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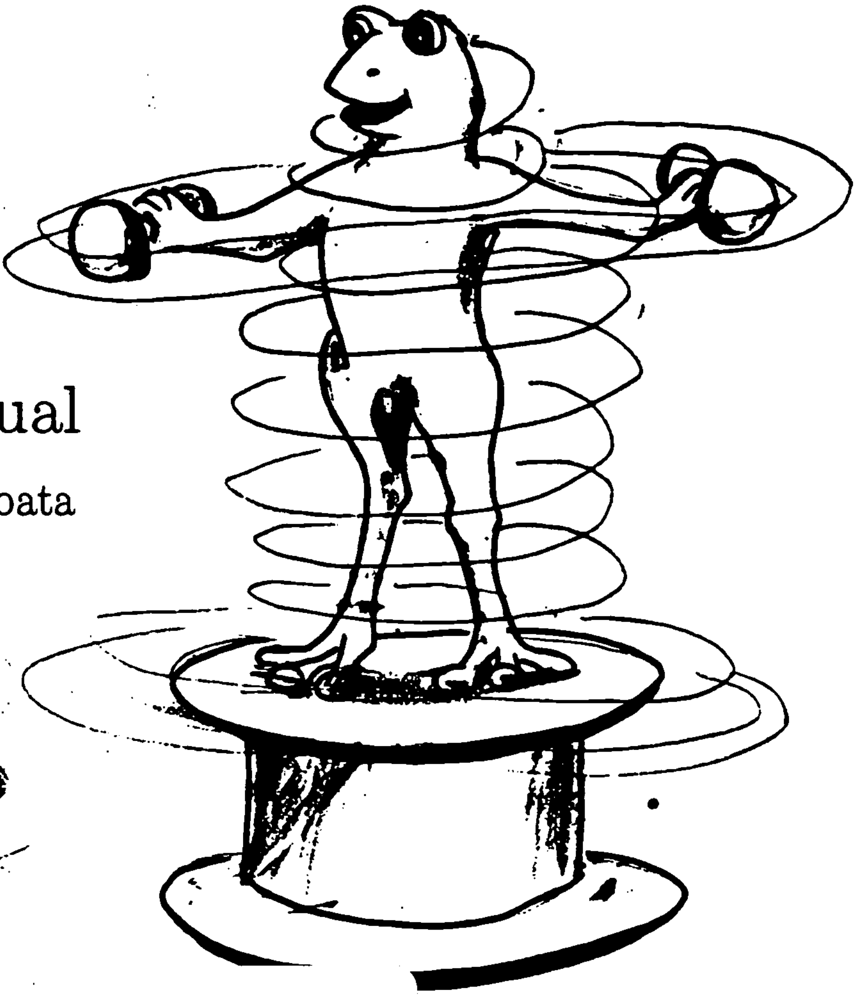
ABSTRACT

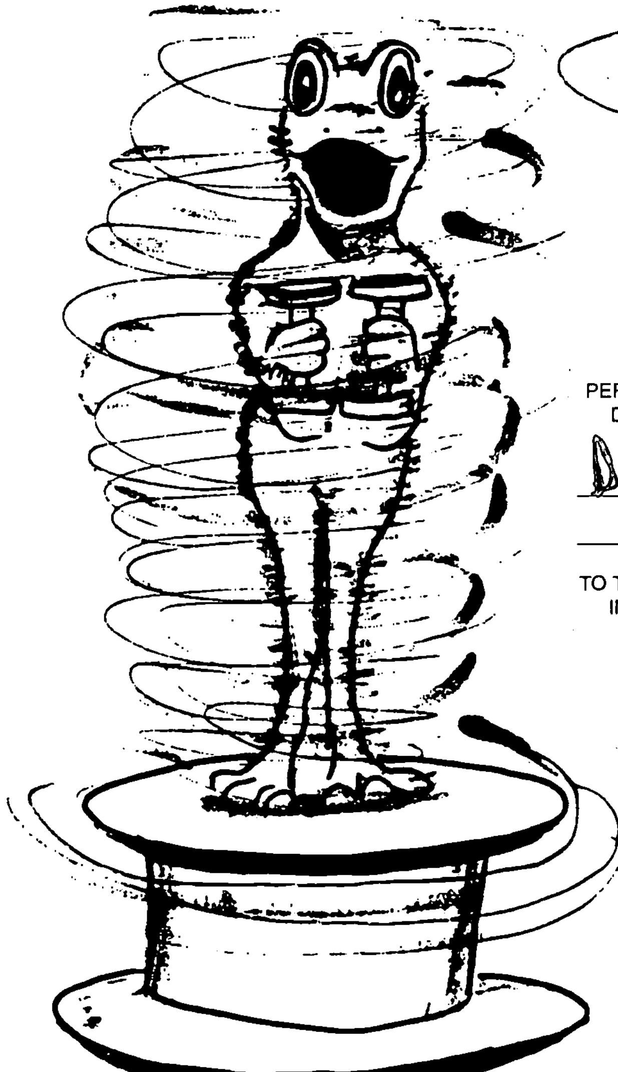
This document is the laboratory manual for the Physics 152 course at Purdue University. It includes a laboratory introduction, hardware and software guide, and laboratory report guide. Labs include: (1) "Measurement Uncertainty and Propagation"; (2) "Introduction to Computer Data Acquisition and Relationships between Position, Velocity, and Acceleration"; (3) "Newton's Second Law, Work, and Kinetic Energy"; (4) "Graphical Analysis and Least Squares Fitting"; (5) "Conservation of Mechanical Energy"; (6) "Impulse and Momentum"; (7) "Rotation Dynamics"; and (8) "Simple Harmonic Motion and the Torsion Pendulum". (JRH)

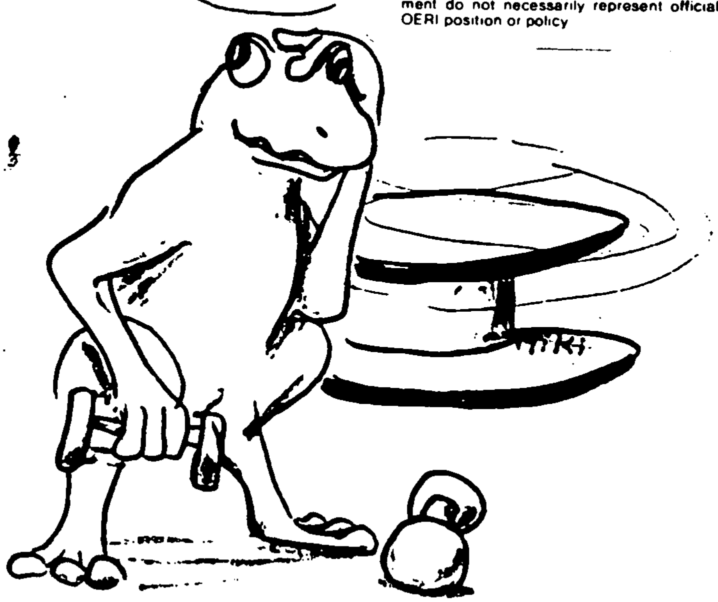
Eighth Edition

Physics 152 Laboratory Manual

MacIsaac • Pekarek • Shibata







Department of Physics
Purdue University

Physics 152
Laboratory Manual

Eighth Edition

Dan MacIsaac
Thomas M. Pekarek
Edward I. Shibata

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Dan MacIsaac – Thomas M. Pekarek – Edward I. Shibata

Cover art by Miki McDonagh. The cover art depicts a sequence of three events involving a frog on one of the rotating platforms used in Experiment E5.

- Initially the frog rotates slowly on one of the rotating platforms.
- When the frog pulls the barbells close to its body, angular velocity increases in order to conserve angular momentum.
- Finally, the frog is quite dizzy, and may have fallen off the rotating platform. One should sit, not stand, on the rotating platform when performing Experiment E5!

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Acknowledgments

We wish to acknowledge the many contributions of all of our students to this curriculum — many students have given us suggestions and comments which have improved the activities performed as part of this course. Student assistance has been generous and has been the main contributor to improvement in PHYS 152L.

We also want to acknowledge the extensive contributions of PHYS 152L employees to this curriculum — the Development Crew, the Instruction/Grading Crew, the Graduate Teaching Assistants of Physics 152L, and the instructional faculty of Physics 152. In particular, we have benefitted from written critiques by Lucy Bekofske, Sarah Bekofske, Rachel Bigsby, Dan Bodony, Jeff Crowder, Binh Do, Christopher Dunn, Cassandra Forthofer, Kathleen Horgan, Kathleen Falconer, Richard Flack, Sarah Fuchs, Aggie Hamburger, Steve Howell, Gretchen Hygema, Mohammed Ibrahim, Mike Idlewine, John Judd, Kirby Lee, TakHee Lee, Erich Lincoln, David Loy, Belinda Marchand, John Moffett, Andrew Oxtoby, Monica Perez, Scott Sadowski, David Schleef, Scott Thompson, Waleska Soto, Jessica Tomochek, Shu-Ju Tu, Luiz Valentini, Uma Vohra, Hans Wwissker, and Jianan Yang.

In the spring and summer of 1996 herculean efforts were made to prototype, construct, test and integrate new materials, World Wide Web reference materials, and apparatus by the following individuals: Natasha Bralts, Christos Christodoulou, Jeff Crowder, Kathleen Falconer, Elizabeth Hingley, Mike Howard, Steve Howell, Tom Jagatic, Tak Hee Lee, Andrew Lewicki, Eric Lincoln, Belinda Marchand, Michael Michaelides, John Moffett, Scott Sadowski, Jessica Tomochek, Michael Walker, Tracy Wiegand, Dick Webeck, and Jim White. We are thankful to Kirk Arndt of the Purdue High Energy Physics group for showing us how to improve the mounting of strain gauges on our experiment E4 force probes.

We give special thanks to Kathleen Falconer, who has served as the graduate student aide in Physics 152L for several semesters. In addition to serving as one of the laboratory instructors, she has been heavily involved in the development of hardware, software, and teaching materials. Special thanks also go to Tracy Wiegand, who is the Laboratory Technician for the Physics 152 laboratory. Kathleen and Tracy have been responsible for the day-to-day operation of the laboratory and have implemented many improvements of the laboratory and its teaching activities. Many of their suggestions have been incorporated in this laboratory manual and in the new hardware that has been designed and built for the laboratory.

Elton Graugnard helped to produce revised exercises for this edition of the laboratory manual. He and Cassandra Forthofer helped us with final editing of the manuscript. Cover art for this manual is by Miki McDonagh. Julie Peterson and Angie Fenter Halicki have done past cover art.

Several activities in this edition have been inspired by the *Workshop Physics* curriculum developed for use at Dickinson College in Carlisle, PA by P. Laws, R. Boyle, J. Leutzelschwab, D. Sokoloff and R. Thornton. We continue to be inspired by their work.

Finally, we wish to recognize the assistance and contributions of several organizations: the Purdue University Department of Physics, the Purdue University Computing Center, the Purdue University Class of 1941, the AT&T Research Foundation, the National Science Foundation, and National Instruments Corporation.

Comments

We welcome your written comments and observations regarding this manual or any part of the Physics 152 Laboratory curriculum, materials, and presentation. Please forward written comments by campus mail to Tom Pekarek (Department: PHYS, Building: PHYS) or by electronic mail to *pekarek@physics.purdue.edu* on the Internet. The Purdue Physics 152 Laboratory staff can also be reached through the World Wide Web at URL: *http://www.physics.purdue.edu/* under the links for courses or by surface mail in care of:

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West Lafayette, IN, U.S.A.
August 1996

READ ME FIRST !

Your fee statement shows the date on which you will perform your first Physics 152L experiment in Room 18 of the Physics Building. **Even though you are not scheduled to perform an experiment the first week, you should start your lab work in the very first week — there are prelaboratory and measurement analysis activities due when you first go to lab.** We suggest that you fill in the Physics 152L Assignment Schedule on page 5 as you find the due dates for Physics 152L assignments, and that you commence work on MA1 immediately.

There are two Physics 152L activities scheduled for you during the week of August 19–23, 1996:

1. A Measurement Analysis 1 (MA1) lecture. If your fee statement shows that your first PHYS 152L lab session is scheduled during August 26–30, your MA1 is scheduled on Thursday, August 22, 8:30-10:00 PM in CL50 Room 224. If your fee statement shows that your first PHYS 152L lab session is scheduled during September 2–6, your MA1 is scheduled for Wednesday, August 28, 8:30-10:00 PM in CL50 Room 224.

   Attending this lecture is optional, but the MA1 assignment is NOT optional and must be handed in at the start of your first lab for credit. If you cannot attend your scheduled session or want to review parts of MA1, you can see the essential points of it on the World Wide Web (Start at `http://www.physics.purdue.edu`, look under *Course Homepages*, go to *PHYS 152 Lab*, and finally go to *MA1*.)

   Your MA1 exercises are due at the start of your very first lab (E1). In addition, there are a set of prelaboratory exercises due at the start of E1, and one of the exercises require you to use plotting software, such as *KaleidaGraph* (preferred) or *Excel*. They are available at public Purdue University Computing Center (PUCC) labs throughout campus. We suggest PHYS 14, right next door to your Physics 152 lab — PHYS 18. A *KaleidaGraph Summary* is available on the World Wide Web. (Start at `http://www.physics.purdue.edu`, look under *Course Homepages*, go to *PHYS 152 Lab*, and finally go to *KaleidaGraph Summary*.)

   Prelaboratory exercises are due at the beginning of each of your scheduled experiments. These exercises are worth 25% of the credit for an experiment. Late prelaboratory exercises are worth zero points. Therefore, you should get busy on the Prelaboratory exercises for experiment E1 as soon as possible.

2. During August 19–23 (first week of classes) there will be PHYS 152L orientation sessions in Physics Room 18 to give you information on your Purdue University computer accounts and to familiarize yourselves with the laboratory. If your fee statement shows that your first PHYS 152L lab session is scheduled during August 26–30, your orientation session starts at the beginning of your normal 2-hour laboratory day and time. If your fee statement shows that your first PHYS 152L lab session is scheduled during September 2–6, then your orientation session starts one hour later than the starting time of your normal 2-hour laboratory day and time.

ACTIVITY	POINT VALUE	YOUR SCORE	WHERE PERFORMED	DATE PERFORMED	DATE DUE
Orientation	0		PHYS 18	Aug. 19-23	
MA1 *Measurement Uncertainty...* Worksheet (approx 2 hours)	20		CL50 224 8:30-10:00 pm Aug. 22, 28	*MA1 lecture attendance optional but strongly encouraged.	
E1 *Introduction to Computer Data Acquisition...* Prelab Questions (2 hours) Laboratory (2 hours) Report Writeup (3 hours)	35		PHYS 18		
E2 *Newton's Second Law...* Prelab Questions (1.5 hours) Laboratory (2 hours) Report Writeup (3 hours)	35		PHYS 18		
MA2 *Graphical Analysis and Least Squares Fitting* Worksheet (approx 2.5 hours) Requires Spreadsheet software	20		CL50 224 8:30-10:00 pm Sept. 11	*MA2 lecture attendance optional but strongly encouraged.	
E3 *Conservation of Mechanical Energy* Prelab Questions (2.5 hours) Laboratory (2 hours) Report Writeup (3 hours)	35		PHYS 18		
E4 *Impulse and Momentum* Prelab Questions (1.5 hours) Laboratory (2 hours) Report Writeup (3 hours)	35		PHYS 18		
E5 *Rotational Dynamics* Prelab Questions (1.5 hours) Laboratory (2 hours) Report Writeup (3 hours)	35		PHYS 18		
E6 *Simple Harmonic Motion-Torsion Pendulum* Prelab Questions (1.5 hours) Laboratory (2 hours) Report Writeup (3 hours)	35		PHYS 18		
Total Points	250				

Please Note: Prelaboratory Questions are due at the start of your laboratory session.
Measurement Analysis One (MA1) is due at the start of your scheduled Experiment 1.
Measurement Analysis Two (MA2) is due at the start of your scheduled Experiment 3 (requires computer plotting and spreadsheets).
All other assignments are due in to the drop slot outside PHYS 144 seven days after they are scheduled to take place.
Late assignments may be penalized or may not be graded. To make up activities, see your TA.

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Laboratory Introduction

Welcome to the Physics 152 laboratory (Physics 152L). It is important that you understand what your instructors are trying to teach you in these mechanics labs, and why they teach in the particular ways they do. Hence, we offer this explanation of the goals and philosophy of Physics 152L, along with some insights into some difficulties that some students experience.

During this semester your laboratory lecturers and instructors will help you meet the three major goals of this course.

1. Development of your laboratory skills. These skills are marketable, professional skills required of all practicing engineers and scientists. The skills you will practice and master include collecting, manipulating, critically analyzing and presenting data. You will prepare formal written reports upon your experiences. You will practice taking measurements, determining their uncertainties, working with modern computer assisted data acquisition and reduction, and working with a group of other researchers to collect, interpret and report data. Your learning will require repetitive practice with frequent feedback — as does developing any new skill. You will be required to practice using computer software to plot and analyze data – you must use a scientific plotting package and a spreadsheet in Physics 152; these will be discussed during the Measurement Analysis lectures.

2. Illustration of principles and phenomena described in Physics 152 lectures. You will be exposed to and have the opportunity to manipulate apparatus intended to provide insights into several of the major topics in Physics 152. This is the experiential part of the laboratory — here you have the opportunity to enrich your insights into various phenomena through guided exposure.

3. Promotion of conceptual change (changing the way you think) about the major topics of Newtonian mechanics.

These goals are listed in their order of difficulty. It is quite easy for you to learn to write reports and use apparatus, somewhat harder to learn and apply mathematical measurement analysis, and extremely difficult to change the way you interpret the world.

1 Physics 152L activities

The course grade is divided amongst eight activities: six laboratories and two measurement analysis assignments. Together, these are intended to represent a work load comparable to other first year lab courses with ten or so labs for the entire semester. The grade breakdown follows:

Activity	Points	Notes
MA1: Introduction...	20	worksheet due at start of E1.
E1: Laboratory Introduction and...	35	PLQs due at start of E1.
E2: Newton's Second Law...	35	PLQs due at start of lab.
MA2: Graphical Methods...	20	worksheet due at start of E3.
E3: Conservation of...	35	PLQs due at start of lab.
E4: Impulse & Momentum	35	PLQs due at start of lab.
E5: Rotational Dynamics	35	PLQs due at start of lab.
E6: Simple Harmonic...	35	PLQs due at start of lab.
Maximum possible points	250	

Here MA stands for Measurement Analysis, E*n* for Experiment *n*, and PLQ for Prelaboratory Questions.

1.1 Time requirements

Each laboratory is broken down into three major parts. First, you must answer a set of prelaboratory questions (PLQs) that must be prepared in advance and turned in at the start of your laboratory data collection session. The prelaboratory activities typically require 2–4 hours of effort preparing graphs, deriving equations, performing practice calculations and answering questions regarding the theory for each experiment. During the two-hour laboratory data collection session you will observe phenomena, manipulate your apparatus, collect data and ask questions in the laboratory room – PHYS 18. Finally, you will prepare a detailed laboratory report due SEVEN CALENDAR days later. The report typically requires about 2-5 hours of effort.

The breakdown of a typical laboratory report including the point assignment follows:

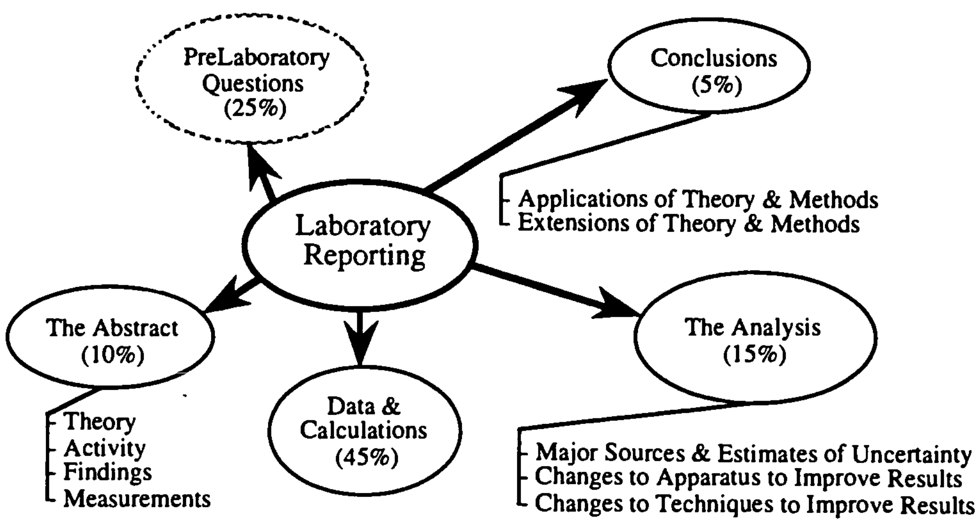


Figure 1. Breakdown of a Physics 152L Report.

The two measurement analysis exercises are designed to introduce and practice skills used throughout the experiments. Each exercise typically requires about two hours to complete. A one-hour optional evening lecture will review each assignment one week before it is due.

Note that you will require additional out-of-class time for your first session using the computer graphing and spreadsheet software, but later you will save time on all following activities involving plotting. The time required for the initial session varies widely depending upon personal computing background and experience, but most students require 1–2 hours for an initial session. Afterwards, high-quality graphs can be created, stored and printed out in less than 5-10 minutes each. Most students use their computer graphing and spreadsheeting skills in many of their courses thereafter.

1.2 Grading practice and philosophy

Physics 152L is a mastery course, and most students master the majority of the skills set forth in the curriculum; hence a typical semester average grade will be about 80%, with a standard deviation for student grades of less than 10%. Most students will do well. The most common problem preventing students from doing well is poor time management with subsequent poor preparation and low quality work.

Grading is done by several instructors — Graduate Teaching Assistants will grade the written and analytic portions of your laboratory reports, and undergraduate instructors will grade the remaining numerical calculations, prelaboratory questions and measurement analysis assignments. These people will all be present in PHYS 18 during your data collection session to answer questions you have regarding the exercises and grades, and the GTA will also be available in the Physics Learning Center and at other posted times for your assistance. Monitors will also be available in PHYS 14 and other Purdue computer labs to help with computer use.

1.3 Due dates

Physics 152L laboratory reports and measurement analysis exercises are due SEVEN CALENDAR DAYS after you have performed the experiment or the scheduled measurement analysis talk. *Those reports which fall due on a major holiday or an official university closure are due on the first official day that classes have resumed after the original due date.* Attach a green Physics 152L cover sheet (available in the lab or window of PHYS Room 144) to the front of your exercise or report and put it in the "Room 144 Drop Slot for Physics Lab Reports" located under the mailboxes across from Physics Room 149 before 10:00 p.m. of the day it is due.

If an experiment is missed for a valid reason (e.g., illness), you must give written documentation to your **laboratory** Graduate Teaching Assistant (GTA) within 10 days of the missed experiment and come to a mutual agreement about making it up. Late labs will be penalized. Unexcused absences may result in a final grade of incomplete for PHYS 152L. Check your lab record periodically (your GTA will post it in lab). If a discrepancy is found, resolve it with your GTA as soon as possible. At the end of the officially posted records check week, your laboratory record becomes final. Changing your lab grade will require a formal appeal after records check week is over.

Note that you must complete and turn in a satifactory report for all PHYS 152L activities or you may receive a grade of INCOMPLETE.

1.4 Pitfalls in PHYS 152L

Most students do well in PHYS 152L — average lab scores are about 80%, but some students still have difficulty with some activities. These events are fortunately rare, but you should be aware that they do occur and that with preparation you can avoid most of them. The most common problems (in order of prevalence) include:

1. Poor or no student prelaboratory preparation resulting in a delayed start and frequent delays during the experiment while students stop to read and interpret the manual.

2. Uneven progress where students seem inactive, aimless, or nonproductive for lengths of time. Lack of concurrent activity and adopted roles, or inability to interrelate in a worthwhile manner with partners.

4. Not submitting work on time or at all. (This will result in a grade of incomplete or F.)

5. Confusing, insufficient or misleading manual instructions or laboratory software.

6. Erratic computer behavior and associated delays.

7. Confusing, insufficient, or misleading instructor guidance.

These problems arise in most instructional laboratories and we update our materials, methods, and equipment every term to reduce these difficulties; therefore, we need your assistance locating and solving them.

We welcome your suggestions at any time concerning possible improvements to PHYS 152L. The current activities are the result of much student input. Please feel free to comment on these activities to your GTA or directly to the authors listed at the front of this manual.

2 Active learning in Physics 152L

In order to achieve real insights into Newtonian mechanics in the laboratory and change the way you think about the physical world, you must take responsibility for your own learning and instruction. If you do so, you will find Physics 152L to be intellectually stimulating, challenging and rewarding. Our experience with Physics 152L has been that students who take an active role in their learning not only enjoy their laboratory experience, but they get better grades.

Research into human learning and science education has clearly demonstrated that students learn by taking an active role in their own learning experiences. People learn by relating what they already know to new perceptions and experiences. People also learn by negotiating new understandings and relationships with themselves and with others. Real science is performed the same way — a community of scientists research and learn by experience and negotiation with their own beliefs and those of the scientific community. Science is not done by people working alone, nor is science worthwhile unless it is communicated to and interpretations are negotiated by a group of people.

Similarly, in Physics 152L we will make every effort possible to encourage you to think about, predict, observe, describe and explain mechanics phenomena. You will predict in writing what will happen before you go into the lab, and you will later describe in your own words what actually happened. You will negotiate meanings and interpretations with your fellow students whenever possible, even to the point of preparing group reports. You will explain and interpret measurements from the activities, and you will prepare recommendations and critiques of the measurement procedures and instruments basing your arguments upon numerical data. Whenever possible, you will be asked to explain overtly what you are thinking to make you re-examine your own interpretations and beliefs.

To become an active learner in Physics 152L, there are several things that you must do and several things that your instructors try to support during the activities.

1. **Know the goals of the activity, and constantly relate them to what you are doing.**

Each activity has clearly listed goals which state the most important skills and concepts you should be able to use at the end of the activity. These goals will be used to grade your efforts. These goals are also used in future activities (For instance, skills learned in the first activity will be used in all of the following activities in the course.) The goals are presented to you so that you can actively monitor and control your own learning. If you feel that you do not adequately understand the concepts listed as goals at the end of an activity, start asking questions and make an extra effort.

2. **Be prepared in advance for the activities. Get the most of your laboratory time. Tinker in the laboratory. Adopt roles with your partners.**

To be an active learner, you must take responsibility for arriving prepared at the laboratory. You will have very limited time actually working with the apparatus, and it is important that this time is not spent reading the theory and introduction to the experiment, figuring out how the apparatus works, etc. We have tried to encourage you to prepare by assigning 25% of the laboratory points for each experiment to a set of prelaboratory questions. These prelaboratory questions will review the theory and calculations that you will require during the laboratory session. They will also start you thinking about the major lab concepts examined in the lab, and will require you to make predictions regarding laboratory phenomena. *Prelaboratory questions are to be done before you step into the laboratory, and* *will be collected at the door when you enter. These questions will not be accepted after the start of the experiment.*

Reading the lab manual, closely examining the diagrams, thinking about and discussing the theory with others, and completing these prelaboratory questions before you arrive will prepare you well for the laboratory. When you are well prepared, you can then spend the maximum amount of time in the lab collecting and making sense of your data. You will have time to think about what you are doing and to discuss the experimental relationship behind the numbers with your partners and instructors. You will have adequate time to formulate and test hypotheses regarding unusual data and to compare your data to that of other students. You owe it to yourself and your partners to be prepared. You need your partners and they need your insight. You will learn much more together than you will individually.

Sometimes unprepared students run into difficulties when they cannot complete a part of an activity and are unsure what to do next. If you find that you are not getting anything done in lab — you have run into a stone wall or become stuck during an activity, do not hesitate to call on your TA for help. If you cannot proceed, do not waste valuable time — go on to another activity (say the next part of the lab) and do other necessary work while getting sorted out. Adopt laboratory roles with your partners (e.g., one person acts as computer operator, another as theoretician, and another as apparatus mechanic, etc.) so you will benefit from concurrent activity. If everyone has something to do, momentary confusion will not shut everything down. Check one another's work frequently and switch roles periodically.

3. **Discuss your lab work with your partners and other students. Ask your laboratory instructors questions regarding the material. Work with your peers — you are required to prepare group laboratory reports for all labs.**

Meaningful learning requires you to think, communicate, and negotiate. Scientific knowledge is useless unless it can be shared with, understood, and validated by other human beings (a scientific community). In the lab you have many different people with which you can discuss your interpretation of data and theory. The best people to speak with are fellow students, as they are thinking about the same things you are in much the same way you do. Purdue students have been selected for their academic excellence, and are intelligent people with worthwhile things to say. Thus, your partners in the lab are an invaluable source of expertise and critical thought.

If you require more assistance or further discussion, you should ask your instructors. Each laboratory has instructors who are interested in discussing your questions with you, but there are only two of them and about 29 other students in your laboratory section. To get the best use of them as resources, you should try your questions on your classmates before asking the instructors whenever possible. The instructors often will not answer questions directly, but will try to guide you so that you can answer your inquiries yourself. You can discuss experimental theory and results with your laboratory instructors outside of the laboratory during their regular weekly office hour or by appointment. There is also the Physics Learning

Center in PHYS Room 234, where there are regular Physics 152L instructors scheduled along with others for consultation. Finally, there will be a limited number of open lab hours available for you to come in and explore the apparatus and talk to a TA about the lab activities in advance. Schedules of these opportunities will be made available after the beginning of the semester.

When completing your experimental report, you will be asked to discuss in writing your understanding of the theory and data examined during the activity. We deliberately require PHYS 152L students to prepare a single group report together and receive a single mark (to be combined with the individual prelaboratory scores). **All lab reports are to be group reports. All prelaboratories and the Measurement Analysis activities are to be your own individual work.**

Groups collect their data as partners, meet separately outside the lab afterwards to discuss their calculations, and prepare their single combined report. Our experience has been that the quality of laboratory reports prepared by groups of people are almost always superior to those of individuals alone.

Working in small groups is the way scientific and industrial research is actually conducted. To actively learn physics, you must do more than just solve written problems or sit in lecture — you must think in the language of physics and practice speaking it with others. When working in groups, all students can benefit regardless of their ability — students with poorer understanding benefit by getting assistance from others in their group, while students with richer understanding benefit by having the opportunity to present or teach their ideas to others. We also encourage you to form a study group of your peers to work together on the lecture material as well as for laboratory reports.

There are a number of things that you can do to make your lab group more successful. Research by D. B. Young and F. M. Pottenger III [*Foundational Approaches in Science Teaching (FAST) Instructional Guide, 2nd Ed., 1992* CRDG, University of Hawaii at Honolulu] has shown that an effective group:

- knows why it is formed and what it is to do.
- openly communicates feelings and attitudes because group members recognize that open communication helps the group function.
- makes sure that all members express themselves on important decisions; it is not dominated by the ideas of any member.
- listens attentively to ideas expressed by all members.
- recognizes that all members and their ideas are important.
- accepts that each member is unique and has personal needs that must be met.
- recognizes the different talents and abilities of each member and tries to make use of them.
- recognizes that success is the responsibility of each member, not just the leader.

- shares responsibility for leadership, with each member assuming leadership at one time or another.

4. **Acquire basic computing skills.**

Learn to do elementary tasks using a computer graphing package and spreadsheet skills as soon as possible. Learning to use word processors, spreadsheets and graphing packages will make you much more productive during your time at Purdue, and will be of benefit throughout your career. These skills are required in Physics 152L. You should also investigate the facilities afforded to you on the Internet through the World Wide Web network protocols. Both the regular lecture and laboratories for Physics 152 regularly post information on the web, all of your instructors can be reached via the web, and course notices will be made available on the web. You should get an electronic mail account (free to all Purdue students) through the SSINFO system.

Further information on Physics 152 is available on the World Wide Web:

- Physics 152 Lecture URL: *http://www.physics.purdue.edu/* under Course Homepages.
- Physics 152 Lab URL: *http://www.physics.purdue.edu/* under Course Homepages.

Hardware and Software Guide

Goals of this guide

This section describes the Macintosh computer hardware and LabVIEW data acquisition software used throughout Physics 152L experiments. The information provided on this sheet will make your laboratory experience easier, save you time, and increase your confidence.

By the end of this handout you should be able to describe how to use the mouse and LabVIEW buttons to control LabVIEW programs, how to enter text and numbers into LabVIEW programs, how to edit using *wipe-and-type,* how to change the scales on graphs, how to print copies of the monitor screen at the printer, and how to use the cursors by dragging, using buttons, and using *wipe-and-type.*

The computer and its associated hardware

You will be using one of fifteen Apple Macintosh Quadra 800 microcomputers located in PHYS 18 for these experiments. Each consists of the following components:

1. The main computer body or chassis, which houses the motherboard, hard and floppy disk drives, and interface cards. (In hacker jargon, these computers have 24 megabytes (MB) of random access memory (RAM), 1 M read only memory (ROM), a Motorola 68040 microprocessor with a built-in floating point processor running at 33 MHz and run the Macintosh operating system version 7.5.)
2. A keyboard and mouse for your input of information.
3. A National Instruments Lab-NB multifunction board (located inside the computer case) for the input and output of data. This board contains circuitry to drive and read electrical signals in as digital or analog data.
4. A 16" color monitor.
5. A high speed laser printer located in the back corner (opposite from the main door) in PHYS 18.

External to the Mac (short for Macintosh) are sensors (ultrasonic transducers, strain gauges, and optical switches) to read position and force data from the experiments plus the associated electronics to preamplify and preprocess that data for input to the National Instruments multifunction board.

Software: the Macintosh GUI and LabVIEW

The Mac operating system makes use of a Graphical User Interface (GUI) in contrast to the Command Line Interface (CLI) typically used by UNIX or MS-DOS (MicroSoft Disk Operating System) computers. CLIs require the user (you) to type in command words on individual lines that the computer performs when you press the **return** key. GUIs use multiple screen areas called *windows* for various tasks, small pictures called *icons* to represent files or logical devices, and a pointer called a *mouse.* The user (you again) tells the computer what to do by pointing with the mouse and pressing the button upon it. There are also versions of GUIs available for UNIX and DOS (e.g., XWindows, Windows, and OS/2).

In Physics 152L, you will be using the Mac GUI to run laboratory software known as LabVIEW, a graphical programming tool by National Instruments of Austin, TX. Many of the skills you learn will be of use with Macs anywhere. LabVIEW provides users with *buttons* for data acquisition, *text fields* in which to enter names and station numbers, and graphs that can be used to display data. An example of a LabVIEW display is shown in Figure 1.

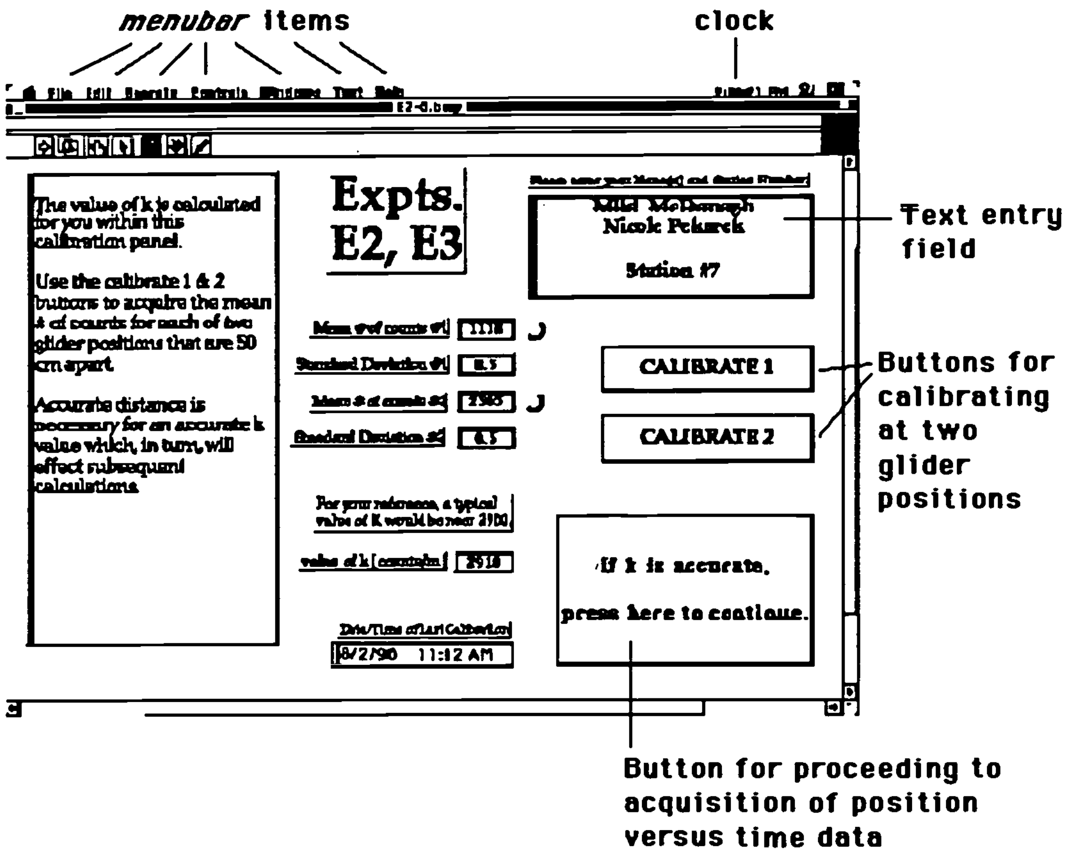


Figure 1. An example of a LabVIEW screen display on the Macintosh. In it students Miki McDonagh and Nicole Pekarek have entered their names and station number in the text entry field. To take calibration data at the first glider position, they would *click* on the button labeled CALIBRATE 1. Likewise, they would *click* on the button labeled CALIBRATE 2 for calibration data at the second glider position. If they were satisfied with the collected calibration data, they would then *click* on the large square button in the lower right corner of the screen.

Pointing, clicking, and dragging with the mouse

You will operate the Mac for experiments by using the mouse to position the cursor at different places on the screen. This is done by moving the mouse across the mousepad on the table beside the keyboard until the cursor is at the desired point and then pushing the button on the top of the mouse, a process called *clicking*. You can click on various pictures of button displayed on the screen to tell the Mac to start collecting data, analyze the data in detail and many other actions. Most of your commands to LabVIEW will be *point-and-click* commands.

You can also issue commands to the Mac using the *menubar* of commands always displayed at the top of the screen. To use them, select a menu title from the bar, point to it with the mouse and hold down the mouse button. A pull-down menu will appear, and you can select any command by keeping the mouse button down and *dragging* down over the menu until you are at the command you want. Then release the mouse button. Whenever you move the mouse with its button depressed, you are performing the operation known as *dragging*. In Figure 2 the issuing of the **Print...** command is illustrated.

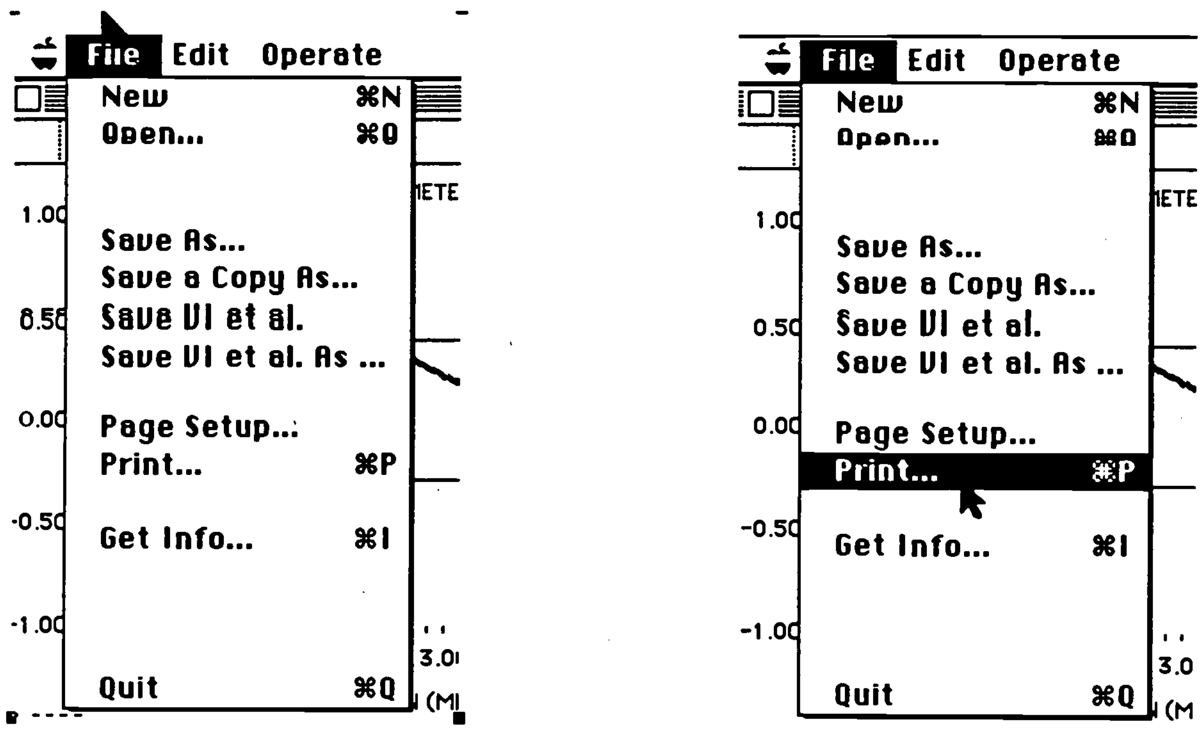


Figure 2. Lefthand side: Pull-down menu under File. Righthand side: Dragging down over the **File** menu to **Print**... and releasing the button at this point will initiate the printing process.

