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ABSTRACT

This paper addresses the evidence that secondary school students use to imply that they understand something in mathematics. As part of a larger study, an open-ended questionnaire probing several aspects of metacognitive self-knowledge, was administered to students (N=72) in four schools. Two previously identified types of understanding were identified in the students' responses: (1) instrumental (knowing how to do a piece of mathematics) and (2) relational (knowing why it works). Analysis of the data revealed that students who associated understanding with explaining also reported engaging in frequent mathematical discussions with other students. This result suggests a connection between metacognitive functioning and social interaction consistent with Vygotsky's views on learning. Observations of students in one of the classrooms participating in the study are used to add depth to the questionnaire data and suggest implications for teaching. Contains 17 references. (DDR)

How Do You Know When You Understand? Using Explanation to Monitor and Construct Mathematical Understanding

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Abstract. Knowing-when-you-understand is an important element of metacognitive knowledge, vital for keeping track of progress while studying a new concept or working on a mathematical task. This paper deals with the evidence that secondary school students use to infer that they understand something in mathematics. As part of a larger study, an open ended questionnaire, probing several aspects of metacognitive self-knowledge, was administered to 72 students in four schools. Two previously defined types of understanding (Skemp, 1987) were identified in the students' responses: *instrumental* (knowing how to do a piece of mathematics) and *relational* (knowing why it works). One response category consistent with relational understanding, and indicating high quality metacognitive knowledge, described evidence of understanding as being able to explain ideas to another person. Closer analysis of this and other data revealed that students who associated understanding with explaining also reported engaging in frequent mathematical discussion with other students. The latter result is of particular interest, as it suggests a connection between metacognitive functioning and social interaction consistent with Vygotsky's (1978) views on learning. Observations of students in one of the classrooms participating in the study are used to add depth to the questionnaire data, and suggest implications for teaching.

Metacognition, or knowledge about and control over one's own cognitive processes, is often considered to be critical to effective mathematical thinking and problem solving (Garofalo & Lester, 1985; Schoenfeld, 1992; Silver & Marshall, 1990), and the ability to monitor one's learning and problem solving behaviour distinguishes novices from experts in the domain (e.g. Schoenfeld, 1987; Venezky & Bregar, 1988). Knowledge about one's state of understanding can influence metacognitive control decisions either during initial learning of a concept or procedure, or while working on mathematical tasks that apply the learned procedure. For example, detecting a lack of understanding could signal the need for further study of the material to be learned, or trigger a decision to review one's working on a problem. Thus the role of understanding in driving metacognitive activity makes it important for mathematics students to be able to recognise when they do, and do not, understand.

Some of the more successful approaches to improving students' metacognitive capabilities have been based on Vygotsky's (1978) sociocultural theory of learning, which claims that higher mental processes have their origins in social interactions with either expert adults or peers. Students' individual self-knowledge and self-regulatory capacities may be extended if they initially operate within a zone of proximal development, where interaction with others elicits their emerging intellectual skills. The role of the teacher in providing *adult guidance* has been exemplified in Schoenfeld's (1985) work with college level mathematics students. The teacher provides expert scaffolding by structuring the task so as to support the learner's efforts, while pressing for increasingly complex strategic behaviour to prepare the learner for independent performance. As the learner begins to direct his or her own thinking, the teacher is able to relinquish the "expert" role.

Students may also be able to scaffold each other's thinking during collaborative problem solving; however, less is known about the processes of *peer collaboration* that might contribute to metacognitive development. It is possible that collaborative interaction, during which students propose and defend their own ideas and explore their partners' reasoning and viewpoints, may create a bi-directional zone of proximal development (Forman, 1989; Forman & McPhail, 1993) that enriches and extends the thinking of all participants.

The general aim of the research on which this paper is based is to identify the features of adult guidance and peer collaboration that help secondary school mathematics students to develop metacognitive knowledge and control. The paper focuses specifically on the role of mathematical understanding in guiding metacognitive activity, and uses data from the study to examine two questions:

1. What evidence do students use to decide whether or not they understand something in mathematics?

2. What classroom practices help students to monitor their understanding so that they can effectively regulate their mathematical thinking?

