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ABSTRACT

This report identifies 13 instructional strategies for teaching adult numeracy skills that address issues of assessment, development of mathematical skills, and development of problem-solving skills. The rationale and suggestions regarding the following 13 instructional principles are described: address and evaluate attitudes and beliefs about learning and using math; determine what students already know about a topic before starting instruction; develop understanding by providing opportunities to explore ideas with representations and hands-on activities; encourage development and practice of estimation skills; emphasize mental math as a legitimate alternative computational strategy and encourage development of mental math skills; view computation as a tool for problem solving; encourage use of multiple solution strategies; develop students' calculator skills and foster familiarity with computer technology; provide opportunities for group work; link numeracy and literacy instruction; situate problem-solving tasks within meaningful, realistic contexts; develop students' skills in interpreting numerical or graphical information in documents and text; and assess a broad range of skills, reasoning processes, and dispositions, using a range of methods. A final section discusses implications, namely that their implementation will necessitate a reevaluation and redefinition of teachers' roles within the classroom and will require both collegial and institutional support. (Contains 33 references.) (YLB)

NATIONAL CENTER ON ADULT LITERACY

INSTRUCTIONAL STRATEGIES FOR TEACHING ADULT NUMERACY SKILLS

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NCAL TECHNICAL REPORT TR96-02
May 1996

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The National Center on Adult Literacy (NCAL) was established in 1990 by the U.S. Department of Education, with co-funding from the Departments of Labor and Health and Human Services. The mission of NCAL addresses three primary challenges: (a) to enhance the knowledge base about adult literacy; (b) to improve the quality of research and development in the field; and (c) to ensure a strong, two-way relationship between research and practice. Through applied research and development and dissemination of the results to researchers, policymakers, and practitioners, NCAL seeks to improve the quality of adult literacy programs and services on a nationwide basis. NCAL serves as a major operating unit of the Literacy Research Center at the University of Pennsylvania.

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TABLE OF CONTENTS

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Abstract

Many adult educators want to improve the ways they teach mathematics, particularly in light of K–12 national mathematics education reform efforts, but they are unsure of what changes to make. This report identifies 13 instructional strategies that address issues of assessment, development of mathematical skills, and development of problem-solving skills. The strategies reflect research on how adults learn, the cognitive processes involved in learning mathematics, and the mathematical concepts that are important for adults to learn for educational and real life purposes. Implementation of these strategies within adult numeracy classrooms will necessitate a reevaluation and redefinition of teachers' roles within the classroom and will require both collegial and institutional support.

INTRODUCTION

During the past 20 years, the field of mathematics education has been enriched by extensive research into the nature of the cognitive processes involved as people learn mathematics (for example, see Siegler, 1991; Wearne & Hiebert, 1988) and into teaching techniques that are particularly effective in helping students learn mathematics (see Ball, 1993; Lampert, 1986). The National Council of Teachers of Mathematics (NCTM) has drawn implications from this research as well as from teachers' experiences and produced the *Curriculum and Evaluation Standards for School Mathematics* (1989), a vision of math education that is revitalizing and revolutionizing the teaching of math in the United States.

Although the focus of the *Standards* is on the K–12 curriculum, adult numeracy educators can certainly identify with and learn from NCTM's articulation of the broad goals of math education and the descriptions of instructional practices aimed at achieving those goals. However, the goals and instructional practices of adult numeracy education may not always be the same as those in the K–12 realm (Gal, 1993); the life situations, experiences, and motivations of adult students are very different from those of the children and teens in traditional schooling environments.

The motivation, prior educational and life experiences, and perceptions of the usefulness of new learning that adults bring to their studies become extremely important when considering adult education, since these may have an impact on adults' willingness to come to, invest in, or stay in educational programs. In particular, adults return to educational settings when they feel that they will benefit from the education, in that it will help them perform tasks or deal with real world problems that they are presently unequipped to handle. Furthermore, they come with a wealth of experiences that are closely tied to their self-identity. While these experiences may differ among students and contribute to heterogeneous classes, they are also a rich resource for enhancing instructional episodes (Knowles, 1990). On the other hand, years of experiences may also have created deep-seated negative beliefs and attitudes (towards learning in general and towards mathematics learning in particular) and ingrained strategies and ideas that may be counter productive or actually wrong (Ginsburg, Gal, & Schuh, 1995).

Theories of adults' learning processes tend to be general in nature and do not address learning in specific content domains. This report suggests specific instructional practices and strategies that are aimed at improving numeracy education for adults. The suggestions reflect what we have learned about how people learn math, what is important math learning (important both in the sense of building a foundation upon which further math learning can be constructed and in the sense of real world utility to individuals), and what we know about how adults learn. These instructional practices and strategies aim to help students develop those particular numeracy practices and skills that are important and useful to adults as they function in the world. They also keep in mind how and when adults actually *do* mathematics away from school and develop conceptual understandings that will support further mathematics learning.

Since the community of adults learning math is diverse and there is a multiplicity of educational contexts in which adults study, the instructional principles are presented in broad terms. They are designed to provide an instructional framework within which any curricular content can be addressed. Practitioners are expected and encouraged to select, modify, and apply the ideas to their own local situations.

