DOCUMENT RESUME

ED 396 933 SE 058 598

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TITLE Teaching Mathematical Problem Solving to Students with Limited English Proficiency.
PUB DATE 8 Apr 96
NOTE 19p.; Paper presented at the Annual Meeting of the American Educational Research Association (New York, NY, April 8, 1996).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Bilingual Students; *Classroom Communication; *Elementary School Students; Grade 6; Intermediate Grades; *Limited English Speaking; Mathematics Instruction; Mathematics Teachers; *Problem Solving

ABSTRACT

Many mainstreamed students with limited English proficiency continue to face the difficulty of learning English as a second language (ESL) while studying mathematics and other content areas framed in the language of native speakers. The difficulty these students often encounter in mathematics classes and their poor performance on subsequent assessments of their learning of mathematics, therefore, is often unrelated to their potential for learning and understanding mathematics concepts and procedures. This study describes a program that attempts to utilize a blend of techniques from the fields of ESL or bilingual education and those of current practices in mathematics education focusing on communication in order to develop better mathematical problem solving approaches among upper elementary grade language minority students. An ethnographic study of one mathematics teacher and his approximately 30 sixth-grade students found that students with greater English competency did not demonstrate significant differences in their scores for problem solving in English and Spanish although there was a strong trend f r scores to be higher in Spanish both at pretest and posttest times. (MKR)

Teaching Mathematical Problem Solving to Students with Limited English Proficiency

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Paper presented at the American Educational Research Association, April 8, 1996, New York, NY.

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Among its position statements, the National Council of Teachers of Mathematics (NCTM, 1995) has indicated its commitment to equity in mathematics education for all students including language minority students. Specifically the statement indicates that "cultural background and language must not be a barrier to full participation in mathematics programs" (p. 20). But what exactly should that statement mean in practical terms? Clearly it is not enough simply to change the textbooks and provide grade level instruction for all students.

Research on achievement of language minority students tells us that although it takes only two years to achieve conversational competence in a second language, it takes up to seven years to attain sufficient second language competence to achieve in academic areas at the level of native speakers (Cummins, 1986). Thus, many mainstreamed students with limited English proficiency (LEP) continue to face the difficulty of learning English as a second language (ESL) while studying mathematics and other content areas framed in the language of native speakers. *The difficulty these students often encounter in mathematics classes and their poor performance on subsequent assessments of their learning of mathematics, therefore, is often unrelated to their potential for learning and understanding mathematics concepts and procedures. Rather many language minority students are forced into a pattern of failure because they do not yet understand the language in which mathematics problems and concepts are embedded.* THIS IS THE BASIC PREMISE OF

OUR STUDY.

Logic dictates that if we are to take the NCTM's recommendation for mathematics for everyone seriously (NCTM, 1989), we need to have some very specific adaptations in instructional methods for language minority students in American school systems. In general, these adaptations should be a blend of techniques from the fields of English as a second language or bilingual education and those of current practices in mathematics education focusing on communication. The study that we are presenting today describes a program that attempts to utilize this blend in the service of developing better mathematical problem solving approaches among upper elementary grade language minority students.

The method that we developed focused on problem solving because of the importance of linguistic competence in this area and because it is one of the essential standards and goals addressed by the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). By problem solving we refer to the students' efforts to make sense of the information (given or inferred) in word problems or stories that involve mathematical relationships. Our study investigated the effectiveness of the instructional method we developed for teaching problem solving to students with limited English proficiency. Students' solution activities and their explanations for such problems were the variables of interest in the study. The method itself developed as a collaborative effort between Rodrigo Patino, a bilingual teacher in the Paterson New Jersey public school district, and myself, a teacher educator at William Paterson College in New Jersey.

The Instructional Method

The research methodology we used began with an ethnographic examination of the techniques Rodrigo regularly utilized in teaching mathematics to his Spanish/English bilingual sixth grade class of approximately 30 students. This class contained students who had been in this country anywhere from a few weeks to just under three years, from a variety of Spanish speaking countries of origin, and with a wide range of English competence from none to near mainstream levels. The more advanced students were considered Level II English users as rated on the Maculaitis inventory of language skills. However, because many of the students were not in this country long enough to take the test, they were considered Level I.

We began this phase of the investigation by selecting several problems from the district's sixth grade mathematics textbook that was normally used in this class. Rodrigo then taught these problems to the students in his natural way utilizing bilingual/ESL techniques while I videotaped these sessions. Each problem took a minimum of one full class period to complete.

Following the videotaping, I reviewed the observed sequence with Rodrigo and abstracted a generic sequence of essential activities and processes in which he engaged across problems. This sequence became the starting point for the instructional model that eventually formed the basis of our empirical validation.

