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ABSTRACT

A major goal of instruction is for students to develop competence with numbers. This study attempts to determine the effect of instructional materials that develop number concepts and connect these concepts to mental computation procedures. The study used 16 classes of eighth-grade students and a pretest-posttest design with two treatment groups. An experimental group used a set of author-written instructional materials as a supplement to the regular curriculum. Instructions to teachers stressed that student discussion of invented procedures was an important component of the instructional process and alternative methods of computation were to be encouraged. Results indicated that students showed a tendency to use standard procedures when whole numbers were involved, but when the problems involved decimals, fractions, and percents, students used methods which indicated a change away from the use of standard procedures to non-standard procedures. Significant gains were made in mental computation achievement. (MKR)

Filling in the Gaps

An Experimental Study on Mental Computation Achievement and Strategies

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Introduction

Children begin formal schooling with the ability to do complex mental computations. Research has found that children invent these procedures based on their understanding of numbers and quantities acquired through working with concrete objects in their play and exploration of the environment (Ginsburg, 1977; Carpenter, Hiebert, & Moser, 1981; Carpenter & Moser, 1984). However, an examination of middle grade students' achievement in mental computation reveals that students lack the skills to compute mentally and choose not to compute mentally even when the numbers are not extremely large (NAEP, 1983; Reys, Reys, & Hope, 1993; Markovits & Sowder, 1993).

Middle grade students often view numbers as symbols to manipulate rather than quantities rich with internal relationships. They do not see the relationships between the quantities involved but compute using methods that are mental analogs of written procedures as if they had "blinders on" (Hope, 1986). Students who have a high ability in mental computation tend to view numbers as quantities that bristle with relationships that contribute to the ability to compute mentally.

This change in mental computation achievement that occurs as students progress from the primary grades to middle school is believed to be related to the mathematics instruction received in school. When formal mathematics instruction begins in school two problems often arise: (1) instruction is based on manipulation of written symbols and memorization of procedures and is not connected to students' conceptual knowledge of numbers and quantities; and (2) instruction does not focus on building new mathematical understanding. Hiebert (1984) argued that these two problems lead to the use of mechanical and meaningless procedures that result in low student achievement. If mathematics instruction in school was based children's prior understanding of numbers and operations, it seems likely that students' ability to compute mentally could be improved.

It is this conjecture that led to the formulation of the problem for this study -- to determine the effect of instructional materials that develop number concepts and connect these concepts to mental computation procedures. These specific questions addressed were:

1. Is instruction that specifically connects conceptual and procedural mathematical knowledge an effective means of teaching students to compute mentally?
    A. Do students show an increased tendency to use mental computation when asked to compute by a method of their choice?
    B. Does student achievement in mental computation improve when they must compute mentally?
2. What is the effect of instruction that specifically connects conceptual and procedural mathematical knowledge on students' thought processes about number, the operations, and quantities; and specifically on their mental computation procedures?

Method

Subjects

The subjects for this study came from a public school located in a small midwestern city. Sixteen intact classes of eighth grade students and six teachers participated in the study. The sixteen classes that participated represented all students in grade eight in this school.

The study used a pretest -- posttest design with two treatment groups. An experimental group used a set of author written instructional materials as a supplement to the regular curriculum with the control group completing the regular mathematics curriculum. The regular mathematics curriculum in grade eight concentrated on algebraic concepts and skills. Classes were assigned to experimental and control groups to ensure equal numbers of students and comparable ability levels.

Procedures

Prior to the study, the investigator met with the Director of Secondary Education and the teachers who would use the instructional materials in their classes. The teachers were given lesson plans, assessment instruments, and sample lessons. The discussion focused on the purposes of the study, the use of the instructional materials, specific instruction on the use of mathematical models throughout the unit, the importance of class discussion, and the relationship of the instructional materials to the state designed mathematics curriculum. Teachers were provided a set of guidelines with teaching suggestions that would serve as a reference throughout the study. It was stressed that student discussion of invented procedures was an important component of the instructional process and alternative methods of computation should be encouraged.

The study began in September 1993 with interviews conducted with randomly selected students. Instruction began in late September and continued through the end of May 1994 with the post interviews. The timeline for the study is given in Figure 1.

