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ABSTRACT

With the advent of the computer and user-friendly statistical software packages, factor analysis has become accessible to most researchers. However, conventional factor analysis, or R-technique, is only useful for research concerning types or groups of variables. Educational and psychological researchers are often interested in types of people, and R-technique is often incorrectly used in these research situations. However, appropriate factor analytic models exist to address research questions related to people, occasions, or other entities. This paper discusses the six basic factor analytic models and when they are appropriate, and it cites specific examples with the research questions they addressed. The six models discussed are: (1) R-technique, the usual factor analysis; (2) Q-technique, used for assessing types of people or groups of people; (3) P-technique, used to show changes in scores for the same person at different times; (4) O-technique, used to identify similarities in occasions for a particular individual; (5) S-technique, used to factor individuals across occasions; and (6) T-technique, used to factor occasions across individuals. The research situation and the research questions should determine the choice of factor analytic technique. (Contains 1 figure, 3 tables, and 24 references.) (Author/SLD)

Running Head: Two-Mode Factor Analysis

The Six Two-Mode Factor Analytic Models

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Abstract

With the advent of the computer and user-friendly statistical software packages, factor analysis has become accessible to most researchers. However, conventional factor analysis, or R-technique, is only useful for research concerning types or groups of variables. Often, educational and psychological researchers are interested in types of people, and R-technique is often incorrectly used in these research situations. However, appropriate factor analytic models exist to address research questions related to people, occasions, or other entities. The present paper discusses the six basic factor analytic models, when they are appropriate, and cites specific examples with the research questions they addressed.

Factor analysis was first developed in the early part of the twentieth century by Spearman (1904) and independently by others, but remained inaccessible to many researchers until the advent of both the computer and user-friendly statistical software packages (Thompson & Dennings, 1993). Factor analysis embodies a variety of analytic techniques aimed at examining or summarizing the relationships among variables or other factorable entities (Carr, 1992; Gorsuch, 1983). As Kerlinger (1986) stated,

> Factor analysis serves the cause of scientific parsimony. It reduces the multiplicity of tests and measures to greater simplicity. It tells us, in effect, what tests or measures belong together... It thus reduces the number of variables with which the scientist must cope. (p. 569)

R-technique

The "usual" factor analysis is known as R-technique. In this analysis, the data matrix has variables as columns and subjects as rows. Variables are factored and subjects are replicates of the relationships among the variables. This analysis is used to reduce the number of variables, to explore new theory, or to confirm theory regarding the underlying structure of the variables. An example of such a study can be found in Thompson and Borrello (1987). However, there are other research questions that cannot be answered using R-technique, such as questions concerning types of people. Thus, a broader conception of factor analysis is needed.

According to Cattell (1966), the Covariation Chart was

> introduced primarily as a means of classifying and extending possibilities of correlational researches... 1) a 'nursery example' of learning to think

multidimensionally in research planning, and 2) historically as illustrating the manner of birth of new factor analytic techniques (P, O, S, and T techniques) which demonstrated the utility of relational systems.

The three-axis covariation chart consists of three "dimensions" or modes: subjects, variables, and occasions. These three modes form a box, referred to as the Data Box, with each face of the box consisting of two modes, such as subjects and variables. Figure 1 illustrates the Data Box. This conception of relationships between modes suggests "forms of valuable research which might otherwise be overlooked." Cattell (1966) further developed the Data Box as what he called the Basic Data Relation Matrix (BDRM). He includes 10 modes: person, stimulus, response, background, observer, and five "occasion" terms related to each of the others. For the sake of simplicity, the following discussion is limited to the three modes of the covariation chart, since these are the modes most commonly used. However, the concepts and techniques generalize to include any modes.

It is possible to employ factor analyses that simultaneously consider three modes, though it is very uncommon to do so (Gorsuch, 1983; Thompson & Miller, 1978). Gorsuch (1983) reported that multimode factor analysis procedures are insufficiently developed to be accessible to most researchers. Thus, almost all researchers use two-mode techniques, and the following discussion will be limited to these.

