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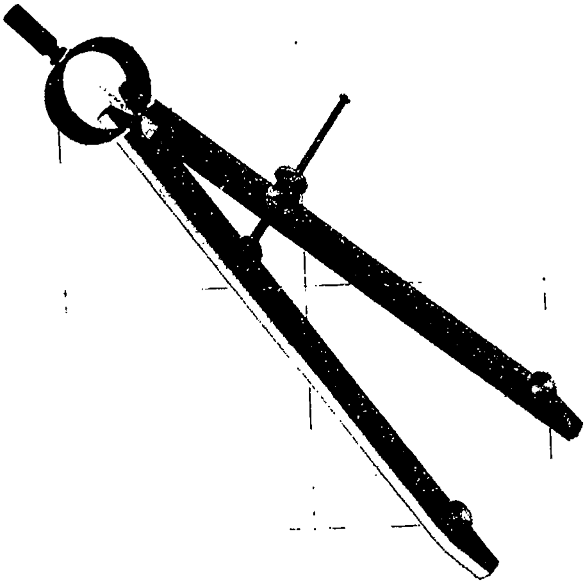
ABSTRACT

This study was conducted to obtain information related to mental computation from students in Years 3, 5, 7, and 9 in Western Australia. The numbers of subjects were 163, 163, 163 and 152 in years 3, 5, 7, and 9, respectively. The research was designed to provide three different perspectives of mental computation: (1) a survey of the kinds of computations which students in Years 3, 5, 7, and 9 prefer to do mentally; (2) a measure of attitude towards mental and written computation of students in Years 5, 7, and 9; and (3) an assessment of mental computation performance of students in Years 3, 5, 7, and 9. Major findings revealed that: (1) the great majority of students in Years 5, 7, and 9 felt it was important to be good at mental computation; (2) students tended to consider that written computation was mostly learned at school but mental computation was mostly learned outside of school; and (3) the more able the students, the better they could predict whether or not they could undertake a computation mentally. This self-confidence varied considerably both within and across year levels. Implications for curriculum development and teaching practice in the mathematics classroom are discussed. They include: (1) The curriculum needs to be much more flexible to cater to the wide range of ability; (2) Emphasizing skills at the expense of understanding is unlikely to prove effective; and (3) Real life computation involves mental computation and/or calculator use, so classroom teaching should emphasize these aspects rather than traditional paper and pencil algorithms. (MKR)



Mental Computation in School Mathematics

Preference, Attitude and Performance of Students in Years 3, 5, 7 and 9

Alistair McIntosh
Jack Bana
Brian Farrell

MASTEC Monograph Series

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4

Table of contents

Preface

Since 1988 teams of researchers in the United States and Japan have been involved in a collaborative research project to assess the mental computation and the number sense of their students. The principal researchers have been Professors Robert and Barbara Reys of the University of Missouri and Professor Nobuhiko Nohda of the University of Tsukuba.

The focus of this project has not been so much the international comparison of achievement as an attempt to look more deeply into the curricular and instructional factors that underlie the results.

Commencing in 1989 a team consisting of Alistair McIntosh, Paul Swan and Ellita de Nardi at Edith Cowan University, Perth, Western Australia, probed the strategies used by children of primary school age when calculating mentally, with a view to developing and promoting more appropriate and effective mental computation.

At an international seminar on "Needed Research in Computation" organised by Queensland University of Technology and held at Gwinganna, Queensland in August 1991, those involved in the two projects exchanged information and decided to work collaboratively on joint research into the mental computation ability and number sense of students in Japan, the United States and Australia.

This present monograph, which looks at the mental computation of Western Australian students in Years 3, 5, 7 and 9, is one of the outcomes of this collaborative research project.

A further publication will examine the number sense of the same cohort of students.

CHAPTER ONE

Background of the study

In primary schools in Australia, as in most schools worldwide, written computation has traditionally held a more important and more central place in the primary (elementary) school curriculum than mental computation.

Until recently the prior claims of written computation have been for the most part taken for granted. This view has not been unopposed. In 1908 Benchara Branford wrote of the "incomparable superiority of mental over written calculations" (Branford, 1908). "Written arithmetic", he said, "is not to be introduced until the pupil's mind sees the necessity for it in the difficulty of registering more than a certain amount in the memory".

In more recent times the prior importance of mental over written computation has been put more frequently and forcefully (see for example Wandt and Brown, 1957; Plunkett, 1979; Maier, 1980; McIntosh, 1980; McIntosh, 1990; Hope, 1986; Reys, 1984; Reys and Reys, 1986). Wandt and Brown (1957) showed that, even before the advent of calculators, adults used mental computation for 75 per cent of their calculations, while they used written calculations no more than 25 per cent of the time. Plunkett (1979) contrasted the passive and automatic nature of written computation with the fluid and creative nature of mental computation. Maier (1980) contrasted the use of mental computation in "folk mathematics" with the concentration on written computation in "school mathematics". In the age of the electronic calculator Girling (1977) and others have stressed the importance of mental computation as a check on results obtained by calculators.

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) in the United States, *Mathematics in the National Curriculum* (DES, 1991) in the United Kingdom, and the *National Statement on Mathematics for Australian*

Schools (Australian Education Council, 1991) have all stressed the equal importance of ability with the calculator and with mental and written computation and emphasised the need to choose a sensible computation method depending on the circumstances. McIntosh, Reys and Reys (1992), Markovitz and Sowder (1994), and others have singled out mental computation as both an important ingredient of number sense and an important indicator of its presence.

However, in spite of this recent increase in attention to mental computation, little research has been done into the mental computation ability of Australian children beyond the basic number facts. Standard testing procedures have not been established which would allow secure comparisons to be made. Many questions lack answers founded on a secure research base, including whether mental computation tests should be presented orally or visually or both, what the amount of time allowed for calculating should be, and what are appropriate expectations at different stages.

Apart from the work of Hunter (1962) who studied the mental computation strategies of adults who were exceptional at mental calculations, little had been done before the 1980s to discover the ways in which people calculate mentally. Recently much more interest has been shown in this area, particularly in the thinking strategies used by young children in the acquisition of basic facts and in whole number mental computations (Rathmell, 1978; Bana & Korbosky, 1995; Swan, 1991; McIntosh, Swan & De Nardi, 1994).

As to instruction in mental computation, apart from the pioneering efforts of Flournoy (1954), little has been done to assess the effectiveness of traditional methods. However Biggs (1967) showed, in a study of some 5000 primary school students in the United Kingdom, that the allocation of time (varying from 0 to over 11 minutes per day) to traditional mental arithmetic in which speed of response is encouraged bore "no relation at all to achievement", but that number anxiety "tends to increase slightly with more time devoted to mental arithmetic". The possibility of more effective and creative approaches to mental computation has been shown by a number of researchers (McIntosh, 1980; McIntosh, Swan & De Nardi, 1994; Hope, Reys & Reys, 1987).

The term "year" in this paper is the Australian equivalent of "grade". However because of the differences in the age of starting school in Western Australia, the United States and Japan, the average age of a "Year 3" Australian student in this study is approximately that of a "Grade 2" student in Japan and the

United States. For this reason the tests given in America and Japan to students in grades 2, 4, 6 and 8 were given in Western Australia to students in Years 3, 5, 7 and 9 respectively. The average age of the youngest cohort of students at the time of testing in Australia, Japan and the United States was 8.2, 8.4 and 8.2 years of age respectively.

The term "primary school" corresponds to the American "elementary school", except that in Western Australia Year 7 students are still in primary school.

Purpose of the study

This study was conducted to obtain information from students in Years 3, 5, 7 and 9 in Western Australia related to mental computation. The research was designed to provide three different perspectives of mental computation as follows:

1. A survey of the kinds of computations which students in Years 3, 5, 7 and 9 prefer to do mentally.
2. A measure of attitude towards mental and written computation of students in Years 5, 7 and 9.
3. An assessment of mental computation performance of students in Years 3, 5, 7 and 9.

These perspectives taken collectively should provide a useful data set for a better understanding of mental computation in mathematics classrooms. In addition, it was anticipated that this study would provide some valuable benchmarks for future research in the areas of mental computation and number sense, including comparisons of the Australian data with that obtained from students in Japan and the USA.

CHAPTER TWO

Design of the study

Three different survey instruments were developed for the study. Construct validity was established through a series of reviews and trials. Drafts of the instruments were developed jointly with American and Japanese mathematics educators for use in all three countries and were reviewed by prospective researchers in those countries. The resulting instruments were then field tested with students and further revised as a result of the pilot studies.

The three survey instruments developed for and used in this study were as follows:

Preference Survey	(PS)
Attitude Survey	(AS)
Mental Computation Test	(MCT)

All the instruments were administered in the order listed above during one 50-minute period for all classes in Years 3, 5, 7 and 9, except that the Attitude Survey (AS) was not used in Year 3.

The Mental Computation Test (MCT) consisted of two parts—a set of items presented orally (items read individually by the administrator) and a set of items presented visually (items presented individually using an overhead projector). Half of the classes in the sample took the first half of the test through an oral administration format followed by the second half of the test by way of a visual administration format. The administration format was reversed with the other half of the sample (visual administration for the first half; oral administration for the second half). Table 1 describes the administration format for the Year 3 MCT. This plan provided an opportunity to examine any mode-of-presentation effect, as well as any learning effect of the test form. Each of the other year-level tests followed the same pattern.

Table 1: Administration Pattern of MCT Forms A and B for Year 3 Students

Items	Administration Format Form A	Form B
1-15	Oral	Visual
16-30	Visual	Oral

Sample

Four schools (three primary and one secondary) in Western Australia participated in the study. The set of schools was chosen from a "typical" metropolitan region. Such a region was "typical" in that it reflected the setting of many Australian schools. The secondary school was selected together with three of its major "feeder" primary schools (Years K-7) to enable more meaningful between-year comparisons to be made.

Within each primary school, two classes were randomly selected at each of the year levels 3, 5 and 7. Students in all classes were heterogeneously grouped as is the custom in most Australian primary schools. One class in each pair was randomly assigned Form A of the MCT while the other was assigned Form B of the test. In the secondary school where students were streamed on ability, as is the case in many Australian secondary schools, stratified random sampling was used to select three pairs of classes, with each pair at a different level of ability as previously determined by the school. One class from each pair was assigned Form A of the MCT, while Form B was assigned to the other three classes in the pairs. The total numbers of subjects were 163, 163, 163, and 152 in Years 3, 5, 7, and 9 respectively.

Instruments

Preference Survey (PS)

Numerical calculations can be carried out by three main methods: mental computation, written computation, or with a calculator. Among these alternatives, the curriculum for Western Australian schools places most emphasis on written computation (Curriculum Branch, Ministry of Education, 1989). This is also true of other Australian states.

The Preference Survey (PS) began with a reminder that different types of computational methods existed, and that each person needed to choose which method to use for a particular computation. The Preference Survey provided a series of numerical computations and asked participants if they would choose to do these computations mentally. For example, Year 7 students were asked to respond "yes" or "no" to whether they would mentally compute items such as 58 + 34, 60 x 70, 264 - 99, and 6 - 4.5. Table 2 shows an excerpt from the beginning of the PS for Year 9 students.

Table 2: Excerpt from Year 9 Preference Survey

Computation is often involved in solving real-world problems. When solving problems, several computational methods exist:

- Sometimes people use a calculator.
- Sometimes people use paper and pencil.
- Sometimes people compute mentally without writing anything down.

We want to learn which problems you prefer to do mentally.
Please look at each problem below and decide if you prefer to do it mentally.

Circle YES or NO to indicate your response. It is not necessary for you to work the problems.

	Problem	I would do this problem mentally	
1.	165 + 99	Yes	No
2.	7 x 25	Yes	No
3.	14 x 83	Yes	No
4.	945 x 1000	Yes	No

The participants were not asked to carry out the particular computation but only to decide if they would do the computation mentally if allowed a choice. Students indicated their answer by marking "yes" or "no". The same instructions were used for each year level. The survey items at each year level were selected to coincide with items commonly found in the mathematics curriculum at that particular year level. Four "checker" items (14 x 83, 35 x 55, $^{4}/_{7}$ + $^{2}/_{5}$ and 0.35 x 567) that would be tedious to compute mentally were included to determine how discriminating the students were in their responses to the preferences. A few items, such as 165 + 99, were used in more than one year level to provide a profile of preferences across years. Some items in the PS were also included in the MCT so that data reporting preferences could be compared with actual performances on the same item. Copies of the PS for each year level are included in Appendix A.

