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ABSTRACT

This exploratory study had two goals: (1) to identify signs of abstraction and consolidation of abstractions of functional concepts in students in a situation-based curriculum; and (2) to use theoretical arguments as well as experimental evidence in order to shape notions of abstraction and consolidation of abstractions. The methodology was consciously circular because the primary goal of the work was to clarify the notion of consolidation. An instrument was designed to make observations on abstraction and consolidation. Data were used to reshape ideas about the theoretical notion of consolidation and to conclude how consolidation would manifest itself in students' actions. Finally, data were analyzed again to find these "signs of consolidation." An interview was administered to students from classes using CompuMath, a curriculum which involves interactive computer software. Students were asked to use a functions software program with which they were familiar to draw the graph of a quadratic function. The theme of the interview was development of a number of animal populations in a park. Consolidation did occur for some students in the CompuMath curriculum. Furthermore, consolidation of abstract knowledge did not usually occur suddenly; consolidation processes may proceed on a continuum. Appendices include the interview administered and student transcripts. (PVD)

CONSOLIDATION OF MATHEMATICAL ABSTRACTIONS IN A SITUATION BASED FUNCTIONS CURRICULUM

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PRELIMINARY REMARK

This is a draft version prepared for presentation at the AERA conference, Chicago, March 24-28, 1997. Please do not quote this draft.

On the other hand, comments and suggestions are welcome. Please feel free to e-mail them to any or all of the authors:
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INTRODUCTION: Abstract versus Situated

Many recently developed mathematics curricula are based on problem situations, in the expectation that students will most easily associate meaning with mathematical concepts and processes which they meet in contexts taken from outside of mathematics. One might thus see abstract formulations of mathematical concepts and relationships as diametrically opposite to situated or contextualized presentations of analogous concepts and relationships. In a recent paper, Greeno's (1997) makes the point that the question is not simply one of abstract versus situated presentation but that the framing assumptions underlying the cognitive perspective and the situative perspective are fundamentally different. For example, where the cognitive perspective would ask whether knowledge does transfer, a corresponding question from the situative perspective would be something like: "Does success of participating in an activity in one kind of situation lead to success in others?" Greeno also specifically relates to abstraction. He sees the question "What are the relative advantages and disadvantages of abstract instruction as opposed to instruction for specific activity, especially for jobs?" as relevant from both, the situative and the cognitive perspectives. The difference between the perspectives is in the way abstraction is conceived: Is the abstract simply the opposite of concrete or is it part of the issue of representation, involving the portability of symbolic or iconic representations that can be interpreted apart from their referents. Greeno concludes that abstract representations can contribute to meaningful learning but only if their meanings are understood, and that it is consistent with the situative perspective that abstract representations can facilitate learning.

With respect to abstraction, both perspectives take meaning as their focus. The cognitive perspective places this meaning in concepts, structures and relationships. The ability to abstract from concrete features of situations is seen as crucial to the realization of the power of mathematics: The same concepts, structures and relationships appear and are useful in different but structurally similar tasks; these tasks thus acquire some common meaning which resides in the abstract elements of the underlying structure. As a result, the mathematical structures underlying the situations become a central object of curricular consideration.

The situative perspective takes into consideration links between objects, events, actions currently dealt with, other situations, the ways activities are presented to students, and the ways students interact when considering a situation. Knowledge is thus always situated. Often students learn to index situations in such ways as to be usable in other similar tasks although no abstraction occurred. On the other hand, meaningful abstract representations and abstract language may arise from this use in different tasks in a way similar to the development of the meaning of words in conversation.

Not surprisingly, implementations in the field allow for interpretations consistent with both perspectives. Bransford et al. (1996) (for example) "have shown that mathematical thinking and learning are highly context dependent, and that children learn mathematics far better if it occurs in situations that are meaningful to them" (Sternberg, 1996, p. 312). Goldin (1996), on the other hand, has pointed to the dangers of favoring in-context understandings at the expense of abstract understandings. After describing decontextualized mathematics as the practice of introducing mathematical notations or methods, without context, as rules of procedure to be learned for their own sake, he claims that "... it is equally dangerous to react to it by insisting that all mathematics be 'in context' - especially in those 'common sense' contexts that pose obstacles to abstraction. *It is essential that we see the autonomous stage of mathematical representation, in which 'the same' concept applies with great facility to new semantic domains, as our goal.*" (p. 81, italics in original).

In situation based curricula, abstract concepts are rarely objects of explicit attention, even though abstraction from situations and thinking with abstract concepts may be considered an important goal. Abstract knowledge, if it arises at all, may thus remain passive, inert or fragile, at least initially. Before recognizing and using the power of abstract reasoning, students need to make another step, in addition to abstraction, in order to overcome this passive, inert or fragile state. This step will be discussed below and called consolidation of the abstract knowledge. Consolidation occurs when concepts and processes become a mental ability or mental function of each individual because the individual feels some need to have and use them. These processes may first have been constructed in interactive ways (with the material tools and partners), while solving problems.

In this paper we will

1. elaborate the meaning of abstraction and consolidation in a situation based mathematics curriculum;
2. identify some student behaviors which might indicate that abstraction or consolidation has occurred, generally in mathematics and more specifically in the content domain of functions;
3. report on a first observation of such behaviors in students who have learned in a situation based functions curriculum.

CONSOLIDATION

We take the position that any knowledge is contextualized; however, the context may be a realistic situation or otherwise generally accessible to students of the age group under consideration, or the context may be purely mathematical. The following terminology will thus be used: We reserve the term "situated" to mean that the students had meaningful access to non-mathematical objects during the mathematical activity. The term "abstraction" from such a situation refers to a process leading to quantities, relations and structures that describe the situation but that are considered independently of it. In fact, abstraction refers to a process situated in the mathematical realm indicating that the use of "mathematical abstract entities" is meaningful.

In order to formulate questions about students' abstractions in situation based curricula which are sufficiently specific for empirical investigation, a tighter theoretical framework is needed, as well as a narrowing down of the content domain. In principle, we do not and cannot exclude the possibility that things will look differently in other content domains and age groups.

The experimental part of this research was carried out in the framework of a situation based functions curriculum called CompuMath, to be described in more detail below. Like in many other situation based curricula, a great part of CompuMath students' actions and thinking is based on the realistic situations under consideration at any given moment. The question arises to what extent their thinking is situation-bound or in other words, to what extent their thinking transcends (or at least potentially transcends) the situations in which it is embedded and takes place at an abstract level. To give an example: Do students think in terms of increasing (decreasing, maximal, equal, ...) animal populations or increasing (decreasing, maximal, equal, ...) volume, or have they abstracted the meaning of the notion increasing (decreasing, maximal, equal, ...) and are able to think in terms of increasing (decreasing, maximal, equal, ...) functions?

The aims of CompuMath include students' ability to think about abstract concepts, for example functions, and their properties and relationships, and to do so with the same confidence as they think about them in situations. One of the reasons for choosing this as an aim is the power and usefulness of mathematics in general, and of functions in particular to describe (model) many different but similarly structured situations by means of the same tools (functions). This power does not stem from easily recognizable similarities in the surface features of the different situations but from deep structural analogies between them.

CompuMath students are encouraged to make use of many signs and tools, as well as interpersonal interactions. The interpersonal interactions consist of collaboration in small groups with the possible scaffolding of the teacher (for example when students ask for help) and of summary sessions. The main tool is the computer with appropriate software. The signs are the language used by the teacher, and the different embodiments of mathematical concepts in different settings; these embodiments have been called "representatives" elsewhere; for example a particular graph of $y=x^2$ is a representative of the function which assigns to each real number its square; together with all other graphs of the same function, it belongs to and constitutes the graphical representation of this function (Schwarz & Dreyfus, 1995).

The use of the same tools in a succession of problem-situations having similar abstract structures (progressively more complex, though) in the CompuMath project, provides the learners with cues about the purposes of the developers. It is hoped that the similarity between the successive problem-situations indicates to the students that they are expected to behave the same in the different of activities. Moreover, the activities often end by asking the student to manipulate the algebraic objects used to model the problem-situation. This manipulation is formal; the student are often explicitly told that the objects under consideration (the functions) do not necessarily refer to a problem-situation. Therefore, students are habituated to consider formalizations in relation to problem-situations. The question is, of course, whether the recurrent use of these formalizations triggers any internalization of the above behaviors.

Because they act the same in an environment in which the teacher demonstrates certain behaviors when dealing with a particular situation, it is expected that students develop mental constructs which enable them to cope with new similar situations. They learn to use "abstract" terms such as functions, domain of increase, image or preimage. Also, they operate in three "abstract" settings embedded in the computer tool (graph, table of values, formula). In which sense the use of such terms and operations corresponds to a mental construction that can be called abstract, is an matter of point of view. This question is not new. It deals with the relationships between external and internal activities in general and the problem of internalization in particular.

Both Piaget and Vygotski have treated this problem in theory and in many examples. Piaget linked internalization (the passage from external actions on objects to internal mental actions) to reflective abstraction (Piaget,1977). Reflective abstraction characterizes the formal operational phase and involves an operation abstracted from one's action to be reflected onto a higher plane. The item abstracted at the higher level is to be integrated into a new structure via a process of reconstruction. Moreover, this reconstruction widens the scope of the lower level, making it more general (Kitchener, 1986).

Vygotski's approach to the relationship between internal and external activity is well rendered through his perspective about the development of word meaning (Vygotski, 1981, 1986; Wertsch & Stone, 1985). Vygotski's analysis of this issue begins with the axiom that categorization or generalized word meaning is inextricably tied to human social interaction. Therefore, generalization becomes possible with the development of social interaction. Any new level in the child's generalization signifies a new level in the possibility of social interaction. Vygotski's genetic analysis of word meaning begins outside the realm of the meaning itself. It begins by the agreement on reference. He cautioned that this agreement on reference can easily be mistaken for agreement on meaning. He argued that the correct use of words is the beginning and not the end of the development of concepts. This development of the meaning of words is achieved through a process of internalization. Internalization is the formation of an internal plane of functioning when the child comes to master social and hence external sign forms.

Vygotski's description of the development of word meaning is highly relevant to concept meaning in rich environments, and thus to the use of mathematical terminology and of multiple representatives. For example, students learn practices using computer keys for defining the boundaries of the x and y scales of the graph of a function, walking on the graph with the cursor, or the passage from one setting to another one. Another important practice is to model concrete features of a situation: finding properties of an algebraic entity that models a situation; recognizing the mathematical structure underlying the situation becomes the main object of consideration.

In addition, students are encultured to some social practices. One of them which is central to this project is a summary discussion at the end of an activity. During such a summary discussion, the teacher attempts to legitimatize knowledge, to provide normative language and to help the students work out those aspects of the underlying structure to which he or she (or the curriculum developer) decides to give predominance at that stage of the learning process, i. e. to "institutionalize the knowledge" (Yackel and Cobb, 1996; Margolinas, 1992).

The general question we raise in this paper is which position abstract concepts take in the students' knowledge structure and what meanings the students associate with them. Although the assumptions as to the emergence of the abstract concepts may differ according to whether one takes the Piagetian or the Vygotskian perspective, the question arises and is important in any case: It concerns the outcome rather than the process. More specifically, we ask whether students do construct meaningful abstractions, and if so, whether they can meaningfully act on these abstract structures? These two questions are different in essence.

Given that abstract concepts are rarely, if ever, the explicit focus of attention for the students' activity, and that they occur explicitly only in summary discussions, one might hypothesize that many students are at the end of the activity loosely acquainted with the abstract concepts which were involved but their knowledge is very fragile. This does not necessarily mean that this knowledge is unavailable to the students for solving problems but that when using it, they might easily be uprooted from their trains of thought, and feel uncertain about whether their actions are correct and useful for solving the problem at hand. They might recognize the terms but feel very uncertain when they are asked to verbalize what these concepts mean or to carry out operations in an abstract setting without recourse to a concrete situation. They may tend to fall back as quickly as possible into the certainty and the security they feel in a concrete setting. They associate meaning with the concepts only via a concrete situation. Other students might have a fairly solid grasp of some of the concepts in their abstract form; concepts like increase might be meaningful to them even if they are not directly linked to a concrete situation; such students will be able to understand questions which are asked in terms of the abstract objects and answer these question by acting with or on these or other abstract objects. They might in fact have noticed the power that is inherent in the abstraction and thus prefer to work in an abstract form, even if the problem they deal with is given in a situation. We will use the term *consolidation of the abstract knowledge* for the process of transition from the fragile to the stable state. Consolidation thus refers to a further development of the process of internalization, which leads to the ability to explicitly act upon the abstract structures to lead to a new internal construct. Conceivably, Vygotski's distinction between pseudo-concept and concept (Sierpinska, 1994) could account at least partially for the difference between non-consolidated and consolidated abstractions.