By holding one mode constant, the Data Box collapses onto a face. There are two techniques for each face of the box. The mode held constant is the limit to generalizability, while the other two modes provide either the factorable entities or the replicated entities. Table 1 displays the six two-mode techniques, delineating the modes being factored, replicated, or held constant. Applications of most of these techniques have been published.

However, R-technique is the most commonly used (Gorsuch, 1983; Tinsley, 1992), followed by Q-technique and P-technique.

The two techniques from each face of the box are related. Consider R- and Q-techniques from the Variables and Subjects face of the box as an example. The data matrix factored in Q-technique has subjects as columns and variables as rows. This is the transpose of the data matrix used in R-technique. In most research applications of these techniques, the comparison could not be made because the research design considerations for each technique are different. However, for theoretical purposes, if the data matrix is double-standardized (i.e. across both rows and columns) and the same number of factors are extracted, the factor scores from one analysis will be equivalent to the factor pattern coefficients from the other analysis for the unrotated factors (Gorsuch, 1983). It is important to note, however, that if each method is analyzed in the usual manner, the results will not be the same.

In R-technique, more subjects than variables are needed, since subjects are the replicates. In Q-technique, variables become the replicates. Thus, we need more variables than subjects (Thompson, 1980). To generalize, more replicates (or rows of the data matrix) than entities to be factored (or columns) are always needed.

Statistical significance tests are sometimes performed on R-technique results. This is acceptable, if done correctly, because subjects can be sampled randomly. However, when interpreting results of Q-technique, statistical significance tests should not be used because they assume random sampling of variables which is difficult to attain. This caution should be heeded for other factor analytic models, as well.

To compare individuals in Q-technique, the variables need to have comparable score units. Thus, the variables need to be standardized to take out the mean difference between individuals. If not, a general factor, known as the "species factor" will result, indicating which individuals are most like the "typical individual in that sample," not an effective typology (Gorsuch, 1983). This standardization of variables is perhaps not necessary in R-technique. Hence, the difference between R- and Q-technique results is that R will include information on mean differences between individuals.

Q-technique

In 1917, Sir Cyril Burt proposed factoring people over a series of tests, though at the time he did not label the technique "Q-technique" (Cattell, 1978). In 1935, the British factorist, Sir Godfrey Thomson, published a paper outlining the possibilities of computing correlations between persons rather than tests. Thomson named this technique "Q" to distinguish the technique from the traditional R-technique. However, for various reasons, Thomson was pessimistic about Q-technique and did not pursue it further (Brown, 1980).

At virtually the same time (independent of Thomson), William Stephenson was writing about the possibilities of person correlations and intrapersonal relationships (Brown, 1980; Stephenson, 1935). Stephenson introduced Q-methodology as a means of investigating what he called human subjectivity. Stephenson elaborated on the theory and techniques of Q-methodology in his classic text, The study of behavior (Stephenson, 1953).

Q-technique is used to assess types of persons or to obtain an in-depth understanding of a smaller group of people, e.g., the people in a given therapy group. It is important to note the results of a Q-technique factor analysis differ from a traditional typology in which each person fits one, and only one, discrete category. Unless exceptionally simple structure is

Q-technique has been used in a variety of investigations including the study of artistic judgments (Meltzoff & Kornreich, 1970), counseling session semantic content (Levitov, 1981), clustering of psychotherapists (Fiedler, 1950), evaluators categorized by their perceptions of evaluation issues (Thompson, 1980), and counselors typed by their views of love (Gillaspy, Campbell & Thompson, 1996).

As an in-depth example, consider Campbell (1996) in which 24 applicants to a hypothetical Ph.D. psychology program were ranked with regard to desirability for acceptance into the program. Eight raters ranked each applicant. The raters consisted of three full professors, three associate professors, one assistant professor and one "composite" student. The researcher was interested in exploring the types of raters. In this example, the variables, or replicates, are the 24 applications being reviewed. Notice the small number of raters, or columns in the data matrix, as compared to the number of applications, or rows.