INSTRUCTIONAL PRINCIPLES AND STRATEGIES

The 13 instructional principles described below address issues of assessment, development of mathematical skills, and development of problem-solving skills for dealing with real life contexts in which there is a math component. These principles assume acceptance of a broad conception of what numeracy education encompasses and suggest ways to bring about meaningful and useful numeracy learning.

1. Address and evaluate attitudes and beliefs regarding both learning math and using math.

RATIONALE

Many students come to the classroom with fears about their own abilities in the area of math ("I can't do math," "I can't remember how to do math even though I've learned it so many times," etc.). In addition, students often carry with them nonproductive beliefs about what it means to "know math" or what learning math should look like, such as "there is always only one correct answer," "there is one right way to solve problems," or "you should always work alone on math problems," and so forth. (McLeod, 1992). These negative attitudes and beliefs often hold students back from engaging in math tasks in meaningful ways and from trusting their own mathematical intuitions. Limiting self-images and beliefs are particularly harmful because adult students may inadvertently communicate them to their own children.

For many, negative attitudes towards mathematics initially develop in response to early school experiences of failing to understand. A student's initial confusion may be followed by a failure to receive further explanations or assistance from the teacher, leading to a loss of confidence and panic over the loss of control of his or her own learning. As these experiences are repeated, the student feels frustrated and assumes it is futile to expect understanding. Finally, the student "switches off" to distance him/herself from the situation (Allan & Lord, 1991; Tobias, 1994).

SUGGESTIONS

It is important to discuss openly with students how traditional K–12 methods of teaching math and stated or implied messages from parents and teachers may have caused them to develop negative beliefs and attitudes. If this is not done, some students may continue to blame themselves and may not approach classroom-based and real world mathematical tasks in a productive way. Have students freely talk or write in a trusting environment about their attitudes and beliefs. You can share your own fears and experiences. Point out and encourage students to look for manifestations of their existing (even if informal) mathematical understandings, of which they may be unaware, to encourage the development of feelings of comfort and control. This process of exploration and reflection should occur throughout instruction, not only at the outset, since different negative attitudes and beliefs may be tied to different areas of the curriculum.

2. Determine what students already know about a topic before instruction.

RATIONALE

Adult students have a rich background of real life experiences. Even if they have learned little formal math, they are likely to have engaged in counting, sorting, measuring, playing games of chance, and, most importantly, handling money (Bishop, 1991). Through such experiences, adults most likely have developed various skills and their own (partially) formulated conceptual understandings, some correct and some incorrect. For example, Ginsburg et al. (1995) investigated adult literacy students' understandings of five basic concepts having to do with percent (such as 100% means "whole" or "all," percentages of the parts of a "whole" add up to 100%, percents lie on an ordinal scale, etc.). Almost all students had some reasonable ideas about some of the concepts but these were accompanied by gaps in understanding of others of the concepts, with different students displaying different gaps.

Identifying students' partial understandings and intuitions is important because new learning will be filtered through and become integrated with prior knowledge. Each student's informal knowledge should be identified so that new instruction can be designed to link with what already has meaning to the student. At the same time, attention must also be paid to incorrect ideas or "patchy" knowledge so that these do not distort new learning or cause confusion.

SUGGESTIONS

An informal discussion of students' real world and school experiences should be used to explore "what do we know already?" when starting a new area of instruction. This gives students opportunities to think about and discuss mathematical issues and begin linking bits of disconnected prior knowledge into a structure without the competing demands of computation. A computation pretest (either a standardized test, such as the TABE or ABLE, or one constructed by a teacher) should not be used as a primary source of information. Besides producing anxiety for the student, it reinforces a sense that mathematical tasks involve only computation. In addition, such tests rarely provide teachers with much information on a student's underlying thinking processes and informal knowledge.

3. Develop understanding by providing opportunities to explore mathematical ideas with concrete or visual representations and hands-on activities.

RATIONALE

Being able to create a physical model enables students to visualize the concrete reality underlying abstract symbols and processes. Students will find the math they are learning more meaningful if they can link the ideas, procedures, and concepts to realistic situations, concrete representations, or visual displays; these can help students "see" and "feel" how and why computational algorithms work. In addition, the use of concrete materials or visual displays may provide students with a means for monitoring their own computations and procedures, helping them to pinpoint gaps in understanding when they might have difficulty identifying and describing these gaps using more abstract language. The process of working with hands-on activities may help students develop backup strategies that can be used when they become confused with the mechanisms of newly learned strategies or when they want to be certain that computations are indeed correct. However, Ball (1993) asserted that while the particular concrete objects or representations the teacher selects should highlight important mathematical content, the teacher should also be aware that there may be some limitations within the model. For example, "Using bundling sticks to explore multidigit addition and subtraction directs attention to the centrality of *grouping* in place value, but may hide the importance of the positional nature of our decimal number system" (p. 163).

