DOCUMENT RESUME

ED 395 830 SE 058 592

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TITLE	Mental Computational Skills of Students in a Reform Mathematics Curriculum.
SPONS AGENCY	National Science Foundation, Arlington, VA.
PUB DATE	Apr 96
CONTRACT	ESI-9252984
NOTE	12p.; Paper presented at the Annual Meeting of the American Educational Research Association (New York, NY, April 8-12, 1996).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Arithmetic; *Cognitive Ability; Demonstration Programs; *Elementary School Students; Grade 5; Intermediate Grades
IDENTIFIERS	*Mental Computation; *University of Chicago School Mathematics Project

ABSTRACT

This paper presents research conducted as part of the field test of the University of Chicago School Mathematics Project's (UCSMP) curriculum, Fifth Grade Everyday Mathematics. The UCSMP curriculum is one of the current reform programs funded by the National Science Foundation to implement the Standards of the National Council of Teachers of Mathematics. In this study a timed mental computation test was given to (n=78) fifth-grade students who had been in the UCSMP curriculum since kindergarten. The UCSMP students outperformed the baseline group on all but one of the 25 items. During interviews UCSMP students rarely used a mental version of a paper-and-pencil method and tended to employ some of the methods used by good mental calculators. (MKR)

Mental Computational Skills of Students in a Reform Mathematics Curriculum

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Paper presented at the Annual Meeting of the American Educational Research Association, New York City, April, 1996.

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The research reported in this article was supported by the National Science Foundation under Grant No. ESI-9252984. The opinions expressed in this article are those of the authors and do not necessarily reflect the views of NSF.

In recent years, there has been reviewed interest in mental computation as an important component of the elementary mathematics curriculum. There are several reasons for this. First, in everyday situations, mental calculations and estimations are generally more common and often more useful than are paper-and-pencil computations. Second, with the availability of calculators, there is less need for extended practice of tedious arithmetic computation. However, quick estimation skills are still needed, for example when considering the meaning of a quotient obtained on a calculator or estimating the order of magnitude of a product. Third, mental computation, when used in conjunction with estimation and problem solving activities, should help to build number sense. One reason is that good mental computation requires more flexible thinking about numbers and operations. For example, 102-79 be solved easily by adding up, e.g. 79 to 80 (1), 80 to 100 (20), 100-102 (2) so 23, but this strategy would not work well for 4000 - 25. Similarly, the multiplication problem 25 * 28 can be reformulated as a simpler fraction problem (1/4 of 28 * 100) while other problems would require a different method. Good mental computation relies on these relationships rather than rigid algorithms (Hope and Sherrill, 1987).

Because mental computation relies on these number and operation relationships, it is one method for assessing number sense. For example, when faced with the problems like 102 - 79 or 4000 - 25, a student with a weak number sense may apply the standard subtraction algorithm mentally, quite probably with some regrouping error. In contrast, students with a stronger number sense will use methods that fit the problem, perhaps adding up on the first problem and reformulating the second problem, e.g., (100 - 25) + 3900.

While some people develop their own strategies for mental computation, this topic has generally been neglected, and sometimes discouraged in school ("Show your work if you want credit,"). Instead, standard pencil-and-paper algorithms have been at the center of the elementary school curriculum. Perhaps for this reason, research has generally found that students are poor at mental computation and estimation. This deficit is not limited to the United States but is fairly widespread (Reys & Nobuhiko, 1994). A recent study of third, fifth, and seventh graders attempted to set benchmarks for current levels of performance in this area (Reys, Reys, and Hope, 1993). Second, fifth, and seventh grade students in two districts (n = 250 in fifth grade) were administered a whole-class test of problems suited for mental calculation. Some problems were presented orally, some visually (on the overhead) Students were allowed 8 seconds to solve each of the problems and were instructed to solve all problems mentally. A narrow piece of paper was distributed for recording the answer, but no written work. In the fifth-grade test, discussed in this report, problems focused on addition, subtraction, multiplication, and division or whole numbers. Although all items were suitable for mental computation, results confirmed a generally weak ability to calculate mentally. For example, only about one-third of the students correctly added 47 + 29 or 28 + 75. Students scored low on problems on where strategic use of relationships would be useful (e.g., 265 - 98 ⇒ (265 - 100) + 2 or 426 + 75 ⇒ (425 + 75) + 1).