Attitude Survey (AS)

A series of statements designed to document students' attitudes toward mental computation were developed, field tested, refined and utilised in the Attitude Survey (AS) for Years 5, 7 and 9. The Attitude Survey was not considered to be suitable for Year 3 students. Students at this level were unlikely to have the required comprehension. The final statements resulted from reviews of earlier versions of the attitude instruments by a number of mathematics educators in Australia, Japan and the USA, as well as pilot information from Japanese teachers and students. The Attitude Survey included 28 statements clustered by five dimensions. The dimensions and a sample of the statements are shown in Table 3.

Table 3: Framework and Sample Items from Attitude Survey Instrument

Interest and Enjoyment
Written computation is more interesting than mental computation.
Mental computation is more interesting than written computation.

Perception of Competence
I am good at written computation.
I am good at mental computation.

Perception of Value
It is more important to be good at written than mental computation.
It is more important to be good at mental than written computation.

Perception of Use
I think I will do written computation more than mental computation as an adult.
I think I will do mental computation more than written computation as an adult.

Perception of Source of Instruction
I learned to do written computation at school.
I learned to do mental computation by myself.

Two types of statement were included within each dimension of the framework. One type provided a parallel mental computation statement to accompany each statement related to written computation. For example, the parallel statements, "It is important to be good at mental computation" and "It is important to be good at written computation" were both included in the survey. Another type of statement required a response to a judgemental statement such as, "I am better at written than mental computation". Each judgemental statement was accompanied by a parallel statement, in this case, "I am better at mental than written computation". These pairings provided a further means of checking on the consistency of student responses. A copy of the complete AS instrument is included in Appendix B.

Mental Computation Test (MCT)

The MCT was designed by the researchers for group administration. The Year 3 and Year 5 MCT versions contained 30 items, 15 administered orally and 15 administered visually. The Year 7 and Year 9 versions contained 40 items — 20 administered orally, and 20 administered visually. Two different forms (A and B) were developed for each year level. Each form contained the same set of items but differed in the presentation format as was illustrated in Table 1. Prior research has documented the difficulty in obtaining valid and reliable measures of mental computation (Reys, B., 1985; Reys, R., 1985; Reys, Reys & Hope, 1993; Sachar, 1978; Shigematsu, Iwasaki & Koyama, 1994).

In order to provide an accurate assessment of mental computation, several steps were taken. First, the Mental Computation Test (MCT) included only non-contextual computational items. This allowed students to focus exclusively on the required computation, thereby eliminating the need for students to decide from the context of the question which operation was appropriate.

Second, the MCT was composed of oral and visual items, with half of the items presented orally (read aloud by the administrator) and half presented visually (via an overhead projector). To investigate the order effect of the administration, half of the classes were given the oral section of the test first followed by the visual section, while the other half experienced the visual section of the test followed by the oral section.

Third, all items on the mental computation test were given one at a time and the time allotted for each item was carefully controlled. This was done to guard against the possibility of students writing down items and using written rather than mental computation techniques. Items were individually paced by the examiner with 20-second intervals between item presentations. Pilot testing confirmed that 20 seconds was very generous for some students and yet adequate for nearly everyone to attempt the computation mentally. The visually-presented items were individually displayed on an overhead screen for a period of 20 seconds. The orally administered items were read twice with a brief pause (2-3 seconds) between readings followed by a 20-second wait period between items. The test items were selected to best match the mathematics curricula of the three countries for each year level. Several items used in earlier research studies (Reys, Reys and Hope, 1993; Shigematsu, Iwasaki & Koyama, 1994) were also included to provide some comparative benchmarks.

Fourth, a specially constructed answer sheet provided room only for a written answer, thereby discouraging copying of the problem onto the paper. In addition, students were specifically instructed not to write anything down but the answer. Every response to the MCT was evaluated and coded as either correct or incorrect.

Finally, in addition to providing a profile of student mental computation performance at each year level, the tests were designed to monitor the development of mental computation skills over the year levels. A set of common items across year levels was embedded within the tests. Several sets of "nested" items (items related in mathematical structure) were also included (see Appendix C for a complete listing of items in each year level of the MCT). Table 4 shows the distribution of items by operation and domain of numbers for each year.

Table 4: Mental Computation Test (MCT) Item Distribution

Number Type	Operation	Year 3	Year 5	Year 7	Year 9
Whole	Addition	12	6	4	2
Numbers	Subtraction	12	6	4	2
	Multiplication	4	6	6	4
	Division	2	6	6	6
Fractions	Addition	-	2	4	4
	Subtraction	-	2	4	4
	Multiplication	-	-	2	4
	Division	-	-	-	2
Decimals	Addition	-	2	2	1
	Subtraction	-	-	2	2
	Multiplication	-	-	2	2
	Division	-	-	-	2
Percentages	Multiplication	-	-	4	5
Total		**30**	**30**	**40**	**40**

CHAPTER THREE

Analysis of results

Preference Survey results

The Preference Survey (PS) focused on computations which students preferred to do mentally and provided one perspective of mental computation. Most items in the PS were also included in the Mental Computation Test (MCT), but four very difficult "checker" items ($^4/_7 + ^2/_5$, 14 x 83, 35 x 55 and 0.35 x 567) were included to provide a check on the validity of the PS data.

The results from the PS for each year level are shown in Table 5. For the 17 items that were used across year levels, all but two (0.1 x 45 and 7 x 25) show an increasing preference for mental computation in the higher years. For example, the item 165 + 99 shows preferences of 39, 60, 70 and 89 per cent in the four Years 3, 5, 7 and 9 respectively. Some of the increases were considerable; for example, the item 264 - 99 has percentages of 43 and 76 in Year 7 and Year 9 respectively. However this trend is not true of the one "checker" item included in more than one grade. The percentage of students opting to do 14 x 83 mentally fell over the grades from 18 in Year 5 to seven in Year 9.

It is reasonable to hypothesise that those younger students who expressed a preference to perform this calculation mentally misunderstood the level of difficulty. This is supported by Table 6 which shows that the lower ability students were more likely to choose to do 14 x 83 mentally. Overall, 40 per cent of Year 9 students indicated a preference for computing $^4/_7 + ^2/_5$ mentally but 63 per cent of the fifth quintile (lowest 20 per cent of students) did so. It may well be that some of these students were misinterpreting the computation required and would perform the calculation by separately adding the numerators and the denominators.

More than 40 per cent of Year 5 students would not do the item 100 x 35 mentally and between one third and one quarter of Year 7 and Year 9 students would not calculate 945 x 1000 mentally. This suggests that many students lack conceptual understanding rather than computational skill. For example, a student who has conceptual understanding of the decimal system of numeration would see 945 x 1000 as simply 945 thousand.

Table 5: Computational Preferences Reported by Students in Years 3, 5, 7 and 9 as Percentages

Item	Year 3 (n = 163)	Year 5 (n = 163)	Year 7 (n = 163)	Year 9 (n = 152)
6 + 8	88			
60 + 80	69	94		
36 + 9	63			
58 + 34	42	78	96	
47 + 54 + 23			72	78
265 + 100	57			
500 + 300	80	99		
165 + 99	39	60	70	89
74 - 30	48	74		
100 - 68	45			
73 - 23	58			
80 - 24	55	77		
264 - 99			43	76
6 - 4.5			72	89
$^1/_2 + ^3/_4$			75	78
$1 - ^1/_3$		61	69	
Double 26	61	88		
60 x 70		60	86	
100 x 35		57		
945 x 1000			68	74
7 x 25		57	72	63
$^1/_{10}$ of 45			47	60
0.1 x 45			48	45
$90 \div ^1/_2$				88
25% of 48				55
'Checker' Items				
$^4/_7 + ^2/_5$				40
14 x 83		18	10	7
35 x 55			18	
0.35 x 567				1

This lack of conceptual understanding was also apparent in the results for the item 0.1 x 45, for which fewer than half of Year 7 or Year 9 students would use mental computation in spite of the easy computation involved. One item in the PS (945 x 1000) was also included as part of one of the earlier National Assessment of Educational Progress (NAEP) (1983) mathematics assessments. NAEP reported that about 35 per cent of the American 13-year-olds would do the computation mentally, with the remainder opting to use either paper and pencil or a calculator. However in our research over two thirds of the Year 7 and Year 9 students indicated they would do this computation mentally.

Preference versus performance

In order to determine whether students scoring high and low on the MCT differ in their selection of items to compute mentally, student responses were sorted by first (high), third (middle) and fifth (low) quintiles according to their total score on the MCT. The results are reported in Table 6. An examination of Table 6 indicates that, with the exception of the "checker" items, the higher the MCT score the greater is the preference for mental computation, and there is a marked difference in the preferences of high and low performers on the MCT. Thus, students who are more skilled at mental computation tend to prefer this method over others, while less skilled students tend not to opt for a mental computation approach.

Table 6 shows that for almost all items that are common across years, markedly more students in the high performance quintile for any year level opt for mental computation than do those in the low performance quintile in the higher year level. For example, for the item 165 + 99, 61 per cent of the high performance Year 3 group would use mental computation compared to 30 per cent of the low performance Year 5 group; 82 per cent of the high Year 5 group compared to 55 per cent of the low Year 7 group; and 88 per cent of the high Year 7 compared to 77 per cent of the low Year 9 group.

Gender differences

Table 7 shows computational preferences according to gender. In Year 3 the percentages of boys preferring a mental computation approach were higher than for girls for all items, with the differences being very marked for most items. In Year 9 the boys' preferences for mental computation were greater than for the girls in all but one of the items, and the differences were generally of the order of 10-15 points. One possible explanation is that mental computation is seen as an approach involving risk-taking, and that girls are less inclined to take risks in mathematics than boys. However, while this may be true at both Year 3 and Year 9 levels it does not explain the lack of gender differences at Year 5 and Year 7 levels, where the results show little overall differences between boys and girls.

Table 6: Percentages of Year 3, 5, 7 and 9 Students in First, Third and Fifth Quintiles Preferring Mental Computation for the Given Calculations *

Item	Year 3			Year 5			Year 7			Year 9		
	L	M	H*	L	M	H	L	M	H	L	M	H
6 + 8	76	88	94									
60 + 80	39	73	91	82	97	100						
36 + 9	36	64	82									
58 + 34	21	42	76	67	79	87	88	100	100			
47+ 54 + 23							58	82	88	63	87	87
265 + 100	30	52	91									
500 + 300	55	88	94	94	100	100						
165 + 99	24	42	61	30	67	82	55	70	88	77	93	100
74 - 30	42	52	70	52	79	94						
100 - 68	27	42	73									
73 - 23	36	64	76									
80 - 24	42	61	73	61	82	91						
264 - 99							27	39	64	63	73	93
6 - 4.5							36	76	97	73	90	100
$^1/_2 + ^3/_4$							67	73	85	60	80	90
$1 - ^1/_3$				48	58	88	61	58	91			
Double 26	36	67	97	64	88	100						
60 x 70				45	55	91	70	85	100			
100 x 35				42	58	85						
945 x 1000							30	79	97	43	87	97
7 x 25				42	52	76	58	79	82	47	53	87
$^1/_{10}$ of 45							30	36	76	37	63	97
0.1 x 45							33	52	67	17	50	77
$90 \div ^1/_2$										73	87	97
25% of 48										40	53	83
'Checker' items												
$^4/_7 + ^2/_5$										60	30	40
14 x 83				24	30	21	18	6	9	7	7	10
35 x 55							24	21	15			
0.35 x 567										0	0	0

* L, M, H designate low, middle and high quintile MCT groups.

Table 7: Percentages of Year 3, 5, 7 and 9 Students by Gender Preferring Mental Computation for the Given Calculations

Item	Year 3		Year 5		Year 7		Year 9	
	F	M	F	M	F	M	F	M
6 + 8	87	89						
60 + 80	63	[illegible]	89	99				
36 + 9	59	67						
58 + 34	28	54	75	81	95	96		
47 + 54 + 23					76	69	73	83
265 + 100	50	64						
500 + 300	78	82	99	99				
165 + 99	33	44	55	66	69	71	84	95
74 - 30	40	55	70	77				
100 - 68	35	55						
73 - 23	55	60						
80 - 24	47	62	76	78				
264 - 99					41	45	65	86
6 - 4.5					69	76	80	97
$^1/_2 + ^3/_4$					76	73	76	79
$1 - ^1/_3$			58	65	68	70		
Double 26	56	66	86	91				
60 x 70			62	52	91	81		
100 x 35			61	59				
945 x 1000					71	65	69	78
7 x 25			60	54	75	69	59	67
$^1/_{10}$ of 45					49	46	53	67
0.1 x 45					51	45	35	55
$90 \div ^1/_2$							88	87
25% of 48							50	60
'Checker' items								
$^4/_7 + ^2/_5$							41	40
14 x 83			20	16	10	10	3	10
35 x 55					23	14		
0.35 x 567							0	1

Attitude Survey results

Appendix B shows that the statements in the Attitude Survey (AS) were randomly ordered for presentation to the students. In order to facilitate the review and analysis of the attitude data, these statements were grouped within clusters as illustrated in

Table 3. The categories were Interest and Enjoyment, Perception of Competence, Perception of Value, Perception of Use, and Perception of Source of Instruction. All the AS results are presented in Table 8.