SUGGESTIONS

During instruction, students should move between "real objects" (e.g., apples, coins, cups), "representations" that can stand for real objects (e.g., drawings, diagrams, charts), and numbers. At the beginning of a new unit, have students solve a number of related problems using real objects or a representation. Encourage students to talk about these examples, make observations, and explain how processes work before introducing formal notations or mathematical rules. After students are comfortable with new concepts in a concrete context, a discussion generalizing and formulating more abstract principles is appropriate.

Many commercial publishers sell "manipulatives" such as pattern blocks, base ten blocks, fraction circles, Cuisenaire rods, geoboards, and so forth, for use as representative objects in teaching specific topics. Transparent versions of these objects are also available for use with an overhead projector, and can greatly help in demonstrating work with manipulatives to groups of students. These materials are handy to use because the sizes, shapes, or color schemes are consistent and "user friendly." In addition, manipulatives can also be inexpensive everyday objects such as toothpicks, beans, cards, nails, buttons, cut-to-size pieces of stiff paper, and so forth.

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4. Encourage the development and practice of estimation skills.

RATIONALE

Many everyday or work tasks do not require precise, computed answers, but rather quick approximations of numbers, distances, timeframes, and so forth, based on some known information. The demands of the situation will determine how precise or imprecise an estimate has to be and whether it can be made mentally or requires written work or a calculator. When shopping, it is often more reasonable to approximate a total cost mentally instead of using a calculator or doing a written computation to determine if one is carrying enough cash to pay for groceries or confirm that the total at the cash register is correct. Computing a 15% tip at a restaurant does not have to be precise yet can be an overwhelming task to someone who can only conceive of calculating percentages by using a formula. Generating quick, approximate answers to math questions on standardized tests is often sufficient to discount all but one or two of the response choices. Good estimation skills can also be used to catch gross computational errors from misplaced decimal points or from errors while using a calculator (Schoen & Zweng, 1986).

Sowder and Wheeler (1989) found that there is a developmental progression in accepting that there can be multiple good estimates and that computational estimation requires "rounding-then-computing" rather than "computing-then-rounding." Both of these initial responses seem to be grounded in students' beliefs that there should be only one "correct" response for a questions involving mathematics.

SUGGESTIONS

Have students identify everyday and work situations for which estimates may be more appropriate than exact answers, reinforcing the notion that estimating is a valuable skill, not merely something you do when you don't know how to compute an answer the "right" way. Discuss reasons for the need to estimate and the inevitable tradeoff between the benefits of estimating and the possible cost in error due to lack of precision. Stress that there are no "right" or "wrong" estimates, only ones that are closer or farther from a computed answer, and that the importance of the degree of exactness (which may determine the estimation strategy to use) depends on the requirements of the situation.

Encourage students to share with each other the estimation strategies they use (such as "multiplying by 10 instead of 9 and then subtracting a little"), and supplement the class repertoire with strategies that you use in your everyday life. Differentiate between rounding numbers as a strategy for estimating and rounding off an answer that has been computed. Encourage students to adopt a reflective attitude towards estimation, asking themselves questions such as (a) How reasonable (close) is the estimate? (b) Is this degree of accuracy appropriate for purposes of the situation? (c) Was the method or strategy appropriate? (d) Does the situation suggest making a high or low estimate (when making sure I have enough money for groceries, I may want my estimate to err on the high side)?

For classroom estimation practice, students can be asked to estimate amounts as they would in the everyday contexts for which an exact answer is not normally required (i.e., tips, number of miles per tank of gas, approximate cost of a basket of groceries, number of miles between New York and Chicago as represented on a map and time it will take to drive from one to the other) as well as to estimate answers to traditional arithmetic exercises before carrying out the computations. Using sample standardized test items with multiple choice responses for estimation practice gives students an opportunity to see that often a good estimate can replace tedious computation and point to a reasonable response.

5. Emphasize the use of "mental math" as a legitimate alternative computational strategy and encourage development of mental math skill by making connections between different mathematical procedures and concepts.

RATIONALE

Mental math involves conceiving of and doing mathematical tasks without pencil and paper (or a calculator) by using "in my head" mathematical procedures that may be quite different from school-based, written procedures. In everyday life, we often want to know answers, not estimates, to questions involving numbers but do not want to stop to write a computation, or we realize that the written procedure is cumbersome while a solution is easily attainable in another way. (Ask your students and colleagues *how* they multiply $3.99 times 4 in their heads, and *why* they do it differently than on paper.) Mental math is not simply carrying out the mental equivalent of paper and pencil algorithms but rather using strategies determined by the properties of particular numbers and relationships between quantities within problems (Hope, 1986).

There are two reasons for encouraging the development of mental math skill: its practical usefulness and its educational benefits. Often a quick response to an everyday or workplace number-laden situation is expected or required. While computation using standard algorithms or procedural rules is efficient using paper and pencil, applying those same algorithms to mental computation is often awkward at best and at worst creates confusion and a memory overload (particularly, for example, long division, multiplication of two digit numbers, subtracting with complex borrowing, etc.). Our short-term memory has limited capacity, and keeping numerous intermediate results of calculations in mind simultaneously is very difficult. Mental math strategies aim to reduce the amount of information necessary to keep in mind at once, thus reducing errors and improving accuracy.

