times 4 = 240$	$135 \times 4 = 540$	$240 \times 4 = 960$

I think that I got the right answer on this problem!

You take the rectangle or rectangle, times it by 4, and you get a rectangle that is coming up!

The capacity of an elevator is either 20 children or 15 adults. If 12 children are currently on the elevator, how many adults can still get on?

Explain your thinking at each step and your answer(s).

M Q8

N

At first I tried to figure out the ratio of 20 and 15 it was 4 children for 3 adults. When I got those two numbers made on list, I put 20 and 15 on the top and subtracted 4 from 20 and ~~two~~ three from 15 each time down to 4 and 3. I looked and saw that 12 and 9 were together. I thought I was the answer at first. I even tried a proportion and got 9 as the answer. I later figured out that 9 wasn't the answer but the capacity of adults if the capacity of children was 12. I then tried something simpler I subtracted 12 from 15 and got ~~three~~ 3 as my answer. So 3 more adults can still get on the elevator if 12 children are on the elevator.

20 to 15

$$\frac{20}{15} = \frac{4}{3}$$

can hold 20c 15a

12 to 9

$$\frac{12}{x} = \frac{20}{15} = 9$$

$$\begin{array}{r|l} 20 & 15 \\ 16 & 12 \\ \hline 12 & 9 \\ 8 & 6 \\ 4 & 3 \end{array}$$

$$\begin{array}{r} 15 \\ -12 \\ \hline 3 \end{array}$$

**3** adults

Frank jogged an average of 2.5 miles a day for 3 days. He jogged 4 miles on the first day and 1.25 miles on the second day. How many miles did he jog on the third day?

Explain your thinking at each step and your answer(s).

MRK

SP

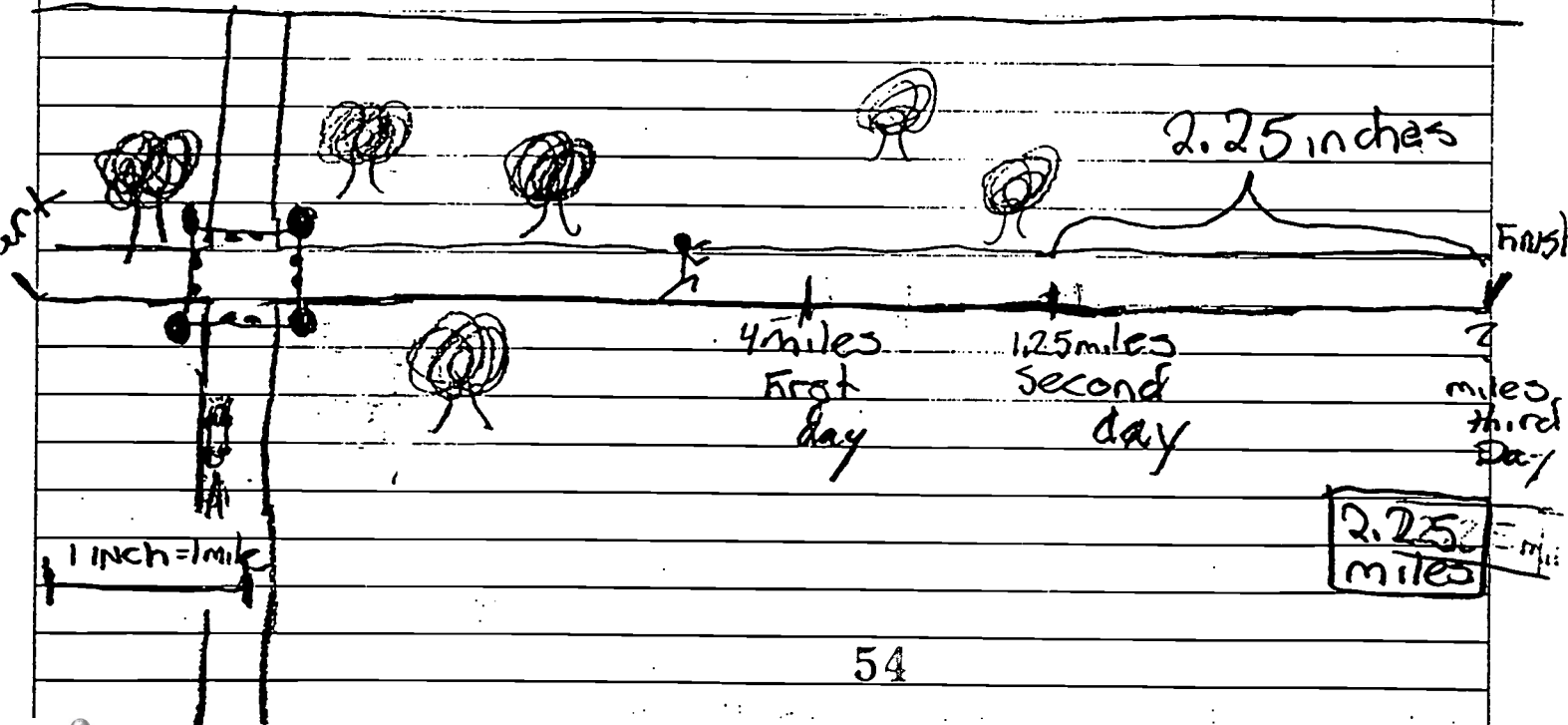
$$\begin{array}{r} 4 \\ + 1.25 \\ \hline 5.25 \end{array}$$

$$\begin{array}{r} 2.5 \\ \times 3 \\ \hline 7.5 \end{array}$$

$$\begin{array}{r} 7.5 \\ - 5.25 \\ \hline 2.25 \end{array}$$

on the third day  
Fred jogged 2.25 miles.

First of all you have to think "if his average is 2.5 miles and he jogged 3 days then you must multiply 2.5 by 3. That would give you 7.5 miles over all. Then you would add 1.25 and 4 because that is the amount of miles he had jogged the first two days, you should now have 5.25. subtract 5.25 from 7.5 and you will get the answer, which represents the the amount of miles Fred jogged on the third day. 2.25 is your answer.