Text entry with the mouse and keyboard

You can change the comments displayed upon the Mac monitor by *editing* the *text fields* shown. For instance, to type your own names into the text field, simply click on that text field (see Figure 3) and start typing at the keyboard. You can erase old names by either clicking at the end of the text shown and using the **delete** key on the keyboard to gobble them up in the style of Pac-Man, or you can drag the mouse across the old text and just start typing. Your new text will overwrite the old text automatically. Dragging over old text and then typing new text to replace it is also called *wipe-and-type*.

Every experiment you perform will require you to enter your own and your partners' names and your computer station ID number. If you skip this step and leave the names of the students from a previous class entered, your TA may not accept the data as yours.

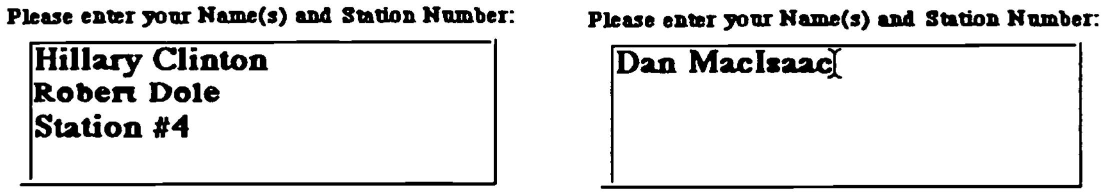


Figure 3. Text field editing. Click on the text field to enable editing. Left: Names left in the text field by a previous class. Right: By *wiping-and-typing* over the leftover names, Dan MacIsaac has entered his name, and will now hit the **return** key and type in the name of his partner, hit the **return** key again, and finally enter the station number.

Printing screen dumps

If you want a printout or hardcopy of any image (e.g., a graph of your data) on the computer monitor, use your mouse to select the **File** entry from the menubar, drag down over the menu and stop at the **Print...** entry. If you release the mouse button while the mouse points to the **Print** entry, the computer will prepare to print the screen image.

A *Dialog Box* as shown in Figure 4 will appear on the screen, and you must respond to it. You can either click on either the **Print** button to go ahead with printing or the **Cancel** button if you change your mind. Note that there is also a **Preview** option in case you want to review your data.

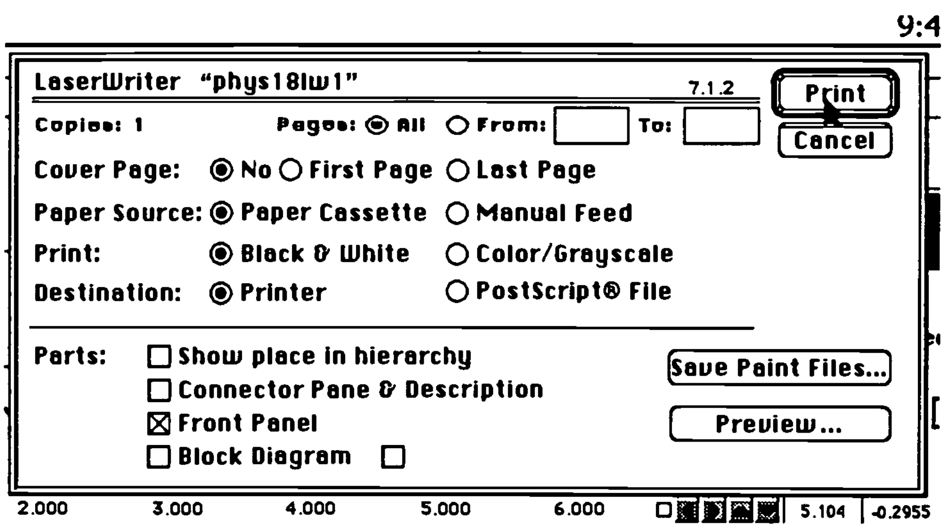


Figure 4. The Print dialog box. Clicking on Print will send the output to LaserWriter No. 1, denoted as "phys18lw1".

LabVIEW graph scaling

LabVIEW graphs can be *rescaled* to see more detail in your data. In general, the initially plotted points will occupy a small portion of the graph and it will be hard to deduce accurate numerical values from them, as illustrated in Figure 5. By changing the upper and lower limits of the horizontal and vertical scales, one can fill the available area in a graph with

data, as shown in Figure 6. The upper or lower limits on graph axes can be changed by double-clicking on the value to be changed, typing in the new value, and then hitting the *enter* key on the keyboard. *Wipe-and-typing* in the new value will also work. CAUTION: Do not attempt to change values other than the leftmost, rightmost, topmost, or bottommost values on a horizontal or vertical axis; bizarre effects can occur if this is attempted.

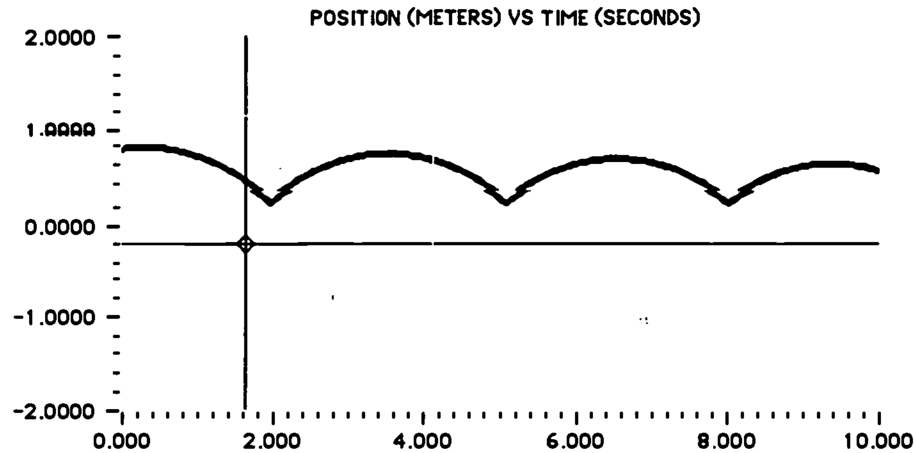


Figure 5. Data as they initially appear in a graph of position versus time. Only a small vertical region contains the data, so that position values read from the vertical axis will be crude.

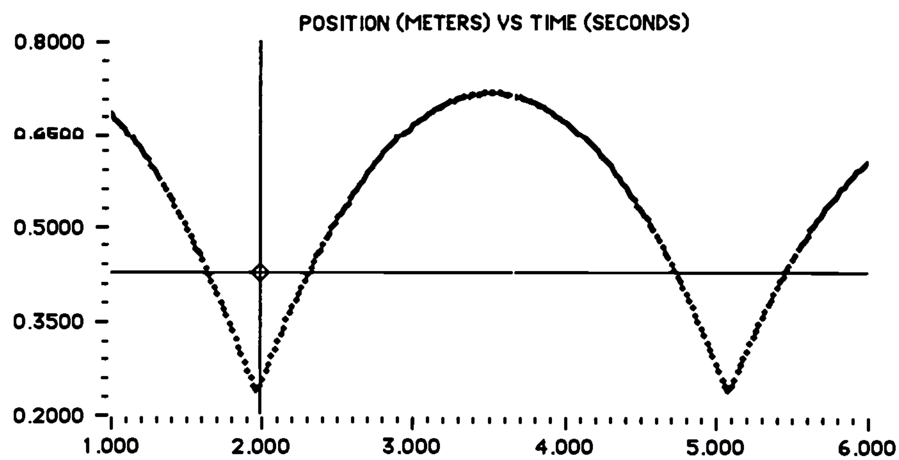


Figure 6. Rescaled graph showing the same data as in Figure 5. The vertical limits are now 0.2 m and 0.8 m, so that details are more apparent. The horizontal axis has been changed to have a lower limit of 1 s and an upper limit of 6 s to look in more detail in a region of interest.

LabVIEW graph cursors

LabVIEW data analysis graphs have user-controlled *measurement cursors* which can be used when making measurements. The coordinates and displacement between these two cursors are displayed alongside the graph. You can move a measurement cursor by double-clicking on the cursor coordinate to be changed, entering the new value, and hitting the **enter** key on the keyboard. You can also use the mouse to click on the measurement cursor you want moved and then dragging it to the new location.

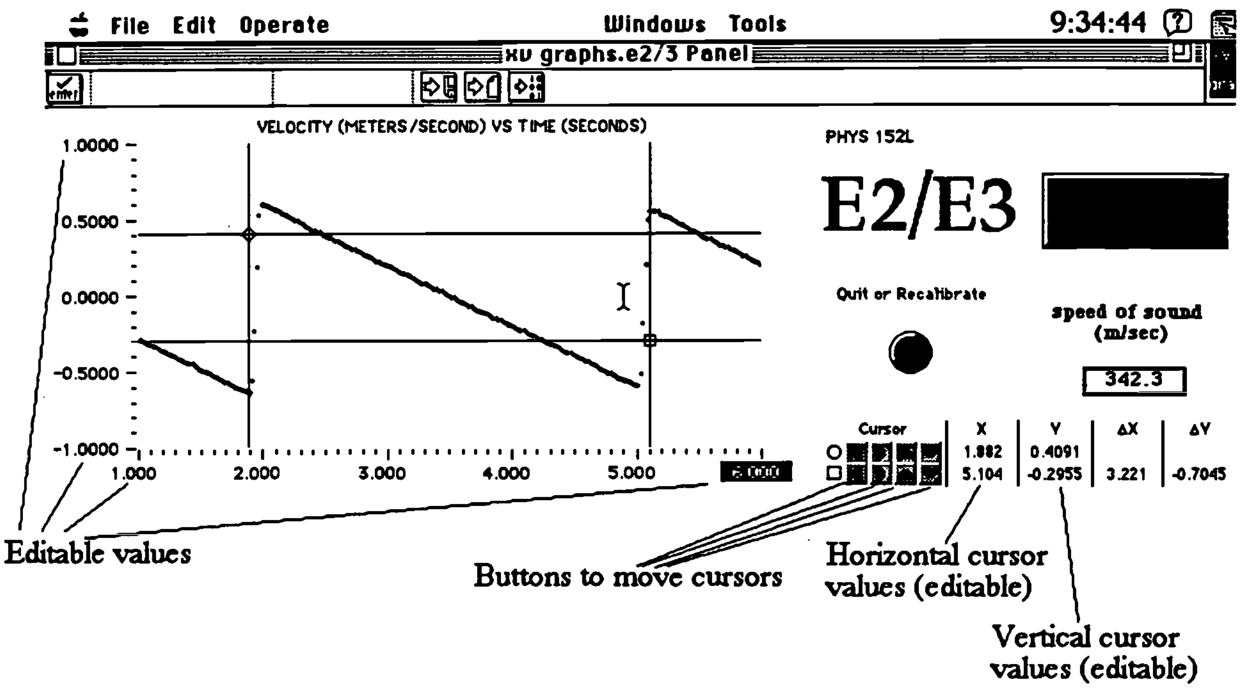


Figure 7. Two measurement cursors are shown on this LabVIEW graph. The positions of these cursors are can be changed by using the buttons, by *dragging* a cursor with a mouse, or by double clicking on value. The round cursor is at $(x_1, y_1) = (1.882, 0.4091)$, and the square cursor is located at $(x_2, y_2) = (5.104, -0.2955)$. Also, note that $\Delta x = x_2 - x_1 = 5.104 - 1.882 = 3.221$ and $\Delta y = y_2 - y_1 = -0.2955 - 0.4091 = -0.70445$ are displayed at the lower right of the figure.

Estimating measurement uncertainty

In the Physics 152L experiments you are asked to estimate the uncertainties in measurements. For the hardware used in the experiments you will make these estimates by looking at the typical scattering of data points. The cursors can then be used to give numerical values for these uncertainties. In Figure 8 an example of a reasonable estimate of uncertainties is given for a set of data points.

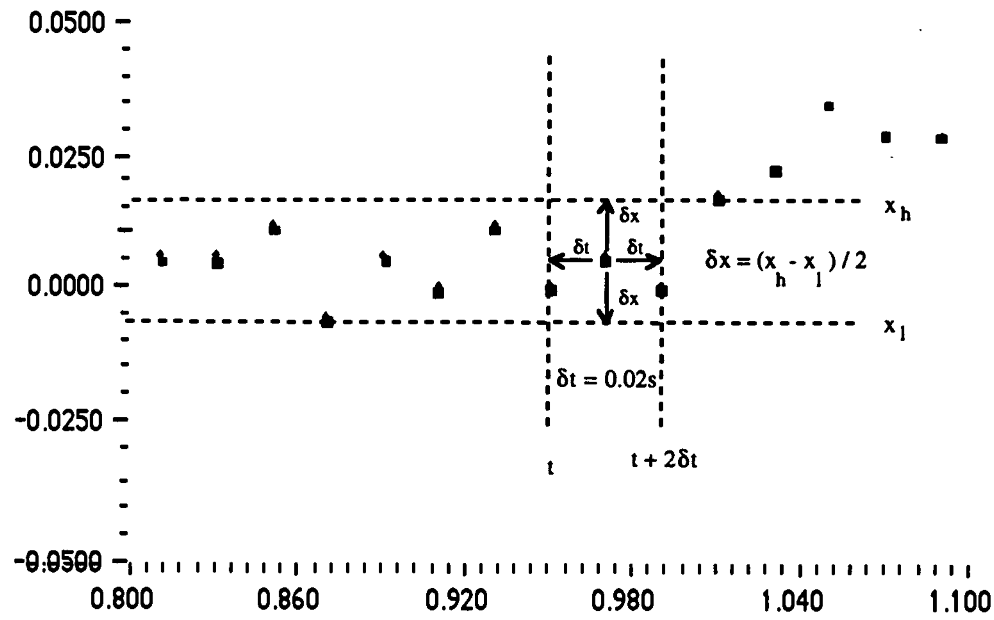


Figure 8. A graph has been rescaled enough to see the individual data points. By looking at the typical scatter of the data, one can estimate the uncertainties δt along the horizontal axis and δx along the vertical axis.

Conclusions

Make an effort to familiarize yourself with these LabVIEW and Macintosh GUI skills during laboratory open hours and during experiments E1 and E2. You will have an opportunity to experiment with the software during your first labs that will not be available due to time restrictions during other, more demanding experiments. E1 and E2 are intended to give you this opportunity to explore LabVIEW, and adequate time has been provided for this purpose. Feel free to ask your laboratory instructors for assistance with any of these features. Some suggested exercises to familiarize you with the Physics 152L software and hardware are listed on the next page.

Some suggested exercises follow. Exercises 7 and 8 will give you a headstart on Experiment E1.

1. Enter your names and station number in the text entry box by using *wipe and type.*

2. With air flowing out of the small holes in the air track give the glider a moderate push and take some data with the E1 data acquisition software. Take note of the features of the data, and be able to identify when the glider hits the bumpers on each end of the air track. with a glider in motion. Repeat this as many times as you wish to make sure that you understand what is being plotted on the graph. You can also move the glider by hand, or put your hand in various places along the top of the air track.

3. Rescale the axes of the graph to see the region between the bumper hits on each end of the air track. Do this by changing the upper and lower limits on each of the graph axes by one of the methods described earlier.

4. Position the two cursors to read the time and distance (in units of clock counts) between the two bounces from the cursor position displays. The easiest way is to double click on the appropriate cursor position display and then typing in the desired value of the cursor coordinate. Confirm that Δt and Δx are given correctly by the position display.

5. Rescale the E1 graph enough to see individual data points. Put one on the cursors on one of the data points and then *click* on either the right or left cursor movement buttons. The cursor should jump from one point to the next each time that you *click* on the right or left cursor movement buttons. This feature is very useful for part of Experiment E1.

6. Print out a graph of interest to you.

7. Figure out how to convert clock counts to a distance. How to do this is described in the materials for experiment E1.

8. Deduce how accurately the ultrasonic ranging system measures the position of a stationary glider.

Laboratory Report Guide

Goals of this guide

Writing reports is a major part of the work of practicing engineers and scientists, and the quality of the reports is an indication of the competence of an author and his/her institution. Thus, a major goal Physics 152L is to help you to develop professional reporting skills.

At the end of this guide you should be able to describe the contents of the five major parts of a Physics 152L Report. You should also be able to describe the marking criterion for each part of the report, and describe how marks are allocated for group laboratory reports.

Laboratory format

Your completed report should include the following sections:

1. **Prelaboratory Questions** (25 % — The Prelab Questions Sheet). These questions are due a week before the remainder of the report is due: namely, at the start of the actual laboratory session. You will be asked to hand in the prelab questions at the door, and they will NOT be accepted later.

2. **Abstract** (10 % — one paragraph). This is a brief summary of your experimental results.

3. **Data and calculations** (45 % — the Laboratory Data Sheet). Here you will show your original numeric data and associated uncertainties, and calculations based on these data.

4. **Analysis** (15 % — four paragraphs maximum). Here you will numerically describe difficulties and sources of uncertainty encountered, and use this evidence to suggest and support modifications to the laboratory apparatus and methods.

5. **Conclusions** (5 % — two paragraphs maximum) State the main ideas, relevance and practical applications of the laboratory methods and techniques in this section.

This adds up to 100% for experiments E1–E6. Be concise in your lab reports; we have tried to indicate the desired length of the various parts of the lab reports by the amount of inserted blank space in the laboratory report forms. Reports of unreasonable length may not be marked by your GTA due to time constraints. *GTAs may choose to only mark your responses until they reach the recommended lengths, leaving the remainder unmarked.* **If you wish, you may paste word-processed text into your laboratory reports for your abstract, analysis, and conclusions.**

The Laboratory Abstract

Your abstract is intended to be a brief summation and introduction to your entire report, just as it is in professional research journals. When research scientists review the work of others, they decide whether a particular journal paper or report is applicable to their interests from a quick glance at the abstract, which summarizes the entire paper. The abstract briefly describes the single main theory behind the experiment, introduces the activity, gives the major numerical data findings, and discusses whether these agree with theoretical predictions within experimental uncertainty or describes a percentage discrepancy in cases of disagreement.

Your abstract must contain all four of the above elements: theory, activity, findings and agreements. You will have to examine your laboratory activity carefully and decide which data and calculations are important enough to be included in your abstract, and which are trivial.

At the start of E2 you will be given a detailed example of the preparation of an abstract – an abstract for E1 will be posted in lab. In the meanwhile, here is an abstract describing an experiment measuring the speed of sound:

Measurement of the speed of sound using an ultrasonic transducer

Using time-of-flight techniques, the speed of sound was measured. The experimental apparatus consisted of an ultrasonic transducer, a flat aluminum plate reflector placed at known distances from the transducer, and a clock which counts every 0.5 μs. By making a least-squares fit to a set of time-of-flight measurements corresponding to roundtrip pathlengths ranging from 1 to 4 m, the speed of sound in $(20 \pm 1)°C$ *air with a relative humidity of* $(35 \pm 3)\%$ *was measured to be* (339 ± 5) *m/s, which is about 1% less than the value of 343 m/s given by R. A. Serway for 20°C air.*

Laboratory Data and Calculations

All of your data and all calculations must appear upon the printed sheets provided to you, and data must be initialed by your laboratory instructor *before you leave the laboratory* to confirm that it is your own work. Be as neat as possible the first time. Do not recopy data. You should first attempt this writing on scrap paper, and then transfer it to your report sheets.

The Laboratory Analysis

In this section of the laboratory report you will discuss your experimental results and comment on them critically. In particular, you should quantitatively discuss the consistency or inconsistency of your results with regard to predictions, and why you and the readers of your report should or should not believe your experimental results. Using propagation of uncertainty techniques from Measurement Analysis 1. you should examine the relative contributions of the various measurement uncertainties, and identify any dominant contribution. You should give real or estimated numerical sizes for the most important sources of uncertainty in the activity, rather than giving no idea of their magnitude or guessing wildly at possible sources. Mention difficulties and shortcomings that you encountered during your experiment, and suggest apparatus redesigns or changes in experimental techniques to eliminate or overcome these troubles. Use numbers and observations to justify these suggested changes to the activity. Discuss how the measurement methods themselves can be changed to reduce or eliminate uncertainty. A small diagram might be appropriate when describing the exact situation.

We are interested primarily in shortcomings in measurement techniques and not in your personal errors (mistakes). However, if a particular method encourages personal errors, you should describe this as well.

Thus, your analysis must contain the following three elements: major sources of uncertainty and true or estimated numerical sizes for them, suggested changes to apparatus and/or measurement techniques to reduce or eliminate measurement uncertainties. You must identify the most important parts of your experiment and analyze them because you cannot discuss everything in the limited space provided.

The Laboratory Conclusion

Your conclusions are expected to take your laboratory activities and extend them outside of the laboratory—you must describe their relevance and application to real-world phenomena and activities. You must first briefly comment on whether the main physical theories you examined in the lab were supported by your observations and calculations. For Physics 152 we require that you choose one of the major mathematical relationships addressed in the laboratory experiment and explain how this relationship can be used to analyze non-laboratory physical behavior. You have to choose a situations where your chosen equations apply, then use either estimated or actual numbers to SHOW that these equations do describe the phenomenon. In short, you have to demonstrate an ability to mathematically transfer the theory you used in lab to a real-world situation.

Your conclusions require these two elements: a general concluding statement that compares you laboratory findings to the accepted theory, and a mathematical application of the theory to a real world, non-laboratory situation. Again, you must discriminate and discuss only key ideas due to space restrictions. Note that there is little or no overlap between your conclusions and abstract in the laboratory report. Also, recall that the abstract should be the section you write last, as it reflects the total and final contents of the entire report.

Measurement Analysis 1: Measurement Uncertainty and Propagation

You should read Sections 1.6 and 1.7 on pp. 12–16 of the Serway text before this activity. Please note that while attending the MA1 evening lecture is optional, the MA1 assignment is NOT optional and must be turned in before the deadline for your division for credit. The deadline for your division is specified in READ ME FIRST! at the front of this manual.

At the end of this activity, you should:

1. Understand the form of measurements in the laboratory, including measured values and uncertainties.
2. Know how to take measurements from laboratory instruments.
3. Be able to discriminate between measurements that agree and those that are discrepant.
4. Understand the difference between precision and accuracy.
5. Be able to combine measurements through addition, subtraction, multiplication, and division.
6. Be able to properly round measurements and treat significant figures.

1 Measurements

1.1 Uncertainty in measurements

In an ideal world, measurements are always perfect: there, wooden boards can be cut to exactly two meters in length and a block of steel can have a mass of exactly three kilograms. However, we live in the real world, and here measurements are *never* perfect. In our world, measuring devices have limitations.

The imperfection inherent in all measurements is called an *uncertainty*. In the Physics 152 laboratory, we will write an uncertainty almost every time we make a measurement. When we say the word "measurement" in this laboratory, we are really referring to three pieces of information: a measured value, the corresponding uncertainty, and the units. Our notation for measurements takes the following form:

$$measurement = (measured\ value \pm uncertainty)\ proper\ units$$

where the $\pm$ is read 'plus or minus.'

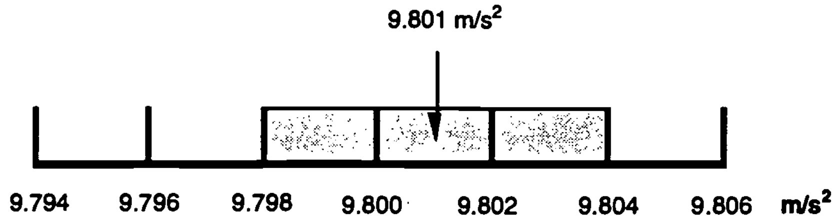


Figure 1: Measurement and uncertainty: $(9.801 \pm 0.003)\ m/s^2$

Consider the measurement $g = (9.801 \pm 0.003)$ m/s^2. We interpret this measurement as meaning that the experimentally determined value of g can lie anywhere between the values $9.801 + 0.003$ m/s^2 and $9.801 - 0.003$ m/s^2, or 9.798 m/s$^2 \leq g \leq 9.804$ m/s^2. As you can see, a real world measurement is not one simple measured value, but is actually a *range* of possible values (see Figure 1). This range is determined by the uncertainty in the measurement. As uncertainty is reduced, this range is narrowed.

Here are two examples of measurements:

$$v = (4.000 \pm 0.002) \text{ m/s} \qquad G = (6.67 \pm 0.01) \times 10^{-11} \text{ N·m}^2/\text{kg}^2$$

Look over the measurements given above, paying close attention to the number of decimal places in the measured values and the uncertainties (when the measurement is good to the thousandths place, so is the uncertainty; when the measure is good to the hundredths place, so is the uncertainty). You should notice that they always agree, and this is most important:

> — *In a measurement, the measured value and its uncertainty must always have the same number of digits after the decimal place.*

Examples of nonsensical measurements are (9.8 ± 0.0001) m/s^2 and (9.801 ± 0.1) m/s^2; writing such nonsensical measurements will cause readers to judge you are either incompetent or sloppy. Avoid writing improper measurements by always making sure the decimal places agree.

Sometimes we want to talk about measurements more generally, and so we write them algebraically (i.e. without actual numbers). In these cases, we use the lowercase Greek letter *delta*, or δ to represent the uncertainty in the measurement. Algebraic examples include:

$$(X \pm \delta X) \qquad (Y \pm \delta Y)$$

Although units are not explicitly written next to these measurements, they are implied. We will use these algebraic expressions for measurements when we discuss the propagation of uncertainties in Section 4.

1.2 Taking measurements in lab

In the laboratory you will be taking real world measurements, and you will record both measured values and uncertainties. Getting values from measuring equipment is usually as simple as reading a scale or a digital readout. Determining uncertainties is a bit more challenging since you—not the measuring device— must determine them. When determining an uncertainty from a measuring device, you need to first determine the smallest quantity that can be resolved on the device. Then the uncertainty in the measurement is taken to be this value. For example, if a digital readout displays 1.35 g, then you should write that measurement as (1.35 ± 0.01) g. The smallest division you can clearly read is your uncertainty.

On the other hand, reading a scale is somewhat subjective. Suppose you use a meter stick that is divided into centimeters to determine the length $(L \pm \delta L)$ of a rod, as illustrated in Figure 2. First, you read your measured value from this scale and find that the rod is 6 cm. Depending on the sharpness of your vision, the clarity of the scale, and the boundaries of the measured object, you might read the uncertainty as ± 1 cm, ± 0.5 cm, or ± 0.2 cm; all would be acceptable. However, an uncertainty of ± 0.1 cm or smaller is dubious because the ends of the object are rounded and it is hard to resolve ± 0.1 cm. Thus, you can record your measurement as $(L \pm \delta L) = (6 \pm 1)$ cm, $(L \pm \delta L) = (6.0 \pm 0.5)$ cm, or $(L \pm \delta L) = (6.0 \pm 0.2)$ cm, and all three measurements would be regarded as reasonable. *For the purposes of discussion and uniformity in this laboratory manual, we will use the largest reasonable uncertainty. For our example, this is ± 1 cm.*

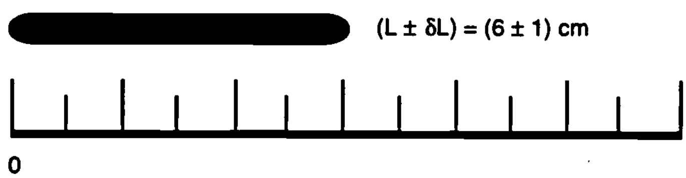


Figure 2: A measurement obtained by reading a scale. Acceptable measurements range from 6.0 ± 0.1 cm to 6.0 ± 0.2 cm, depending on the sharpness of your vision, the clarity of the scale, and the boundaries of the measured object. Examples of unacceptable measurements are 6 ± 2 cm and 6.00 ± 0.01 cm.

1.3 Percentage uncertainty of measurements

When we speak of a measurement, we often want to know how reliable it is. We need some way of judging the relative worth of a measurement, and this is done by finding the percentage uncertainty of a measurement. We will refer to the *percentage uncertainty* of a measurement as the ratio between the measurement's uncertainty and its measured value multiplied by 100%. You will often hear this kind of uncertainty or something closely related used with measurements – a meter is good to ± 3% of full scale, or ± 1% of the reading, or good to one part in a million.

The *percentage uncertainty* of a measurement $(Z \pm \delta Z)$ is defined as $\dfrac{\delta Z}{Z} \times 100\%$.

Think about percentage uncertainty as a way of telling how much a measurement deviates from "perfection." With this idea in mind, it makes sense that as the uncertainty for a measurement decreases, the percentage uncertainty $\frac{\delta Z}{Z} \times 100\%$ decreases, and so the measurement deviates less from perfection. For example, a measurement of (2 ± 1) m has a percentage uncertainty of 50%, or one part in two. In contrast, a measurement of (2.00 ± 0.01) m has a percentage uncertainty of 0.5% (or 1 part in 200) and is therefore the better measurement. If there were some way to make this same measurement with zero uncertainty, the percentage uncertainty would equal 0% and there would be no deviation whatsoever from the measured value—we would have a "perfect" measurement. Unfortunately, this never happens in the real world.

1.4 Implied uncertainties

When you read a physics textbook such as Serway's, you may notice that almost all the measurements stated are missing uncertainties. Does this mean that Serway is able to measure things perfectly, without any uncertainty? Not at all! In fact, it is common practice in textbooks not to write uncertainties with measurements, even though they are actually there. In such cases, the uncertainties are *implied.* We treat these implied uncertainties the same way as we did when taking measurements in lab:

> — *In a measurement with an implied uncertainty, the actual uncertainty is written as ± 1 in the smallest place value of the given measured value.*

For example, if you read $g = 9.80146$ m/s^2 in a textbook, you know this measured value has an implied uncertainty of 0.00001 m/s^2. To be more specific, you could then write $(g \pm \delta g) = (9.80146 \pm 0.00001)$ m/s^2.

1.5 Decimal points — don't lose them

If a decimal point gets lost, it can have disastrous consequences. One of the most common places where a decimal point gets lost is in front of a number. For example, writing .52 cm sometimes results in a reader missing the decimal point, and reading it as 52 cm — one hundred times larger! After all, a decimal point is only a simple small dot. However, writing 0.52 cm virtually eliminates the problem, and writing leading zeros for decimal numbers is standard scientific and engineering practice.

2 Agreement and Discrepancy

In the laboratory, you will not only be taking measurements, but also comparing them. Most often you will compare your experimental measurements (i.e. the ones you find in lab) to some theoretical, predicted, or standard measurements (i.e. the type you calculate or look up in a textbook). We need a method to determine how accurate our measurements are; in other words, a way to determine how closely our experimental measurements match with standardized, agreed-upon measurements. To simplify this process, we adopt the following notion: two measurements, when compared, either *agree* within experimental uncertainty or they are *discrepant* (that is, they do not agree). Before we illustrate how this classification is carried out, you should first recall that a measurement in the laboratory is not made up of one single value, but a whole range of values. With this in mind, we can say,

> Two measurements are in *agreement* if the two measurements share values in common; that is, their respective uncertainty ranges partially (or totally) overlap.

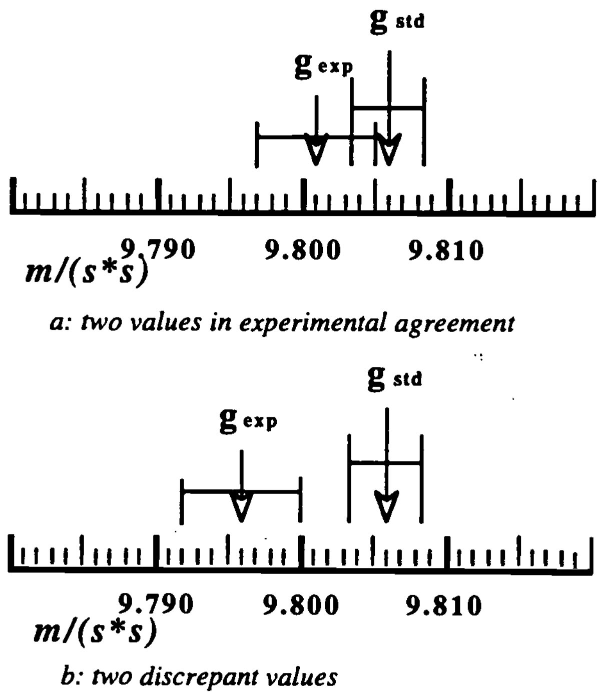


Figure 3: Agreement and discrepancy of gravity measurements

For example, a laboratory measurement of $(g_{exp} \pm \delta g_{exp}) = (9.801 \pm 0.004)$ m/s^2 is being compared to a scientific standard value of $(g_{std} \pm \delta g_{std}) = (9.8060 \pm 0.0025)$ m/s^2. As illustrated in Figure 3(a), we see that the ranges of the measurements partially overlap, and so we conclude that the two measurements agree.

Remember that measurements are either in agreement or are discrepant. It then makes sense that,

> Two measurements are *discrepant* if the two measurements *do not* share values in common; that is, their respective uncertainty ranges do not overlap.

Suppose as an example that a laboratory measurement $(g_{exp} \pm \delta g_{exp}) = (9.796 \pm 0.004)$ m/s^2 is being compared to the value of $(g_{std} \pm \delta g_{std}) = (9.8060 \pm 0.0025)$ m/s^2. From Figure 3(b) we notice that the ranges of the measurements do not overlap at all, and so we say these measurements are discrepant.

When two measurements being compared do not agree, we want to know by how much they do not agree. We call this quantity the *discrepancy* between measurements, and we use the following formula to compute it:

The discrepancy Z between an experimental measurement $(X \pm \delta X)$ and a theoretical or standard measurement $(Y \pm \delta Y)$ is:

$$Z = \frac{X_{experimental} - Y_{standard}}{Y_{standard}} \times 100\%$$

As an example, take the two discrepant measurements $(g_{exp} \pm \delta g_{exp})$ and $(g_{std} \pm \delta g_{std})$ from the previous example. Since we found that these two measurements are discrepant, we can calculate the discrepancy Z between them as:

$$Z = \frac{g_{exp} - g_{std}}{g_{std}} \times 100\% = \frac{9.796 - 9.8060}{9.8060} \times 100\% \approx -0.10\%$$

Keep the following in mind when comparing measurements in the laboratory:

1. If you find that two measurements agree, state this in your report. Do NOT compute a discrepancy, because the measurements are NOT discrepant.
2. If you find that two measurements are discrepant, state this in your report and then go on to compute the discrepancy.

3 Precision and accuracy

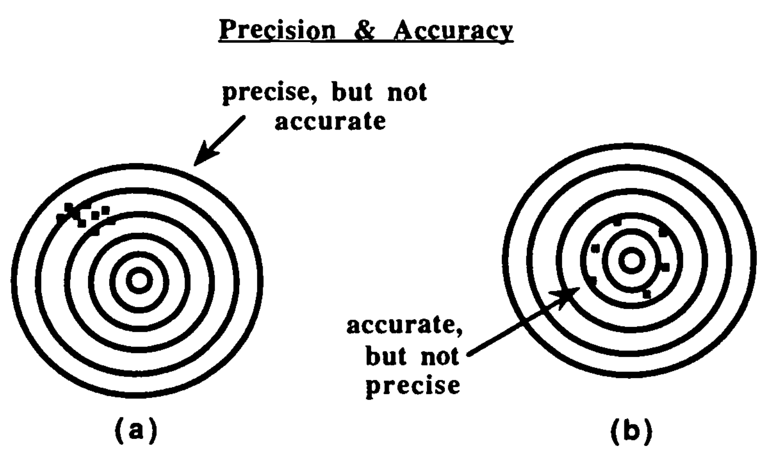


Figure 4: Precision and accuracy in target shooting.

In everyday language, the words precision and accuracy are often interchangeable. In the sciences, however, the two terms have distinct meanings:

Precision describes the degree of certainty one has about a measurement.

Accuracy describes how well measurements agree with a known, standard measurement.

Let's first examine the concept of precision. Figure 4(a) shows a precise target shooter, since all the shots are close to one another. Because all the shots are clustered about a single point, there is a high degree of certainty in where the shots have gone and so therefore the shots are precise. In Figure 5(b), the measurements on the ruler are all close to one another, and like the target shots, they are precise as well.

Accuracy, on the other hand, describes how well something agrees with a standard. In Figure 4(b), the "standard" is the center of the target. All the shots are close to this center, and so we would say that the targetshooter is accurate. However, the shots are not close to one another, and so they are not precise. Here we see that the terms "precision" and "accuracy" are definitely not interchangeable; one does not imply the other. Nevertheless, it is possible for something to be both accurate and precise. In Figure 5(c), the measurements are accurate, since they are all close to the "standard" measurement of 1.5 cm. In addition, the measurements are precise, because they are all clustered about one another.

Note that it is also possible for a measurement to be neither precise nor accurate. In Figure 5(a), the measurements are neither close to one another (and therefore not precise), nor are they close to the accepted value of 1.5 cm (and hence not accurate).

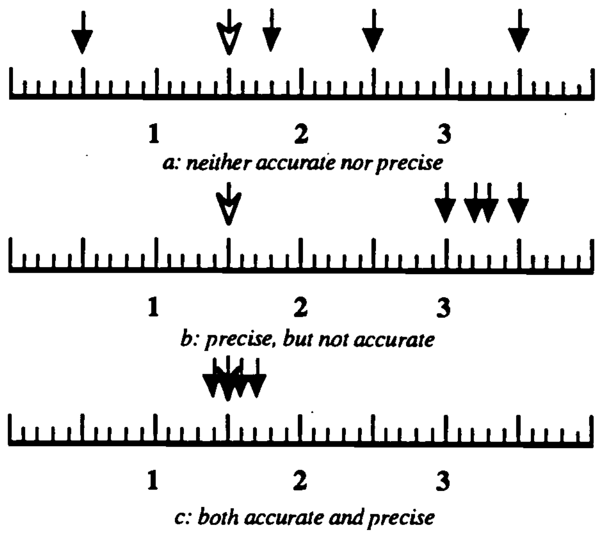


Figure 5: Examples of precision and accuracy in length measurements. Here the hollow headed arrows indicate the 'actual' value of 1.5 cm. The solid arrows represent measurements.

You may have noticed that we have already developed techniques to measure precision and accuracy. In Section 1.3, we compared the uncertainty of a measurement to its measured value to find the percentage uncertainty. The calculation of percentage uncertainty is actually a test to determine how certain you are about a measurement; in other words, how precise the measurement is. In Section 2, we learned how to compare a measurement to a standard or accepted value by calculating a percent discrepancy. This comparison told you how close your measurement was to this standard measurement, and so finding percent discrepancy is really a test for accuracy.

It turns out that in the laboratory, precision is much easier to achieve than accuracy. Precision can be achieved by careful techniques and handiwork, but accuracy requires excellence in experimental design and measurement analysis. During this laboratory course, you will examine both accuracy and precision in your measurements and suggest methods of improving both.

4 Propagation of uncertainty (worst case)

In the laboratory, we will need to combine measurements using addition, subtraction, multiplication, and division. However, measurements are composed of two parts—a measured value and an uncertainty—and so any algebraic combination must account for both. Performing these operations on the measured values is easily accomplished; handling uncertainties poses the challenge. We make use of the *propagation of uncertainty* to combine measurements with the assumption that as measurements are combined, uncertainty increases—hence the uncertainty *propagates* through the calculation. Here we show how to combine two measurements and their uncertainties. Often in lab you will have to keep using the propagation formulae over and over, building up more and more uncertainty as you combine three, four or five set of numbers.

1. **When adding two measurements**, the uncertainty in the final measurement is the sum of the uncertainties in the original measurements:

$$(A \pm \delta A) + (B \pm \delta B) = (A + B) \pm (\delta A + \delta B) \tag{1}$$

As an example, let us calculate the combined length $(L \pm \delta L)$ of two tables whose lengths are $(L_1 \pm \delta L_1) = (3.04 \pm 0.04)$ m and $(L_2 \pm \delta L_2) = (10.30 \pm 0.01)$ m. Using this addition rule, we find that

$$(L \pm \delta L) = (3.04 \pm 0.04)\ \text{m} + (10.30 \pm 0.01)\ \text{m} = (13.34 \pm 0.05)\ \text{m}$$

2. **When subtracting two measurements**, the uncertainty in the final measurement is again equal to the sum of the uncertainties in the original measurements:

$$(A \pm \delta A) - (B \pm \delta B) = (A - B) \pm (\delta A + \delta B) \tag{2}$$

For example, the difference in length between the two tables mentioned above is

$$\begin{aligned}(L_2 \pm \delta L_2) - (L_1 \pm \delta L_1) &= (10.30 \pm 0.01)\ \text{m} - (3.04 \pm 0.04)\ \text{m} \\ &= [(10.30 - 3.04) \pm (0.01 + 0.04)]\ \text{m} \\ &= (7.26 \pm 0.05)\ \text{m}\end{aligned}$$

Be careful not to subtract uncertainties when subtracting measurements— uncertainty ALWAYS gets worse as more measurements are combined.