Perhaps more important is the educational benefit from working with mental math since it requires developing a facility for moving between equivalent representations of quantities and on understanding the implicit connections between procedures. That facility requires knowledge of how and why procedures work and an expectation that there are meaningful connections between concepts. Traditionally, we have assumed that students will intuitively make connections between the different topics encompassed in math classes, but in fact students often think of them as self-contained units. This view of math as a series of discrete and unrelated topics is reinforced by individual "topic" workbooks (such as "fractions" or "percents"). Students who expect there to be connections between different mathematical concepts will be less fearful of learning math since they will expect new mathematical ideas to be extensions of what they already understand.

Adult students, in particular, may benefit from developing mental math skills as alternatives to the computational algorithms that are typically taught since they may have difficulty remembering multi-step procedures that are not really meaningful to them. Plunkett (1979) suggests that standard computational algorithms do not correspond to the ways people naturally think about numbers and therefore often require suspended understanding. In contrast, mental math procedures are flexible and can and should be adapted to suit the particular numbers involved; they demand the student be involved in selecting an appropriate procedure; they focus attention on entire numbers rather than only on selected digits; they are intertwined with understanding and cannot be used to demonstrate proficiency without the underlying understanding; and they often yield an early approximation of the answer because usually the leftmost digits are used first in the calculations.

SUGGESTIONS

Some students are fearful of or uncomfortable with doing mental math ("This is not the real math I learned in school"), and may not spontaneously generate mental math strategies. Therefore, mental math strategies should be identified, discussed, and then practiced over time in numerous situations so that students will trust their abilities enough to use them when appropriate. Since some numbers are less amenable to mental math than others (the exact answer to $3.99 times 4 can easily be determined using mental math, $3.62 times 4 is more awkward), students should become aware of and monitor their own decisions over strategy use. It is important that students are able to discuss and demonstrate their understanding of the differences and similarities between what they figure out in their heads and what they do with paper and pencil, and that they develop a repertoire of mental math strategies with which they feel comfortable.

Refer frequently to previously studied material to help students see the connections between different mathematical concepts, such as fractions, decimals, and percents, so that students become flexible switching from one "system" to another when performing "mental math" (for example, 25% is one fourth, so divide by four). Describe mental math strategies you use yourself and spend time practicing mental math skills first with small numbers and then with larger or more complex numbers. Discuss "why" you can get the same answer using different computational procedures or representations and elicit opinions about the relative advantages of one representation or mental or written procedure over another.

6. View computation as a tool for problem solving, not an end in itself.

RATIONALE

While the acquisition of computational skills is important, it is of little use unless students also develop the ability to determine *when* certain computations are appropriate and *why*. Time spent mastering computational skills should be balanced with time spent talking about the applications of computations and enabling students to grapple with the application of their skills in both familiar and less familiar situations. By situating learning of skills in contexts, the skills mastered are generalizable and useful (Strasser, Barr, Evans & Wolf, 1991).

One approach to problem solving in which the complementary role of computation becomes apparent is suggested by Brown and Walter in their book, *The Art of Problem Posing* (1990). They encourage approaching a problem-solving situation as an opportunity to explore the ramifications of "What ifs?" They suggest that it is important to explore the reasons a question was asked in the first place and then explore ways in which small changes in the problem (asking "What if ...?) might affect solutions. Through this approach to problem solving, computations are done to answer specific questions. As the problem space is expanded, computational procedures may be examined and investigated to see why and how an answer or conclusion was reached. By so extending the problem-solving experience, critical thinking skills and deeper understanding are developed. Clearly here, computation is but a tool for answering "important questions" just as word decoding is a tool in the process of finding meaning in written communication.

In addition, students need to develop a sense of *why* a particular computational procedure is appropriate in a particular situation. This requires an understanding of what is happening as a computational procedure is being used and of the differences and commonalities between procedures. The understanding described here is not equivalent to a recitation of the steps involved in performing a computational procedure but rather reflects an internalized vision of a model (which can be described in concrete or representational terms) supporting the procedure.

SUGGESTIONS

When teachers (and textbooks) focus on computational procedures in isolation from meaningful contexts, students assume they "know" the mathematical content and that this "knowledge" will be useful to them. However, it is a rare situation (perhaps only standardized tests) for which mere computational skill is useful. Rather, most real world situations require computation only within the context of problem solving. Take, for example, the case of fractions. Often students "learn" how to add, subtract, multiply, and divide fractions, completing numerous work sheets with context-free numbers separated by designated operations. Once the procedural skills are mastered, fractions have been "learned." Yet, when the student has to determine the quantity of eight-foot boards that must be ordered so that five bookcase shelves of two and three-fourths feet can be cut, they ask, "Do we add, subtract, multiply, or divide?" By focusing a significant part of instructional time on situational questions, students will have opportunities to analyze problem situations and appreciate the function of computational procedures as one component of the problem-solving process. A variety of computational procedures can be practiced when problems are extended. The above problem can be extended by asking "What if?" questions such as "What if we want to make the five shelves as long as possible using 2 eight-foot boards?" "What if the boards are 10 feet long?" or "What might a bookshelf look like if we want to use all the wood from 2 eight-foot boards for five shelves that do not have to be the same size and will be supported by cinder blocks?" This approach has an additional benefit of providing contexts for discussions of alternative solutions.