The first part of the paper addresses the above questions by reporting on students' responses to questionnaires that investigated the nature of their self-knowledge and their perceptions of school mathematics practices. As the results suggest a connection between viewing understanding as the ability to explain ideas to another person, and learning activities that involve mathematical discussion between peers, the last part of the paper draws on classroom observational data to illustrate how the students explained their thinking to each other in order to achieve understanding. Before describing the conduct and results of the study, however, it is necessary to consider the nature of mathematical understanding itself.

MATHEMATICAL UNDERSTANDING

Hiebert and Carpenter (1992) have presented a framework that describes mathematical understanding in terms of the structure of an individual's internal knowledge representations. They define understanding as "making connections between ideas, facts, or procedures" (p. 67), where the extent of understanding is directly related to the characteristics of the connections. In considering these connections, it is helpful to distinguish between two kinds of mathematical understanding: instrumental and relational (Skemp, 1987). Instrumental understanding is knowing *what* to do in order to complete a mathematical task, while relational understanding is knowing both what to do and *why* the particular piece of mathematics works. The actions of students with instrumental understanding are driven by the goal of getting the correct answer. Because these students learn mathematics as a set of fixed, minimally connected rules whose applicability is limited to a specific range of tasks, they cannot adapt their mental structures to solve novel or non-routine problems. On the other hand, students who have relational understanding construct richly connected conceptual networks that enable them to apply general mathematical concepts to unfamiliar problem situations.

Because "understanding" cannot be observed directly it is difficult to determine the kind of understanding that a student possesses. Skemp (1987) addresses the issue of evidence by describing abilities that correspond to the different kinds of understanding:

> **Instrumental understanding** [is evidenced by] the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.
> **Relational understanding** [is evidenced by] the ability to deduce specific rules or procedures from more general mathematical relationships (p. 166).

However, the problem of inferring students' state of understanding is not solved by these formulations, as performance on a mathematical task is not a reliable indicator of the kind of understanding to which students have access. (As an illustration, consider the possibility that a student who possesses relational understanding may still demonstrate instrumental functioning on a routine task.)

Chi, Bassok, Lewis, Reimann and Glaser (1989) have proposed that additional evidence of understanding can be found in the explanations that students generate while learning from worked examples. As well as confirming that understanding exists, self-explanations have the advantage of revealing the processes that create understanding. Chi et al. obtained verbal protocols from eight undergraduate students while they studied worked-out examples of mechanics problems. Differences between the protocols of Good and Poor students (a post hoc classification based on performance on isomorphic and far-transfer problems attempted after the example study) highlighted two mechanisms through which understanding took shape: Good students produced more *explanations* and more *monitoring statements* when studying examples than did Poor students. Good students' self-generated explanations elaborated on and justified the actions contained in the examples by specifying how, and under what conditions, principles were applied—information that is essential for relational understanding, but typically missing from worked examples in mathematics. When monitoring their state of understanding Good students were also more likely than Poor students to detect comprehension failure, and used this signal to guide subsequent self-explanations.

The various forms of evidence discussed above might allow teachers or researchers to assess the extent of students' understanding, but how do students themselves decide whether or not they understand? This question, together with several others probing metacognitive knowledge, beliefs about mathematics, and perceptions of school mathematics practices, was included in questionnaires administered to a group of secondary school students. The next section reports on the evidence these students use to infer that they understand, and examines links between metacognitive knowledge-of-understanding and the learning activities in which the students participate.

THE QUESTIONNAIRE STUDY

Method

Subjects

One mathematics class from each of four secondary schools participated in the first year of a two year study investigating metacognitive development in senior students. Three schools were located in Brisbane (two Government schools and one independent school) and one in a provincial city (independent). The sample consisted of 72 students: three Year 11 classes (one Mathematics A, two Mathematics B) and one Year 12 class (Mathematics C).