Early September	Pre-instruction Interviews
Late September	Instruction begins - Lessons on Whole Numbers
Early December	Lessons on Decimals
Late January	Lessons on Fractions
Early March	Lessons on Percents
Mid May	Post-instruction Interviews

Figure 1. Timeline for the Study

Each teacher was observed by the researcher several times during the course of the study. After each observation, a short debriefing session was held with the teacher to discuss the lesson's strengths, suggestions for improvement, how the teacher felt the materials worked, and suggestions for improvement of the materials.

Instructional Materials

The mental computation instructional materials written by the investigator consisted of 83 lessons related to whole numbers, decimals, fractions, and percents. A model developed by Payne and Rathmell (1975) and expanded by Payne, Towsley, and Huinker (1990) was used to design the instructional materials in this study (see Figure 2). This model stresses the connection of mathematical symbols to referents as a way to build conceptual knowledge. As children connect written mathematical symbols to the other three representations, they begin to develop competence with symbols. Through experiences with each of four representations, mathematical ideas become part of conceptual knowledge. Connections between mathematical symbols and quantities or referents become paths between form (procedural knowledge of the symbol) and understanding (the conceptual knowledge of the quantity) (Hiebert, 1984).

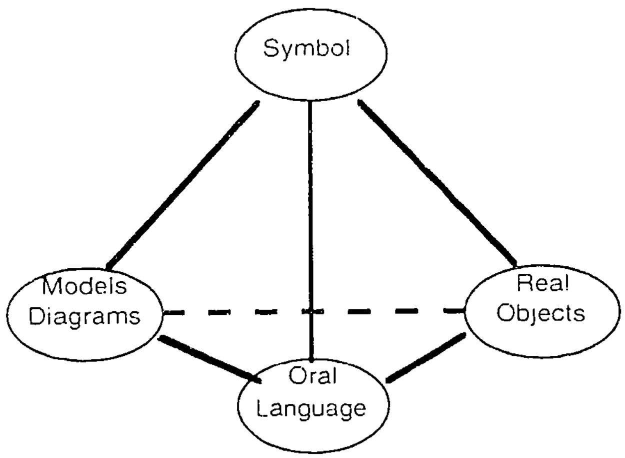


Figure 2. Conceptual Knowledge Connections

Lesson titles and the sequence of the instruction are shown in Figure 3. The initial lesson in each subtopic took most of a 40-minute class period and provided an in-depth study of the content. On subsequent days, students were given the lesson when they entered the room and began work immediately. These lessons were completed and discussed in the first 10 minutes of class that took little if any time away from the usual mathematics instruction.

For each lesson, problems were presented in written form. Students were to do all the work mentally and record only their final answers on the lesson worksheet. The instructional lessons were taught using large group discussion with mathematical models presented on an overhead projector. Students did hands-on learning including the use of models and the drawing and shading of diagrams. In some lessons students worked in small groups to check their work and discuss various strategies used in mental computation.

Day 1-3	Representations of Whole Numbers
Day 4-7	Addition & Subtraction of Whole Numbers
Day 8-9	Addition of Compatible Whole Numbers
Day 10-11	Subtraction of Whole Numbers - Regrouping
Day 12-14	The Meaning of Multiplication
Day 15-16	Multiplication by Multiples of 10
Day 17-19	Multiplication - Distributive Property
Day 20-21	Multiplication - Compatibles
Day 22-23	The Meaning of Division
Day 24-25	Dividing by Multiples of 10

Day 1-3	The Meaning of Decimals
Day 4-7	Addition and Subtraction with Decimals
Day 8-10	Multiplication by Powers of 10
Day 11-12	Division by Powers of 10

Day 1-3	The Meaning of Fractions
Day 4-6	Equivalent Fractions
Day 7-10	Finding Fractional Parts of Whole #'s
Day 11-13	Multiplying Fractions
Day 14-16	Dividing Fractions
Day 17-19	Add - Subtract Like Fractions
Day 20-22	Add - Subtract Related Fractions

Day 1-4	The Meaning of Percent
Day 5-7	Equivalent Fractions and Part/Whole
Day 8-11	Fractions and Percents
Day 12-16	Percent of a Number
Day 17-20	Percents, Fractions, and Decimals
Day 21-24	Fractions and Decimal of a Number

Figure 3. Lesson Titles and Suggested Time Allocations

The initial lessons for each content area stressed the development of conceptual knowledge by connecting individual mathematical symbols to referents. Models, diagrams, and oral language were integral components of each lesson and were used to develop a strong conceptual understanding of whole numbers, decimals, fractions, and percents (see Appendix A).