The method underwent weekly formative evaluation and evolved as we went along. Several processes remained central to the method throughout, but several other components emerged as we refined Rodrigo's techniques. Five essential components, each with some degree of elaboration, finally took shape. These included:

SHOW OVERHEAD 1 HERE - OVERHEAD OF KEY COMPONENTS OF TEACHING STRATEGY

1. Providing a **linguistic warm-up to the problem** - This component was included from the outset of the study and has its roots in bilingual educational techniques. It requires that the vocabulary and the situational context of the problem be discussed before any mathematics is addressed. The warm-up provides a cognitive-linguistic set for students and, most importantly, enables them to attach some personal meaning to the problem.

2. **Breakdown of the problem into natural grammatical phrases** - This component was one of the last adaptations in the method and should occur before students attempt to solve the problem. It is a key element in our technique and goes beyond simple vocabulary building. Instead it focuses on the breaking down of longer sentences into natural grammatical phrases - a process in which the students took on increasing responsibility as the instruction progressed. The idea here was to teach the students a technique for word problem analysis - a way of understanding the meaning of the context and mathematical relationships expressed in the problem - and then to encourage them to use that process independently.

During the natural grammar phrase analysis process, the meaning of the problem is derived from the utilization of a variety of non-verbal techniques adapted from ESL/bilingual education. These enable students to derive the linguistic and mathematical meanings embedded in the problems by using **graphic representations** (diagrams, charts), gestures and **physical enactments** and or role playing of the relationships in the problem, and by **rephrasing the problem components in their own words**.

SHOW A SAMPLE OF THE USE OF THE NATURAL GRAMMATICAL PHRASES HERE - USE OVERHEAD 2 - MOTORBOAT PROBLEM

3. **Students work out the problem in pairs**. During this process students are expected to provide both a solution and an explanation for why they did what they did. During many problem periods, students were provided with Texas Instruments Explorer calculators to be used as needed.

4. **Students present their own solutions** to the group. Problem solutions and explanations are put on overhead transparencies to be shared with the group upon completion of the work. Two or three student pairs share their work each day.

5. **Students create problems with similar structures** which are subsequently shared with the rest of the class and solved. These too may be conveniently placed on overhead transparencies.

The following videotaped vignette of one of the problem solving sessions used during the student illustrates the technique.

SHOW VIDEOTAPE OF BOAT PROBLEM HERE

The Data Collection Procedure

In order to assess the effectiveness of this technique, once developed, we utilized a pretest-posttest method of validation. To do this we selected a group of 20 mathematically simple (or so we thought at first pass) problems from a supplemental problem solving book and from among these problems selected 8 of them to serve as pretest/posttest assessment items. The other 12 problems were utilized for instructional purposes. What we wanted to do was very pointedly to focus the students' attention on how to analyze long word problems into their meaningful units of expression so that the important facts and questions of the problem could be understood without direct instruction from the teacher. Therefore, we deliberately selected problems that would not present an arithmetic challenge, but would present a communication challenge.

Prior to conducting the formal instruction with the revised model, we had students take a pretest with 8 problems, first in English and then in Spanish, to serve as a baseline for assessing their growth as a function of the instructional period. On the pretests, students were instructed to obtain solutions and to offer explanations about how and why they got their answers. At these times as during some of the instructional periods, students were provided were provided with calculators. Following an 8-week instructional program, the same problems were again administered (with numbers changed), first in English and then in Spanish. On the assessments, two problems were given per day until all items were completed. Each administration took approximately 2 weeks to complete. After students moved or were transferred to other classes, 24 students remained who participated in the complete program of instruction and assessment, 14 of whom were at Level II in English and 10 of whom were at Level I.

Following the administration of the posttest items, performance on pretest and posttests was compared. The assessments were scored for accuracy and for quality of explanations, each on a four point scale.

SHOW OVERHEAD OF SCORING CRITERIA - OVERHEAD 3

SHOW EXAMPLES OF THREE KINDS OF RESPONSES OF STUDENTS - OVERHEAD 4, OVERHEAD 5, AND OVERHEAD 6

Utilizing t-tests for related samples, comparisons were made between scores within each language and across the languages. We did these analyses for the class as a whole, for the 14 members of the class who had the highest levels of English proficiency, and for the 10 students who had minimal or essentially no competence in English.

Today we will present only those results pertaining to the 14 students with the greater English proficiency. We found that:

SHOW OVERHEAD WITH TABLE OF RESULTS - OVERHEAD 7

1. Students with greater English competency did not demonstrate significant differences in their accuracy scores for problem solving in English and Spanish although there was a strong trend for scores to be higher in Spanish both at pretest and posttest times (Pretest $t(13) = 1.77$, $p = 0.097$; Posttest $t(13) = 1.73$, $p = 0.104$).