The question now arises how we can identify to what extent students have acquired abstract and knowledge and consolidated it. On the basis of the above theoretical considerations, we propose several observable phenomena which we will take as evidence of abstraction and/or consolidation of the abstract knowledge.

Situation boundness of a student's reasoning is a strong indication of lack of abstraction. If a student's reasoning is guided by the situation to such an extent as to disregard the data because of preconceptions about the situation, this student is a long way from abstracting that knowledge. This, however, will be a rare occurrence, and we usually must assess cases where students do use the given data and correctly use them; we then need to try and assess to what extent their thinking is independent of the situation in which the data were embedded, and to what extent the procedures the student uses are situation independent and thus generally applicable to questions and problems with an analogous structure.

Students' confidence in their actions or procedures could be taken as an indication of the solidity of concepts and processes and thus of consolidation. Obviously, confidence can have other roots but a particular student might be relatively confident in some actions and lack confidence in others. We will take this to indicate that the processes with respect to which the student is confident have been consolidated more than the others.

The fragility of a student's knowledge of certain processes or concepts, and thus the lack of their consolidation, could also manifest itself through the inconsistent use of these concepts or processes. The ability to access and use concepts, on the other hand, is a sign of consolidation. This ability may be deeply intertwined with the student's ability to model a situation by abstract representations.

Since CompuMath is a functions curriculum, students were asked to deal with situations which can be described by functions. Four functional settings have been used: verbal description, tables of values, graphs of functions and algebraic expressions. In most cases, each of these settings can be used to describe a situation, but they are not equivalent. Some students' cognitive style might lead them to prefer certain settings over others (Dolev, 1996). Some settings may be more efficient than others to solve a particular problem, especially with the limited knowledge available to the student. (Example: An intersection point between two graphs might not be accessible to algebraic treatment). But in any case, the curriculum and the available computer tools give the students opportunities (a) to operate and act in each of the settings, (b) to translate from any of the four settings to any other one, (c) to integrate between the settings via modelling tasks, (d) to give extended meaning to the integration of settings through the summaries. Recurring (a) (b) (c) and (d) enables the student to consolidate internalized knowledge. If a student uses these internalizations more and more in new situations, we will interpret this as evidence of consolidation. If a student can use abstract representations but only some, or even a single one of them, i. e. if the student's cognitive preference is so strong as to constrain this student and prevent transitions to other representations, we will take this as a sign of limits on consolidation.

The nature of the concept of function admits a further step on the ladder of abstraction, namely abstraction from the specificity of the settings towards an integrated view, according to which the different settings, and the representatives of a function in the different settings are multiply linked and connected among each other and with their common referent (Schwarz & Dreyfus, 1995). This additional step, just as the previous one, requires not only abstraction but also consolidation. A flexible use of several representations in dealing with functional problems is a sign that this additional step of abstraction and consolidation has occurred.

The consolidation of the internalized knowledge gives confidence, security, and with it the ability to make flexible use of the abstract notions in varied situations which might arise later. Flexibility means to know where a concept can and should be applied and to have that concept available for application when it is needed or useful (Alves Dias & Artigue, 1995; Dreyfus & Eisenberg, 1996).

METHOD

This study was carried out within the framework of a curriculum development project which takes a situations based approach to functions. One of the aims of the curriculum is that students develop an trans-representational abstract concept of function. This research comes to investigate to what extent the use of situations may support or prevent the formation of such a function concept.

The CompuMath project

This is a large scale development and research program (Hershkowitz & Schwarz, in press). Its main idea is to use existing interactive software, in order to create an environment for meaningful learning of mathematics. The long term goal is to produce a "complete" mathematics curriculum, based on learning and teaching in an interactive computerized environment.
The project has three components:
i) the development of sets of activities in various mathematical topics to cover the official syllabus in Israel;
ii) research on learning processes and on the roles of teacher and students within a dynamic environment;
iii) implementation at different levels.
The learning materials are being developed as a sequence of open-ended problem situations, in which the students are encouraged to make their own decisions about:
- which tools to use (computerized tools, paper and pencil, discussion in groups, etc.);
- what representation/s (i.e. numerical, algebraic, graphical) to choose, and if, when and how to link them.

The computer tools facilitate dynamic linkages between representations and actions within each of them (e.g., drawing, scaling, plotting, numerical and algebraic computing). Most of the work is done collaboratively and is evaluated by the students' reports and other products. These learning characteristics lead the students to take more and more responsibility for their own learning processes, with the teacher acting as problem poser, facilitator and synthesizer.

One component of the CompuMath project consist of a ninth grade functions curriculum. The problem situations are engineered in five phases: (0) An individual assignment (mostly given as a homework) that prepares students for the activity; (1) The problem solving activity itself is done in groups of two to four students having a multirepresentational tool at their disposal; (2) The writing of a group report on the ideas raised and steps carried out in phase (1); (3) A teacher initiated whole class discussion/synthesis in which group work is reported, evaluated, agreed upon or refuted. In addition, the teacher and students sum up together the ideas, as well as the learning processes; (4) An individual assignment designed ad hoc by the teacher, based on ideas raised by the groups and according to what the teacher feels as important or needs strengthening. Phases 0 and 1 engage students in "doing", namely in the strategies and processes of solving within the problem situation in small groups. Phases 2, 3, and 4 invite students to distance themselves from previous actions and to reflect on them. This design of multiphased activities has a double aim, one related to the learning itself, and the other to the research of the learning as it occurs. In other words, it provides a better foundation for learning the function concept and it also sets the scene for investigating concept learning.

Interview

For the purpose of the present study, an interview was designed. The interview was administered to students working alone, with a computer available at all times during the interview. In addition to the student and the interviewer, a second researcher was present in the room to videotape the interview and take notes; the second researcher did not intervene. The computer was open at the beginning of the interview and a functions software was loaded with which the students were familiar; in fact, each class had previously worked with this software during at least twenty sessions in a computer lab. At the beginning of the interview, the students' attention was drawn to the computer by asking them to use it to draw the graph of a (quadratic) function. None of the students had any problems with this. One student was from a class where the CompuMath curriculum was taught using graphic calculators, and in her case the computer was replaced by her calculator.

The theme of the interview is the development of a number of animal populations in a park. The measure of reality of the situation was chosen so as to reflect the measure of reality of situations which were common in the CompuMath curriculum: A scenario was chosen, all of whose components had concrete meaning for the students; they were familiar with all beings and objects mentioned and could easily make sense of all the non-mathematical notions that were used. On the other hand, no biological reasons were given for the particular development of the animal populations nor did such reasons constitute a topic of discussion. In fact, it would probably be hard to find a convincing biological background for the particular functions that were used, and in this sense, the situation is not realistic.

The interview is structured into four parts. Each of the first three parts introduces the development of an animal population over the first ten years of the park's existence. In the first part, the function is linear, decreasing and the information is given graphically, in the second part the function is linear, increasing and the information is given verbally, and in the third part the function is quadratic, with a maximum and the information is given by a formula. The first two parts gave numerical information explicitly, the third implicitly. After some introductory questions about the size of the various populations at various times, and about different ways to describe the populations and their development, students were asked to compare and/or equate[1] the numbers of animals in the populations during the ten year period as well as their rates of growth. Part four (which due to time limitations was reached with only a part of the students) asked for making changes in one of the populations which would lead to the existence of a point in time when all populations are equal. The exact questions which were presented in the interview are listed in Appendix A.

Population

[1] The interview was conducted in Hebrew. In Hebrew, there is no distinction between the words for "compare" and "equate"; this has led students to interpret some questions as relating to "equate" when our intention was to ask about the more general "compare".

Although over 30 students were interviewed, most came from the same class and one might thus see teacher dependent effects. In this paper, we therefore concentrate on four students, named A, L, N and S. The letter I will be used to denote the interviewer. The interview transcripts of the four students A, L and S are listed in Appendices B, C and D respectively. The transcript of student N has not yet been translated into English.

It should be noted that the four students whose interviews are used to illustrate the claims of the paper came from four different classes with four different teachers in three different schools of very different nature (L from a religious, girls only, grades 9-12 school in a large city, N from a secular, grades 7-9 school in a small town, and A and S from a grade 8-12 integrated secular kibbutz school.) The chance that similarities between the students' behavior has been induced by the environment or the teachers has thus been minimized.

Final Remark on Methodology

Our methodology is consciously circular: We use the data to get ideas how the theoretical notion of consolidation could possibly manifest itself in students' actions; and then we look at the same data again to find these "signs of consolidation".

RESULTS AND DISCUSSION

Situation-boundness

Situation-boundness is a prime sign of lack of abstraction. Out of the four students whose interviews were analyzed for this study, one was strongly situation-bound whereas the three others were not. One indication of this were the opposite reactions of A and S to the zebra graph in part 1 of the interview: S's reaction is dictated by the given data; he does comment that 'a population increases usually, isn't it?' (S008, see Appendix D) but accepts that this particular population decreases, and even proposes possible reasons. He immediately and correctly interprets the zebra graph although it contradicts his intuition that a population should increase. A's reaction, on the other hand, is dictated by what he expects from the context. His preconception that the zebra population increases was stronger than the consolidation of his graph reading, and made him read the graph from right to left, pointing to the right end of the graph when asked how many zebras there were when the park first opened , and answering 200. He would presumably read any graph which is not context bound from left to right. This can be seen towards the end of his interview; in A232, after having worked with the lions graph and produced his variant of the zebra graph (y=200+20x) on the screen, he himself interprets the point (0, 400) on the original zebra graph as 'initial point', and to Tommy's question, what 'initial point' means, he answers (A234, see Appendix B): 'How many there were at the beginning'.

A shows a similar lack of abstraction thinking about when the populations are equal; this is answered purely from within the situation, with some numbers, but no reference to an intersection point of graphs. There is a weak reference to formulas ('you can substitute here and then get it there') but when asked to do this, he doesn't get anywhere. Nor, after getting what he calls a 'common point' (A138) is he able to interpret this; he cannot answer the question '... this common point, what does it give us?'. Moreover, when asked to check somehow that both populations are indeed equal to 320, he does not propose to use the table but rather points to the computer which, at best could produce more precise graphs.

S, on the other hand, picks up some abstract properties of functions such as the symmetry axis and domain of increase of the corresponding parabola while answering other questions, even though he was never asked to find them (S146). He might have gained this knowledge about the parabola while rescaling the windows in order to see the parabola.

Generality of method

S has two ways of generating the formula for the zebras. First he arrives at the (correct) expression 400-20x intuitively (S020: 'I found -20x, and it makes sense: at the beginning there were 400 and one subtracts this from 400.') Shortly thereafter, he reconstructs the formula from two points criticizing his first way as follows: 'But I did this by error (sic!), just like this, not according to some kind of method.' He seems to connect well between the intuitive and the formal way of doing things. However, he also realizes that there are formal, generally applicable ways of doing things and he has such a formal way at his command; the formal way is context-independent, and he is able to easily and confidently apply it correctly showing that in this case the abstract method has been consolidated.