The process of using conceptual knowledge to develop mental computation procedures was the focus of the remaining instructional lessons for each content area (see Appendix B). Computation problems were presented in written form and required students to interpret symbols based on their conceptual understanding. The development of mental computation procedures was connected to the conceptual knowledge by using the same models, diagrams, and oral language that had been used to develop the meaning of the symbols. This was accomplished by having students manipulate the referents (either physically or by using mental models) and observing the results. Since the referents represent the symbols, the result is the same for operations with referents and the symbols.

Since each students' conceptual knowledge was unique, the mental computation procedures based on conceptual knowledge was also unique. The use of invented procedures was encouraged through students' verbal explanations of their methods. This sharing helped students clarify and reorganize their own conceptual understanding of numbers and mental computation procedures and led to discussion of alternative approaches.

Assessment Instruments

Several assessments were used to determine the effect the instructional materials had on student's mental computation achievement, choice of computation method, and thinking strategies (see Appendix C).

Achievement Test

A mental computation achievement test was given before instruction began and again after instruction was complete. Test items were structurally similar on the pretest and posttest. Each test contained 59 items that measured student's ability to compute mentally with whole numbers, decimals, fractions, percents, and to solve world problems mentally. The items for these tests were written to correspond with the mental computation objectives given in *The Michigan Essential Goals and Objectives for Mathematics Education* (MSBE, 1988).

Stated Computation Method

A pretest and post test were given in which students stated which method of computation (mental computation, calculator, paper-pencil) they would use to find the answer to exercises involving whole numbers, decimal fractions, fractions, and percents. The items on these tests were identical to the items on the achievement tests.

Student Interviews

Randomly selected students from the experimental group participated in a pre-interview and post-interview to determine their conceptual understanding of numbers and the methods used to compute mentally. Each interview was audio taped with notes taken by the interviewer concerning the student's use of diagrams in explaining thinking and other reactions that were not able to be recorded on tape. The tapes were transcribed and the notes incorporated into the transcript. Interview questions and mental computation problems are listed in Figure 4.

All numbers and problems were presented on index cards (except for the last four that were read orally by the interviewer) and shown to each subject one at a time.

I: Can you read this fraction? ($\frac{7}{10}$)
I: When would you ever use this fraction in real life?
I: How would you explain the meaning of this fraction to a boy or girl in a younger grade who does not understand this fraction?

I: Can you read this decimal? (.26)
I: When would you ever use this decimal in real life?
I: How would you explain the meaning of this decimal to a boy or girl in a younger grade who does not understand this decimal?

I: Can you read this? (84%)
I: When would you ever use this percent in real life?
I: How would you explain the meaning of this percent to a boy or girl in a younger grade who does not understand this percent?

I: On these cards are some problems and I want you to read the problem out loud and then talk out loud how you would find the answer to each problem.

1) 276 + 38	2) 52 - 28	3) 4 x 49	4) 20 x 60
5) 230 ÷ 100	6) .07 + .2	7) 4 - .9	8) $\frac{5}{8} + \frac{1}{2}$
9) 5 - $\frac{7}{10}$	10) $\frac{1}{3}$ x 15	11) $\frac{1}{7}$ of 63	12) 5 ÷ $\frac{1}{3}$
13) $\frac{3}{5}$ = ___%	14) 25% of 40		

I: These next problems I am going to read out loud and I want you to find the answer and then tell how you figured out the answer.

1) 53 x 6	2) $\frac{1}{4} + \frac{3}{8}$	3) 10% of 80	4) $\frac{1}{6}$ of 54

Figure 4. Interview Protocol

Results

Mental computation achievement pretests were given in early September 1993 with posttests given in mid May 1994. Because the instructional sequence lasted over the entire school year, the posttests provided a measure of retention of mental computation skills and conceptual understanding of mathematics. In most instances posttests were given several months after students completed the instructional lessons on a particular topic.