After the factor analysis was run, three factors accounted for 69.9% of the variability in rankings. Table 2 displays the factor pattern/structure matrix or the correlations of each rater with each rater/person factor. Table 3 reports the factor scores (in standardized form) which determine which applicants helped define each of the rater/person factors. Since lower rankings were more favorable, negative factor scores are more favorable. Consider factor scores with absolute value greater than one indicating that they are more than one standard deviation from the mean.

To interpret the factors, look at attributes of those applications with negative factor scores and attributes common but opposite to those with positive factor scores. In this example, Factor I was determined to be research oriented, Factor II to be clinically oriented, and Factor III explicator raters, or focused on an area of interest. Then, these factors can be achieved with the factor analysis, each person may be related to more than one typological factor in Q-technique (Gorsuch, 1983).

The methodology for Q-technique differs from R-technique. Again, more variables than people are needed. In fact, it was shown that the most people to use in relation to the number of variables is one less than half the number of variables, or $p = (v/2) - 1$. Variables are randomly sampled as replicates, but people are chosen to respond differently to the variables. According to Kerlinger (1986) "One tests theory on small sets of individuals carefully chosen for their 'known' or presumed possession of some significant characteristic or characteristics" (p.521).

Since Q-technique seeks to type people, the variables need to be standardized. This is usually achieved through using a Q-sort. A Q-sort is a sample of stimuli from a specifiable universe of content which is given to an individual for sorting. The sample typically consists of 60 to 100 cards. To eliminate the mean difference between individuals, each subject's responses must have the same mean, standard deviation, and shape. Often this is accomplished by asking each respondent to sort the stimuli into a given number of piles, placing a specified number of stimuli into each pile. Usually, there are from seven to nine piles with the stimuli distributed through the piles approximately normally. Carr (1992) provides an in-depth discussion of these procedures.

Since statistical methods do not know the difference between subjects and variables, the actual factor analysis used in Q-technique is the same as in R-technique. The interpretation of the results differs. In Q-technique, people have pattern coefficients on each factor, while variables have factor scores. These scores are used to interpret which variables identify and differentiate the factor clusters of people.

confirmed by checking the hypothesized definitions of the factors against what is known of the raters.

Variables and Occasions

P-Technique

Gorsuch (1983) deemed P-technique to be "... the forerunner of and multivariate approach to what is currently called single-subject design or N of 1 analysis" (p. 312). P-technique is a method used to show changes in scores for the same person at different points in time or for the average score of a group of people at different points in time (Gorsuch, 1983; Nunnally, 1978). The technique factors variables over occasions with subjects held constant. It is used to generalize to the population of occasions. This technique is particularly useful in the analysis of the psychotherapy process (Cattell & Birkett, 1980; Mintz & Luborsky, 1970).

Consider Mintz and Luborsky (1970) as an example. The researchers sampled 60 segments from nine hours of transcribed therapy sessions. Each session was rated on 18 variables related to outcomes of psychotherapy. Three raters sorted the segments into a quasi-normal distribution for each variable. The three rater scores were pooled. The 18 variables were intercorrelated across the 60 occasions, and this correlation matrix was factor analyzed.

Four factors emerged from the factor analysis. To interpret the factors, the researchers considered the variables with the highest pattern coefficients on each factor. They determined that two factors related to the patient and two factors related to the therapist. The researchers then used the mean factor score in a one-way ANOVA to determine if the factors differentiated between the sessions; the patient factors did. They then explored the nature of the patient factor trends over time using polynomial contrasts; one factor correlated with the cubic, and the other with both the linear and quadratic.

O-Technique

O-Technique seeks to identify the similarities in occasions for a particular individual (Gorsuch, 1983; Sells, 1963). It factors occasions over variables with subjects held constant, thereby seeking to generalize to the population of variables. Thus, it is important to ensure that the variables are representative of the population.

As an example, Jones, Thompson and Miller (1980) sought to simplify the instructional choices confronting science teachers when considering models of teaching. In the study, there were 16 models of teaching which were the conditions or occasions. There were 33 indicators of instructional parameters, or variables. In this case, subjects were not held constant, but pooled; 142 subjects rated five or six of the instructional models on each of the 33 variables, and the median rating was used to calculate the intercorrelations.