Similarly, when presented with the formula for the eagle population in part 3 of the interview, S immediately focuses on the algebraic, functional aspects of the abstract formula (S078) - this clearly is the primary object for him; he handles it well, not only realizing it is a quadratic but quickly drawing the qualitatively correct shape of a quadratic with a maximum (S080). He knows that these formulae may be interpreted, and he is able to do it (for instance comparing growth rates in S144/S146), but he doesn't really need the interpretation - it is secondary. His thinking is independent of the context. Later he also shows that although he was never asked to find the symmetry axis and domain of increase of the corresponding parabola he became conscious of them while answering other questions (S146). He might have gained this knowledge about the parabola while rescaling the windows in order to see the parabola; in fact, during the rescaling, his choice of windows seems to have been experimental rather than guided by analysis of the formula.

Fragility of knowledge and confidence

Knowledge which hasn't been consolidated remains fragile. Evidence of such fragility of knowledge are abundant in A's transcript; one example is the one mentioned earlier of his uncertainly about reading graphs from right to left or form left to right. Similarly, when constructing the lion graph in part 2 of the interview (A074), he at first chooses to represent the number of lions on the horizontal axis, and time on the vertical one, although he had already interpreted the zebra graph which was done in the more conventional way. By the way, L also drew a graph with time on the vertical axis but did this on purpose, after having used the conventional graph with time on the horizontal axis, and upon the interviewer's request to provide other descriptions of the same development of population. This idea of inverting the axes and thus presenting the inverse function is, in fact, quite original, and none of the researchers would have expected it, or even thought of it as a possible response to the request for other descriptions. Let us add that L might not have been able to do the same operation out of context.

Inconsistencies

When a notion is not consolidated, inconsistencies are unlikely to disturb a student. A is a case in point: Early in the interview he expects a straight line (A092), but proposes to plot ten points (A082). In response to the question 'How many points do I really need to get a straight line?', he answers 'two' (I133/A134); just a few minutes later, he again wants to enter 'all of them' into the computer (A176).

Similarly, when comparing the zebra and lion populations at time t=10 in part 2 of the interview, A looks at the graphs, and gives the lion population as 600 (there is a point (10,600) on the graph) and the zebra population as 400 (there is a point (0,400) on the graph and an entry 10/400 in his table). One could have expected that at this point he realizes the inconsistency of having the lion population after ten years above x=10 but the zebra population after 10 years on the vertical axis - but this did not seem to disturb him. Certainly, the idea that a vertical line in a graph contains all points for which $x=x_0$ does not seem consolidated. This is further confirmed by the fact that A interprets "when it gets to ten" (his own expression) as "all the time" (A106).

Toward the end of the interview, the zebra graph (y=200+20x) is obtained from the computer and looks quite different from the original one. A shows no reaction, and certainly does not seem surprised. The interviewer tries to get A to realize there is some inconsistency (I225 to I249) - but to no avail: A sees a difference but not an inconsistency.

Concepts

One sign of consolidation (or at least of abstraction) is that a student can answer questions asked in a situation independent manner. For example, in I201 to A208, the interviewer attempts to make A interpret what an intersection point means (with example on the screen), but the best he can get out of him are the two numbers 5 and 300. On the other hand, to N it appears patently obvious that intersection point means equality of functional values, and she seems almost offended to be asked such a trivial question (N142).

N appears to think in terms of functions or graphs, intersection points, slopes etc. but not in terms of the specific situation that was presented to her. This does not mean that the situation is inaccessible to her. She can and does interpret intersection points in terms of equal populations and slopes in terms of rates of change. Similarly, S has a well consolidated notion of slope; at the beginning, this seems to be a purely computational algebraic notion (S062/S064) but in the background, he is conscious of its rate of change interpretation; in fact, when asked where his claims about rate of growth come from, he says (S086): 'On the basis of the slope' and is able to point to it in the graphs. An interesting event is the 'slope 100' which he gives in S088; this is correct but probably a lucky stroke.

Settings

The four students' ways and reasons for choosing settings are very different. A's choices seem very fragile and somewhat accidental. When asked how else he could describe the zebra population (which was given graphically), he mentions story, formula and table, and shows a preference for finding the table (A026), thus choosing a less powerful setting than the graph which he already has, at a time when a more powerful one, for example formula could easily have been obtained (though maybe not by him). The table might give him a feeling of security which he doesn't seem to have with the graph or formula, or even the story. On the other hand, when asked to compare the zebra and the lion populations, at a time when he already has graphs and tables ready (at least partially), he chooses the most difficult and least efficient setting, namely formula (A046).

S has a clear cognitive preference for doing things generally, for working with a formula rather than the tabular setting or direct numerical calculation. He could presumably have reasoned that the zebra population starts at 400 and decreases by 20 each year thus quickly arriving at 340 for year 3; instead - he derived the formula with which he seems to feel more at ease. Also, when he was asked to compare the zebra and the lion populations (S042), and later the eagle and the zebra populations (S136), he immediately turns to writing equations (even when he cannot solve these) which shows a high degree of abstraction, at least with respect to this topic of points of intersection. There is no doubt that he could easily answer analogous questions, with or without context in almost any situation. This level of abstraction and generality is another sign of consolidation. However, S is clearly conscious of the other settings (see, for example S030, S096/S098, S142), and he chooses settings appropriate to the task at hand with a great degree of confidence.

There were as many opportunities for L to choose a setting as for the other students but she systematically remained in the graphical setting. She did quite well in most aspects of the graphical setting including passage to the graphical setting from verbal (L086, see Appendix C) or formula setting (L226), but avoided, even resisted using other settings. When building the lions graph, for example, she plots points. Her realization that the graph is a straight line arises slowly out of the observation of the points she plots (following L102). She does not, like S or N, realize that a constant growth rate implies a straight line; but once she has the straight line, she realizes that it implies a constant growth rate (L116). She can handle graphical information better than any other and seems to understand many concepts via the graphical and only the graphical setting. This is not only a cognitive preference but also a limitation. If there is such a thing as consolidation within a setting only (with the very limited degree of abstraction which this implies) - L shows this phenomenon. Further on, another example will be shown of a concept that has been consolidated in one setting only (N, tabular setting).

N is extremely quick to grasp the constant growth rate of the lions and to express it in several settings. She immediately corrects the interviewer in N116, and quickly writes down y=60x without even being asked to do so. She flexibly chooses appropriate settings to answer questions she is being asked; for example, the term 'compare' immediately makes her think of graphs (N128). When asked to propose other descriptions of the zebra population, for example, she (like L) first proposes other graphs (rescaling), but unlike L she is flexible enough to also propose a table (N044): She can interpolate the (0, 400) and (10, 200) values to give (5, 300) explaining that it decreases 'in jumps of 100' (N068). Her quick and correct proposal for the modified lion graph y=80x is impressive, and has clearly surprised the interviewer. In fact, it relies not only on her fully consolidated and fully integrated knowledge of linear functions (between algebraic and graphical setting) but also on a clever problem solving strategy: Pick the point where zebras and eagles meet, and change the lions function so that it passes through the same point. She is able to explain all the details of how she found the solution.

Abstraction without consolidation

Abstraction does not imply consolidation. For example, L's formulation that a straight line graph has a constant growth rate is made independent of context; this appears to be a well consolidated piece of knowledge. However, it is partial in the sense that she cannot reverse it; although she realized that there was a constant growth rate, she did not conclude that it must be a straight line. Nor does she seem to realize that two points suffice to draw a straight line (L120).

The possibility that abstraction has occurred without consolidation, is also seen in N: Her thinking is very independent of the context; this is true (i) when describing the zebra population, (ii) when describing the lion population, (iii) when comparing lion and zebra population, (iv) when comparing rates of growth, (v) when changing the function for the lion population in order to generate a triple intersection point, and (vi) when changing the function for the zebra population in order to generate a triple intersection point. A clear sign of this comes when she is on the process of comparing the zebra and the lion populations graphically, but does not even remember what animals are being talked about (N132): she thinks in terms of functions or graphs (or maybe abstract populations) but not the specific context that was presented to her. This is true when her confidence indicates that abstraction and consolidation have occurred as in (ii), (iii), (v), but also when she feels uncertain as in (i), (iv), (vi), and consolidation may still be lacking.

Consolidation of wrong conceptions

Wrong notions can be consolidated! For example, L fails to distinguish between negative and decreasing (L210 versus L216; and again L302 and L310). She consistently uses the word decreasing when she discovers negative values of f(x) and also draws a positive and decreasing graph when asked in L310. But this mistake seems to be context free; context does neither help nor hinder this (wrong) relationship she has established. So this is an interesting case of consolidation of a wrong relationship. In fact, in L310 she produces a graph different from the one which the calculator has produced: The calculator graph is negative for $x>20$ and she drew one which is positive and decreasing for $20<x<30$. This does not seem to disturb her because it fits her image that the graph must be decreasing. The interviewer then poses her an analogous problem in a different context: Which rate of growth is bigger at $x=2$, that of the lions or of the eagles (I323); in spite of the fact that she has the graphs in front of her, she does not look at slopes but proposes to plug $x=2$ in order to find the values (L326). This shows how context independent (consolidated) her misconception is.

A similar confusion of slope and value occurs when N is asked to compare rates of growth. This is difficult, even for the most talented among our ninth graders. N has trouble distinguishing and swings forth and back between talking about the size of the populations and their growth rate: for example N196, or N210 - N214 where she tries to run away from the confusion she experiences by zooming out and looking what happens later on the eagles, although this seems quite irrelevant at the moment; (well, maybe it is not irrelevant in the sense that she is confused by local notions and might feel that a global view could help her 'get the big picture'). She mixes the two notions until I's insistence on the term 'rate of change' (3 times in I233) provokes her to propose generating a table in which she is able to identify the growth rate (difference quotients). It should be noted that during this entire difficult phase N did not explicitly refer to the context. Like L, she finds it difficult to distinguish growth rate and size; like L, this difficulty does not seem to be made better or worse by context; unlike L, N finally generates a way out of the confusion: In N240, she uses herself the term 'rate of change' and correctly explains how she plans to find it. She can recognize it in the table, but not in the graph (N264: 'it is impossible to see this'); this is context independent but not across settings. In N256 and N258 she clearly got the point and can herself give a verbal description of the development of the rate of change. The notion of rate of change appears to be partially consolidated, namely within the tabular setting only.

Disturbing effects of context

It seems that some of N's difficulties to distinguish between slope and value of a function might have been exacerbated by the fact that these notions occurred in a situation which added the term like population, rate of change, rate of growth etc. Although this cannot be established beyond doubt, there are other moments like N276 where she explicitly expresses discomfort with the context, and some where the context quite clearly seem to disturb her; one of these occurs in N150 - N154 where she has trouble stating what x stands for, and the second in N323-N336 where she has a lot of trouble interpreting the requirement 'do not change the living conditions' as 'do not change the slope'.

The overall impression is that N might have had less trouble in a curriculum which started from abstract notions and then presented situations as occasions to demonstrate the power of these abstract notions. This impression is confirmed by her superb analysis of the eagles graph which is completely context independent (N160); she provides a description of the variation with time much like S did (S080-1092) but she does it in terms of the algebraic expression, whereas S used the shape of the graph.

Process of consolidation

The process of consolidation is presumably a lengthy one, and therefore we are likely to observe in most cases its results: Notions and processes which have been consolidated; behaviors which point to consolidation which has already occurred. One exception in our data is S's consolidation of the notion that an intersection point between two graphs points to equal values of the two functions, or rather that different points cannot signify that at a given time values are equal. Although many, probably dozens of opportunities to realize this have occurred in class, for S, this realization seems to have come during the interview. At the two previous occasions when he was asked to compare populations, he immediately turned to writing down equations, even if he could not necessarily solve these. But when asked whether there was a time when all three populations were equal, a minor mistake in visually identifying the intersection point at (5,320) instead of (4,320) led to an interesting situation where he was convinced that there is a time when all three populations are equal (his 'yes' in S164 is immediate and very strong) while seeing on the screen that there is no intersection point of all three graphs. In S156 he 'knows' that all three points are at x=5. He sees, however, that there is more than one point. He concludes that these points must be above one another! Things do make sense to S; they are consistent for him, and he talks about them with confidence although they are not internally consistent; for example, populations are equal at time 5 years (S166) but also at many points (S168); here S seems to forget about the fixed time and concentrate on equal y-values (S174, S176: 'It's not the same year, only the population is equal'); when asked whether there is a year when all populations are equal, he comes back twice to his claim that at x=5 the populations are equal (S180, S186) before realizing, without further help, 'It's a mistake' (S190). The sorting out of this mistake, can be considered as a process of consolidation of the notion of 'two functions have the same value'.