While traditional mathematics curricula have largely ignored mental computation. it is hoped that curricula and instruction influenced by the NCTM Standards (NCTM 1989, 1991) would show more positive results. Students with a stronger number sense and better problem solving abilities should also be better at mental computation. The purpose of this report is to investigate the mental calculation, and the number sense, of fifth-graders in a reform mathematics curriculum. This research was conducted as part of the field test of The University of Chicago School Mathematics Project's (UCSMP) curriculum, *Fifth Grade Everyday Mathematics* (Bell et. al, 1994). The UCSMP curriculum is one of the current reform programs funded by the National Science Foundation to implement the NCTM Standards (1989). There are several reasons to expect that these students would do well on the test used in the Reys et. al study (Reys, Reys, and Hope 1993). First, during the primary grades, students in this program are not taught traditional algorithms. Instead, students are encouraged to "invent" their own solution procedures - often mental procedures, chosen to fit the problem at hand. Even in the middle grades, algorithm invention is encouraged, and the algorithms introduced in the curriculum are generally analogous to the mental solutions developed earlier. Second, students are encouraged to share their solution procedures, both in small groups and in whole class discussions. Thus, students focus on process and may be introduced to a variety of methods that can be utilized on different problems. Third, activities are often aimed at developing number sense and understanding rather than applying a set of rules to a problem. Finally and most importantly, computation is generally done as part of some larger activity - i.e., opportunities to apply mental calculation are integrated throughout the curriculum.

However, there are also reasons to expect that the UCSMP students may not do as well, or at least may show no major improvement in mental computation. First, few specific mental strategies are taught in the curriculum through fourth grade[1]. While mental computation is encouraged, it is not an actual instructional strand, and spontaneously "inventing" mental algorithms may be difficult for some students. Second, more time is devoted to geometry, data, and use of representations and tools and less time to computation relative to traditional curricula.

Method

Participants

Students in four fifth-grade classes took part in the testing (n = 78). With the exception of transfer-ins, students in all classes had been in the UCSMP curriculum since kindergarten. Classes were in 3 suburban public schools and 1 urban parochial school, a sample similar to the Reys et. al schools (Reys et. al, 1993).

Procedure

Twenty five items from the Reys test were presented to the whole class, and on each item students were allowed 8 seconds to answer. Students were instructed to solve all problems mentally and provided with a narrow piece of paper on which to record their answers. The procedures used were identical to those used in the Reys study (see Reys, Reys, and Hope, 1993 for details) with one exception - the four number stories were

[1] This study was conducted during the field test year of *Fifth Grade Everyday Mathematics*. While practice in multiplication and division of powers of ten were included as part of the curriculum (3500 / 70), other mental computation strategies were not. In the published version, partially in response to this research, some mental calculation strategies (e.g., 99 * 72 = 100 * 72 - 72) were introduced as warm-ups to lessons.

shown visually and read aloud. Following the test, students in two of the classes were administered a solution preference survey in which they were asked how they would solve selected problems (mentally, with pencil and paper, or with a calculator)[2]. Five students in one class were interviewed individually and videotaped to help analyze the methods they used as they answered 10 questions suited for mental computation.

Results

The UCSMP students outperformed the baseline group on all but one item (see Table 1). On most questions, this difference was significant (Chi square, $p < .05$). Across the 25 problems, the UCSMP mean correct score (47%) was nearly twice that of the Reys et. al sample (24%). UCSMP classes mean correct scores ranged from 41% to 49%. and these differences were not significant.

The tests indicated certain areas of strength and relative weakness. UCSMP students scored much higher than the baseline students on all multiplication and division problems involving powers of 10 (e.g., 60 multiplied by 70) as well as on all number stories. UCSMP students also scored higher on problems involving multiple additions and subtractions (e.g., 75 + 85 + 25 + 2000) as well as simple addition problems that required regrouping (e.g., 47 + 29). As will be discussed below, this differences are not simply due to more practice at mental computation. but to approaches that did not require mental "borrowing" or "carrying". During interviews, UCSMP students rarely used a mental version of a paper-and-pencil method.