Table 8: Percentages of Responses of "Yes", "No" and "Not Sure" to all AS Items by Students in Years 5, 7 and 9

	Yr 5 (n=163)			Yr 7 (n=163)			Yr 9 (n=152)		
Interest and Enjoyment	Y	N	NS	Y	N	NS	Y	N	NS
1. I enjoy doing written computation	63	10	27	60	12	24	41	33	26
15. I enjoy doing mental computation	60	23	17	60	25	15	47	36	17
2. I think written computation is interesting	49	25	26	36	30	34	37	42	21
23. I think mental computation is interesting	60	23	17	56	25	19	47	31	22
14. Written computation is more interesting than mental computation	41	33	26	42	36	22	32	39	29
27. Mental computation is more interesting than written computation	38	32	30	38	25	37	32	33	35
Perception of Competence									
21. Written computation is challenging to me	35	41	24	35	45	19	38	38	24
16. Mental computation is challenging to me	54	24	21	66	18	16	72	10	18
12. I am good at written computation	77	7	16	64	10	26	68	11	21
17. I am good at mental computation	52	21	27	45	20	35	50	24	26
13. I think written computation is more challenging than mental computation	23	56	21	23	57	20	9	76	15
3. I think mental computation is more challenging than written computation	63	24	13	70	14	15	80	9	11
4. I am better at written than mental computation	51	29	19	57	19	24	60	18	21
22. I am better at mental than written computation	29	44	27	32	39	29	30	49	22
Perception of Value									
19. It is important to be good at written computation	70	6	24	64	13	24	76	11	12
25. It is important to be good at mental computation	75	5	19	77	7	16	87	4	9
26 It is more important to be good at written than mental computation	27	29	44	14	41	45	8	41	41
8. It is more important to be good at mental than written computation.	49	21	30	47	21	32	53	18	30

Table 8 continued opposite

	Yr 5 (n=163)			Yr 7 (n=163)			Yr 9 (n=152)		
Perception of Use									
9. I think I will do written computation more than mental computation as an adult	30	47	23	22	41	38	20	41	39
24. I think I will do mental computation more than written computation as an adult.	48	27	25	44	19	37	40	29	31
18. At school I do mental computation more than written computation	24	40	36	21	47	32	30	55	15
5. At school I do written computation more than mental computation.	52	18	30	45	18	37	59	24	17
11. I do written computation more than mental computation away from school	34	43	23	22	50	28	34	54	12
6. I do mental computation more than written computation away from school.	41	38	21	46	29	25	57	32	11
Perception of Source of Instruction									
20. I learned to do mental computation at school	56	29	15	54	25	21	39	39	22
28. I learned to do written computation at school.	65	23	12	68	14	18	78	12	10
10. I learned to do written computation by myself	40	42	18	24	52	24	17	64	19
7. I learned to do mental computation by myself	42	37	21	44	33	24	59	18	23

The Interest and Enjoyment cluster suggests that equal percentages of students in Years 5, 7 and 9 think that both mental and written computation are equally interesting. However Table 9 shows that whereas support for written computation was spread fairly evenly across abilities, interest in mental computation was much more closely correlated with ability.

Table 9: Distribution by Quintiles of Percentages of Students Giving Positive Responses to PS Items 1 and 15

	Year 5					Year 7					Year 9				
Quintile	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1. I enjoy doing written computation	58	56	76	56	70	61	53	61	56	73	47	45	53	29	20
15. I enjoy doing mental computation	76	72	67	44	42	76	69	64	50	45	77	52	40	42	23

In regard to statements classified under the Perception of Competence heading about one-third of the students at each year level said written computation is challenging, while a majority at each year level said mental computation is challenging. Over half of the students at each year level felt they were better at written than mental computation while less than a third said they were better at mental computation.

The statements relating to Perception of Value of mental computation illustrate the similarity of responses across primary and secondary school. For example, about three quarters or more of the students at each year level felt it was important to be good at mental computation while slightly less felt it was important to be good at written computation. Likewise, less than a quarter of the students felt it was more important to be good at written computation than mental computation, and about half of the students agreed that it was more important to be good at mental than written computation.

Did students see mental and written computation as equally useful? The Perception of Use cluster revealed that less than one half of all students felt they would do more mental computation than written computation as an adult, while only about one-quarter of the students said they would do more written computation than mental computation as an adult. This is in line with research by Wandt and Brown (1957) which indicated that adults in non-occupational tasks use mental computation three times as often as they use written computation. About a half of the students at each year level said they would do more written than mental computation at school, whereas similar percentages said that they would do more mental than written computation away from school.

The Perception of Source of Instruction cluster shows that a majority of students (65, 68 and 78 per cent of Years 5, 7, and 9 respectively) reported learning written computation at school whereas about a half (56, 54 and 39 per cent of Years 5, 7, and 9 respectively) reported learning mental computation at school. At each year level, more students reported learning mental than written computation by themselves.

Attitude profile

Many messages are suggested in the attitude data but most could be confirmed only from case studies, involving careful observation and/or interviews with students. Nevertheless, there are some common themes that seem to cut across primary and secondary school. Based on these data, a possible characterisation of the attitude of a Year 9 student might read as follows:

> I enjoy doing both mental and written computation. I learned to do written computation at school but learned to do mental computation by myself. I spend more time at school doing written computation than mental computation but experience the opposite when away from school. I find mental computation far more challenging than written computation, but feel that I am better at doing written computation than mental computation. I think it is important to be good at both mental and written computation, but mental computation will be used more when I'm an adult, so it is more important than written computation.

Mental Computation Test

Overall performance

The Mental Computation Test (MCT) at each year level was composed of items that the researchers felt were reasonable for a significant percentage of students in the year level to compute mentally. There were 30 items for each of Years 3 and 5, and 40 items for each of Years 7 and 9.

Figures 1-4 display frequency distributions of the students' performances at each year level.

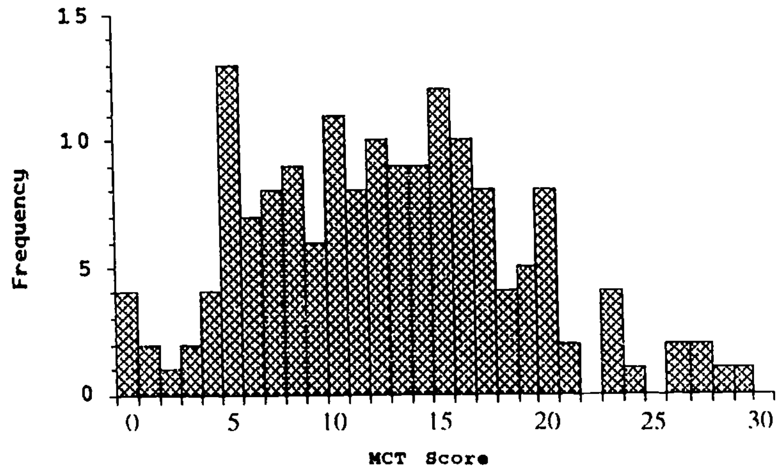

Figure 1: Frequency Distribution of MCT Performances for all Year 3 Students

These histograms provide visual evidence of the range of performance with 13, 13 and 2 students in Years 3, 5 and 7 respectively scoring less than five, suggesting that for these students the MCT was very difficult. On the other hand four Year 7 students and eight Year 9 students answered all 40 items correctly.

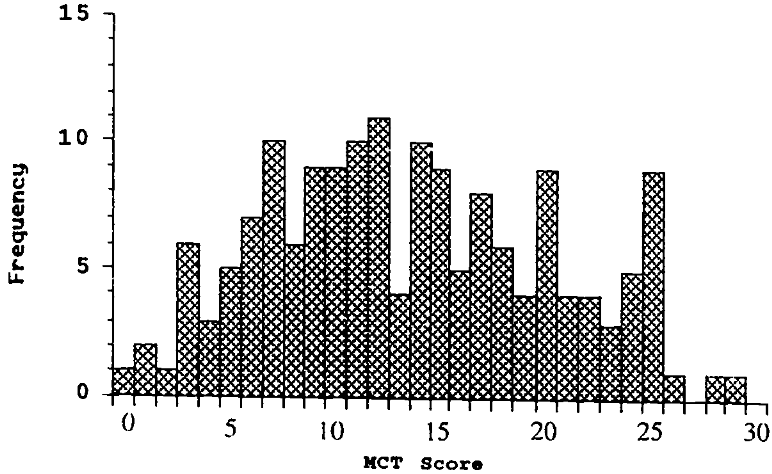

Figure 2: Frequency Distribution of MCT Performances for all Year 5 Students

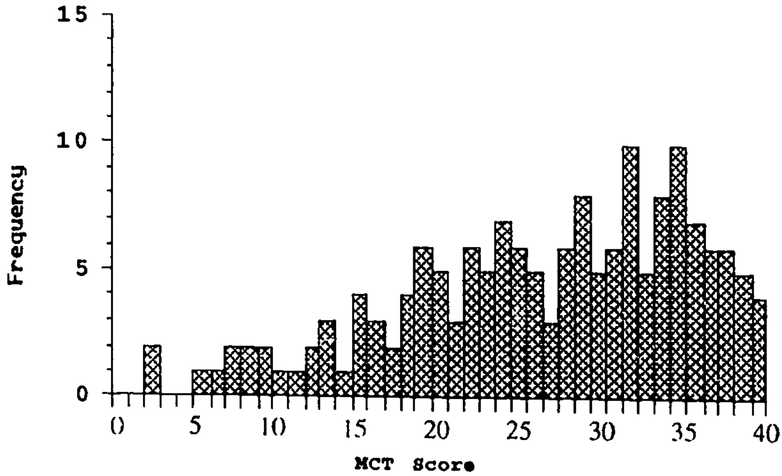

Figure 3: Frequency Distribution of MCT Performances for all Year 7 Students

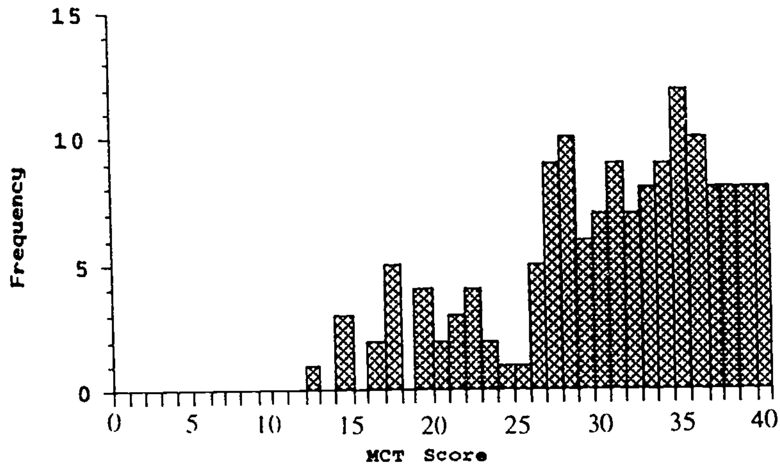

Figure 4: Frequency Distribution of MCT Performances for all Year 9 Students

The overall results confirm the researchers' belief that the items included in the MCT were reasonable to compute mentally at the specified year levels. A summary of the MCT total scores is reported in Table 10. Each of the year level tests was unique although some common items across year levels were included. Therefore, year level comparisons of group performance are inappropriate, except in the cases of individual common items.

Table 10: Summary of Student Performance on MCT

	Year 3	Year 5	Year 7	Year 9
Number of Students	163	163	163	152
Number of Items	30	30	40	40
Minimum Score on MCT	0	0	2	12
Maximum Score on MCT	29	29	40	40
Mean	12.27	13.55	26.55	30.63
Standard Deviation	6.21	6.67	9.13	6.85
Error of Measurement	0.49	0.52	0.72	0.56

A summary of the MCT results by classroom is reported in Table 11. There was considerable variation within classes in performance on the MCT at every year level except for Year 5. For example, at Year 9 the ranges of scores for a particular class were as small as 32 to 40 and as large as 12 to 35. A review of Table 11 reveals similar ranges of extreme scores at each year level, which produced means with a minimum range of 12.58 to 14.36 in Year 5 to a maximum of 23.67 to 36.56 in Year 9.