CONCLUSION

This paper reports on an exploratory study with two aims:

* Identifying signs of abstraction and consolidation of abstractions of functional concepts in students having learned in the framework of a situation based curriculum;
* Using theoretical argument as well as experimental evidence in order to shape the notions of abstraction and consolidation of abstractions.

Methodologically, our procedure was circular. On the basis of field experience and a preliminary theoretical analysis, we designed an instrument to make observations on abstraction and consolidation. We then used the data to reshape our ideas about the theoretical notion of consolidation and to conclude how consolidation would manifest itself in students' actions; finally, we looked at the same data again to find these "signs of consolidation". Since at this stage, the primary aim of our work is not to find out whether the students have consolidated knowledge but rather to clarify the notion of consolidation, this circularity is to some extent justifiable. To establish these "signs of consolidation" more firmly, we would need to analyze independent interviews for the same signs: first other students with same interview (we have the data), then other interviews (we don't have the data yet).

There are other limitations to our study, including

(i) The small number of students, and their streaming: L, N and S all learned in the A-stream (which comprises about 60% of the population). A learned in the B-stream.

(ii) The curriculum: CompuMath is not an extreme curriculum from the point of view of situating activities; for example, many activities were followed by an extension in which the limitations of the situation were lifted and the same function was investigated in its maximal domain. This is done with the idea of triggering abstraction and supporting consolidation.

(iii) The lack of modelling: Situations may be distinguished by their "degree of reality". Real situations may contain irrelevant information, and lack relevant information, and thus require critical thinking before a formulation in mathematical terms (a model) even becomes possible. Our choices and formulations for the interview were determined in large measure by the fact that our primary goal was to look into consolidation rather than abstraction. We therefore designed the situation so that the main hurdle would not be abstraction. This led to the choice of a situation with a rather low "degree of reality", and low requirements in terms of modelling.

(iv) The lack of non-situated questions: If we were to design the interview now, we would do it differently because our theoretical views on what consolidation is, and how it can be observed, have been shaped on the basis of the analysis of the data from this study. For example, we would also want to find out whether students are able to answer questions which are given in a non-situated formulation, and to what extent they would themselves generate situations (or make connections to situations they have worked with earlier) when presented with such questions. One indication for lack of consolidation would be if a student was asked questions out of context and would build a context in order to think about these questions.

Under these limitations, the experimental evidence shows that consolidation does occur, at least for some students in the CompuMath curriculum - we have an existence statement. On the other hand, the situations based curriculum seems not to have helped student A along very far on the road to solid applicable mathematical knowledge.

It also appears that consolidation of abstract knowledge does not usually occur suddenly but is a process which might take a long time; consolidation processes for different mathematical concepts and processes may or may not be linked. Thus there is a continuum of consolidation stages. One example of a consolidation process has been found but this may have been a stroke of luck, since we do not have efficient means to observe consolidation processes.

We were also able to observe that there are cases of partial consolidation (within one setting) and that there exists a danger of consolidating wrong conceptions.

On the theoretical side, two tentative conclusions are that abstraction is a prerequisite for consolidation but not the opposite; and that the situative-interactive and the cognitive-structural view can coexist in a pluralistic theoretical framework that takes into consideration both cognitive and social processes.

In future work, it would be interesting not only to lift the methodological problems with which this study was beset but to carry out parallel studies in other mathematical content domains as well as to investigate whether there exist parallels with content domains outside of mathematics such as history, language, or physics.

REFERENCES

Alves Dias, M. & Artigue, M. (1995), Articulation problems between different systems of symbolic representations in linear algebra. In L. Meira (ed.), Proceedings of the 19 th International Conference on the Psychology of Mathematics Education, Vol. 2 (pp. 34-41). Recife, Brazil.

Bransford, J. D., Zech, L., Schwartz, D., Barron, B. and Vye, N. (1996), Fostering Mathematical Thinking in Middle School Students: Lessons from Research. In R. J. Sternberg & T. Ben-Zeev (eds.), The Nature of

Mathematical Thinking (pp. 203-250). Mahwah, NJ, USA: Lawrence Erlbaum.

Dolev-Cohen, O. (1996). The passage between representations as an action of characterizing students' learning in an interactive computer based environment. Unpublished M. A. thesis. Jerusalem, Israel: The Hebrew University of Jerusalem.

Dreyfus, T. & Eisenberg, T. (1996), On Different Facets of Mathematical Thinking. In R. J. Sternberg & T. Ben-Zeev (eds.), The Nature of Mathematical Thinking (pp. 253-284). Mahwah, NJ, USA: Lawrence Erlbaum.

Goldin, G. A. (1996), Contextualization and Abstraction in Mathematics Teaching and Learning. In C. Keitel et al. (eds.), Mathematics Education and Common Sense: The Challenge of Social Change and Technological Development, Proceedings of the 47th CIEAEM meeting (pp. 78-82). Berlin, Germany: Freie Universität.

Greeno, J. G. (1997), On Claims That Answer the Wrong Questions. Educational Researcher 26 (1), 5-17

Hershkowitz, R. & Schwarz, B. (in press), Unifying Cognitive and Sociocultural Aspects in Research on Learning the Function Concept. In E. Pehkkonen (ed.), Proceedings of the 21st International Conference on the Psychology of Mathematics Education. Lahti, Finland, July 1997.

Kitchener, R. F. (1986), Piaget's theory of knowledge. Hartford, CO, USA: Yale University Press.

Margolinas, C. (1992), Eléments pour l'analyse du rôle du maître: les phases de conclusion. Recherche en didactique des mathématiques 12 (1), 113-158.

Piaget, J. ...

Schwarz, B. & Dreyfus, T. (1995), New actions upon old objects: A new ontological perspective on functions. Educational Studies in Mathematics 29 (3), 259-291.

Sierpinska, A. (1994), Understanding in Mathematics. London, UK: Falmer Press.

Sternberg, R. J. (1996), What Is Mathematical Thinking? In R. J. Sternberg & T. Ben-Zeev (eds.), The Nature of Mathematical Thinking (pp. 303-316). Mahwah, NJ, USA: Lawrence Erlbaum.

Vygotski, L. (1981), The Develoopment of Higher Forms of Attention in Childhood. In J. V. Wertsch (ed.), The Concept of Activity in Soviet Psychology. Rmonk, N.Y.: Sharpe.

Vygotski, L. (1986), Thought and Language. Cambridge, MA, USA: MIT Press.

Wertsch, J. V. & Stone, A. C. (1985), The Concept of Internalization. In J. V. Wertsch (Ed.), Culture, Communication and Cognition, Vygotskian perspectives (pp. 162-182). Cambridge, UK: Cambridge University Press.

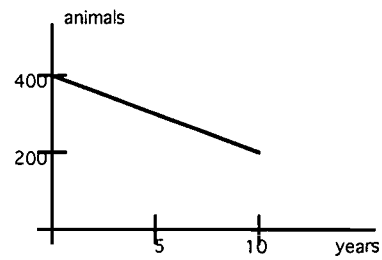
Yackel, E. & Cobb, P. (1996), Sociomathematical Norms, Argumentation and Autonomy in Mathematics. Journal for Research in Mathematics Education 27 (4), 458-477.

APPENDIX A - The interview

Animal Park

Ten years after its opening, the board of the Belangoo animal park ordered a survey of the development of various animal populations in order to be better able to plan for the future.

I. The variation of the park's zebra population is described in the following graph:



a) How many zebras were there in the park when it first opened?

b) How many zebras were there in the park after three years?

c) Could you describe the variation of the zebra population over the years in a different manner?

d) Do you know still other ways of description?

II. When the park opened, there were no lions. In the course of the first year, 60 lions were brought in, and then the lion population continued to grow at the constant rate of 60 lions per year.

a) How many lions were there in the park after three and a half years?

b) Compare the number of lions to the number of zebras during the first ten years.

c) What can the planners say about the two populations in the future?

III. The eagle population in the park varied according to the expression $f(x) = 5x\,(20-x)$ (x denotes the time, in years).

a) Do you think the living conditions for the eagles in the park are good?

b) Is the rate of growth of the eagle population larger or smaller than that of the lion population?

c) Is your conclusion valid for the entire first ten year period?

d) Compare the number of eagles to the number of zebras during the first ten years.

e) What can the planners say about the two populations in the future?

f) Is there any time at which the three populations (zebras, lions and eagles) were equal?

IV. A park is being planned which will be similar in all aspects to the existing one, except that the planners want the three populations to be exactly equal at some point in time.

a) The first planner proposed to change the living conditions of the lions so that the rate of growth of this population will change. What exactly did he propose?

b) The second planner proposed to achieve the aim by means of a change in the number of zebras present at the time the park is opened. What exactly did he propose?

APPENDIX B - Transcript of Student A

I001: You don't have to use the computer but you may use it whenever you like, according to the questions I am going to ask you. Now, here we talk about an animal park - here you have a pencil, eraser, whatever you want - we talk about a new animal park, it's not so new any more, ten years, we talk about the development of the animals, of the number of animals in the park during the ten years that have passed and want to plan its development for the next ten years. All questions relate to this animal park. Let's begin with zebras. [points to graph.] This, this graph is supposed to describe the development of the zebra population during the first ten years. Now, according to the graph, can you tell me how many zebras were in the park when it was opened, at the beginning?

A002: 200.

I003: How do you see this?

A004: [points to right (!) end of the graph]

I005: Here. Ok. Now, this, what is this x? {3:00}

A006: The number of years.

I007: So what's the number of years, 5 [points to 5 on x-axis], and 10 [points to 10 on x-axis]. How do you decide it starts from here [pointing to right end of graph] and not from there [pointing to left end of graph]?

A008: [10 seconds] Each year ...[2 seconds]

I009: Each year?

A010: Each year it grows.

I011: It grows. Because you want that each year the zebra population grows?

A012: Yes.

I013: Ok. All right. So, how many zebras were there after, say, three years

A014: three? [22 seconds]. 250. {4:00}

I015: About 250. How did you decide that?

A016: The difference here is 200 [points to the interval from 200 to 400 on the zebra axis], and then each five years it's 100, so in three years this must come out to about 250.

I017: Ok. All right. This is certainly a nice approach. Now, here I gave you the graph of the zebra population. The question is whether you can think of other ways to describe the same zebra population?

A018: [20 seconds] Again. {5:00}

I019: Again; the question you want again, right? Ok. Here we worked on a graph and you described for me the zebra population, its development during the first ten years on the basis of the graph which I brought. Now I ask whether it is possible to describe the zebra population also differently: instead of by means of a graph, by means of something else.

A020: Formula.

I021: Formula.

A022: Story.

I023: Story, yes, nice. Can you try to do this?

A024: Table.

I025: Choose whatever is most convenient for you.

A026: Table. *(Remark: While T was speaking (I023-I025), A said, in a very low voice, 'table' (A024). T did not realize that. Therefore, R (a math educator who operated the camera) interjected 'T - he also said table'; to which T reacted with 'Table - that's nice'. It is right upon this exchange that A026 was said. It is unclear whether A026 was a reaction to the I025 or to the exchange between R and T.)*

I027: You want a table - whatever you want. {6:00}

A028: [starts drawing a table, vertically, with x on the left and y on the right.] Is this the formula connected to that? [points to the formula 3x-7 that happened to remain on the same sheet of paper from the preparatory phase to the interview, and then points to the table he had prepared.]

I029: No, no, no - this is just some formula - forget it. we only used it to exercise that story [pointing to the computer]

A030: [writes number pairs into the table: 5, 300; 0, 200; 10, 400.] ... 200; when it's 10, it's 400.

I031: Yes. Maybe do another one!

A032: Until 10? Can I more than 10?

I033: Beyond 10? {7:00}

A034: [writes 15, 500]

I035: Now, can you do another one between 0 and 10?

A036: 2, 220

I037: How did you get this?