Procedure

Two questionnaires were administered by the researcher during regular mathematics lessons. The *Beliefs Questionnaire* consisted of statements to which students were asked to respond on a four or five point Likert scale. Most statements were based on those found in similar instruments used by Clarke, Waywood and Stephens (1993), McDonagh and Clarke (1994), and Schoenfeld (1989), while others were constructed for the purpose of the present study. The questionnaire was divided into four sections: (1) attributions for success and failure; (2) beliefs about mathematics; (3) perceptions of classroom practice, and (4) mathematics achievement and perceptions of ability and effort. The *Self-Knowledge Questionnaire* contained open-ended questions probing students' metacognitive knowledge; for example, *What do you do when you are stuck on a problem? What kinds of problems are you best at? Why? How do you know when you understand something in maths?* Some items were drawn from Schoenfeld's (1989) questionnaire, while others were constructed from Garofalo's (1987) suggestions for questions that teachers could put to their students to help develop their metacognitive awareness.

Results

Of the questionnaire data, only that part concerning students' perceptions of "understanding" and classroom learning activities is dealt with in this paper.

Knowing when you understand

Students expressed overwhelming agreement with the Beliefs Questionnaire statement *The best way to learn maths is to make sure you understand why things work* (Strongly Agree 63%, Agree, 35%), but only lukewarm support for the contrasting proposal that *The best way to learn maths is to memorise all the formulae* (Strongly Agree 17%, Agree 43%). Thus the majority of students seemed to believe that understanding is important in mathematics. However, what they mean by "understanding" only becomes clear when their responses to the Self-Knowledge Questionnaire are examined. Because this instrument invited open ended responses, categories were created to allow similar responses to be identified and grouped. The following categories (arranged in increasing order of metacognitive quality) emerged from students' responses to the question *How do you know when you understand something in maths?*

	Category	Examples
I	Correct answer	• When I get it right. • You can do heaps of them without mistakes. • You are able to go and do questions that are the same.

II	Affective response	• I get interested. • I feel confident when doing it. • I have a good feeling.
III	Makes sense	• It fits in with my previous knowledge. • When it makes sense, and I'm not asking myself why it is so. • You realise *why* you use the formula, what reasons.
IV	Application/transfer	• When I can apply it to something else outside school. • When I can understand a complex problem and do all the related problems. • You can do complicated problems doing the basic things you've learned.
V	Explain to others	• When I can explain it to other people without confusing myself. • I can explain the theory to other students. • I can explain ideas to other people and know they understand what I'm talking about.

Students whose responses fell into Category I (correct answer) offered the kind of evidence consistent with instrumental understanding. Category II responses (affective) may also point to instrumental understanding if confidence and enjoyment are the immediate rewards for obtaining the right answer. While responses in Category III (makes sense) clearly imply relational understanding, the evidence here is not as specific as that mentioned in Categories IV (application/transfer) and V (explain to others), both of which refer to an observable product or process from which relational understanding can be inferred.

Response frequencies and proportions for each of the five categories relating to evidence of understanding are shown in Table 1. Three-quarters of the sample claimed that they knew they understood something in mathematics if they could do the associated problems and get the correct answer (Category I), a result which suggests these students had quite poor metacognitive knowledge, as it is possible to apply a learned rule to solve a problem without understanding why the rule works or how to use it in unfamiliar situations. The most sophisticated form of self-knowledge was displayed by the seven students who knew they understood a mathematical idea when they could explain it to another person (Category V). This response category is especially interesting because explaining, although described here as a product of understanding, was previously identified as a learning process, guided by metacognitive self-monitoring, that can generate relational understanding in the first instance (Chi et al., 1989). The question of learning processes is taken up in the next section, which considers students' reported participation in classroom activities, and how those activities may be related to the perception of understanding as the ability to explain.