Table 11: Summary of Ranges, Standard Deviations and Mean Scores on the MCT for Students in Year 3, 5, 7 and 9 Classrooms

Year	School	Classroom	N	Range	SD	Mean
3	A	1	26	7 - 29	5.32	14.46
	A	2	25	4 - 27	6.31	14.92
	B	3	27	0 - 17	5.21	8.52
	B	4	27	0 - 23	6.14	10.93
	C	5	29	5 - 28	6.68	13.59
	C	6	29	1 - 27	5.54	11.45
		Total	163	0 - 29	6.22	12.27
5	A	1	26	3 - 26	6.92	13.89
	A	2	27	1 - 24	6.30	14.04
	B	3	27	3 - 25	5.54	12.67
	B	4	26	1 - 25	7.46	12.58
	C	5	28	4 - 29	5.81	14.36
	C	6	29	0 - 28	8.03	13.69
		Total	163	0 - 29	6.67	13.55
7	A	1	26	6 - 39	9.23	27.77
	A	2	25	8 - 40	9.33	27.76
	B	3	26	2 - 38	8.46	21.96
	B	4	27	8 - 38	7.86	24.37
	C	5	30	2 - 40	9.33	28.73
	C	6	29	5 - 39	9.23	28.31
		Total	163	2 - 40	9.13	26.55
9	D	1	26	16 - 36	5.35	25.00
	D	2	21	12 - 35	7.53	23.67
	D	3	29	16 - 40	5.45	31.10
	D	4	24	20 - 39	4.50	30.17
	D	5	27	32 - 40	2.23	36.56
	D	6	25	28 - 40	3.76	35.80
		Total	152	12 - 40	6.85	30.63

In order to test if the classes at each year level were significantly different in their performance on the MCT, a one-way ANOVA was performed at each year level on the MCT total score. These results are reported in Table 12. The analysis confirms that the classes did differ significantly ($p < 0.05$) from each other for all year levels except Year 5. Differences between the Year 9 classes were the most significant because these classes had been streamed on ability.

Table 12: ANOVA of MCT Total Scores by Classroom at Each Given Year Level

Year	Source	DF	SS	MS	F-test	p
3	Between	5	799.92	159.80	4.60	0.001
	Within	157	5455.10	34.75		
	Total	163				
5	Between	5	73.81	14.76	0.33	0.90
	Within	157	7132.60	45.43		
	Total	163				
7	Between	5	983.80	196.76	2.47	0.035
	Within	157	12532.51	79.83		
	Total	163				
9	Between	5	3470.27	694.05	28.04	0.00001
	Within	146	3613.36	24.75		
	Total	152				

Order of presentation

To examine the order effect of the different modes of presentation, two different forms of the MCT were used at each year level. In Form A the first half of the items were presented orally, the second half presented visually. For Form B which used the same sequence of items, the first half was presented visually, the second half orally. Table 13 reports the means of the two forms of the MCT for each year and shows that the MCT total scores on the two forms were not significantly different in any year. The order of presentation did not have any significant effect on the results.

Mode of presentation

In order to investigate the mode-of-presentation effect on the MCT, a t-test was conducted between the first half of Form A and the first half of Form B (identical items which were presented orally in Form A and visually in Form B), and between the second half of Form A and the second half of Form B (like items which were presented visually in Form A and orally in Form B). The t-test for Year 3 showed a significant difference ($p < 0.5$) between the two means (5.70 visual and 6.75 oral). For the other classes there were no significant differences between the means of the visually presented items and the orally presented items. The mode of presentation did not affect the students' results on the MCT. The complete analysis is reported in Table 14.

Table 13: t-test Comparing Total Scores on Form A and Form B of the MCT

Year	Form	N	Mean	SD	DF	t-ratio	p
3	A	82	11.72	6.09	161	-1.15	0.26
	B	81	12.83	6.32			
5	A	82	13.87	6.57	161	0.62	0.54
	B	81	13.22	6.79			
7	A	81	26.00	9.69	161	-0.77	0.45
	B	82	27.10	8.57			
9	A	82	30.96	6.52	150	0.66	0.51
	B	70	30.23	7.24			

Table 14: t-test Comparing Performance on MCT by Mode of Presentation

Year	Form	Half	Mode	N	Mean	SD	DF	t-ratio	p
3	A	1st	O	82	6.02	3.57	161	-0.09	0.93
	B	1st	V	81	6.07	3.69			
	A	2nd	V	82	5.70	3.00	161	-2.24	0.03
	B	2nd	O	81	6.75	3.04			
5	A	1st	O	82	7.33	3.85	161	0.51	0.61
	B	1st	V	81	7.03	3.78			
	A	2nd	V	82	6.54	3.23	161	0.66	0.51
	B	2nd	O	81	6.20	3.36			
7	A	1st	O	81	13.51	5.37	161	-1.36	0.18
	B	1st	V	82	14.57	4.64			
	A	2nd	V	81	12.49	4.87	161	-0.04	0.97
	B	2nd	O	82	12.52	4.32			
9	A	1st	O	82	15.90	3.18	150	-0.19	0.85
	B	1st	V	70	16.00	3.27			
	A	2nd	V	82	15.06	3.75	150	1.29	0.20
	B	2nd	O	70	14.23	4.22			

Tables 15-18 each shows the results for one of the four operations for both oral and visual modes of presentation for all four year levels. These tables also indicate the development of mental computation skill across the year levels for all the items used at more than one level. The order of difficulty of the test items for both presentation modes is given in Appendix E.

Table 15: Percentage Scores of Students in Years 3, 5, 7, and 9 on MCT Addition Items Administered Orally and Visually

	Year 3		Year 5		Year 7		Year 9	
Item	O	V	O	V	O	V	O	V
6 + 8	91	79						
16 + 9	78	62						
36 + 9	67	72						
20 + 70	73	95						
36 + 20	61	70						
60 + 80	30	44	85	88				
68 + 32	37	40	68	89				
25 + 27	22	38						
79 + 26	17	16	59	73	81	81	86	93
25 + 99	29	30						
58 + 34	17	16	66	88	80	85		
182 + 97	4	2	21	51	58	70		
165 + 99			46	54	64	79	80	89
$^1/_2 + ^1/_4$			55	19	72	73	91	80
$^1/_2 + ^3/_4$			36	29	75	68	70	80
$2^1/_2 + 3^1/_2$					83	83	100	91
$2^1/_2 + 3^3/_4$					58	64	69	76
0.5 + 0.75			12	13	59	57		
6.2 + 4.9			35	38	59	80	84	91

For the majority of addition items, students performed better in the visual than in the oral presentation mode, as shown in Table 15. The most notable exception was $^1/_2 + ^1/_4$ in Year 5 where almost three times as many students were correct with the oral presentation compared with the visual presentation. A similar ratio is evident for the corresponding subtraction item of $^3/_4 - ^1/_2$ as seen in Table 16. It seems that for Year 5 in the cases of simple fractions the visual presentation distracts the students — perhaps by encouraging some instrumental approach rather than a common-sense one.

Table 16: Percentage Scores of Students in Years 3, 5, 7, and 9 on MCT Subtraction Items Administered Orally and Visually

	Year 3		Year 5		Year 7		Year 9	
Item	**O**	**V**	**O**	**V**	**O**	**V**	**O**	**V**
14 - 6	59	51						
36 - 9	30	26						
36 - 10	52	58						
90 - 70	58	43						
74 - 30	25	17	64	45				
73 - 23	30	32						
140 - 60	16	26	77	67				
80 - 24	4	.12	55	53	84	78		
100 - 25	27	32						
100 - 68	11	10	54	53	77	84		
105 - 26	6	4	46	38	68	69	82	86
105 - 97	10	6	52	40				
264 - 99					37	48	76	74
$1 - \frac{1}{3}$			27	38	73	62	77	93
$\frac{3}{4} - \frac{1}{2}$			59	17	74	69		
$4\frac{1}{2} - 3$					91	91		
$6 - 4\frac{1}{2}$					73	74	91	80
$3 - \frac{5}{6}$							66	67
$5\frac{1}{4} - 2\frac{3}{4}$							34	52
6 - 4.5					75	78	94	90
4.5 - 3					83	80	89	98

Table 17: Percentage Scores of Students in Years 3, 5, 7, and 9 on MCT Multiplication Items Administered Orally and Visually

	Year 3		Year 5		Year 7		Year 9	
Item	O	V	O	V	O	V	O	V
Double 8	79	71						
Double 50	89	72						
Double 15	59	60						
Double 26	34	35	80	79	94	94		
60 x 70			19	41	73	73	77	80
100 x 35			44	52				
300 x 40			21	17	64	74		
7 x 25			34	40	72	74	87	91
38 x 50			6	9	26	36	46	66
7 x 49					37	48	82	74
$^1/_{10}$ of 45					60	49	84	79
$4 \times 3^1/_2$					41	46	80	74
$^2/_3$ of 90							94	91
$^1/_2 \times 6^1/_2$							50	44
0.5 x 48					35	38		
0.1 x 45					49	44	67	65
1.5 x 20							73	83
50% of 48					84	94	95	97
100% of 48					89	83	94	97
25% of 48					78	84	90	100
10% of 45					58	47	89	82
75% of 48							77	77

Table 18: Percentage Scores of Students in Years 3, 5, 7, and 9 on MCT Division Items Administered Orally and Visually

	Year 3		Year 5		Year 7		Year 9	
Item	O	V	O	V	O	V	O	V
Half of 16	65	49						
Half of 30	55	60						
Half of 52			74	67				
300 ÷ 5			56	48				
3500 ÷ 35			22	36	56	74	78	86
4200 ÷ 60			20	20	59	52	74	88
450 ÷ 15			17	15	58	58	74	87
150 ÷ 25			42	39	80	77	86	85
440 ÷ 8					28	41	47	55
12 000 ÷ 40					46	53	66	81
$90 \div {}^1/_2$							34	23
$6^1/_2 \div 2$							79	78
3.5 ÷ 0.5							63	50
90 ÷ 0.5							38	31

Gender differences

Do boys and girls differ in their ability to do mental computation? An examination of the means of the MCT total scores in Table 19 shows that, although the mean for the boys was higher than the mean for the girls in each year, only in Years 5 and 9 were the means significantly different beyond the 0.05 level.

Table 19: t-test of MCT Total Scores for Males and Females

Year	Gender	N	Mean	SD	DF	t-ratio	p
3	Females	78	11.68	5.67	161	-1.16	0.25
	Males	85	12.81	6.66			
5	Females	84	12.17	6.73	161	-2.78	0.01
	Males	79	15.01	6.32			
7	Females	80	26.31	9.67	161	-0.33	0.74
	Males	83	26.78	8.64			
9	Females	74	29.45	6.90	150	-2.09	0.04
	Males	78	31.74	6.65			

Difficulty levels for selected items

Items in each year test were ordered by percentages of students answering each item correctly (oral and visual forms combined). The complete tables are given in Appendix E. In Year 3, it is noticeable that almost all addition items are in the upper (i.e. easier) half of the table in Appendix E, while almost all subtractions are in the lower half. That subtraction is a much more difficult operation for these children than addition is shown by contrasting addition items and their inverse subtraction items in Table 20.

Table 20: Year 3 Performance on Selected Addition Items and their Inverses

Item	% correct
6 + 8	85
14 - 6	55
20 + 70	84
90 - 70	50
68 + 32	37
100 - 68	10

However when "doubling" items and their inverses are compared, results were much more similar to each other as shown in Table 21. It should be stressed that these students were attempting mentally many computations that were beyond those normally taught as written computations in the classroom. Success in these items almost certainly implies an understanding and mastery of the numbers and operations not acquired through normal school mathematics teaching.

Table 21: Year 3 Performance on Selected 'Doubling' Items and their Inverses

Item	% correct
Double 15	60
Half of 30	58
Double 8	74
Half of 16	57

The most difficult of the four basic number fact items was 14 - 6 (55% of Year 3 students correct). There were seven items involving larger numbers which Year 3 students found easier than this item. These were 20 + 70 (84%), double 50 (80%), 16 + 9 (70%), 36 + 9 (69%), 36 + 20 (65%), double 15 (60%) and half of 30 (58%).

In Year 5, less difference was observed between the success rates for items and their inverses, as shown in Table 22. It is surprising that the success rates for these last two items ($^3/_4 - ^1/_2$ and $^1/_2 + ^1/_4$) dropped from 59 per cent and 55 per cent to only 17 per cent and 19 per cent respectively when presented visually. It is tempting to conclude that intuition takes precedence over taught procedures when the item is heard but not seen, and that the items are then perceived as relatively easy. However when the item is presented visually the student is more explicitly reminded of the written algorithm, which is probably inhibiting because of its perceived complexity. In this case learning the written algorithm may therefore inhibit rather than encourage success.