3. **When multiplying two measurements**, the uncertainty in the final measurement is found by summing the percentage uncertainties of the original measurements and then multiplying that sum by the product of the measured values:

$$(A \pm \delta A) \times (B \pm \delta B) = (AB)\left[1 \pm \left(\frac{\delta A}{A} + \frac{\delta B}{B}\right)\right] \tag{3}$$

A quick derivation of this multiplication rule is given below. First, assume that the measured values are large compared to the uncertainties; that is, $A \gg \delta A$ and $B \gg \delta B$. Then, using the distributive law of multiplication:

$$\begin{aligned}(A \pm \delta A) \times (B \pm \delta B) &= AB + A(\pm\delta B) + B(\pm\delta A) + (\pm\delta A)(\pm\delta B) \\ &\cong AB + A(\pm\delta B) + B(\pm\delta A) \end{aligned} \tag{4}$$

Since the uncertainties are small compared to the measured values, the product of two small uncertainties is an even smaller number, and so we discard the product $(\pm\delta A)(\pm\delta B)$. With further simplification, we find:

$$\begin{aligned} AB + A(\pm\delta B) + B(\pm\delta A) &= AB + B(\pm\delta A) + A(\pm\delta B) \\ &= AB\left[1 \pm \left(\frac{\delta A}{A} + \frac{\delta B}{B}\right)\right] \end{aligned}$$

Now let us use the multiplication rule to determine the area of a rectangular sheet with length $(l \pm \delta l) = (1.50 \pm 0.02)$ m and width $(w \pm \delta w) = (20 \pm 1)$ cm $= (0.20 \pm 0.01)$ m. The area $(A \pm \delta A)$ is then

$$\begin{aligned} (A \pm \delta A) &= (l \pm \delta l) \times (w \pm \delta w) = (lw)\left[1 \pm \left(\frac{\delta l}{l} + \frac{\delta w}{w}\right)\right] \\ &= (1.50 \times 0.20)\left[1 \pm \left(\frac{0.02}{1.50} + \frac{0.01}{0.20}\right)\right] \text{ m}^2 \\ &= 0.300[1 \pm (0.0133 + 0.0500)] \text{ m}^2 = 0.300[1 \pm 0.0633] \text{ m}^2 \\ &= (0.300 \pm 0.0190)\text{m}^2 \\ &\approx (0.30 \pm 0.02) \text{ m}^2 \end{aligned}$$

Notice that the final values for uncertainty in the above calculation were determined by multiplying the product (lw) outside the bracket by the sum of the two percentage uncertainties $(\delta l/l + \delta w/w)$ inside the bracket. Always remember this crucial step! Also, notice how the final measurement for the area was rounded. This rounding was performed by following the rules of significant figures, which are explained in detail later in Section 5.

Recall our discussion of percentage uncertainty in Section 1.3. It is here that we see the benefits of using such a quantity; specifically, we can use it to tell right away which of the two original measurements contributed most to the final area uncertainty. In the above example, we see that the percentage uncertainty of the width measurement $(\delta w/w) \times 100\%$ is 5%, which is larger than the percentage uncertainty $(\delta l/l) \times 100\% \approx 1.3\%$ of the length measurement. Hence, the width measurement contributed most to the final area uncertainty, and so if we wanted to improve the precision of our area measurement, we should concentrate on reducing width uncertainty δw by changing our method for measuring width.

4. **When dividing two measurements**, the uncertainty in the final measurement is found by summing the percentage uncertainties of the original measurements and then multiplying that sum by the quotient of the measured values:

$$\frac{(A \pm \delta A)}{(B \pm \delta B)} = \left(\frac{A}{B}\right)\left[1 \pm \left(\frac{\delta A}{A} + \frac{\delta B}{B}\right)\right] \tag{5}$$

As an example, let's calculate the average speed of a runner who travels a distance of (100.0 ± 0.2) m in (9.85 ± 0.12) s using the equation $\overline{v} = D/t$, where $\bar{v}$ is the average speed, D is the distance traveled, and t is the time it takes to travel that distance.

$$\begin{aligned}
\bar{v} &= \frac{D \pm \delta D}{t \pm \delta t} = \left(\frac{D}{t}\right)\left[1 \pm \left(\frac{\delta D}{D} + \frac{\delta t}{t}\right)\right] \\
&= \left(\frac{100.0 \text{ m}}{9.85 \text{ s}}\right)\left[1 \pm \left(\frac{0.2}{100.0} + \frac{0.12}{9.85}\right)\right] \\
&= 10.15\,[1 \pm (0.002000 + 0.01218)\,] \text{ m/s} = 10.15\,[1 \pm (0.01418)\,] \text{ m/s} \\
&= (10.15 \pm 0.1439) \text{ m/s} \\
&\approx (10.2 \pm 0.1) \text{ m/s}
\end{aligned}$$

In this particular example the final uncertainty results mainly from the uncertainty in the measurement of t, which is seen by comparing the percentage uncertainties of the time and distance measurements, $(\delta t/t) \approx 1.22\%$ and $(\delta D/D) \approx 0.20\%$, respectively. Therefore, to reduce the uncertainty in $(\overline{v} \pm \delta\overline{v})$, we should change the way t is measured.

5. **Special cases—inversion and multiplication by a constant:**

   (a) If you have a quantity $X \pm \delta X$, you can invert it and apply the original percentage uncertainty:

$$\frac{1}{X \pm \delta X} = \left(\frac{1}{X}\right)\left[1 \pm \frac{\delta X}{X}\right]$$

(b) To multiply by a constant,

$$k \times (Y \pm \delta Y) = [kY \pm k\delta Y]$$

It is important to realize that these formulas and techniques allow you to perform the four basic arithmetic operations. You can (and will) combine them by repetition for the sum of three measurements, or the cube of a measurement. Normally it is impossible to use these simple rules for more complicated operations such as a square root or a logarithm, but the trigonometric functions $\sin\theta, \cos\theta,$ and $\tan\theta$ are exceptions. Because these functions are defined as the ratios between lengths, we can use the quotient rule to evaluate them. For example, in a right triangle with opposite side $(x \pm \delta x)$ and hypotenuse $(h \pm \delta h)$, $\sin\theta = \frac{(x\pm\delta x)}{(h\pm\delta h)}$. Similarly, any expression that can be broken down into arithmetic steps may be evaluated with these formulas; for example, $(x \pm \delta x)^2 = (x \pm \delta x)(x \pm \delta x)$.

6. **Finding the uncertainty of a square root**

The method for obtaining the square root of a measurement. uses some algebra coupled with the multiplication rule.

Let $(A \pm \delta A)$ and $(B \pm \delta B)$ be two measurements. Further, assume that the square root of $(A \pm \delta A)$ is equal to the measurement $(B \pm \delta B)$. Then,

$$\sqrt{(A \pm \delta A)} = (B \pm \delta B) \tag{6}$$

Squaring both sides, we obtain

$$(A \pm \delta A) = (B \pm \delta B)^2$$

Using the multiplication rule on $(B \pm \delta B)^2$, we find

$$\begin{aligned} (A \pm \delta A) &= (B \pm \delta B)^2 \\ &= B^2\left[1 \pm \left(\frac{2\delta B}{B}\right)\right] \\ &= (B^2 \pm 2B\delta B) \end{aligned}$$

Thus,

$$(A \pm \delta A) = (B^2 \pm 2B\delta B) \text{ which means } B = \sqrt{A} \text{ and } \delta B = \frac{\delta A}{2B} = \frac{\delta A}{2\sqrt{A}}.$$

Making this substitution into Equation 6, we arrive at the final result

$$\sqrt{(A \pm \delta A)} = \left(\sqrt{A} \pm \frac{\delta A}{2\sqrt{A}}\right) \tag{7}$$

This technique for finding the uncertainty in a square root will be required in E4 — E6.

7. **Another example: a case involving a triple product.** The formula for the volume of a rod with a circular cross–section (πr^2) and length l is given by $V = \pi r^2 l$. Given initial measurements $(r \pm \delta r)$ and $(l \pm \delta l)$, derive an expression for $(V \pm \delta V)$. Note that π has *no* uncertainty.

   Using the derivation of the worst case multiplication propagation rule (Equation 4) as a guide, we start with

   $$(V \pm \delta V) = \pi(r \pm \delta r)(r \pm \delta r)(l \pm \delta l)$$

   and expand the terms involving r on the left hand side.

   $$(V \pm \delta V) = \pi[r^2 + r(\pm\delta r) + r(\pm\delta r) + (\pm\delta r)(\pm\delta r)](l \pm \delta l)$$

   We can discard terms involving products of measurement uncertainties such as $(\delta r)(\delta r)$ since they are small to obtain

   $$(V \pm \delta V) = \pi[r^2 + 2r(\pm\delta r)](l \pm \delta l)$$

   Next we multiply out the final product on the left.

   $$(V \pm \delta V) = \pi[r^2 l + r^2(\pm\delta l) + 2rl(\pm\delta r) + 2r(\pm\delta r)(\pm\delta l)]$$

   Again we discard terms involving products of measurement uncertainties such as $(\delta r)(\delta l)$ to obtain

   $$(V \pm \delta V) = \pi[r^2 l + r^2(\pm\delta l) + 2rl(\pm\delta r)]$$

   Finally, we can factor out $r^2 l$ to obtain

   $$(V \pm \delta V) = \pi r^2 l(1 \pm \frac{\delta l}{l} \pm 2\frac{\delta r}{r})$$

5 Rounding measurements

The previous sections contain the bulk of what you need to take and analyze measurements in the laboratory. Now it is time to discuss the finer details of measurement analysis. The subtleties we are about to present cause an inordinate amount of confusion in the laboratory. Getting caught up in details is a frustrating experience, and the following guidelines should help alleviate these problems.

An often-asked question is, "How should I round my measurements in the laboratory?" **The answer is that you must watch significant figures in calculations *and then* be sure the number of decimal places of a measured value and its uncertainty agree.** Before we give an example, we should explore these two ideas in some detail.

Measured value	Number of significant figures
123	3
1.23	3
1.230	4
0.00123	3
0.001230	4

Table 1: Examples of significant figures

5.1 Treating significant figures

The simplest definition for a *significant figure* is a digit (0 - 9) that actually represents some quantity. Zeros that are used to locate a decimal point are not considered significant figures. Any measured value, then, has a specific number of significant figures. See Table 1 for examples.

There are two major rules for handling significant figures in calculations. One applies for addition and subtraction, the other for multiplication and division.

1. **When adding or subtracting quantities**, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum (or difference).

   Examples:

   $$51.4 - 1.67 = 49.7 \qquad 7146 - 12.8 = 7133 \qquad 20.8 + 18.72 + 0.851 = 40.4$$

2. **When multiplying or dividing quantities**, the number of significant figures in the final answer is the same as the number of significant figures in the least accurate of the quantities being multiplied (or divided).

   Examples:

   $$2.6 \times 31.7 = 82 \;\; \textit{not} \;\; 82.42 \qquad 5.3 \div 748 = 0.0071 \;\; \textit{not} \;\; 0.007085561$$

5.2 Measured values and uncertainties: Number of decimal places

As mentioned earlier in Section 1.1, we learned that for any measurement $(X \pm \delta X)$, the number of decimal places of the measured value X must equal those of the corresponding uncertainty δX.

Below are some examples of correctly written measurements. Notice how the number of decimal places of the measured value and its corresponding uncertainty agree.

$$(L \pm \delta L) = (3.004 \pm 0.002) \text{ m} \qquad (m \pm \delta m) = (41.2 \pm 0.4) \text{ kg}$$

5.3 Rounding

Suppose we are asked to find the area $(A \pm \delta A)$ of a rectangle with length $(l \pm \delta l) = (2.708 \pm 0.005)$ m and width $(w \pm \delta w) = (1.05 \pm 0.01)$ m. Before propagating the uncertainties by using the multiplication rule, we should first figure out how many significant figures our final measured value A must have. In this case, $A = lw$, and since l has four significant figures and w has three significant figures, A is limited to three significant figures. Remember this result; we will come back to it in a few steps.

We may now use the multiplication rule to calculate the area:

$$\begin{aligned}
(A \pm \delta A) &= (l \pm \delta l) \times (w \pm \delta w) \\
&= (lw)\left[1 \pm \left(\frac{\delta l}{l} + \frac{\delta w}{w}\right)\right] \\
&= (2.708 \times 1.05)\left[1 \pm \left(\frac{0.005}{2.708} + \frac{0.01}{1.05}\right)\right] \ \mathrm{m}^2 \\
&= (2.843)\left[1 \pm (0.001846 + 0.009524)\right] \ \mathrm{m}^2 \\
&= 2.843\ (1 \pm 0.011370) \ \mathrm{m}^2 \\
&= (2.843 \pm 0.03232) \ \mathrm{m}^2
\end{aligned}$$

Notice that in the intermediate step directly above, we allowed each number one extra significant figure beyond what we know our final measured value will have; that is, we know the final value will have three significant figures, but we have written each of these intermediate numbers with four significant figures. Carrying the extra significant figure ensures that we will not introduce round-off error.

We are just two steps away from writing our final measurement. Step one is recalling the result we found earlier—that our final measured value must have three significant figures. Thus, we will round 2.843 m^2 to 2.84 m^2. Once this step is accomplished, we round our uncertainty to match the number of decimal places in the measured value. In this case, we round 0.03233 m^2 to 0.03 m^2. Finally, we can write

$$(A \pm \delta A) = (2.84 \pm 0.03) \ \mathrm{m}^2$$

6 References

These books are on reserve in the Physics Library (PHYS 291). Ask for them by the author's last name.

1. Bevington, P. R., *Data reduction and uncertainty analysis for the physical sciences* (McGraw-Hill, New York, 1969).

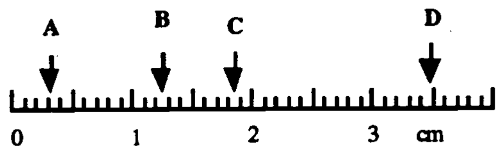
2. Young, H. D., *Statistical treatment of experimental data* (McGraw-Hill, New York, 1962).

Measurement Analysis Problem Set MA1

Name ____________________ Lab day/time ____________________

Division ____________________ GTA ____________________

Write your answers in the space provided. *For full credit, show all essential intermediate steps, include units and ensure that measured values and uncertainties agree in the number of decimal places.*



1. Using the above figure, write the measured values and uncertainties for the following:

   (a) $A = (\qquad \pm \qquad)$ cm.

   (b) $B = (\qquad \pm \qquad)$ cm.

   (c) $C = (\qquad \pm \qquad)$ cm.

   (d) $D = (\qquad \pm \qquad)$ cm.

   (e) How did you determine these uncertainties?

2. In the laboratory, your partner uses a digital balance to find the mass of a small object. He/she tells you (correctly) that the digital readout shows 43 g.

   (a) Write the correct mass measurement and its uncertainty.
   $(M \pm \delta M) = (\qquad \pm \qquad)$ g.

   (b) What is the percentage uncertainty of this measurement?

3. In lab, one of your partners determines (correctly) that the surface area of an object is 25.97 cm^2. All of the figures in this measurement are significant.

   (a) Your HP-9000 calculator tells you that the uncertainty is 0.04382361 cm^2. Write the appropriate measurement and its uncertainty.
   $(A \pm \delta A) = (\qquad \pm \qquad)\ cm^2$

   (b) Alternatively, your HP-9000 told you that the uncertainty is 0.0012543797 cm^2. Write the appropriate measurement and its uncertainty.
   $(A \pm \delta A) = (\qquad \pm \qquad)\ cm^2$

   (c) Finally, the calculator told you that the uncertainty is 0.017386642 cm^2. Write the appropriate measurement and its uncertainty.
   $(A \pm \delta A) = (\qquad \pm \qquad)\ cm^2$

   (d) Briefly explain how you determined these three numerical uncertainties.

4. In the laboratory you determine the gravitational constant $(g_{exp} \pm \delta g_{exp})$ to be $(9.824 \pm 0.006)\ m/s^2$. According to a geophysical survey, the accepted local value for $(g_{acc} \pm \delta g_{acc})$ is $(9.802 \pm 0.002)\ m/s^2$.

   (a) Draw a diagram like Figure 3 showing whether your measurement agrees with the accepted value within the limits of experimental uncertainty or not.

   (b) If these measures do not agree, what is the actual discrepancy?

5. For a laboratory exercise, you determine the masses of two airtrack gliders as $(m_1 \pm \delta m_1) = (484.9 \pm 0.3)$ g and $(m_2 \pm \delta m_2) = (314.2 \pm 0.1)$ g. Determine the following combinations of the measurements, and show your work.

   (a) $(m_2 \pm \delta m_2) - (m_1 \pm \delta m_1) = ($ $\pm$ $)$ g.

   (b) Determine the precisions of the measurements $(m_1 \pm \delta m_1)$ and $(m_2 \pm \delta m_2)$. Which calculated precision is the larger? Using this information, determine which is the better measurement.

   (c) $(m_1 \pm \delta m_1) + (m_2 \pm \delta m_2) = ($ $\pm$ $)$ g.

6. (a) Laboratory measurements performed upon a rectangular steel plate show the length $(l \pm \delta l) = (2.41 \pm 0.25)$ cm and the width $(w \pm \delta w) = (8.30 \pm 0.10)$ cm. Determine the area of the steel plate and the uncertainty in the area.
   $A \pm \delta A = ($ $\pm$ $)$ cm^2.

   (b) As Director of the National Science Foundation, you must decide what is the best means to improve the precision of the area measurement of the steel plate. You can spend money on a space gizmotron to better measure length, or on a superconducting whizbang to better measure width, but not both. On which measurement should you spend the money? Justify your decision with numbers.

7. During another airtrack experiment, you determine the initial position of a glider to be $(x_i \pm \delta x_i) = (0.482 \pm 0.001)$ m and its final position to be $(x_f \pm \delta x_f) = (0.633 \pm 0.003)$ m, with an elapsed time $(t \pm \delta t) = (0.48 \pm 0.01)$ s during the displacement.

   (a) Find the total displacement of the glider $(D \pm \delta D) = (x_f \pm \delta x_f) - (x_i \pm \delta x_i)$.
   $(D \pm \delta D) = ($ $\pm$ $)$ m.

   (b) Find the average speed of the glider $(\bar{v}_{glider} \pm \delta\bar{v}_{glider}) = (D \pm \delta D)/(t \pm \delta t)$.
   $\bar{v}_{glider} \pm \delta\bar{v}_{glider} = ($ $\pm$ $)$ m/s.

   (c) Calculate the percentage uncertainties for the two quantities $(D \pm \delta D)$ and $(t \pm \delta t)$. Based on these precisions, determine which of the two quantities contributes most to the overall uncertainty $\delta\bar{v}_{glider}$.

   (d) If we could change the apparatus so as to measure *either* distance *or* time ten times more accurately (but not both), which should we change and why?

8. In the laboratory, a measurement for $(x \pm \delta x)$ was taken as $(x \pm \delta x) = (33.4 \pm 0.5)\ \mathrm{s}^2$. Write a value for its reciprocal. Show calculations, and ensure that you handle significant figures properly.

   $\frac{1}{(x \pm \delta x)} = (\qquad \pm \qquad)\ \frac{1}{\mathrm{s}^2}$.

9. The formula for the volume of a box with height h , base b , and length l is given by $V = bhl$. Given initial measurements $(h \pm \delta h)$, $(b \pm \delta b)$, and $(l \pm \delta l)$, derive an expression for $(V \pm \delta V)$. Do NOT use the multiplication rule (Equation 3) in deriving this equation.

   Hint: Use the *derivation* of the worst case multiplication propagation rule (Equation 4) as a guide. Start with $(V \pm \delta V) = (b \pm \delta b)(h \pm \delta h)(l \pm \delta l)$. Expand, show intermediate steps, and regroup and simplify your solution as much as possible. Discard products of measurement uncertainties, such as $(\delta h)(\delta b)(l)$ and $(\delta h)(\delta b)(\delta l)$.

Experiment E1: Introduction to Computer Data Acquisition and Relationships Between Position, Velocity and Acceleration

Prelaboratory Questions are due at the start of this activity.

Goals of this activity

The goals of this experiment are as follows:

1. **Understanding of the relationships between position, time, velocity, for uniformly accelerated motion.** You will calculate average velocity and average acceleration from position and time data collected in the lab, and will describe the graphical relationships between position, time and acceleration.

2. **Learning how a ranging system works.** You will learn how such a system is calibrated, how to collect data with it, and what the various system limitations are.

3. **Using standard laboratory procedures and reporting techniques,** such as the determination and propagation of measurement uncertainties, the use of plotting software and standard laboratory reporting procedures. **You will use computer plotting software (preferably *KaleidaGraph* or less desirably *Excel*) to generate high quality plots.**

1 Theory

1.1 The Physics 152L sonar data acquisition system

In this experiment, you will first familiarize yourself with the ranging system we use in the Physics 152L laboratory. The system makes use of sound waves propagating in air, and the technique is known as sonar, which is an acronym for SOund NAvigation Ranging. You will this sonar system to study the motion of a glider under a constant gravitational acceleration.

At each station in the laboratory, you will find hardware similar to that illustrated in Figure 1. The three critical components to this setup are the airtrack, the glider with a reflector, and an ultrasonic transducer. Since we will often want to study the motion of this glider without worrying about frictional effects, we use an airtrack upon which the glider can float with very little friction.

The key component in the sonar measuring system is an ultrasonic transducer, which is simply a membrane which acts (1) as a speaker to emit pulses of high frequency sound and (2) as a microphone to detect the reflected pulses. The transducer sends out an ultrasonic pulse towards the glider's reflector. Immediately upon sending this pulse, a fast counter, which increments by one count every 2 $\mu s = 2 \times 10^{-6}$ s, is started. The pulse propagates through the air, hits the glider's reflector, and is reflected back towards the transducer. When the reflected pulse is received by the transducer, the counter is stopped. You can think of this process as timing how long it takes your friend to run away from you, touch a wall, and

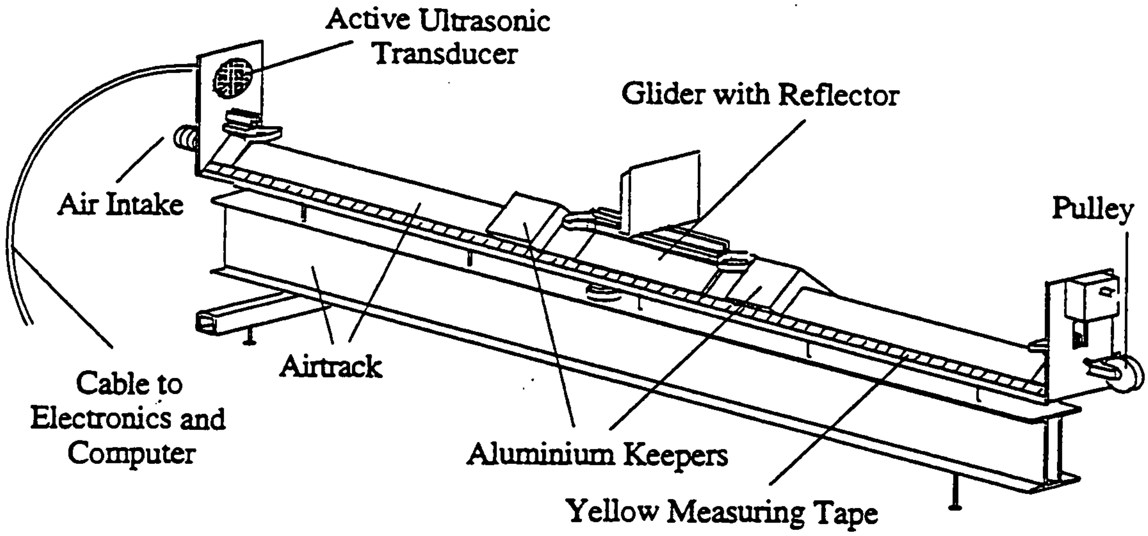


Figure 1: Air track with a glider and associated hardware.

run back to you. You start your stopwatch (a "counter") when your friend (analogous to the "pulse") runs away from you, and you keep your stopwatch running until your friend finally gets back to the starting position.

All distance measurements taken by the sonar apparatus are performed in this manner. However, the sonar measures distance in units of clock counts, which are not very useful for our purposes, because we generally measure distance in units of length, such as meters. Therefore, we must find a way to convert the sonar's counts into the more useful units of meters. Fortunately, this conversion is readily performed in the laboratory, and is what you will do for Part I of the experiment.

1.2 Calibration of the ultrasonic system

It turns out that the number of counts n is proportional to the distance $D + d$ between the transducer and the reflector. In other words,

$$n = k(D + d) \tag{1}$$

where k is a constant with the units of counts per meter. As shown in Figure 2, D is the distance between the reflector surface and the face of the ultrasonic transducer mounting plate, and d is the distance between the face of the mounting plate to the active membrane of the transducer. In principle, to determine k, and hence calibrate the system, we only need to find n corresponding to a single measured distance. The problem with this in practice is that d is not well known. However, we can get around this problem by taking data for two different positions of the glider. Then, the calibration will depend only on the distance between the two positions of the glider, and d does not matter. Algebraically, $n_1 = k(D_1 + d)$

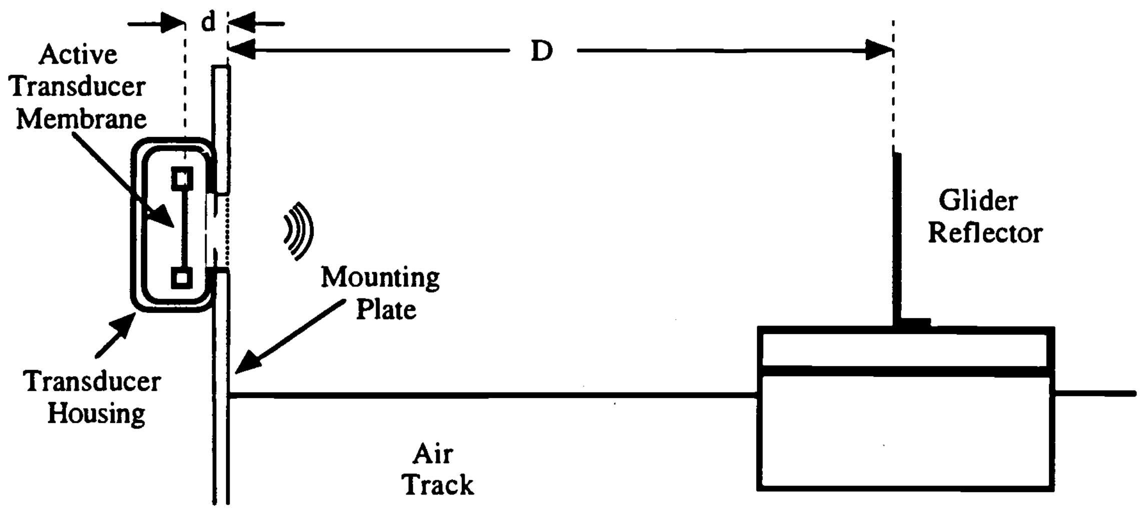


Figure 2: Definition of D and d in Equation 1.

for position 1 and $n_2 = k(D_2 + d)$ for position 2, which yields,

$$k = \frac{n_2 - n_1}{D_2 - D_1} \tag{2}$$

1.3 Average velocity and acceleration from (t, x) measurements

Given an initial time and position (t_i, x_i) and a final time and position (t_f, x_f) of an object, we define the *average velocity* by

$$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \tag{3}$$

In a similar manner, if we are given an initial time and velocity (t_i, v_i) and a final time and velocity (t_f, v_f), we define the *average acceleration* by

$$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \tag{4}$$

These equations are also described in Serway sections 2.1–2.3. Remember that these average values are associated with the midpoint of the motion of an object. Unless the velocity is constant $\bar{v}$ occurs neither at the time t_i nor at t_f, but at the average (or midpoint) of these times, $\frac{t_i+t_f}{2}$. A similar argument holds for $\bar{a}$.

Time t (s)	Position x (m)	Disp. Δx (m)	Interval Δt (s)	Avg Vel $\bar{v} = \frac{\Delta x}{\Delta t}$ (m/s)	Ch in $\bar{v}$ $\Delta\bar{v}$ (m/s)	Interval Δt (s)	Avg Acc $\bar{a} = \frac{\Delta\bar{v}}{\Delta t}$ (m/s^2)
0.000	0.250	......	......	......	......	......	......
0.125	......	-0.090	0.250	-0.360	......	......	......
0.250	0.160	......	......	......	-0.028	0.250	-0.112
0.375	......	-0.097	0.250	-0.388	......	......	......
0.500	0.063	......	......	......	......	......	......

Table 1: Example of the use of a table to determine $\bar{v}$ and $\bar{a}$.

Using this information, we can calculate values for the average velocity $\bar{v}$ and average acceleration $\bar{a}$ from original time and position (t, x) data. Table 1 demonstrates this procedure. Suppose that we collected three (t, x) data points in the laboratory; namely (0.000 s, 0.250 m), (0.250 s, 0.160 m), and (0.500 s, 0.063 m). We record the time data in the first, third, and fifth rows under the column heading "Time" (we will get back to the second and fourth row entries soon) and the position data in the first, third, and fifth rows under the column heading "Position." With this information, we can calculate the remainder of the table. Our first step is to calculate values for Δt, the difference in time between two consecutive (t, x) data points, and Δx, the displacement (or difference in position) between two consecutive (t, x) data points. Now we can find two average velocity values $\bar{v}$, since $\bar{v} = \frac{\Delta x}{\Delta t}$.

At this point, it is critical to realize that our average velocity values $\bar{v}$ occur at *different* times than our original (t, x) data—the values $\bar{v}$ occur at a time in between, or at the average, of the original (t, x) data. The table shows us that the first value for $\bar{v}$, -0.360 m/s, appears on the second row; the time for that row is written in italics to point out that it is an *average* time. Thus, on a velocity vs. time (v vs. t) graph, you would plot -0.360 m/s at the average time 0.125 s.

The procedure for finding the average acceleration $\bar{a}$ is quite similar to that used for finding the average velocity $\bar{v}$. Table 1 illustrates the necessary steps. Notice that the value for $\bar{a}$ in the table occurs at 0.250 s, the average of the "average times" 0.125 s and 0.375 s.

We will follow this algorithm when calculating average velocities and accelerations with the computer throughout this course — we require at least TWO position data points (t, x) to determine ONE average velocity data point $(t, \bar{v})$ which occurs out of sync with the original position data. We require at least TWO velocity points $(t, \bar{v})$ to calculate ONE average acceleration data point $(t, \bar{a})$, which is out of sync with the velocity data. Hence, we need THREE position data points (t, x) to determine ONE average acceleration data point $(t, \bar{a})$, which is back in sync with the original position data.

This seems simple, but is actually quite profound — to get higher-order information we must lose some lower-order information: two positions for one velocity; three positions for one acceleration. This is characteristic of both the problem and the mathematics developed to address this kind of motion; that is, a differential 'calculus of infinitesimals' developed by

Newton, Leibnitz, and others.

2 Experimental method

Items on the Data and Calculations section of your laboratory report which you should complete in the laboratory are indicated by the (√) symbol.

To effectively gather the necessary data in lab, you should review the *LabVIEW graph scaling* and *LabVIEW graph cursors* sections of the Hardware and Software Guide in this manual.

Part I. Calibration of the ultrasonic system

In this part of the experiment you will determine the constant k in Equation 2.

1. Set the glider at ~ 0.2000m (200.0 mm) by using the yellow tape measure on the air track. Use the aluminum keepers to hold the glider still. Record this measured value D_1 in your lab book. Next, determine the uncertainty in the yellow tape measure δD_1 associated with this measurement. Record $(D_1 \pm \delta D_1)$ on your laboratory data sheet.

2. Determine n_1 by taking data using the sonar measuring system. Click the *ACQUIRE DATA* button on the screen to collect data; your computer should show the data acquisition in real time, and after ten seconds it will return control to you. Now use the on-screen cursors to determine the value for n_1 and the associated uncertainty δn_1.

3. Move the glider to ~ 0.7000m (700.0 mm). Record $(D_2 \pm \delta D_2)$.

4. Acquire data again, and record $(n_2 \pm \delta n_2)$.

5. Determine the value of k from your two data runs and Equation 2. Be careful with rounding and significant figures (see Section 5 in MA1). Your TA will ask you to complete and show this calculation for k before you leave the laboratory. If your k value is reasonable, record it in the *Input k* box on the screen.

Part II. Precision and Accuracy of the distance measurements

In Equation 1, n is an integer and this limits the precision of our measurements. This phenomenon is called digitization error. Of course, in this apparatus there are other factors which will limit the precision, but ultimately the fact that n can only be an integer determines the theoretical best you can do. In this part you will investigate the actual experimental uncertainty in n for a fixed position, and the resulting uncertainty in the position measurement. Again, be careful with your significant figures.

1. Use the data run from which you recorded $(n_2 \pm \delta n_2)$. This time rescale the horizontal axis of your counts vs. time plot to view a smaller time interval, so that you can see

individual points. Most of your points should be at some constant value with the remainder of the points deviating up or down by a few counts. Choose a typical region of data (not the noisiest but with some noise) and enlarge it. The usual settings show a one to two second range on the horizontal (time) axis and a five to ten count range on the vertical (counts) axis.

2. Using the on-screen cursors, record the time and counts data (t_i, n_i) for the twenty consecutive points you choose.

3. Print out this graph, ensuring that the limits of the horizontal (time) and vertical (counts) axes show a reasonable sampling of points. (See *Printing screen dumps* in the Hardware and Software Guide for information on making printouts). Be sure to title this printout (by hand).

4. Using the table, determine the average number of counts and standard error of the mean of the position measurement. This determines the precision of the system.

5. Enter your value for k in the *Input k* box on the screen. Ensure that the position values now plotted on the position vs. time plot (the lower plot) seem reasonable.

6. Check to see that your air track is reasonably level by placing the red glider on it without the aluminum keepers. If level, there should be little motion or very slow movement of the glider. If the track does not appear level, call your TA.

7. Place the glider at several locations on the airtrack, and use the computer to measure the position of the glider. Record the yellow measuring tape values for glider position and compare the computer values to the measuring tape values. Comment on systematic and random uncertainty in the accuracy of the sonar glider position measurements.

Part III. Motion of a glider on a level airtrack

In this part of the lab you will collect and observe position and time data for a glider moving from end to end of the track.

1. Practice giving the glider an initial velocity so that it bounces from endpost to endpost smoothly, without jumping off the airtrack when it strikes an endpost.

2. When you succeed in obtaining the motion described above, click the *ACQUIRE DATA* button. The computer should gather ten seconds worth of this data.

3. You will now collect (t_n, x_n) data from the position vs. time plot (the lower plot). Find one complete end-to-end bounce on this plot, and pick your beginning time t_0 about 1.0 seconds before the bounce. Use the cursors to find the time and position of this point; record that data in Table 5. Also determine an uncertainty for t_0.

4. Complete the (t_n, x_n) columns in Table 5 by measuring the position every 0.20 s. *This means you will record the data from every tenth sample taken by the computer.* Be sure you are taking data at the correct intervals of 0.20 s, otherwise your subsequent $\bar{v}$ and $\bar{a}$ values will be in error.

5. Print out a copy of your x vs. t graph to hand in with your lab report. This plot should be cropped by changing the vertical (position) axis and horizontal (time) axis to show the region of interest (i.e., the data you recorded in the table). Be sure to title this plot.

6. Click the *Go to a,v,x graphs* button to switch the display to the position, velocity, and acceleration versus time plots.

7. Set the horizontal (time) axes for all three plots to show the entire range of motion; that is, set them from zero to ten seconds.

8. Ensure that the vertical axes for each plot have been set correctly. The entire range of motion should be visible for these axes as well; all high points, low points, peaks, and valleys.

9. Make a print out of this screen and title it.

Part IV. Motion of a glider on an inclined plane

In this part of the experiment you will collect position and time data for a glider starting with an initial velocity from the bottom end of the inclined plane. We want the glider to go up the inclined plane, be slowed to a stop by the gravitational acceleration, and then come back to the bottom of the incline. We do not want the glider to touch the right endpost.

1. Incline the airtrack by placing the block under the single leg of the airtrack.

2. Practice giving the glider an initial velocity so that it goes *at least* three-quarters along the length of the track before coming to a stop, *and* does not hit the right endpost.

3. When you succeed in obtaining the motion described above, click the *ACQUIRE DATA* button. The computer should gather ten seconds worth of this data.

4. You will now collect (t_n, x_n) data from the position vs. time plot (the lower plot). Find the first complete parabola on this plot, and pick your beginning time t_0 as close to the left side of this parabola as you can without being at a "bounce." Use the cursors to find the time and position of this point; record that data in Table 6. Also determine an uncertainty for t_0.

5. Complete the (t_n, x_n) columns in Table 6 by measuring the position every 0.40 s. *This means you will record the data from every twentieth sample taken by the computer.* Be sure you are taking data at the correct intervals of 0.40 s, otherwise your subsequent $\bar{v}$ and $\bar{a}$ values will be in error.

6. When gathering this data, be sure not to take points beyond a "bounce." If for some reason you anticipate going beyond a bounce, consult your TA.

7. Print out a copy of your x vs. t graph to hand in with your lab report. This plot should be cropped by changing the vertical (position) axis and horizontal (time) axis to show the region of interest (i.e., the data you recorded in the table). Be sure to title this plot.

8. Click the *Go to a, v, x graphs* button to switch the display to the position, velocity, and acceleration versus time plots.

9. Set the horizontal (time) axes for all three plots to show the entire range of motion; that is, set them from zero to ten seconds.

10. Ensure that the vertical axes for each plot have been set correctly. The entire range of motion should be visible for these axes as well; all high points, low points, peaks, and valleys.

11. Make a print out of this screen and title it.

Final checks before you leave

1. Be sure you have completed all items on your Data and Calculations section that have been marked by the ($\surd$) symbol.

2. Be sure you have five printouts; one for Part II and two each for Parts III and IV. If you want to make additional printouts you may do so.

3. Be sure you calculated k properly, both in its measured value and its uncertainty. Have your TA look over this value if he or she has not done so already.

4. Have your TA check over and initial your data sheet before you leave the laboratory.

Prelaboratory Questions for E1

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

Your answers are due at the start of your laboratory session, and will be worth zero if not turned in at that time. Write your answers in the space provided, showing the essential steps that led to them.

1. In an air track experiment students obtained the following calibration data: $D_1 = (0.2000 \pm 0.0005)$ m, $n_1 = (1021 \pm 2)$ counts, $D_2 = (0.7000 \pm 0.0005)$ m and $n_2 = (2720 \pm 5)$ counts. Use Equation 2 and calculate $k \pm \delta k$. Show all work.

   $k = ($ $\pm$ $)$ counts/m.

2. During an experiment with a different glider and the air track inclined at an angle the following data were obtained. Complete the table to analyze the motion of the glider:

Time t (s)	Position x (m)	Disp. Δx (m)	Interval Δt (s)	Avg Vel $\bar{v} = \frac{\Delta x}{\Delta t}$ (m/s)	Ch in $\bar{v}$ $\Delta \bar{v}$ (m/s)	Interval Δt (s)	Avg Acc $\bar{a} = \frac{\Delta \bar{v}}{\Delta t}$ (m/s^2)
0.000	0.200	......	......	......	......	......	......
0.125	......		0.250		......	......	......
0.250	0.269	......	......	......		0.250	
	......		0.250		......	......	......
0.500	0.325	......	......	......		0.250	
	......		0.250		......	......	......
0.750	0.369	......	......	......		0.250	
	......		0.250		......	......	......
1.000	0.399	......	......	......		0.250	
	......		0.250		......	......	......
1.250	0.419	......	......	......		0.250	
	......		0.250		......	......	......
1.500	0.425	......	......	......		0.250	
	......		0.250		......	......	......
1.750	0.419	......	......	......		0.250	
	......		0.250		......	......	......
2.000	0.399	......	......	......	......	......	......

Table 2: Laboratory data from a glider on an inclined track.

(a) Complete Table 2. Note the skipped entries indicate we cannot know x, $\bar{v}$, and $\bar{a}$ all simultaneously.

(b) Calculate the mean of all the table values for average acceleration, and then write the values for $\bar{a} \pm \sigma_{\bar{a}}$. Note that there are no theoretical grounds for believing in a nonconstant acceleration here.

$\bar{a}$ = ($\qquad\qquad$ ± $\qquad\qquad$) m/s^2

(c) Calculate the percentage uncertainty $\frac{\sigma_{\bar{a}}}{\bar{a}} \times 100\%$.

$\frac{\sigma_{\bar{a}}}{\bar{a}} \times 100\% = ($ $) \%$.