2. Students significantly increased their accuracy scores in both English and Spanish from pretest to posttest times, (Spanish ($t(13) = 4.66$, $p < .01$; English ($t(13) = 2.53$, $p < .05$). Interestingly, this finding applied to the Level 1 students only in English and not in Spanish (English pretest mean =

0.92, English posttest mean = 1.81, t(9) = 5.4[illegible]. $p < .01$; Spanish pretest mean = 1.60, Spanish posttest mean = 2.14, $t(9) = 2.03$, $p = 0.071$).

3. At posttest time, English accuracy scores were higher than Spanish pretest accuracy scores, though not quite significantly so ($t(13) = 1.73$, $p = 0.105$).

4. Students demonstrated vir[illegible]ally equivalent explanation scores in English and in Spanish at pretest and posttest times (Pretest mean for Spanish and English = 1.01; Posttest = 1.20)

5. Students did not significantly increase their explanation scores in either English or Spanish from pretest to posttest time, although there was a trend toward an increase in these scores in both languages (Spanish $t(13) = 1.16$, $p = 0.265$; English t(1[illegible]) = 1.[illegible], $p = 0.317$).

6. At posttest time, English explanation scores tended to be higher than Spanish pretest explanation scores, though not significantly so ($t(13) = 1.32$, $p = 0.209$).

Discussion

So what do we think these findings mean? We believe it provides a strong indicator of the effectiveness of utilizing ESL/bilingual teaching techniques for teaching students with limited English proficiency to enable them to become more successful mathematical problem solvers. On the one hand the techniques enabled these students to apply those arithmetic procedures with which they were familiar to contexts that went beyond rote computation. It strengthened their confidence in themselves as mathematical problems solvers and provided them with tools for independent work in mathematics.

On another level, the problem solving activity itself provided these students with another channel for raising their use of English in academic contexts - it acted as a language building tool.

We believe that the techniques that were applied here can be applied to other subject areas as well. We also believe that these techniques can provide the monolingual teacher with an improved method for communicating with language minority students and a context analysis strategy that can be incorporated into lessons at varying levels for students at different levels of linguistic competence. Further, the method might be adaptable and useful for working with native English speaking students who have difficulty with mathematics word problem analysis for reasons that do no directly involve English language usage. In particular, it may have value in mathematics education for native speakers who have limited literacy skills.

We now would like to expand the piloting of this technique and evaluate its effectiveness with monolingual teachers who have ESL students mainstreamed in their classes. We would also like to extend the range of mathematical problems used so that more mathematically challenging open-ended problems are included. Finally we would be interested in training a cadre of teachers in the Paterson School District in using this technique and assessing the extent to which it impacts on students' performance on a statewide competency test taken in eighth grade.

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OVERHEAD 1 - PROBLEM SOLVING PROCEDURE UTILIZING NATURAL GRAMMAR PHRASES

1. **Linguistic Warm-Up: Focus discussion on special vocabulary words and problem context**

2. **Natural Grammatical Phrases Breakdown**
   A)Teacher reads the whole problem through slowly and fluently, as it is projected on the overhead.
   B)Teacher has students repeat the fluent reading exercise (meaning not addressed at this point)
   C)Teacher models phrase-by-phrase reading of the problem for meaning - (use gestures, pictures, translation, personal experience, other English words; spiral the reading of phrases until the whole problem has been discussed in terms of meaning)
   D)Students repeat phrase-by-phrase reading of the full problem
   E)Students take responsibility for phrase-by-phrase analysis
   F)Ask students to rephrase problem in their own words
   G)Ask students to record the facts and the question of the problem

3. **Have students solve problems in pairs including explanations** (have some pairs write their solutions on overhead transparencies)

4. **Have students present problem solutions on overhead transparencies and discuss**

5. **Ask students to write problems that are similar to the one done and share their created problems and solutions**

OVERHEAD 2 - MOTORBOAT PROBLEM: AN EXAMPLE OF "NATURAL GRAMMAR PHRASES"

A motorboat travels 25 miles in the time a sailboat travels 10 miles. At that rate, how far will the motorboat travel when the sailboat has traveled 60 miles?

A motorboat travels 25 miles

in the time

a sailboat travels 10 miles.

At this rate,

how far will the motorboat travel

when the sailboat

has traveled 60 miles?

OVERHEAD 3 - SCORING CRITERIA FOR PRETEST AND POSTTEST

Computational Accuracy Score

0 Answer is missing or completely irrelevant to problem

1 Answer is completely incorrect

2 Answer is almost correct except for a small computational error

3 Answer is completely correct

Quality of Explanation **(Scored regardless of accuracy of answer and appropriateness of solution strategy)**

0 No explanation or diagram

1 Student provides only a sketchy explanation of the solution strategy

2 Student provides a detailed step-by-step account of what was done *without telling why* the strategy was used

3 Student provides an accurate, complete, and reasonably clear explanation describing step-by-step what was done *and why* the solution strategy was used (even if the answer has a computational error)

six cans of soda cost $2.49. If you buy 4 cans, how much will you pay?