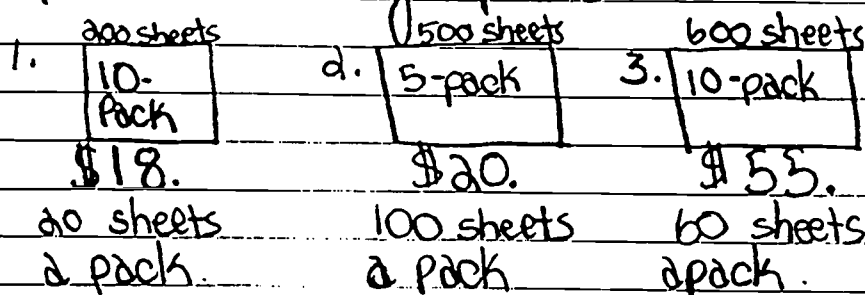
Allison is ordering copy machine paper for the office. She can purchase a 10-pack box with 200 sheets in a pack for \$18, or she can get a 5-pack box with 500 sheets in a pack for \$20. The third option is to buy a box containing 10 packs of 600 sheets each for \$55. Which is the better buy?

Explain your thinking at each step and your answer(s).

MC11

MP

The first thing I want to do is find the answer to this problem. What I want to do is draw a picture of my problem.



$$10 \overline{) 180} = 18$$

$$5 \overline{) 200} = 40$$

$$10 \overline{) 550} = 55$$

What I just found out was how many sheets are in each pack of papers. In the first pack of 10 there is 20 sheets in each pack, in the second pack there is 100 sheets in each, and in the third there is 60 sheets in each sheet. The way I found throughs three answers was by dividing the number of packs by the numbers of sheets.

1. <sup>1 sheet</sup> [ ] = .90¢	2. <sup>1 sheet</sup> [ ] = .20¢	3. <sup>1 sheet</sup> [ ] = .92¢
-------------------------------------	-------------------------------------	-------------------------------------

Now that I know how much 1 sheet cost I know now that Allison should buy the 5-pack for \$20. and get 500. Thats a better buy because each sheet cost less then the other to deals. Also you get 10 sheets in each pack for only 5 dollars.

BEST COPY AVAILABLE

Allison is ordering copy machine paper for the office. She can purchase a 10-pack box with 200 sheets in a pack for \$18, or she can get a 5-pack box with 500 sheets in a pack for \$20. The third option is to buy a box containing 10 packs of 600 sheets each for \$55. Which is the better buy?

Explain your thinking at each step and your answer(s).

M XII

MP

Well first I am going to find out the total number of sheets in each pack.

- 1- 10 pack - 2,000 sheets for \$18
  - 2- 5 pack - 2,500 sheets for \$20
  - 3- 10 pack - 6,000 sheets for \$55
- Then I'll find out how much per paper. (Divide total cost (\$) by total paper)

- 1-  $\frac{9}{10}$  of a penny per paper
- 2-  $\frac{8}{10}$  of a penny
- 3-  $\frac{9}{10}$  of a penny

The first two came out even tenths of a penny, but the third came out a little more than  $\frac{9}{10}$ .

From the results it clearly shows that buying the 5-pack box with 500 is the best buy.

Above I found out which was the best according to each individual sheet. That may have been the most accurate way to figure it out but I'm going to find out how much (\$) per 10 pieces of paper.

- 1- 10 pack - 2,000 for \$18
  - 2- 5 pack - 2,500 for \$20
  - 3- 10 pack - 6,000 for \$55
- I will first use the same first step as before.

Then I will divide each \$ by the 10<sup>th</sup> (maybe) of the total.

1-  $\frac{2,000}{10}$  then  $\frac{18}{200}$  = same as above

I started to do it that way but it didn't work the way I expected it and it's kind of a waste of time so I'll

try it a different way.

- 1- 2,000 for \$18
- 2- 2,500 for \$20
- 3- 6,000 for \$55

Well I will eliminate the possibility of #3 because twice #2 then you get 5,000 for 40 \$ then

if you add half of #2 (1,250/\$10) you get 6,250 sheets of paper for 50 dollars and that a way better deal than 6,000 for \$55.

So now I have the first two to determine the difference or which one's the better buy

I'm going to take 500 (from #2) and take the equal \$ amount away.

500 sheets (in #2) cost \$4.50 cents so to make the number of sheets even I'll make it -

- 1- 2,000 for \$18
- 2- 2,000 for \$20 - 4.5 (\$15.50)

That means that number 2 is the best deal once again

The Answer is that the \$18 5-pack with 500 in each pack is the better buy for Allison.



## Middle Level Scores and Commentaries

PAPER ID CU PS C V  
CORRECTNESS

**MT51**      **4**      **4**      **3**      **3**

**PRECISELY CORRECT - the total is \$5.00 and 100 coins (60 pennies, 39 dimes, and 1 half dollar)**

- 4 The work shows only using pennies, dimes, and half dollars; making combinations of \$5.00, and finding 100 coins
- 4 The work uses a table to record guess and check - a reasonable process/strategy
- 3 The reasoning is partially displayed with gaps that have to be inferred (why the student used multiples of 10 in the penny column; did the rows of the chart add to \$5.00; how did the student decide the next guess? what is the pattern of guesses?)
- 3 There is evidence the student looked back at their work ( $60 + 3.90 + 50 = 5.00$ ); there would be no reason to state this unless the student had looked back at what the task was asking. However, the work doesn't show the rows add to \$5.00 - in fact 50,25,5 doesn't.

**MT91**      **5**      **4**      **4**      **3**

**ESSENTIALLY CORRECT - 792 ft should be 792 square feet**

- 5 The work shows understanding of the difference between the area including the pool and the area of the pool being the area of the deck; the work also shows the need to double the 6 for each dimension
- 4  $14 \times 40 = 560$ ;  $26 \times 52 = 1352$ ;  $1352 - 560 = 792$
- 4 The "why" of the first approach to the problem was clearly organized and described, however, the "why" of the second approach has gaps to be inferred
- 3 The second look at the task (the back of the sheet) supports the original approach, but contains errors which are not detected or corrected ( $1352 \div 560 = 720$ ; 720 is not the same as 792, twice)

## Middle Level Scores and Commentaries

PAPER ID CU PS C V  
CORRECTNESS

**MTK1**      **5**      **5**      **5**      **5**

**PRECISELY CORRECT - 2.25 miles the 3rd day**

- 5 The work shows thorough understanding of average; the difference of total provided and total jogged is the 3rd day miles
- 5 Charting the total miles and then breaking it into givens is thoroughly developed support for the original strategy to find the missing number of miles jogged
- 5 The reasoning behind the algorithmic process of solving the task is clearly described on the front of the page, the reasoning behind the “map” is clearly described on the back of the page.
- 5 The second look at the task (the “map”) uses a different perspective to verify or support the first approach (the algorithmic).