3. Use plotting software (such as *KaleidaGraph*) to make three separate graphs of the gliders' position $x(t)$, average velocity $\bar{v}(t)$, and average acceleration $\bar{a}(t)$ as a function of time from these data (Table 2). Use the curve fitting capability of your plotting software to fit $x(t)$, $\bar{v}(t)$, and $\bar{a}(t)$ with a low order ($\leq$ second order) polynomial.

   (a) Label your plots, and attach them to this prelab report. Why are $(t, \bar{v})$ points plotted at different instants of time than (t, x) and $(t, \bar{a})$?

   (b) Briefly interpret these graphs in your own words. How do the fitted curves compare to each other and what is physically happening at the critical points on the plots? (Be especially careful when discussing the acceleration plot. The acceleration data are quite noisy, so don't try to interpret noise as something meaningful – look instead for the trend or average and comment.)

E1 Laboratory Report

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

Laboratory reports are due one week [SEVEN CALENDAR DAYS] from the day you perform the experiment. Attach a green Physics 152L cover sheet (available in the lab or window of PHYS Room 144) to the front of your exercise or report and put it in the 'Room 144 Drop Slot for Physics Lab Reports' located under the mailboxes across from Physics Room 149 before 10:00 p.m. of the day it is due.

Abstract (10 points)

Write your Abstract in the space provided below. Devote 1-2 paragraphs to briefly *summarize and describe the experiments in terms of theory, activity, key findings and agreements.* Include actual numerical values, agreements and discrepancies with theory. Especially mention your values for k, and the precision and accuracy of the sonar system. Summarize the relationship between $x(t)$, $\bar{v}(t)$, and $\bar{a}(t)$. Write the abstract AFTER you have completed the entire report, not before.

Data and Calculations (45 points)

Part I. Calibration of the ultrasonic system

Record the following data, determining appropriate uncertainties by eye.

1. (√) $D_1 =$ (±) m.
2. (√) $n_1 =$ (±) counts.
3. (√) $D_2 =$ (±) m.
4. (√) $n_2 =$ (±) counts.
5. (√) Value of $k =$ (±) counts/m. Show your work for calculating $(k \pm \delta k)$ here, and have your TA check your calculations before you leave the room. A reasonable value for k is about 2900 counts/m.

Part II. Precision of the distance measurements

i	Time T_i	Counts	Deviation	Deviation^2
	(s)	n_i	$(\bar{n} - n_i)$	$(\bar{n} - n_i)^2$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
Totals	----	$\sum_{i=1}^{N} n_i$	$\sum_{i=1}^{N} (\bar{n} - n_i)$	$\sum_{i=1}^{N} (\bar{n} - n_i)^2$
----	----			

$$\text{Here } \bar{n} = \frac{1}{N}\sum_{i=1}^{N} n_i, \quad N = 20, \quad s_n^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\bar{n} - n_i)^2, \quad \sigma_{\bar{n}} = \frac{s_n}{\sqrt{N}}$$

Table 3: Statistical analysis of counts

1. (√) Choose 20 continuous points from a typical section of your graph of a stationary glider. Complete the 'Time' and 'Counts' columns in Table 3.

2. (√) Print out your 'enlargement of the twenty consecutive points' graph, and title it "Part II." Indicate the points you chose, and label any other interesting features. Attach this printout to this page when you hand in your report.

3. Complete the remainder of Table 3. Use this information to complete the following items.

4. Mean value $\bar{n}$ of 20 consecutive counts measurements $\bar{n} = (\qquad)$ counts.

5. Standard deviation s_n of 20 consecutive counts measurements $s_n = (\qquad)$ counts.

6. Standard Error of the Mean $\sigma_{\bar{n}}$ of 20 consecutive counts measurements $\sigma_{\bar{n}} = (\qquad)$ counts.

7. Use k and $\bar{n} \pm \sigma_{\bar{n}}$ to determine $D_{glider} = (\qquad \pm \qquad)$ m.

8. The computer measures position in counts, which are always whole numbers (integers). Therefore, the smallest *change* in position that the computer can measure corresponds to one count.

   (a) Without using uncertainties, convert one count into a distance measurement (in meters) using your value for k.

(b) If you changed the position of the glider by an amount less than what you found in 8a, would the sonar system be able to detect this change? Explain in terms of precision.

9. (√) Value of k from Part I: $k = (\qquad \pm \qquad)$ counts/meter. Enter your k value now. A plot of $x(t)$ should appear.

10. (√) Move the glider to various locations on the airtrack and complete the following table:

sonar Position Readout (m)	Tape Measure Position (m)	Difference (m)

Table 4: sonar-measuring tape comparison

11. Discuss these data (from Table 10) in terms of sonar measurement accuracy.

Part III. Motion on a level track

1. (√) Time t_0 at which you are beginning your study of the motion of your glider:
   $t_0 = ($ $\pm$ $)$ s.

2. (√) Complete the 'Time' and 'Position' columns in Table 5 by using the cursors on the position vs. time data on the computer screen. Record enough data to complete all of the time intervals shown. *Note that you are recording data every* 0.20*s, which corresponds to every tenth data point. Also note that you need not fill in the* average *time values in the laboratory; that is, you should fill in only the first, third, fifth, etc. rows of the 'Time' and 'Position' columns in the laboratory.*

Time t (s)	Position x (m)	Disp. Δx (m)	Interval Δt (s)	Avg Vel $\bar{v} = \frac{\Delta x}{\Delta t}$ (m/s)	Ch in $\bar{v}$ $\Delta\bar{v}$ (m/s)	Interval Δt (s)	Avg Acc $\bar{a} = \frac{\Delta\bar{v}}{\Delta t}$ (m/s^2)
		......	......	......	......	......	......
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......		0.20	
	......		0.20		......	......	......
		......	......	......	......	......	......

Table 5: Motion of a glider on level track

3. (√) Print out your sonar position vs. time plot for this section of data and attach it to this page when you hand in your laboratory report. Title this printout "Part III."

4. Complete the remaining entries of the table.

5. (√) Use the LabVIEW software to plot values of $x(t)$, $\bar{v}(t)$ and $\bar{a}(t)$ from the table. Label the plots clearly. Remember that your values for $\bar{v}$ will occur at different times than those of $\bar{a}$.

6. Use a pen to label your plotted $x(t)$, $v(t)$ and $a(t)$ graphs as necessary to illustrate your answers to the following questions:

   (a) Describe how the physical motion of the glider relates to the plot of $x(t)$ for these plotted data. Tell what the glider is doing at the different points on the graph — describe and indicate on the plot. Label the following: the nearest and farthest distance from the active left hand transducer, glider striking an endposts.

   (b) Describe how the plot of $\bar{v}(t)$ changes and what the glider is doing for your data.

   (c) Describe how the plot of $\bar{a}(t)$ changes and what the glider is doing for your data. What is happening at the exact instant of the bounces?

   (d) Where does $\bar{v}(t)$ have a consistent value on this plot? Why?

   (e) Where does $\bar{a}(t)$ have a consistent value on this plot? Why?

Part IV. Motion on an inclined track

1. (√) Time t_0 at which you are beginning your study of the motion of your glider:
   $t_0 = (\quad \pm \quad)$ s.

2. (√) Complete the 'Time' and 'Position' columns in Table 6 by using the cursors on the position vs. time data on the computer screen. Record enough data to complete all of the time intervals shown. *Note that you are recording data every* 0.40s, *which corresponds to every twentieth data point. Also note that you need not fill in the* average *time values in the laboratory; that is, you should fill in only the first, third, fifth, etc. rows of the 'Time' and 'Position' columns in the laboratory.*

Time t (s)	Position x (m)	Disp. Δx (m)	Interval Δt (s)	Avg Vel $\bar{v} = \frac{\Delta x}{\Delta t}$ (m/s)	Ch in $\bar{v}$ $\Delta \bar{v}$ (m/s)	Interval Δt (s)	Avg Acc $\bar{a} = \frac{\Delta \bar{v}}{\Delta t}$ (m/s^2)
		......	......	......	......	......	......
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......		0.40	
	......		0.40		......	......	......
		......	......	......	......	......	......

Table 6: Motion of a glider on an inclined track

3. (√) Print out your position vs. time plot for this section and attach it to this page when you hand in your laboratory report. Title this printout "Part IV."

4. Complete the remaining entries of the table.

5. Calculate a value and standard error of the mean for the average of the tabulated average accelerations. $\bar{a} = ($ $\pm$ $)$ m/s^2

6. (√) Use the LabVIEW software to plot your values of $x(t)$, $\bar{v}(t)$ and $\bar{a}(t)$ from the table. Label the plots clearly. Remember that your values for $\bar{v}$ will occur at different times than those of $\bar{a}$.

7. Use a pencil to label your $x(t)$, $v(t)$ and $a(t)$ graphs as necessary to illustrate your answers to the following questions:

   (a) Describe how the physical motion of the glider relates to the plot of $x(t)$ for your plotted data. Tell what the glider is doing at the different points on the graph — describe and indicate on the plot. Label the following: the nearest and farthest distance from the active left hand transducer, glider striking an endposts.

   (b) Describe how the plot of $\bar{v}(t)$ changes and what the glider is doing for your data.

   (c) Describe how the plot of $\bar{a}(t)$ changes and what the glider is doing for your data.

Analysis (15 points)

Write your Analysis in the space provided. *Using numerical examples and analysis, discuss the larger sources of uncertainty in your calculations.* Use your numbers to justify possible improvements in apparatus and methods. Comment on δk and its sources ($\delta D_1 + \delta D_2$) and ($\delta n_1 + \delta n_2$). Which contributed most to δk? How could we reduce δk?

Conclusions (5 points)

Write your Conclusions in the space provided. Did the experiment tend to confirm the theoretical physical relationships examined? Choose a significant mathematical relationship discussed in this experiment, and use it to numerically explain an activity or phenomenon from outside of the classroom. Illustrate your example by using genuine, approximate or estimated numbers.

Experiment E2: Newton's Second Law, Work, and Kinetic Energy

Prelaboratory Questions are due at the start of this activity.

Goals of this experiment

In this experiment you will predict the acceleration of a glider on an air track due to forces acting on it, and compare it to the measured acceleration. In addition, you will compare the final kinetic energy of a glider staring from rest with the amount of work performed on the glider by a force acting on it.

1 Theory

1.1 Newton's Second Law

Newton's second law states that the acceleration **a** of an object is directly proportional to the net force $\Sigma\,\mathbf{F}$ acting on it and is inversely proportional to its mass m.

This is usually written as

$$\Sigma\,\mathbf{F} = m\mathbf{a} \tag{1}$$

as in Serway's Equation 5.2. In this experiment, net force is generated by the gravitational attraction between objects and the earth, given by

$$\mathbf{F}_{weight} \equiv \mathbf{w} = m\mathbf{g} \tag{2}$$

where m is the mass of the object and **g** points vertically downward and has a magnitude of (9.80146 ± 0.00002) m/s^2 on the main campus of Purdue University. Note that Serway defines the weight of an object as **w**. This is Serway's Equation 5.7.

In this experiment, we will use gravitational attraction to create net forces that will result in motion, but these net forces will be slightly more complex than that given by Equation 2. We will create a net force upon a glider on an inclined plane as shown in Figure 1 and Serway's Example 5.7, such that an angle θ is formed by raising an incline of length L by a height h.

Here the geometry dictates that

$$\sin\theta = \frac{h}{L} \tag{3}$$

and the component of gravitational force parallel $F_{\parallel}$ to the surface of the inclined plane is given by

$$F_{\parallel} = mg\sin\theta = mg\frac{h}{L} = \frac{mgh}{L}. \tag{4}$$

The predicted acceleration is given by

$$a_{pred} = g\sin\theta = \frac{gh}{L}. \tag{5}$$

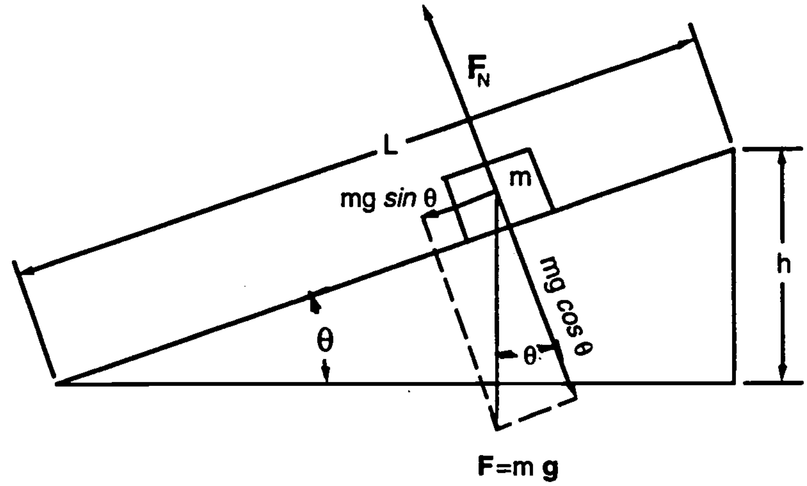


Figure 1: Free-body diagram for an object of mass m at rest on an inclined plane.

A second, even more complex situation occurs when a hanging mass is linked to a glider on a level airtrack using a pulley and thread as shown in Figure 2. Here the **W** is due to m_w but the net force exerted on the hanging mass m_w is diminished by the tension in the string, and the resulting motion depends upon the mass of the combined glider-hanging mass system, not simply the hanging mass. You will derive the exact relationship for the Σ **F** of the glider-hanging mass system in your Prelaboratory Questions.

The predicted acceleration of this glider–hanging mass system is given by

$$a_{pred} = \left(\frac{m_w}{M + m_w}\right) g. \tag{6}$$

In this experiment, you will first predict the acceleration for the two systems described above, and then calculate an experimental value for these systems. To determine the experimental average acceleration $\bar{a}$, you will use the following equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \tag{7}$$

where (t_i, v_i) and (t_f, v_f) are initial and final points measured from a velocity versus time plot. This is Serway's Equation 2.5, and is the technique you will use in this experiment to measure acceleration.

1.2 Work

If a force moves an object a distance s, mechanical work W has been done on that object. If the force is constant, the work W is equal to the product of the component of the force in the direction of the displacement ($F \cos\theta$) and the magnitude of the displacement (s):

$$W = Fs\cos\theta, \tag{8}$$

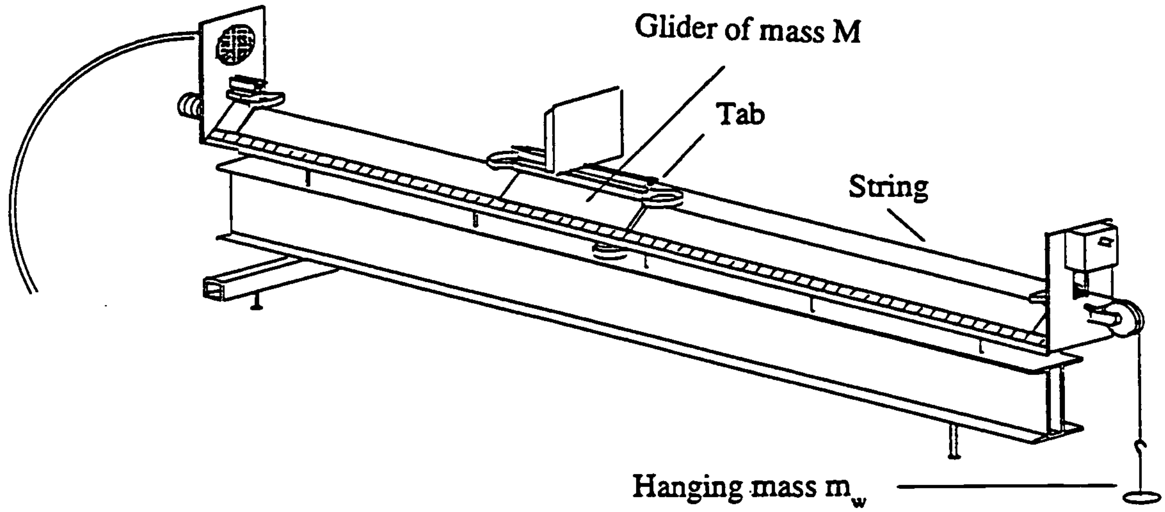


Figure 2: Part III: Airtrack with hanging mass.

This is Serway's Equation 7.1.

If the force and displacement are parallel to the direction of the force, $\cos\theta = 0$, and work W done on the object is simply

$$W = Fs \text{ if the force and displacement are parallel.} \quad (9)$$

In this experiment, work will always be done by forces parallel to the direction of displacement and Equation 9. In Part II, the force of gravity acts on the glider with mass M through a distance s along the track. Using the force expressed in Equation 4, we find

$$W = Fs = \left(\frac{Mgh}{L}\right) s. \quad (10)$$

In Part III, the weight of the hanging mass m_w provides the force which displaces the glider a distance s. Thus, $F = m_w g$, and so

$$W = Fs = (m_w g)s. \quad (11)$$

1.3 Kinetic Energy

The kinetic energy of an object with mass m is defined as

$$K \equiv \frac{1}{2}mv^2. \tag{12}$$

This is Serway's Equation 7.14. The *change* in kinetic energy of a system is given by

$$\Delta K = K_f - K_i. \tag{13}$$

In our experiments, the glider will always start from rest, so $v_i = 0$ and thus $K_i = 0$ and

$$\Delta K = K_f \text{ if } K_i = 0. \tag{14}$$

In Part II, the system is just the glider of mass M, so the change in kinetic energy takes the form

$$\Delta K = K_f = \frac{1}{2}Mv_f^2 \text{ if } K_i = 0. \tag{15}$$

In Part III, the system includes the glider of mass M *and* the hanging mass of mass m_w, so

$$\Delta K = K_f = \frac{1}{2}(M + m_w)v_f^2 \text{ if } K_i = 0. \tag{16}$$

1.4 The Work–Energy Theorem

The work–energy theorem states that the work done on a system equals the change in kinetic energy of that system, or

$$W = \Delta K. \tag{17}$$

It is this relation that you will investigate in lab for Parts II and III. Keep in mind that our use of the work-energy theorem does not account for non-conservative forces such as friction.

Applying the work–energy theorem to the inclined plane experiment in Part II, we expect

$$\left(\frac{Mgh}{L}\right)s = \frac{1}{2}Mv_f^2 \text{ if } v_i = 0 \tag{18}$$

where W and ΔK are given by Equations 10 and 15, respectively.

For the glider–hanging mass experiment in part III, we expect

$$(m_w g)s = \frac{1}{2}(M + m_w)v_f^2 \text{ if } v_i = 0 \tag{19}$$

where W and ΔK are given by Equations 11 and 16, respectively.

In the laboratory you will measure both W and ΔK and compare them to see if the equality predicted by the work–energy theorem holds.

2 Experimental Method

An air track, a glider, and associated hardware will be used in this experiment to study the motion of gliders under constant acceleration. An ultrasonic ranging system is used to determine the position of the reflector on a glider on the air track. In Experiment 1 you learned how to obtain velocity versus time (v vs. t) from a position versus time (x vs. t) plot. As before, you can take an unlimited number of runs quickly with this program. However, print out only those plots that you will be turning in with your lab report.

In Part I you will calibrate the software, make sure that the air track is leveled, and determine the masses of the glider and some weights using one of the two scales provided. In Part II of the experiment the glider will move down a tilted air track; you will predict and calculate its acceleration, and then compare the work done on the glider to its final kinetic energy. In Part III the track will be level and the glider will be pulled by a weight hanging on a very light string; you will predict and calculate the acceleration, and then compare the work done on the glider – hanging mass system to its final kinetic energy. In Part IV, you will investigate a possible source of measurement uncertainty.

Part I. Set up and calibration

The first three steps in this part of the experiment do not have to be done in order.

1. Measure the mass M of the glider and then the combination of the 5 g mass hanger and 10 g mass with one of the scales available in the laboratory. This combined mass m_w should be approximately 15 g. You can delay measuring the masses until the waiting line for the scales is short, and can even proceed to Parts II and III as long as you measure the mass before you leave. Determine the uncertainty of the balance by finding the smallest amount measurable on the balance and dividing this value by two. (Refer to "Taking measurements in lab," in Measurement Analysis 1.)

2. Set up your air track so that the end with the pulley hangs over the edge of the table. Check to see that your air track is level by placing the glider at rest on the air track. If the track is reasonably level, there should be little or no motion. If the track does not appear to be level, ask for assistance.

3. Calibrate the ultrasonic transducer by placing the glider at the two specified positions along the air track and taking a calibration sample at each position with the *CALIBRATE 1* and *CALIBRATE 2* buttons, using the two aluminum bumpers to keep the glider in fixed positions. The 0.5000 m difference in the two positions should be measured as exactly as possible using an edge of the glider and the yellow ruler taped to the air track. Check your value of k — does it seem to be an appropriate value — does it agree with what you know about k from E1?

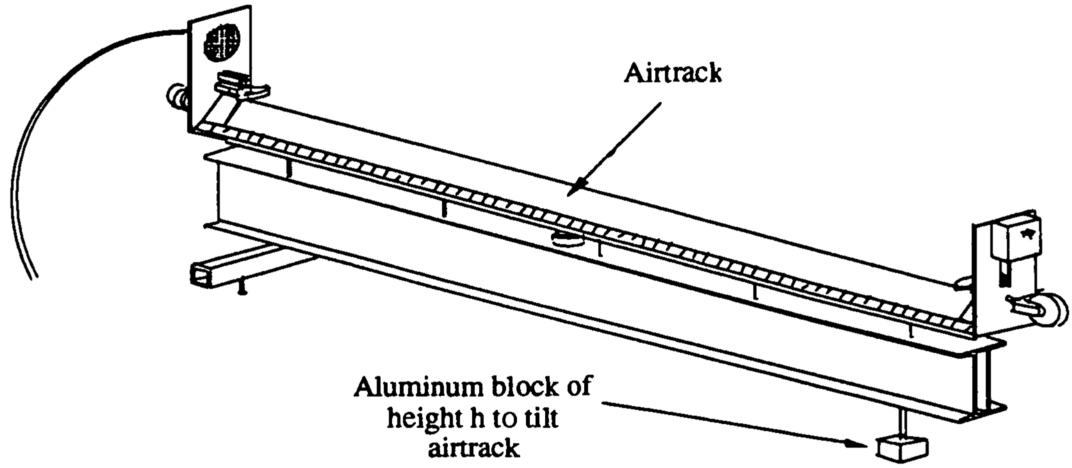


Figure 3: Part II. Tilting the airtrack with an aluminum block of height h

Part II. Motion on an inclined plane

1. Incline the air track by placing the ~ 0.0254 m thick aluminum block under the single foot on one end of the air track as shown in Figure 3.

2. Hold the glider at rest at the elevated end of the air track, and click the *ACQUIRE DATA* button. Wait until two seconds of data have been taken by the computer, then release the glider smoothly. The idea here is to let it fall away from rest, or to achieve a release velocity v_i of zero.

3. After the computer returns control to you, examine the velocity vs. time plot first. Identify the release point on this graph; the coordinates of this point are (t_i, v_i). This point is where the velocity vs. time plot changes from a horizontal line with no slope to one with some slope. Record these two values in your lab manual on page 88.

4. Next identify the point just before the glider bounces; the coordinates of this point are (t_f, v_f). Again, record these values in your lab manual.

5. Now find the point (t_i, x_i) on the position vs. time plot. Use the value of t_i you obtained in step 3 above to find x_i. Note that the time values on the position vs. time plot are shifted a certain amount from those on the velocity vs. time plot; be sure to include an uncertainty δt that compensates for this effect. Record the value of x_i.

6. Find the point (t_f, x_f). Use the value of t_f you obtained to find x_f. Record this value.

7. Make a printout of the velocity and position vs. time plots.

8. Calculate a predicted acceleration for the glider.

9. Determine an experimental value for the acceleration of the glider, and compare it with your predicted value.

10. Calculate the work done on the glider. Then calculate the change in kinetic energy of the glider. Compare these two values.

Part III. Motion due to a hanging weight

1. Remove the aluminum block from underneath the single foot of the air track.
2. Attach the hanging mass m_w to the glider. Hook a thread to the tab attached to the glider, threat it through the hole in the ultrasonic transducer mounting plate and over the pulley as shown in Figure 2. Finally attach the weight hanger to the end of the thread. Be sure that the total hanging mass m_w is 15 g—the 5 g mass hanger plus an additional 10 g mass.
3. Hold the glider at rest at the "no-pulley" end of the airtrack , and click the *ACQUIRE DATA* button. Wait until two seconds of data have been taken by the computer, and release the glider smoothly.
4. After the computer returns control to you, examine the velocity vs. time plot first. Identify the release point on this graph; the coordinates of this point are (t_i, v_i). Record these two values in your lab manual on page 94.
5. Next identify the point just before the glider bounces; the coordinates of this point are (t_f, v_f). Again, record these values in your lab manual.
6. Now find the point (t_i, x_i) on the position vs. time plot. Use the value of t_i you obtained in step 4 above to find x_i. Record the value of x_i.
7. Find the point (t_f, x_f). Use the value of t_f you obtained to find x_f. Record this value.
8. Make a printout of the velocity and position vs. time plots.
9. Calculate a predicted acceleration for the glider– hanging mass system.
10. Determine an experimental value for the acceleration of the glider–hanging mass system, and compare it with your predicted value.
11. Calculate the work done on the glider–hanging mass system. Then calculate the change in kinetic energy of the glider–hanging mass system. Compare these two values.

Part IV. Evaluating sources of uncertainty

For this part, you will hypothesize a possible source of measurement uncertainty, design a method for measuring this effect, and collect data using your new method. You will then compare your results to an ordinary data collection on a typical run with the apparatus; in other words, you will compare your experimental data to a control.

For full credit, you must state what source of uncertainty you were investigating, how you went about measuring this source (that is, what you did to measure this source), and what you found out (that is, whether the source of uncertainty you examined is a significant one). Remember to get a printout for this part.

Below are examples of possible investigations:

- **Air resistance.** Repeat Part II of the experiment, but place a larger sail on the glider. Compare accelerations. (Acceleration is independent of mass in Part II if we neglect track friction. Therefore, adding an extra mass such as a larger sail to the glider should not affect its acceleration and so any change in acceleration can be attributed to the effects of air resistance.)

- **Track–glider friction.** Repeat either Parts II or III but increase the friction between the track and the glider. Possible methods for increasing the friction include placing a piece of paper underneath the track or partially removing the air hose from the nozzle on the air track.

- **String slipping on pulley.** In Part III we assumed that the string never slipped on the pulley. You may investigate what would happen if the string did slip. Repeat Part III but pin the pulley in place so that it cannot rotate, thus forcing the string to slip on the pulley.

- **Acoustic noise.** Repeat either Parts II or III while you increase the noise around the transducer by clapping or some other means. Be sure to note if the noise affects the measurement uncertainties in position and velocity.

- **Mechanical vibration.** Set up an experiment similar to that described for acoustic noise, but create a vibration in the system. For example, you could lightly tap the transducer housing with a pencil.

CAUTION: Do not jolt the laboratory table or computer. This could cause a hard disk crash in the computer.

For further suggestions and clearances for your investigations, consult your TAs.

Final checks before you leave

Verify that you have completed all items marked by the (√) symbol before leaving the laboratory. Also, you should have a total of three printouts, one each for Parts II, III, and IV. It is especially important to label your printout from Part IV before you leave because it may closely resemble one from an earlier part of the experiment.

3 Measuring g at Purdue: The Inside Story

In this experiment, to verify the observed value of the acceleration of a glider on an airtrack you must use a standard, accepted numeric value for g. This value was derived from a set of measurements described in a paper by J. C. Behrendt and G.P. Woollard in *Geophysics* Vol. 26, pp. 57–75 (1961). In this article, the authors describe two measurements of g made in West Lafayette:

$$g_{airport} = (9.801469 \pm 0.000001)\frac{\mathrm{m}}{\mathrm{s}^2}$$

measured at Purdue University airport in the terminal waiting room in the corner opposite the door to the field, and:

$$g_{campus} = (9.801456 \pm 0.000001)\frac{\mathrm{m}}{\mathrm{s}^2}$$

measured at the Purdue University Chemical Engineering Building (CHME), entrance to Room 4.

The difference in these two measurements is $0.000013\frac{\mathrm{m}}{\mathrm{s}^2}$, more than 10 times the uncertainties in the individual measurements. We must assume that the difference in location dominates the uncertainty in g, which is known to vary from location to location depending upon height above ground, underground ore bodies and objects, etc. (In fact, looking for minute changes in g via aerial survey is a method of prospecting for oil and minerals.)

Therefore, the final selected standard value for g used in PHYS 152L is taken as

$$g_{Purdue} = (9.80146 \pm 0.00002)\frac{\mathrm{m}}{\mathrm{s}^2}$$

which is precise enough (has enough digits) for our requirements and agrees in accuracy (overlaps the accepted values) with the two measurements given in the journal article.

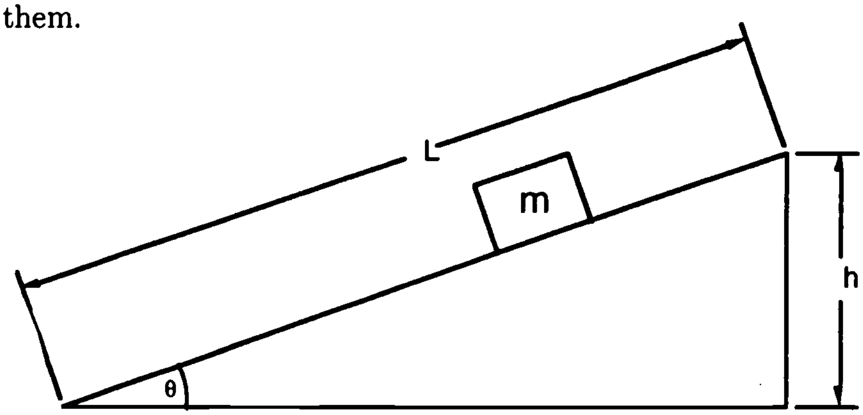
Thanks to Professors E. Fischbach (Physics) and W. J. Hinze (Earth and Atmospheric Sciences) for bringing this article to our attention.

Prelaboratory Questions for Experiment E2

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

Your answers are due at the start of your laboratory session, and will be worth zero if not turned in at that time. Write your answers in the space provided, showing the essential steps that led to them.



1. The above diagram shows an object of mass m on an inclined plane. This is the basic set up used in Part II in which the object is a glider and the inclined plane is a slightly tilted air track. Assume that there is no friction between the object and the inclined plane.

   (a) Suppose that $L = 1.063$ m, $h = 0.027$ m, $g = 9.80146$ m/s^2, and $m = 0.183$ kg. What is the acceleration of the object (ignoring uncertainty)?

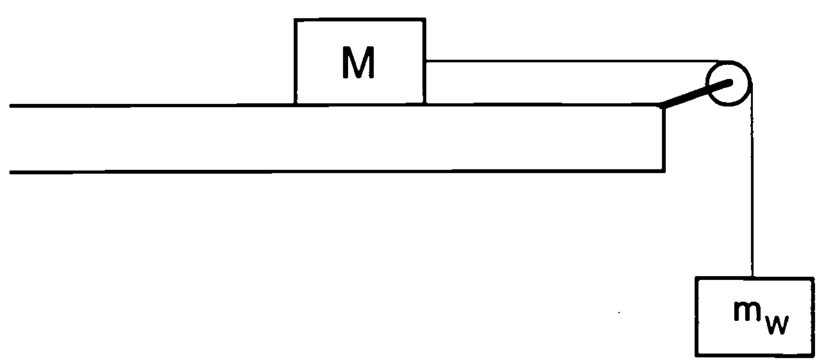
   (b) If the object were twice as massive, what acceleration do you predict for the object?

(c) If the object is released from rest and then travels a distance of 0.800 m along the incline, how much work is done on the object?

(d) What is the kinetic energy of the object when its velocity is 0.240 m/s?

(e) What causes the change in kinetic energy of this object?

2. As shown below, mass M is on a horizontal table, connected to a string that passes over a frictionless pulley. Mass m_w is connected to the other end of the string and hangs vertically.



(a) Using two free body diagrams - one representing each block, derive Equation 6 for this glider–hanging mass system in terms of M, m_w, and g. Assume that there is no friction between mass M and the table, and that the string is massless. Don't forget to accelerate the entire system — that is $M + m_w$. (See Serway Examples 5.8 and 5.9 for similar situations.)

(b) This is the basic set up for Part III, where mass M is the glider and mass m_w is a 5.0 g mass hook with an additional 25.0 g mass. Calculate the value of the acceleration for the above system if the mass of the glider is $M = (516.4 \pm 0.1)$ g and the total hanging mass m_w is (30.0 ± 0.1) g. Use the the value of g_{Purdue} given at the end of Experiment E1.

3. Qualitatively describe the effects of the following changes to the apparatus upon the motion of the hanging mass-glider system. Use a free body diagram to analyze each situation and include it here. Assume that the initial and final positions of the system remain unchanged, and that the velocities and accelerations change.

   (a) Case 1: Suppose friction were increased between the glider and the airtrack (e.g., placing sand on the track). How does this change the system's acceleration?

   (b) Case 2: The string has a considerable mass per unit length (e.g., a chain). How does the acceleration of the system change? (Hint: M is the mass on (or above) the track, m_w is the mass hanging off the pulley. Do these masses remain constant over the time of the systems motion?)

4. Design three short, simple investigations that you could use to measure the following possible uncertainty sources for this apparatus. Use rough sketches. Tell how you could collect data for the following sources of uncertainty using the same apparatus (or slight variations) you used in E1.

   (a) Glider/airtrack friction

   (b) Air resistance (on sail)

   (c) Acoustic noise in the room

E2 Laboratory Report

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

Laboratory reports are due one week [SEVEN CALENDAR DAYS] from the day you perform the experiment. Attach a green Physics 152L cover sheet (available in the lab or window of PHYS Room 144) to the front of your exercise or report and put it in the 'Room 144 Drop Slot for Physics Lab Reports' located under the mailboxes across from Physics Room 149 before 10:00 p.m. of the day it is due. Include all of your data in your laboratory report.

Abstract (10 points)

Write your Abstract in the space provided below. Devote 2–3 paragraphs to briefly summarize and describe the experiments in terms of theory, activity, key findings and agreements. Include actual numerical values, agreements and discrepancies with theory. Especially comment on whether your measurements in Parts II and III agreed with the predictions made by the work-energy theorem. Write the abstract AFTER you have completed the entire report, not before.

Part I. Set up and calibration

1. (√) What is the glider's mass and the uncertainty in this mass? $M = (\quad \pm \quad)$ g $= (\quad \pm \quad)$ kg.

2. (√) What is the combined hanging mass of the mass hook and 10 g mass? $m_w = (\quad \pm \quad)$ g $= (\quad \pm \quad)$ kg.

3. What is the combined mass of the entire system that is to be accelerated? $M + m_w = (\quad \pm \quad)$ g $= (\quad \pm \quad)$ kg.

4. (√) What is L and its uncertainty? (L is the distance between the single foot of the air track and a line connecting the other two feet). $L = (\quad \pm \quad)$ m.

5. (√) What is h and its uncertainty? $h = (\quad \pm \quad)$ m.

Part II. Motion on an inclined plane

1. (√) When did the motion under study begin and end? Determine values for the following quantities and their uncertainties. You may wish to refer to the **Hardware and Software Guide** in this manual to help determine these uncertainties.

   $t_i = (\quad \pm \quad)$ s

   $x_i = (\quad \pm \quad)$ m

   $v_i = (\quad \pm \quad)$ m/s

   $t_f = (\quad \pm \quad)$ s

   $x_f = (\quad \pm \quad)$ m

   $v_f = (\quad \pm \quad)$ m/s

2. (√) Make a printout of the velocity and position vs. time plots. Label this printout "Part II. Motion on an inclined plane." Enlarge the scale of your plot to clearly show how you determined δx_f and δv_f. Use the last few pages of the *Hardware and Software Guide* in this manual if you require an example. Label this printout 'Part II. Determining uncertainties in motion measurements."

3. Using Equation 5, determine the predicted acceleration of the glider. Use $g = (9.80146 \pm 0.00002)$ m/s^2.

$a_{pred} = (\qquad \pm \qquad) \text{ m/s}^2.$

4. What was the experimental acceleration a_{exp} and uncertainty you determined for the glider?

   $a_{exp} = (\qquad \pm \qquad) \text{ m/s}^2$

5. Compare your predicted and experimental values for the acceleration. Indicate agreement or calculate discrepancy. Justify your answer.

6. Using Equation 10, determine the work done on the glider in moving it from x_i to x_f along the track.

   $s = x_f - x_i = (\qquad \pm \qquad) \text{ m}$

   $W = (\qquad \pm \qquad) \text{ J}$

7. Using Equation 15, determine the change in kinetic energy of the glider as it moved along the track.

   $\Delta K = K_f - K_i = (\qquad \pm \qquad) \text{ J}$

8. Compare the work W done on the glider and the change in kinetic energy ΔK. Indicate agreement or calculate discrepancy. Justify your answer.

Part III. Motion due to a hanging weight

1. (√) When did the motion under study begin and end? Determine values for the following quantities and their uncertainties. You may wish to refer to the **Hardware and Software Guide** in this manual to assist the determination of these uncertainties. Check that these numbers are reasonable and consistent with your TA.

   $t_i = (\qquad \pm \qquad)$ s

   $x_i = (\qquad \pm \qquad)$ m

   $v_i = (\qquad \pm \qquad)$ m/s

   $t_f = (\qquad \pm \qquad)$ s

   $x_f = (\qquad \pm \qquad)$ m

   $v_f = (\qquad \pm \qquad)$ m/s

2. (√) Make a printout of the velocity and position vs. time plots. Label this printout "Part III. Motion due to a hanging weight."

3. Using Equation 6, determine the predicted acceleration of the glider–hanging mass system. Take $g = (9.80146 \pm 0.00002)$ m/s^2.

   $a_{pred} = (\qquad \pm \qquad)$ m/s^2

4. What was the experimental acceleration a_{exp} you measured?

   $a_{exp} = (\qquad \pm \qquad)\ \mathrm{m/s^2}$

5. Compare your predicted and experimental values for the acceleration. Do your results agree or indicate a discrepancy? Justify your answer.

6. Using Equation 11, determine the work done on the glider–hanging mass system in moving it from x_i to x_f along the track.

   $s = x_f - x_i = (\qquad \pm \qquad)\ \mathrm{m}$

   $W = (\qquad \pm \qquad)\ \mathrm{J}$

7. Using Equation 16, determine the change in kinetic energy of the glider–hanging mass system as it moved along the track.

   $\Delta K = K_f - K_i = (\qquad \pm \qquad)\ \mathrm{J}$

8. Compare the work W done on the glider–hanging mass system and the change in kinetic energy ΔK. Do your results agree or indicate a discrepancy? Justify your answer.

Part IV. Evaluating sources of uncertainty

Present your data and calculations examining a source of measurement uncertainty here. Describe what source of uncertainty you were investigating, and summarize both your laboratory investigation and your findings.

(√) Get a printout for this part and label it "Part IV. Evaluating a source of uncertainty." Also record all necessary data here before you leave the laboratory.

Analysis (15 points)

Write your Analysis in the space provided. Using numerical examples and analysis, discuss the larger sources of uncertainty in your calculations. Use your numbers to justify possible improvements in apparatus and methods. Especially compare your uncertainties in W and ΔK for Parts II and III. Which uncertainties were the largest? The smallest? [Hint: Calculate the precisions of these measurements and compare.] Do not repeat **Analysis** material from previous experiments.

Conclusions (5 points)

Write your Conclusions in the space provided. Did the experiment tend to confirm the theoretical physical relationships examined? Choose a significant mathematical relationship discussed in this experiment, and use it to numerically explain an activity or phenomenon from outside of the classroom. Illustrate your example by using genuine, approximate or estimated numbers.

Measurement Analysis 2:
Probabilistic Uncertainty, Least Squares Fitting, and Graphical Analysis

Goals of this activity

After completing this activity you should be able to

- Interpret and calculate the mean, standard deviation, and standard error of the mean (SEM) of a set of repeated measurements.
- Use a spreadsheet to perform a Least Squares Fit statistical reduction of data to determine the statistically best values (and uncertainties) for the slope and y-intercept of a linear relationship.
- Know the standards for clear, professional, easily-readable graphs (complete with error bars).