Mental Computation Achievement

Descriptive statistics for the pretest and posttest scores are given in Table 1. The descriptives show that there were initial differences on the pretest achievement test between the experimental and control groups with the experimental group scoring higher on the achievement pretest than the experimental group.

Table 1
Achievement Tests: Means and Standard Deviations for Experimental and Control Groups

	Experimental (n=183)		Control (n=230)	
	Mean	SD	Mean	SD
Achievement Pretest	37.6	11.0	24.8	11.8
Achievement Posttest	43.2	10.9	26.2	12.1

To evaluate the effectiveness of the instructional materials on mental computation achievement, the initial differences in pretest scores were controlled for. An analysis of covariance (ANCOVA) was used with pretest and gender used as covariates. Controlling for the initial differences between the two treatment groups, analysis found that the in 'ructional materials had a positive affect on mental computation achievement, $F(3, 409) = 331.9$, $p < .001$ (see Table 2). The instructional materials accounted for 35% of the variance in the achievement post test ($R^2 = .359$). Gender did not have a significant impact on mental computation achievement .

Table 2
Analysis of Covariance for Achievement Posttest : Controlling for Pretest and Gender

	SS	df	MS	F
Regression	60058.23	3	20019.41	331.9*
Residual	24673.49	409	60.33	

Variables in Equation	B	t	R sq.
Treatment	7.641	8.64*	.359
Gender	.725	1.88	.003
Pretest	.744	22.17*	.358

* $p < .0001$

Stated Computation Method

One of the assessment instruments asked students to state the method of computation they would use to find the answer to problems that contained whole numbers, decimals, fractions, and percents. Students were asked to state if they would use mental computation, paper - pencil, or calculator to compute the answer to each problem. The total number of items that the student stated they would solve by mental computation were recorded. Descriptive statistics for the pretest and posttest scores are given in Table 3. There were differences between the two groups on the pretest with the experimental group indicating they would use mental computation more often than the control group.

Controlling for initial group differences, the analysis showed that the instructional materials had a positive affect on the students use of mental computation, $F(3, 386) = 185.6$, $p < .001$ (see Table 4). The instructional materials accounted for 27% of the variance in the achievement post test ($R^2 = .267$). Again, there were no gender differences.

Table 3
Method of Computation Tests: Means and Standard Deviations for Experimental and Control Groups

	Experimental (n=160)		Control (n=230)	
	Mean	SD	Mean	SD
Method Pretest	25.8	12.5	14.7	10.0
Method Pretest	34.7	14.5	19.7	9.4

Table 4
Analysis of Covariance for Method of Computation Posttest : Controlling for Pretest and Gender

	SS	df	MS	F
Regression	48265.87	3	16088.62	185.6*
Residual	33451.81	386	86.66	

Variables in Equation	B	t	R sq.
Treatment	6.601	6.17*	.267
Gender	-.123	-.26	.0007
Pretest	.747	17.42*	.323

$p < .0001$

Interview Results

Interview data were analyzed to determine the methods students used when computing mentally. It was hypothesized that the development of the underlying concepts of numbers would result in a change to more flexible methods based on number relationships and conceptual knowledge.

There were few changes in the procedures students used to compute mentally from pre-interview to post-interview on items that involved whole numbers. One possible explanation for this may be that students had well-established standard procedures that work for whole numbers and they found them easy to use. A second explanation is that students had used standard procedures so often that their use has become autonomized.

However, one student showed a dramatic change in the methods he used to compute mentally. Student S10 used a standard method on all but one of the whole number items in the pre-interview. One the post-interview, S10 used a non-standard method on all the whole number items. The following excerpts from the interviews illustrates the change in S10's thought process:

276 + 38

Pre-interview

S10: Three hundred fourteen. Six plus eight is fourteen carry over the one. Eight plus three is eleven carry over the one and two plus one is three.

Post-interview

S10: Three fourteen. I knew I first just kept the two by itself and then added the seven and the three and that made it three hundred and eight plus my six is fourteen.