Table 22: Year 5 Performance on Selected Addition and 'Doubling' Items and their Inverses

Item	% Correct
60 + 80	87
140 - 60	72
Double 26	79
Half of 52	71
$^3/_4 - ^1/_2$	38
$^1/_2 + ^1/_4$	37

In Year 5 only 13 per cent correctly computed the item 0.5 + 0.75, compared with 33 per cent for the related item $^1/_2 + ^3/_4$. In Year 7 there was also a marked difference with results being 58 and 72 per cent respectively. In Year 7 a lack of understanding of decimal operations was apparent. Table 23 shows that although at least 89 per cent of students recognised that 50% of 48 was equal to half of 48, only 36 per cent recognised the item 0.5 x 48 as equivalent to this.

Table 23: Year 7 Performance on Selected Decimal and Percentage Items

Item	% Correct
50% of 48	89
0.5 x 48	36

Table 24 shows that the relationship between 45 and 4.5 was operationally understood by only half of the students regardless of the form in which the item was presented. However the overall impression is that the conceptual understanding of the Year 7 students was much higher than that of the Year 5 students, and that their difficulties were more often technical ones associated with the size of the number rather than conceptual ones (Table 25). That all three items were more successfully answered when presented visually suggests that a written algorithm was seen as the simplest method of calculating each of these. In each case a relatively simple mental transformation of the computations (such as 265 - 100, 110 ÷ 2, 19 x 100) was less often successfully used.

Table 24: Year 7 Performance on Related Fraction, Decimal and Percentage Items

Item	% Correct
$^1/_{10}$ of 45	55
10% of 45	52
0.1 x 45	47

Table 25: Year 7 Performance on Selected Technically Complex Whole Number Items

Item	% Correct
264 - 99	42
440 ÷ 8	34
38 x 50	31

In Year 9 a major feature was the generally higher rate of success on the whole test. All but four of the 40 questions (when presented orally) were successfully answered by at least 50 per cent of students. Division or multiplication by a fraction or decimal less than one was found to be the main conceptual difficulty The item $6^1/_2 \div 2$ was found to have a 78 per cent success rate but other similar items proved more difficult as shown in Table 26.

Table 26: Year 9 Performance on Selected Fraction and Decimal Multiplication and Division Items

Item	% Correct
$3.5 \div 0.5$	56
$^1/_2 \times 6^1/_2$	47
$90 \div 0.5$	35
$90 \div {}^1/_2$	29

In only three items was visual presentation markedly advantageous. These are shown in Table 27. It is not clear in any of these cases whether seeing the items has helped the conceptual approach or the algorithmic process.

Table 27: Items where Year 9 Performance was Higher in the Visual than the Oral Mode of Presentation

	% Correct	
Item	**Oral**	**Visual**
$1 - {}^1/_3$	77	93
38×50	46	66
$5^1/_4 - 2^3/_4$	34	52

Preference versus performance

In the PS students were presented with a number of computations and asked whether or not they would prefer to do them mentally. At each year level, a sub-section of these items was also included in the MCT. If we assume that those students who stated that they preferred to do the calculation mentally thought they would calculate the answer correctly, then for each of these items (nine items for each of Years 3 and 9 and ten for each of Years 5 and 7) it is possible to classify all students into four categories as follows:

- Those who thought they could do the calculation, and could.
- Those who thought they could do the calculation, but could not.

- Those who thought they could not do the calculation, but could.
- Those who thought they could not do the calculation, and could not.

In order to discover to what extent ability was a factor, the results were analysed by quintiles, according to the performance of each student on the complete MCT. The full set of results is given in Appendix F.

It is not surprising that overwhelmingly the more able students were more likely to be in the first category of students — those who correctly predicted that they could perform the calculation mentally. On the other hand, the less able students were much more likely to assert correctly that they could not perform the calculation mentally. These less able students were also more likely to be unrealistically optimistic about their ability, giving incorrect answers to calculations which they thought they could do. However this was not true of Year 3 students, a large percentage of whom thought incorrectly that they could add and subtract two digit numbers mentally. Seven of these items were included and the percentages of students giving this response ranged from 32 to 51, with these students being spread across all ability quintiles. The reasons for this are not clear, though they are presumably related to the fact that the students are at this stage learning about the written algorithms for addition and subtraction.

It is particularly interesting to note that those students who were more competent than they gave themselves credit for, correctly performed calculations which they had not felt confident they could do. These students were spread quite evenly across the quintiles. A typical case is given in Table 28, showing the results for Year 7 for the item $^1/_2 + ^3/_4$.

Table 28: Year 7 Preference Versus Performance Percentages by Quintiles for Item $^1/_2 + ^3/_4$

	Thought could and could do mentally	Thought could but couldn't do mentally	Thought couldn't but could do mentally	Thought couldn't and couldn't do mentally
1st Quintile	79	3	18	0
2nd Quintile	78	3	19	0
3rd Quintile	70	9	18	3
4th Quintile	34	25	22	19
5th Quintile	18	39	6	36
All Year 7	56	16	17	12

About half of the students correctly predicted that they could calculate this item mentally, the percentage rising markedly with ability. Only 12 per cent correctly predicted that they could not do the calculation, and 16 per cent incorrectly thought they could; the percentages in both cases decreasing with ability. However some 20 per cent in each quintile, except those in the fifth quintile succeeded where they thought they would fail.

The item which produced the highest degree of diffidence in students about their performance (that is, the item for which the greatest percentage of students thought they couldn't calculate the item, but could) was 25% of 48. Of the Year 9 students, 41 per cent gave this response. Some 30 per cent of Year 9 students gave similar responses for the items $^1/_{10}$ of 45, and 0.1 x 45, indicating a lack of confidence in the area of fractions, decimals and percentages.

The patterns of Table 28 are fairly typical of most of the items in all year levels. The more able the students the better they were at correctly predicting success on items of mental computation. However, the less able the students the better they tended to be at correctly predicting failure on items. In terms of incorrect prediction of failure, there was no definite trend, except that results were often fairly similar for all ability levels.

Overall, a majority of students had a true impression of their ability or inability to calculate each item mentally, the percentage increasing with age. Thus maturity appears to bring with it a better sense of one's strengths and limitations in calculation, and this self-awareness is an important aspect of number sense The only exception was the item $90 \div {}^1/_2$, for which 62 per cent of students incorrectly indicated that they could calculate the answer, presumably equating the calculation with half of 90.

The responses of each quintile were separated by gender to see whether the pattern of boys' responses differed from that of the girls. Table 29 shows the number of cases where the responses of boys and girls in a quintile in each of the four response categories differed from each other by 20 or more percentage points. It appears that boys more often accurately predicted their ability to succeed at mental computation, whereas girls more often unnecessarily expressed diffidence concerning their ability to calculate mentally.

Table 29: Number of Cases for which Boys' and Girls' Predictions of Mental Computation Ability Differed by More than 20 Percentage Points

Response	G > B	B > G
Thought could, and could	12	21
Thought could, but couldn't	19	18
Thought couldn't, but could	13	6
Thought couldn't, and couldn't	19	13

CHAPTER
FOUR

Discussion of results and implications

We begin this section by summarising the major aspects of the study and its limitations. The key findings are then discussed. Finally, some implications are proposed for curriculum and teaching, and for further research.

Overview of the study

This research is part of an international study of mental computation in Australia, Japan and the USA, with the same design and instruments, and comparable subjects used in all three countries.

Three "typical" primary (K-7) schools and a secondary school in the same region in Perth suburbs were chosen for the study. The subjects were the students in 24 classrooms. There were two classes in each of Years 3, 5 and 7 from each of the three primary schools; and six Year 9 classes from the secondary school. The numbers of student participants were 163, 163, 163 and 152 in the four respective year levels.

Three instruments were developed specifically for the international study. One measured attitudes to mental computation including level of interest; and perceptions of competency, value, use, and source of instruction. This attitude questionnaire was not given to Year 3 students. The second instrument determined computational preferences (mental verses paper/pencil or calculator) for a series of items, 17 of which were used later in the mental computation tests.

The mental computation tests consisted of 30, 30, 40, and 40 items for Years 3, 5, 7 and 9 respectively. Many items were repeated for two or more year levels to measure skill development. Half the test was administered orally and the other half visually to compare performances in the two modes. In each pair of matching classes the test forms were administered in reverse sequence to guard against any order effect.

The study had several limitations. It involved a small number of schools and results therefore have limited generalisability. It was designed to provide only quantitative data regarding attitudes, preferences and performance in mental computation. No interviews or case studies were conducted to help explain the findings. No data were collected on the mental computation strategies used. However, this latter aspect had already been documented extensively by McIntosh, De Nardi and Swan (1993).

Discussion of results

The great majority of students in Years 5, 7 and 9 felt it was important to be good at mental computation. About half the students thought being good at mental computation was more important than being good at written computation, while less than a quarter thought the reverse should be the case. Students tended to consider that written computation was mostly learned at school but mental computation was mostly learned outside school. Thus, despite the lack of emphasis on mental computation in the classroom students still seem to manage to learn its skills, and be aware of its importance.

In the Preference Survey (PS) 17 items were used in more than one year level, and all but two showed increasing preference by students for a mental computation approach with increasing year level. At each year level 10-12 items from the PS were included in the Mental Computation Test (MCT). Preferences were closely related to ability, as one might expect. Thus, the more able the students, the better they could predict whether or not they could undertake a computation mentally. This self confidence varied considerably both within and across year levels. For example, 61 per cent of the top quintile of Year 3 students preferred to compute 165 + 99 mentally but only 55 per cent of the bottom quintile of Year 7 students indicated this preference. This illustrates vast differences in confidence. Gender differences favoured boys in Years 3 and 9 but were not significant in Years 5 and 7. It is not clear why this was so.

The MCT results showed no significant difference between oral and visual presentation of mental computation items overall at any year level. However, for some individual items there was a marked difference. It may be that in these cases, the varied presentations encouraged different strategies, thus resulting in different performances. Boys performed significantly better than girls in Years 5 and 9. However, the difference could be attributed to three items (out of 30) in Year 5 and only two items (out of 40) in Year 9. Performances for Years 3 and 5 were distributed approximately in normal fashion. However, in Years 7 and 9 distributions were skewed because of easier items and/or the peaking of mental computation ability. Increases in performances across year levels were much higher from Years 3 to 5 than from Years 7 to 9.

Students performed much better on addition than on the equivalent subtraction items. For example, 85 per cent of the Year 3 students computed 6 + 8 correctly but only 55 per cent were successful with 14 - 6; and only 37 per cent correctly completed the related item 60 + 80. It seems that too many students are not aware of number relationships, and neither are they making connections between related expressions. For pairs of items involving doubling and halving the difference in performance was minimal. The results also reveal other conceptual difficulties. For example, less than half of the Year 5 students correctly computed 100 x 35 which shows a lack of understanding of place value and general numeration concepts.

Implications

This study suggests the following implications for curriculum development and teaching practice in the mathematics classroom.

- The curriculum needs to be much more flexible to cater for the wide range of ability.
- Despite the fact that students see mental computation as being more important than written computation it is the latter that gets the majority of teachers' attention. However, it may not be desirable to teach particular mental computation strategies. Rather, teachers should foster mental computation skills by encouraging strategies that are suited to the individual student.
- Emphasising skills at the expense of understanding is unlikely to prove effective. Students need to develop a sound understanding of the numeration system, and need to be made aware of relationships between number facts.
- Some mental computation items are found to be easier if presented visually rather than orally. For some others the opposite is true, so both methods of presentation should be used by the teacher at various times.

- Finally, it should be stressed that real life computation involves mental computation and/or calculator use, so classroom teaching should emphasise these aspects rather than the traditional paper and pencil algorithms.

The results of this study suggest a number of questions that need to be addressed in further research.

- What effect does a teaching/learning emphasis on relationships between number facts have on mental computation performance?
- What types of mental computation items do students prefer to be presented visually/orally?
- What is the explanation for gender differences found in this study?
- If all the time currently spent on written algorithms in classrooms is devoted to mental computation and calculator use what differences would this make?
- If the mental computation items were contextually based what difference would this make to performance?
- What differences are there between strategies used in oral versus visual presentation?
- What error patterns are there in the results?
- What are the upper limits of computation that can be performed mentally by Year 7 and Year 9 students?
- What is the relationship between children's mental computation skill and their overall number sense?