Another way to reinforce connections between computational skill and applications is by asking students to write their own problem stories (word problems) targeting a particular procedure (i.e., multiplying fractions). Student-generated problems can be shared with other class members, mixing problems suggesting different computational procedures so that students will have opportunities to select appropriate solution methods rather than think, "This week we have been learning to multiply fractions, so you must have to multiply to solve each of these fraction problems."

The opportunity to use manipulatives and other visual aids while solving problems will help students develop a sense of why one procedure works while another is inappropriate. Students need time to question and discuss the meaning of answers or methodologies rather than leave a problem as soon as a correct answer has been reached.

7. Encourage use of multiple solution strategies.

RATIONALE

Some students come to instruction believing that there is only one proper or best way to solve a math problem. When students feel that they do not have the specific knowledge or skill needed for a particular problem, they may feel anxious, frustrated, and quickly give up rather than persevering and looking for alternative paths to solutions. Students will become flexible problem solvers by developing webs of interconnected ideas and understandings; instruction should include explanations and demonstrations of many ways to arrive at a good solution to a problem, and continuously point to connections between mathematical representations, concepts, and procedures (Hiebert & Carpenter, 1992). In addition, different strategies or representations may be more meaningful to some students than others. By having alternatives available, more students will be able to connect new learning with their individual experiences and perspectives.

SUGGESTIONS

Rather than leaving a problem as soon as it has been solved and rapidly moving to the next one, ask if any student can think of another way to approach it. Frequently ask students *why* they did what they did and what they could have done as an alternative. Include mental math strategies when enumerating alternative solution paths. Continually cycle back to previous topics to show connections between new and old skills and concepts.

Develop computational algorithms logically so students see that the algorithms are simply shortcuts for time-consuming procedures (such as multiplication for repeated addition and division for repeated subtraction) or alternatives for other representations (as percents for fractions). While exposing them to new methods, allow students to continue to use "lower level" strategies (such as finger counting when adding or subtracting and multiple addition rather than multiplication) if they need to do so; most will eventually see that certain strategies are cumbersome and, as they feel more secure with new strategies, will probably begin using the "higher level" strategies. If they never get to that point, they will at least be able to use a strategy that is dependable and meaningful to them.

8. Develop students' calculator skills and foster familiarity with computer technology.

RATIONALE

These days, there is less need for skill in fast calculation given the availability of calculators. This is not to say that calculators should replace computation skills, but rather that the goals of adult numeracy education will be enhanced by encouraging judicious calculator usage. Students should have opportunities to become skilled at using what has become an accepted and essential workplace tool.

Some teachers fear that allowing students to use calculators will reduce their opportunities to perfect traditional computational skills. Although calculators may take the place of tedious computation and ensure accuracy, they do not replace deep understanding of mathematical concepts and procedures, and they cannot make decisions or solve problems. It is becoming ever more important to know *what* a procedure does, *why* it works, and *how* the results can be evaluated to make certain they are appropriate responses to the original task, demands that are essential whether students learn to compute with or without calculators. In fact, calculators can be used as an instructional tool to support the development of conceptual understanding; students can quickly observe the results of many calculations, see patterns, make generalizations about mathematical processes, and focus on understanding without getting bogged down in lengthy calculations.

A review of 79 research studies of students ranging from kindergarten through twelfth grade showed that, in almost all grades, the "use of calculators in concert with traditional mathematics instruction apparently improves the average student's basic skills with paper and pencil, both in working exercises and in problem solving" (Hembree & Dessart, 1986, p. 83). In addition, for students in all grades and ability levels, attitudes towards mathematics and self-concept in mathematics were better for those who used calculators than for those who did not.

Beyond issues related to calculators, adult educators must consider the broader issue of the use of computers. Since most jobs now require some familiarity and facility with technology, many students are eager to use computers. Numerous commercial programs are available that create simulations that target computational, problem solving, and workplace-oriented skills. Aside from having opportunities to encounter mathematical content in simulations that would be too complex to create without a computer, students can become computer literate and develop computer-related skills that are valued by the community and the workplace.

SUGGESTIONS

Provide opportunities to use calculators and set aside time to make certain that each student knows how to use his/her calculator, being wary that sometimes functions work differently on different calculators. The calculators can then be used for various types of activities including (a) checking mental or written calculations; (b) as a discovery tool when numerous manual computations would be time consuming or tedious (e.g., investigating what happens when two decimal fractions less than 1 are multiplied together or what 10% of any number is), (c) keeping records of experiments and then drawing conclusions from patterns; or (d) as a problem-solving tool to be used if and when a student feels it would be a helpful aid. By posing realistic, extended problems, students will have opportunities to determine for themselves when using a calculator (rather than estimating, using mental math strategies, or written computation) is the most appropriate thing to do. The comparative benefits of these options for different problem situations can also be discussed in class.