There were some questions that indicated weaknesses in the UCSMP students mental computational abilities. Students scored relatively low on 265 - 98 and 8 × 99, problems that could be solved easily by transforming the 99 (or 98) to 100 and then readjusting the answer. Further, only 3% of the UCSMP students solved 25 × 28, a problem that could be solved easily using money knowledge (25 cents is $1, so 28 quarters is $7) or fraction knowledge (100 × 1/4 × 28). Results suggest that while students have some good mental strategies, they would benefit from experience (or instruction) in certain areas.

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[2] See *Report on the Field Test of Fifth Grade Everyday Mathematics* (Carroll, 1995) for details.

Student interviews

Poor mental computers often employ paper-and-pencil methods, mentally picturing columns of numbers and borrowing and carrying as in the standard written algorithms (Hope & Sherrill, 1987). None of the UCSMP students employed these methods, instead using methods similar to the invented algorithms observed at earlier grades. For example, on the problem 68 + 32, all students used some type of left to right addition, e.g., 60 + 30 = 90; 90 + 8 + 2 = 100.

Interviews also supported the idea that mental computational abilities were based on their earlier experiences (e.g., sharing alternative solutions) rather than specific classroom instruction (see Table 2 for a listing of component skills observed and/or hypothesized). For example, in solving 426 + 75, four different approaches were correctly used, only the first being one that might be taught in typical mental computation instruction.

$$426 + 75 \Rightarrow 426 + 5 \Rightarrow 431 + 70$$

$$426 + 75 \Rightarrow 70 + 20 \Rightarrow 490 + [illegible] + 5)$$

$$426 + 75 \Rightarrow 70 + 30 \Rightarrow 100 + 1 \Rightarrow 101 + 400$$

$$426 + 75 = (425 + 75) + 1$$

Figure 1: Four mental approaches to 475 + 75

Use of relationships between operations were also apparent during the interviews. Four of the students reformulated 100 - 65 as an addition problem, 65 + ____ = 100, and found the missing addend by counting up, trial and error, or some other method. Similarly, three of the students correctly solved "Double 84" by addition (80 + 80 + 4 + 4). Interview results indicated that most students knew and used effective mental methods.

However, interviews also indicated some weaknesses. For example, the problem 265 - 98 (which only 8% solved correctly on the whole-class tests) was solved correctly by only one of the students, using a counting down method:

"I took 265 and minused 90 from it. That took me down to 175. And then I minused another 8."

None employed the relatively easy strategy of subtracting 100 and adding back 2. Similarly only one of the student correctly solved 8 × 99 by changing it to 8 × 100 and readjusting. The others skipped the problem or used some faulty procedure.

Interviews suggested that students used their own methods, generally built upon algorithms invented during the primary grades. They also indicate that while these methods are successful on many problems, there are certain areas where students might profit by strategic instruction. This was confirmed during interviews. Following the first problem (265 - 98), unsuccessful students were shown how to simplify the problem by use of 100. All students successfully transferred this strategy to similar problems presented.

Discussion

UCSMP students scored much higher than the baseline group on the test of mental computation. Follow-up interviews with students showed that students tended to employ some of the methods used by good mental calculators, e.g., chaining sums rather than adding or subtraction columns, changing operations to simplify computation, and recognizing compatibility of numbers, despite the fact that these methods were not explicitly taught. Rather these procedures were similar to the algorithms invented in the primary grades, e.g., adding left to right and counting up for subtraction. These results, as well as students response to solution preference survey, confirm expectations of the NCTM Standards and similar reform initiatives that early number exploration in problem-solving situations, rather than algorithm practice, will result in better number sense.

Results also indicated some areas in which UCSMP students might benefit from explicit instruction. For example, most students did not spontaneously use a "nines strategy" on 265 - 98 or 8 × 99, a strategy that would turn a tedious calculation into an easy mental computation. This raises the question of how to integrate problem solving, exploration and instruction. For example, it might be hypothesized that the UCSMP students strength in mental computation is based on earlier experiences in which they were allowed to explore various solution meth ls - whether or not these solution methods were the most effective. Instruction of efficient strategies might interfere with development of number sense and a flexibility in solving problems. For example, 265 - 98 can be solved just as efficiently by a counting up strategy: 98 (+2): 100 (+165) 265, and this strategy is much more generalizable. However, there are some good strategies that most students will not "invent" spontaneously. How to best treat this dilemma is an issue, to allow number sense to develop but to also provide students with powerful strategies, might not be easily resolved.