**MTK3**      **5**      **4**      **3**      **0**

**PRECISELY CORRECT - 2.25 miles the 3rd day**

- 5 The work shows thorough understanding of average; the difference of total provided and total jogged is the 3rd day miles
- 4  $2.5 \times 3 = 7.5$   $7.5 - 5.25 = 2.25$
- 3 The reasoning is partially displayed with gaps that have to be inferred (5.25 came from where?); the work does not follow a clear sequence (it begins with the solution, and then describes a justification)
- 0 No evidence of looking back or checking the original solution is provided. 0

**MT31**      **4**      **4**      **4**      **2**

**ESSENTIALLY CORRECT - the student has found a pattern for the changing areas (although the pattern is different from what most students were looking for)**

- 4 The work shows an understanding of area, doubling, tripling, and patterns
- 4 Pictures, models, and symbols are used to solve the task using reasonable mathematical processes/strategies - doubling, tripling, multiplied by 4, then by 5, and so on; finding areas, looking for patterns.
- 4 The reasoning behind the process of solving the task (what the student did when and why s/he decided on the next step) is clear throughout the organized work.
- 2 There is evidence of approaching the task of finding the pattern from a variety of directions, but there is no review of the original strategy (the checking focuses on the solution)

## Middle Level Scores and Commentaries

**PAPER ID CU PS C V**  
**CORRECTNESS**

**MQ8 3 2 3 0**

**NOT CORRECT - 6 adults could fit on the elevator**

- 3 The work shows a partial understanding of ratio  $12:x = 20:15$ , but not complete enough to know what to do with the results
- 2 The strategy of subtracting 12 from 15 was ineffective
- 3 The reasoning is partially displayed with gaps that have to be inferred (why wasn't 9 the correct answer?)
- 0 No evidence is provided

**MQK 5 5 4 4**

**PRECISELY CORRECT - 2.25 miles on the third day**

- 5 The work shows thorough understanding of average; the difference of total provided and total jogged is the 3rd day miles
- 5 Charting the total miles (the line is actually 7.5 inches long) and then breaking it into givens is thoroughly developed support for the original strategy to find the missing number of miles jogged
- 4 The reasoning behind the process of solving the task is clear throughout the organized work.
- 4 The second look at the task (the "map") uses a different perspective to verify or support the first approach (the algorithmic).

**MC11 3 2 3 0**

**PRECISELY CORRECT - Allison should buy the 5-pack**

- 3 An understanding of some of the major concepts (best buy; price per sheet) is displayed. A lack of understanding of sheets in a pack is also displayed.
- 2 Mathematical processes/strategies are complete, but are not all reasonable, (Why divide the number of sheets in a pack by the number of packs?) and detract from solving the task
- 3 The reasoning is partially displayed with gaps that have to be inferred. (Also you get 10 sheets in each pack for only \$5.00 - where did this come from?)
- 0 No evidence is provided.

## Middle Level Scores and Commentaries

PAPER ID CU PS C V  
CORRECTNESS

**MX11 5 4 4 5**

**PRECISELY CORRECT - Allison should buy the 5-pack**

- 5 The work shows an understanding of sheets in a pack; price per sheet; best buy; equivalent sizes; estimation of prices
- 4 10 pack with 200 sheets = 2000 sheets;  $\$18/2000$  sheets =  $9/10\text{¢}$ ; estimation
- 4 The reasoning behind the process of solving the task is clear throughout the organized work. (i.e. I am going to take 500 sheets from #2 and take the equal \$ amount away . . . to make the number of sheets even . . comparing prices)
- 5 The second look at the task uses a different perspective to verify or support the first approach. "Above I found out which was the best according to each individual sheet. That may have been the most accurate way to figure it out, but I'm going to find out how much (\$) per 10 pieces of paper . . . . I started to do it that way, but it didn't work the way I expected it and it's kind of a waste of time so I'll try it a different way. . . . that means that number 2 is the best deal once again."

# Scoring Guide Packet

## How to Use the Mathematics Scoring Guide

The intent of the Mathematics scoring guide is to provide students with feedback related to each of the four dimensions as well as the overall accuracy of their solution. The four dimensions isolate distinct elements of the problem solving process on which student progress can be measured. **Conceptual Understanding (CU)** is the “what” of solving a task. The student will be showing an understanding of the mathematical concepts related to the task. **Processes and Strategies (PS)** is the “how” of solving a task. The student will be choosing processes/strategies that can work, and carrying them out. **Communication (C)** is the student connecting the what and the how of solving a task. The student will be explaining the reasoning (the “why”) at each step, using diagrams, symbols, and/or vocabulary. **Verification (V)** is the “proof” of solving a task. After solving the task, the student will be reviewing the work in relation to the task, and showing that the solution(s) is/are reasonable.

Additionally, students need feedback regarding the overall accuracy of their work. There are three possible responses. The work may be **Precisely Correct** : the solution is accurate with no misuse of terminology or improper labels. Another option is if the work is **Essentially Correct** : there is enough evidence that the student knows what s/he is doing although there are flaws. No additional instruction appears necessary for the student to improve the work. A brief note (i.e., did you show why you know you have the correct solution?) to the student would allow him/her to improve the paper. If the work doesn't meet either of these descriptions, then it is **Not Correct**.