1 Probabilistic (or Statistical) Uncertainty — mean, standard deviation and SEM

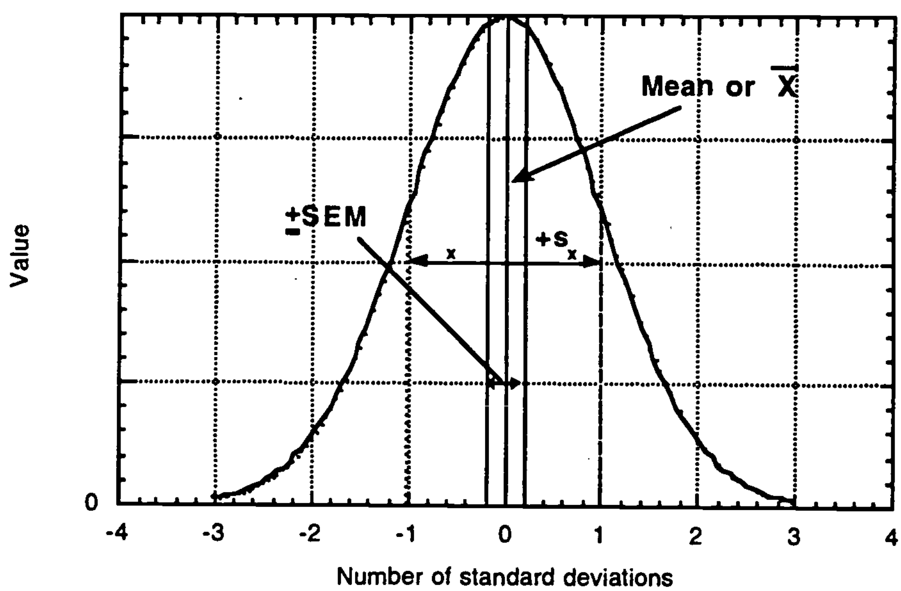


Figure 1: A Gaussian Distribution

When we make a repeated measurement and experience random uncertainties due to resolution limitations, we can treat these uncertainties probabilistically — we assume that the distribution of uncertainty will follow the Gaussian or Normal distribution as shown in Figure 1. This procedure gives a better uncertainty calculation than the propagation methods discussed earlier: those methods always assumed the *worst or maximum uncertainty* in any situation, while statistical treatments give the *most likely uncertainties.*

We begin our discussion with the idea of a statistical mean value. The statistical *mean value* is exactly equivalent to the quantity we knew in high school as the simple average for a set of repeated measurements. For N repetitions of a measurement X_i, the statistical mean is written as

$$\bar{X} = \frac{1}{N}[X_1 + X_2 + X_3 + \cdots + X_N]$$

or written more compactly,

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \tag{1}$$

For example, suppose that we measure the speed of sound five times in the laboratory and collect the following data: 341 m/s, 344 m/s, 338 m/s, 340 m/s and 343 m/s. The mean value would be:

$$\begin{aligned} \bar{v} &= \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{5}(v_1 + v_2 + v_3 + v_4 + v_5) \\ &= \frac{1}{5}(341 + 344 + 338 + 340 + 343)\ \text{m/s} = \frac{1}{5} \times 1706 = 341.2 \approx 341\text{m/s}. \end{aligned} \tag{2}$$

Often we wish to know by how much measurements deviate from the mean value. The quantity that relays this information is known as the *standard deviation* and is indicated by the lowercase letter s with an appropriate subscript, as in s_X. (Note that there are several different standard deviations in statistics; here we intend the sample standard deviation.) To determine the standard deviation, first the mean value $\bar{X}$ must be calculated, then s_X^2 is calculated by taking the sum of the squares of the deviations of each point from the mean and dividing that sum by $N - 1$:

$$s_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\bar{X} - X_i)^2 \tag{3}$$

Then the standard deviation s_X is

$$s_X = \sqrt{s_X^2} \tag{4}$$

Note that Equation 1 for the mean contains a $\frac{1}{N}$ term while Equation 3 for standard deviation squared contains $\frac{1}{N-1}$. The mean and standard deviation for ANY Gaussian curve entirely define the curve and are CONSTANTS.

Recall the speed of sound data we collected in the laboratory. Having already found the mean value of 341 m/s for this data, we now wish to calculate the standard deviation of this sample.

Using Equation 3, we first find s_v^2:

$$\begin{aligned}
s_v^2 &= \frac{1}{N-1}\sum_{i=1}^{N}(\bar{v}-v_i)^2 \\
&= \frac{1}{5-1}[(\bar{v}-v_1)^2+(\bar{v}-v_2)^2+(\bar{v}-v_3)^2+(\bar{v}-v_4)^2+(\bar{v}-v_5)^2] \\
&= \frac{1}{4}[(341-341)^2+(341-344)^2+(341-338)^2+(341-340)^2+(341-343)^2]\ \mathrm{m^2/s^2} \\
&= \frac{1}{4}[0+9+9+1+4]\ \mathrm{m^2/s^2} = \frac{1}{4}\times 23\mathrm{m^2/s^2} \qquad (5) \\
&\approx 5.8\ \mathrm{m^2/s^2} \qquad (6)
\end{aligned}$$

Then we use Equation 4 to find the standard deviation:

$$s_v = \sqrt{s_v^2} = \sqrt{5.8\ \mathrm{m^2/s^2}} \approx 2.4\ \mathrm{m/s}$$

After we know the mean value $\bar{X}$ and the sample standard deviation s_X of a set of measurements, we can determine the *Standard Error of the Mean* ($\sigma_{\bar{X}}$ or sometimes SEM) calculated from our limited set of data as:

$$\mathrm{SEM} = \sigma_{\bar{X}} = \frac{s_X}{\sqrt{N}} \qquad (7)$$

With $\bar{X}$ and $\sigma_{\bar{X}}$ known, we can write our measurement in the usual form: $(\bar{X} \pm \sigma_{\bar{X}})$. Note that the SEM is very sensitive to reduction by taking more data: the more data, the less uncertainty in the measure.

To find the standard error of the mean $\sigma_{\bar{v}}$ of our speed of sound data, we use Equation 7:

$$\mathrm{SEM} = \sigma_{\bar{v}} = \frac{s_v}{\sqrt{N}} = \frac{2.4\ \mathrm{m/s}}{\sqrt{5}} = 1.07\ \mathrm{m/s} \approx 1.1\ \mathrm{m/s}$$

Finally, we would conclude that our measured speed of sound is $(\bar{v} \pm \sigma_{\bar{v}}) = (341 \pm 1)$ m/s. Notice that the number of decimal places in the measured value and its uncertainty agree (in this case, both have zero decimal places).

2 Least Squares Fitting

Often you will determine a quantity by examining a linear plot of N collected data points (x_1, y_1), (x_2, y_2), ... (x_i, y_i), ... (x_N, y_N). We will call these (x_i, y_i) where i may have any value from 1 to N.

Because of measurement uncertainties, the plot of these points will not exactly define a straight line, and a number of different lines can be drawn through the data points. The problem becomes one of 'goodness of fit' — which of the many possible different lines we can possibly draw is the 'best' fitting one?

The line with the best fit can be fairly easily determined if we make three basic assumptions about the nature of our measurement uncertainties:

1. The absolute uncertainties are nearly the same for all data points (we can use single uncertainty values σ_x and σ_y).
2. The uncertainties are principally in the dependent variable – the measured y_i, with effectively trivial uncertainties ($\sigma_x \sim 0$) in the independent variable x_i.
3. The uncertainties are random in nature (not systematic or due to human error).

Given these three assumptions, we can statistically determine the equation for the best line by calculating the sum of the squares of the distances between the theoretical line and our data points, and then by minimizing this sum of squares. This is done by taking partial derivatives of that theoretical quantity, and so we will not derive the formulas here (although the derivation will be shown in lecture to interested parties). This whole process of minimizing the squares of the distances (the uncertainties) is widely known as the *Least Squares Fit* algorithm. The algorithm is given below:

$$m = \frac{N(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{\Delta} \tag{8}$$

$$b = \frac{(\sum x_i^2)(\sum y_i) - (\sum x_i)(\sum x_i y_i)}{\Delta} \tag{9}$$

where $$\Delta = N(\sum x_i^2) - (\sum x_i)^2$$

You should be aware that $(\sum x_i)^2 \neq \sum x_i^2$.

2.1 Uncertainties in Least Squares Fitting

When calculating the uncertainties in a least squares fit, we must first calculate σ_y, which gives the standard deviation of the y values from the straight line. This value is given by:

$$\sigma_y^2 = \frac{1}{N-2} \sum (y_i - b - mx_i)^2. \tag{10}$$

Knowing σ_y^2, the values for the uncertainties in m and b can be found from:

$$\sigma_{\bar{m}}^2 = \frac{N\sigma_y^2}{\Delta} \tag{11}$$

$$\sigma_{\bar{b}}^2 = \frac{\sigma_y^2 \sum x_i^2}{\Delta} \tag{12}$$

Note that $\Delta = N(\sum x_i^2) - (\sum x_i)^2$ is a very useful quantity to evaluate early and have at hand when calculating least square fits. Also, you will need to take $\sqrt{\sigma_{\bar{m}}^2}$ and $\sqrt{\sigma_{\bar{b}}^2}$ to find the final uncertainties and write $(m \pm \sigma_{\bar{m}})$ and $(b \pm \sigma_{\bar{b}})$.

Example

1. Data collected by a Physics 152L student to describe the motion of a skier who moved from a level section of a ski run onto a smooth slope at $t = 0$ seconds is compiled in Table 1.

Time t (s)	Uncertainty in time δt (s)	Velocity v (m/s)	uncertainty in v δv (m/s)
0.50	0.01	7.10	0.20
0.75	0.01	7.90	0.20
1.00	0.01	8.50	0.40
1.25	0.01	9.50	0.30
1.50	0.01	10.00	0.40
1.75	0.01	10.50	0.40
2.00	0.01	11.20	0.30

Table 1: Velocity and time data describing a skier's descent

If we assume that the velocity data can be fitted by the linear equation:

$$v(t) = v_i + at, \tag{13}$$

we can determine the initial velocity v_i and the acceleration a of the skier on the slope.

(a) Perform a least squares fit upon the data, and determine v_i, a, and their associated uncertainties $\sigma_{\bar{v}_i}$ and $\sigma_{\bar{a}}$.

(b) Graph these data following the standards for graphical presentation of laboratory data.

(c) What was the skier's initial velocity as she started into the slope?

(d) What was her acceleration on the slope?

The Solution

(a) First we need to calculate a series of statistics from our data. We will use Table 2 to assist in the least squares fitting. (Note that we cannot calculate the final column until *after* we determine the values for b and m from our least squares fit).

i	x_i	y_i	x_i^2	$x_i y_i$	$(y_i - b - mx_i)^2$
	time t (s)	velocity v (m/s)	t^2 (s^2)	vt (m)	$(m/s)^2$
1	0.50	7.10	0.2500	3.5500	0.0115
2	0.75	7.90	0.5625	5.9250	0.0002
3	1.00	8.50	1.0000	8.5000	0.0041
4	1.25	9.50	1.5625	11.8750	0.0661
5	1.50	10.00	2.2500	15.0000	0.0062
6	1.75	10.50	3.0625	18.3750	0.0100
7	2.00	11.20	4.0000	22.4000	0.0062
N	$\sum x_i$	$\sum y_i$	$\sum x_i^2$	$\sum x_i y_i$	$\sum(y_i - b - mx_i)^2$
7	8.75	64.70	12.6875	85.6250	0.1043

Table 2: A least squares fit table

Always carry extra (non-significant) decimal places during LSQ fit calculations, then round to the correct number of significant digits when you finally interpret the results. Otherwise, each time you make a calculation you will introduce a round-off error. In this example, we will carry at least two non-significant digits throughout, and will dispose of these at the end.

Then using the values from Table 2:

$\Delta = N(\sum x_i^2) - (\sum x_i)^2 = (7)(12.6875) - (8.75)^2 = 12.2500$ seconds^2

$m = \frac{N(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{\Delta} = \frac{(7)(85.6250) - (8.75)(64.70)}{12.2500} = 2.7143 \sim 2.71$ m/s^2

$b = \frac{(\sum x_i^2)(\sum y_i) - (\sum x_i)(\sum x_i y_i)}{\Delta} = \frac{(12.6875)(64.70) - (8.75)(85.6250)}{12.2500} = 5.8500 \sim 5.85$ m/s

Then to calculate the uncertainties:

$\sum(y_i - b - mx_i)^2 = 0.1043$ (m/s)^2

$\sigma_y^2 = \frac{1}{N-2}\sum(y_i - b - mx_i)^2 = \frac{0.1043}{7-2} = 0.0209$ (m/s)^2

$\sigma_{\bar{m}} = \sqrt{\frac{N\sigma_y^2}{\Delta}} = \sqrt{\frac{(7)(0.0209)}{12.2500}} = 0.1092 \sim 0.11$ m/s^2

$\sigma_{\bar{b}} = \sqrt{\frac{\sigma_y^2 \sum x_i^2}{\Delta}} = \sqrt{\frac{(0.0209)(12.6875)}{12.2500}} = 0.1470 \sim 0.15$ m/s

Therefore $m = (2.71 \pm 0.11)$ m/s^2 and $b = (5.85 \pm 0.15)$ m/s .

(b) The complete plot of this data was previously seen in this activity as Figure 2. Note that, as required in Physics 152L, the slope and y-intercept have been labelled and their meanings have been noted.

(c) The skier's initial velocity as she started into the slope was $v_i = 5.85$ m/s, $\sigma_{\bar{v}_i} = 0.15$ m/s.

(d) Her acceleration on the slope was $a = 2.71$ m/s^2, and $\sigma_{\bar{a}} = 0.11$ m/s^2. Therefore $(a \pm \sigma_{\bar{a}}) = (2.71 \pm 0.11)$ m/s^2 and $(v_i \pm \sigma_{\bar{v}_i}) = (5.85 \pm 0.15)$ m/s .

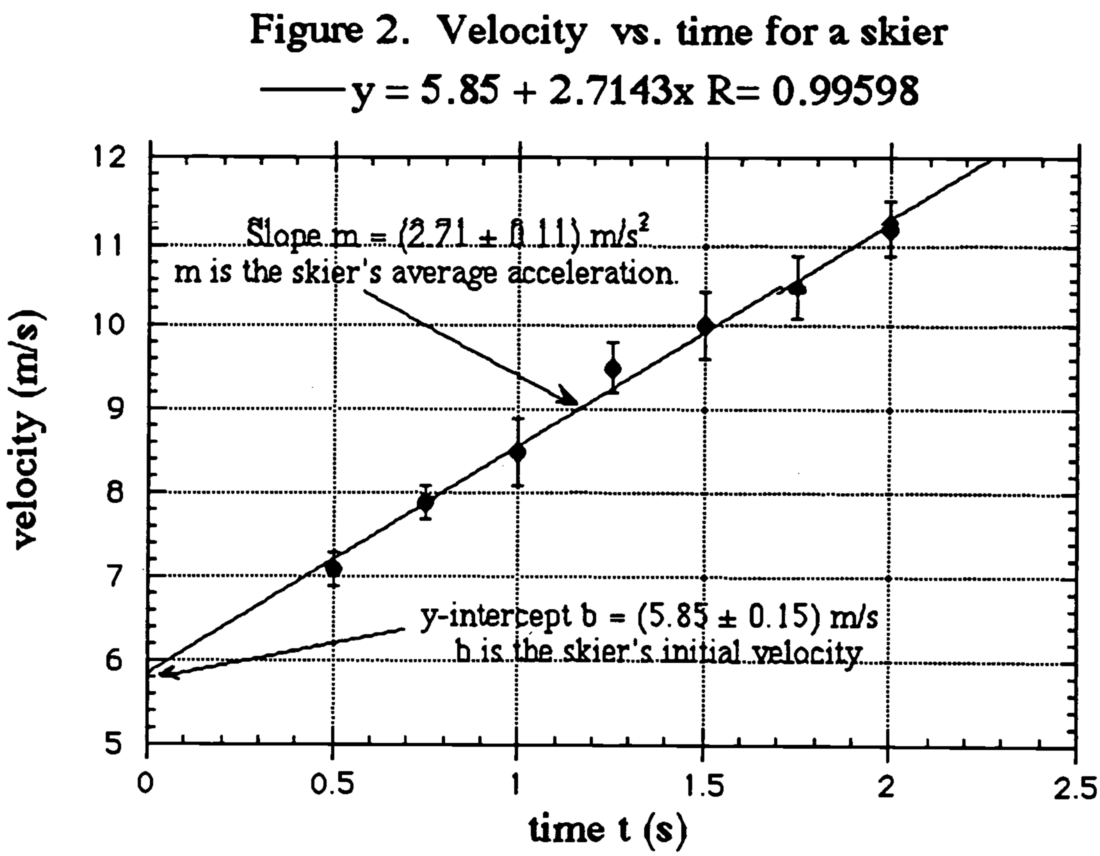


Figure 2: Example of a graph meeting all of the Physics 152L standards described in the next section. The uncertainties in the slope m and the y-intercept b are from the least squares fit performed above. The equation for the line and the value of the Linear Correlation Coefficient (R) displayed under the main title was generated by *Kaleidagraph* is optional; it serves as a quick check of whether the least squares calculations are correct.

All of the enclosed graphs and statistical fits were computer-generated using hardware and software available for your use in PHYS 14 (next to your laboratory room). There are samples and explanation files for PHYS 152L graphs and fits on those machines as well as knowledgeable student monitors paid to assist you with your work during weekly open hours. Please explore and make use of these facilities. *You must use computer software to generate plots and least squares fits as a requirement for completing PHYS 152L, and this activity (MA2) must be completed using computer software for the calculations and the plots.* We suggest *Kaleidagraph* and *Excel.* These skills are extremely useful to science and engineering practitioners and students.

3 Graphing standards

All laboratory graphs must follow the guidelines listed below.

1. **Titles and labels.** All graphs should be clearly labeled with both a figure number and/or an explanatory title directly above the graph. The title text should briefly explain the graph independently of other text elsewhere.

2. **Axes.** Both the horizontal and vertical axes of graphs should be clearly labeled with variable names and units, and numerical values at major tickmarks should be given. The conventional way of deciding what quantity goes with which axis is to plot the dependent or measured value is on the upright axis and the independent or controlling variable on the horizontal axis. For example, a plot of velocity vs. time (i.e., $v(t)$) would show velocity on the vertical axis and time on the horizontal axis.

3. **Error bars.** When plotting points with known uncertainties, error bars should be included. If your plotting software does not allow error bars, pencil them in on your graph.

4. **Slope and y-intercept.** When plotting linear relationships, the slope and y-intercept of the line are of interest. These values and their units should also be clearly displayed on the graph. Sometimes the x-intercept is also of interest; if this is the case, it should also be labeled and its value indicated. In Physics 152L the physical significance of the the slope and y-intercept should be stated, as shown in Figure 2.

5. **Size and clarity.** In order to express all of this information clearly and legibly, you should chose axis limits so that the region of interest occupies most of your graph. Labelling should be done with reasonably large size numbers and letters.

4 Expressing Linear Relationships

Often you will determine a quantity by examining a plot of collected (x, y) data points. If the quantity plotted on the vertical axis depends linearly on the quantity plotted along the horizontal axis, ideally (ignoring measurement uncertainties) these data points will fall on a straight line whose equation can be written in the form:

$$y = mx + b \tag{14}$$

where b is known as the y-intercept and m is the slope. The slope can be further defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \tag{15}$$

where (x_1, y_1) and (x_2, y_2) are two points on the line. This is illustrated in Figure 3.

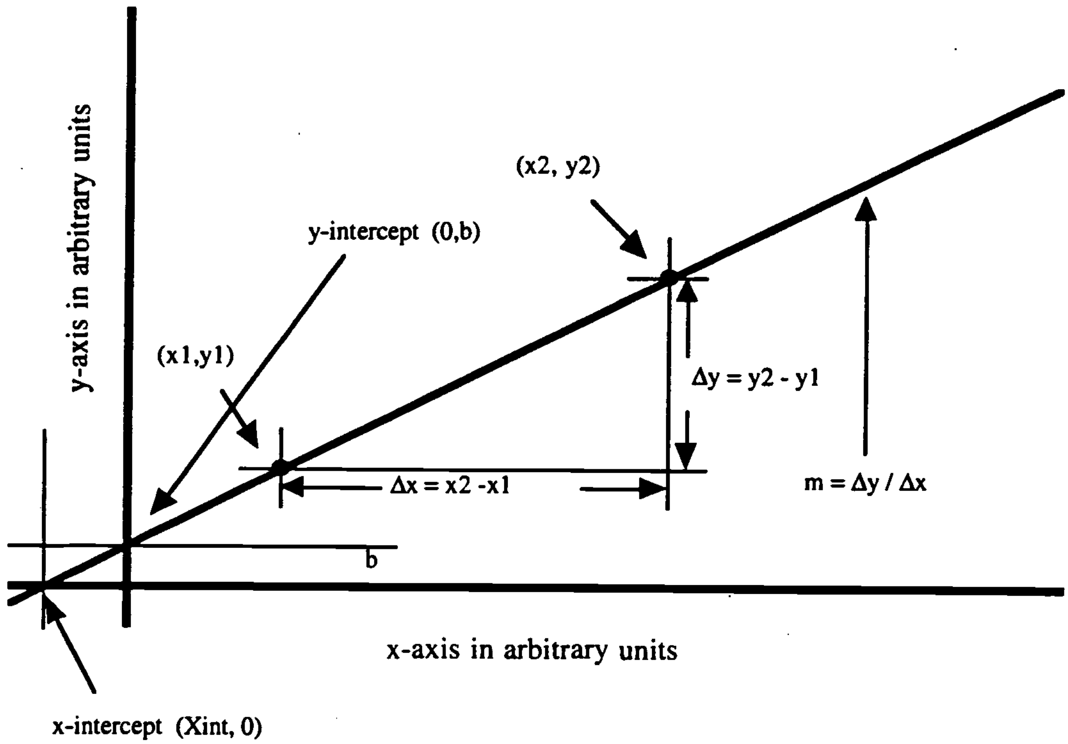


Figure 3: Illustration of the slope of a straight line.

The y-intercept is the position where a linear graph crosses the y-axis and corresponds to the value for which the value of x is zero. Sometimes we need to know the x-intercept rather than the y-intercept; if this is the case we can apply the fact that $y = 0$ at the x-intercept to our equation for a straight line as follows:

$$y = 0 = mx_{int} + b,$$

$$mx_{int} = -b$$

Thus,

$$x_{int} = -\frac{b}{m} \tag{16}$$

and we can readily determine the x-intercept knowing the y-intercept and the slope.

5 References

1. Bevington, P. R., *Data reduction and uncertainty analysis for the physical sciences* (McGraw-Hill, New York, 1969).

2. Taylor, J. R., *An introduction to uncertainty analysis: the study of uncertainties in physical measurements* (University Science Books, Mill Valley, CA, 1982).

3. Young, H. D., *Statistical treatment of experimental data* (McGraw-Hill, New York, 1962).

Measurement Analysis Problem Set MA2

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

This assignment is due at the start of E3 for your division. Some portions of this assignment must be done by hand, and other parts must be done with a spreadsheet plus graphing software for full credit.

1. The mass of an object was measured eight times on a balance readable to ±1 g, and the following values were recorded: 416 g, 423 g, 399 g, 410 g, 410 g, 417 g, 402 g, and 431 g. Find $\bar{m}$, s_m, and $\sigma_{\bar{m}}$ by hand. Carry extra digits through calculations and rationalize your numbers at the end, showing all work.

   (a) $\bar{m} = (\qquad)$ g

   (b) $s_m = (\qquad)$ g

   (c) $\sigma_{\bar{m}} = (\qquad)$ g

   (d) Write the final measurement $(\bar{m} \pm \sigma_{\bar{m}}) = (\qquad \pm \qquad)$ g

   (e) What is the percentage uncertainty of the measurement? $\frac{\sigma_{\bar{m}}}{\bar{m}} \times 100\% =$ %

2. The following data were collected by a Physics 152 student to describe the motion of a motorcycle racer who applied the brakes at $t = 0$ seconds:

Time t (s)	Uncertainty in time δt (s)	Velocity v (m/s)	uncertainty in v δv (m/s)
0.10	0.01	14.8	0.4
0.20	0.01	14.0	0.4
0.30	0.01	13.2	0.4
0.40	0.01	12.6	0.4
0.50	0.01	12.1	0.4
0.60	0.01	11.0	0.3
0.70	0.01	10.3	0.3
0.80	0.01	9.7	0.3
0.90	0.01	8.8	0.3

If we assume that the velocity data can be fitted by the linear equation:

$$v(t) = v_i + at \qquad (17)$$

we can determine the initial velocity v_i and the acceleration a of the motorcyclist.

(a) How does Equation 17 relate to the standard equation for a line? Identify the y-intercept and the slope of Equation 17.

(b) Create an appropriate table and perform a least squares fit upon the data, and determine the slope and intercept of the graph and their associated uncertainties. Perform all calculations for this first LSQ fit by hand – write your answers on a separate sheet of paper, showing all calculations. Carry at least two extra decimal places through all statistical calculations, and round appropriately when expressing the final results. Use the correct units.

(c) Using a software plotting package (preferably *KaleidaGraph*; *Excel* is fine but harder to use for graphing with labels, etc.), graph these data following the standards for graphical presentation of laboratory data. Label the slope and y-intercept and say what information they provide about the motion of the motorcyclist. Attach your plot to this page.

(d) What was the driver's initial velocity when the brakes were first applied?

(e) What was the driver's acceleration (deceleration)?

3. The linear relationship between the restoring force applied by a spring and its distortion is known as Hooke's Law and is written as:

$$\mathbf{F}_{restoring} = -k\mathbf{x} \tag{18}$$

where $\mathbf{F}_{restoring}$ is the restoring (resisting) force applied by the spring, k is a positive quantity known as the spring constant and $\mathbf{x}$ is the amount the spring is stretched from its natural equilibrium position x_e. If the total length of the stretched spring is x_s, $x = x_s - x_e$. The negative sign means that the force is opposite in direction to the displacement—the spring tries to return to its original size by pulling against the direction of the stretch. Typical values for k for lab springs are on the order of one Newton per meter (1 N/m).

In the laboratory, it is fairly easy to apply a force to the spring and the displacement or stretch of the spring. This applied force $\mathbf{F}_{applied}$ is equal in size and opposite in direction to the restoring force $\mathbf{F}_{restoring}$ such that:

$$\mathbf{F}_{applied} = -(\mathbf{F}_{restoring}) = -(-k\mathbf{x}) = k\mathbf{x} \tag{19}$$

Therefore, you will plot $F_{applied}$ vs. x, where $x = x_s - x_e$ is the stretch of the spring.

The following data were collected to describe the stretch of a spring in a Physics 152 lab:

Force F applied (N)	Amount of stretch $x = x_s - x_e$ (m)
0.203	0.250
0.409	0.455
0.596	0.676
0.810	0.882

(a) Use a spreadsheet to prepare an appropriate table for performing a least squares fit on the data and determine the best values for the slope and y-intercept along with their uncertainties. Print out your spreadsheet, showing the intermediate sums and calculations on the sheet so the essential steps are obvious. *Note: this same spreadsheet can be re-used to solve the LSQ fits in E3 lab and E6 lab.*

(b) Plot *Applied force F vs. Stretch* $x = x_s - x_e$ from this data according to laboratory graphing standards. Label the slope and y-intercept and interpret them. Note that you have no uncertainty bars for this example.

(c) What is the value of the spring constant and its uncertainty?

(d) What should the value for the y-intercept be according to Equation 18? Does the data agree with theory within the uncertainties given by the LSQ fit?

Experiment E3: Conservation of Mechanical Energy

Prelaboratory Questions are due at the start of this activity.

Goals of this experiment

In this experiment you will verify the conservation of total mechanical (kinetic plus potential) energy in a mechanical system. You will investigate two different methods of storing potential energy in mechanical systems using gravitational potential and spring distortion, and you will calculate the amount of energy stored in a stretched spring by determining the spring constant.

1 Theory

As discussed in Chapter 8 of Serway, the law of conservation of energy states that the total energy in an isolated system is a constant and does not change with (or is *independent with respect to*) time. In this experiment we will be considering two types of energy, potential energy U and translational kinetic energy K. Neither U or K is conserved separately, but the sum of the two, the total energy E, is conserved such that:

$$E = K + U = \text{constant} \tag{1}$$

Energy conservation can be used as an alternative to the equations relating position x, velocity v and acceleration a to predict the motion of a body. In some cases it is much easier to use the conservation of energy than to consider all the forces in a problem, especially when the forces change with time.

In this experiment we will consider two systems in which potential energy and kinetic energy values change while their total is conserved. One system experiences uniform acceleration, and the other experiences nonuniform acceleration.

Translational kinetic energy K is defined for an object of mass m and speed v as Serway's Equation 7.14:

$$K = \frac{1}{2}mv^2 \tag{2}$$

Potential energy will be calculated differently for Parts II and III of this experiment. In Part II, the potential energy of the system is due to *gravitational potential,* which is a function of the vertical position of an object. A body of mass m at a height y has potential energy:

$$U = mgy \tag{3}$$

where y is glider height in our experiment. We actually measure x, y_i and x_i and then assume $\Delta x = \Delta y$. Thereafter we calculate y from x. Recall that on the Purdue West Lafayette campus, $g_{Purdue} = (9.80146 \pm 0.00002)$ m/s^2.

In Part III of this experiment, potential energy is stored in a stretched spring. When a spring is displaced an amount **x** beyond its natural length x_e and released, its potential energy is converted into kinetic energy of the glider to which it is attached. To hold a spring stretched by a displacement x requires a force F_s that is directly proportional to x such that

$$\mathbf{F_s} = -k\mathbf{x} \tag{4}$$

where k is the spring constant. (Notice that F_s is the restoring force exerted by the spring on the glider and opposite in direction to the displacement **x**, hence the negative sign. Also note that **x** is a *displacement,* not a simple position — usually we need to know an equilibrium position x_e, then find the current length x_s of the spring to determine this displacement. Sometimes the force required to stretch the spring a given distance is measured instead; this force is equal in magnitude to F_s but opposite in direction — it exactly opposes F_s. The results are the same but often makes graphing easier.) Equation 4 is Serway's Equation 7.9 and is known as Hooke's Law — it is accurate as long as the displacement **x** is not too great. The stretched spring stores a potential energy U described as nonlinear with position:

$$U = \frac{1}{2}k\mathbf{x}^2 = \frac{1}{2}k(x_s - x_e)^2 \tag{5}$$

where x_e is the equilibrium (or unstretched) position of the spring. Notice that in each of these two cases (Parts II and III) the amount of potential energy in the system is related to the *position* of the glider while the amount of kinetic energy in the system is related to the *velocity* of the glider. Therefore, you will require measurements of *both position and velocity* to determine total energy for each portion of the experiment.

2 Expressing Linear Relationships

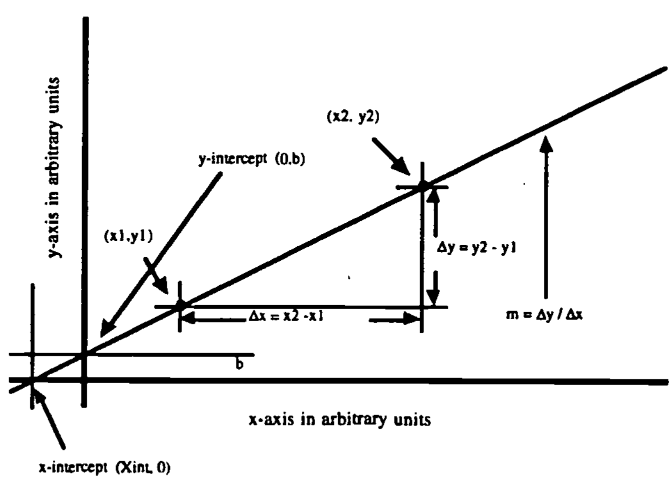


Figure 1: Illustration of the slope of a straight line.

Often you will determine a quantity by examining a plot of collected (x, y) data points. If the quantity plotted on the vertical (y) axis depends linearly on the quantity plotted along the horizontal (x) axis, ideally (ignoring measurement uncertainties) these data points will fall on a straight line whose equation can be written in the form:

$$y = mx + b \tag{6}$$

where b is known as the y-intercept and m is the slope. The slope can be further defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \tag{7}$$

where (x_1, y_1) and (x_2, y_2) are two points on the line. This is illustrated in Figure 1.

The y-intercept is the position where a linear graph crosses the y-axis and corresponds to the value for which the value of x is zero. Sometimes we need to know the x-intercept rather than the y-intercept; if this is the case we can apply the fact that $y = 0$ at the x-intercept to our equation for a straight line as follows:

$$y = 0 = mx_{int} + b,$$

$$mx_{int} = -b$$

Thus,

$$x_{int} = -\frac{b}{m} \tag{8}$$

and we can readily determine the x-intercept knowing the y-intercept and the slope. *Kaleidagraph* or other plotting software can be easily used to fit lines to data – both for determining the equations and plotting the best fit lines. To obtain the uncertainties in a fitted line, you will have to complete a least squares fit with as spreadsheet such as *Excel.*

3 Experimental method

In this experiment you will use an air track, a blue glider, and the ultrasonic position measuring system. For best accuracy use the yellow ruler taped to the air track only for initial calibration. Afterwards use the computer display for your distance measurements. Best accuracy is obtained by determining the time of release points or bounce points (t_i and t_f) upon the v vs. t plot.

You should enter your data directly into the Laboratory Data Sheet, which your TA will initial as you leave the laboratory. Once you leave the lab, it is difficult, if not impossible, to gather the proper information. If time permits, you could calculate the sum of potential and kinetic energy at two points in each data run to confirm that your data do agree with Equation 1 before you leave the lab.

Part I. Initial set up of the experiment

Complete the following items for this part of the experiment:

1. Calibrate the ultrasonic position measuring system by following the on-screen instructions and using the yellow ruler tape on the air track.
2. Measure the mass M of the blue glider. Remember to determine the uncertainty in the mass measurements.
3. Measure the mass m_w which consists of the 5 gram mass hanger and an additional 15 grams of mass disks. Thus, $m_w \approx 20$ g.

After Part I is completed, the remaining steps (Parts II and III) of the experiment do not have to be done in order.

Part II. Conservation of energy in a system with a falling weight

The experimental goal of this part is to simultaneously take position vs. time $x(t)$ and velocity vs. time $\bar{v}(t)$ data for the glider when it is released at rest from a position no less than 60.0 cm from the pulley. The $\bar{a}(t)$ data will not be required for the initial parts of the experiment.

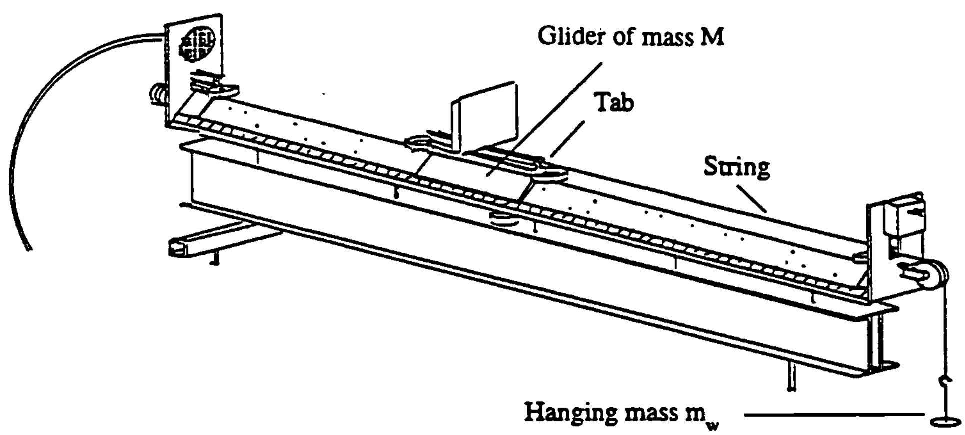


Figure 2: Set-up for Part II

1. Attach the hook on the thread to the metal tab on the glider, pass the thread through the hole in the transducer mounting plate, and then over the pulley at the end of the air track. Finally, attach the mass m_w (mass hanger and mass disks) to the thread loop. The set-up for this part of the experiment is shown in Figure 2.

2. Move the glider along the track so that the mass m_w is approximately one meter from the floor. Then use an aluminum bumper to hold the glider at this fixed position. Use the meter stick to determine the height of m_w, and record this measurement as y_i on your data sheet. Also, determine the uncertainty δy_i from the meter stick.
3. Remove the aluminum bumper and hold the glider in place. Click on the *ACQUIRE DATA* button until two seconds of data have been taken. Then, release the glider.
4. Using the computer generated v vs. t data graph, determine the exact release time t_i and initial velocity v_i. Also determine the corresponding uncertainties δt_i and δv_i. Record these measurements on your data sheet.
5. Using the two cursors and their coordinate readouts, measure the *velocity* every 0.4000 s from the release point until you either fill Table 3 or reach the time of collision with the endpost or bumper. Record your values in Table 3 of the Data and Calculations section. You will use these values to complete a table of K vs. t in your report.
6. Now move to the x vs. t plot on the computer, and relocate the exact release time t_i. Locate the position value at the time t_i and record it as x_i. Using the two cursors and their coordinate readouts, start from the release point x_i and measure the *position* every 0.4000 s until you fill the table or reach the time of collision with the endpost. Record your values in Table 4 of the Data and Calculations section. You will use these values to complete a table of U vs. t in your report.
7. Scale this graph and place the cursors appropriately to display the region of data that describes the motion of interest, namely the time interval between glider release and the collision with the endpost. Make a printout of this graph, and indicate the release point and the contact point on the printout. Clearly label the graphs with titles that include the part of the experiment to which they belong, and label any physical events or discontinuities on the plots as well.

Note: Be careful when using Equations 2 and 3 that you use the correct masses; Equation 2 describes the kinetic energy K of the SYSTEM — that is, BOTH masses moving together at velocity v. Equation 3 refers to the change in potential energy of the SYSTEM as a whole, but in our case only the hanging mass ALONE experiences a change in gravitational potential. Therefore, both masses need to be accounted for when calculating K, but only the hanging mass is required when calculating U.

Part IIIa. Determining the spring constant k

The first step is to measure the spring constant k. This will be done by pulling the spring with a known force and measuring the stretch of the spring beyond its normal unstretched length. You will use four different masses m_i to stretch the spring by varying amounts. The known force applied to the spring will be the gravitational force $W = m_i g$.

1. As shown in Figure 3, hook one end of the spring to the tab on the aluminum extrusion at the non-pulley end of the air track and the other end of the spring to one of the tabs on the glider. Put the hook on the thread through the tab on the other end of the glider, pass the thread through the hole in the transducer mounting plate, and then over the pulley at the end of the air track. Finally, place a 5.0 gram mass hook on the end of the thread. Add another 20.0 g for a total m_i of 25.0 g.

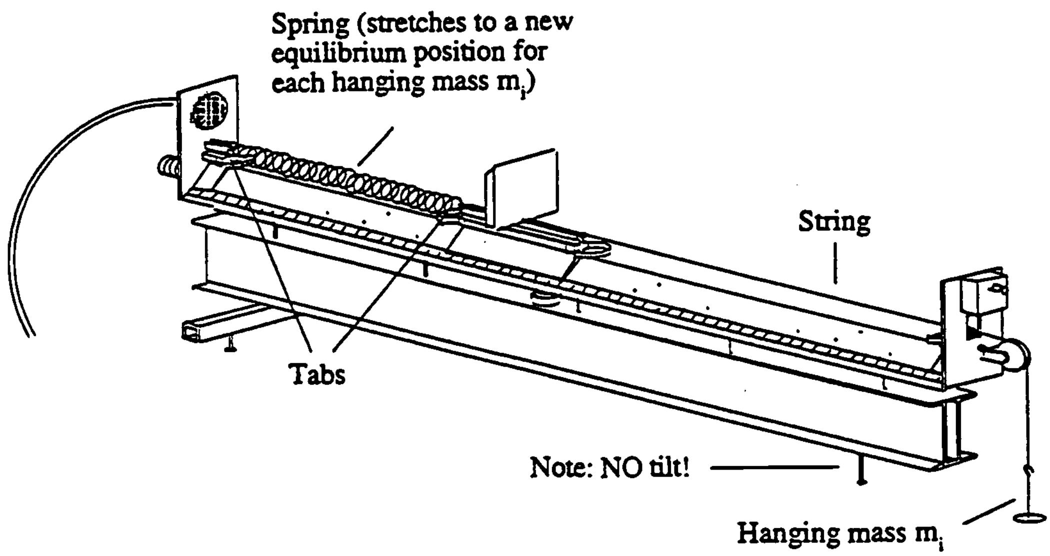


Figure 3: Set-up for Part IIIa

2. When the glider reaches equilibrium (i.e., it either stops or moves by an exceedingly small amount), measure the position of the glider with the ultrasonic position measuring system — *with the computer.* Complete Table 6 of the Data and Calculations section.

3. Repeat this procedure for the remainder of the masses m_i listed in Table 6.

4. Use a spreadsheet to perform a least squares fit upon these four (x, W) points, and determine $\bar{k}$, $\sigma_{\bar{k}}$ and the theoretical equilibrium position x_e of the system with no hanging mass. The equilibrium is NOT the y-intercept of this graph; it is the x-intercept and you will have to calculate it from your fitted values of m and b. *Your TA will demonstate this procedure in the laboratory, and you can use the same spreadsheets you wrote for MA2 in this lab.*

5. Use Table 6 in the Data and Calculations section and make a plot of W vs. x using plotting software such as *Kaleidagraph.* We choose to plot $W = +kx$ instead of $F_s = -kx$ to place our graph in the 1st quadrant of our graph paper. Indicate k and x_e and

their uncertainties on this graph, and follow the MA2 graphing standards. *You MUST use computer software for both the LSQ fit and plot to receive full credit.*

Part IIIb. Conservation of energy in a system with a stretched spring

1. When you are finished collecting data for the determination of the spring constant, remove the string, mass hook, and weights from the glider. Move the glider slowly to stretch the spring until the spring is roughly 30 - 50 cm from the edge of the air track (see Figure 4) and get ready to take data. Here the goal is to acquire x vs. t and v vs. t data which includes the motion of the glider from the time it is released to when it hits the endpost or bumper at the end of the track. To acquire data, just proceed as you did with the hanging mass in Part II.