Through factor analysis, three factors were identified, becoming super-models of teaching. To interpret the factors, the researchers considered the pattern coefficients to determine which models made up each factor. Then, they calculated the factor scores to determine which instructional parameters (variables) provided the basis for the subjects' assigning the models to the categories.

Occasions and Subjects

For both S- and T-techniques, the variable mode is held constant. S-technique factors individuals across occasions, while T-technique factors occasions across individuals. There were no published applications of S-technique found in the ERIC database. According to Gorsuch (1983),

> S and T techniques are virtually never used because the generalization would be limited to one variable. One suspects that the lack of S and T applications is a function of the fact that factor analysis comes, historically, from the individual difference approach to psychology and therefor experimentalists interested in only one characteristic have not been aware of its possibilities. (pp. 312-313)

T-Technique

Frankiewicz and Thompson (1979) provide an interesting example of a T-technique application. The study sought to determine if there were different types of teacher brinkmanship behaviors. There were 12 teacher brinkmanship behaviors as conditions or occasions. There were 168 subjects who rated each of the 12 behaviors on each of 18 adjectival scales or semantic differentials (variables). This study has data for all three modes of the Data Box.

The actual T-technique analysis was not performed, but recommended after the researchers had performed a confirmatory R-technique factor analysis to verify that the 18 adjectival scale variables were representative and exhaustive of the population of variables or attitudinal domain. This analysis allowed the 18 data matrices consisting of the 12 behaviors as columns and 168 subjects as rows to be pooled, or made constant. The T-technique would then produce factors, or types, of brinkmanship behaviors. These would be interpreted by considering which behaviors had the largest pattern coefficients on each factor.

Conclusion

Factor analysis can be used to factor other modes than just variables. As Cattell (1966) stated, ". . . the question prompting the study will determine which mode should be factored." It is inappropriate (though common), for example, to use R-technique methods to investigate questions about types of people. Consider the research situation and research questions in order to determine which of the six two-mode techniques of factor analysis is appropriate to use.

References

Brown, S. R. (1980). Political subjectivity. New Haven: Yale University Press.

Campbell, T.C. (1996). Investigating structures underlying relationships when variables are not the focus: Q-technique and other techniques.

Carr, S. C. (1992). A primer on the use of Q technique factor analysis. Measurement and Evaluation in Counseling and Development, 25, 133-138.

Cattell, R. B. (1966). The data box: Its ordering of total resources in terms of possible relational systems. In R. B. Cattell (ed.), Handbook of multivariate experimental psychology (pp. 67-128). Chicago: Rand McNally.

Cattell, R. B., & Birkett, H. (1980). Can P-technique diagnosis be practicably shortened?: Some proposals and a test of a 50-day abridgment. Multivariate Experimental Clinical Research, 5, 1, 1-16.

Fiedler, F. E. (1950). A comparison of the therapeutic relationships in psychoanalytic, non-directive, and Adlerian therapy. Journal of Consulting Psychology, 14, 436-445.

Frankiewicz. R. G., & Thompson, B. (1979, April). A comparison of strategies for measuring teacher brinkmanship behavior. Paper presented at the annual meeting of the American Educational Research Association, San Francisco. (ERIC Document Reproduction Service No. ED 171 753)

Gillaspy, A., Campbell, T., & Thompson, B. (1996, January). Types of lovers: A Q-technique factor analysis of counselor person-factors defined by counselors' love attitudes. Paper presented at the annual meeting of the Southwest Educational Research Association, New Orleans. (ERIC Document Reproduction Service No. ED forthcoming)

Gorsuch, R. L. (1983). Factor Analysis (2nd ed.). Hillsdale, NJ: Erlbaum.

Jones, H. L., Thompson, B., & Miller, A. H. (1980). How teachers perceive similarities and differences among various teaching models. Journal of Research in Science Teaching, 17, 321-326.

Kerlinger, F. N. (1986). Foundations of behavioral research (3rd ed.). New York: Holt, Rhinehart and Winston.