Table 1. Evidence of Understanding Reported by Students

	Category	Frequency	Proportion
I	Correct answer	55	.76
II	Affective response	7	.10
III	Makes sense	12	.17
IV	Application/transfer	7	.10
V	Explain to others	7	.10

Notes. 1. Frequencies represent the numbers of responses in each category. Proportions were calculated as frequency + sample size (n=72).

2. The sum of frequencies exceeds 72, and the sum of proportions 1.00, because students' open-ended responses could contain evidence belonging to more than one category.

Learning activities associated with understanding-as-explaining

In Section (3) of the Beliefs Questionnaire students were asked to indicate how often they were likely to be engaged in the following activities when they were doing maths at school:

1. talking about maths to the teacher
2. talking about maths to other students
3. copying notes from the blackboard
4. working on my own
5. doing problems from the textbook
6. listening to the teacher
7. listening to other students.

Table 2. Classroom Learning Activities

Learning Activity	Frequency (Proportion)				
	Always	Often	Sometimes	Seldom	Never
Talking about maths to the teacher	**2** (.03)	**13** (.18)	**28** (.39)	**26** (.36)	**3** (.04)
Talking about maths to other students	**12** (.17)	**24** (.33)	**30** (.42)	**6** (.08)	**0** (.00)
Copying notes from the blackboard	**13** (.18)	**19** (.26)	**26** (.36)	**13** (.18)	**1** (.01)
Working on my own	**2** (.03)	**25** (.35)	**36** (.50)	**8** (.11)	**1** (.01)
Doing problems from the textbook	**6** (.08)	**38** (.53)	**25** (.35)	**2** (.03)	**1** (.01)
Listening to the teacher	**9** (.13)	**38** (.53)	**23** (.32)	**1** (.01)	**1** (.01)
Listening to other students	**8** (.11)	**21** (.29)	**35** (.49)	**6** (.08)	**2** (.03)

Note. Frequencies represent the numbers of students who responded. Proportions were calculated as frequency ÷ sample size (n=72).

Table 2 shows the frequencies and sample proportions for the range of responses from which students could choose (Always, Often, Sometimes, Seldom, Never). As it was expected that all these activities would play some part in students' classroom experience, it was not surprising that responses from the whole sample tended to cluster around the Sometimes and Often anchors (see Table 2).

However, a different perspective could emerge if one asked whether any of the learning activities (Beliefs Questionnaire) were differentially associated with the various perceptions of understanding (Self-Knowledge Questionnaire), particularly understanding-as-explaining. Response patterns of the seven students who judged their state of understanding by the ability to explain (hereafter labelled "explainers") were therefore analysed separately and compared with the distributions that occurred within the whole sample. It was not appropriate to carry out chi-square tests to measure the degree of association, as expected frequencies fell below permissible levels (Minium, 1978). Nevertheless, an inspection of the proportionate distributions suggested an association that merits closer attention.

Table 3. Association Between Learning Activities and Understanding-as-Explaining ("Always" Responses)

Learning Activity	Expected Proportion (Whole Sample)	Obtained Proportion (Explainers)
Talking about maths to the teacher	.03	.00
Talking about maths to other students	.17	.57
Copying notes from the blackboard	.18	.00
Working on my own	.03	.00
Doing problems from the textbook	.08	.00
Listening to the teacher	.13	.00
Listening to other students	.11	.43

Table 3 shows expected and obtained proportions (i.e. for the whole sample and explainers respectively) for "Always" responses to the range of nominated learning activities, as it was within the latter response category that the most striking differences emerged. Each expected proportion was calculated by dividing the "Always" response frequency for the whole sample by the sample size (n=72). Obtained proportions were calculated by dividing the "Always" response frequency for the explainers by number of explainers (7).

From Table 3, it is clear that explainers were much more likely than other students to state that they *always* spent time *talking about maths to other students* (four out of seven students, or 57%, compared with 17% within the whole sample) and *listening to other students* (three out of seven students, or 43%, compared with 11% in the whole sample). Additional analysis of "Always" response patterns for the other categories of understanding (Categories I to IV) found no comparable differences between the observed and expected distributions of responses for these, or any other, learning activities. The possibility of a connection between mathematical discussion and testing one's understanding via explanation is explored more fully in the next part of the paper.