Explore the possibility of using computers with math education software to help students develop specific mathematical skills, but keep in mind that computer usage should be integrated with other classroom activities and needs to be accompanied by classroom discussions. Encourage students to use simple "integrated" software (with graphing, database, or spreadsheet capabilities) as an aid in planning, managing, and presenting results of group projects (timetables, attendance lists, graphs, conclusions, activity logs). Even simple word processing programs can be used to write math journals and explanations of solutions to extended problems, giving students opportunities to improve their keyboarding, writing, and word-processing skills. Overall, such practices should enrich student experiences with literacy-numeracy connections and help them integrate their skills. (We realize that many programs do not have much computer equipment and may not provide teachers with much training; yet, some goals must be established in this area, even if the beginning is very humble.)

9. Provide opportunities for group work.

RATIONALE

The SCANS Commission (1991) suggests that those joining the workforce must be competent in working with others on teams, teaching others, and negotiating. Often the contexts in which these skills are demanded include problem solving that concerns numerical information. Traditionally, math has been studied alone and communal work was relegated to other disciplines such as science or social studies. Yet in the real world, people regularly have to communicate about numerical issues (negotiating a contract, making business or purchasing decisions, defending an estimate, etc.). Furthermore, students often benefit from their peers' observations or explanations because one student may be able to identify another student's point of confusion or explain a concept with examples that are especially helpful for that particular student (Slavin, 1990). Indeed, a study involving 77 adult college students (mean age of 28 years) in remedial mathematics classes showed that those using cooperative learning performed as well or better on all measures than did a control group, with better performance on tasks involving higher cognitive skills (Dees, 1991).

SUGGESTIONS

It is difficult to expect students to develop realistic group work skills in the context of isolated, brief tasks of the kind espoused in most textbooks. There is a need to create an atmosphere in which students frequently have to work together and help or teach each other. Periodically develop longterm, realistic projects for which heterogeneous and extended group efforts are appropriate, such as organizing a group trip, arranging a party or a meal (including planning, deciding on and managing schedules, budget, supplies, materials, division of labor, etc.), or conducting a survey about a meaningful issue (including collecting, analyzing, and reporting on findings and implications).

Groupings may vary according to the particular tasks involved and may consist of pairs of students, small groups of three or four students, or even a large group with subgroups, each of which would address a particular aspect of a complex task and report findings to the larger group to inform a decision-making process. Ground rules for behavior within groups should be established as a class, with students involved in determining appropriate interactions.

10. Link numeracy and literacy instruction by providing opportunities for students to communicate about mathematical issues.

RATIONALE

Many workplace and real world situations require individuals not only to solve mathematical problems, but also to communicate their reasoning and the results or implications of their work to others (Carnevale, Gainer, & Meltzer, 1990). Adults also frequently find themselves discussing mathematical concepts with their school-aged children as they help with homework assignments or studying for tests. Communicating mathematically might include drawing a diagram (of a room to plan carpeting), writing a letter about an error on a utility bill, calling someone to report that a shipment arrived with less than the ordered amount, or negotiating terms of a sale, and so forth. Thus, reading, writing, and communicating are activities within which math is found and should be taught and practiced with mathematical content. "Talking about math," whether verbally, schematically, or in written form, enables students to clarify and structure their thinking so that a target audience will clearly understand their information or argument. A focus on communication issues in the context of learning mathematics contributes to and should be seen as an integral part of developing general literacy skills.

SUGGESTIONS

In the course of mathematical problem solving, encourage students to put into words for others what they are doing and why, using both written and oral formats. Journals enable students to reflect on and describe successes and points of confusion to themselves and their teacher. Extend activities to encompass a range of literacy experiences and the creation of literary products, including

- letters of complaint to companies clearly detailing billing problems;
- letters to the editor of a newspaper or magazine, or to the chair of a civic group explaining an opinion based in part on some numerical information;

24

- dialogue journals, editorials, or essays in which students respond to one another, arguing another side of an issue by interpreting the same numerical information in a different way;
- word problems or more extended math stories for others in the class to solve; or
- a detailed explanation of how and why some mathematical procedure is used, which could then be saved to create a "resource book" for the class or the individual student.

Written and verbal communication skills can be developed by presenting problems to the class that do not dictate a single solution process or lead to a single right-or-wrong answer (such as "How can we measure the area of an irregularly shaped lake?") and by giving students opportunities to present and discuss possible solutions. Alternatively, encourage students to conduct a survey; write a report detailing a description of what was done, results, and implications; and then make a verbal presentation with visuals to the class.

11. Situate problem-solving tasks within meaningful, realistic contexts in order to facilitate transfer of learning.

RATIONALE

Educators hope that the skills that students develop in the classroom will be used effectively and appropriately in out-of-school environments. Unfortunately, researchers have found that skills learned in one environment may not be easily transferred to or applied effectively within another environment and the farther the learning context is from the target context, the less likely it is that transfer will occur (Mikulecky, Albers, & Peers, 1994; Nunes, Schliemann, & Carraher, 1993; Perry, 1991). For example, Ginsburg et al. (1995) found that some adult students who were able to compute solutions to context-free percent exercises (such as "25% of 20=") by applying a standard school-based multiplication procedure, were unable to determine how much money they would save if an $80 coat were on sale at "25% off."