To use the mathematics scoring guide, teachers will usually read through the student work and assign a score of Precisely Correct, Essentially Correct, or Not Correct. Teachers will then typically read the descriptor of a 3 for one dimension, consider the student work, then read the 4 and 5 descriptors or the 1 and 2 descriptors prior to assigning a score. The process is then repeated for the other dimensions. It is important when using the scoring guide to treat each dimension as a separate score rather than a holistic approach. A weakness in the work should only reduce a score in the dimension in which the weakness occurred. Likewise, a paper with a Precisely Correct answer still needs careful consideration for success in each of the four dimensions. The bullets within each dimension are helpful when identifying specific strengths or weaknesses of the work. Some teachers have found the version with the letters replacing the bullets to be helpful in communicating with students.

Because a single scoring guide is used for a variety of tasks, the student work is not always expected to fit each of the criteria (identified with bullets or letters) for any score point. It will however, have characteristics similar to those described in the criteria, or will meet most of the criteria. This non-exact matching allows the scoring guide to fit a variety of tasks. Not all tasks can allow the student to demonstrate each of the characteristics within a dimension score. On the mathematics scoring guide, the level 0 identifies when no evidence is present for that dimension. The level 6 is not yet described but creates the opportunity to go beyond what is described in the 5, allowing the students to show their strengths. To describe a 6 for mathematics will require the review of more evidence from student performances.

For a student to meet the proposed standard, using the “official scoring guide” s/he would need to receive a score of at least a 4 in the four dimensions, and the work would need to be rated either precisely correct or essentially correct. This is a two-part requirement.

**Note: these edits to the scoring guide are the result of the field tests. This is the version of the scoring guide which will be used to measure student results during the 1996-1997 statewide assessment.**

## MATHEMATICS PROBLEM SOLVING STUDENT DIRECTIONS

Today you will be given one task to solve. Please show all of your work. Your paper will be read and scored by someone other than your teacher. Please be sure to make it clear to the readers of your paper how you solved the problem and what you were thinking. The two people who will read your paper will give you four scores, looking for these things:

1. How well you **understood the problem** and the kind of mathematics you used.
2. How well you carried out more than one **problem solving strategy** (if appropriate).
3. How you **communicated** your mathematical reasoning in arriving at your **solution**.
4. How you reviewed your work to be sure it made sense.

**You may use manipulatives or a calculator to work on your task.**

*Please, show all of your work (first efforts and those that worked) in the spaces provided, so everything you do is visible to the readers.*

### Guide To Completing The Task

To receive the highest score in each of the four areas, you will want to find evidence of each of these parts of a successful solution.

#### 1. CONCEPTUAL UNDERSTANDING

- I used the important information to solve the task, changing it to mathematical ideas.
- My work and solution fit what was requested in the task/question.

#### 2. PROCESSES & STRATEGIES

- I used diagrams, pictures, models and/or symbols to solve the task.
- I used problem solving skills/strategies that showed good reasoning.

#### 3. COMMUNICATION

- I explained what I was thinking while working the task, including using pictures, charts or diagrams to help explain "the why" of my steps.
- In my solution, one step seems to flow to the next.

#### 4. VERIFICATION

- I showed that I reviewed my solution process (and checked my calculations) to verify my work makes sense.

In addition, you will know if you were Precisely Correct, Essentially Correct (a helpful hint will get you to be precisely correct), or Not Correct.

# Mathematics Scoring Guide

\_\_\_\_\_ Precisely Correct      \_\_\_\_\_ Essentially Correct (no additional instruction appears necessary)      \_\_\_\_\_ Not Correct

	<b>Conceptual Understanding</b>	<b>Processes &amp; Strategies</b>	<b>Communication</b>	<b>Verification</b>
	Showing an understanding of the mathematical concepts related to the task (the "what")	Choosing strategies that can work, and then carrying out the strategies chosen (the "how")	Explaining the reasoning (the "why") at each step, using diagrams, symbols, and/or vocabulary	After solving the task, reviewing the work & showing that the solution process is reasonable in relation to the task (the "proof")
<b>6</b>				
<b>5</b>	A) The task is translated into thoroughly developed mathematical concepts B) All relevant information and/or data from the task is used	A) Pictures, models, diagrams, and/or symbols are clearly used to solve the task B) Thoroughly developed mathematical processes/strategies are completed	A) The reasoning behind the process of solving the task is clearly displayed throughout and enhanced through the use of graphics and/or examples B) A logical & coherent presentation allows the reader to move easily from one thought to another	A) The review of the work (strategies and calculations) uses a different perspective to verify or support the reasonableness of the first approach
<b>4</b>	C) The task is translated into adequate mathematical concepts D) Relevant information and/or data from the task is used	C) Pictures, models, diagrams, and/or symbols are used to solve the task D) Reasonable mathematical processes/strategies are completed	C) The reasoning behind the process of solving the task is clear throughout D) The presentation of the reasoning is clear and organized	B) The review of the work (strategies and calculations) supports the original approach, but may not present a different perspective
<b>3</b>	E) An understanding of some of the major concepts is displayed F) Fragments of the information and/or data presented in the task are used	E) Pictures, models, diagrams, and/or symbols may be only partially useful or used to solve the task F) Reasonable mathematical processes/strategies are partially completed (or partially recorded)	E) The reasoning behind the process is partially displayed with gaps that have to be inferred F) The presentation of the reasoning does not follow a clear pattern or sequence	C) The review of the work (strategies and calculations) supports the original approach, but an error is not detected or corrected
<b>2</b>	G) Part of the task is translated into inappropriate concepts H) Assumptions about the information and/or data presented in the task may be flawed	G) Pictures, models, diagrams, and/or symbols used may partially detract from solving the task H) Processes and strategies are ineffective	G) Communication focuses solely on the solution and does not indicate a clear path to the solution	D) The checking of the work focuses on the solution and/or E) The checking may be in conflict with the original approach, but the conflict is not addressed
<b>1</b>	I) The task is translated into inappropriate concepts J) Inappropriate information and/or data presented in the task is used	I) Pictures, models, diagrams, and/or symbols conflict with their solution	H) The reasoning detracts from the work and may include irrelevant ideas	F) The checking of the work is ineffective ( i.e., misses the errors)
<b>0</b>	K) No evidence is provided	J) Strategies are not recorded	I) The reasoning is not completed or provided	G) No evidence is provided





\_\_\_\_ Precisely Correct

\_\_\_\_ Essentially Correct (no additional instruction appears necessary)