4 x 49

Pre-interview

S10: One hundred ninety-six. Nine times four would be ...thirty-six put the six down in my head and then carry over the four -- three, then four times four is sixteen plus three is nineteen.

Post-interview

S10: Fifty-two...not fifty-two...five...one ninety-six. I times my forty times my four and timed my nine times my four and then added the two numbers.

The thought processes of several other students changed over the course of the study but on only one or two items. Examples of these changes follow:

52 - 28

Pre-interview

S3: You'd have to carry, take one out of the five and that would be four. Carry one over the two, then twelve minus eight is four and then four minus two is two, so twenty-four.

Post-interview

S3: Twenty-five, no twenty-four. I rounded the twenty-eight to thirty and I rounded the fifty-two to fifty-four. I just brought it up two then subtracted.

Pre-interview

S8: Twenty-four. Cross out the five and get four and bring the one over to the two and minus the eight from the twelve and that would be four. Minus the two from the four and that would be two ... twenty-four.

Post-interview

S8: Twenty-four. I just added it to twenty-eight. I added twenty to it and that was forty-eight and then I just added another to it which is fifty-two. I added instead of subtracting.

4 x 49

Pre-interview

S7: One ninety-six. Four times nine is thirty-six carry the three. Four times four is sixteen add the three.

Post-interview

S7: One hundred and ninety-four ... ninety-six. I rounded forty-nine up to fifty and I timesed it by four and then minused four.

I: Why did you round forty-nine up to fifty?

S7: It's easier to multiply by fifty than forty-nine.

Student's thought processes did show some change when problems involved decimals, fractions, and percents with the thought processes changing from standard procedures to invented strategies based on understanding of the numbers.

For the decimal items in the pre-interviews, students generally computed using rules and procedures. When asked to explain why they used a certain method, most students continued to give a procedural answer. The post-interviews found more students gave explanations based on a conceptual understanding of decimals as illustrated in the following excerpts from the interviews:

.07 + .2

Pre-interview

S8: Huh... twenty-seven hundredths. If it was lined up add the zero and the two and the zero would be over here so just add it up.

I: You said if they were lined up. If what is lined up?

S8: Like the points.

I: Why do you line up the decimal points?

S8: So at the bottom it will be correct.

Post-interview

S8: Twenty-seven hundredths. I added the zero and the two which is two tenths and then I brought the seven over and put a zero behind the two and then added it to the seven.

I: Why did you add the two to the zero and not the two to the seven?

S8: Because the decimal point, because that's in the tenths column and that's in the hundredths.

Pre-interview

S10: Decimal two seven. I would just add the decimal two to the decimal seven.

I: Why do you add the decimal two to the decimal zero and not the decimal two to the decimal seven?

S10: The decimal two, when we do math we always line up the decimals.

I: Why do we line up the decimals?

S10: The teachers never really explained, they just said it was easier to keep track of your numbers that way.

Post-interview

S10: Two tenths and seven hundredths. Twenty-seven hundredths. Just added a zero at the end of the two and then zero and your seven is seven and zero and the two is two.

I: Why do you add the zero and the two and not the seven and the two?

S10: You can't add like hundredths and tenths together.

4 - .9

Pre-interview

S3: You'd add a decimal and a zero so it would be four point zero minus point nine. Then you would take one away from the four and add it to the one, then ten minus nine would be point one. Then three point one.

Post-interview

S3: Three point one. I just rounded the nine tenths to one and then I subtracted and got three and added the tenth that I took away.

Pre-interviews indicated the mental computation methods used with fractions were very procedural, reflecting the use of the standard paper-pencil algorithms. The post interviews found some students changed to using non-standard methods to compute mentally. Changes were especially noteworthy for students S1, S3, and S7. The use of conceptual knowledge was apparent when students were asked to divide a whole number by a fraction.

$\frac{1}{3}$ x 15

Pre-interview

S3: Oh, OK. You put the fifteen on top and a one below it. Then you can change the three and fifteen - three goes into three once and three goes into fifteen five times. One times five is five and one times one is one. Five over one.

Post-interview

S3: Five. A third of. three goes into fifteen five times. so one of the three parts of the whole would be fifteen parts so one third of that would be five.

