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ABSTRACT

The purpose of this exploratory study was to examine the problem-solving behaviors and perceptions of (n=27) seventh-grade students as they worked on solving a mathematical problem within a small-group setting. An assessment system was developed that allowed for this analysis. To assess problem-solving behaviors within a small group a Group Problem Solving Assessment Instrument was developed. To assess student perceptions a stimulated-recall interview was used. Data were obtained through videotapes of the students working in small groups and audiotapes of stimulated-recall interviews of the individual students. The results suggest the importance of higher level processes used when students talk about the problem with their peers. A continuous interplay of talking about the problem and doing the problem appeared to be necessary for successful problem solving and maximum student involvement. Perceptions, particularly those of higher and lower-ability students, provided an understanding of the observed problem-solving behaviors within groups. The assessment system shows promise of being a powerful tool for examining the internal dynamics of mathematical problem solving in a small-group setting. Contains 25 references. (Author)

# ..ssessing the Internal Dynamics of Mathematical Problem Solving in Small Groups

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## Abstract

The purpose of this exploratory study was to examine the problem-solving behaviors and perceptions of 27 seventh-grade students as they worked on solving a mathematical problem within a small-group setting. An assessment system was developed that allowed for this analysis. To assess problem-solving behaviors within a small group a Group Problem Solving Assessment Instrument was developed. To assess student perceptions a stimulated-recall interview was used. Data were obtained through videotapes of the students working in small groups and audiotapes of stimulated-recall interviews of the individual students. The results suggest the importance of higher level processes used when students talk about the problem with their peers. A continuous interplay of talking about the problem and doing the problem appeared to be necessary for successful problem solving and maximum student involvement. Perceptions, particularly those of higher and lower-ability students, provided an understanding of the observed problem-solving behaviors within groups. The assessment system shows promise of being a powerful tool for examining the internal dynamics of mathematical problem solving in a small-group setting.

A recent meta analysis (Qins, Johnson, & Johnson, 1995) has confirmed the effectiveness of cooperative learning approaches for improved problem solving. Research however, has not yet identified the internal dynamics that are responsible for these results. The purpose of this study is to apply an assessment system that studies the individual problem-solving behaviors and perceptions of students as they engage in mathematical problem solving within a small-group setting. In this way, we hope to gain a better understanding of the specific ways in which the interplay of cognitive and affective processes influence the problem-solving processes that occur within a small group.

The kinds of competencies underlying mathematical power include the ability to communicate mathematically, and a self confidence and disposition to solve complex problems. In the last decade, numerous research findings from cognitive psychology and mathematics education have identified the nature and level of cognitive processes that underlie problem solving (Artzt & Armour-Thomas, 1992; Garofalo & Lester, 1985; Schoenfeld,1987; Silver,1987). There is also a growing body of research that has explored the importance of affective issues such as beliefs, emotions, and attitudes, in mathematical problem solving (McLeod & Ortega, 1993). Some mechanism is needed though, to understand how the cognitive and affective processes interact when students work within a small-group setting. Toward this end we have developed an assessment system designed to examine the problem-solving behaviors and perceptions of students as they work in a small-group setting.

The *Assessment Standards for School Mathematics* (NCTM,1995) has emphasized the importance of aligning assessment with instruction and using multiple sources for obtaining assessment information. The types of behaviors and attitudes that teachers assess and later evaluate is a reflection of what they value for student learning. If teachers believe it is important for their students to become good problem solvers, then they must use assessment strategies specifically designed to examine problem-solving processes. Similarly, if teachers believe it is important for their students to solve problems in a small-group setting, then they must use assessment strategies specifically designed to examine problem-solving processes as students communicate within a small group. This alignment of assessment and instruction is critical for the promotion of effective problem solving among students. Students' perceptions of themselves and their group members influence how they communicate within their groups (Artzt & Armour-Thomas, 1992; Artzt, 1994). Therefore, to promote optimum problem solving within a small-group setting, teachers must use an assessment strategy specifically designed to examine the perceptions of students regarding themselves and their group members.

In this article, we analyze the problem-solving behaviors and perceptions of students as they worked on solving a mathematical problem within a small group. The assessment system we used was based on our earlier work (Artzt & Armour-Thomas, 1992) and was designed to examine: (1)the interplay of cognitive processes observed in the problem-solving behaviors of individual students working in small groups; and (2) the perceptions of individual students about themselves as problem solvers and about working in a small group. The assessment system is derived from research on problem solving within a small group, cognitive processes in mathematical problem solving, and affective processes in mathematical problem solving. Through this study, we hope to learn more about the levels of thinking of individual students in a group problem-solving task and their perceptions of themselves as problem solvers within a group. Together, these data should shed light on the internal dynamics that occur when students solve a mathematical problem in a small-group setting.