\_\_\_\_ Not Correct

	<b>Conceptual Understanding</b>	<b>Processes &amp; Strategies</b>	<b>Communication</b>	<b>Verification</b>
<b>6</b>	Showing an understanding of the mathematical concepts related to the task (the "what")	Choosing strategies that can work, and then carrying out the strategies chosen (the "how")	Explaining the reasoning (the "why") at each step, using diagrams, symbols, and/or vocabulary	After solving the task, reviewing the work & showing that the solution process is reasonable in relation to the task (the "proof")
<b>5</b>	<ul style="list-style-type: none"> <li>The task is translated into thoroughly developed mathematical concepts</li> <li>All relevant information and/or data from the task is used</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols are clearly used to solve the task</li> <li>Thoroughly developed mathematical processes/strategies are completed</li> </ul>	<ul style="list-style-type: none"> <li>The reasoning behind the process of solving the task is clearly displayed throughout and enhanced through the use of graphics and/or examples</li> <li>A logical &amp; coherent presentation allows the reader to move easily from one thought to another</li> </ul>	<ul style="list-style-type: none"> <li>The review of the work (strategies and calculations) uses a different perspective to verify or support the reasonableness of the first approach</li> </ul>
<b>4</b>	<ul style="list-style-type: none"> <li>The task is translated into adequate mathematical concepts</li> <li>Relevant information and/or data from the task is used</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols are used to solve the task</li> <li>Reasonable mathematical processes/strategies are completed</li> </ul>	<ul style="list-style-type: none"> <li>The reasoning behind the process of solving the task is clear throughout</li> <li>The presentation of the reasoning is clear and organized</li> </ul>	<ul style="list-style-type: none"> <li>The review of the work (strategies and calculations) supports the original approach, but may not present a different perspective</li> </ul>
<b>3</b>	<ul style="list-style-type: none"> <li>An understanding of some of the major concepts is displayed</li> <li>Fragments of the information and/or data presented in the task are used</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols may be only partially useful or used to solve the task</li> <li>Reasonable mathematical processes/strategies are partially completed (or partially recorded)</li> </ul>	<ul style="list-style-type: none"> <li>The reasoning behind the process is partially displayed with gaps that have to be inferred</li> <li>The presentation of the reasoning does not follow a clear pattern or sequence</li> </ul>	<ul style="list-style-type: none"> <li>The review of the work (strategies and calculations) supports the original approach, but an error is not detected or corrected</li> </ul>
<b>2</b>	<ul style="list-style-type: none"> <li>Part of the task is translated into inappropriate concepts</li> <li>Assumptions about the information and/or data presented in the task may be flawed</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols used may partially detract from solving the task</li> <li>Processes and strategies are ineffective</li> </ul>	<ul style="list-style-type: none"> <li>Communication focuses solely on the solution and does not indicate a clear path to the solution</li> </ul>	<ul style="list-style-type: none"> <li>The checking of the work focuses on the solution and/or</li> <li>The checking may be in conflict with the original approach, but the conflict is not addressed</li> </ul>
<b>1</b>	<ul style="list-style-type: none"> <li>The task is translated into inappropriate concepts</li> <li>Inappropriate information and/or data presented in the task is used</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols conflict with their solution</li> </ul>	<ul style="list-style-type: none"> <li>The reasoning detracts from the work and may include irrelevant ideas</li> </ul>	<ul style="list-style-type: none"> <li>The checking of the work is ineffective (i.e., misses the errors)</li> </ul>
<b>0</b>	<ul style="list-style-type: none"> <li>No evidence is provided</li> </ul>	<ul style="list-style-type: none"> <li>Strategies are not recorded</li> </ul>	<ul style="list-style-type: none"> <li>The reasoning is not completed or provided</li> </ul>	<ul style="list-style-type: none"> <li>No evidence is provided</li> </ul>

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# MATHEMATICS PROBLEM SOLVING STUDENT DIRECTIONS

The two people who will read your paper will give you four scores, looking for these things:

1. How well you **understood the task**.
2. The **steps you used** (and ways you) solved the task.
3. How well you **told the thinking** behind each step of solving the task.
4. How you **reviewed your work to be sure it made sense**.

**You may use manipulatives or a calculator to work on your task.**  
Please, show all of your work (first efforts and those that worked) in the spaces provided, so everything you do is visible to the readers.

## Guide To Completing The Task

To get the highest score in each of the four areas, you will want to ✓ each of the 's.

### 1. CONCEPTUAL UNDERSTANDING

- I changed the important information into math ideas to work the task.
- I used all the necessary information given in the task.

### 2. PROCESSES & STRATEGIES

- I used pictures, charts or symbols to help solve the task (if it helps).
- I picked and used the best strategy I could think of to solve the task.

### 3. COMMUNICATION

- My work shows the readers what I was thinking on each part of the task.
- My thinking is easy to follow from one step to the next.

### 4. VERIFICATION

- I showed that I reviewed my solution (and checked my calculations) to verify my solution makes sense.

In addition, you will know if you are Precisely Correct, Essentially Correct (almost correct) or Not Correct.

# Mathematics Scoring Guide

\_\_\_\_ Precisely Correct

\_\_\_\_ Essentially Correct (no additional instruction appears necessary)      \_\_\_\_ Not Correct