$5 \div \frac{1}{3}$

Pre-interview

S3: You put five over one and switch the three and the one I think.... So five times three is fifteen over one so it would be fifteen. No, that's not right...Yea it would be.

I: Why do you switch that one number?

S3: You don't divided fractions, you flip the numbers and multiply.

I: Do you know why you do that?

S3: No. I learned it that way in class.

Post-interview

S3: Fifteen. If there are three parts in each whole then three times five would be fifteen.

Pre-interview

S11: That is like the inverse of the one third times fifteen. If you divide a whole number by a fraction the answer gets larger.

Post-interview

S11: Fifteen. Basically it is like saying how many one thirds are there in five?

I: Can you explain that?

S11: Say ten divided by two, is five. Because five sets of two is ten. So five divided by one third, there are fifteen sets of one third in five.

Pre-interview

S7: Put the five over a one and then cancel, it's two. I mean one remainder two.

Post-interview

S7: Fifteen. Five times three. You want to see how many thirds are in five.

$\frac{5}{8} + \frac{1}{2}$

Pre-interview

S7: Six eighths. Added the top two and then the bottom two I made an eight.

I: Why do you need the bottom number the same?

S7: You just don't add the bottom numbers.

I: Why?

S7: I have no idea. Because that is the way I was taught.

Post-interview

S7: One and one eighth. Four eighths would be half and I added that and that made one and I had one eighth left over.

I: Can you explain that again?

S7: Four eighths is a half, so one half plus one half is one and that one eighth is left over.

In both interviews, the procedures students followed when working with percents were mostly non-standard. Pre-interview results found a majority of students had a fairly strong understanding of percents. Students used this understanding to develop mental computation procedures. Many students also appeared to have an adequate understanding of the relationship between fractions and percents and used this understanding in the solution of the problems presented in the interviews. Examples of the methods students used to solve percent problems and related explanations follow:

25% of 40

<u>Pre-interview</u>

S1: Twelve. Half of forty is twenty and twenty is half which is fifty percent and then half of twenty would be ten. Ten would be the answer.

<u>Post-interview</u>

S1: Ten. half of forty is twenty and take half of that.

<u>Pre-interview</u>

S4: Ten. One hundred percent of forty is forty. seventy-five percent is thirty, and so on.

I: How do you know seventy-five percent of forty is thirty?

S4: Twenty-five percent is one fourth and one fourth of forty is ten ... just subtract.

<u>Post-interview</u>

S4: Ten. Twenty-five percent is equal to one fourth. One fourth of forty is ten.

Discussion

A major goal of mathematics instruction is for students to develop competence with numbers. Hiebert (1986) proposed that the difficulties students have with written numbers occur because students have developed procedures before they have connected the symbols to referents. This study focused on the development of conceptual knowledge of numbers by connecting symbols to referents as a way to develop mental computation procedures. The instructional lessons helped students build connections by emphasizing concrete, pictorial, and symbolic, and real world representations of numbers.

The instructional sequence in this study produced significant gains in mental computation achievement. The emphasis placed on developing understanding of numbers and then connecting this understanding to mental computation procedures is important for learning meaningful mental computation procedures and improving achievement. This was especially evident on decimals, fractions, and percents.

The instructional sequence also had an impact on student's stated choice of computation methods. When students understand the numbers involved in the computation they are more likely to choose mental computation because they can more easily retrieve procedures from their conceptual network. This was very evident on problems containing decimals, fractions, and percents. Instruction was necessary to bring about this change; it did not occur incidentally.

Students showed a tendency to use standard procedures when the problems involved whole numbers. The methods students used to compute when the problems involved decimals, fractions, and percents indicated a change away from the use of standard procedures to non-standard procedures. Many of the students who switched from using a standard to a non-standard method based their solution processes on their understanding of numbers and demonstrated this understanding in the interviews.

In this study, students' understanding of decimals, fraction, and percents can be described as limited. Instruction emphasizing conceptual knowledge helped students to develop a more complete and stronger understanding of decimals, fractions, and percents. Efforts at remediation required extensive time and effort. Instruction that emphasizes conceptual knowledge must begin in the early grades and continue throughout formal mathematics instruction.