Summary

Students' written responses to the question *How do you know when you understand something in maths?* were grouped into five categories, corresponding to different levels of metacognitive knowledge. The category of most interest referred to the ability to explain to others, a process that not only assesses one's understanding, but may also play a part in creating understanding. Those students who were labelled as explainers were more likely than other students to report that they always spend time in class discussing mathematics with their peers. Although it is unwise to draw firm conclusions from the testimony of such a small number of students, these responses are consistent with the view, derived from sociocultural models of learning, that metacognitive functioning can be developed through social interaction, particularly that which occurs between peers. The next part of the paper draws on extensive observation of one of the classrooms participating in the research study to show how students explained their ideas to each other as they monitored and tested their understanding.

Classroom Observation

Two of the students referred to as explainers were members of a Year 12 Mathematics C class that was observed and videotaped for 90 minutes per week over a period of thirteen weeks. Although they were the only students in the class who claimed they tested their understanding through explanation, it was apparent that explanation and justification of ideas featured strongly in classroom social interactions.

The teacher regularly asked students to work together on problems that were designed to develop understanding of new concepts. However, students frequently initiated discussion between themselves without the teacher's prompting, and it is these spontaneous interactions that will be illustrated here. They occurred in three settings: the study and interrogation of worked examples, whole class discussion led by the teacher, and individual practice on problems. Although many examples of interactions in these settings were observed, those that follow are taken from two lessons on simple harmonic motion. (The speakers include the two explainers, "Rob" and "Belinda".)

Studying worked examples

The teacher believes in the importance of allowing time in class for students to study worked examples so that they learn to find their way independently through mathematical text. The examples, which also introduce students to the formal reasoning involved in applying new concepts, then become the subject of whole class discussion. Although students initially read in silence, after a short time they invariably turn to their neighbours either to seek clarification or to confirm their individual interpretations of the example.

In a lesson introducing the principles of simple harmonic motion, the teacher asked the class to read an example involving a disc of radius 0.6m that rotated, with its diameter pointing directly at the sun, at a rate of one revolution per second. The example showed how to calculate the position, velocity and acceleration of the shadow cast by a point on the rim of the disc, using the equations $x = r\cos\omega t$, $\dot{x} = -r\omega\sin\omega t$, and $\ddot{x} = -r\omega^2\cos\omega t$ respectively (r is the radius, ω the angular velocity and t time). After a few minutes of silent reading, students began to form pairs and small groups to ask each other questions about parts of the example they did not understand; that is, to *explain* the example to each other. The following snatch of conversation was overheard as a group of three boys read through the part of the example shown in Figure 1:

Duncan:	Why is omega [the angular velocity] 2π?
Rob:	Because one revolution per second is 2π per second.
Ben:	Where does 1.2π come from?
Duncan:	It's 0.6 [the radius] times 2π.

$$(i)\ x = r\cos\omega t = 0.6\cos 2\pi t = \textit{displacement}$$

$$(ii)\ \dot{x} = -r\omega\sin\omega t = -1.2\pi\sin 2\pi t = \textit{velocity}$$

Figure 1. A worked example of simple harmonic motion calculations

In these brief explanatory exchanges the boys have elaborated on the solution steps in much the same way that a student working alone might generate self-explanations to overcome the incompleteness of an example. But instead of each silently asking himself for an explanation when understanding failed, they asked each other.

Student-student talk during whole class discussion

During whole class discussion, the teacher expects students to clarify and justify the ideas they contribute, as well as critique the contributions of other students. In contrast with traditional classrooms where such public talk must be channelled through the teacher, students in the classroom under study frequently direct their comments to each other without the teacher's mediation, thus sparking the kind of spontaneous argumentation that might otherwise be restricted to more private, small group interactions. The following instance comes from a lesson introducing Hooke's Law.