	<b>Conceptual Understanding</b>	<b>Processes &amp; Strategies</b>	<b>Communication</b>	<b>Verification</b>
6	Showing an understanding of what the task is asking (the "what")	Choosing strategies that can work, and then carrying out the strategies chosen (the "how")	Explaining the reasoning (the "why") at each step, using pictures, symbols, and/or vocabulary	After solving the task, reviewing the work & showing the solution process is reasonable, in relation to the task (the "proof")
5	A) The task is changed into complete mathematical ideas B) All necessary information and/or data from the task is used	A) Pictures, models, diagrams, and/or symbols are clearly used to solve the task B) Math skills/strategies that fit the task are used to complete a solution	A) The thinking behind each step is clearly explained using words, pictures and/or examples B) The thinking behind the work is shown in a logical manner that is easy to follow	A) The work is checked (verified) using a different strategy to arrive at the same solution
4	C) The task is changed into useable mathematical ideas D) Necessary information and/or data from the task is used	C) Pictures, models, diagrams, and/or symbols are used to solve the task D) Reasonable math skills/strategies are used to find a solution	C) The thinking used to solve the task is clearly shown D) The presentation of the thinking is clear and organized	B) The work is checked (verified) a second time using the same strategy
3	E) An understanding of some of the ideas is shown F) Only part of the information given in the task is used	E) Pictures, models, diagrams, and/or symbols may only partly fit the task F) Math skills/strategies are partially completed (or recorded)	E) The thinking is partially shown with gaps that have to be filled in F) The work does not follow a clear pattern or sequence	C) The work is checked (verified) but an error is missed
2	G) Part of the task is changed into incorrect math ideas H) Assumes information that is not correct	G) Pictures, models, diagrams, and/or symbols may partially detract from solving the task H) Math skills/strategies do not work	G) The communication explains the solution, but not the path to the solution	D) The checking focuses on the calculations and/or E) The checking results in a different answer, but the differences are ignored
1	I) The task is changed into incorrect math ideas J) Incorrect information given in the problem is used	I) Pictures, models, diagrams, and/or symbols do not fit with their solution	H) The thinking explained detracts from the solution	F) The checking of the work is ineffective (ex. misses the errors)
0	K) No evidence is provided	J) Skills/strategies are not shown	I) The reasoning is not completed or provided	G) No evidence is provided

# Mathematics Scoring Guide

\_\_\_\_\_ Precisely Correct

\_\_\_\_\_ Essentially Correct (no additional instruction appears necessary) \_\_\_\_\_ Not Correct

	<b>Conceptual Understanding</b>	<b>Processes &amp; Strategies</b>	<b>Communication</b>	<b>Verification</b>
	<i>Showing an understanding of what the task is asking (the "what")</i>	<i>Choosing strategies that can work, and then carrying out the strategies chosen (the "how")</i>	<i>Explaining the reasoning (the "why") at each step, using pictures, symbols, and/or vocabulary</i>	<i>After solving the task, reviewing the work &amp; showing the solution process is reasonable, in relation to the task (the "proof")</i>
<b>6</b>				
<b>5</b>	<ul style="list-style-type: none"> <li>The task is changed into complete mathematical ideas</li> <li>All necessary information and/or data from the task is used</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols are clearly used to solve the task</li> <li>Math skills/strategies that fit the task are used to complete a solution</li> </ul>	<ul style="list-style-type: none"> <li>The thinking behind each step is clearly explained using words, pictures and/or examples</li> <li>The thinking behind the work is shown in a logical manner that is easy to follow</li> </ul>	<ul style="list-style-type: none"> <li>The work is checked (verified) using a different strategy to arrive at the same solution</li> </ul>
<b>4</b>	<ul style="list-style-type: none"> <li>The task is changed into useable mathematical ideas</li> <li>Necessary information and/or data from the task is used</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols are used to solve the task</li> <li>Reasonable math skills/strategies are used to find a solution</li> </ul>	<ul style="list-style-type: none"> <li>The thinking used to solve the task is clearly shown</li> <li>The presentation of the thinking is clear and organized</li> </ul>	<ul style="list-style-type: none"> <li>The work is checked (verified) a second time using the same strategy</li> </ul>
<b>3</b>	<ul style="list-style-type: none"> <li>An understanding of some of the ideas is shown</li> <li>Only part of the information given in the task is used</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols may only partly fit the task</li> <li>Math skills/strategies are partially completed (or recorded)</li> </ul>	<ul style="list-style-type: none"> <li>The thinking is partially shown with gaps that have to be filled in</li> <li>The work does not follow a clear pattern or sequence</li> </ul>	<ul style="list-style-type: none"> <li>The work is checked (verified) but an error is missed</li> </ul>
<b>2</b>	<ul style="list-style-type: none"> <li>Part of the task is changed into incorrect math ideas</li> <li>Assumes information that is not correct</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols may partially detract from solving the task</li> <li>Math skills/strategies do not work</li> </ul>	<ul style="list-style-type: none"> <li>The communication explains the solution, but not the path to the solution</li> </ul>	<ul style="list-style-type: none"> <li>The checking focuses on the calculations and/or</li> <li>The checking results in a different answer, but the differences are ignored</li> </ul>
<b>1</b>	<ul style="list-style-type: none"> <li>The task is changed into incorrect math ideas</li> <li>Incorrect information given in the problem is used</li> </ul>	<ul style="list-style-type: none"> <li>Pictures, models, diagrams, and/or symbols do not fit with their solution</li> </ul>	<ul style="list-style-type: none"> <li>The thinking explained detracts from the solution</li> </ul>	<ul style="list-style-type: none"> <li>The checking of the work is ineffective (ex. misses the errors)</li> </ul>
<b>0</b>	<ul style="list-style-type: none"> <li>No evidence is provided</li> </ul>	<ul style="list-style-type: none"> <li>Skills/strategies are not shown</li> </ul>	<ul style="list-style-type: none"> <li>The reasoning is not completed or provided</li> </ul>	<ul style="list-style-type: none"> <li>No evidence is provided</li> </ul>

# **Samples of Open-ended Tasks**

# Elementary

75

Your Name \_\_\_\_\_

P E N C U   P S   C   V

Your Teacher's Name \_\_\_\_\_


Grade Level \_\_\_\_\_

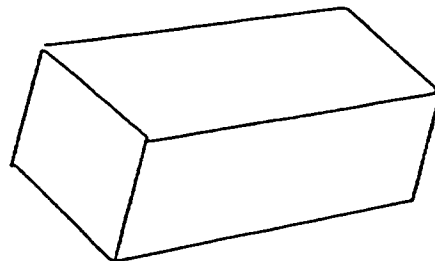
**E - G** At a factory, a baseball is put in a box that has a volume of  $512 \text{ in}^3$ . Each boxed baseball is then put in a larger box for shipment. The shipping box holds 24 boxed baseballs in each layer and contains a total of 96 boxed baseballs. What is the volume of the shipping box? Explain your thinking at each step and your answer(s).