A038: The difference here is 200 [pointing at 200 and 400 on the vertical axis]; so, somehow, when it got to 5 it was 300. between 200 and 300 there are 100, and then I divided it - I divided by 5. {8:00}

I039: Ok. That's fine. Now just to continue - I intended to talk to you about other populations. ... [there is a break in the video recording; I estimate it at maybe half a minute.] In the course of the first year, they brought 60 lions. And then the population continues to grow, all the time at the same rate: each year 60 more lions. Ok? Now, if so, can you tell me after, say, 3 and a half years, how many lions do you think there were? Maybe you want another sheet - the lions' sheet.

A040: [after 8 seconds] 210.

I041: 210 - how did you get this?

A042: 6 uuh, 60 times three years, yes and 180 ... [inaudible; moves hands in a cutting motion and presumably said something like:] half of it.

I043: Now, let's think about the entire ten year period. If we want to compare these populations, the lions, the zebras, in the course of the ten years, can we compare them? Were there more lions? Were there more zebras? What can you tell me about this question? {9:00}

A044: One can ask.

I045: How?

A046: [8 seconds] One substitutes, one makes a the formula.

I047: Do you have a formula?

A048: The lions, for the lions it's 60 x [writes]

I049: I want to see what you wrote: ah - the lions, yes, nice, 60x. And the zebras? {10:00}

A050: the zebras it's - 200x [writes]

I053: Maybe you can explain the 60x to me, where you got it from, and then the 200x, from where you got it.

A054: 60 x is every year there are 60 more; this is the x of the years.

I055: Mhmm

A056: And here it is 200. Is that ok?

I057: [5 seconds] And this is expressed in the same way? Here you wrote the 200 just like you wrote the 60 there; but in the description you gave me it sounded quite different: you say each year there are 60 more therefore the 60 must multiply the x, and here you say it starts from 200. {11:00}

A058: [erases 200x]

I059: Are you more certain about the 60?

A060: Yes, 60.

I061: It seems to me that you are more certain about the 60 - the 200 you erased.

A062: Yes. {12:00}

I063: [17 seconds pause] You try ... I feel that you try to remember something. Maybe let's try to build instead of remember. Do you know how to build a formula?

A064: [10 seconds pause]

I065: Or, if you want, maybe you have a different way to compare the populations.

A066: In a graph.

I067: That's possible. Will this be easier or more difficult for you? {13:00}
A068: Easier.
I069: So, why to choose a formula if it's easier with a graph?.
A070: Let's do it in a graph.
I071: Ok; that's acceptable to me.
A072: Do it here? [points to computer]
I073: That's possible, definitely, if you want.
A074: [Starts drawing on the paper]
[There is a break in the video recording; I estimate it at only a few seconds]
I075: ... the number of years [pointing to the vertical axis in S's graph] and here the number of animals [pointing to the vertical axis]
A076: this .. here ... [inaudible] [takes the eraser and erases the labels from the axes] {14:00}
I077: It's possible this way and it's possible the other way. It is more convenient the other way, you're right.
A078: [continues labeling axes] Here it is 60, 220. {15:00}
I079: Now what does one do?
A080: [inaudible] [starts plotting points].
I081: How many points will you do?
A082: Ten.
I083: From scratch, do you know something ...?
A084: That it goes this way [showing a straight line with positive slope]
I085: Does it surprise you that it goes this way?
A086: No! It is systematic.
I087: It is? Systematic?{16:00}
A088: Yes.
I089: In which sense is it systematic?
A090: That it all the time grows by sixty.
I091: What does this cause that it all the time grows by 60? How does this express itself in the graph?
A092: straight line.
I093: A line, straight line.
A094: I can draw it.
I095: Now I am returning to my question, yes? How can we compare the zebra population and the lion population. We want to know of which there are more.
A096: How?
I097: How do I ask? The question again?
A098: Mhmm.
I099: There is a zebra population. Is it possible to say something about this on the basis of the data?

A100: More zebras
I101: More zebras? {17:00}
A102: Uuuh lions, more lions.
I103: More lions. How do you know that there are more lions?
A104: Because I see when it gets to 10, it is 600, and here when it gets to 10 it is 400.
I105: That means ... What does this mean 'when it gets to ten'? Is this conclusion of yours that there are more lions, is it correct for all the years or is it valid at a specific time?
A106: [12 seconds] All the time.
I107: All the time there are more lions? Is that what you say?
A108: Yes.
I109: For example, after 8 years, are there also more lions?
A110: Yes.
I111: Yes? Can you check this?
A112: Let's multiply 8 by 60. {18:00}
I113: That's 480.
A114: [inaudible]
I115: And this is more than that?
A116: Yes.
I117: And after two years?
A118: After two years ... twenty ... this [points to the graph of the zebras]
I119: What does this mean? What did you intend?
A120: This is larger.
I121: Ah! So, after two years there are more zebras? {18:00}
A122: Yes.
I123: And how is it possible, maybe, can you try to give me a general picture how it develops, the zebras versus the lions?
A124: The zebras, that starts from 200, and the lions from zero. And here, after two years it's 120, and there after a year it is already 200.
I125: Yes; and how does this develop in the course of time? At the end, you said yourself there are more lions. How does this come about?
A126: The number of lions that are brought here, this is more than that. {19:00}
I127: And is there a time when the populations are equal?
A128: [17 seconds] yes. You can substitute here and then get it there.
I129: How would you do this? Maybe, let's try this. Ok.
A130: Here?
I131: Wherever you want. You are free to use the entire paper that's here.
A132: [Writes, erases, writes] {20:00}

I133: Uum. Before, when you sketched here - I earlier remembered the question and I forgot it again, I now repeat it: You drew and you said it is a straight line and you said this is because each year 60 lions are added. Now, how many points do I really need to get a straight line, if I know that it is a straight line.
A134: Two.
I135: Maybe you can save yourself some work. I simply try to make you work less hard.
A136: [Continues to write, plot points on the lions graph {21:00, 22:00}. In particular, plots the point (5, 300)]. Here.
I137: Mhmm. What's this here?
A136: 320.
I137: What's this the 320?
A138: Emm. Common. A common point.
I139: How would you describe, let's say, with some details, this point?
A140: It's a meeting between the two axes. {23:00}
I141: What does it symbolize for us this common point? What does it give us?
A142: That the ...
I143: Maybe I'll interpret you. You'll tell me whether I interpret you correctly. You say that after 6 years, here are 320 zebras.
A144: Right.
I145: And also 320 lions. Ok. Do I have another any other source of information where I can now go and check whether I really have 320?
A146: I could try here [pointing to the computer, which in the meantime 'fell asleep'].
I147: Mhmm, if you want. Do you know how to ... - you just press anywhere and it comes, if you want.
A148: Needs a formula.
I149: Do you need a formula? First of all, for the lions, you have a formula. Right? {24:00}
A150: 60x. [types this into the computer.] Need boundaries.
I151: What is needed?
A152: Boundaries.
I153: Uum, yes. You can with F5 choose the limits.
A154: Right.
I155: Do F5 ... Ok?
A156: [incomprehensible]
I157: Did you do this? What's ten?
A158: Years
I159: [in order to get the graph on the screen] you need F ...
A160: I know {25:00}

I161: You know better than me.
I163: Does this look good this to you?
A164: Yes.
I165: That's exactly what you drew here [meaning the graph on paper]. Now, really the problem, you said, for the zebras you do not yet have a formula. Right? Is this the only manner to draw here a graph of a straight line.
A166: Yes.
I167: Didn't you learn how to do this with points? Maybe you didn't do it.
A168: We did.
I169: Did you? Come on, I'll explain it to you - never mind. You can enter points here, in the green one - where are the points? {26:00} I don't remember ... here, it's this. Now you can enter points and then you can ask it to connect the points and then you have the graph of the zebras.
A170: Ah! We didn't do this.
I171: Hmmm?
A172: We didn't do this on the computer.
I173: You didn't do this on the computer? So, I'll show you. Which points would you like to enter?
A174: UUUm, one, ..., sixty, ... (1, 60).
I175: Not lions, lions you have; zebras; for the lions you have a formula, you made a formula, yes?
A176: [inaudible] ... all of them.
I177: How many do you need, how many points?
A178: Two; two's enough.
I179: Enough.
A180: What to do here?
I181: You need a comma, I think if you leave a blank, this way, simply make a blank, that way, and write 200, I think that will work, and do enter, here, try enter ... {27:00}
A182: It's a comma.
I183: A comma? Aaaah, ok, one needs a comma, I need to learn this story; try enter again: Ah here, you have a point.
A184: Five [enters the data for the second point: (5,300)]
I185: Oh, you learned it very fast.
A186: Enough.
I187: Right. Now you transfer the points to the graph, I think with Alt F2. Press Alt, and F2. You have the two points. Now there is a possibility to connect them. This also I need to look for ... {28:00}

A188: It's here! [points to the screen item: connect points and presses the corresponding key]

I189: Ah, the starting point is not (0,0), write there P1.

A190: Here?

I191: Yes, simply P1, that tells it to take the point P1 as first point ...

A192: pointing to the enter key] Here?

I193: Yes, you may press enter. Slope we don't know, length of the interval we don't know, end point! At the bottom, here there is an end point - put P2, P ... 2 ...

A194: [enters the data. Generally, it should be noted that during this entire episode S sits there, head in one hand, the other hand on the keyboard, and enters data without saying anything beyond an occasional one word question like "here?" in a very low voice]

I195: Now, which F again? Drawing?

A196: [enters command to graph]
[Computer draws the wrong line: y=200] {29:00}

I197: This he did already once to me. He is not ok! I don't know, this also happened with another student. He shouldn't have done this. It's really not logical. And what did we do then, we simply did it again. Excuse me, I'm coming now and I'll try to get this line ... [works on the keyboard]. Terminate without drawing ..., draw the segment! Here, now, now he did it! Yes? Now, we'll simply ignore the violet one. It can be erased, but, uuhm, ... here, this I want to erase, how do I erase, erase all functions - I do not want to erase all functions; he doesn't erase it! He doesn't erase it! Well, ok, we'll just simply ignore it, yes?

A198: [Nods]

I199: This is what we obtained for the zebras [pointing to the segment (0,200) to (5, 300)] Let's take another five minutes and talk about that, yes? [exchange between R, the camerawoman, and T about the fact that the computer also gave the equation of the incorrect horizontal line] {31:00} [Addressing S again:] So, you said earlier that you wanted to draw the graphs here in order to check the intersection points, yes?

A200: Yes.

I201: Where is now the intersection point?

A202: [points to (5, 300)]

I203: Just the P2 which you have already found. How do you interpret this?

A204: [inaudible]

I205: How can you interpret this?

A206: [inaudible]

I207: This, right? How much is it?

A208: 300.
I209: It's very close to what you thought here, from the hand drawing. And when?
A210: Five. five.
I211: After five years, good. Now I want to ask you the following: I want to ask you still something else. If I want to check what happens after two years, yes?
A212: Yes. {32:00}
I213: After two years there are 180 lions, right, where do I see this ... oh, no, sorry, 120. Where do I see this?
A214: Here ... [points to 2 on the horizontal axis]
I215: Fine. How many zebras are there after two years?
A216: 120 [appears to have read this number from the lions graphs] - ah that's lions.
I217: Hmm? Yes, lions 120.
A218: Can I do it according to the graph?
I219: Yes, sure! [incomprehensible]
A220: [Generates point on screen with intention to 'walk on graph'; they collaborate to make the point walk indeed on the graph]
I221: Oh! How did you do it?
A222: [?]
I223: Yes?
A224: [walks the point to x=2] 245
I225: 245 - This is already very close; at the beginning you said 250 - 100%. Now if we return for a moment to what I gave you - the graph of the zebras: Does it look like the graph you received? [gives him the original graph on the sheet] {33:00}
A226: [looks forth and back between computer screen and sheet]
I227: Hm?
A228: [shakes head]
I229: In what is it different?
A230: The initial point is different?
I231: What is the initial point?
A232: Here it is 200 [pointing to screen] and here it is 400 [pointing to sheet]
I233: What does it mean, the initial point ...? {34:00}
A234: How many there were at the beginning.
I235: At the beginning. Do you remember what you told me here [pointing to the sheet] - you told me something very reasonable here.
A236: [15 seconds]
I237: Don't you remember? What did you write?
A238: In the table?