## Development and Description of Assessment System

### Assessing Cognitive Processes in Mathematical Problem Solving

Within the past twenty years, cognitive psychologists have been conducting research on cognitive processes that has focused on the knowledge, monitoring, evaluation, and overseeing that individuals use as they engage in problem solving. In the psycholcgical literature the common terms used for these processes is *metacognition* (Brown, 1978; Brown, Bransford, Ferrara, & Campione, 1983; Flavell, 1981; Jacobs & Paris, 1987). A definition of *metacognition* was provided by Flavell (1981): "knowledge or cognition that takes as its object or regulates any aspect of cognitive endeavor. Its name derives from this 'cognition about cognition' quality" (p.37). Important aspects of metacognition therefore are the reflection on cognitive activities and also the regulation of these activities at any time or position during a problem-solving endeavor. Flavell introduces the idea of the duality of cognitive processes needed for successful problem solving: " We develop cognitive actions or strategies for *making* cognitive progress and we also develop cognitive actions or strategies for *monitoring* cognitive progress. The two might be thought of as cognitive strategies and metacognitive strategies" (p. 53).

In mathematics Polya (1945) was the first to conceive of problem solving as a four-phase heuristic process (understanding, planning, carrying out the plan, and looking back). This framework has served as a template for assessing competence in problem solving. Schoenfeld (1983) built on Polya's work and that of cognitive psychologists to expand Polya's model and assess mathematical problem solving in five episodes: reading, analysis, exploration, planning/implementation, and verification. Additionally, he focused on what he called the "executive decision points" which he identified as the mechanisms through which the problem-solving process was kept on course. Building on Polya's and Schoenfeld's structures, Garofalo and Lester (1985) devised a framework for analyzing metacognitive and cognitive aspects of problem-solving performance. They used the following four broad categories (Schoenfeld's related categories are in parentheses) - *orientation* (reading and analysis), *organization* (planning), *execution* (implementation), and *verification* (verification). Summaries of research studies that assess problem solving from this perspective (Garofalo & Lester, 1985; Schoenfeld, 1987; Silver, 1987) indicate that monitoring and self-regulation are metacognitive behaviors that are critical for successful problem solving in mathematics. They suggest that a main source of difficulty in problem solving is in students' inabilities to actively monitor and subsequently regulate their cognitive processes during problem solving.

These results were recently supported in our previous study (Artzt & Armour-Thomas, 1992) which examined the mathematical problem-solving behaviors of individual students as they worked within small groups. There is a growing research interest in the use of small groups to study problem solving It appears that the small-group format provides opportunities for students to engage in social and cognitive behaviors in ways that promote or constrain successful problem solving. For example, a number of researchers have suggested that grouping strategies that ensure positive interdependence and individual accountability serve as a motivation for students to actively participate in discussions within the group. (Slavin, 1989-1990). During such discussions students can engage in the critical examination of one another's ideas, and this may be the process that accounts for successful problem solving.

The framework we developed (1992) for analyzing the mathematical problem-solving behaviors of individual students as they work within small groups, synthesized the problem-solving steps identified in mathematical research by Garofalo and Lester, Polya, and Schoenfeld, and the cognitive and metacognitive levels of problem-solving behaviors studied within cognitive psychology, in particular, by Flavell (1981). This framework partitioned the problem solving into what Schoenfeld (1985) called episodes: "...a period of time during which an individual or a problem-solving group is engaged on one large task" (p. 292). The episodes were categorized as *reading, understanding, exploring, analyzing, planning, implementing, verifying,* and *watching and listening*. In addition, each of these behaviors was categorized as predominantly *cognitive* or *metacognitive*, where the working technical distinction of cognition and metacognition is that cognition is involved in doing, and metacognition is involved in choosing and planning what to do and monitoring and regulating what is being done. The episodes were assigned the following cognitive levels: metacognitive alone (*understanding, analyzing, planning*); cognitive alone (*reading*); metacognitive or cognitive (*exploring, implementing, verifying*); no cognitive level assigned (*watching and listening*). ( For a justification of the assignment of these cognitive levels see Artzt and Armour-Thomas (1992)). The results of this study and that by Curcio and Artzt (1992) supported the results of the previous studies indicating the importance of metacognitive processes. Specifically, the interpersonal monitoring and regulating of group members' goal directed behaviors, that seems to occur naturally within a small-group setting, appears to account for a group's success in problem solving. In fact, a continuous interplay of cognitive and metacognitive behaviors seems to be necessary for successful problem solving and maximum student communication. In addition, in the successful, most communicative groups, the problem-solving behaviors occurred intermittently as students returned several times to *read, understand, explore, analyze, plan, implement,* or *verify.* These behaviors were similar to the self-communication behaviors of the expert mathematical problem solvers who worked alone in Schoenfeld's study (1987).