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APPENDIX A. Preference Surveys (PS)

STUDENT PREFERENCE SURVEY - YEAR 3

Name: ______________________ School: ____________________
(first) (last)

Computation is often involved in solving real-world problems. When solving problems, several computational methods exist:

- Sometimes people use a calculator.
- Sometimes people use paper and pencil.
- Sometimes people compute mentally without writing anything down.

We want to learn which problems you prefer to do mentally. Please look at each problem below and decide if you prefer to do it mentally. Circle YES or NO to indicate your response. It is **not** necessary for you to work the problems.

	Problem	**I would do this problem mentally.**	
1.	6 + 8	Yes	No
2.	36 + 9	Yes	No
3.	58 + 34	Yes	No
4.	500 + 300	Yes	No
5.	60 + 80	Yes	No
6.	74 - 30	Yes	No
7.	100 - 68	Yes	No
8.	80 - 24	Yes	No
9.	73 - 23	Yes	No
10.	165 + 99	Yes	No
11.	What is double 26?	Yes	No
12.	265 + 100	Yes	No

STUDENT PREFERENCE SURVEY- YEAR 5

Name: ______________________ School: ____________________
(first) (last)

Computation is often involved in solving real-world problems. When solving problems, several computational methods exist:

- Sometimes people use a calculator.
- Sometimes people use paper and pencil.
- Sometimes people compute mentally without writing anything down.

We want to learn which problems you prefer to do mentally. Please look at each problem below and decide if you prefer to do it mentally. Circle YES or NO to indicate your response. It is **not** necessary for you to work the problems.

	Problem	**I would do this problem mentally.**	
1.	500 + 300	Yes	No
2.	What is double 26?	Yes	No
3.	58 + 34	Yes	No
4.	60 + 80	Yes	No
5.	74 - 30	Yes	No
6.	80 - 24	Yes	No
7.	60 x 70	Yes	No
8.	14 x 83	Yes	No
9.	100 x 35	Yes	No
10.	$1 - \frac{1}{3}$	Yes	No
11.	165 + 99	Yes	No
12.	7 x 25	Yes	No

STUDENT PREFERENCE SURVEY - YEAR 7

Name: ______________________ School: ____________________
(first) (last)

Computation is often involved in solving real-world problems. When solving problems, several computational methods exist

- Sometimes people use a calculator.
- Sometimes people use paper and pencil.
- Sometimes people compute mentally without writing anything down.

We want to learn which problems you prefer to do mentally.
Please look at each problem below and decide if you prefer to do it mentally.
Circle YES or NO to indicate your response. It is **not** necessary for you to work the problems.

	Problem	**I would do this problem mentally.**	
1.	58 + 34	Yes	No
2.	47 + 54 + 23	Yes	No
3.	264 − 99	Yes	No
4.	14 x 83	Yes	No
5.	60 x 70	Yes	No
6.	7 x 25	Yes	No
7.	165 + 99	Yes	No
8.	945 x 1000	Yes	No
9.	35 x 55	Yes	No
10.	What is $\frac{1}{10}$ of 45?	Yes	No
11.	$1 - \frac{1}{3}$	Yes	No
12.	$\frac{1}{2} + \frac{3}{4}$	Yes	No
13.	6 − 4.5	Yes	No
14.	0.1 x 45	Yes	No

STUDENT PREFERENCE SURVEY - YEAR 9

Name: ______________________ School: ____________________
(first) (last)

Computation is often involved in solving real-world problems. When solving problems, several computational methods exist:

- Sometimes people use a calculator.
- Sometimes people use paper and pencil.
- Sometimes people compute mentally without writing anything down

We want to learn which problems you prefer to do mentally.
Please look at each problem below and decide if you prefer to do it mentally. Circle YES or NO to indicate your response. It is **not** necessary for you to work the problems.

	Problem	I would do this problem mentally.	
1.	165 + 99	Yes	No
2.	7 x 25	Yes	No
3.	14 x 83	Yes	No
4.	945 x 1000	Yes	No
5.	264 - 99	Yes	No
6.	$\frac{1}{2} + \frac{3}{4}$	Yes	No
7.	$\frac{4}{7} + \frac{2}{5}$	Yes	No
8.	6 - 4.5	Yes	No
9.	0.35 x 555	Yes	No
10.	What is 25% of 48?	Yes	No
11.	47 + 54 + 23	Yes	No
12.	What is $\frac{1}{10}$ of 45?	Yes	No
13.	$90 \div \frac{1}{2}$	Yes	No
14.	0.1 x 45	Yes	No

APPENDIX B. Attitude Survey (AS)

STUDENT ATTITUDE SURVEY

Name: ______________________ Year: ______ School: ______________________
(first) (last)

Here are some statements about written and mental computation. Tick the space which best describes your feeling about each statement.

Statement	YES	NO	NOT SURE
1. I enjoy doing written computation.			
2. I think written computation is interesting.			
3. I think mental computation is more challenging than written computation.			
4. I am better at written than mental computation.			
5. At school I do written computation more than mental computation.			
6. I do mental computation more than written computation away from school.			
7. I learned to do mental computation by myself.			
8. It is more important to be good at mental than written computation.			
9. I think I will do written computation more than mental computation as an adult.			
10. I learned to do written computation by myself.			
11. I do written computation more than mental computation away from school.			
12. I am good at written computation.			
13. I think written computation is more challenging than mental computation.			
14. Written computation is more interesting than mental computation.			
15. I enjoy doing mental computation.			
16. Mental computation is challenging to me.			
17. I am good at mental computation.			
18. At school I do mental computation more than written computation.			
19. It is important to be good at written computation.			
20. I learned to do mental computation at school.			
21. Written computation is challenging to me.			
22. I am better at mental than written computation.			
23. I think mental computation is interesting.			
24. I think I will do mental computation more than written computation as an adult.			
25. It is important to be good at mental computation.			
26. It is more important to be good at written then mental computation.			
27. Mental computation is more interesting than written computation.			
28. I learned to do written computation at school.			

APPENDIX C. Mental Computation Tests (MCT)

MENTAL COMPUTATION TEST 1/13/92
YEAR 3

Form A: Oral then Visual

Presentation Format: Oral

Examples: (i) 20 and 3
(ii) 45 take 10

1. 36 and 9
2. 20 and 70
3. 36 and 20
4. 68 and 32
5. 25 and 27
6. 25 and 99
7. 36 take 9
8. 36 take 10
9. 73 take 23
10. 80 take 24
11. 100 take 68
12. 105 take 26
13. What is double 15?
14. What is double 26?
15. What is half of 30?

Presentation Format: Visual

Examples: (iii) 10 + 15
(iv) 18 - 8

16. 6 + 8
17. 16 + 9
18. 60 + 80
19. 79 + 26
20. 58 + 34
21. 182 + 97
22. 14 - 6
23. 90 - 70
24. 74 - 30
25. 140 - 60
26. 100 - 25
27. 105 - 97
28. What is double 8?
29. What is double 50?
30. What is half of 16?

MENTAL COMPUTATION TEST
YEAR 3

1/13/92

Form B: Visual then Oral

Presentation Format: Visual

Examples; (i) 20 + 3
(ii) 45 - 10

1. 36 + 9
2. 20 + 70
3. 36 + 20
4. 68 + 32
5. 25 + 27
6. 25 + 99
7. 36 - 9
8. 36 - 10
9. 73 - 23
10. 80 - 24
11. 100 - 68
12. 105 - 26
13. What is double 15?
14. What is double 26?
15. What is half of 30?

Presentation Format: Oral

Examples: (iii) 10 and 15
(iv) 18 take 8

16. 6 and 8
17. 16 and 9
18. 60 and 80
19. 79 and 26
20. 58 and 34
21. 182 and 97
22. 14 take 6
23. 90 take 70
24. 74 take 30
25. 140 take 60
26. 100 take 25
27. 105 take 97
28. What is double 8?
29. What is double 50?
30. What is half of 16?

MENTAL COMPUTATION TEST
YEAR 5

Form A: Oral then Visual

Presentation Format: Oral

Examples: (i) 20 times 3
(ii) 45 and 35

1. 58 and 34
2. 68 and 32
3. 165 and 99
4. 80 take 24
5. 100 take 68
6. 105 take 26
7. What is double 26?
8. 300 times 40
9. 7 times 25
10. What is half of 52?
11. 3500 divided by 35
12. 450 divided by 15
13. 1/2 and 1/4
14. 3/4 take 1/2
15. 6.2 and 4.9

Presentation Format: Visual

Examples: (iii) 60 - 8
(iv) 50 ÷ 5

16. 60 + 80
17. 79 + 26
18. 182 + 97
19. 74 - 30
20. 140 - 60
21. 105 - 97
22. 60 x 70
23. 100 x 35
24. 38 x 50
25. 300 ÷ 5
26. 4200 ÷ 60
27. 150 ÷ 25
28. 1/2 + 3/4
29. 1 - 1/3
30. 0.5 + 0.75

MENTAL COMPUTATION TEST
YEAR 5

Form B: Visual then Oral

Presentation Format: Visual

Examples: (i) 20 x 3
(ii) 45 + 35

1. 58 + 34
2. 68 + 32
3. 165 + 99
4. 80 - 24
5. 100 - 68
6. 105 - 26
7. What is double 26?
8. 300 x 40
9. 7 x 25
10. What is half of 52?
11. 3500 ÷ 35
12. 450 ÷ 15
13. 1/2 + 1/4
14. 3/4 - 1/2
15. 6.2 + 4.9

Presentation Format: Oral

Examples: (iii) 60 take 8
(iv) 50 divided by 5

16. 60 and 80
17. 79 and 26
18. 182 and 97
19. 74 take 30
20. 140 take 60
21. 105 take 97
22. 60 times 70
23. 100 times 35
24. 38 times 50
25. 300 divided by 5
26. 4200 divided by 60
27. 150 divided by 25
28. 1/2 and 3/4
29. 1 take 1/3
30. 0.5 and 0.75

MENTAL COMPUTATION TEST
YEAR 7

Form A: Oral then Visual

Presentation Format: Oral

Examples: (i) 25 times 3
(ii) 45 and 35

1. 58 and 34
2. 165 and 99
3. 100 take 68
4. 105 take 26
5. What is double 26?
6. 300 times 40
7. 7 times 25
8. 3500 divided by 35
9. 450 divided by 15
10. 12000 divided by 40
11. 1/2 and 1/4
12. 2 1/2 and 3 1/2
13. 3/4 take 1/2
14. 6 take 4 1/2
15. 4 times 3 1/2
16. 6.2 and 4.9
17. 6 take 4.5
18. 0.5 times 48
19. What is 50% of 48?
20. What is 25% of 48?

Presentation Format: Visual

Examples: (iii) 100 - 70
(iv) 60 ÷ 5

21. 79 + 26
22. 182 + 97
23. 80 - 24
24. 264 - 99
25. 60 x 70
26. 7 x 49
27. 38 x 50
28. 150 ÷ 25
29. 4200 ÷ 60
30. 440 ÷ 8
31. 1/2 + 3/4
32. 2 1/2 + 3 3/4
33. 1 - 1/3
34. 4 1/2 - 3
35. What is 1/10 of 45?
36. 0.5 + 0.75
37. 4.5 - 3
38. 0.1 x 45
39. What is 100% of 48?
40. What is 10% of 45?

MENTAL COMPUTATION TEST
YEAR 7

Form B: Visual then Oral

Presentation Format: Visual

Examples: (i) 25 x 3
(ii) 45 + 35

1. 58 + 34
2. 165 + 99
3. 100 - 68
4. 105 - 26
5. What is double 26?
6. 300 x 40
7. 7 x 25
8. 3500 ÷ 35
9. 450 ÷ 15
10. 12000 ÷ 40
11. 1/2 + 1/4
12. 2 1/2 + 3 1/2
13. 3/4 - 1/2
14. 6 - 4 1/2
15. 4 x 3 1/2
16. 6.2 + 4.9
17. 6 - 4.5
18. 0.5 x 48
19. What is 50% of 48?
20. What is 25% of 48?

Presentation Format: Oral

Examples: (iii) 100 take 70
(iv) 60 divided by 5

21. 79 and 26
22. 182 and 97
23. 80 take 24
24. 264 take 99
25. 60 times 70
26. 7 times 49
27. 38 times 50
28. 150 divided by 25
29. 4200 divided by 60
30. 440 divided by 8
31. 1/2 and 3/4
32. 2 1/2 and 3 3/4
33. 1 take 1/3
34. 4 1/2 take 3
35. What is 1/10 of 45?
36. 0.5 and 0.75
37. 4.5 take 3
38. 0.1 times 45
39. What is 100% of 48?
40. What is 10% of 45?