I239: Yes
A240: Here?
I241: Yes
A242: It's 400
I243: What's this 400?
A244: 400 zebras
I245: When?
A246: In ten years
I247: And at the beginning?
A248: 200
I249: The graph, you simply read it differently, ... [continues explanation and thanks A]. {35:00}

APPENDIX C - Transcript of Student L

I001: ... the curriculum still needs to be improved and so we asked you, T (the teacher), and Y (another student) to collaborate, and simply to discuss with us a problem situation just as you do it in class, but here in a quieter place ... [noise from electricians who enter the room in order to carry out a repair; some discussion with the electricians ensues. They leave but the noise continues throughout a good part of the interview.] {1:00}
L002: [Reacts to electricians.]
I003: Well, I brought here all I thought you might need: pencil, eraser, calculator. So, when you want, use the calculator, ok?
L004: Ok.
I005: The situation we'll talk concerns an animal park ...
L006: A park?
I007: An animal park that's supposed to have opened ten years ago; and now one looks back in order to plan it for the future.
L008: [Nods]
I009: And the, the ... one speaks about all kinds of animal populations. We'll start with this here [gives Limor the graph of the zebra population.]
L010: [Nods]
I011: This is a description of the zebra population in the course of the first ten years.
L012: [Looks intently at the graph; nods]
I013: And I'll begin by asking you questions about the zebras and then we'll turn to other populations. {2:00}
L014: [Nods]

I015: According to this description [graph], how many zebras were there at the beginning, when they opened the park?
L016: 200.
I017: [Nods] 200. And, say, after three years?
L018: About 300.
I019: About 300. How do you get to this number?
L020: [Points to the graph, height 300] [inaudible]
I021: [Gives orders to close the window in order to lower the noise level from outside.] Excuse me. How did you get to 300?
L022: Because I get to over here [with her finger shows a horizontal line at height about 300 on the graph].
I023: And this looks about like 300?
L024: No, it looks more, but ...
I025: More?
L026: Yes. {3:00}
I027: [Another discussion about the noise.] Now, this is a certain description of the development of the zebra population. Can you think of another description?
L028: What do you mean?
I029: [incomprehensible]
L030: Yes, if here, here are the zebras here [pointing to the horizontal axis] here the years [pointing to the vertical axis].
I031: That's certainly possible. Do you want to try this?
L032: With the same numbers?
I033: Yes. {4:00}
L034: Ok. [Neatly draws a separate coordinate system, with 200 and 400 marked on the horizontal axis, and 5 and 10 on the vertical one; she puts a straight line graph through the points (400, 0), 300,5), and (200, 10) in this order.]
I035: Do you want to explain to me how you chose those points? {6:00}
L036: In the first year was [points to the origin and then to (400,0), and after five years [external noise becomes overwhelming; points to 5 on vertical axis and then to (300,5).
I037: Let me tell you: You just solved a very difficult problem which I wouldn't have dared to give you, and you solved it well.
L038: [incomprehensible]
I039: And you did something very original; you are, I didn't tell you, the eighth girl, boy or girl, whom we interview like this and you are the first to come up with this idea. It's a nice idea.
L040: What did they all do?
I041: Hmmm?
L042: What did they all do?

I043: Ok, now I'll ask you more; we don't want to reveal what all others did. But maybe you have another idea. What kind of description ...? How would you characterize all these descriptions? [pointing to the sheet with the two graphs.] {7:00}

L044: [10 seconds pause]

I045: Well, [incomprehensible] graph. This also is a graphical description. Can you think of other descriptions?

L046: Yes.

I047: For example?

L048: One can prepare graphs but different ones.

I049: Different graphs. And also something which is not ...? Here you did graphs, right?

L050: No, not something like of this form; let's say diagrams like this [making a rectangular motion]

I051: Diagrams are possible. What else is possible?

L052: Maybe, possibly circles [making a circular motion, presumably meaning pie diagrams.]

I053: How ... The diagram which you said, what kind of diagram would you have made? {8:00}

L054: This [starts drawing a histogram]

I055: Only in big lines.

L056: That way.

I057: Ah - I understood. Fine. A histogram. That's a technical term which you don't need to know. You also proposed a circle, what's called a pie diagram. Did you call this a pie diagram?

L058: Yes.

I059: Did you do this ... [external noise]

L060: We did this in computers, not in mathematics.

I061: Aha, in the computer lessons, not in mathematics. Fine. Now, when they opened the graph (sic!), they brought lions there. [Holds a sheet of paper high and presents it to S.] {9:00}

L062: [laughs]

I063: Now with the lions it's as follows: At the beginning, there were no lions. In the course of the first year, they brought 60 lions. Then, later, the lion population grew, at the same rate all the time, each year 60. Ok? Now I'm going to ask you questions about the lions. say, after three ad a half year, how many lions do you think were there?

L064: Uuum, 210 [very low voice].

I065: What? [Looks at S with a question mark in his eyes.]

L066: 210.

I067: Hmmm?

L068: 210.

I069: 210. Ok. 100%. I would also have said 210. Now, if we want to compare the zebra population with the lion population: you know: whether there more of these or of those, when, and so on. Do you have a proposal how to do this?

L070: Maybe one could, say, do it with a graph of the two of them [pointing to her graphic calculator].

I071: [Makes an inviting gesture with his hand] If you want, take your calculator, please. {10:00}

L072: [Takes out the calculator and starts working.] Which ... [incomprehensible]

I073: Whatever you like, you do.

L074: [Clearly doesn't like this answer and asks another question which cannot be heard.]

I075: Hmmm?

L076: [incomprehensible; probably says something about an expression for the lion population]

I077: That's a possibility.

L078: And it's possible to compare this. [Building noise; during this time S thinks, looking at the calculator.]

I079: Can you, can you think of the expression for the lions? Can you build it? {11:00}

L080: Uuum. ... Yes.

I081: Yes?

L082: Yes! But, for example, ... were there lions already at the beginning?

I083: On the day they opened the park, no. But after a year here were 60. And so on, as you said after three and a half years there were 210.

L084: Can I sketch? Without ... [meaning the calculator]

I085: [Nods.]

L086: [Sets the calculator aside and easily sketches a coordinate system marking 1, 2, 3, 4, 5 on the horizontal axis and, after some hesitation, 50, 100, 150, 200, 250 on the vertical one. Plots points at (1, 60), (2, 120), (3, 180), (4, 240), (5, 300). The first three points look fairly linear, the following two look sublinear.]

I087: How many points did you draw?

L088: Five.

I089: How did you decide on five? {13:00}

L090: Just like this.

I091: Just like this? Could you have drawn twelve?

L092: Yes. [Marks the origin!]

I093: Could you have done less?

L094: Yes!

I095: How many?
L096: Until three and a half.
I097: Until three and a half years?
L098: Yes.
I099: Well, quite accidentally I talked about three and a half. The park is ten years old. Yes?
L100: [inaudible, drowned in construction noise. During this time S connects the points she had plotted.]
I101: Which, which graph did you get?
L102: Uuum, straight [barely audible]
I103: Straight. [confirming this with a nod].
L104: Not really; it looks straight but there is ... [inaudible] here and here
I105: Yes
L106: So it isn't straight; if here there was one and here there was two [pointing approximately to some points within the coordinate system] it would be straight.
I107: But ...
L108: But, in fact, yes, yes, because all the time [showing a straight line with her hand and arm] {14:00}
I109: [interrupts S] How do I decide whether it is or isn't a straight line?
L110: If it's all the time straight, if, let's say no deviations.
I111: Mhmm. And how do you see this? You entered this by means of numbers ...
L112: No I thought that ...
I113: ... and I wanted to understand how you drew this conclusion out of the numbers.
L114: ... no I thought that only if there were here [pointing to the y-axis] the same numbers as there [pointing to the x-axis], then there is a straight line; but I now understood that this is not so.
I115: So what does need to be the case in order for it to be a straight line?
L116: That if it all the time, say, increases at the same rate.
I117: That it all the time increases at the same rate. How do I see the rate?
L118: I see that every year there are 60 more.
I119: Aha. And so it gives a straight line, a graph of a straight line. How many points do I need to draw?
L120: At least nine.
I121: At least nine. Fine. Ok, let's now return to the question. The question was to compare: more lions, more zebras, when, etc. Can you now say something about this story? {15:00}
L122: You mean, compare?

I123: Mhmm.
L124: I can draw this also here [pointing to the graph she has just drawn]. [incomprehensible; looks at T with questioning eyes]
I125: Mhmm.
L126: Yes.
I127: If you need this in order to compare, that's fine with me.
L128: [starts plotting, probably intending to plot (0,400)]
I129: Which graph do you draw? From here [pointing to the sheet which has the two zebra graphs, the given one and the inverse one]
L130: [points to the given one]
I131: This, ok, fine. {16:00}
L132: [plots (0, 400); after about 60 seconds marks additional numbers on the horizontal axis: 6, 7, 8, 9, 10; after another 30 seconds plots a point close to (3.5, 300), then quickly (10, 200) and the line connecting the three points.]
I133: Where this [pointing to (3.5, 300)] is which point?
L134: [points to 300 on the vertical axis and from there over to the graph] five, three hundred.
I135: [incomprehensible]
L136: [shows the segments from (5,0) to the graph]
I137: Five, three hundred. Fine, 100%, ok. Now, what do we ..., what can you say about the comparison of the... {18:00}
L138: ... populations.
I139: Of which are there more?
L140: Of which are there more?
I141: Mhmm.
L142: Zebras.
I143: More zebras. All the time there are more zebras?
L144: [It should be noted tat the two graphs S drew barely intersect. The zebra graph happened to go through the 'end' of the lions graph. S now points to this intersection point and says something incomprehensible.]
I145: There are times ...?
L146: ... when it's the same.
I147: ... when it's the same. What does it mean 'there are times' when it's the same?
L148: In this year [showing a segment from the horizontal axis up to the intersection point.]
I149: Which, how do you determine the year when it's the same? According to what here in the drawing?
L150: I can [incomprehensible; shows the same segment, this time from the intersection point down to the axis.]
I151: According to this point [pointing to the intersection point]

L152: [shows a continuation of the lion graph beyond year 5] ... continuation, because every year there are 60 more.
I153: Aha [incomprehensible] What does it mean 'sometimes it's equal'? {19:00}
L154: In the fourth year there was the same [incomprehensible]?
I155: In the fourth year there was the same number? And in addition? What else can you say? In ..., after ...
L156: After the fourth year the zebras were less and the lions more.
I157: Fine. Now, can you somehow check this ..., this result that it was in the fourth year? Note, the graph, we just like this drew it, not very precisely.
L158: I can put the two formulas in the calculator [take the calculator into her hand] and walk on the graph to the same point. {20:00}
I159: [nods] And here, on paper, it's difficult, we do not have this tool to walk on the graph. But maybe we have another tool ...
L160: what?
I161: ... which allows us to check
L162: Maybe it's possible to do, uuuh, say, uuh, here [pointing to the intersection point]
I163: Mhmm. Can you try this?
L164: Yes. But there is a little bit of a problem; I need to compute precisely for each year.
I165: Let's start and see.
L166: [puts the calculator precisely in front of her and computes by hand 60 times 4 = 240] This is what I ... [incomprehensible]
I167: Why [incomprehensible]
L168: Because I obtained that ...
I169: Oh - you computed the fourth year ...
L170: [incomprehensible]
I171: and now, how did you conclude that the 240 which you obtained ...
L172: Because I see that - 240 is about here, yes? [shows a horizontal segment from 240 on the vertical axis to the graph] about the fifth year.
I173: About the fifth year? [small pause] Is it possible to check this?
L174: Yes, ... [incomprehensible]
I175: Hmmm?
L176: But I don't know how to do it.
I177: You don't know how to check?
L178: [Questioning look at T]
I179: What do you know, and what don't you know? I think that you do know. [5 secs] You already started, no? {22:00}

L180: Yes, but in the one of the zebras I don't have ... I have it approximately.