To assess the problem-solving behaviors and cognitive processes of students as they work in small groups, we have extended our previous work to develop a system that can be used by researchers, teachers, and students in their mathematics classes. (A brief report of an application of part of this system appears in Artzt (in press).

## Group Problem Solving Assessment Instrument

The system for assessment of mathematical problem solving in small groups was developed to provide a means through which the individual behaviors of students could be assessed as they work within small groups. Our system attempts to make it possible to identify, record, and understand the problem-solving behaviors of individuals and their respective groups as they work on solving a mathematical problem within a classroom setting.

In our previous work, one of the main purposes was to distinguish between cognitive and metacognitive behaviors. Although conceptually, one can distinguish the dual nature of cognitive processes, as pointed out in our other work, operationally the distinction is not always so clear. We used a working distinction of cognition and metacognition that was similar to Garofalo and Lester's (1985, p.164) : "Cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done." Metacognitive behaviors are therefore manifested in statements made about the problem or statements made about the problem-solving process. Whereas cognitive behaviors are manifested by verbal or nonverbal actions that may indicate actual information processing. In accordance with these ideas, and in efforts to further operationalize the distinction for use by observers, we have divided student behaviors into two categories: **talking about the problem** and **doing the problem.**

In our previous work, the only time that a behavior could be coded as *metacognitive* was when we heard the verbalization of the students. That is, no inference could be made about the nature of student thought processes when the ideas were not expressed out loud. Therefore, students heard **talking about the problem** may be engaged in any of the following metacognitive problem-solving processes: *understanding, analyzing, exploring, planning, implementing, verifying.* Note that these behaviors include the monitoring and regulation of the problem-solving process. Students seen either reading, writing, or engaging in some calculating activity can be categorized as **doing the problem.** They may be engaged in one of the following cognitive problem-solving processes. *reading, exploring, implementing,* or *verifying.* The overlapping of the categorization of the problem-solving processes *exploring, implementing* and *verifying* points to an important distinction. In his 1987 work Schoenfeld described how exploration at the cognitive level alone is likely to result in unchecked "wild goose chases" (p.210). However, guided by the monitoring of either oneself or one's groupmate, the exploration process can be controlled and focused. Therefore, only when students can be heard commenting on their own or a group member's exploration is it coded as a metacognitive process. A similar analysis applies for *implementation* and *verification*, which also can occur with or without monitoring and regulation.

As students work in groups they may also engage in **watching and listening.** As in our other study, there is no cognitive level attached to this behavior since there is no audible verbalization or visible work on the problem. However, as we showed in the other study, **watching** and **listening** can play an important role in the problem-solving process when students are working in a small group. Students may also be coded as being **off task.** (See Artzt and Armour-Thomas (1992) for a detailed description of the problem-solving processes and their cognitive assignments.)

Figure 1 illustrates the variety of sequences of behavior that might occur within a small group as they work on solving a problem.

Insert Figure 1 about here

**Assessing Problem-Solving Behaviors**

In this study we transformed the cognitive-metacognitive framework for protocol analysis of problem solving in mathematics, described in our previous study (1992), into a group problem solving assessment instrument. There was a one to one correspondence between the *metacognitive, cognitive*, and *watching and listening* categories of the Cognitive-Metacognitive Framework and the respective categories of *talking about the problem, doing the problem* and *watching and listening* of the group problem solving assessment instrument. The assessment instrument contained the one new additional category that allowed for the recording of off-task behavior: *off-task*. We viewed the videotapes of the six groups to record any instances of off-task behaviors.

**Group Problem-Solving Task**

The following problem was given to the students: "A banker must make change of one dollar using 50 coins. She must use at least one quarter, one dime, one nickel, and one penny. How many of each coin must she use to do this?"