The class had again interrogated a worked example demonstrating how to describe the motion of a mass executing simple harmonic motion while suspended from a spring. During the ensuing whole class discussion, some students questioned the change of notation from $x = r\cos\omega t$ (as used in the lesson mentioned above) to $x = a\cos nt$ (a more general form that applies to all kinds of simple harmonic motion, not just that derived from a projection of uniform circular motion on a diameter of the circle). Rather than providing a rationale, the teacher withdrew from the discussion to allow students to resolve the issue for themselves:

Rob:	Why did they suddenly skip to a ?
Belinda:	Because x is equal to $a\cos nt$.
Ben:	Why use a and n, when we have the exact same formula with r and ω ? Does it refer to ω involving radians?
Rob:	On this side [referring to the handout containing the example—also used in the lesson mentioned earlier] they said $x = r\cos\omega t$, on the other side $x = a\cos nt$.
Belinda:	Excuse me, I have a point to make here! You can't always use r because—(to teacher) Oh, sorry! (Teacher indicates she should continue.) I don't know if anyone will agree with me—because you're not always using a circle, it's not always going to be the radius.
Rob:	Radius, yeah.

Belinda: So the amplitude's not always the radius.

By ceding control of the debate the teacher provided another opportunity for students to ask for, and receive, explanations from each other until they were satisfied that they understood.

Informal discussion while working on problems

In the observation classroom it is rare to find students working individually on textbook problems, as most cluster into informal groups so that they can discuss their progress with each other. Although such interactions often involve little more than periodic checking of results and procedures, the discussion reaches a deeper level if a student is unable to resolve a difficulty or if a disagreement occurs. One such instance occurred towards the end of the Hooke's Law lesson mentioned above. Rob, Ben and Duncan had been working together on the task shown in Figure 2.

A mass M is attached to the end of an elastic of natural length a and reaches its equilibrium position when the string is extended by l. The mass is then displaced downwards a further distance d and released. Find the period and amplitude of the motion for each set of data:

(a) $M = 6$ kg, $l = 1$ m, $d = 0.5$ m

(b) $M = 1$ kg, $l = 0.4$ m, $d = 0.3$ m

(c) $M = 10$ kg, $l = 0.5$ m, $d = 0.1$ m

(d) $M = 10$ kg, $l = 0.5$ m, $d = -0.1$ m

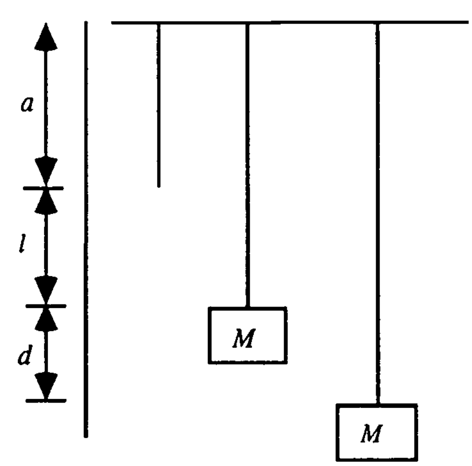

Figure 2. The Elastic Problem

After the trio had completed parts (a) to (c), Ben noticed the unusual conditions for part (d), in which the initial displacement of the mass is negative rather than positive (d = - 0.1):

Ben:	What do you do for the next one?
Rob:	What's this?
Duncan:	The next one?
Ben:	It's *negative*.
Duncan:	Does that mean—that it—
Ben:	The amplitude's still got to be—
Duncan:	They've pushed it up then (using his hand to indicate upward displacement).