One solution would be to encourage algorithm invention and discussions of student procedures during the primary grades. By the early elementary grades, some types of problems (99 × 17) might be introduced and students encouraged to come up with their own methods. Special strategies could then be introduced if no student suggests them spontaneously. However, this requires strong teacher knowledge of mental computation and student thinking. A less cumbersome method would be to provide occasional instruction in a few strategies while more generally encouraging students to come up with their own methods. This is more the approach the UCSMP curriculum has taken in its revision of *Fifth Grade Everyday Mathematics*. A third alternative would be to include a regular mental computation instructional strand. Relative benefits of these and other approaches warrant further investigation.

This study examined only mental computation in a timed situation. Additional research is needed to examine the interplay between mental computation, estimation, and written computation. For example, on end-of-the-year written tests, the UCSMP students tended to do about as well as traditional students on multidigit multiplication and weaker on division problems, although division problems on this test were limited to powers of tens, e.g., 3500 ÷ 35. However, on this test, UCSMP students did much better, in these areas. Is this because these problems were more suited to a mental solution or because UCSMP students are accustomed to mental solutions? Current investigations are looking at the solution methods used by UCSMP students on mental and written tests - how the solution method used is affected by the presentation mode Research is also needed to examine how to enhance students' use of number sense. While students were adept at holding and transforming numbers mentally and often explicitly stated place value, there was little evidence that estimation skills were used. and some answers (e.g., $84 \times 2 = 1604$) indicated that, at times, students did not consider the reasonableness of their answers. How to best facilitate this as part of the computation and problem solving procedure is worth investigating.

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Table 1: Mean Correct Score on Mental Math Items

Presentation and Problem	UCSMP fifth graders (n = 78)	Fifth graders from Reys et al. (n = 250)
	Mean correct (standard deviation)	
ORAL PRESENTATION		
47 plus 29	.62 (.49)	.35 *
28 plus 75	.63 (.49)	.34 *
265 minus 98	.08 (.27)	.06
Double 84	.76 (.43)	.50 *
30 multiplied by 70	.63 (.49)	.33 *
4000 multiplied by 100	.50 (.50)	.17 *
8 times 99	.24 (.43)	.26
5 times 125	.31 (46)	.15 *
6 times 55	.45 (.50)	.33
5 multiplied by 54	.33 (.47)	.20 *
3800 divided by 10	.72 (.45)	.12 *
VISUAL PRESENTATION		
68 + 32	.81 (.40)	.53 *
325 + 25 + 75	.62 (.49)	.39 *
75 + 85 + 25 + 2000	.38 (.49)	.01 *
426 + 75	.58 (.50)	.37 *
470 - 300	.67 (.47)	.64
$20.00 - $11.98	.19 (.40)	.10 *
7000 - 4000 - 300	.42 (.50)	.18 *
25 × 28	.03 (.16)	.01
2 × 27 × 5	.24 (.43)	.12 *
3500 ÷ 35	.53 (.50)	.16 *
STORY PROBLEMS		
Linda had $20. How much will she have left if she buys this scarf? (Picture of scarf for $12.85).	.31 (.46)	.09 *
Chuck's family lives 100 km from Chicago. They stop after driving 65 km. How much farther do they have to go?	.86 (.35)	.32 *
Kevin delivers 38 newspapers each day. How many newspapers does he deliver in 5 days?	.49 (.50)	.26 *
Five identical tapes cost $10.30. What does each tape cost?	.21 (.41)	.04 *
TOTAL	.47 (.19)	.24 *

Note: * Chi-square test indicated a significant difference, $p < .05$.

Table 2
Some components used by UCSMP students during mental computation

Skill or process	Example
Place value knowledge	68 + 32 = 6 **tens** plus 3 **tens** plus 10.
Experience with invented procedures	100 - 65 ⇒ 65, 75, 85, 95, 100.
Operation relationships	86 × 2 = 86 + 86
Number relationships	426 + 75 = 425 + 75 + 1
Estimation skills	8 * 99 is about 8 * 100 (not apparent during interview).
Metacognitive processes	Recognition of appropriate and alternative solution procedures; appropriateness of answer.