There were several reasons for the selection of the banking problem. First, because there is no algorithm available to students at this level, it lends itself to a variety of less structured approaches. Second, the teachers felt that the topic of money was within the ability level and interest of their students. Third, it offered the opportunity for students to organize information systematically, keeping track of units and working with decimals. (We recognize that the type of problem selected influences the type of problem-solving behaviors students will use. That is, the banking problem lends itself to a trial-and-error approach which would contribute to a high incidence of exploratory behaviors.)

To better understand the application of the group problem solving assessment instrument, we present an outline of several approaches that could be used, some of which were used, and how they would be assessed.

***Talking About the Problem***

*Understanding the problem.* The student makes a statement showing an understanding that three conditions must be met if the problem is to be solved.

1. There must be a total of 50 coins.
2. The value of the coins must be one dollar.
3. There must be at least one quarter, one dime, one nickel, and one penny

*Analyzing the problem.* The student makes a statement showing an attempt to analyze the problem. The following list indicates several ways this may be done.

1. Using the condition that one of each type of coin must be used, the problem can be reformulated. That is, four coins (one quarter, one dime, one nickel, and and one penny) have the value 25 + 10 + 5 + 1 or 41 cents. This constitutes a reduction of the original problem with one less restriction. That is, now one must find any 46 coins that total 59 cents. There is no longer a restriction about the type of coins that must be used.
2. Quarters, dimes, and nickels are all multiples of five. Therefore, their sums, in any combination, will also be a multiple of five. Since the total sum must be 100, also a multiple of five, the number of pennies used must be a multiple of five.
3. For fifty coins to be worth only $1.00, most of the coins selected will have to be pennies.
4. Not many quarters can be used, because four quarters are equivalent to one dollar and 50 coins must be used.

*Planning.* The student makes a statement showing an attempt to plan an approach for solving the problem. The following list indicates several ways this may be done.

1. Create a chart that has headings: quarter, dime, nickel, penny. Start by using one coin of each type and continue adding coins until there are 50 coins whose total value is $1.00.
2. Begin with 50 pennies, and exchange the remaining pennies with different coins.
3. Start with four quarters, and keep breaking down each quarter until there are 50 coins.
4. Use actual coins to get an idea of how to solve the problem.

*Exploring.* The guess-and-test problem-solving approach is a good strategy for solving this problem. The is a form of exploration. The student makes statements showing that he or she is making guesses, testing the guesses, and then making new guesses based on the results of the old ones. The student may also be making comments or suggestions to other students regarding the exploration they may be involved in. These type of statements reveal the monitoring and regulation of the exploration.

*Implementing.* When the student or group has devised a plan for solving the problem, an implementation is likely to occur. The student makes statements showing that while the plan is being implemented he or she is checking that the implementation is leading him or her or the group toward a solution. If it is not, recommendations may be made that the plan be relinquished and another plan be devised.

*Verifying.* To effectively verify the solution, the student must be able to take his or her final solution and check that the number of coins is 50, that the total value of the coins is $1.00, and that one of each type of coin is used. The student makes statements showing that he or she is monitoring the results to check that they meet the conditions of the problem.

***Doing the Problem***

*Reading.* The student is observed reading or listening to someone else read the problem.

*Exploring.* The student is observed making, what appear to be, aimless calculations based on a series of random guesses.

*Implementing.* The student is observed making calculations that appear to be the result of a plan. However, no comments or written evidence suggests that he or she is checking that the calculations are leading to a solution.

*Verifying.* A solution has been proposed and the student is adding the numbers. However, no comments or written evidence suggests that he or she is checking that the numbers they add satisfy the conditions of the problem.

***Watching and Listening***

The successful solution of this particular problem is facilitated through the communication that occurs between the group members. For this to happen, students must be observed listening to each other's ideas and watching each other work.

*Off Task*

The student is observed not engaging in, or paying attention to, the process of problem solution.

See Artzt (in press) for an example of discourse and the application of the Assessment Instrument for Group Problem Solving. See Artzt and Armour-Thomas (1992) for other examples of discourse and the application of the Cognitive-Metacognitive Framework from which the present instrument was adapted.

## Assessment of Students' Perceptions

The purpose of the interview was to learn about students' beliefs, emotions, and attitudes about working on a mathematical problem and working within a small-group setting. Research has documented the importance of students' beliefs, attitudes, and emotions in mathematics learning (Marshall, 1989; Reys, 1984; Schoenfeld, 1985; Stodolsky, 1985; Wagner, Rachlin, & Jensen, 1984). Specifically, students' perceptions about mathematics and themselves as learners of mathematics can strengthen or weaken their ability to solve mathematical problems. It is therefore important to study students' perceptions with regard to themselves as mathematical problem solvers within a small-group setting. As documented in our previous study, the perceptions of students before and during their group work play an important role in determining the discourse and problem-solving behaviors that occur as the group members work together in solving a problem. In line with the recommendations for using multiple sources to assess student work, we examined the perceptions of students regarding their group and themselves in relation to that group.