MENTAL COMPUTATION TEST
YEAR 9

Form A: Oral then Visual

Presentation Format: Oral

Examples: (i) 1.5 times 2
(ii) 75 and 75

1. 165 and 99
2. 105 take 26
3. 7 times 25
4. 7 times 49
5. 3500 divided by 35
6. 450 divided by 15
7. 12000 divided by 40
8. 1/2 and 1/4
9. 2 1/2 and 3 1/2
10. 3 take 5/6
11. 6 take 4 1/2
12. What is 2/3 of 90?
13. 4 times 3 1/2
14. 90 divided by 1/2
15. 6 take 4.5
16. 1.5 times 20
17. 90 divided by 0.5
18. What is 100% of 48?
19. What is 50% of 48?
20. What is 25% of 48?

Presentation Format: Visual

Examples: (iii) 125 - 15
(iv) 1000 ÷ 20

21. 79 + 26
22. 264 - 99
23. 60 x 70
24. 38 x 50
25. 150 ÷ 25
26. 4200 ÷ 60
27. 440 ÷ 8
28. 1/2 + 3/4
29. 2 1/2 + 3 3/4
30. 1 - 1/3
31. 5 1/4 - 2 3/4
32. What is 1/10 of 45?
33. 1/2 x 6 1/2
34. 6 1/2 ÷ 2
35. 6.2 + 4.9
36. 4.5 - 3
37. 0.1 x 45
38. 3.5 ÷ 0.5
39. What is 10% of 45?
40. What is 75% of 48?

MENTAL COMPUTATION TEST
YEAR 9

Form B: Visual then Oral

Presentation Format: Visual

Examples: (i) 1.5 x 2
(ii) 75 + 75

1. 165 + 99
2. 105 - 26
3. 7 x 25
4. 7 x 49
5. 3500 ÷ 35
6. 450 ÷ 15
7. 12000 ÷ 40
8. 1/2 + 1/4
9. 2 1/2 + 3 1/2
10. 3 - 5/6
11. 6 - 4 1/2
12. What is 2/3 of 90?
13. 4 x 3 1/2
14. 90 ÷ 1/2
15. 6 - 4.5
16. 1.5 x 20
17. 90 ÷ 0.5
18. What is 100% of 48?
19. What is 50% of 48?
20. What is 25% of 48?

Presentation Format: Oral

Examples: (iii) 125 take 15
(iv) 1000 divided by 20

21. 79 and 26
22. 264 take 99
23. 60 times 70
24. 38 times 50
25. 150 divided by 25
26. 4200 divided by 60
27. 440 divided by 8
28. 1/2 and 3/4
29. 2 1/2 and 3 3/4
30. 1 take 1/3
31. 5 1/4 take 2 3/4
32. What is 1/10 of 45?
33. 1/2 times 6 1/2
34. 6 1/2 divided by 2
35. 6.2 and 4.9
36. 4.5 take 3
37. 0.1 times 45
38. 3.5 divided by 0.5
39. What is 10% of 45?
40. What is 75% of 48?

Distribution of items by year level

The numbers in columns two to five indicate the question number for that item at each level.

Item	Year 3	Year 5	Year 7	Year 9
36 + 9	1			
20 + 70	2			
36 + 20	3			
68 + 32	4	2		
25 + 27	5			
25 + 99	6			
36 - 9	7			
36 - 10	8			
73 - 23	9			
80 - 24	10	4	23	
100 - 68	11	5	3	
105 - 26	12	6	4	2
Double 15	13			
Double 26	14	7	5	
Half of 30	15			
6 + 8	16			
16 + 9	17			
60 + 80	18	16		
79 + 26	19	17	21	21
58 + 34	20	1	1	
182 + 97	21	18	22	
14 - 6	22			
90 - 70	23			
74 - 30	24	19		
140 - 60	25	20		
100 - 25	26			
105 - 97	27	21		
Double 8	28			
Double 50	29			
Half of 16	30			
165 + 99		3	2	1
300 x 40		8	6	
7 x 25		9	7	3
Half of 52		10		
3500 ÷ 35		11	8	5
450 ÷ 15		12	9	6
$^1/_2 + ^1/_4$		13	11	8
$^3/_4 - ^1/_2$		14	13	
6.2 + 4.9		15	16	35
60 x 70		22	25	23
100 x 35		23		
38 x 50		24	27	24

Table continued overleaf

Item	Year 3	Year 5	Year 7	Year 9
300 ÷ 5		25		
4200 ÷ 60		26	29	26
150 ÷ 25		27	28	25
$\frac{1}{2} + \frac{3}{4}$		28	31	28
$1 - \frac{1}{3}$		29	33	30
0.5 + 0.75		30	36	
12000 ÷ 40			10	7
$2\frac{1}{2} + 3\frac{1}{2}$			12	9
$6 - 4\frac{1}{2}$			14	11
$4 \times 3\frac{1}{2}$			15	13
6 - 4.5			17	15
0.5 x 48			18	
50% of 48			19	19
25% of 48			20	20
264 - 99			24	22
7 x 49			26	4
440 ÷ 8			30	27
$2\frac{1}{2} + 3\frac{3}{4}$			32	29
$4\frac{1}{2} - 3$			34	
$\frac{1}{10}$ of 45			35	32
4.5 - 3			37	36
0.1 x 45			38	37
100% of 48			39	18
10% of 45			40	39
$3 - \frac{5}{6}$				10
$\frac{2}{3}$ of 90				12
$90 \div \frac{1}{2}$				14
1.5 x 20				16
90 ÷ 0.5				17
$5\frac{1}{4} - 2\frac{3}{4}$				31
$\frac{1}{2} \times 6\frac{1}{2}$				33
$6\frac{1}{2} \div 2$				34
3.5 ÷ 0.5				38
75% of 48				40

APPENDIX D. Protocols for PS, AS and MCT

Australian Data Collection - MC Research
August, 1992
Administration Instructions

1. Enter classroom
2. Introduce yourself and briefly state the purpose of the research. For example, "My name is ---. I am from Edith Cowan University. My colleagues and I are studying how students at your year level do calculations mentally. We need your help and we appreciate your cooperation today. I will be administering two short questionnaires and a mental computation test. The results will be kept confidential and we hope you will do your best in this effort. Thank you."

Preference Survey

3. Ask children to take out a pencil and clear everything else from their desk.
4. Distribute Preference Survey to children.
5. Ask them to write their name and school at the top of the survey.
6. Give directions for Preference Survey and show children the transparency illustrating different ways of calculating the same problem.
7. Ask students to turn their paper over on their desk when they are finished.
8. Collect the Preference Survey when everyone is finished.

Attitude Survey (not year 3)

9. Distribute Attitude Survey to children.
10. Ask them to write their name, year and school.
11. Give directions for Attitude Survey.
12. Give the teacher a copy of the Teacher Attitude Survey. Ask the teacher to complete it as the students are completing their survey.
13. Ask students to turn their paper over when finished.
14. Collect the Attitude Survey when everyone is finished.

Mental Computation Test

15. Distribute Answer sheets for mental computation test.
16. Ask students to fill in all the spaces at the top of the answer sheet.
17. Give directions for the Mental Computation Test.
18. Administer examples (i) and (ii) (Form A: orally; Form B: visually).
19. Administer the first half (Form A: oral portion, Form B: visual portion) of the Mental Computation Test.
20. At the conclusion of the first half of the test, allow students to stretch and relax for a minute or two.
21. Explain that for the second half of the test, the items will be administered differently (Form A: visually, Form B: orally). Administer examples (iii) and (iv) (Form A: visually, Form B: orally).
22. Administer the second half of the Mental Computation Test.
23. Collect answer sheets.
24. Thank the teacher and students for participating.

After leaving the classroom

25. Arrange papers in three separate bundles in alphabetical order.
26. Mark Mental Computation Test, placing 1 for correct, 0 for incorrect, to the immediate left of each question number.

Instructions for Administering the Mental Computation Test

To the test administrator: This test consists of two parts. The first part contains 15 (20) items which are to be presented orally (visually). The second part contains 15 (20) items which are to be presented visually (orally). Ask students to get a pencil or pen and to clear everything else from the top of their desk. Hand out the answer sheet, one per student. Check to see that students write their name on the answer sheet as well as the other information requested. Read the directions below to the class. Ask students to write their answer to each problem after the proper number. Remind students that:

* they should compute the answer mentally,
* they should not copy the numbers down,
* they should make no marks on their answer sheet except for their answer,
* for items 1-15 (20) you will say the problem aloud once and repeat it once (show the problem on the overhead screen),
* for items 16-30 (21-40) reverse of above,
* once you start the test you will not stop to answer questions or repeat a problem.

For the oral portion of the test. Read aloud the item number then the item. Repeat the item, wait 20 seconds, then go to the next item.

For the visual portion of the test. Display each item on the overhead projector screen one item at a time. Say only the item number as each item is displayed for 20 seconds.

To the students:

• "Please print your name on the answer sheet. Fill in all other information requested at the top of the answer sheet."(Wait for students to complete this information and direct as needed)

• "Today I am going to give you some arithmetic(maths) problems that I want you to do mentally. You should not copy the problem or do any written computation (working on paper). All computations(working) should be done in your head. As soon as you have computed(worked) an answer in your head, write your answer on the answer sheet. If you cannot get an answer, just put a cross in that answer space and wait for the next problem. Do not copy the item(question) on your paper or make any other marks on your paper except for your answer. If you have to correct an answer put a line through it and write the correct answer alongside."

(For oral portion: "I will say the problem slowly, and then repeat it once, so you must listen carefully.")

•"Once we start we will not stop until the list is finished. First, we will do two practice examples. Write your answers to the practice examples on the top of your answer sheet at at Ex (i) and (ii)." (Administer the two practice examples as you will the test items.)

Example (i): 20 x 3 or as appropriate
Example (ii): 45 + 35 or as appropriate

• "Any questions?"
(For the visual administration, encourage students to adjust their seat so they can clearly see the screen but do not provide feedback on the example items or show how they might be done. Provide no instruction.)

• "Let's begin."

• "Number 1"
(Read the first item. Wait 2-3 seconds. Read it again.) OR Display first item.
Wait 20 seconds.

• "Number 2"
Read the second item. Wait 2-3 seconds. Read it again. OR Display second item.
Wait 20 seconds.

Continue with items 3 - 15 (20).

Give students an opportunity to relax for a short time (2-3 minutes) before proceeding with the second part of the test.

• "In this part of the test the items will be shown on the screen (read). Here are two examples. Please write your answers to the practice examples on your answer sheet at Ex (iii) and (iv)"

Example (iii): 60 - 8 or as appropriate
Example (iv): 50 ÷ 5 or as appropriate

Items 16 -30 (21-40) are to be visually (or orally) presented.

Display item number 16 (21) OR Read item number 16 (21). Wait 2-3 seconds. Repeat.
Wait 20 seconds

Display item number 17 (22) OR Read item number 17 (22). Wait 2-3 seconds. Repeat.
Wait 20 seconds

Continue in this manner for items 18-30 (23-40)

• "The test is now finished. Please pass your papers to the front of the room. Thank you very much for helping us."

Sample Overhead Transparency for Visual Presentation of MCT Item

(25)

140 - 60

MENTAL COMPUTATIONAL TEST - ANSWER SHEET

TEST FORM: ______

Name: ______________________ (first) (last)   School: ______________________

Year: ________ Sex: ________   Teacher: ______________________

Example (i): ____________   Example (iii): ____________

Example (ii): ____________   Example (iv): ____________

1. ________	21. ________
2. ________	22. ________
3. ________	23. ________
4. ________	24. ________
5. ________	25. ________
6. ________	26. ________
7. ________	27. ________
8. ________	28. ________
9. ________	29. ________
10. ________	30. ________
11. ________	31. ________
12. ________	32. ________
13. ________	33. ________
14. ________	34. ________
15. ________	35. ________
16. ________	36. ________
17. ________	37. ________
18. ________	38. ________
19. ________	39. ________
20. ________	40. ________

APPENDIX E. Order of Difficulty of MCT Items at Each Year Level

Columns two to four indicate the percentage of correct responses to each item, when presented orally, visually and orally and visually combined. Items are ordered by percentage of combined correct responses.