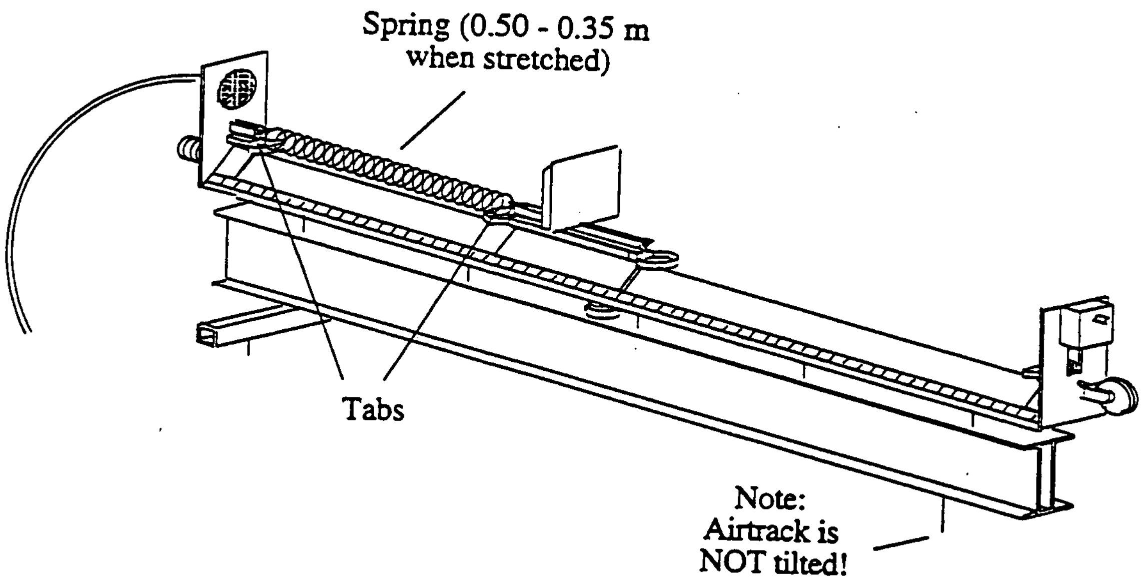


Figure 4: Set-up for Part IIIb

2. You need to obtain the same type of x vs. t and v vs. t data as you did with the hanging mass, and Tables 7 and 8 are provided for this purpose. You will use the data to again calculate values for K, U and E.

3. Print out a copy of the motion graph to hand in with your lab report. This graph should be cropped to show the region of interest, namely the time interval between glider release point and the spring returning to its normal length. Clearly label the graph with titles that include the part of the experiment to which it belongs, and label all physical events or discontinuities on the graph as well.

4. Write a paragraph comparing the x and v vs. t plots for the hanging mass system (Part II) and the spring system (Part IIIb). How do these two systems differ?

Final checks before you leave

You should have printouts of velocity and postion versus time for Parts II and III. You should check the items in the Data and Calculations section to make sure that you can fill in all of the information requested for your laboratory report. Also, verify that you have completed the items marked by the ($\surd$) symbol. Your TA will demonstrate a LSQ fit in *Kaleidagraph* and *Excel*, and you may also have time to do the LSQ fit and plot in the lab before you leave as well.

Prelaboratory Questions for Experiment E3

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

Your answers are due at the start of your laboratory session, and will be worth zero if not turned in at that time. Write your answers in the space provided, showing the essential steps that led to them.

1. As shown below, a glider of mass $M = 300.0$ g is held at rest on a horizontal air track. The glider is connected to a hanging weight of mass $m_w = 40.0$ g by a massless string. When the glider is released it experiences a uniform acceleration due to the hanging mass. At $t_i = 0.000$ s, the stationary glider is 0.150 m from one end of the track and the hanging weight is 0.850 m from the floor. At $t_f = 1.000$ s, the moving glider is 0.726 m from the same end of the track and the hanging mass is 0.274 m from the floor.

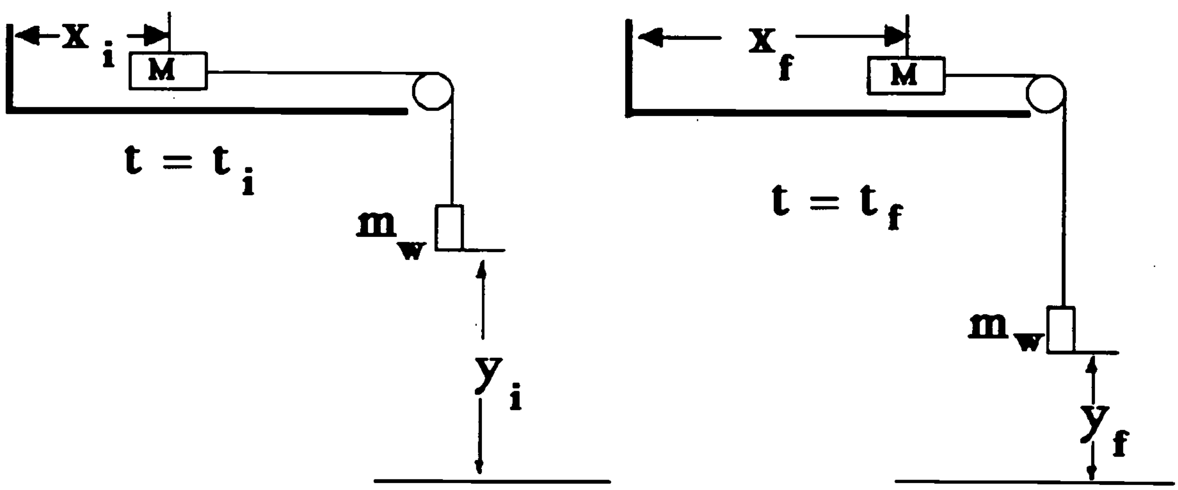


Figure 5: Diagram of a glider and hanging mass system.

(a) Complete Table 1 for our example.

(b) Use plotting software like *Kaleidagraph* to plot $K(t)$, $U(t)$, and their sum $E(t)$ vs. t for the two-mass system all together on a single graph (same axes) for the times given in Table 1.

Time Elapsed	Horizontal Position	Vertical Height	Horizontal Velocity	Kinetic Energy	Potential Energy	Total Energy
t	$x(t)$	$y(t)$	$v_x(t)$	$\frac{1}{2}(M+m_w)v^2$	$m_w gy$	$K+U$
(s)	(m)	(m)	(m/s)	(J)	(J)	(J)
0.000	0.150	0.850	0.000			
0.125	0.160		0.140			
0.250	0.185		0.288			
0.375	0.235		0.432			
0.500	0.289		0.576			
0.625	0.375		0.721			
0.750	0.476		0.862			
0.875	0.590		1.009			
1.000	0.726	0.274	1.153			

Table 1: Motion of the glider–hanging mass system

(c) Describe the relationship among $K(t)$, $U(t)$ and $E(t)$ vs. t plots.

2. Here is a situation that concerns how potential energy U is defined. The experiment described in question 1 is repeated, but instead of taking all the height measurements y relative to the floor in the laboratory, we take them relative to the floor in the basement, one flight below. Thus, all our height measurements increase by about 6 meters.

   (a) Do the values for potential energy U of the hanging mass in this trial differ from those gathered in question 1? Explain.

(b) Do the values for kinetic energy of the glider–hanging mass system differ from those gathered in question 1? Explain.

(c) Does the motion of the glider–hanging mass system change? Explain.

(d) Below, hand sketch a new graph for this increased potential energy situation. Compare this with your plot for part 1b. Indicate which of the following has changed: U, K, and/or E vs. t.

3. A glider of mass $M = 0.350$ kg is attached to an endpost of an airtrack by a spring whose $k = 0.800$ N/m. The spring is stretched to a point 0.600 m beyond its equilibrium position and released. The following table describes the resulting motion of the glider-spring system.

Time Elapsed t (s)	Spring stretch $x_s - x_e$ (m)	Horizontal Velocity $v_x(t)$ (m/s)	Kinetic Energy $\frac{1}{2}Mv^2$ (J)	Potential Energy $\frac{1}{2}k(x_s - x_e)^2$ (J)	Total Energy E = K + U (J)
0.000	0.600	0.000			
0.125	0.585	−0.175			
0.250	0.560	−0.335			
0.375	0.506	−0.487			
0.500	0.440	−0.622			
0.625	0.349	−0.735			
0.750	0.254	−0.822			
0.875	0.149	−0.875			
1.000	0.035	−0.906			

Table 2: Motion of the glider–spring system

(a) Complete Table 2. Why are the velocities negative?

(b) Use plotting software like *Kaleidagraph* to plot K(t), $U(t)$ and their sum $E(t)$ vs. t for the two-mass system on the same graph (one set of axes) for the times given in Table 2. Describe the relationship among $K(t)$, $U(t)$ and $E(t)$.

E3 Laboratory Report

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

Laboratory reports are due one week [SEVEN CALENDAR DAYS] from the day you perform the experiment. Attach a green Physics 152L cover sheet (available in the lab or window of PHYS Room 144) to the front of your exercise or report and put it in the 'Room 144 Drop Slot for Physics Lab Reports' located under the mailboxes across from Physics Room 149 before 10:00 p.m. of the day it is due.

Abstract (10 points)

Write your Abstract in the space provided below. Devote 1-2 paragraphs to briefly summarize and describe the experiments in terms of theory, activity, key findings and agreements. Include actual numerical values, agreements and discrepancies with theory. Especially comment on whether total mechanical energy was conserved for Parts II and IIIb. Write the abstract AFTER you have completed the entire report, not before.

Data and Calculations (45 points)

Part I. Set-up

1. (√) Mass of the blue glider:
   $M = (\quad \pm \quad)\text{g} = (\quad \pm \quad)\text{kg}.$
2. (√) Mass of the hanging weight:
   $m_w = (\quad \pm \quad)\text{g} = (\quad \pm \quad)\text{kg}.$
3. Mass of combined system:
   $M + m_w = (\quad \pm \quad)\text{g} = (\quad \pm \quad)\text{kg}.$

Part II. Conservation of energy in a system with a falling weight

1. (√) Hanging mass initial release height: $y_i = (\quad \pm \quad)$ m.
2. (√) Glider initial release time: $t_i = (\quad \pm \quad)$ s.
3. (√) Glider initial release velocity: $v_i = (\quad \pm \quad)$ m/s.
4. (√) Glider initial release position: $x_i = (\quad \pm \quad)$ m.
5. (√) Complete the 'Time' and 'Velocity' columns in Table 3 using the values from your v vs. t plot.

Note that these data are taken at intervals of 0.40s — *every 20th sample is recorded.*

Time (s)	Velocity (m/s)	Kinetic Energy $K = \frac{1}{2}(M + m_w)v^2$ (J)
$t_i =$		
$t_i + 0.40 =$		
$t_i + 0.80 =$		
$t_i + 1.20 =$		

Table 3: Time, velocity, and kinetic energy K for Part II

6. Complete the column 'Kinetic Energy K' in Table 3 using Equation 2. Remember that the mass is $M + m_w$.
7. Calculate the uncertainty in K at t_i + 0.80 using the uncertainty for v_i.
   $K = (\quad \pm \quad)$ J.

Note that these data are taken at intervals of 0.40s — *every 20th sample is recorded.*

Time (s)	x (m)	y (m)	Potential Energy U $m_w g y$ (J)
t_i =			
t_i + 0.40 =			
t_i + 0.80 =			
t_i + 1.20 =			

Table 4: Time, x, y, and potential energy U for Part II

Time (s)	Potential energy U (J)	Kinetic energy K (J)	Total energy E (J)
t_i =			
t_i + 0.40 =			
t_i + 0.80 =			
t_i + 1.20 =			

Table 5: Time, U, K and total mechanical energy E for Part II

8. (√) Complete the 'Time' and 'Position' columns in Table 4 using the values from your x vs. t plot.

9. (√) Print out and clearly label the plots of x and v vs. t graph and title it 'Part II.' Label the release and bounce points upon the plot.

10. Complete the column 'Potential Energy U' in Table 4 using Equation 3. Remember that the mass is m_w here. Also note that you will require the value y_i to calculate y.

11. Calculate the uncertainty of U at t_i + 0.80 using the uncertainty you found for y_i as the uncertainty in y.

    U = (±) J.

12. Complete Table 5.

13. Calculate the uncertainty of total mechanical energy E at t_i + 0.80:
    $E = U + K = ($ $\pm$ $)$ J.

14. Complete the following to determine if total mechanical energy is conserved:

    (a) Use the uncertainty δE from item 13 to calculate the allowable uncertainty range in total energy E.
    $E_{max} = E + \delta E = ($ $)$ J
    $E_{min} = E - \delta E = ($ $)$ J

    (b) Do all your total energy values in Table 5 fall into this range?

    (c) Based on this measurement analysis, is total mechanical energy conserved in the glider–hanging mass system?

Part IIIa. Determining the spring constant k

Determine the spring constant k by stretching the spring with known masses and measuring the distance of the stretch. See Figure 3 for the set up. For each of the entries take a separate data run.

1. Complete the "x from transducer" column of Table 6 by stretching the spring with the known masses m_i.

2. ($\surd$) Complete the remainder of Table 6.

    (a) Use Table 6 and a spreadsheet like *Excel* to perform a LSQ fit to the points and determine the slope $\frac{\Delta W}{\Delta x}$, which is equal to k. A reasonable value for k of these springs is on the order of 1 N/m.
    $k = ($ $\pm$ $)$ N/m.

Mass of hanging weight m_i (kg)	$W = m_i g$ (N)	x from transducer (m)
0.0250		
0.0300		
0.0350		
0.0400		

Table 6: Mass vs. spring displacement for Part IIIa.

(b) Determine the equilibrium position of the glider x_e which corresponds to the (normal) unstretched spring. The value for x_e is not determined by direct measurement, but is calculated from the values you obtain from a LSQ fit using a spreadsheet. Note that x_e is the x-intercept of the plot W vs. x; that is, the value of x where $W = 0$. Reasonable values for x_e are approximately 5 to 20 cm.

$x_e = ($ $\pm$ $)$ m.

(c) Use *Kaleidagraph* or similar plotting software to plot W vs. x based on the data in Table 6 and your least squares fit. Label your plot according to graphics standards.

Part IIIb. Conservation of energy in a system with a stretched spring

Your labeled v vs. t and x vs. t plot for this part of the lab must be attached to the end of this report.

1. (√) Glider initial release time: t_i = (±)s.
2. (√) Glider initial release position: x_i = (±)m.
3. (√) Glider initial release velocity: v_i= (±)m/s.
4. (√) Complete the 'Time' and 'Velocity' columns in Table 7 using the values from your v vs. t plot.

Note that these data are taken at intervals of 0.10*s — every 5th sample is recorded.*

Time (s)	Velocity (m/s)	Kinetic Energy $K = \frac{1}{2}Mv^2$ (J)
t_i =		
t_i + 0.10 =		
t_i + 0.20 =		
t_i + 0.30 =		
t_i + 0.40 =		
t_i + 0.50 =		

Table 7: Time, v, and K for Part IIIb.

5. Complete the column 'Kinetic Energy K' in Table 7 using Equation 2.
6. Calculate the uncertainty in K at t_i + 0.30 — you may use the uncertainty for v_i from Part II if necessary.

   KE = (±) J.

7. (√) Complete the 'Time' and 'Position' columns in Table 8 using the values from your x vs. t plot.

Note that these data are taken at intervals of 0.10s — *every 5th sample is recorded.*

Time (s)	x (m)	Stretch of spring $x_s - x_e$ (m)	$U = \frac{1}{2}k(x_s - x_e)^2$ (J)
$t_i =$		$x - x_e =$	
$t_i + 0.10 =$		$x - x_e =$	
$t_i + 0.20 =$		$x - x_e =$	
$t_i + 0.30 =$		$x - x_e =$	
$t_i + 0.40 =$		$x - x_e =$	
$t_i + 0.50 =$		$x - x_e =$	

Table 8: Time, spring stretch x, and U for Part IIIb.

8. (√) Print out and clearly label the plots of x and v vs. t graph and title it 'Part IIIb.' Label the release and bounce points upon the plot.

9. Complete the column 'Potential Energy U' in Table 8 using Equation 5.

10. Calculate the uncertainty in U at t_i + 0.30:

    U = (±) J.

11. Complete Table 9.

Time (s)	Potential Energy (J)	Kinetic Energy (J)	Total Energy (J)
$t_i =$			
$t_i + 0.10 =$			
$t_i + 0.20 =$			
$t_i + 0.30 =$			
$t_i + 0.40 =$			
$t_i + 0.50 =$			

Table 9: Time, U, K, and E for Part IIIb.

12. Calculate the uncertainty of total mechanical energy E at t_i + 0.30:

    $E = U + K = ($ $\pm$ $)$ J.

13. Complete the following to determine if total mechanical energy is conserved:

    (a) Use the uncertainty δE from item 12 to calculate the allowable uncertainty range in total energy E.

    $E_{max} = E + \delta E = ($ $)$ J

    $E_{min} = E - \delta E = ($ $)$ J

    (b) Do all your total energy values in Table 9 fall into this range?

    (c) Based on this measurement analysis, is total mechanical energy conserved in the glider–spring system?

14. In Part II, the force of gravity accelerated the glider. In Part IIIb, however, the force from a stretched spring accelerated the glider. Compare your graphs of velocity vs. time for Part II and IIIb. Judging from the overall shape of these graphs,

    (a) Is the acceleration constant over time for Part II (gravitational force)? Explain why you think this is so.

    (b) Is the acceleration constant over time for Part IIIb (stretched spring providing the force)? Explain why you think this is so.

Analysis (15 points)

Write your Analysis in the space provided. Using numerical examples and analysis, discuss the larger sources of uncertainty in your calculations. Use your numbers to justify possible improvements in apparatus and methods. Especially compare and comment on the uncertainties for total energy in Parts II and III(b).

Conclusions (5 points)

Write your Conclusions in the space provided. Did the experiment tend to confirm the theoretical physical relationships examined? Choose a significant mathematical relationship discussed in this experiment, and use it to numerically explain an activity or phenomenon from outside of the classroom. Illustrate your example by using genuine, approximate or estimated numbers.

Experiment E4: Impulse and Momentum

Prelaboratory Questions are due at the start of this activity.

Goals of this activity

At the end of this activity you will be able to define, calculate and describe the linear momentum of a moving solid object. From Newton's second and third Laws, you will be able to derive conservation of momentum and quantitatively verify it in the laboratory. You will describe and measure impulse I and quantitatively examine the impulse-momentum theorem. You will describe in detail impulse and momenta in a typical collision. You will integrate a fit to your experimental data of force versus time to obtain the impulse for a collision and quickly check your work using an area counting approximation. The majority of these measurements will be collected using the SONAR and strain gauge (force probe) systems, which you will be able to describe in detail.

1 Theory

1.1 Defining Momentum

The fundamental quantity of object motion (known as momentum in modern terminology) was developed by Galileo, Oresme, and Buridan in the 16th century. Momentum is a vector quantity – the product of the mass (scalar) and velocity (vector) of an object. In SI units, momentum has the dimensions of kg m/s. Serway writes this relation in equation 9.1:

$$\vec{p} \equiv m\vec{v} = m\frac{d\vec{x}}{dt}. \tag{1}$$

1.2 Force as rate of change in Momentum

Newton developed his concept of force and his three Laws of mechanics in terms of momentum change. Einstein subsequently showed that all mechanical interactions (including those at at relativistic speeds) conserve momentum even if the total mass of the system changes dramatically due to particle creation or annihilation. Thus the original form of Newton's Second Law $\vec{F} \equiv d\vec{p}/dt$ was also relativistically correct! However, for most mechanical interactions we observe, the total mass is constant. We can then expand the derivative and drop the term with dm/dt since it is zero for constant mass

$$\vec{F} \equiv \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{v} = m\frac{d\vec{v}}{dt}\bigg|_{m=\text{constant}} = m\vec{a}|_{m=\text{const}} \tag{2}$$

to obtain the famous equation $\vec{F} = m\vec{a}$. We again stress that this form is only true for the special case of constant total mass.

1.3 Conservation of Momentum

By combining Newton's second and third Laws we can say something profound about the way the universe works. The second law defines force as a time change in momentum, and the third states that two interacting objects always exert equal and opposite forces:

$$\underbrace{\frac{d\vec{P}_1}{dt}}_{\text{2nd Law}} = \underbrace{\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}}_{\text{3rd Law}} = \underbrace{-\frac{d\vec{P}_2}{dt}}_{\text{2nd Law}}. \tag{3}$$

The far right and left parts of this equation indicate that momentum is conserved during 2-particle interactions (collisions). This can be seen by moving both terms to the same side of the equation:

$$0 = \frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt} = \frac{d}{dt}(\vec{P}_1 + \vec{P}_2). \tag{4}$$

Since the derivative of $(\vec{P}_1 + \vec{P}_2)$ is zero, $(\vec{P}_1 + \vec{P}_2)$ must be a constant or the total linear momentum $\vec{P}$ of the 2-particle system is conserved. Therefore:

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = \text{constant}. \tag{5}$$

Momentum conservation can be shown for any number of particles in a closed system as a result of Newton's Laws. Alternately, if we obtain conservation of momentum without Newton's laws, we can reverse the above derivation to obtain Newton's second and third Laws. Serway discusses this in section 9.1. Momentum conservation is a symmetry law – linear momentum is something that our universe always seems to conserve, and physicists have never observed an interaction in which total linear momentum is lost or destroyed. Momentum conservation is a fundamental relationship in nature, and even holds (with relativistic corrections to mass) at relativistic speeds.

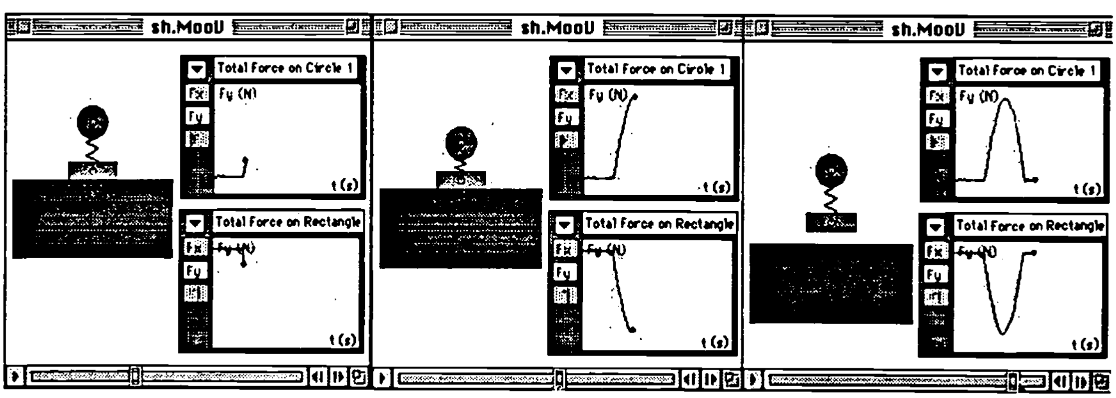


Figure 1: Computer model of a reformable object striking a non-reformable object

1.4 Forces during Collisions

When two objects collide, the forces exerted by each object on the other are always equal and opposite (Newton's third law), but the forces typically change with time. These forces are caused by small deformations in the objects, just like the large deformations we see in a spring. (See Serway Figures 9.1 and 9.8 showing collision forces.) Figure 1 shows pictures from a computer model of two objects colliding. The objects have been designed so that all of their deformation is visible in the spring that holds the modeled objects together. Note that for the collisions we will study in this experiment, the impulse curve looks something like the Gaussian or normal 'bell' curve.

In real materials, this flexure takes place within the fibers, the matrix composite, and even at the level of the electrostatic bonds between the atoms in the material. All real world materials undergo some flexure during collisions, even though it is microscopic in many cases. This kind of flexure is usually very hard to record because it is both very small and rapid. In this lab we can perform this flexure measurement using a fast data acquisition system and a force probe. We can then determine the impulse $\vec{I}(t)$ from our $\vec{F}(t)$ measurement by integration or by plotting the data on a grid and simply counting the number of squares. (This is analogous to integration by the Trapezoid or Simpson's method).

1.5 Impulse

The term impulse predates modern mechanics. Impulse was the term originally used by ancient philosophers to describe something invisible that was imparted to an object and made it move, but then 'wore off' as the object slowed to a stop. Newton was the person who finally banished this idea with his First and Second Laws, which accounted for these phenomena with a frictional force.

The modern meaning of impulse is based upon the $\vec{F}(t)$ plot. Today we define impulse as the area under the $\vec{F}(t)$ plot. This integral calculates impulse $\vec{I}$ as a vector quantity like momentum, in the same units. If this definition of impulse is applied to the second law, we obtain a relation known as the impulse-momentum theorem (Serway Section 9.2):

$$\vec{I} \equiv \int_{t_i}^{t_f} \vec{F}(t)dt = \vec{p}_f - \vec{p}_i = \Delta\vec{p}. \tag{6}$$

In this experiment, you will do this integral by fitting a polynomial to the experimental data and integrating with respect to time. LabVIEW will conduct this fit and return the coefficients for $F(t)$ to you. Recall:

$$\int t^n dt = \frac{t^{n+1}}{n+1}. \tag{7}$$

You can quickly check your results by counting grid squares under the plot. This is done by multiplying the amount of impulse per square by the total number of squares under $F(t)$.

1.6 Elastic, Inelastic, Perfectly Inelastic Collisions

When describing a collision, linear and angular momentum are the quantities most often conserved. Kinetic energy conservation, on the other hand, is far from generally assumed. In fact, whenever a collision involves objects breaking or deforming, kinetic energy is typically lost and the collision is termed inelastic. The limit of maximum kinetic energy loss occurs when the colliding objects permanently stick together in a perfectly inelastic collision. However, if the objects spring back to their original shapes, the collision is elastic and kinetic energy is conserved.

1.7 The Strain Gauge Force Probe

A strain gauge is a length of very fine wire that changes resistance when its dimensions are changed. The figure below shows a typical strain gauge at about four times actual size. The fine wire is laid out as a grid and then is bonded to piece of thin plastic film. Two leads connect the gauge to external measuring circuitry (Figure 2).

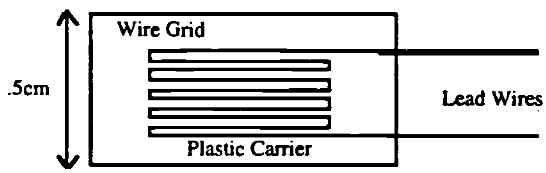


Figure 2: Details of strain gauge design (after Vernier, 1993)

Our strain gauges are glued to a strip of spring steel. Consider two strain gauges that are bonded to a metal bar with the grid wires running parallel to the length of the bar.

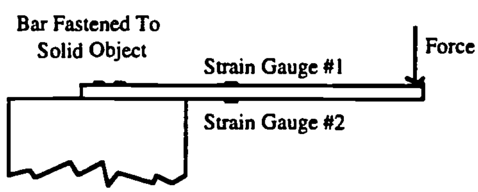


Figure 3: Details of strain gauge installation (after Vernier, 1993)

When a force is applied to the bar, it bends down in proportion to the force (after Hooke's Law). As the bar bends downward, the top of the bar stretches. The top strain gauge is also stretched, and its resistance increases slightly because the length of wire in the gauge increases and the width of the wire in the strain gauge decreases. For the strain gauge on the bottom of the bar, all of these effects are reversed (see Figure 3). Typical resistance changes are small, usually much less than 1%, but they can be measured accurately with the proper circuit (a Wheatstone Bridge is typically used). It is this change in resistance which is used to calculate the force. The calibration of the strain gauge allows us to calculate the proportionality constant needed to convert the measured resistance into a force.

The concept of monitoring microscopic amounts of bend in a metal bar to measure force can be used in a wide variety of situations. If you use a thin, metal bar that bends very easily (a spring steel bar) you have a force probe that can study collisions of dynamics carts in a physics lab. The other extreme would be to use a very large steel beam. With strain gauges mounted on a large beam, objects as heavy as vehicles or buildings can be weighed. Strain gauges are often placed on structural members in grain elevators, then calibrated to indicate how full the elevator is.

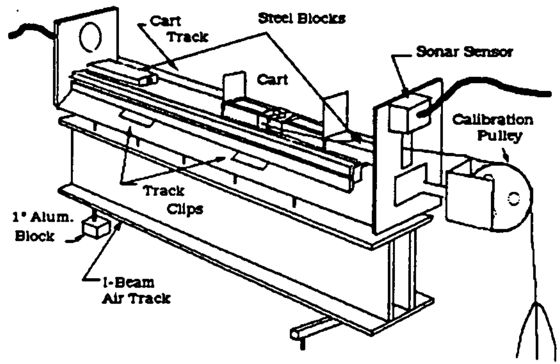


Figure 4: Low friction wheeled cart, force probe and cart track apparatus

2 Experimental method

The apparatus for this experiment uses SONAR techniques seen in E1-E3 to measure the position of two rolling low-friction carts on a cart track (placed on top of the airtrack used in E1-E3). This is shown in Figure 4.

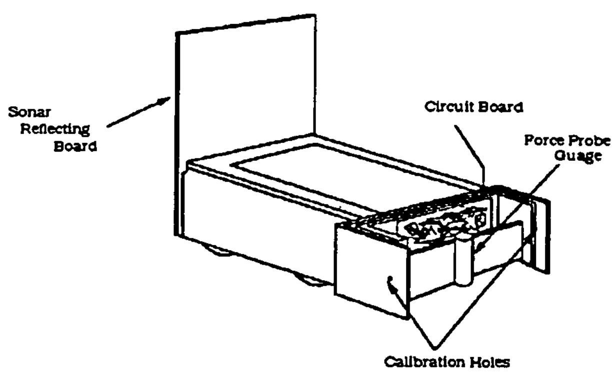


Figure 5: Low friction wheeled cart and force probe detail

The apparatus also allows force probe data to be collected from the strain gauges mounted on the front of the carts (Figure 5). These gauges are mounted on a thin strip of spring steel and can be calibrated so that the output voltage of the strain gauge null detector accurately reflects the force applied to the strip. This makes the gauges and steel strip useful as a force probe. When calibrating the probe, you will apply two known forces to the strip. The computer will compute the necessary calibration information in a manner exactly equivalent to the SONAR calibration.

To calculate $\vec{I}$ in this experiment, you will calculate the area under a plot of your experimental $\vec{F}(t)$ data collected by the force probe during a collision. Since your real-world data may not be perfectly smooth, you will do this by fitting your data to a polynomial and integrating.

You should also check your integration by quickly counting the blocks under the $\vec{F}(t)$ plot as shown in Figure 6. One method of doing this rough calculation of the area beneath the curve is to enlarge the graph and superimpose a grid. Note that force data has been collected every 0.001 sec. After counting the full blocks, quickly examine each partial block along the edge and estimate how much (e.g. 0.00, 0.25, 0.50 0.75 or 1.00 of a block) should be added for each. Finally, to determine the area you simply multiply the number of blocks by the area represented by a single block. An example of this fitting is shown in Figure 6.

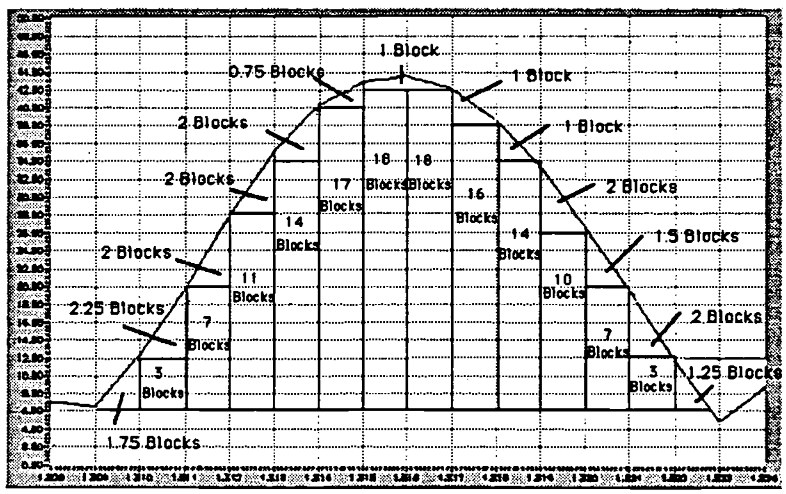


Figure 6: Impulse $\vec{F}(t)$ curve with area approximation by blocks

Part I. Calibration

In the first part of the experiment we need to calibrate the apparatus and determine the mass of both wheeled force probe carts. You must also calibrate the sonar ranger and the left and right force probes.

1. Enter your name and station number in the usual places. The left portion of the calibration screen contains the sonar calibration buttons that operate the same way as in earlier experiments.

2. First place the left cart at the left end of the track and click "Left SONAR Zero." Place the cart on the track at the 20 cm and 70 cm marks and click the "SONAR Calibrate 1" and "SONAR Calibrate 2" buttons, respectively. Don't forget to enter 0.500 m in

for the "Position Difference." The constant k will be automatically calculated for you. Remember that its value should be around 0.1 m/V. Repeat this for the right cart on the right end of the track. As soon as you are happy with your sonar calibration values, you can move on to the force probe calibrations.

3. To start the force probe calibration, CAREFULLY unclip the cable from both carts. Measure the mass of each cart as well as the calibration tray and two calibration masses using the electronic balance. Ensure the airtrack is level (not tilted with the block).

4. Reconnect the cable for the left cart and hang the tray with the first calibration mass from the string over the pulley. Hold the cart in place when you connect the hanging mass. Enter the total initial hanging mass of the tray plus the first calibration mass in the "First Calibration Mass"field. First hold the cart away from the bumper so that nothing is in contact with the force probe and click on the zero force probe calibration button. Let the force probe rest against the bumper, damp out any swinging in the mass, and click the "Left Force Probe Calibrate 1"button. This sets the first calibration point for the left force probe.

5. Now, add the second calibration mass to the tray and enter the total final hanging mass (the mass of the tray plus both calibration masses) in the "Second Calibration Mass"field. Click the "Left Force Probe Calibrate 2"button. This sets the second calibration point for the left force probe. The force constant for the force probe will be automatically calculated for you and displayed in the Force/Volt area. You should get a value ranging from 5 to 25 N/V. If you do not get a value in this range, redo this calibration (steps 4 and 5). If you are having problems, ask your TA for help.

6. Repeat steps 4 and 5 for the right force probe.

7. Once you obtain acceptable calibration values, you are ready to start the lab. Hit the button at the bottom right hand corner and move to the next screen.

Part II. Momentum and Impulse for a single cart collision

In this part of the experiment, you are expected to take SONAR and force probe data on a low-friction wheeled cart as it rolls down an incline and collides with the bumper pad on the steel blocks. The idea is to measure the velocity of the cart from the derivative of position immediately before and after a collision (pre- and post-impact velocities), then calculate p_i and p_f. You will simultaneously acquire force data from the strain gauge to obtain a plot of $F(t)$. Finally, you will determine the impulse delivered by the steel block to the cart during the collision. This is done by integrating $F(t)$. Then you will compare your I from the strain gauge data with Δp from the SONAR data.

1. Place the **RIGHT** cart on the track and pull it back to the top of the ramp (as far to the right as possible) and note its location. Make sure that the track is tilted for this experiment (use the 2.55 cm metal block under the single support end of the airtrack). Then click on "Acquire Data,"check the monitor to ensure that data acquisition has

begun and release the cart. After the data run, zoom in on the $v(t)$ plot and measure v_i and v_f immediately before and after the collision. Calculate values for p_i, p_f, and Δp.

2. Enlarge the collision $F(t)$ plot at the first collision. Enlarge it so that you have a plot similar to Figure 6. Obtain a plot suitable for determining the impulse and print a copy.

3. LabVIEW will conduct a polynomial fit to your $F(t)$ data for the cart and return the polynomial coefficients to you. You may check this polynomial fit if you wish by using a spreadsheet or graphics program or you can take advantage of the polynomial fitting capability of many calculators. The fit can also be done by hand using an appropriate statistical method (however, this can be very tedious and I don't recommend it!).

4. Print a graph showing the experimental $F(t)$ data along with your polynomial fit as a solid line.

5. Integrate the polynomial to find the impulse as a function of time $I(t)$ and the total impulse of the collision I for each cart.

6. Finally, compare your Δp calculated from the SONAR data with the value of the impulse I calculated from the $F(t)$ strain gauge data. Do they agree?

Part III. Momentum and Impulse for a two cart collision

In this part of the experiment, you are expected to simultaneously take SONAR and force probe data on two low-friction wheeled carts during a collision. This collision will involve a stationary left cart and a more massive cart moving in from the right. You will investigate this collision in detail. The change in momentum for each cart during the collision can be obtained from SONAR measurements. This is accomplished by measuring each cart's total mass and taking the derivative of the $x(t)$ sonar data. You can then determine if momentum was conserved for this collision. You can also calculate the change in momentum for each cart. The impulse for each cart can be determined using from the force probe data. This is done by integrating $F(t)$ (calculating the area under the curve). Finally, you will will compare the total impulse I from the strain gauge data with Δp from the SONAR data.

1. Place the left cart ($M_{C_{left}}$) on the track 50 cm from the left end of the track with the force probe facing right. Place the right cart along with an additional mass ($M_{C_{right}+MetalWeight}$) at the far right end of the track with the force probe facing left. Make sure that the track is level for this experiment. Click on "Acquire Data,"check the monitor to ensure that data acquisition has begun. Give the right cart a soft push. Do this a few times to get a feel for how hard to push the right cart before acquiring your final data set.

2. Zoom in on the collision region of the $v(t)$ plot. Calculate values for the initial (p_{1i} and p_{2i}), final (p_{1f} and p_{2f}), and change in momentum (Δp_1 and Δp_2) for both carts.

3. Enlarge the $F(t)$ plot so that you have a plot similar to Figure 6. Obtain a plot suitable for determining the impulse and print.

4. LabVIEW will conduct a polynomial fit to your $F(t)$ data for each cart and return the polynomial coefficients to you. You may check this polynomial fit if you wish by using a spreadsheet or graphics program or you can take advantage of the polynomial fitting capability of many calculators. The fit can also be done by hand using an appropriate statistical method (however, this can be very tedious and I don't recommend it!).

5. Print a graph showing experimental $F(t)$ data sets for each cart (using different symbols e.g. circles and squares) along with your polynomial fits (using different line types e.g. solid and dashed lines).

6. Integrate to find the impulse as a function of time $I(t)$ and the total impulse of the collision I for each cart.

7. Finally, compare your Δp calculated from the SONAR data with the value of the impulse I calculated from the $F(t)$ strain gauge data. Do they agree?

Part IV. Force Probe static experiment on Newton's Third Law

This is a simple but effective demonstration of Newton's Third Law. Place both carts against one end of the track in contact with each other so that their force probes are in contact. Use your finger to vary the force between the two carts and acquire data. Qualitatively describe what you see in terms of Newton's Third Law.

Part V. Design a force probe experiment to investigate Newton's Third law

This part allows you to design and carry out a short investigation of Newton's Third law. Be creative! Use two force probe carts and have them interact in a way different from those investigated already. Remember to conduct a complete experiment which obtains data that you can use to perform calculations leading to a direct test of Newton's Third Law.

Print plots as necessary showing your experimental data and attach these to your report.

Final checks before leaving the lab

Be sure that you have completed all items marked by the (√) symbol. Check also that you have at least four force probe data printouts – one each from Parts II through IV.

Prelaboratory Questions for E4

Name ______________________ Lab day/time ______________________

Division ____________ GTA ____________ Estimated Time for Completion ____________

Show your work in the spaces provided. Write a draft on scrap paper first. Ignore uncertainties unless specifically requested to calculate them.