Procedures are very powerful. Once a procedure is learned and practiced, students tend to use that procedure exclusively even though other procedures may be more efficient. This was found in the interviews with problems that contained whole numbers. When procedures are not routinized by students, as was the case for decimals, fractions, and percents, instruction that emphasizes conceptual knowledge is effective in the development of non-standard algorithms. Instruction based on understanding can influence the methods students use to compute and can help them develop more efficient procedures.

The increase in achievement for whole numbers was small, which raises the question of including any whole number work beyond grade six. The types of mental computation problems with whole numbers in this study were fairly easy, and mental computation could easily be done using a standard method. Perhaps different types of problems involving whole numbers should be included that would require the use of a non-standard method of computation.

The achievement for decimals, fractions, and percents was far from mastery level at the end of grade eight. This brings into question the current middle school curriculum that often does not include any specific instruction on decimals, fractions, and percents.

Teachers should view mental computation as an integral component of a total mathematics curriculum with instruction beginning in the primary grades and extending through high school. The development of conceptual knowledge of numbers is critical in helping students develop mental computation procedures. The approach taken in this study shows promise for helping students becoming competent with written symbols.

REFERENCES

Carpenter, T. P., Hiebert, J., & Moser, J. M. (1981). Problem structure and first-grade children's initial solution processes for simple addition and subtraction problems. Journal for Research in Mathematics Education, 12, 27-39.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. Journal for Research in Mathematics Education, 15, 179-202.

Ginsburg, H. (1977). Children's arithmetic: The learning process. New York: Van Nostrand.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. The Elementary School Journal, 84, 497-513.

Hope, J. A. (1986). Mental calculation: Anachronism or basic skill? In H. L. Schoen & M. J. Zweng (Eds.), Estimation and Mental Computation (pp. 45-54). Reston, VA: National Council of Teachers of Mathematics.

Markovits, Z., & Sowder, J. (1994). Developing number sense: An intervention study in grade seven. Journal for Research in Mathematics Education, 25, 4-29.

National Assessment of Educational Progress [NAEP] (1983). The third national mathematics assessment: Results, trends, and issues. Denver: Educational Commission of the States.

Payne, J., & Rathmell, E. C. (1975). Number and numeration. In J. N. Payne (Ed.), Mathematics learning in early childhood (pp. 125-160). Reston, VA: National Council of Teachers of Mathematics.

Payne, J. Towsley, A., & Huinker, D. (1990). Fractions and decimals. In J. N. Payne (Ed.). Mathematics for the young child (pp. 175-202) . Reston, VA: National Council of Teachers of Mathematics.

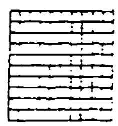
Reys, B. J., Reys, R. E., & Hope, J. A. (1993) Mental computation: A snapshot of second, fifth, and seventh grade student performance. School Science and Mathematics, 93, 306-315.

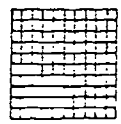
Appendix A

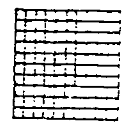
Mental Computation

Meaning of Decimal Fractions - Lesson #1

For each diagram, give the decimal fraction it represents.

1)  ________

2)  ________

3)  ________

Write the decimal fraction.

4) 6 tenths ________ 5) 34 hundredths ________ 6) 3 hundredths ________

7) 1 and 3 tenths ________ 8) 1 tenth + 5 hundredths _____ 9) 54 hundredths ____

Tell if each decimal is closer to 0, 1/2, or 1 whole. Circle the answer.

10) .9 0 1/2 1 11) .07 0 1/2 1

12) .53 0 1/2 1 13) .87 0 1/2 1

14) .1 0 1/2 1 15) .97 0 1/2 1

Challenge Problems:

Draw a circle around the decimal fractions that are close to one half. Put a square around the decimal fractions that are close to zero. Draw a triangle around the decimal fractions that are close to 1 whole.

.52 .49999 .998
.05 .102 .1
.9 .513 .910

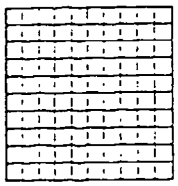
Appendix B

Mental Computation

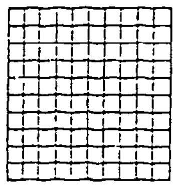
Addition and Subtraction of Decimal Fractions - Lesson #5

Shade a diagram to solve each problem.