		Year 3	
Item	Oral	Vis.	All
6 + 8	91	79	85
20 + 70	73	95	84
Double 50	89	72	80
Double 8	79	71	74
16 + 9	78	62	70
36 + 9	67	72	69
36 + 20	61	70	65
Double 15	59	60	60
Half of 30	55	60	58
Half of 16	65	49	57
36 - 10	52	58	55
14 - 6	59	51	55
90 - 70	58	43	50
68 + 32	37	40	37
60 + 80	30	44	36
Double 26	34	35	34
73 - 23	30	32	31
25 + 27	22	38	30
25 + 99	29	30	30
100 - 25	27	32	29
36 - 9	30	26	28
74 - 30	25	17	21
140 - 60	16	26	20
79 + 26	17	16	17
58 + 34	17	16	17
100 - 68	11	10	10
80 - 24	4	12	8
105 - 97	10	6	8
105 - 26	6	4	5
182 + 97	4	2	3

Item	Oral	Year 5 Vis.	All
60 + 80	85	88	87
Double 26	80	79	80
68 + 32	68	89	79
58 + 34	66	88	77
140 - 60	77	67	72
Half of 52	74	67	71
79 + 26	59	73	66
74 - 30	64	45	55
80 - 24	55	53	54
100 - 68	54	53	53
300 ÷ 5	56	48	52
165 + 99	46	54	50
100 x 35	44	52	48
105 - 97	52	40	46
105 - 26	46	38	42
150 ÷ 25	42	39	40
$^3/_4 - {^1/_2}$	59	17	38
7 x 25	34	40	37
$^1/_2 + {^1/_4}$	55	19	37
6.2 + 4.9	35	38	37
182 + 97	21	51	36
$^1/_2 + {^3/_4}$	36	29	33
$1 - {^1/_3}$	27	38	33
60 x 70	19	41	30
3500 ÷ 35	22	36	29
4200 ÷ 60	20	20	20
300 x 40	21	17	19
450 ÷ 15	17	15	16
0.5 + 0.75	12	13	13
38 x 50	6	9	7

Item	Oral	Year 7 Vis.	All
Double 26	94	94	94
$4^1/_2$ - 3	91	91	91
50% of 48	84	94	89
100% of 48	89	83	86
58 + 34	80	85	83
$2^1/_2 + 3^1/_2$	83	83	83
79 + 26	81	81	81
80 - 24	84	78	81
25% of 48	78	84	81
4.5 - 3	83	80	81
100 - 68	77	84	80
150 ÷ 25	80	77	79
6 - 4.5	75	78	77
$6 - 4^1/_2$	73	74	74
60 x 70	73	73	73
7 x 25	72	74	72
$^1/_2 + ^1/_4$	72	73	72
$^1/_2 + ^3/_4$	75	68	72
165 + 99	64	79	71
$^3/_4 - ^1/_2$	74	69	71
6.2 + 4.9	59	80	70
300 x 40	64	74	69
105 - 26	68	69	68
$1 - ^1/_3$	73	62	67
3500 ÷ 35	56	74	65
182 + 97	58	70	64
$2^1/_2 + 3^3/_4$	58	64	61
450 ÷ 15	58	58	58
0.5 + 0.75	59	57	58
4200 ÷ 60	59	52	55
$^1/_{10}$ of 45	60	49	55
10% of 45	58	47	52
12000 ÷ 40	46	53	49
0.1 x 45	49	44	47
$4 \times 3^1/_2$	41	46	43
264 - 99	37	48	42
7 x 49	37	48	42
0.5 x 48	35	38	36
440 ÷ 8	28	41	34
38 x 50	26	36	31

Item	Year 9 Oral	Vis.	All
$2\frac{1}{2} + 3\frac{1}{2}$	100	91	96
50% of 48	95	97	96
25% of 48	90	100	95
100% of 48	94	97	95
4.5 - 3	89	98	93
$^2/_3$ of 90	94	91	93
6 - 4.5	94	90	92
79 + 26	86	93	89
7 x 25	87	91	89
6.2 + 4.9	84	91	88
$^1/_2 + ^1/_4$	91	80	86
150 ÷ 25	86	85	86
$1 - ^1/_3$	77	93	86
$6 - 4\frac{1}{2}$	91	80	86
10% of 45	89	82	85
105 - 26	82	86	84
165 + 99	80	89	84
3500 ÷ 35	78	86	82
4200 ÷ 60	74	88	82
$^1/_{10}$ of 45	84	79	82
450 ÷ 15	74	87	80
60 x 70	77	80	79
$4 \times 3\frac{1}{2}$	80	74	78
7 x 49	82	74	78
1.5 x 20	73	83	78
$6\frac{1}{2} \div 2$	79	78	78
75% of 48	77	77	77
$^1/_2 + ^3/_4$	70	80	76
264 - 99	76	74	75
12000 ÷ 40	66	81	73
$2\frac{1}{2} + 3\frac{3}{4}$	69	76	72
0.1 x 45	67	65	66
$3 - ^5/_6$	66	67	66
38 x 50	46	66	57
3.5 ÷ 0.5	63	50	56
440 ÷ 8	47	55	51
$^1/_2 \times 6\frac{1}{2}$	50	44	47
$5\frac{1}{4} - 2\frac{3}{4}$	34	52	44
90 ÷ 0.5	38	31	35
$90 \div ^1/_2$	34	23	29

APPENDIX F. Preference and Performance by Quintiles on Items Common to the PS and MCT Tests.

Percentages are given for each category in the tables below. Students in the 'Yes Yes' category predicted that they could mentally calculate that item and were successful, and so on for the other three categories.

Year 3	Yes Yes	Yes No	No Yes	No No	Yes Yes	Yes No	No Yes	No No	Yes Yes	Yes No	No Yes	No No
		6 + 8				36 + 9				58 + 34		
Top Quintile	94	3	3	0	45	39	6	9	30	42	15	12
2nd Quintile	84	9	3	3	38	38	19	6	16	28	13	44
3rd Quintile	73	15	12	0	15	45	9	30	3	36	6	55
4th Quintile	78	9	13	0	3	56	3	38	0	31	0	69
5th Quintile	52	24	15	9	3	33	0	64	0	21	0	79
All Year 3	76	12	9	2	21	42	7	29	10	32	7	52
		60 + 80				74 - 30				100 - 68		
Top Quintile	76	18	3	3	42	24	24	9	21	52	12	15
2nd Quintile	31	44	13	13	3	47	19	31	6	50	3	41
3rd Quintile	27	55	6	12	9	36	3	52	0	42	6	52
4th Quintile	13	44	13	31	0	34	3	63	0	28	3	69
5th Quintile	0	39	0	61	0	42	0	58	0	27	0	73
All Year 3	29	40	7	24	11	37	10	42	5	40	5	50
		80 - 24				73 - 23				Double 26		
Top Quintile	21	52	9	18	45	30	18	6	88	9	3	0
2nd Quintile	0	53	9	38	31	28	16	25	34	34	9	22
3rd Quintile	0	58	0	42	21	45	9	24	9	58	9	24
4th Quintile	0	50	0	50	6	44	3	47	9	28	6	56
5th Quintile	0	42	0	58	0	36	6	58	0	36	0	64
All Year 3	4	51	4	41	21	37	10	32	28	33	6	33

Year 5	Yes Yes	Yes No	No Yes	No No	Yes Yes	Yes No	No Yes	No No	Yes Yes	Yes No	No Yes	No No
	Double 26				58 + 34				60 + 80			
Top Quintile	97	3	0	0	97	0	3	0	100	0	0	0
2nd Quintile	84	13	3	0	72	9	13	6	94	6	0	0
3rd Quintile	76	9	15	0	58	18	15	9	97	3	0	0
4th Quintile	75	25	0	0	50	22	22	6	69	19	13	0
5th Quintile	33	27	15	24	33	30	21	15	52	30	9	9
All Year 5	73	15	7	5	62	16	15	7	82	12	4	2
	74 - 30				80 - 24				60 x 70			
Top Quintile	85	12	3	0	91	3	6	0	61	27	3	9
2nd Quintile	41	34	16	9	59	19	9	13	34	31	16	19
3rd Quintile	48	30	9	12	48	30	15	6	3	45	15	36
4th Quintile	28	34	6	31	16	56	6	22	13	47	6	34
5th Quintile	18	36	18	27	15	48	3	33	0	39	0	61
All Year 5	44	30	10	16	46	31	8	15	22	38	8	32
	100 x 35				$1 - {}^{1}/_{3}$				165 + 99			
Top Quintile	79	6	12	3	70	18	6	6	67	12	21	0
2nd Quintile	50	13	22	16	38	28	3	31	41	9	25	25
3rd Quintile	30	24	12	33	18	42	0	39	42	24	9	24
4th Quintile	13	25	22	41	6	31	6	56	19	56	13	13
5th Quintile	0	45	3	52	12	42	3	42	6	24	9	61
All Year 5	34	23	14	29	29	32	4	35	35	25	15	24
	7 x 25											
Top Quintile	70	6	18	6								
2nd Quintile	41	25	16	19								
3rd Quintile	18	24	9	48								
4th Quintile	6	50	0	44								
5th Quintile	0	45	6	48								
All Year 5	27	30	10	33								

Year 7	Yes Yes	Yes No	No Yes	No No	Yes Yes	Yes No	No Yes	No No	Yes Yes	Yes No	No Yes	No No
	58 + 34				264 - 99				60 x 70			
Top Quintile	97	3	0	0	55	15	18	12	91	6	3	0
2nd Quintile	97	3	0	0	34	16	28	22	97	0	0	3
3rd Quintile	88	12	0	0	15	24	24	36	61	24	12	3
4th Quintile	78	13	9	0	13	19	16	53	44	38	16	3
5th Quintile	36	52	9	3	0	24	9	67	33	36	6	24
All Year 7	79	16	4	1	23	20	19	38	65	21	7	7
	7 x 25				165 + 99				1/10 of 45			
Top Quintile	79	3	18	0	79	9	12	0	70	6	24	0
2nd Quintile	78	3	19	0	69	9	19	3	44	13	31	13
3rd Quintile	70	9	18	3	61	9	18	12	24	15	33	27
4th Quintile	34	25	22	19	41	19	34	6	16	19	13	53
5th Quintile	18	39	6	36	15	39	9	36	3	27	15	55
All Year 7	56	16	17	12	53	17	19	12	31	16	23	29
	1 - 1/3				1/2 + 3/4				6 - 4.5			
Top Quintile	88	0	12	0	85	0	15	0	94	0	6	0
2nd Quintile	81	0	13	6	78	6	16	0	94	0	6	0
3rd Quintile	55	6	18	21	70	3	15	12	73	6	15	6
4th Quintile	34	16	16	34	44	22	9	25	38	22	25	16
5th Quintile	18	45	0	36	18	48	9	24	12	24	21	42
All Year 7	55	13	12	20	59	16	13	12	62	10	15	13
	0.1 x 45											
Top Quintile	61	9	27	3								
2nd Quintile	34	16	28	22								
3rd Quintile	21	27	21	30								
4th Quintile	9	25	16	50								
5th Quintile	12	24	3	61								
All Year 7	28	20	19	33								

Year 9	Yes Yes	Yes No	No Yes	No No	Yes Yes	Yes No	No Yes	No No	Yes Yes	Yes No	No Yes	No No
	165 + 99				7 x 25				264 - 99			
Top Quintile	93	3	0	3	87	0	13	0	83	7	7	3
2nd Quintile	84	10	6	0	71	0	26	3	71	10	19	0
3rd Quintile	80	13	7	0	50	3	40	7	47	27	23	3
4th Quintile	61	26	10	3	52	10	29	10	55	16	19	10
5th Quintile	67	10	13	10	40	3	37	20	47	17	3	33
All Year 9	77	12	7	3	60	3	29	8	60	15	14	10
	$^{1}/_{2} + ^{3}/_{4}$				6 - 4.5				25% of 48			
Top Quintile	87	3	10	0	100	0	0	0	80	0	20	0
2nd Quintile	77	10	13	0	90	3	6	0	61	0	35	3
3rd Quintile	67	10	17	7	87	3	10	0	50	7	43	0
4th Quintile	52	19	19	10	81	6	13	0	39	3	58	0
5th Quintile	27	37	10	27	57	17	17	10	37	0	50	13
All Year 9	62	16	14	9	83	6	9	2	53	2	41	3
	$^{1}/_{10}$ of 45				$90 \div ^{1}/_{2}$				0.1 x 45			
Top Quintile	97	0	3	0	83	13	3	0	70	3	23	3
2nd Quintile	68	0	32	0	32	61	3	3	52	3	32	13
3rd Quintile	60	3	33	3	7	77	3	13	27	17	37	20
4th Quintile	32	6	48	13	3	90	0	6	16	23	35	26
5th Quintile	23	10	27	40	3	67	7	23	7	10	30	53
All Year 9	56	4	29	11	26	62	3	9	34	11	32	23

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