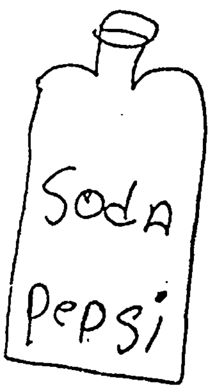
1

FACT: six cans of soda cost $2.49 If you buy 48 cans.

Question: how much will you pay

PLAN:

cadaSODA cuesta $2.49 igual a 14
14.94



48 SodA cuesta cada una $2.49
cuanto costaran todas.
119.52

answer. 119.52

OVERHEAD 5 - 2/3 RESPONSE

Six cans of soda cost $2.39. If you buy 42 cans, how much will you pay?

Facts: 6-can-pack ~~$2.39~~ — 42cans

Question: How much will you pay?

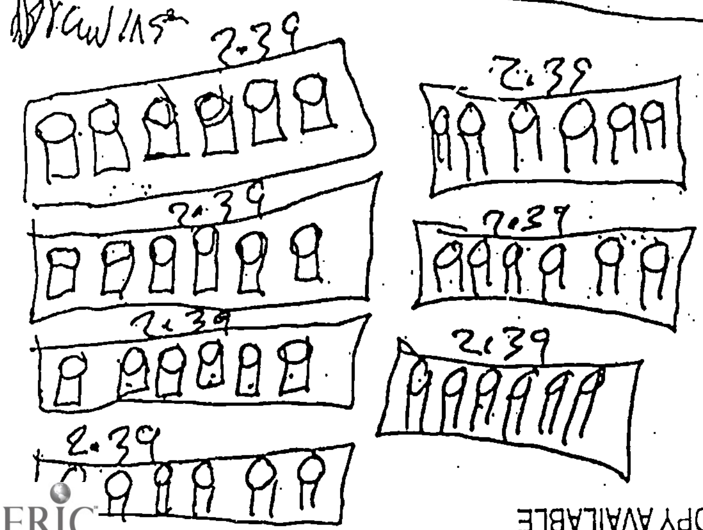
Plan:

subtract	adding	multiply	dividing
=	$2.39 2.39 2.39 2.39 2.39 + 2.39	— —	=

Solve: $2.39+2.39+2.39+2.39+2.39+2.39+2.39=$16.46

Answer: I will pay $16.46¢ because there are 7 pack of soda each pack cost $2.39 I add 2.39 7 times that why the answer is $16.46

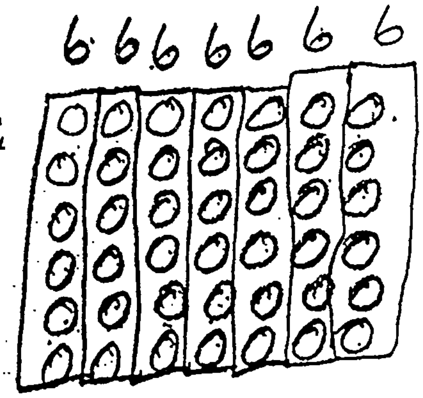
Drawings:



42 cans
7 pack

2.39
2.39
2.39
2.39
2.39
2.39
+ 2.29
$16.46

$16.46



OVERHEAD 6 - 3/3 RESPONSE

Six cans of soda cost $2.39. If you buy 42 cans, how much will you pay?

Facts: $2.39 each pack
- 6 can. pack
- 42 cans

Question: How much will you pay for 42 cans?

plan:

1 pack 2.39
1 pack 2.39
1 pack 2.39
1 pack 2.39
1 pack 2.39
1 pack 2.39
1 pack 2.39
16.73

$$\begin{array}{r} 2.39 \\ \times\ 7 \\ \hline 16.73 \end{array} \qquad \begin{array}{r} 7 \\ \times 6 \\ \hline 42 \end{array}$$

ANSWER:
Se pagan 16.73 por 42 cans

yo primero conté las sodas de 6 entre 42 y me dio completo no sobro entonces cada paquete valo 2.39 y lo sume

OVERHEAD 7 - RESULTS OF THE STUDY

Table 1.

Comparison of Mean Pretest and Posttest Scores for Accuracy and Explanations on Problem Solving Assessment of Level II Students

	Accuracy			Explanation		
	English	Spanish	**t-score**	English	Spanish	**t-score**
Pretest	1.76	1.97	**1.77**	1.01	1.01	-----
Posttest	2.28	2.55	**1.73**	1.20	1.20	-----
t-score	**2.53***	**4.66****		**0.317**	**0.265**	

* $p < .05$

** $p < .01$