It is thus important that students practice using their new skills in environments that are very similar to the life and work environments in which they will have to function, rather than just in context-free environments such as workbooks with extensive isolated arithmetic practice exercises. In addition, interest in learning will be sustained if the students can see clearly that what they are learning will be directly applicable to situations in their own lives.

SUGGESTIONS

Elicit students' experiences of situations in which mathematical issues arise and use them to develop meaningful, realistic contexts for problem-solving tasks. For example, if students live in an area that has some availability of public transportation, a project could be developed around whether or not to buy a car. Considerations of cost, car payments including interest, upkeep, insurance variables, and frequency of usage can be compared with public transportation costs and limitations. Students, as well as teachers, can be involved in posing questions and asking "What if?" to explore consequences of alternative scenarios. In designing tasks, the mathematical content should be appropriate for what one would actually want to know in a particular context (for example, computing and comparing the price per ounce of the same cereal available in small and large quantity boxes is appropriate and useful; finding the average price of 6 items in a grocery cart is not meaningful or useful).

Simulations based on students' experiences can help students practice applications in contexts that are different (in planned ways) from textbook or "school-like" situations. Encourage students to reflect on what is different between school-like and real life problem solving (in terms of skills, beliefs, dispositions, degree of accuracy expected, tools used, etc.).

12. Develop students' skills in interpreting numerical or graphical information appearing within documents and text.

RATIONALE

Numerical information is often embedded in text-rich contexts that are of importance in adults' lives, such as statements of employee benefits, payment schedules, contracts, tax instructions, or maintenance agreements (Kirsch, Jungeblut, Jenkins, & Kolstad, 1993). People often read newspapers or magazines and have to interpret graphs or statistical information presented in tables or text. These tasks are unique in that most often there is little or no computation to do, but only a need to apply conceptual understanding of diverse mathematical topics and combine this understanding with text comprehension. If adults skip pertinent numerical information because they feel uncomfortable or incompetent about processing it, the text loses meaning and people lose access to critical information. The goals of both numeracy and literacy instruction will be most effectively met when we help students develop number sense, statistical literacy, and interpretive skills (Gal, 1993).

SUGGESTIONS

Have students graph information from their lives such as a circle graph showing how they spent the last 24 hours or a bar graph showing how many cigarettes they smoked or cups of coffee they drank each day for a week. Students will see the connections between events in their lives and the graphs they create; these experiences help students understand how other graphs are constructed and the information that can be gleaned from them.

Students could bring in and read newspaper articles or other text-rich materials containing numerical information that must be interpreted but not necessarily "computed" and report a summary of that information orally or in writing to other students. Alternatively, all students in one class could read the same article or document and discuss its implications. In these discussions, it is necessary to be certain that students understand the vocabulary and comprehend any technical terms.

Model and encourage the development of a critical eye for articles or advertisements that draw conclusions from summarized data (for instance, "Nine out of ten doctors ...."). During class discussions, push

26

students to challenge presented information and implications by asking probing questions such as:

- Where did the data on which this statement is based come from? How reliable or accurate are these data?
- Could the study have been biased in some way?
- Are the claims made here sensible and justified from the data? Is there some missing information?

Students may initially feel too intimidated to take a challenging attitude to what is printed or reported on television, but they will eventually begin to feel comfortable in the role if they have opportunities to practice asking these critical questions.

13. Assess a broad range of skills, reasoning processes, and dispositions, using a range of methods.

RATIONALE

Educators communicate their pedagogical priorities to students in part through the assessments they use. Many adult education programs use multiple choice tests to evaluate the mathematical skills of incoming students or to assess learning gains. By using *only* such tests, we communicate that what we value in numeracy education is mostly the ability to compute with decontextualized numbers or solve brief (and sometimes contrived) word problems. Yet, if we accept the curricular goals and instructional principles discussed above, we should significantly extend the scope and methods of assessments used in adult numeracy education.

Assessments should focus on worthwhile content that reflects the instructional goals of the students' program of studies. Problems and tasks used in assessments should yield information that can provide meaningful feedback to the student, as well as inform instructional decisions by the teacher. In addition to mastery of computations and formal procedures, assessments should diagnose the many additional skill and knowledge areas that are part of "being numerate," such as interpreting statistical and quantitative claims, acting upon numerical information in technical documents and forms, applying mathematical reasoning and solving realistic problems, communicating about mathematical issues and explaining one's reasoning, and so forth (see Cumming & Gal, 1996; Lesh & Lamon, 1992; MSEB, 1993; NCTM, 1995; and Webb, 1992 for more comprehensive discussions of assessment).