I181: Aha. First of all, for the lions you do know. You can compute 5 times 60 and that's 300. Ok?

L182: [nods]

I183: Now, is it possible, for these zebras, to get some more numbers than 'approximately'?

L184: [Shakes head, repeatedly, and looks at T to mean: I don't know what you mean]

I185: Ok. So I'll ask you another question. We talked about the park planners. What could they say ... Well, we talked all the time about the first ten years. What could they say beyond the ten years? What about these two populations, the lions and the zebras? {23:00}

L186: The zebras - it's impossible to know. Their number decreases all the time. And the lions continue each year by 60.

I187: If you say 'decreases all the time', why do you also say 'it's impossible to know'?

L188: Because it will end.

I189: Will end. Do you know about when it will end? Or exactly when it will end?

L190: After another five years, that is at fifteen years, about.

I191: About - according to what?

L192: I continue the line. [Shows how the zebra graph continues beyond the sheet in front of her.] This way.

I193: So where does it end? What does it mean that it ends?

L194: Let's say, here in another five years; that's how I draw the line, ... [inaudible; presumably says something about 'zero'].

I195: Yes, fine. Now, the owners of the park brought in eagles, in order for it to be more interesting. Now the eagles - you complained a little here that I didn't give you formulas; now I'll give you a formula [giving her the sheet with the formula for the eagle population]. Here, this is a formula which describes the eagle population. The x, [incomprehensible] the x is the years, and 5x(20-x) is the number of eagles after x years. Ok?

L196: [nods]

I197: Now, just off hand, what do you think about the living conditions of the eagles in the park? Do you think they are good conditions, not so good conditions?

L198: Uuum - what do I know. I don't know. According to their quantity?

I199: Yes.

L200: Seems to me not so good.

I201: Why does it seem to you not so good? {25:00}
L202: I have to multiply by a minus, and ... wait a minute ... [incomprehensible] ... I need to know what x is.
I203: Need to know what?
L204: I need to know what x is.
I205: x is a num... Ah: which number x is or what is its interpretation?
L206: Which number.
I207: So you tell me it depends on x.
L208: [nods affirmatively.]
I209: Can you tell me more?
L210: If, say, it is larger than ... uuuh ... 20, then it will decrease, yes?
I211: If the x is more than 20, then something ... something is not [incomprehensible]
L212: In fact, one multiplies by minus or by zero.
I213: What does it mean if here you get something negative?
L214: That it will be something negative and I will have minus. {26:00}
I215: What will you have minus?
L216: Animals. I will not have animals.
I217: You won't have animals. Minus animals - that's not possible. Ok. Fine ... And when the x is smaller?
L218: There will be.
I219: Mhmm. So this we want to find out about. Do you think there will be any time at all when it increases, the population of ...
L220: Yes.
I221: And this increase, do you think it is stronger than that of the lions or weaker? (Remember: about the lions you said that there was an increase.)
L222: It depends on their number.
I223: How can one find out about this?
L224: By means of a graph.
I225: How do propose to make a graph? {27:00}
L226: On the computer.
I227: [nods]
L228: [takes the calculator and starts working]
I229: I have a problem, that I cannot see this. If you could tell me ...
L230: [positions the calculator so that T can see it]
I231: What did you do? 5x(20-x), yes ...
L232: [graphing command; watches; smiles in a way that shows something is not the way it should be] {28:00}
I233: Now, tell me something ...
L234: I cannot see it, the range
I235: [incomprehensible] Ok, you may change the range.
L236: [works on calculator]

I237: To what do you change it, tell me? Do you change the x now or the y?
L238: The x ... [inaudible] ... hundred
I239: The x to 100, from, from where?
L240: From zero.
I241: Mhmm, from zero.
L242: Ah, no, from ten to 100.
I243: From ten to 100?
L244: The minimum will be zero.
I245: Yes, you'll have from zero to 100.
L246: [asks a question; the background noise is such that absolutely nothing can be understood] {29:00}
I247: [confirms that the answer to her question is affirmative]
L248: [continues working]
I249: Now, are you doing the y?
L250: [inaudible]
I251: I'll get up, excuse me. [Gets up in order to see the screen better.]
L252: [produces the next graph; says something inaudible which expresses some dissatisfaction with the new graph]
I253: Hmmm? [5 secs] You can also ... [inaudible] this is not final, right? {30:00}
L254: [presses one more key, possibly the Graph key]
I255: So what do we have now? x from 0 to 100; and y, how did you choose it?
L256: Also.
I257: Also from 0 to 100?
L256: This is what came out [pointing to the calculator]
I257: Hmmm? This is what came out; and what do we do with this?
L258: [smiles]
I259: What does this really show you?
L260: That it's not good
I261: There is no question of good or no good here. Uuuh, the graph you obtained shows something, right?
L262: Yes. [hesitating] That ... The whole year, the year, uuuh, they have, uuuh, the number ... grows. {31:00}
I263: Where does it grow?
L264: Here I have the x's [with her right hand index, quickly sweeps over the x-axis on the screen, from right to left(!)] so every year it grows [with her left hand index, points to the leftmost part of the graph in an upward motion]
I265: This grows? And what's the, the ... Where does it grow? This grows? [pointing to the same leftmost part of the graph]. It's a little difficult to see here. In fact, why don't you try to change the,

the range, either the x's or the y's or both so that we see maybe a more comprehensive picture of the graph.

L266: [works]

I267: [gets up and tries to see the screen] Ah, you didn't press yet. [sits down again] {32:00}

L268: To ten.

I269: Did you now choose x to ten?

L270: Yes, because ... yes. [inaudible]. From zero.

I271: What do you see now?

L272: That with the number of years, it continues ...[to grow, according to her hand movement]

I273: You said before ... I want to remind you what you said before: You said that if this x here is bigger than 20, we have a problem.

L274: [nods]

I275: Does this show in the graph [pointing to the calculator}

L276: [shakes head]

I277: How can one express this in the graph?

L278: [goes back to work on the calculator] ... in the range ... {33:00}

I279: What do you have? Zero to 20 in the x. What do I see now?

L280: [shows an up and down motion from right to left (again!) with her index finger; says something inaudible]

I281: Ah. Are you missing something?

L282: Yes, the point [inaudible]

I283: Which point?

L284: Where it starts to decrease. [changes the range again; shakes head at result, probably because it still doesn't show the maximum.] Have to change the y. {34:00}

I285: Mhmm.

L286: [Changes the range once more]

I287: How did you stumble onto this 500?

L288: [does not answer; slightly shrugs shoulders]

I289: Do you see now the point where it changes direction? {35:00}

L290: [nods and shows the point on the screen]

I291: There, exactly at 500; you were lucky. Ok. Now, try to describe for us a little what, what happens to this population when we continue.

L292: Uuuh, in the course of the years, uuh, it grows, and then in a certain year, it starts to decrease.

I293: Where exactly does it start to decrease?

L294: [points to the screen] here

I295: How many years is this?

L296: Twenty.

I297: Let's see for a minute what you put in.

L298: [presses keys, probably for seeing the range]
I299: Twenty five? What's written?
L300: Yes, twenty five.
I301: Let's go over to the graph. Where is twenty five? [and without waiting for an answer, points to the maximum and asks:] Does this look like twenty to you?
L302: Uuum, [points to the maximum and from there down to the x-axis] Here it is twenty, and a little further it starts decreasing. {36:00}
I303: Because this is negative? [Pointing to the formula on the sheet of paper.] If the number here [showing a circle around the formula] ... If the number is put here [drawing a new coordinate system on the paper], as if we were to build the graph from scratch, yes? Here 20, here, let's say 30, and here 10 [marking these on the horizontal axis], and here's the x. Now, you said before, and I was very impressed by this, that if you put here an x which is bigger than, let's say it is 30, then this will be negative.
L304: [nods affirmatively]
I305: Can you tell me how this finds its expression in the graph here [pointing to the newly drawn coordinate system] that this f(x) will be negative?
L306: [takes the pencil] Uuum, these are the years, yes?
I307: Yes, these are the years?
L308: [Wants to plot points]
I309: No, there is no need to draw the entire graph; I wanted in particular to relate to the 30. {37:00}
L310: [Draws a positive but decreasing (and concave, from below) line between x=20 and x=30]
I311: Fine. Now, ... let's return to the question which I asked before as the last question. I asked before about the lions. These are the lions [producing from a heap of paper the lion graph], and you have the eagles. And I asked, I think that I asked: If I want to know about the rates of increase of the lions and of the eagles, which population increases more strongly?
L312: The lions, I think.
I313: Why do you think?
L314: They continue to grow.
I315: I remind you what you told me before: You said, it depends on x. It depends on the years. {38:00}
L316: Yes.
I317: Do you remain convinced that it depends on the years?
L318: Yes.
I319: How does it depend on the years?
L320: If, let' say 20 years ago, maybe the eagles grew more than the ...

I321: The eagles grew more? How can one see this?
L322: Maybe here, maybe there [taking the pencil and looking at two sheets of paper: one with the lions and zebras graphs, ad the other one, empty, prepared for the eagles graph]. Here one cannot know it [pointing to the empty graph]. There I can know more because I know the x.
I323: Let's say if I just like this fix x=2, does this help you?
L324: Yes.
I325: How? What would you do in order to compare the growth?
L326: I'll use the x=2 and plug it and I would see. {39:00}
I327: Fine. Ok. Thank you very much.

B (an observer) then asks whether she has a computer or calculator at home. When she says no, he asks whether the other girls do, and she says yes. I asks: all? and she appears to think that all except her have one. B makes sure that she uses the calculator only in the math lessons.

APPENDIX D - Transcript of Student S

Early annotations used in S's transcript:

C: **C**onsolidation, i. e. abstraction ability vs. situation boundness.
S/A: **S**pontaneous use or **A**voidance of a particular representation or of the computer.
M/I: **M**odelling; (lack of) **I**nterpretation in terms of a situation.
V: Cognitively interesting aspects dealing with **V**ariable (e. g. discrete vs. continuous) or other concepts and processes.

I001: [Gives a general introduction about the aim of the interview and makes sure S knows how to use MathematiX to graph a function. He then starts the actual interview.]
02:15 I002: The activity which we want to work on today concerns about an animal park which opened ten years ago; we will talk about changes in the animal populations in order to plan the development of the park in the future.
I003: [Pointing to the zebra population graph] Now we have here the description of the zebra population and I'll start asking you a number of questions about the zebra population.
03:00 S004: Wait a minute; here, at 10, is the image 200?
I005: Yes, that's acceptable. How many zebras were in the park when it opened? at the beginning?
S006: 400.

I007: 400, right; and after three years?

S008: Hmmm. [5 seconds pass with no one speaking.] But there's a problem here. A population increases usually, isn't it?

I009: Aha. And what about this population?

M/I S010: They, like, kill animals.

I011: Or?

S012: Or they bring them somewhere else; but this is a very strange behavior for a population.

I013: Or they die.

S014: But this is [supposed to be] a population!

I015: If you have a population of bacteria in your body, you would like it to decrease, isn't it?

S016: [Returns to the problem of finding out how many zebras there were after three years.] How, much is it here at 5? Or is it just any number? (*Stam mispar*)

04:00 I017: Let's say that at 10, it's 200. Can this tell you something about 5?

S/A S018: Can I write on the paper? [Writes -20, and then 400-20x=y. He puts a box around this equation as if to indicate a result. Then he chooses two points, (0, 400) and (10, 200), computes the equation of the straight line through these two points and writes again 400-20x=y.] (*During this time there was a problem with the camera and maybe a minute or two were not taped; there is, however, a written record of S's computations during this period. Moreover, during the 40 seconds that were taped, nobody spoke.*)

05:00 I019: We missed a bit and I would like to try and recapitulate. You found the same formula, yes? And this gives you some confidence that it is the right formula. I can see exactly what you did here [computation on the paper], but try to explain to me what you did earlier. Earlier, you also got there.