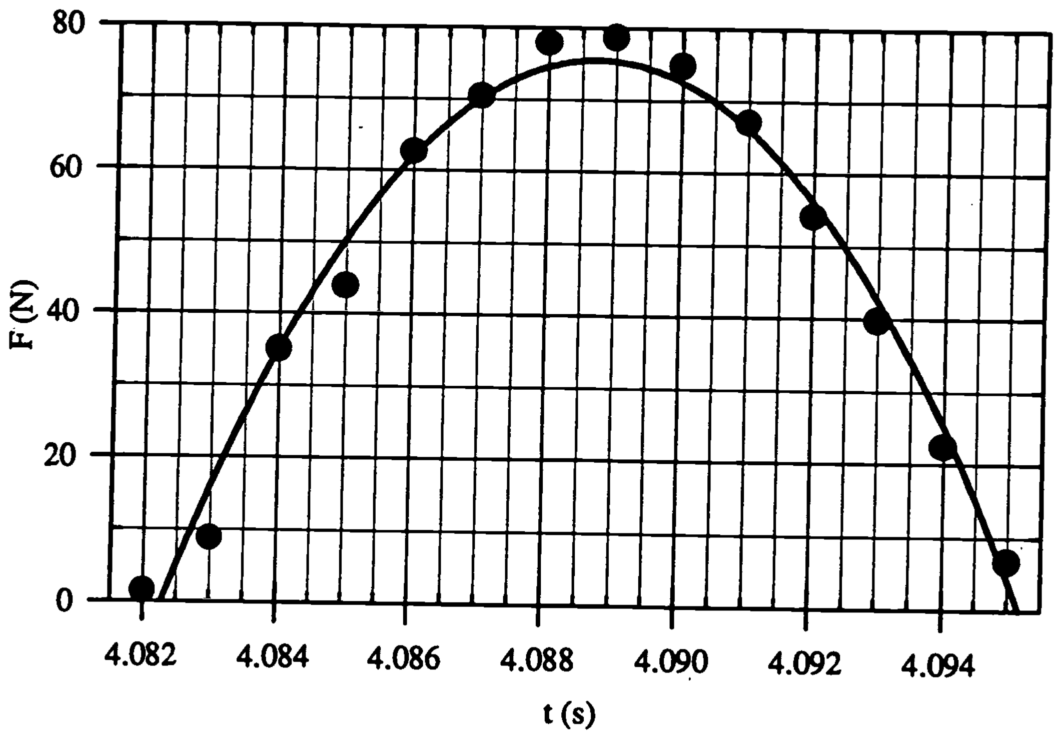


Figure 7: Actual Impulse $F(t)$ curve (Newtons vs. sec)

1. A single cart with a strain gauge collides with a stationary object. The experimental force versus time data are shown in Figure 7. A second order polynomial fit (shown as a solid line) was made to the data. This fit is given by the polynomial

$$F(t) = -3.031 \times 10^7 \text{ (N)} + 1.483 \times 10^7 \text{ (N/s) } t - 1.813 \times 10^6 \text{ (N/s}^2\text{) } t^2. \quad (8)$$

(a) Integrate $F(t)$ to find the polynomial expression for impulse as a function of time.
$I(t) =$

(b) What is the total impulse for this collision?
$I =$ ( ) kg m/s.

(c) Number of squares under the curve = (   ) squares.
Impulse represented by one square = (   ) $\text{kg}\frac{\text{m}}{\text{s}}\frac{1}{\text{square}}$.
What is the total impulse? Impulse = (   ) $\text{kg}\frac{\text{m}}{\text{s}}$.

(d) If the cart and strain gauge assembly has a total combined mass of 0.700 kg, what change in velocity did the cart undergo during the collision?
Δv = (   ) m/s.

(e) At what time during this collision did maximum force occur?
t_{MaxForce} = (   ) s.

(f) How many $\vec{g}$'s does the cart experience at t_{MaxForce}? (Recall: $\vec{g}$ is the acceleration at the Earth's surface.)
$\vec{g}_{\text{MaxForce}}$ = (   ) $\vec{g}$.

2. A physics lab cart of mass $m_1 = 3M$ moving with an initial velocity $v_{1i} = 4$ m/s collides with a second cart of mass $m_2 = M$ which is initially at rest ($v_{2i} = 0$ m/s). Assume that the cars roll without friction and the collision is perfectly elastic.

(a) What is the final velocity of each cart?
v_{1f} = (   ) m/s.
v_{2f} = (   ) m/s.

(b) What impulse did each cart have?
I_1 = (   ) kg m/s.
I_2 = (   ) kg m/s.

(c) If the collision took 0.01 seconds, what average force did the carts feel?
F_{avg} = (   ) N.

3. Now, you call the shots! Design an experiment to directly test Newton's Third Law. What data and calculations will you need to make a logical and complete argument to test Newton's Third Law? Use two force probe carts and have them interact in a way different from those investigated already.

E4 Laboratory Report

Name ______________________ Lab day/time ______________________

Division ____________ GTA ____________ Estimated Time for Completion ____________

Laboratory reports are due one week [SEVEN CALENDAR DAYS] from the day you perform the experiment. Attach a green Physics 152L cover sheet (available in the lab or window of PHYS Room 144) to the front of your exercise or report and put it in the "Room 144 Drop Slot for Physics Lab Reports" located under the mailboxes across from Physics Room 149 before 10:00 p.m. of the day it is due. Include all of your data in your laboratory report.

Abstract (10 points)

Write your Abstract in the space provided below. Devote 1–2 paragraphs to briefly summarize and describe the experiments in terms of theory, activity, key findings and agreements. Include actual numerical values, agreements and discrepancies with theory. Especially mention to what degree $\Delta\vec{p}$ agreed or disagreed with $\vec{I}$. Write the abstract AFTER you have completed the entire report, not before.

Data and Calculations (45 points)

Part I. Calibration

1. Calibrate the SONAR and Force Probes (See the "Calibration" section in the "Experimental Method" portion of this lab).

2. (√) What is the first calibration mass (tray + 1st Calibration Mass)?
   $M_{\text{calibration1}} = (\quad \pm \quad)\text{ g} = (\quad \pm \quad)\text{ kg.}$

3. (√) What is the second calibration mass (tray + both Calibration Masses)?
   $M_{\text{calibration2}} = (\quad \pm \quad)\text{ g} = (\quad \pm \quad)\text{ kg.}$

4. (√) What is the mass of each cart?

   $M_{c_{\text{right}}} = (\quad \pm \quad)\text{ g} = (\quad \pm \quad)\text{ kg.}$

   $M_{c_{\text{left}}} = (\quad \pm \quad)\text{ g} = (\quad \pm \quad)\text{ kg.}$

5. (√) What is the mass of the right cart with an additional metal weight (use one of the Calibration Masses)?

   $M_{c_{\text{right}}+MetalWeight} = (\quad \pm \quad)\text{ g} = (\quad \pm \quad)\text{ kg.}$

Part II. Momentum and Impulse for a single cart collision

1. (√) Release one cart from the top of the inclined track and acquire data for the cart's collision with the bumper pad. Resize and print plots of the SONAR $v(t)$ and force probe $F(t)$ data to closely examine the first impact.

   (√) Make sure that the data list on the right hand side of your $F(t)$ plot shows data through the entire collision which you can analyze later!

   Record the following key points and their uncertainties and label them on your plots:

   Time of Maximum Force: $t_{\text{MaxForce}} = (\quad \pm \quad)\text{ s.}$

   Maximum Force: $F_{\text{max}} = (\quad \pm \quad)\text{ N.}$

   Pre- and Post-Impact velocities of the cart:

   $v_{1i} = (\quad \pm \quad)\text{ m/s.}$

   $v_{1f} = (\quad \pm \quad)\text{ m/s.}$

2. **From your SONAR data**, determine the initial, final, and change in momentum (see Equations 1 and 6):

   $p_i = (\quad \pm \quad)\text{ kg m/s.}$

   $p_f = (\quad \pm \quad)\text{ kg m/s.}$

   $\Delta p = (\quad \pm \quad)\text{ kg m/s.}$

3. Make a polynomial fit for the cart's force probe data ($F(t)$) and print a graph showing the experimental data set with your polynomial fit as a solid line. Record the coefficients of your polynomial fit below:

   $F(t) = ($ $\qquad\qquad)$ (N) + ($\qquad\qquad$) (N/s) t + ($\qquad\qquad$) (N/s^2) t^2 .

4. Integrate to obtain $I(t)$ for the cart (your answer should be a polynomial):

   $I(t) =$

5. What is the total impulse for the collision?

   $I = ($ $\qquad\qquad$) kg m/s.

6. Compare your experimental values for the change in momentum Δp (SONAR data) of the cart and the Impulse I (force probe data) from the $F(t)$ plot. Do your results agree or indicate a discrepancy? Justify your answer.

7. Using the standard mechanics equation for Kinetic Energy (see E3), determine the initial, final, and change in Kinetic Energy for each cart during this collision:

   $KE_i = (\qquad \pm \qquad)$ J.

   $KE_f = (\qquad \pm \qquad)$ J.

   $\Delta KE = (\qquad \pm \qquad)$ J.

8. Was KE conserved in this collision? YES NO

   What kind of collision was this? Elastic Inelastic Perfectly Inelastic

Part III. Momentum and Impulse for a two cart collision

In this section of the lab you will conduct a detailed investigation of a collision of a moving cart with a second cart which is initially at rest.

1. (√) Acquire a few set of data for the two cart collision before obtaining you final data. Resize and print plots of the SONAR $v(t)$ and force probe $F(t)$ data to closely examine the first impact.

   (√) Make sure that the data list on the right hand side of your $F(t)$ plot shows data through the entire collision for both carts which you can analyze later!

   Record the following key points and their uncertainties and label them on your plots:

   Time of Maximum Force: $t_{\text{MaxForce}} = (\quad \pm \quad)$ s.

   Maximum Force: $F_{\text{max}} = (\quad \pm \quad)$ N.

   Pre- and Post-Impact velocities of the carts:

   $v_{1i} = (\quad \pm \quad)$ m/s.

   $v_{1f} = (\quad \pm \quad)$ m/s.

   $v_{2i} = (\quad \pm \quad)$ m/s.

   $v_{2f} = (\quad \pm \quad)$ m/s.

2. **From your SONAR data**, determine the initial, final, and change in momentum for each cart:

   $p_{1i} = (\quad \pm \quad)$ kg m/s.

   $p_{1f} = (\quad \pm \quad)$ kg m/s.

   $\Delta p_1 = (\quad \pm \quad)$ kg m/s.

   $p_{2i} = (\quad \pm \quad)$ kg m/s.

   $p_{2f} = (\quad \pm \quad)$ kg m/s.

   $\Delta p_2 = (\quad \pm \quad)$ kg m/s.

3. To within what percent was momentum conserved in your collision? ( )

4. Make a polynomial fit for each cart's **force probe data** ($F(t)$) and attach a single graph showing both experimental data sets (using different symbols) with your polynomial fits (using different line types e.g. solid and dashed lines). Record the coefficients of your polynomial fits below:

   $F_{\text{right}}(t) = (\quad)$ (N) + ( ) (N/s) t + ( ) (N/s^2) t^2 .

$F_{\text{left}}(t) = ($ $)\ (\text{N}) + ($ $)\ (\text{N/s})\ t + ($ $)\ (\text{N/s}^2)\ t^2$.

5. Integrate to obtain $I(t)$ for both carts (you should obtain polynomials):

   $I_{\text{right}}(t) =$

   $I_{\text{left}}(t) =$

6. What is the total impulse for the collision for each cart?

   $I_{\text{right}} = ($ $)$ kg m/s.

   $I_{\text{left}} = ($ $)$ kg m/s.

7. Compare your experimental values for the change in momentum Δp (SONAR data) of the carts and the Impulse I (force probe data) from the $F(t)$ plot. Do your results agree or indicate a discrepancy? Justify your answer.

8. Using the standard mechanics equation for Kinetic Energy (see E3), determine the initial, final, and change in Kinetic Energy for each cart during this collision:

   $KE_{1i} = ($ $\pm$ $)$ J.

   $KE_{1f} = ($ $\pm$ $)$ J.

   $\Delta KE_1 = ($ $\pm$ $)$ J.

   $KE_{2i} = ($ $\pm$ $)$ J.

   $KE_{2f} = ($ $\pm$ $)$ J.

   $\Delta KE_2 = ($ $\pm$ $)$ J.

9. Was KE conserved in this collision? YES NO

   What kind of collision was this? Elastic Inelastic Perfectly Inelastic

Part IV. Force Probe static experiment on Newton's Third Law

In this section, you will investigate Newton's Third Law by measuring the interaction between two stationary carts in contact. Place both carts against one end of the track so that their force probes are touching. Use you finger to vary the force between the carts and acquire data.

(√) Print you results and label them "Part IV: Force Probe static experiment on Newton's Third Law."

Describe the relationship between the $F(t)$ data from each cart. Also, compare your experimental results with your prediction based on Newton's Third Law.

Part V. Design a force probe experiment to investigate Newton's Third Law

Now, you call the shots! Design an experiment to directly test Newton's Third Law. Present your data and calculations in a logical and complete manner to support your conclusions about Newton's Third Law. Attach plots as necessary.

Analysis (15 points)

Write your Analysis in the space provided. Using numerical examples and analysis, discuss the larger sources of uncertainty in your calculations. Use your numbers to justify possible improvements in apparatus and methods. Address Part III in depth. Do not repeat **Analysis** material from previous experiments.

Conclusions (5 points)

Write your Conclusions in the space provided. Did the experiment tend to confirm the theoretical physical relationships examined? Choose a significant mathematical relationship discussed in this experiment, and use it to numerically explain an activity or phenomenon from outside of the classroom. Illustrate your example by using genuine, approximate or estimated numbers.

Experiment E5: Rotational Dynamics

Prelaboratory Questions are due at the start of this activity.

Due to the unfamiliar and non-intuitive forces involved in these activities, you will be surprised by the forces that you experience. You must personally experience these forces, so ensure that everyone has a turn trying. Be careful when you change the orientation of the bicycle flywheel – it has a lot of angular momentum. Be alert for strong forces and sharp movements in unexpected directions, and keep plenty of free elbow room. Move deliberately and cautiously when handling the spinning flywheel, and do not grab loose flywheels. Use the motor drive provided for spinning up the flywheel. Keep the flywheels on the floor (not on the table) when they are not in use.

Goals of this activity

At end of this activity you will be familiar with the qualitative relationships and quantitative values associated with rotational motion. You should be able to:

- determine the moment of inertia I of rotating objects and systems,
- calculate values of the moment of inertia I, the kinetic energy KE, and the angular momentum $\vec{L}$ for rotating objects, and
- quantitatively and qualitatively describe phenomena associated with conserving angular momentum $\vec{L}$ in rotating objects and systems.

1 Theory

1.1 Torque

The rotational analog to force in Newton's Second Law is torque:

$$\vec{\tau}_{\text{net}} = I\vec{\alpha}, \tag{1}$$

$$\vec{F}_{\text{net}} = m\vec{a}.$$

However, torque can also be expressed as a cross product between an applied force $\vec{F}$ and the vector from the axis of rotation to the applied force:

$$\vec{\tau}_{\text{net}} = \vec{r} \times \vec{F}. \tag{2}$$

Since the direction of cross products are generally non-intuitive, unexpected torques can arise! You will get a chance to experience this yourself using a spinning wheel. Note that $\vec{\tau}_{\text{net}}$ must be defined with respect to an axis of rotation so $\vec{r}$ is the vector position (the radius of application) and $\vec{F}$ is applied force.

1.2 Moment of Inertia

The moment of inertia of an object in rotational problems is analogous to mass in linear motion (compare $\vec{F}_{\rm net} = m\vec{a}$ with $\vec{\tau}_{\rm net} = I\vec{\alpha}$). The moment of inertia depends on the *mass distribution* of the object under rotation, *as well as the axis of rotation* and is defined as (Serway 10.16):

$$I \equiv \int r^2 dm. \tag{3}$$

Since the moment of inertia depends on r^2, I can change dramatically just by moving a constant mass closer to or further from the rotation axis. You will get a chance to feel this for yourself in this lab.

To save you some integration, the moment of inertia for several symmetrical, uniform-density objects are available in standard tables (see Table 10.2 in Serway). For example, the moment of inertia of a disk about the axis of symmetry is

$$I_{\rm disk} = \frac{1}{2}MR^2. \tag{4}$$

But not all objects are simple, regular objects or are rotating about an axis of symmetry. For many real world configurations which can not be accurately modeled by a regular shape, it is easiest to measure the moment of inertial experimentally using $\vec{\tau}_{\rm net} = I\vec{\alpha}$. If you apply net torque $\tau_{\rm net}$ and measure the resulting angular acceleration α, you can directly calculate I.

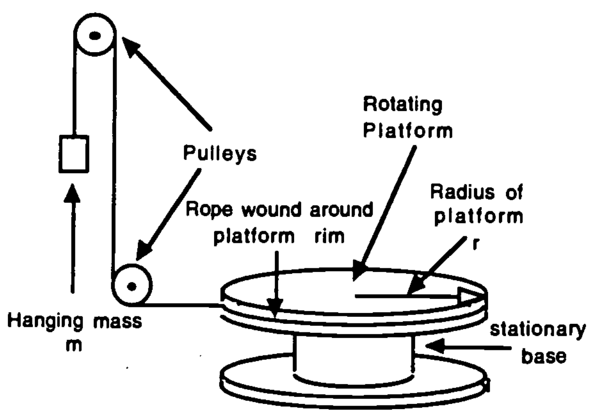


Figure 1: Geometry of the rotational platform and hanging mass

In this experiment, you will use the pulley system shown in Figure 2 to apply a known force (mg due to a hanging mass) on the rim of a rotating circular platform. The applied torque is then rmg. You will measure the resulting angular rotation α of the platform and calculate $I_{\rm Platform}$ or I for any object or system of objects you place on the platform.

1.3 Angular Momentum and Conservation of Momentum

The rotational analog of linear momentum $\vec{p}$ is angular momentum $\vec{L}$. In previous problems in PHYS 152, we found it was convenient to break up the linear momentum into its vector components. Similarly, it is convenient to express angular momentum in terms of vector components. The angular momentum component for a rigid object about the z-axis $\vec{L}_z$ can be derived from the general form $\vec{L} = \vec{r} \times \vec{p}$ where $\vec{r}$ and $\vec{p}$ are in the x-y plane (note: $\vec{r} = r\hat{r}$).

$$\vec{L}_z \equiv \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = \frac{r}{r} r\hat{r} \times mv\hat{v} = mr^2 \frac{v}{r} \hat{r} \times \hat{v} = I_z \omega_z \hat{z}. \tag{5}$$

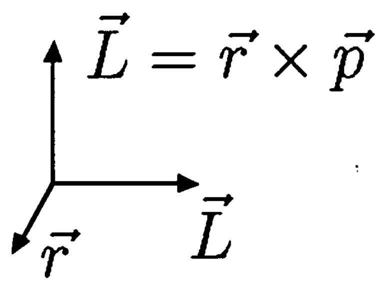


Figure 2: Cross Product for $\vec{L} = \vec{r} \times \vec{p}$

If there is no net external torque on a system, then the total angular momentum is conserved (see Serway section 11.5):

$$\vec{L}_i = \vec{L}_f \tag{6}$$

$$I_i \vec{\omega}_i = I_f \vec{\omega}_f$$

This equation tells us that if the moment of inertia I is varied during rotation, the angular velocity must also change to maintain a constant angular momentum. This is why figure skaters can control their own spin rate by varying the I of their bodies.

The situation is slightly more complex when several rotating bodies form a system (e.g. a rotating bicycle wheel held by a student on a rotating platform). In such a closed system where the external torque is zero, angular momentum is still conserved ($L_{\text{System}i} = L_{\text{System}f}$). However, there are now several terms contributing to the total angular momentum:

$$L_{\text{Wheel}i} + L_{\text{Platform}i} + L_{\text{Student}i} = L_{\text{Wheel}f} + L_{\text{Platform}f} + L_{\text{Student}f}. \tag{7}$$

Be careful-if the rotating bicycle wheel is inverted 180 degrees (π rad), L changes sign! When you flip the wheel, ω_{Wheel} changes sign which carries through to the product $L_{\text{Wheel}} = I_{\text{Wheel}} \omega_{\text{Wheel}}$. Consequently, make sure that you keep track of the direction of rotation and record the correct signs for ω and L!

1.4 Rotational Kinetic Energy

The work and energy concept as developed for linear motion also has an analogous form in rotational motion. The net work done by an external torque to rotate a rigid object about a fixed axis is equal to the change in the rotational kinetic energy of the object. The rotational kinetic energy is:

$$KE_{\text{Rot}} = \frac{1}{2}I\omega^2. \tag{8}$$

Note that for work done by internal forces in a rotating system (like a skater changing the I of their body, or a bicycle wheel being inverted by you on a rotating platform), mechanical energy is being put into the system by chemical reactions in human muscles and rotational kinetic energy will not be conserved. However, since these forces are internal, there is no external torque and angular momentum is conserved.

See Table 10.3 in Serway for a more complete comparison of rotational and linear equations in mechanics.

2 Experimental method

The software for this experiment gives the angular displacement $\Theta(t)$ of the rotating platform as a function of time. The platform has a striped disk attached to its underside as shown in Figure 3. A dual-channel optoelectronic switch senses the stripes and feeds this information to the computer, which converts the information into the angular displacement $\Theta(t)$ of the rotating platform. The software then calculates and displays average angular velocity $\bar{\omega}(t)$ (first derivative) and average angular acceleration $\bar{\alpha}(t)$ (second derivative) using the same kind of algorithms you used for average linear velocity and average linear acceleration in previous experiments.

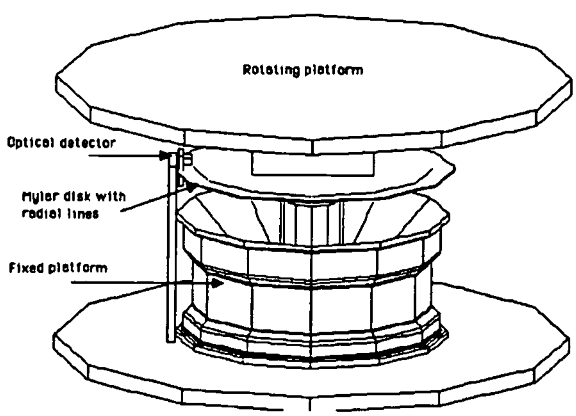


Figure 3: Schematic picture of the rotational platform

Part I. $\vec{L}$ and KE_{Rot} for a bicycle wheel

There is a limited number of photo-tachometers in the laboratory. If all of the photo-tachometers are in use you can begin Parts III and IV first.

Because you will need to calculate $\vec{L}$ and KE_{Rot} for a spinning bicycle wheel, the moment of inertia of the wheel I_{Wheel} has to be determined. Weigh the bicycle wheel and measure its radius. Approximating the wheel as a hoop, we can calculate I_{Wheel} :

$$I_{\text{Wheel}} \sim I_{\text{hoop}} = MR^2. \tag{9}$$

Knowing I_{Wheel}, you can now calculate KE_{Rot} and L for the wheel at any given angular velocity ω. To impart a known amount of angular velocity on the bicycle wheel, a motor rotating at a fixed angular speed is used. Bring the bicycle wheel and the motor into contact and hold the two together as the wheel spins up. Use the photo-tachometer to determine rotational velocity ω. Try twisting the axis of rotation of the spinning wheel to change the direction of the $\vec{L}$. Describe the forces you feel through your arms. Where do these forces come from?

Part II. Conservation of $\vec{L}$ and KE in a complex rotating system

Spin up the bicycle wheel and rotate the wheel so its angular momentum vector $\vec{L}$ points horizontally. Pass the rotating bicycle wheel to your partner seated on the wooden platform. Your partner will then turn the bicycle wheel vertically both ways (see Figure 4). Qualitatively investigate the motion including inverting the wheel in the opposite direction. Describe what happens when you change the orientation of the bicycle wheel. Have fun with it!

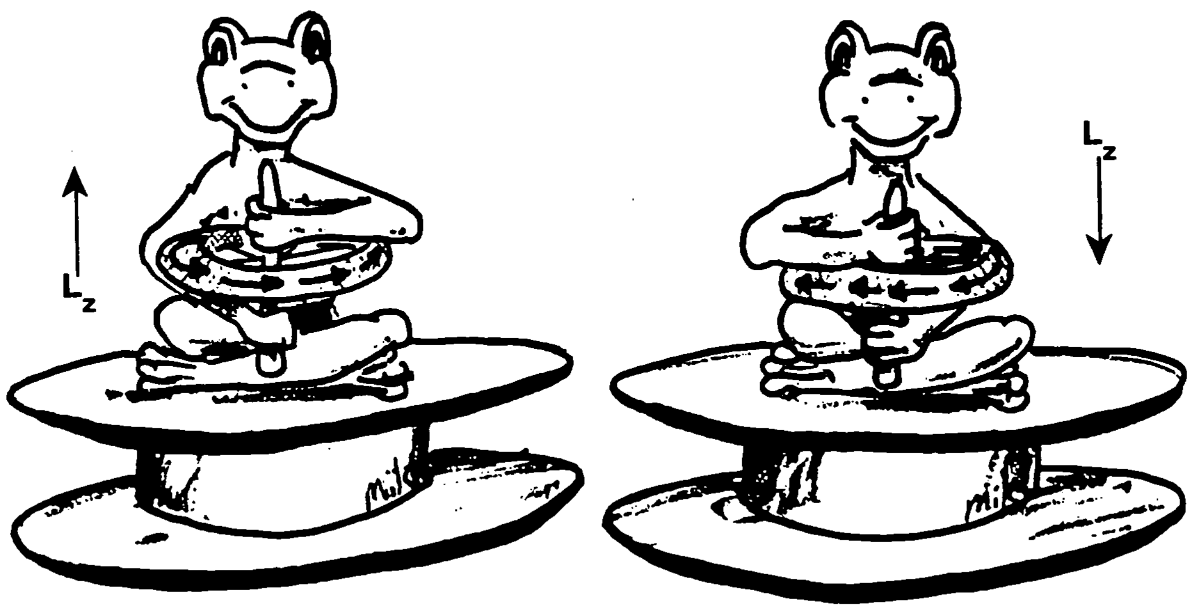


Figure 4: Inverting the spinning bicycle wheel

Next we will examine this phenomenon quantitatively. Use the motor to spin up the bicycle wheel again, turn the spinning bicycle wheel to an upright (vertical) position ($\vec{L}_{Wheel}$ points up), and hand it to one of your partners sitting on the platform, while holding the platform stationary. Using the digital photo-tachometer measure f_{Wheeli} to calculate $\vec{\omega}_{\text{Wheeli}}$ in this orientation (note: $\omega = 2\pi$ rad f). Use this measurement to calculate the initial values I_i, KE_i and $\vec{L}_i$ for the various components of the system.

Release the platform so it can turn freely. Invert the bicycle wheel (turn it by 180 degrees or π rad). Determine $\vec{\omega}_{\text{platform}f}$, and $\vec{\omega}_{\text{person}f}$ from the LabVIEW software (at least two rotations) and calculate $\vec{\omega}_{\text{Wheel}f}$ by stopping the platform and measuring the wheel's frequency with the digital photo-tachometer again. Watch the direction of rotations, as some of the $\vec{L}$'s may change sign.

Is angular momentum conserved in this complex system? For which objects does the angular momentum reverse algebraic sign? Is kinetic energy conserved? Explain why or why not.

Print plots as necessary showing your data and attach these to your report.

Part III. Angular Momentum in a Collision

For our rotating platform, all four major components (the top of the platform, the aluminum/mylar disk, the drive axle and the anchoring platform) rotate about their center axis. In theory we could calculate I for each individual piece and then simply add them together. However, the odd geometry of some of the components used for mounting the platform would make the error in this calculation rather high. Fortunately, we can easily measure I experimentally using $\tau = I\alpha$. We just apply a known net external torque and measure the resulting angular acceleration α as shown earlier in Figure 2 and discussed in section 1.1. You will use this technique to find the moment of inertia for the platform and the platform-disk system. You can then subtract to find the moment of inertia of the disk and compare with a theoretical calculation for a uniform disk.

Spin up the platform (somewhere over 12 rad/sec should work). Start acquiring data. Line up the extra wooden disk and metal peg just above the center hole in the spinning platform. **Make sure the peg falls slightly into the center hole of the platform. If the metal peg is not lined up correctly, the heavy wooden disk could fly off of the platform and hit someone!** If you touch the spinning platform, begin this step over. After about 10 seconds of data from the spinning platform alone, drop the extra wooden disk onto the spinning platform. For this collision, is angular momentum is conserved?

Part IV. $\vec{L}$ and Kinetic Energy under changing I

In this part of the experiment, you will use the same technique you used in Part I to determine the total $I_{\text{Initial}} = I_{\text{Platform}} + I_{\text{StudentBarbellsOut}}$ of a person holding barbells at the end of extended arms and then the total $I_{\text{Final}} = I_{\text{Platform}} + I_{\text{StudentBarbellsIn}}$ with the barbells held close to the body as shown in Figure 5.

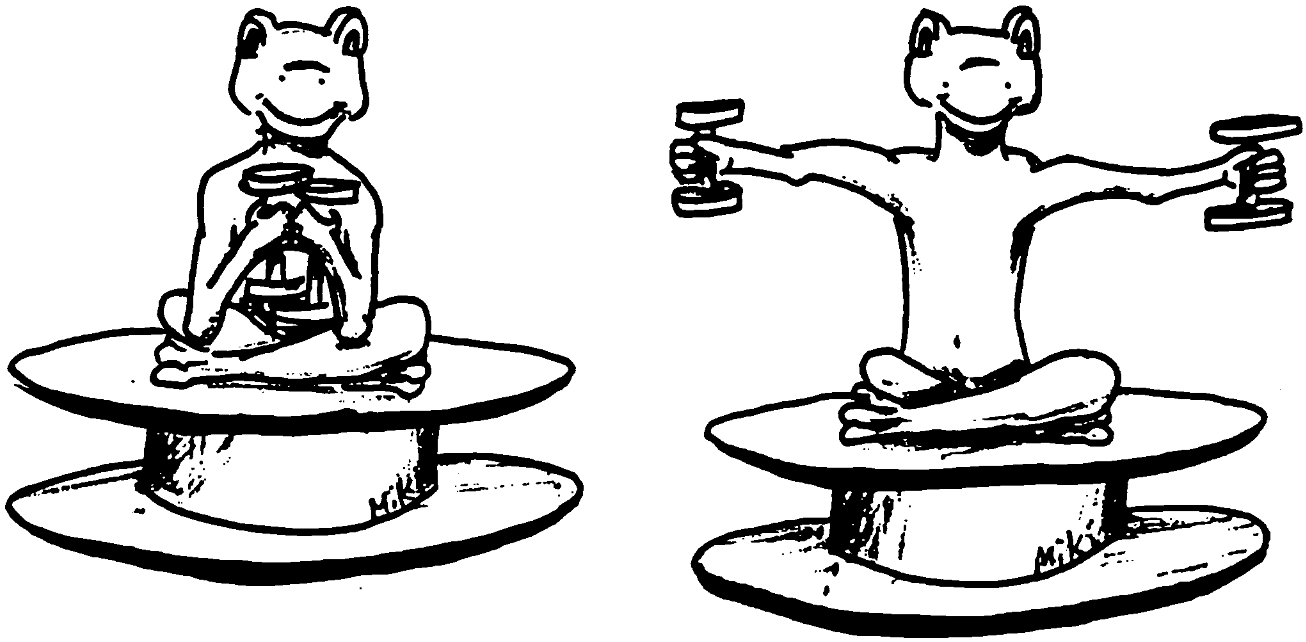


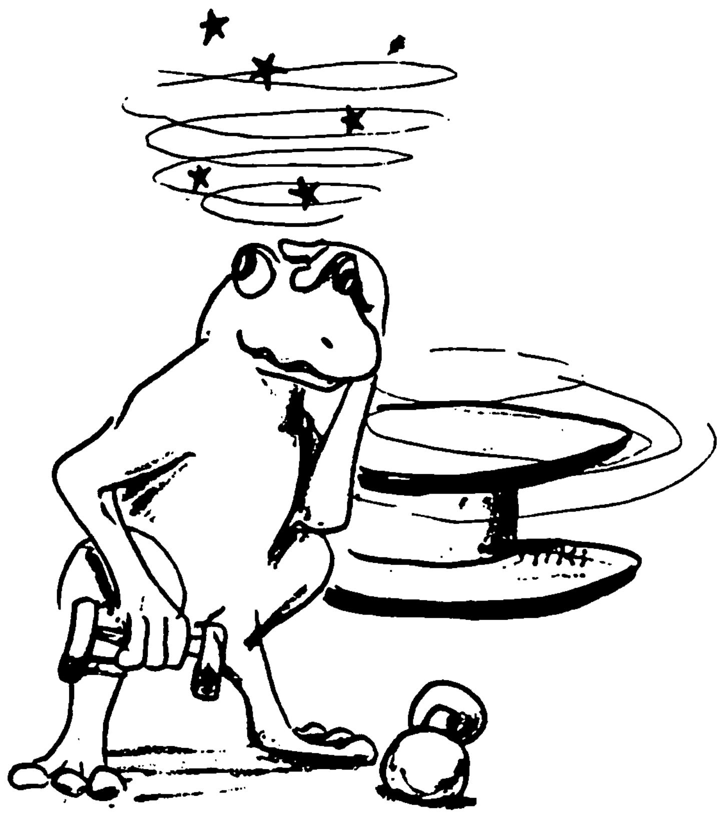
Figure 5: Changing I with barbells for a Frog/Platform system

Sit securely on the rotating platform with your center of mass approximately over the axis of rotation and hold the two barbells at arms length. Your partner will provide a torque to start the platform rotating. Rotate for 1-2 revolutions, pull your arms in close to your body for 1-2 revolutions, and re-extend your arms. Acquire $\omega(t)$ data during the exercise and compare your values of ω_i and ω_f. Is angular momentum conserved as expected? Do you feel a radial force through your arms when you move the barbells in and out? Is rotational kinetic energy conserved? Why or why not? Repeat this several times until you are satisfied with your understanding of the concepts being demonstrated. (Be careful not to overdo it – stop if you become dizzy or disoriented.)

Final checks before leaving the lab

Be sure that you have completed all items marked by the ($\surd$) symbol.

WARNING!!!



Don't be a frog! Remember to sit, not stand, on the platform.

Wear appropriate clothing for doing experiments on rotating platforms-**slacks not skirts.**

Prelaboratory Questions for E5

Name ________________________ Lab day/time ________________________

Division ____________ GTA ____________ Estimated Time for Completion ____________

Show your work in the spaces provided. Write a draft on scrap paper first. *Do not calculate uncertainties unless they are explicitly requested.*

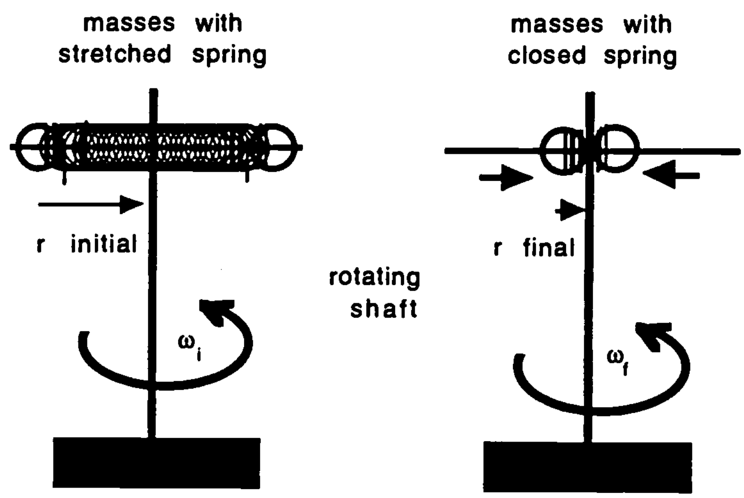


Figure 6: Rotating widget

1. Examine the rotating widget in Figure 6. This apparatus has two masses each of 2.5 kg, initially held $r_i = 1.00$ m away from the axis of rotation, and are set spinning with an initial angular velocity of $\omega_i = 2\pi$ rad/sec ($f = 1.0$ rev/sec). After several frictionless turns, the pins holding the masses out are released (without otherwise disturbing the system) so that the springs are allowed to pull the masses in to a new radius $r_f = 0.05$ m. Assume that all bearings are frictionless, that the horizontal rod is massless, and that the masses slide frictionlessly on the horizontal rod.

   (a) What are the initial I_i and final I_f moments of inertia of the system?
   $I_i = ($ $)$ kg m^2,
   $I_f = ($ $)$ kg m^2.

   (b) What are the initial $\vec{L}_i$ and KE_{Roti} of this system?
   $\vec{L}_i = ($ $)$ kg m^2 rad/sec,
   $KE_{\text{Roti}} = ($ $)$ J.

(c) What is the final angular velocity ω_f of the system?
$\omega_f = (\qquad\qquad)$ rad/sec.

(d) What are the final $\vec{L}_f$ and $KE_{\mathrm{Rot}f}$ of this system?
$\vec{L}_f = (\qquad\qquad)$ kg m^2 rad/sec,
$KE_{\mathrm{Rot}f} = (\qquad\qquad)$ J.

(e) In your own words, explain what has happened to this system in terms of $\vec{L}$ and total kinetic energy of this system. If the system lost or gained angular momentum or kinetic energy, where did it go or come from?

2. When riding a motorcycle down the Interstate to Indianapolis, a physics 152 student ponders how much momentum and kinetic energy are present in her linear and rotational motion. She knows the mass of each wheel is 21 kg, the wheel diameter is 0.8m, and her speed is 126 km/h. Assume you can model her motorcycle wheels as hoops.

   (a) How much L and KE_{Rot} is in the rotation of one of her wheels?
   $L = (\qquad\qquad)$ kg m^2 rad/sec,
   $KE_{\text{Rot}} = (\qquad\qquad)$ J.

   (b) How much L and KE_{Rot} is in the rotation of both of her wheels together? (Hint: twice as much as one alone.)
   $L = (\qquad\qquad)$ kg m^2 rad/sec,
   $KE_{\text{Rot}} = (\qquad\qquad)$ J.

   (c) How much p and KE_{Linear} are in the translational motion of the motorcycle if the motorcycle-physics student system has a total mass of 275 kg?
   $p = (\qquad\qquad)$ kg m/sec,
   $KE_{\text{Linear}} = (\qquad\qquad)$ J.

   (d) Compare KE_{Linear} with KE_{Rot}. Where is most of the kinetic energy? Compare $\vec{p}$ with $\vec{L}$. Does it make sense to compare these two vector quantities? Why or why not?

E5 Laboratory Report

Name ______________________ Lab day/time ______________________

Division __________ GTA __________ Estimated Time for Completion __________

Laboratory reports are due one week [SEVEN CALENDAR DAYS] from the day you perform the experiment. Attach a green Physics 152L cover sheet (available in the lab or window of PHYS Room 144) to the front of your exercise or report and put it in the "Room 144 Drop Slot for Physics Lab Reports" located under the mailboxes across from Physics Room 149 before 10:00 p.m. of the day it is due. Include all of your data in your laboratory report.

Abstract (10 points)

Write your Abstract in the space provided below. Devote 1–2 paragraphs to briefly summarize and describe the experiments in terms of theory, activity, key findings and agreements. Include actual numerical values, agreements and discrepancies with theory. Write the abstract AFTER you have completed the entire report, not before.

Experimental Results and Calculations (45 points)

Part I. $\vec{L}$ and KE of a rotating bicycle wheel

1. (√) Measure the radius and mass of the bicycle wheel.

   $m_{\text{SmallHandle}} = (\quad \pm \quad)$ g,
   $m_{\text{LargeHandle}} = (\quad \pm \quad)$ g,
   $R_{\text{Wheel}} = (\quad \pm \quad)$ m,
   $M_{\text{Wheel}} = (\quad \pm \quad)$ kg.

2. Calculate I of the bicycle wheel (assume it can be approximated by a hoop).

   $I_{\text{Wheel}} = (\quad \pm \quad)$ kg m^2.

3. (√) Use the motor to spin up the bicycle wheel. Use the digital photo-tachometer to measure the frequency f_{Wheel} of the bicycle wheel and calculate the angular velocity ω_{Wheel}.

   $f_{\text{Wheel}} = (\quad \pm \quad)$ rpm $\rightarrow$ $\omega_{\text{Wheel}} = (\quad \pm \quad)$ rad/s.

4. Calculate L_{Wheel} of the bicycle wheel: $L_{\text{Wheel}} = (\quad \pm \quad)$ kg m^2 rad/s.

5. Calculate KE_{Rot} of the bicycle wheel: $KE_{\text{Wheel}} = (\quad \pm \quad)$ J.

6. (√) Move the bicycle wheel around (EVERYONE MUST DO THIS!). In your own words, describe the torques that you feel through your arms when you try and point the bicycle wheel axis in different directions.

7. Describe the physics of what is happening when you change the orientation of the bicycle wheel.

Part II. Conservation of $\vec{L}$ and KE in a complex rotating system

1. (√) Sit in the center of the platform and have one of your partners spin up the bicycle wheel using the motor and hand it to you spinning vertically ($\vec{L}$ is horizontal). Tilt the spinning wheel and describe the resulting motion. Can you control your rotational direction and angular velocity? Remember to make sure everyone gets a chance to work with the spinning wheel.

2. Describe and explain the resulting motion of the system in terms of $\vec{L}$.

3. When you invert the bicycle wheel, do you do work on the system? If so, where does the work go?

4. (√) Spin the wheel up again. Determine initial values for the angular velocity of the bicycle wheel, platform, and student. Use the digital photo-tachometer to measure the frequency and assume the platform is motionless to start. (Watch your signs: + or - !)

$f_{\text{Wheel}i}$ = (±) rpm → $\omega_{\text{Wheel}i}$ = (±) rad/s
$f_{\text{Platform}i}$ = (±) rpm → $\omega_{\text{Platform}i}$ = (±) rad/s
$f_{\text{Student}i}$ = (±) rpm → $\omega_{\text{Student}i}$ = (±) rad/s

5. (√) Determine final values for the angular velocity of the bicycle wheel, platform, and student. Use the LabVIEW software to determine $\omega_{\text{Platform}f}$ and $\omega_{\text{Student}f}$, then stop the platform and measure $\omega_{\text{Wheel}f}$ with the digital photo-tachometer. (Watch your signs: + or - !)