In the discussion that followed the boys clarified their understanding of "amplitude" and agreed that it would be unchanged from part (c). Then, instead of simply carrying out the calculations for (d), they compared the problem conditions for (c) and (d) in order to decide which aspects of the motion would be the same and which different in these two situations—actions that suggest they were striving for relational understanding. A mutually agreed representation of the problem was only established after vigorous debate in which the boys used explanations to challenge each other while testing their own understanding, as the following edited transcript shows:

Rob:	Oh, the same—it's the same: k equals, let me guess ...
Ben:	(pause) No ... no.
Duncan:	The only thing that's going to change is the amplitude.
Ben:	It doesn't change the amplitude.
Duncan:	Yes it does!
Ben:	(after a slight pause) How?
Rob:	Because that's all that changes—the acceleration's the same, because it's the mass that—
Ben:	The amplitude doesn't *change*—
Duncan:	Yes it does!
Ben:	*How?*
Duncan:	See, if you pull it down, it depends on how much you pull it down. You pull it down a *little* bit—
Rob:	—it'll be a *small* amplitude.
Ben:	No no, but isn't the amplitude the amount away, either up or down, from the stationary point? (uses hands to demonstrate)
Duncan:	Yeah—
Ben:	If it goes up point one it's not going to go down point one.
Duncan:	No, I know, but it *should* be. If it was a perfect system.
Ben:	(expression of sudden understanding on his face) No, it's going to be exactly the same, as the last—!
Duncan:	(pause, thinks) Oh, of course, that's just negative (pointing to $d = -0.1$).
Rob:	Why are we doing it? But the, the other thing, the period's going to be the same.

Ben: (confident now) The *period's* going to be the same. *Everything's* going to be the same.

Perhaps the most crucial contributions to the discussion were made by Ben, whose insistence on asking "How?" caused all three boys to critically examine their own and each other's explanations.

CONCLUSION

The research reported in this paper drew on questionnaire and classroom observation data to investigate secondary school students' metacognitive knowledge-of-understanding. One of the aims of the research was to discover the sources of evidence that students use to monitor their state of mathematical understanding. Students' questionnaire responses indicated that the majority relied on fallible information concerning their ability to get the correct answer to a problem. However, a small number of students applied a more exacting criterion, judging their understanding by their ability to explain mathematical ideas to another person. In fact, giving explanations may have dual benefits: as well as establishing one's current level of understanding, the process of explaining can also create understanding by connecting new ideas to existing networks of knowledge.

The second question addressed by the present research concerned classroom practices that might develop metacognitive monitoring of understanding. Significantly, a high proportion of the students dubbed "explainers" also reported that they usually spent time in class talking to other students about mathematics. Thus the questionnaire results suggested that encouraging students to articulate and explain their thinking to each other could help them to become more aware of their state of understanding.

Observation of one of the classrooms participating in the study identified three contexts that provide opportunities for peer discussion beyond those that arise during planned small group work: studying worked examples, whole class discussion, and individual practice on problems. One might expect to find little student talk occurring within these settings, as they usually involve either teacher-student interaction or no interaction at all.

However, the vignettes presented earlier showed this not to be the case. What made the difference? Certainly, the teacher capitalised on his students' natural desire for social interaction and steered their behaviour towards productive collaboration. But he also took a more active role by establishing classroom social norms that emphasised sense making and the communication and justification of mathematical ideas, so that students were expected to convince each other, as well as the teacher, of the validity of their assertions. Impromptu peer discussion flourished under such conditions.

The theoretical question as to how social interaction between peers develops metacognitive habits of mind still needs to be considered; as noted earlier, it is not yet clear how such interaction might create a student-student zone of proximal development that nurtures developing intellectual skills. One possible mechanism that has emerged from this study involves *explanation*. Earlier research showed that individual students working alone regulate their mathematical thinking by monitoring the state of their own understanding and generating self-explanations if comprehension fails (Chi et al., 1989). In the present study it was found that students working *together* monitor and critique their *partners'* thinking and ask *each other* for explanations if they do not understand. If students are encouraged to engage with each other's reasoning by eliciting and offering explanations until mutual understanding is achieved, then these social processes of argumentation may be internalised as self-interrogation and self-explanation. Peer discussion therefore makes visible the processes that individuals could use to monitor and extend their own understanding.

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