**E-G** Scott has 24 meters of fence and wants to make the biggest rectangular area possible for his dog to play in. What length should he make each side of the dog yard? Explain your thinking at each step and your answer(s)

**E-M** "Playo Cubes" are  $2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm}$ .



They are sold in a thin cardboard box  $4 \text{ cm} \times 4 \text{ cm} \times 8 \text{ cm}$ .



How many cubes are in one full box?  
Explain your thinking at each step and your answer(s).

**E-M** Mr. Cross hired the twins to rake his yard, front and back, for \$6.00. It was a large yard, but the front and back were about the same size. When the time came to start, Bill was not home from school, so Barb started alone. Barb was finished with the whole front before Bill showed up. The two of them raked the back together. How should they split the money? Explain your thinking at each step and your answer(s).

**E-M** How many 3-digit numbers can you find using only the digits 1,3,5 & 7 once in each number? Explain your thinking at each step and your answer(s).

Your Name \_\_\_\_\_

P E N C U P S C V

Your Teacher's Name \_\_\_\_\_


Grade Level \_\_\_\_\_

**E-C&E** A chocolate bar is separated into several equal pieces. If one person eats  $\frac{1}{4}$  of the pieces; a second person eats  $\frac{1}{2}$  of the remaining pieces; there are six pieces left over. Into how many pieces was the original bar divided? Explain your thinking at each step and your answer(s).

**E-C&E** In my carpenter shop I make only three-legged stools and four -legged tables. One day I looked at my day's output and counted 31 legs. How many tables and stools could I have completed that day? Explain your thinking throughout and your answer(s).

**E-C&E** Which peanut machine offers the better buy:  
#1: We get 7 peanuts for 3 cents from one machine.  
#2: We get 5 peanuts for 2 cents from the other machine.  
Explain your thinking at each step and your answer(s).

**E-AR** Suppose a baby was born at midnight, and you told one person about this new baby. Then that person told two others within 20 minutes. And then those two others every 20 minutes. And this continued all night long, with every person telling two others every 20 minutes who hadn't yet been told. How many people would know by 4:00 in the morning? Explain your thinking at each step and your answer(s).

**E-AR** Mrs. Newhouse is beginning an exercise program. She plans to walk 2 miles for 2 days, 3 miles for three days, 4 miles for 4 days, and continue the pattern until she is walking 6 miles per day. In how many days will she first walk 6 miles? Explain your thinking at each step and your answer(s)

**E-SP** The fourth graders in Ms. Hartman's class bought some peanuts that come in small bags. Each student in the class reported how many peanuts were in her or his bag. Here are the numbers each student reported:

Number of Peanuts in a Bag

14      17      15      16      16      18      21      13      15      14      15  
15      16      17      17      19      17      16      19      15      19

If someone asked you, "About how many peanuts are in a bag? What would say?  
Explain your thinking at each step and your answer(s).

**E-SP** Miss Springer is arranging books on a library shelf. How many different arrangements can she make using 4 books? Explain your thinking at each step and your answer(s).



# Middle Level

Your Name \_\_\_\_\_

P E N C U P S C V

Your Teacher's Name \_\_\_\_\_


Grade Level \_\_\_\_\_

**M-G** Kyle works at Greenhorn Ranch during vacations. They are building a rectangular corral using an existing fence for one side and 48 feet of new fencing for the remaining sides. How wide and how long should they make the corral to get the largest possible area? Explain your thinking at each step and your answer(s).

**M-G** A rectangle with a perimeter of 24 inches was cut into two squares. What were the dimensions of the rectangle? Explain your thinking at each step and your answer(s).

**M-M** An eighth grade class of 23 students is making book covers out of special matboard with dimensions of 4 feet by 3 feet. The book covers have to be  $9\frac{1}{2}$  inches by  $12\frac{1}{2}$  inches. Determine the number of pieces of matboard that they will need to purchase in order to ensure that they have enough matboard for all the students to have a front and a back cover. Determine the amount and percent of waste on each board. Explain your thinking at each step and your answer(s).

**M-M** Julia is making place cards for the homecoming dance. How many  $1\frac{1}{2}$  x 3-inch cards can she cut from 9-inch square of paper? Explain your thinking at each step and your answer(s).

**M-C&E** Brian's bicycle shop had 48 water bottles for sale. On Monday, Brian marked the water bottles down to \$5 each and sold  $\frac{1}{2}$  of them. On Tuesday, he marked the remaining bottles down to \$4 each and sold  $\frac{1}{2}$  of the remaining bottles. On Wednesday, he marked the bottles down to \$3 and sold  $\frac{1}{3}$  of the remaining bottles. On Thursday, he marked the water bottles he had left down to \$2 and sold them all. If Brian purchased the bottles for \$3 each, how much did he gain or lose? Explain your thinking at each step and your answer(s).

**M-C&E** The planets Earth, Jupiter, Saturn and Uranus revolve around the sun approximately once every 1, 12, 30, and 84 years respectively. A scientific flyby of these planets is being planned for the future but it is important that the planets be relatively lined up. If these four planets were lined up in 1988, in what year will it happen again and be good to plan a flyby? Explain your thinking at each step and your answer(s).

**M-C&E** Sally was given 5 cents for each test she passed. But she had to return 10 cents for each test she failed. At the end of 3 months she passed four times as many tests as she failed and she had earned 20 cents. How many tests did she pass? Explain your thinking at each step and your answer(s).

Your Name \_\_\_\_\_

P E N C U P S C V

Your Teacher's Name \_\_\_\_\_


Grade Level \_\_\_\_\_

**M-AR** There are 8 players in a darts tournament. Each player plays one game against each of the other players. How many darts games will be played in the tournament? Explain your thinking at each step and your answer(s).

**M-AR** Lesley helped put numbers on new lockers at school. The numbers started at 51 and continued on consecutively. She applied the numbers one digit at a time. When finished, she had used 413 digits. How many new lockers were there? Explain your thinking at each step and your answer(s).

**M-SP** If we have 31 flavors of ice cream, how many different double scoop cones can you get? Explain your thinking at each step and your answer(s).

**M-SP** Two students were trying to be clever and invent a new dice game to fool a friend. They made two die that each had the following numbers on it: -3, -2, -1, 0, 1, 2. Before they could invent a game that insured that they would win they did a little problem solving to determine the odds using these new dice. If you were to roll the two dice 100 times, what sum would be rolled most often? Invent a game using either 2 or 3 of these strange dice that will fool their friend. Explain the game and why you think it will fool the friend. Explain your thinking at each step and your answer(s).