SUGGESTIONS

Mathematics assessments should extend well beyond examining students' ability to find the right answer for a computational exercise. They should also include open-ended, extended tasks that may have more than one reasonable solution and/or solution path, and require that students explain their reasoning and the significance of their solutions. These tasks could culminate in diverse products such as a combination of graphs, tables, drawings, written text or oral reports describing a solution process (of an individual or a group), a written recommendation for a course of action, simulated memos aiming to communicate about mathematical issues with specific real world audiences (and demonstrating both appropriate mathematical know-how and literacy skills), or performance on simulations of real world tasks relevant to a particular student population. This form of assessment recognizes that both process and product provide important information about students' understanding of and ability to apply mathematical concepts.

By expanding the notion of what constitutes "assessment," educators can integrate assessment and teaching, and use teaching activities to generate information that can satisfy both diagnostic and instructional needs. To the extent that teachers and students together find such information of value, representative samples of work can be recorded and stored for various uses. However, unless assessment information is put to good and timely use by teachers and students, time spent on assessment is wasted and learning is not enhanced.

IMPLICATIONS: THE CHALLENGE OF TEACHING ADULTS

The instructional principles described above imply that numeracy classes may not often resemble the traditional math class in which the teacher makes a presentation while the students watch or write a sequence of computational steps, followed by a "practice" period during which students practice the specific skill just demonstrated until mastery. Indeed, the activity of teaching or guiding students requires continuous interaction between the teacher and the student(s) as learning activities are selected, presented, modified, and extended. Instruction should focus *both* on "stand-alone" skills and on integrated problem-solving skills. By engaging in the problem-solving process and struggling with solutions, students gain the skills and dispositions needed to apply their numeracy skills appropriately in other problem contexts. They also develop or practice "component skills" in a meaningful context.

Activities within an adult math class should serve to develop mathematical understanding as well as computational skills, and should target generalized problem-solving, reasoning, and communication skills. All too often, students work on one skill at a time and are told what algorithms to apply to contrived or context-free problems (e.g., "do all the fraction problems on this page"). However, in real life contexts, quantitative or quantifiable elements may be interspersed with other information, and it is seldom specified what to do or what knowledge is relevant. People have to *comprehend* a situation, *decide* what to do, and *choose* the right tool(s) from their "mathematical tool chest" that will enable them to reach a *reasonable* solution.

For math education to be effective and meaningful for adults, we must broaden the contexts in which instruction is couched, widening the range of interactions among students and between students and mathematics content. This implies an expansion of the definition of essential math-related skills to include verbal and written communication about numbers, the interpretation of numerical information encountered in the media, and the knowledge to make decisions regarding the level of precision or the most effective mode of response needed for various tasks. Finally, students' attitudes and beliefs about studying and using math both in educational settings as well as in daily, family, and work-related activities should be addressed. These have an impact on motivation to develop mathematical thought processes, to adopt a "mathematical stance" when engaging in real world situations, and to continue studying mathematics.

The framework described here acknowledges the unique challenges of teaching adults. To create learning experiences that enhance instruction, teachers and students will need to identify the knowledge—both formal skills and strategies as well as informal intuitions and ways of thinking—that individual students bring to their studies. By designing instruction with this information in mind, teachers will be able to guide students to "construct" their own numeracy learning based on the premise that new knowledge is related to and builds upon previous knowledge. In this way, numeracy has a cohesiveness and power for the student rather than being a series of separate (and perhaps to the student, unrelated) ideas or skills. Ultimately, instruction should aim to be more obviously useful (keeping students involved and coming) and more cognitively meaningful (so that students will be more likely to leave the classroom with skills that will be retained and applied).

In addition, a significant goal of instruction concerns helping each student realize his or her own personal goals related to mathematics, which often include passing a high school equivalency examination or other "gate-keeping" test, improving job prospects or skills, or feeling able to support his or her children's math education. It is the ongoing responsibility of the instructor to also identify and communicate to the students goals that they may not articulate themselves or information about gaps in their knowledge about which they may be unaware.

As teachers begin the process of instructional change, they often find it difficult to know where to begin. The following are suggestions that teachers can use to initiate the process:

- Seek a partner with whom to exchange ideas and get a broader perspective.
- Involve students by telling them about new pedagogical ideas and asking for their feedback.
- Seek broader support by forming a local "numeracy group" or "inquiry circle." Groups can take different formats. Some may focus on a single "theoretical issue" (such as math anxiety) or one of the instructional principles (for example, linking literacy and numeracy instruction), identifying and reading relevant materials, and then discussing implications for practice. Other groups may focus on an instructional topic (such as teaching fractions), trying out ideas and activities and reflecting on lessons learned.

Such changes in educational emphasis require that teachers reevaluate and redefine their roles within both the classroom and the educational process. Teachers may feel challenged in their own beliefs about the nature of learning math, the nature of math education, how people learn math, how a math class should look, what math skills are important to learn, and what it means to know math. Ultimately, the teacher may no longer view him/herself, or be viewed by students, as the sole "expert" in the class who knows all the answers as well as the "best" ways to arrive at those answers, but rather as a facilitator who participates in the learning process with students by questioning, pushing, explaining, sharing, and also finding his or her own new insights.

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30

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