1) .5 + .34 = _____



2) .67 - .23 = _____



Mentally find each sum or difference.

3) .37 + .20 = _____

4) .75 - .23 = _____

5) 1.3 + 2.4 = _____

6) .8 + .04 = _____

7) 3 - .6 = _____

8) 2.3 + .9 = _____

9) 2.4 - 1 = _____

10) .6 - .28 = _____

11) 5 + .98 = _____

Describe how you would mentally find the answer to each problem.

12) 1.4 + .52 =

____

13) .7 - .02 =

____

Challenge Problem:

The following answers have lost their decimal point. Place the decimal point in each answer to make it correct. *Caution:* the answers have been rounded.

2.3455123 + 1.4567345 = 3 8 0 2

4.3451236 - .90765438 = 3 4 3 7

Appendix C

Method Pretest

Name ____________
School ____________

In each of the following exercises, tell which method of computation you would most likely use to find the exact answer. Answer P if you would use paper and pencil, M if you would find the answer mentally, or C if you would choose to use a calculator.

5477 +1999 = _____	25 + 65 + 25 + 35 = _____	4250 + 30 ÷ 600 = _____
859 - 159 = _____	150 -147 = _____	83 - 27 = _____
4 x 56 = _____	7 x 399 = _____	2 x 23 x 5 = _____
100 x 8 = _____	300 x 40 = _____	1500 ÷ 3 = _____
180 ÷ 10 = _____	600 ÷ 30 = _____	3.2 + 4.31 = _____
12 ÷ .15 = _____	.05 + .6 = _____	6.45 - 3.99 = _____
0.5 - 0.06 = _____	3 - .7 = _____	6 x 1.5 = _____
.5 x 84 = _____	100 x 4.236 = _____	8 x $1.99 = _____
25.4 ÷ 100 = _____	27 ÷ 10 = _____	14 ÷ .5 = _____
$\frac{1}{2} \div \frac{1}{5} =$ _____	$2\frac{2}{5} + 1\frac{3}{10} =$ _____	$\frac{3}{4} + \frac{2}{4} =$ _____
$1\frac{7}{10} - \frac{4}{10} =$ _____	$4 - \frac{3}{4} =$ _____	$\frac{10}{12} - \frac{3}{4} =$ _____
$\frac{2}{3} \times \frac{1}{2} =$ _____	$\frac{1}{3} \times 9 =$ _____	$\frac{1}{4} \times \frac{4}{10} =$ _____
$4 - \frac{1}{4} =$ _____	$\frac{3}{5} \div \frac{1}{5} =$ _____	$\frac{3}{4} \div \frac{1}{2} =$ _____
$\frac{2}{3} = \frac{n}{12}$ n = _____	$\frac{12}{15} = \frac{n}{5}$ n = _____	$2\frac{2}{3} = \frac{n}{3}$ n = _____
50% of 24 = _____	1% of 155 = _____	10% of 80 = _____
25% of 16 = _____	3% of 300 = _____	$\frac{1}{2} =$ _______ %
$\frac{3}{4} =$ _______ %	$\frac{11}{100} =$ _____%	

Your bill at a restaurant comes to $25.40. You want to leave a 10% tip for the waiter. How much should you leave for a tip? ______________

There are 7 buses to take 280 students home from school. Each bus must carry the same number of students. How many students should be on each bus?

____

On a 100 question test, Carol received a 90%. How many questions did Carol get correct and how many did she get wrong?
#correct __________ #wrong __________

I bought $\frac{3}{4}$'s of a pound of candy and then gave $\frac{1}{2}$ of a pound to my friend. How much candy do I still have? __________

Of the 1500 students at Southwestern High School, one third of them are freshman. How many students are freshman?

____

A man quit his job at one company to take another job because it paid $2400 more a year than his former salary of $16,800. What is the man's new salary going to be?

____

If 3 x 74 = 222 then 6 x 74 = __________

Three of your favorite tapes are on sale for $5.99, $7.99 and $4.00. How much would it cost to buy these three tapes? ______________