Levitov, J. E. (1981). A computer-based semantic content analysis of exemplars of five counseling theories. Unpublished dissertation, University of New Orleans.

Meltzoff, J.. & Kornreich, M. (1970). Research in psychotherapy. Chicago: Aldine Publishing.

Mintz, J., & Luborsky, L. (1970). P-technique factor analysis in psychotherapy research: an illustration of a method. Psychotherapy, 7, 13-18.

Nunnally, J. (1978). Psychometric theory (2nd ed.). New York: McGraw-Hill.

Sells, S. B. (1963). Stimulus determinants of behavior. New York: Ronald Press.

Spearman, C. (1904). "General intelligence," objectively determined and measured. American Journal of Psychology, 15, 201-293.

Stephenson, W. (1935). Technique of factor analysis. Nature, 136, 297.

Stephenson, W. (1953). The study of behavior. Chicago: The University of Chicago Press.

Thompson, B. (1980). Validity of an evaluator typology. Educational Evaluation and Policy Analysis, 2, 59-65.

Thompson, B., & Borrello, G. (1987). Concurrent validity of a love relationships scale. Educational and Psychological Measurement, 47, 985-995.

Thompson, B., & Dennings, B. (1993, November). The unnumbered graphic scale as a data-collection method: An investigation comparing three measurement strategies in the context of Q-technique factor analysis. Paper presented at the annual meeting of the Mid-South Educational Research Association, New Orleans. (ERIC Document Reproduction Service No. ED 364 589)

Thompson, B., & Miller, A. H. (1978). Dissonance theory and education students' attitudes toward teachers. Journal of Experimental Education, 47, 55-59.

Tinsley, H. E. (1992). Psychometric theory and counseling psychology research. In S. D. Brown & R. W. Lent (eds.), Handbook of counseling psychology (2nd ed.), pp. 37-30). New York: Wiley & Sons.