C S020: Earlier, I was confused. I thought I could find the formula directly and I found -20x, and it makes sense: at the beginning there were 400 and one subtracts this from 400. But I did this by error (sic!), just like this, not according to some kind of method.

I021: You simply put the 400 because that was at the beginning?

M/I S022: This is the beginning. Usually when it decrease you do that: 400 minus the percent of decrease, somehow ... . And what you asked about 3, it is 340 zebras.

I023: How did you find the 340?

C S024: I plugged 3 into the formula.

06:00 I025: Ok. Really, you already answered my next question: If there are other ways to describe the zebra population; and you already found another way , right?

S026: What do you mean?

I027: You found the formula. Are there still other ways to describe this population?

S028: [pause]. Uuh, 400-y, one can do ...

I029: Maybe I didn't explain myself well. We here [pointing to the graph] have a particular description which I gave you. And then you found another one ...

S/A S030: Each year about 20 zebras disappear from the initial population. And we can do it in a table.

I031: Ok. Now let's assume that when they opened the park, they also brought lions. In the course of the first year, they brought 60 lions, and then the lion population continued to grow ...

07:00 S032: ...multiplied itself ...

I033: ... at the same rate ...

S/A S034: 60x.

I035: What's 60x? if you write 60x?

M/I S036: This is the formula.

I037: You said 'multiplied itself'. Is this related?

S038: No, ..., multiplied itself? Yes! Multiplied itself: If there were 60 ... Multiplied itself? It increased at a constant rate of 60 lions per year. It didn't multiply itself.

I039: Now, after $3\frac{1}{2}$ years, what was it?

08:00 S040: [12 seconds pause] 210 lions.

I041: Now, if you want to compare The zebra population and the lion population in the course of the ten years ...

C S042: The relationships between them or simply that they are equal? (Note: there is only one Hebrew word for 'compare' and for 'make equal'; the word is 'l'hashvot'; this is the word that T used). I do 60x=400-20x.

I043: And then, what do you find?

S044: I find the image where they are equal.

I045: Maybe do this, and then you can expand the question.

09:00 S046: [22 seconds] x=5.

I047: x=5. Now let's see what this is, the x=5which you found.

S048: Wait, I have to substitute it here and then I'll find the point. This is the point we are looking for.

I049: Again! What does it mean? Maybe you can explain it in a sentence, what's the 5 and the 300?

S050: It's the point where the image of the two functions is equal. It's the intersection point of the two functions.
I051: And what does it mean about zebras and lions?
M/I S052: The year and the quantity ... the mapping between... I don't know how to formulate it ... That their population is equal, that's it.
10:00 I053: And how many is that population?
S054: 300.
I055: Ok. What I intended was to somewhat expand this. In fact, you asked whether to only equalize or to compare in the course of time. I also intended to compare over the course of time.
S056: The lion population grows. Can I draw this? Something like that.
I057: And how did you decide on ... [shows various slopes by holding his hand over the paper where S drew] Is it also a straight line, the lions?
S058: The lions, yes, a straight line.
I059: How do you know?
C S060: This is an equation of a straight line.
I061: And how did you decide on the slope?
S062: I didn't do slope here.
I063: No, not numerically, but how ...
C S064: I can find the slope, using the two points (0,60) and (5,300).
M/I I065: What about this point (0,60)? What does it mean, 60?
S066: The initial population of the lions.
12:00 I067: Well, here you wrote 60x.
S068: Ah, no! It is the increase if I wrote here 60x..
I069: There is something strange here, no? This is not exactly the same thing, no?
S070 This is not 60x.
I071: Ok, before you work this out, let me go back to the description: In the course of the first year, they brought, slowly, slowly, 60 lions.
S072: Ah - they did not bring 60, and each time 60, and 60, and 60? Then it must pass this way [points to origin]. But I don't know at which rate they brought the lions. It seems to me that this is a split domain function.
13:00 I073: Now you brought in something really complicated. Our intention was constant rate. This 60x is fine with us, and we'll leave it at that. Let me ask you a last question at this

stage: What can the park planners say about the future, beyond the ten years?

S074: The zebra population will die out and the lion population will grow.

I075: Ok. Now, we'll bring you a third population, to complicate things: an eagle population. The eagle population we described here by means of a formula: By x we denote the time, just as you did, and f(x) is the number of ...

14:00 S076: The number of eagles.

I077: What do you think, as a first impression, about the living conditions of the eagles in the park?

S078: What do you mean?

I079: Are they good? Are they bad?

C M/I S080: The truth is that this looks to me like a quadratic function, and thus it would go like this. [Draws parabolic shape, open towards the bottom.] Thus we can assume that at first they were good and then because of a certain cause, they were not good.

I081: Ok; do you think the rate, the rate of growth of the eagles is bigger or smaller than that of the lions?

15:00 S082: Bigger.

I083: If you say this, do you refer to the entire time period or to certain moments?

S084: If it is really a quadratic function, then here the rate of increase, they'll go down, at a very big rate.

I085: I'll come back to my question, and sharpen it a bit: We compared the rates of growth of the eagles and the lions. You thought that the eagles' was bigger. Why did you think so? Where did you take it from?

M/I S086: On the basis of the slope.

16:00 I087: Do you know this slope [of the quadratic?]

C S088: Yes, here it is 100 [pointing to the sheet with the information about the eagles] and here it is 60 [lions].

I089: And it remains 100, here, for the eagles?

C S090: It is not a constant slope for a quadratic function.

I091: Absolutely right, it is not a constant slope. Now, the slope, how does it appear in the graph, the slope?

S092: It's the angle of increase of the function. If the slope were bigger, it would be like this [draws a narrower parabola.]

I093: Uuh, so you tell me that the rate of change, the rate of increase changes, and at the beginning the rate of increase of the eagles is bigger than that of the lions. From this I conclude that possibly this does not remain so for the whole time.

17:00 S094: It does maybe not remain so but we don't know how to compute rates which aren't constant and it is under assumption that I say that later the rate of increase of the lions will be bigger than that of the eagles.

I095: If you don't know how to compute it, maybe you have another way to find out without computation...

S/A S096: In a table I can find out but if this is about the function I think it is, then it even has a decrease here and then it is clear that ...

I097: How can you find out whether it is the function you think it is?

S/A S098: [turns to the computer and puts his hands on the keyboard.]

I099: If you need a table, I have one; I have prepared one if that helps you.

18:00 S100: Types at the keyboard.

I101: Do you see something? I don't.

S102: Yes, I do, but ...; it drew here.

I103: Oh, that's where it drew; why did it draw there?

S104: Oh, is it so small? what?

I105: What can be the problem here?

C S106: That the scales are too small.

19:00 I107: Which scale or scales will you change?

S108: [S has a technical problem with changing scales that takes some time.]

I109: Now it changed; anyway, we got something else.

S110: I still have to change the y.

21:00 I111: [to the camera] What he does now is he takes x from -1 to 1 and y to 60
[to S] and we already see something slightly different. Does this look like what you expected?

S112: No.

I113: Does that mean it <u>is not</u> what you expected?

C S114: It's not a quadratic function.

I115: Isn't it? How <u>does</u> it look?

S116: Like a linear function.

I117: Now, what did you expect to see? The function is in any case quadratic, right? So you do expect a parabola.

S118: Yes.

I119: What could be the reason? Or, how could you check yourself?

22:00 S120 Maybe I made a mistake.

I121: Maybe you did <u>not</u>.

S122: Maybe one just can't see it.

I123: How can one see it, if one doesn't see it?

C S124: [Turns to the computer and starts rescaling: 0<x<10, 0<y<150]

I125: What did you write? 150 in y, and 10 in x?

S126: [Now rescales to 0<y<300]

I127: 500? What did you write?

S128: 300. Here you see that it starts ... [follows with his hand the beginning of the curvature of the parabola on the screen.]

23:00 I129: Aha! Starts? Let's continue a little.

S130: [Rescales again, this time to 0<y<1000]

I131: 1000.

S132: [Rescales again, this time to 0<x<80]

I133: This is similar to what you thought, yes? Ok, now let's return to the question. We now want to compare these populations. We talked a lot about rates of growth. I now want to ask you as follows: You want to compare the number of eagles and the number of zebras during the first ten years.

24:00 S134: At the point where they are equal or ...?

I135: Whether they are equal at all, or of which there are more.

S/A S136: [Writes an equation on paper.] (also **C, M/I**)

I137: You wrote an equation. Do you know how to solve such equations? Did you learn this?

S138: What do you mean?

I139: Simply, I don't know whether you learned to solve such equations.

25:00 S140: We'll see. [No talking for 45 seconds during which S manipulated the equation.] Hmmm ... That's ... I can't get out of this kind of equation.

I141: That's a little difficult, this equation. I think you'll learn that next year, yes? Now, if one doesn't succeed with the equation, are there other ways?

S/A S142: I can do it in a table or [points to the computer.]

26:00 I143: What do you prefer? [S starts typing into the computer]. Now, let me come back to the question. In course of the ten years which already passed, what can you say about a comparison of the zebra population and the ...

S144: The zebra population decreases and the eagle population increases.

I145: During the entire ten year period?

M/I C S146: Yes, because the symmetry axis of the eagle population is 10.

I147: And which population is bigger?

27:00 S148: Until the point (5,320) there are more zebras but later more eagles.

I149: Does the 5 remind you of something?

S150: 5 years. That was the intersection point between lions and zebras.

I151: And that's my next question: Is there a point where the three populations are equal?

M/I S152: Yes. [adds the lions graph. Thinks a while.] But ... there is a point when they are equal but the intersection point is not uniform.

I153: Ok, I would quite appreciate it if you'd explain this to me.

S154: The intersection between the first function and the second function is higher than that between the first and the third or between the second and the third. They do not all intersect at the same point.

28:00 I155: What does that mean, from the point of view of ...

M/I S156: That not always when they are equal, there is the change.

I157: [repeating, in a low voice] not always when they are equal there is ... [And returning to his regular voice] Which change?

S158: Here, for example, [points to screen] when they are equal there's the change; that means there are less zebras and more lions.

I159: At which point do you mean?

S160: This.

I161: That's between the lions and the zebras? Am I right?

S162: Yes. But it's not the same point between the lions and the eagles.

I163: Ok. then, let me repeat my question: Is there a point, a time when the three populations are equal?

S164: Yes.

I165: Which time is this?

S166: 5 years.

I167: Do you want to check this?

29:00 S168: If I'm not wrong, in fact, ...for the eagles that's ... Ah! they're equal at many points.

I169: At many points? For example, where else?

S170: Uuh ... until the graph, until 400, in fact until the image is 400 they are equal at many points.

I171 Until the number ...

S172 Until the image is 400.
I173: How do you decide they're equal?
S174: I see that the images are identical; the y is identical.
I175: The y is identical. Show me where, for example.
S176: 400. It's not the same year, only the population is equal.
I177: Ah! Not in the same year.
S178: The population is equal, the year is not.
I179: Fine! Now, is there really one year when all the populations are equal?
S180: 5
I181: At 5.
30:00 S182: But wait, ... No, ... [Looks at T, for a second, uncertain]
I183: Yes or no?
S184: All that's small, around 7. I suppose. Also here, there are all three equal.
I185: But here, earlier, you said something that the populations are equal, say 100, 150, but not in the same year. That's what you said.
S186: I have to check; just a second; yes, this is not the same year. Only at 5 its also the same year and the same population.
I187: At five it's the same year. ... How does one see this at 5?
S188: One, I see it 'by eye'. Two, I know. And ... [Looks at the screen.] No! That's a mistake. But at 5 not all the populations are equal.
I189: Anyway not?
S190: It's a mistake; only the zebras and lions are equal.
31:00 I191: And what about the eagles?
S192: The eagles intersect somewhere else.
I193: What happens to the eagles at 5?
S194: Eagles, ... At 5, ... 350.
I195: About; can one check that?
S196: Yes. [Uses 'walk on graph'.] 375.
32:00 I197: And the others were 300, right? That is there were after 5 years more eagles. Now, let me come back to my original question: Is there a time when all three populations are equal?
S198: That also the time is equal? No, I don't think so.
I199: Ok. Fine, S. It was a pleasure to work with you.

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