**M-C&E** Phone chains are used in many circumstances; school groups, friends or in emergency situations. They are used to quickly communicate a message to a large group of people who are geographically separated. Phone chains work basically the same way. The first person on the chain calls two people. Each of those people calls two more people and so on until everyone in the group has been called. In the chain described above each person calls two people. In some chains each person may call, three, four, or five people. You have been hired by the local company to design a phone chain that will contact the 100 people in the company in case of an emergency. Determine how many rounds after you start the chain everyone on the chain would be called if you a two-, three-, four-, or five-person phone chain. Choose the chain you think would have the greatest possibility of reaching the most amount of people in the shortest amount of time. Explain your thinking at each step and your answer(s).

# Secondary

Your Name \_\_\_\_\_

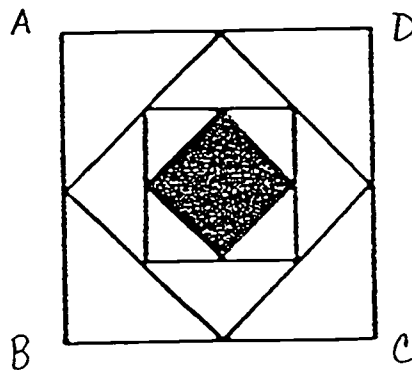
P E N C U P S C V

Your Teacher's Name \_\_\_\_\_


Grade Level \_\_\_\_\_

S-G Kyle works at Greenhorn Ranch during vacations. They are building a rectangular corral using an existing fence for one side and 48 feet of new fencing for the remaining sides. How wide and how long should they make the corral to get the largest possible area? Explain your thinking at each step and your answer(s).

S-G Each smaller square is made by joining the midpoints of the sides of the larger surrounding square. What percent of the area of square ABCD is the shaded square? Explain your thinking at each step and your answer(s).



S-M An eighth grade class of 23 students is making book covers out of special matboard with dimensions of 4 feet by 3 feet. The book covers have to be  $9\frac{1}{2}$  inches by  $12\frac{1}{2}$  inches. Determine the number of pieces of matboard that they will need to purchase in order to ensure that they have enough matboard for all the students to have a front and a back cover. Determine the amount and percent of waste on each board. Explain your thinking at each step and your answer(s).

A-m Start with a 4-inch square. Fold it in half along the diagonal. Now fold the triangle in half along the other diagonal of the original square. What is the perimeter of the final triangular-shaper paper? Explain your thinking at each step and your answer(s).

S-C&E Randy is driving from Prosser to Sunnyside. To get to Sunnyside on time he needs to drive at an average speed of 50 miles per hour. He was halfway to Sunnyside when he noticed that the roads had limited his speed to an average of 25 miles per hour. How fast must he drive to average 50 miles per hour for the whole trip? Explain your thinking at each step and your answer(s).

S-C&E A chocolate bar is separated into several equal pieces. If one person eats  $\frac{1}{4}$  of the pieces; and a second person eats  $\frac{1}{2}$  of the remaining pieces; there are six pieces left over. Into how many pieces was the original bar divided? Explain your thinking at each step and your answer(s).

Your Name \_\_\_\_\_

P E N C U P S C V

Your Teacher's Name \_\_\_\_\_


Grade Level \_\_\_\_\_

**S-C&E** The community of gnomes in the magic forest is upset because their forest is being bulldozed for shopping mall. The little people are moving far away, too far to walk. They are going in boats. Each boat can hold up to 100 grams and still stay afloat. The gnomes, as it happens, come in five different weights: 60 grams (senior citizens), 40 grams (adults), 20 grams (teenagers), 10 grams (children) and 5 gram (infants). If you had to transport 5 senior citizens, 4 adults, 2 teenagers, 3 children, and 4 infants what is the fewest number of boats that you need to transport them? Explain you thinking at each step and your answer(s).

**S-C&E** Laura gave some apples to three sisters. To the youngest sister she gave half of the number of apples plus half an apple. To the second sister she gave half of what she had left plus half an apple. To the oldest sister she gave half of what she had left. All the sisters received whole apples; none of the apples were cut, broken, or divided. How many apples might Laura have had to begin with? Explain your thinking at each step and your answer(s).

**S-C&E** (This problem is from a 16<sup>th</sup> century arithmetic book.) 'If I sold unto you a horse having four shoes, and every shoe 6 nails, with this condition, that you shall pay for the first nail one *ob*, for the second nail two *ob*, for the third nail four *ob*, and so forth, doubling until the end of all the nails: Now I ask you how much would the price of the horse come unto? Explain you thinking at each step and your answer(s).

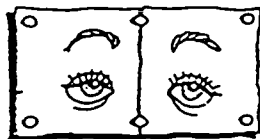
**S-SP** An auditorium has eight doors. In how many ways is it possible to enter by one door and leave by another? Explain your thinking at each step and your answer(s).

**S-P** Your mathematics teacher, Mr. Springer, says you have and 86.5% test average after 8 tests. You will be taking 4 more tests and you hope to raise your overall test average to 90%. What do you have to average on the next four tests in order to have a 90% or better average? Explain your thinking at each step and your answer(s).

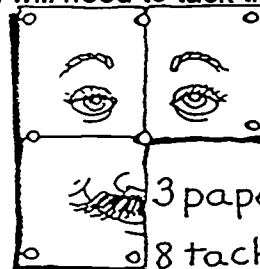
**S-AR** You have 100 congruent square pieces of paper to tack up on a bulletin board. Papers must be tacked at all 4 corners. What is the fewest number of tacks you will need to tack the 100 papers? Explain your thinking at each step and your answer(s).



1 paper:  
4 tacks



2 papers:  
6 tacks



3 papers:  
8 tacks

**S-AR** A clock strikes the hour on the hour. It also strikes twice for each half hour and once for each quarter hour. How many times does the clock strike in each 24-hour period? Explain your thinking at each step and your answer(s).



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*Office of Educational Research and Improvement (OERI)*  
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