Figure 1: The Data Box

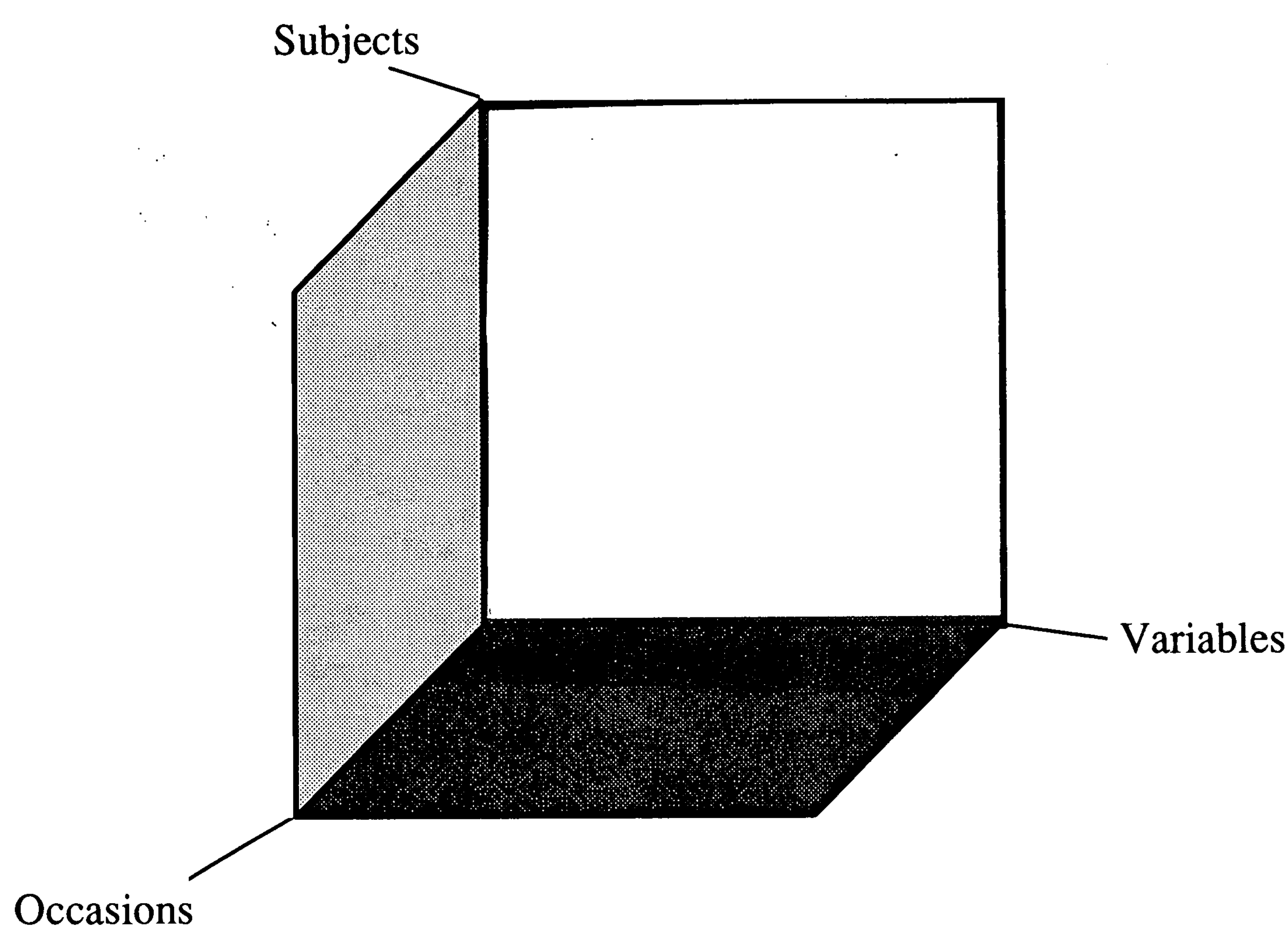

Table 1

Techniques of Two-Mode Factor Analysis

Technique	Factored Mode	Replicated Mode	Constant Mode
O	Occasions	Variables	Subjects
P	Variables	Occasions	Subjects
Q	Subjects	Variables	Occasions
R	Variables	Subjects	Occasions
S	Subjects	Occasions	Variables
T	Occasions	Subjects	Variables

Table 2
Varimax-Rotated Person Factor Pattern/Structure Matrix

Rater	Person Factor I	II	III
1. Full Professor	.83121	-.01939	-.19422
2. Students	.75600	.09377	.44441
3. Full Professor	.67930	.29552	.31966
4. Full Professor	.05542	.83285	-.20431
5. Assoc Professor	-.39483	.73584	.20886
6. Assoc Professor	.28017	.64788	.07693
7. Assoc Professor	.43857	.61117	-.10196
8. Asst Professor	.07005	-.08240	.92966

Table 3
Factor Scores for the 24 Applicants
on the Three Rater/Person Factors

Applicants	Person/Rater Factor I	II	III
1	1.11114	-0.95324	1.42197
2	0.23662	-1.16562	-1.03390
3	1.23519	0.39236	-0.26219
4	-0.23803	-0.36333	-0.85981
5	-0.35986	1.43596	0.50985
6	-0.81097	1.52123	0.12805
7	0.20536	0.39875	2.03652
8	-1.28327	-1.39752	-1.47744
9	-0.75360	0.41634	-0.58937
10	-1.76346	-1.27124	1.50531
11	1.01464	-0.35945	0.22739
12	-0.05614	0.07248	-0.46246
13	-1.01826	0.19900	-0.61641
14	-0.84715	-1.32308	-0.11141
15	1.51502	0.00549	-1.35747
16	0.88398	1.77178	0.30102
17	1.09488	-0.11404	-1.16012
18	-0.65899	0.19688	0.62496
19	-1.24004	-0.24802	1.05219
20	1.29834	-1.87780	-0.14109
21	0.95914	0.12900	0.83400
22	0.27163	-0.08809	1.22346
23	-1.38354	1.35364	-1.45619
24	0.58737	1.26850	-0.33684

Note. SPSS automatically computes factor scores, albeit in *Z* score form, if the "SAVE" command is used in the factor analysis.

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