$f_{\text{Wheel}f}$ = (±) rpm → $\omega_{\text{Wheel}f}$ = (±) rad/s
$f_{\text{Platform}f}$ = (±) rpm → $\omega_{\text{Platform}f}$ = (±) rad/s
$f_{\text{Student}f}$ = (±) rpm → $\omega_{\text{Student}f}$ = (±) rad/s

6. Calculate L of the initial system. $\vec{L}_{\text{System}i}$ points: UP DOWN

$L_{\text{System}i}$ = (±) kg m^2 rad/s.

7. Calculate L of the final system. $\vec{L}_{\text{System}f}$ points: UP DOWN

$L_{\text{System}f}$ = (±) kg m^2 rad/s.

8. Is angular momentum conserved in the system? Explain.

9. Do you expect KE_{Rot} to be conserved in the system? Explain.

Part III. Angular Momentum in a Collision

1. (√) Measure the mass and radius of the extra wooden disk. Calculate I of the wooden disk assuming it can be modeled as a simple regular disk.
   M_{Disk} = (±) kg,
   R_{Disk} = (±) m,
   $I_{\text{DiskCalculated}}$ = (±) kg m^2.

2. (√) Use the hanging mass, rope, and LabVIEW software as shown in Figure 2 to measure τ and α. Calculate I_{Platform}. Attach the plot of $\omega(t)$ used to calculate α.
   τ = (±) N m,
   α = (±) rad/s^2,
   I_{Platform} = (±) kg m^2.

3. ($\surd$) Use the hanging mass, rope, and LabVIEW software as shown in Figure 2 to find τ and α. Determine $I_{\text{Platform+Disk}}$. Attach the plot of $\omega(t)$ used to calculate α.
$\tau = ($ $\pm$ $)$ N m,
$\alpha = ($ $\pm$ $)$ rad/s^2,
$I_{\text{Platform+Disk}} = ($ $\pm$ $)$ kg m^2.

4. Calculate I_{Disk} using data from Parts 2 and 3. $I_{\text{Disk}} = ($ $\pm$ $)$ kg m^2.

5. Do your values for I_{Disk} and $I_{\text{DiskCalculated}}$ agree? Comment.

6. (√) What was the initial, final, and change in angular velocity for the collision?
   ω_i =(±) rad/s,
   ω_f =(±) rad/s,
   $\Delta\omega$ =(±) rad/s.

7. What was the initial, final, and change in angular momentum? (use the experimental values for I_{Platform} and I_{Disk} from steps 2 and 3.)
   L_i =(±) kg m^2 rad/s,
   L_f =(±) kg m^2 rad/s,
   ΔL =(±) kg m^2 rad/s.

8. Was angular momentum conserved for this collision? Justify your answer.

Part IV. L **and Kinetic Energy under changing** I

Note: All students should personally experience these phenomena.

1. (√) Again, use the hanging mass, the rope and the LabVIEW software as shown in Figure 2 to measure τ and α. Determine $I_{\text{Platform}} + I_{\text{StudentBarbellsOut}}$ and $I_{\text{Platform}} + I_{\text{StudentBarbellsIn}}$. Attach plots for $\omega(t)$.

   $\tau = (\quad \pm \quad)$ N m,

   $\alpha_i = \alpha_{\text{Platform}} + \alpha_{\text{StudentBarbellsOut}} = (\quad \pm \quad)$ rad/s^2,

   $I_i = I_{\text{Platform}} + I_{\text{StudentBarbellsOut}} = (\quad \pm \quad)$ kg m^2.

   $\alpha_f = \alpha_{\text{Platform}} + \alpha_{\text{StudentBarbellsIn}} = (\quad \pm \quad)$ rad/s^2,

   $I_f = I_{\text{Platform}} + I_{\text{StudentBarbellsIn}} = (\quad \pm \quad)$ kg m^2.

2. (√) Start rotating slowly (about 1 rev/sec) with barbells out and use LabVIEW to collect $\omega_i(t)$ data for 1-2 complete revolutions, then pull in the barbells and hold them to your chest. Collect LabVIEW $\omega_f(t)$ data for 1-2 revolutions with barbells in. Attach your plot of $\omega(t)$.

   $\omega_i = (\quad \pm \quad)$ rad/sec,

   $\omega_f = (\quad \pm \quad)$ rad/sec.

3. Calculate L_i and L_f. Do the two agree? Comment.

   $L_i = (\quad)$ kg m^2 rad/sec,

   $L_f = (\quad)$ kg m^2 rad/sec.

4. Calculate $KE_{\text{Rot}i}$ and $KE_{\text{Rot}f}$. Do the two agree? Comment.
$KE_{\text{Rot}i} = (\qquad\qquad)$ J,
$KE_{\text{Rot}f} = (\qquad\qquad)$ J.

5. (√) In your own words, describe the effect pulling your arms in has upon your angular velocity.

6. (√) Does pulling your arms in while rotating require more or less effort than when you are stationary? Why?

7. What does this mean in terms of work and KE_{Rot}?

8. If the barbells were omitted, would this enhance or diminish the phenomena? Why?

Discussion and Analysis (15 points)

Write your Analysis in the space provided. Using numerical examples and analysis, discuss the larger sources of uncertainty in your calculations. Use your numbers to justify possible improvements in apparatus and methods. Discuss briefly how your concept of rotational motion changed during this activity. Do not repeat **Analysis** material from previous experiments.

Conclusions (5 points)

Write your Conclusions in the space provided. Did the experiment tend to confirm the theoretical physical relationships examined? Choose a significant mathematical relationship discussed in this experiment, and use it to numerically explain an activity or phenomenon from outside of the classroom. Illustrate your example by using genuine, approximate or estimated numbers.

Experiment E6: Simple Harmonic Motion and the Torsion Pendulum

Prelaboratory Questions are due at the start of this activity.

Goals of this activity

At the end of this activity you should be able to describe and identify features of simple harmonic oscillation. You will also be able to (a) mathematically and physically describe the motion of a mechanical oscillator (a torsion pendulum) and (b) calculate the moment of inertia of an irregular disk by combining it with a ring of known moment of inertia and observing the change in the period of oscillation.

1 Theory

1.1 The torsion pendulum

A disk of moment of inertia I is fastened near the center of a long straight wire stretched between two fixed mounts. If the disk is rotated through an angular displacement θ and released, the twist in the wire rotates the disk back toward equilibrium. It overshoots and oscillates back and forth like a pendulum, and hence it is called a torsion pendulum. An illustration of a torsion pendulum is given in Figure 1.

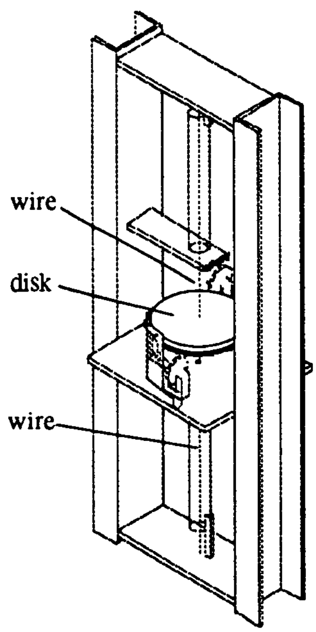


Figure 1: Schematic picture of a torsion pendulum

Torsion pendula are used as the timing element in some clocks. The most common variety is a decorative polished brass mechanism under a glass dome. Likewise, the balance wheel in an old fashioned mechanical watch is a kind of torsion pendulum, although the restoring force is provided by a flat coiled spring rather than a long twisted wire as in this laboratory. Torsion pendula can be made to be very accurate and have been used in numerous precision experiments in physics. In this experiment they are used to study simple harmonic motion because the restoring torque is very nearly proportional to the angular displacement over quite large angles of twist, whereas the restoring force in an ordinary hanging pendulum varies as $\sin\theta$ (Serway section 13.1) rather than θ.

1.2 Simple harmonic motion in the torsion pendulum

When a torsion pendulum disk is twisted away from equilibrium by an angular displacement θ, the twisted wire exerts a restoring torque τ proportional to θ :

$$\tau_{restoring} = -\kappa\theta \tag{1}$$

where κ is called the torsion constant of the support wire. If the wire is thick, short, and/or made of stiff material, κ tends to be large. If the wire is thin, long, or made of soft material, κ tends to be small. This is the rotational analog to Hooke's law for a linear spring. The negative sign indicates that the restoring torque τ is opposite in direction to the angular displacement θ; a clockwise displacement results in a counterclockwise restoring torque and vice versa.

The rotational analog of Newton's Second Law can be written as 'torque τ equals moment of inertia I times angular acceleration α,' or:

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} \tag{2}$$

and in this situation we can set the restoring torque equal to the accelerating torque for the system:

$$\tau = -\kappa\theta = I\frac{d^2\theta}{dt^2} \tag{3}$$

Mathematically solving differential Equation 3 for $\theta(t)$ is beyond the scope of this course, but a solution exists of the form

$$\theta(t) = \theta_0 + A\cos(\omega t + \delta) \tag{4}$$

where θ_o, A, ω, and δ are defined to be constants of the motion determined by the physical situation. Serway's Section 13.1 shows essentially the same form, but it does not include θ_o. θ_o is simply the value of θ about which the simple harmonic motion is centered.

In this experiment, ω and δ in Equation 4 are not easily measurable. Therefore, we make the following substitutions in which ω, the *angular frequency* is given by

$$\omega \equiv \frac{2\pi}{T} \tag{5}$$

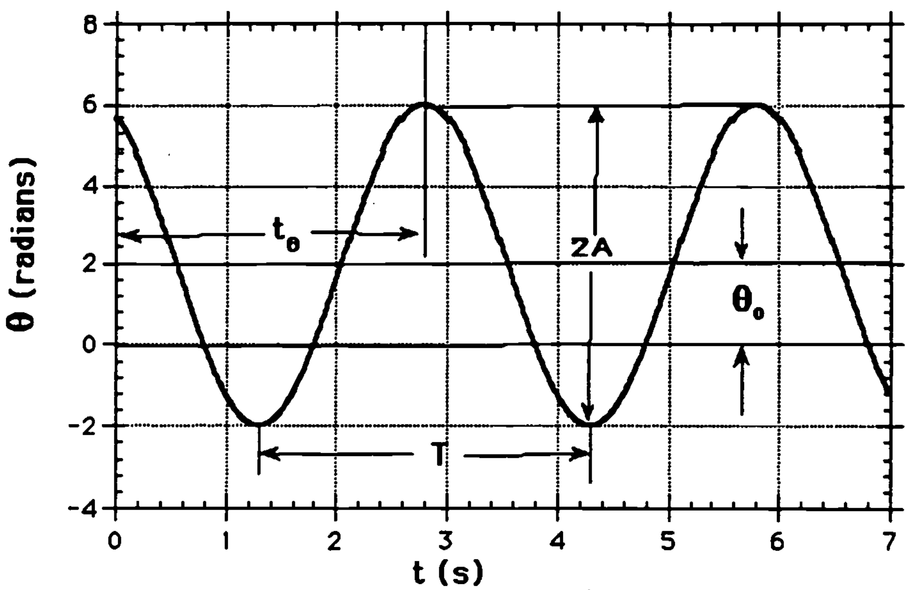


Figure 2: Simple harmonic motion as described by Equation 7, where $\theta_0 = 2.0$ radians, $A = 4.0$ radians, $t_0 = 2.8$ s, and $T = 3.0$ s.

where the period T is easily measurable. Further, we substitute for the phase angle δ by

$$\delta \equiv -\frac{2\pi}{T}\, t_o \tag{6}$$

where the time offset t_o is also easily measurable. Making these substitutions into Equation 4, we obtain

$$\theta(t) = \theta_0 + A\cos\left[2\pi\left(\frac{t - t_o}{T}\right)\right] \tag{7}$$

Here θ_o is the angle about which the simple harmonic motion is centered, and t_o is the time offset from a cosine function that has $\theta = \theta_{max}$ at $t = 0$.

In Equation 7, θ_o, A, t_o, and T are all constants of the motion for a given physical situation. An example is shown in Figure 2. When taking measurements from such a graph, it is easiest to measure $2A$ first and then halve that value to determine A. Finding θ_o involves locating the line of symmetry of the simple harmonic motion, and then measuring the displacement of that line from the zero angular position on the plot. Both the period T and the time offset t_o are measured with respect to the peaks or troughs of the cosine function.

Just as we took time derivatives of linear position with respect to time to find the equations for instantaneous velocity and acceleration in experiments E1–4, we can find instantaneous angular velocity by

$$\omega(t) \equiv \frac{d\theta(t)}{dt} \tag{8}$$

and instantaneous angular acceleration

$$\alpha(t) \equiv \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2} \tag{9}$$

Note that in simple harmonic motion, neither $\theta(t)$, $\omega(t)$, nor $\alpha(t)$ are constant or linear with respect to time. Also, please be aware that the *angular velocity* $\omega(t)$ as defined in Equation 8 is a quantity that describes the rate at which the pendulum disk's position changes with time. Do not confuse it with the *angular frequency* ω defined in Equation 5.

1.3 Predicting T_{pred} from I_{disk} and κ

The period T is the period of time required for the cosine wave to repeat itself (refer to Figure 2). In Part I, you will first experimentally measure the period T_{disk} of the pendulum. Then in Part VI, you will calculate a predicted value T_{pred} of the pendulum based on two measurements—the moment of inertia I_{disk} of the pendulum's disk, and the torsion constant κ of the pendulum's wire. Theory predicts that the period T_{pred} is given by

$$T = 2\pi\sqrt{\frac{I_{disk}}{\kappa}} \tag{10}$$

You will measure both quantities, I_{disk} and κ in the laboratory and will calculate T_{pred} using Equation 10.

Determining I_{disk}

In Part IV of the experiment, you will measure the moment of inertia I_{disk} of the main disk of the pendulum. However, it is rather inconvenient to directly measure I_{disk} for two reasons: first, its irregular geometry poses a problem when calculating a moment of inertia, and second, it would require us to disassemble the pendulum. Therefore, an indirect method is adopted to measure its moment of inertia. First, the period T_{disk} of the torsion pendulum alone is measured. You have done this in Part II already. Next a ring with a calculable moment of inertia is added to the pendulum as shown in Figure 3 According to Serway Table 10.2, the moment of inertia of a ring of mass $M$is given by

$$I_{ring} = \frac{1}{2}M(R_1^2 + R_2^2) \tag{11}$$

where R_1 is the outer radius and R_2 is the inner radius.

The pendulum's new period, T_{new}, is given by

$$T_{new} = 2\pi\sqrt{\frac{I_{disk} + I_{ring}}{k}} \tag{12}$$

By dividing Equation 12 by Equation 10, we obtain

$$\frac{T_{new}}{T_{disk}} = \sqrt{\frac{I_{disk} + I_{ring}}{I_{disk}}}$$

or

$$I_{disk} = I_{ring} \frac{T_{disk}^2}{T_{new}^2 - T_{disk}^2} \tag{13}$$

Since I_{ring} can be calculated and T_{disk} and T_{new} are both measured, I_{disk} can be determined using equation 13.

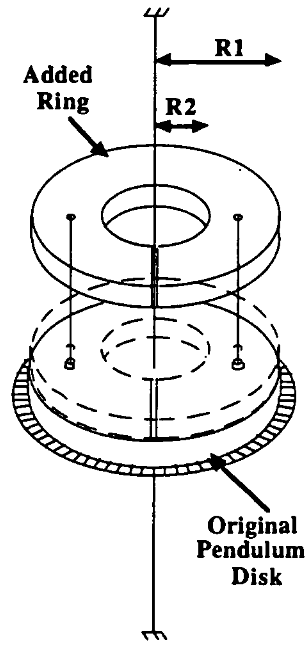


Figure 3: A ring with a known moment of inertia I_{ring} is added to the pendulum to deduce I_{disk}.

Determining κ of the wire

The torsion constant κ of the wire is the second piece of information necessary to use Equation 10 for predicting T_{pred}. The torsion constant κ relates the angular position of the disk to the torque applied, or

$$\tau_{applied} = \kappa\theta \tag{14}$$

You may notice that this equation resembles Hooke's Law $F = kx$ for a spring. Indeed, the relation described in Equation 14 is the rotational analog to Hooke's Law. If you recall from Experiment E3, you determined the spring constant k by first attaching a glider to a spring. You next added specific hanging masses to the glider, thereby creating a force which stretched the spring. You observed the linear displacement **x** of the spring, and then performed a least squares fit on your force F and position x data to find the spring constant k.

You will follow a similar procedure for determining the torsion constant κ of the wire. However, instead of applying forces, you will apply torques $\tau_{applied}$ to the pendulum disk. In addition, you will record *angular* position θ, rather than linear position x. To find the torsion constant κ, you will perform a least squares fit on your applied torque $\tau_{applied}$ and angular position θ data.

The torque that you apply to the pendulum's disk is given by

$$\tau_{applied} = m_{tot} g R_{yoke} \tag{15}$$

where m_{tot} is the total hanging mass applied to the pendulum's disk, $g = (9.80146 \pm 0.00002)$ m/s^2 is the acceleration of gravity, and $R_{yoke} = (0.0571 \pm 0.0001)$ m is the radius of the yoke used to apply torques to the pendulum's disk.

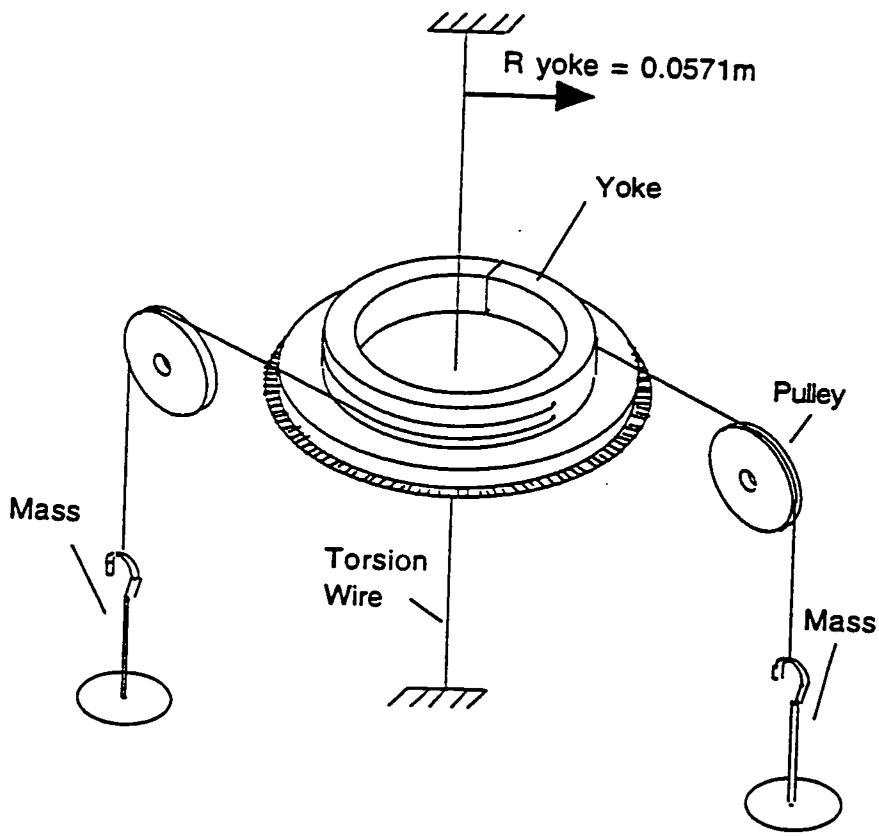


Figure 4: Part V – measuring κ with the yoke

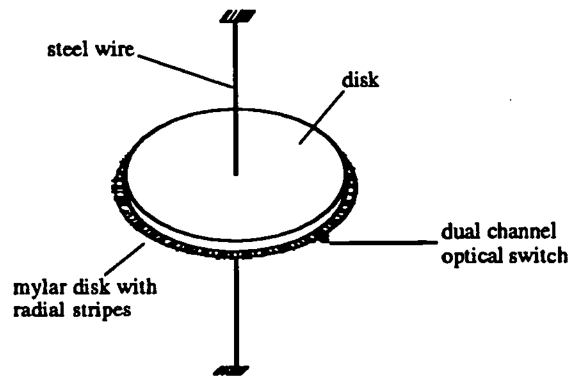


Figure 5: Angular position is measured by counting stripes that pass through an optoelectronic switch

2 Experimental method

The software for this experiment gives the angular position θ of the pendulum as a function of time t. The pendulum has a striped disk attached to its underside as shown in Figure 5. A dual-channel optoelectronic switch senses the stripes and feeds this information to the computer, which converts the information into the angular position of the rotating disk. The software also calculates and displays average angular velocity and average angular acceleration using the same kind of algorithm you have used for linear average velocity and acceleration in Experiments 1 through 3.

When you twist the disk of the pendulum at the beginning of a data run, touch only the top surface of the aluminum disk in order not to put smudges on the striped mylar disk. Such smudges can result in erroneous counting of stripes by the optoelectronic sensors.

Part I. The form and parameters of simple harmonic motion

The purpose of the first part of this experiment is to verify that the torsion pendulum executes simple harmonic motion as described by Equation 7.

1. Give your torsion pendulum disk a twist—about 30°—and check that it oscillates smoothly without rubbing on the detector. If it is rubbing, call your TA.

2. Make a rough measurement of the period of your pendulum. Give it a twist and count 10 complete oscillations while your partner looks at the second hand on one of the wall clocks or a wristwatch. Measure periods by noting when the pendulum reaches a maximum in amplitude on one side. One period has passed when it again reaches a maximum in amplitude on the same side. Also determine an uncertainty in time from your watch. Please note that this is the only part of the experiment where you will *not* use the computer to take measurements.

   CAUTION: Don't count "one" the first time a maximum is reached. This common error will cause you to omit the first swing and get a period which is too small. It may help you to mentally count "zero" the first time the pendulum reaches a maximum and you start timing.

3. Before you start taking data, you have to establish a reference point with respect to which the orientation of the pendulum will be measured. Pressing the button labeled *DEFINE ZERO PT.* when the pendulum is at rest will do the job.

4. Twist the disk approximately 90° (one quarter-turn of the disk.) Release the disk. It will twist back and forth but it may also vibrate from side to side with the wire hitting the sides of the small holes that guide it. To get rid of these vibrations, very gently touch the wire with your finger just above the guide hole platform. When the disk is twisting with little vibration, press the button labeled *ACQUIRE DATA* to start data taking.

5. The angular position θ will be displayed on the screen as a solid black line as a function of time t. The curve should look like a smooth sinusoidal wave described by Equation 7 and illustrated in Figure 2. If your data points don't follow a smooth sinusoidal wave, find out what went wrong, and try again.

6. A blue-crossed sinusoidal wave drawn according to Equation 7 is also displayed. Make this wave match the actual data (black solid line) by changing the values of A, T_0, t_0, and θ_0, that are shown in the lower right of the computer screen. Click on the replot button to replot the blue-crossed sinusoidal wave according to your chosen values of A, T_0, t_0, and θ_0. (Refer to Figure 2 to approximate the values of A, T_0, t_0, and θ_0, and then do some fine-tuning to make the blue-crossed sinusoidal wave match the data as closely as possible.)

7. Does this accurately measured value of $T_0 = T_{disk}$ agree well with the rough value you measured with your wrist watch or wall clock?

8. Make a printout of the graph of θ versus t. Mark this printout by hand to indicate where you extracted your values of A, T_{disk}, t_0, and θ_0 (See Figure 2.)

Part II. Is the period independent of the amplitude?

Whenever the torque that restores a system to equilibrium is exactly proportional to angular displacement of the system from equilibrium, then that system moves with simple harmonic motion, and the frequency or period of that motion is *independent of the amplitude*. At first this may seem paradoxical. It might seem that a pendulum should take longer to swing through a larger arc. But at the larger displacements the torque exerted by the wire is also larger, and the effects cancel. In fact, Equation 10 for the period of oscillation of a torsion pendulum shows that the period depends only on the stiffness of the wire, κ, and the moment of inertia of the disk I_{disk}. It does not depend on how large an oscillation the disk makes. In this part of the experiment you will verify this property of simple harmonic motion.

In Part I you took data for an initial angle close to 90°. Repeat for an amplitude of about 60° and for an amplitude of about 30°.

Part III. Characterizing $\theta(t)$, $\omega(t)$, and $\alpha(t)$ for simple harmonic motion

In this part you will closely examine the plots of $\theta(t)$, $\omega(t)$, and $\alpha(t)$, and use them to draw some general conclusions regarding simple harmonic oscillation and energy. Then you will answer some questions about the energy of the torsion pendulum, and finally you will examine how the maximum pendulum displacement varies with time.

1. Click the "θ, ω, α" button. Scale these plots so that all the maximum and minimum values (peaks and troughs) are clearly shown for at least two periods.
2. Make a printout of these plots, and label them according to the Table in Part III on page 197.
3. Click the "θ" button to return to the $\theta(t)$ screen.

Part IV. The moment of inertia of the torsion pendulum

1. Measure the mass of the ring.
2. As shown in Figure 3, place the ring on top of the torsion pendulum disk. Measure T_{new}.
3. Calculate the moment of inertia of the ring, using Equation 11, $R_1 = 0.0762$ m, and $R_2 = 0.0317$ m.
4. Using Equation 13, calculate the moment of inertia I_{disk} of the original torsion pendulum disk.

Part V. Determining the torsion constant κ of the wire

In this part of the experiment, you will determine the torsion constant κ of the wire.

1. Mount the yoke with the two strings on it over the disk of your torsion pendulum as shown in Figure 4. Wrap the strings around the yoke and then tangentially over the two pulleys. This *must* be set up correctly, else the resulting torques will not match those you calculate in Table 2.
2. First, re-zero the pendulum without any hanging masses. Acquire data with the pendulum at rest to be sure the computer and pendulum are zeroed correctly.
3. Measure the angular twist produced by known applied torques. Use masses of 0.0 grams (i.e., no weight hanger at all), 10.0 grams (i.e., the weight hanger alone on each side), 20.0 grams (i.e., the weight hanger plus 5.0 grams on each side) and 30.0 grams. Note

that all θ values measured in this part of the experiment should be recorded as positive numbers.

(Putting two *equal* weights on the two hangers ($m_i + m_i$) produces a balanced torque on the disk without placing a net translational force on the disk that would pull it to one side or another.)

4. Use Equation 15 to calculate the applied torque $\tau_{applied}$.
5. Perform a least squares fit upon the θ and $\tau_{applied}$ data you collected in Table 2. The slope you calculate from this fit will be equal to κ. You will find that the applied torque $\tau_{applied}$ is quite accurately proportional to the angular displacement or "twist" θ, thereby verifying Equation 14.
6. Make a graph of applied torque $\tau_{applied}$ versus θ.

Part VI. Predicting the period of the pendulum using I_{disk} and κ

In Part I of this experiment you measured the period T_{disk} of the torsion pendulum. In Part III you measured the moment of inertia of the pendulum I_{disk} indirectly. Finally, the torsion constant of the wire κ was measured by applying known torques to it in Part V.

Calculate the period of the pendulum T_{pred} by means of Equation 10 using your previous measurements of I_{disk} and κ. No additional measurements are required in this part. Compare this value to the period T_{disk} measured in Part I.

Final checks before leaving the lab

Be sure that you have completed all items marked by the ($\surd$) symbol. Check also that you have at least two printouts—one of $\theta(t)$ and the other of $\theta, \omega,$ and α vs. t.

Prelaboratory Questions for E6

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

Show your work in the spaces provided.

1. The angular position (θ) vs. time (t) plot shown below has the form:

$$\theta(t) = \theta_0 + A\cos\left[2\pi\left(\frac{t - t_o}{T}\right)\right]$$

By making measurements on the graph, determine the values of the constants θ_0, A, T, and t_0. Indicate where you extracted your values of A, T_{disk}, t_0, and θ_0. Use Figure 2 as a guide.

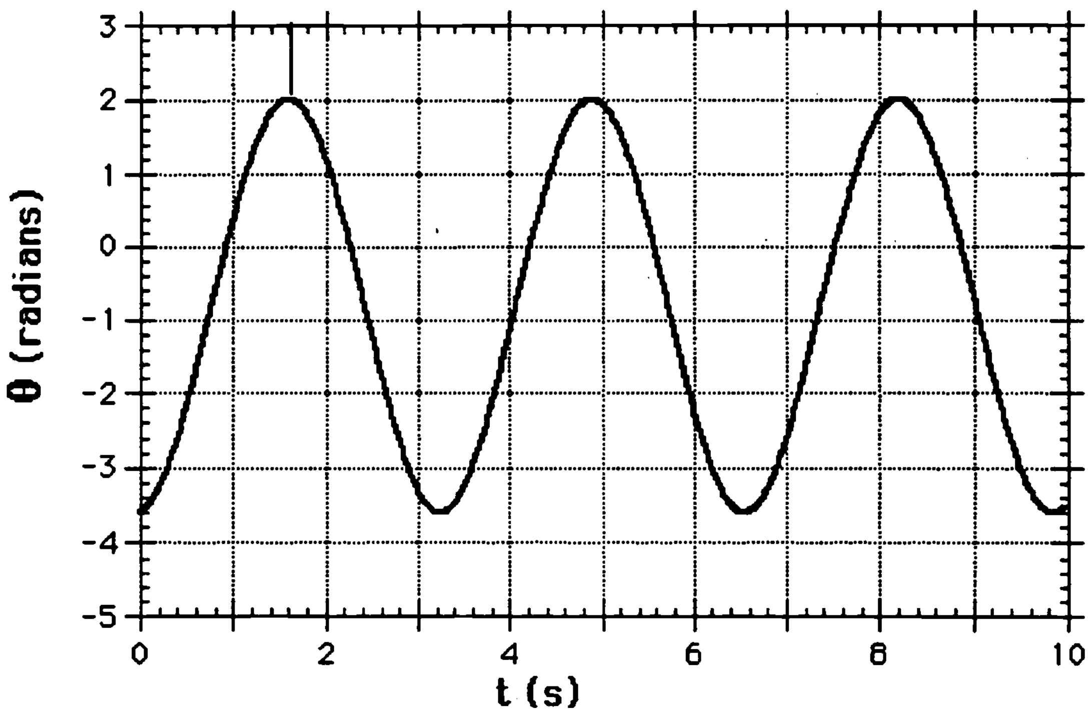


Figure 6: Angular position vs. time (θ vs. t) data

(a) $A = ($ $\pm$ $)$ rad.

(b) $T = ($ $\pm$ $)$ s.

(c) $\theta_0 = ($ $\pm$ $)$ rad.

(d) $t_0 = ($ $\pm$ $)$ s.

2. In Part IV of the experiment you will add a large aluminum ring to the original aluminum disk of the torsion pendulum as shown in Figure 3. By measuring the new period of oscillation T_{new}, it is possible to deduce the moment of inertia of the original disk.

   (a) Assume that the added ring has a mass $M = 0.750$ kg and radii $R_1 = 0.0681$ m, and $R_2 = 0.0435$ m. What is the moment of inertia I_{ring}?

   $I_{ring} = ($ $\pm$ $)$ kg·m^2.

   (b) If the period of oscillation with the original disk is $T_{disk} = 1.50$ s and the new period of oscillation with the added ring is $T_{new} = 2.80$ s, what is the moment of inertia of the original disk I_{disk}?

   $I_{disk} = ($ $\pm$ $)$ kg·m^2.

3. A platform is attached to a vertically mounted stiff wire. Two strings are wrapped around a yoke of radius $R_{yoke} = 0.0571$ m and are hung over two pulleys as shown in Figure 4. Various masses are hung on the ends of the strings and the stationary angular position of the platform is recorded for each mass as given in Table 1. You will have to carry extra digits and use scientific notation to calculate the final column.

i	M_{total} (kg)	θ (radians)	$\tau_{applied}$ (N · m)
1	0.0050	0.173	0.000
2	0.0100	0.329	
3	0.0150	0.485	
4	0.0200	0.650	
N	—	$\Sigma\theta_i$	$\Sigma\tau_{i\ applied}$

Table 1: Table for determining the torsion constant κ

The numbers shown in Table 1 represent the total hanging mass, $(m_i + m_i)$;*half* of this total mass is hung on each string. The applied torque $\tau_{applied}$ is

$$\tau_{applied} = (m_i + m_i)gR_{yoke} = m_{tot}gR_{yoke}$$

where the gravitational acceleration g = 9.80146 m/s^2. Because the platform is at rest, the applied torque and the torque exerted by the twisted wire are equal in magnitude but opposite in sign.

(a) Using a spreadsheet, perform a least squares fit $\tau_{applied}$ vs. θ, using data in Table 1 (watch the order). Attach a copy to this page. The slope you calculate is the torsion constant κ.

(b) Record the torsion constant κ of the wire. A typical value is of the order of about $\frac{1}{100}$ N-m/radian.

κ = (±) N·m/radian

(c) Using scientific plotting software, plot the applied torque $\tau_{applied}$ versus angular position θ of the platform.

4. We have described a solution for simple harmonic motion — the equation $\theta(t)$ describing how the angular position instantaneously varies with time:

$$\theta(t) = \theta_o + A\cos[\omega t + \delta].$$

Use this as your starting equation and take the derivative with respect to time to determine instantaneous angular velocity $\omega(t)$ and instantaneous angular acceleration $\alpha(t)$. After you get these expressions in the form shown in Equation 4, change your solutions (by substitutions) to the notation used in Equation 7 and then rewrite your solutions. E.g., your answers should be written first in terms of ω and δ, then in terms of t_o and T.

   (a) Take the derivative of angular position with respect to time to determine an expression for instantaneous angular velocity. Find $\omega(t) = \dfrac{d\theta(t)}{dt}$. Rewrite this in terms of t_o and T.

   (b) Take the derivative of instantaneous angular velocity with respect to time to determine an expression for instantaneous angular acceleration. Find $\alpha(t) = \dfrac{d^2\theta(t)}{dt^2} = \dfrac{d\omega(t)}{dt}$. Rewrite this in terms of t_o and T.

E6 Laboratory Report

Name ______________________ Lab day/time ______________________

Division ______________________ GTA ______________________

Laboratory reports are due one week [SEVEN CALENDAR DAYS] from the day you perform the experiment. Attach a green Physics 152L cover sheet (available in the lab or window of PHYS Room 144) to the front of your exercise or report and put it in the "Room 144 Drop Slot for Physics Lab Reports" located under the mailboxes across from Physics Room 149 before 10:00 p.m. of the day it is due. Include all of your data in your laboratory report.

Abstract (10 points)

Write your Abstract in the space provided below. Devote 1–2 paragraphs to briefly summarize and describe the experiments in terms of theory, activity, key findings and agreements. Include actual numerical values, agreements and discrepancies with theory. Especially mention if T_{pred} agreed with T_{disk}. Write the abstract AFTER you have completed the entire report, not before.

Data and Calculations (45 points)

Part I. The form and parameters of simple harmonic motion

1. (√) Roughly measure the period of the pendulum using your watch.

   (a) $10\ T_{watch} = (\qquad \pm \qquad)$ s.

   (b) $T_{watch} = (\qquad \pm \qquad)$ s.

2. (√) Measure the following quantities using the curve-fitting software. Estimate your uncertainties in these measurements by moving your measurement cursors around.

   (a) $A = (\qquad \pm \qquad)$ rad.

   (b) $T_{disk} = (\qquad \pm \qquad)$ s.

   (c) $\theta_0 = (\qquad \pm \qquad)$ rad.

   (d) $t_0 = (\qquad \pm \qquad)$ s.

3. (√) Print out the θ versus t graph and title it "Part I." On the printout indicate where you extracted θ_0, A, T_{disk}, and t_0.

4. How does your rough estimate of the period compare to the value you determined from the graph?

   $\left(\frac{T_{disk}-T_{watch}}{T_{disk}}\right) \cdot 100 = \qquad \%$.

Part II. Is the period independent of the amplitude?

1. (√) Measure the period for an amplitude of $\sim 90°$. Remember to use the computer for this part.

   $T_{disk}(\sim 90°) = (\qquad \pm \qquad)$ s.

2. (√) Repeat for an amplitude of $\sim 60°$.

   $T_{disk}(\sim 60°) = (\qquad \pm \qquad)$ s.

3. (√) Repeat for an amplitude of $\sim 30°$.

   $T_{disk}(\sim 30°) = (\qquad \pm \qquad)$ s.

4. Percentage difference between the periods of your largest and smallest amplitude runs:

   $\left[\frac{T_{disk}(\sim 90°)-T_{disk}(\sim 30°)}{T_{disk}(\sim 90°)}\right] \times 100\% = \qquad \%$.

5. Are $T_{disk}(\sim 90°)$, $T_{disk}(\sim 60°)$, and $T_{disk}(\sim 30°)$ the same within your ability to measure them?

Part III. Characterizing $\theta(t)$, $\omega(t)$ and $\alpha(t)$ for simple harmonic motion

1. ($\surd$) What happens to the amplitude of the graph with time? Explain the behavior of the amplitude in terms of both force and energy.

2. ($\surd$) Scale your plot of $\theta(t)$, $\omega(t)$ and $\alpha(t)$ so that you can readily see the entire amplitude of these functions for two complete cycles. Make a printout of these plots and title it "Part III."

3. Indicate and label the following points on all three plots for one cycle and attach the plot to this page.

Symbol	Symbol
A	the beginning of a cycle
B	the end of that same cycle
1	the points where the restoring torque on the pendulum is maximum
2	the points where the restoring torque on the pendulum is minimum
3	the points where the pendulum is at extreme angular displacement
4	the points where the pendulum is at minimal angular displacement
5	the points where the pendulum is instantaneously motionless
6	the points where the pendulum has maximal values of kinetic energy
7	the points where the pendulum has maximal values of potential energy

4. The curves for $\theta(t)$, $\omega(t)$ and $\alpha(t)$ are not *in phase;* that is to say their maxima and minima do not occur simultaneously. Describe the phase relationship between these curves by explaining how $\omega(t)$ and $\alpha(t)$ lead or lag $\theta(t)$ and by how much of the time period T they lead or lag.

5. Where in simple harmonic oscillation is potential energy greatest? Where is it smallest? Explain.

6. Where in simple harmonic oscillation is kinetic energy greatest? Where is it least? Explain.

Part IV. The moment of inertia of the torsion pendulum

1. ($\surd$) Determine the mass of the ring.

   $M = ($ $\pm$ $)$ kg.

2. Calculate the moment of inertia I_{ring} using equation 11. Assume that $R_1 = 0.0762$ m, and $R_2 = 0.0317$ m. See the "Implied Uncertainties" section in Measurement Analysis 1 to determine the uncertainty in these measurements.

   $I_{ring} = ($ $\pm$ $)$ kg·m^2.

3. (√) Measure the period of the pendulum with the added ring.

   $T_{new} = ($ ± $)$ s.

4. Use equation 13 to calculate the moment of inertia of the original torsion pendulum disk.

   $I_{disk} = ($ ± $)$ kg·m^2.

Part V. Measure the torsion constant κ for the wire.

1. (√) Complete the 'θ' and '$\tau_{applied}$' columns of Table 2.

i	M_{total} (kg)	θ (radians)	$\tau_{applied}$ (N · m)
1	0.0000	0.000	0.000
2	0.0100		
3	0.0200		
4	0.0300		
N	—	$\Sigma\theta_i$	$\Sigma\tau_{i\ applied}$

Table 2: Table for determining the torsion constant κ

2. Use a spreadsheet to perform a least squares fit to determine the torsion constant κ. Attach a copy of your spreadsheet here.

   $\kappa = ($ ± $)$ N·m/radian

3. Use scientific plotting software to make a graph of applied torque $\tau_{applied}$ vs. θ and attach it to this page.

Part VI. Predicting the period of the pendulum using I_{disk} and κ

1. Calculate the period of the pendulum using Equation 10 and your previous measurements of I_{disk} (Part IV) and κ (Part V).

   T_{pred} = (±) s.

2. Compare this period T_{pred} to the period T_{disk} you measured in Part I.

   T_{disk} (Part I) = (±) s.

   T_{pred} (Part VI) = (±) s. Does T_{disk} agree with T_{pred}? If so, justify your answer. If not, calculate the percent discrepancy by $\frac{T_{disk}-T_{pred}}{T_{pred}} \times 100\%$.

Analysis (15 points)

Write your Analysis in the space provided. Using numerical examples and analysis, discuss the larger sources of uncertainty in your calculations. Use your numbers to justify possible improvements in apparatus and methods. Determine whether your measurement of the torsion constant κ was better or worse than your measurement of the spring constant k in E3 by comparing the precision of the measurements κ (torsion constant) and k (spring constant). Do not repeat **Analysis** material from previous experiments.

Conclusions (5 points)

Write your Conclusions in the space provided. Did the experiment tend to confirm the theoretical physical relationships examined? Choose a significant mathematical relationship discussed in this experiment, and use it to numerically explain an activity or phenomenon from outside of the classroom. Illustrate your example by using genuine, approximate or estimated numbers.

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