Within one week of the problem-solving session, each student participated in a private interview. In our study the groups were videotaped and each student participated in a private, stimulated-recall interview as he or she viewed the videotape of himself or herself working in the group at six specific moments: (a) before his or her group was given the problem, (b) when his or her group was given the problem, (c) when the student began to work on the problem in his or her group, (d) when he or she was deeply involved in working on the problem in his or her group, (e) when the student thought that his or her group had arrived at the solution, and (f) after his or her group had finished working on the problem. These episodes were located by the investigator who viewed each tape and indicated the number on the VCR counter that corresponded to each episode. Students were asked to recall their thoughts during these times in the problem-solving session.

## Method

### Subjects

The subjects for this study were 27 seventh-grade students (16 boys, 11 girls) attending an urban public middle school in the borough of Queens, New York City. The students were selected from three average-ability mathematics classes in which they had minimal to no experience working in small groups. To familiarize the students with their group members and the process of group work, one week before the study, teachers divided their classes into group of 4 or 5 students of heterogeneous ability in mathematics. In each of the three classes, two groups of students were randomly selected to be observed and videotaped. Thus there was a total of six groups: Groups A,B,C,D,E,F. See Table 1 for a list of the students within each group and their associated national percentile scores on the Metropolitan Achievement Test (MAT) and teacher assigned grades The students within each group differed in ability, sex, race, and ethnic background

With regard to the MAT scores, groups D and F contained only students whose percentile scores fell within the third or fourth quartiles. They were the most homogeneous of all the groups and higher in problem-solving ability. The problem-solving competence of students in Groups A and E were similar and Group B was most unusual since it was the only group that had a student scoring in the lowest quartile whereas all other members scored in the highest quartile.

With regard to the teacher assigned grades, Group E had the smallest range of grades and thus, in terms of classroom performance, was the least heterogeneous group. Group A had the largest range and was the most heterogeneous.

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Insert Table 1 about here
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## Procedure

### Instruments

Group members' mathematical problem-solving competence was estimated from their most recent score on the Metropolitan Achievement Test (intermediate form). The test is given annually in all middle schools in the area. The scores are recorded as percentiles The teachers were given informal interviews regarding their perceptions of each sample member's ability, attitude and classroom behavior. A second index of mathematics achievement was obtained from the teacher-assigned mathematics grades.

### Problem Solving in Small Groups

On the day of the study, students were instructed to sit with their groups and work together to solve a mathematical problem. There was no time limit imposed on the students The teacher stopped the group work when it was clear that most of the groups had solved the problem and the few that had not solved the problem seemed incapable of proceeding further. The problem-solving session lasted between 15 and 20 min.

**Videotape of Group Work**

In each of the three classes, the two randomly selected groups were videotaped for the full time they worked on the problem. The videotape provided a permanent record for coding the problem-solving behaviors. Each videotape was transcribed.

**Stimulated-Recall Interview**

The investigator and a research assistant conducted the interviews within 1 week of the videotaping session. Each interview was audiotaped and transcribed.

## Results

### Assessment of Problem-Solving Behaviors in Small Groups

Charts such as those shown in Figures 2 and 3 were used to record the problem-solving behaviors of the students within each of the six groups. The behavior of each student was categorized in four ways: *talking about the problem, doing the problem, watching and listening*, or *off task*. Each time a student was observed engaging in a certain behavior a tally mark was placed in the appropriate column. The resulting chart yields information about each individual's behavior as well as information regarding the behavior of the group as a whole. At the bottom of the chart anecdotal information is included that further describes the students' behaviors.

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Insert Figures 2 and 3 about here

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The behaviors of each student in the group were further summarized by counting the number of behaviors coded for a particular category and dividing it by the total number of behaviors coded in the group. This provided a profile of each group member's contributions within the group.

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Insert Table 2 about here

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**Categorizing the Groups**

As documented in our previous work (Artzt & Armour-Thomas, 1992), when students work in small groups their interactions can be represented on a continuum ranging from students who work independently with little communication to students who continually interact with one another in the solution of the problem. A depiction of several situations that can occur are given in Figures 4a, 4b, 4c, and 4d called "Cooperative Workers," "Independent Workers," "One-Person Show" and "Combination" respectively. In this study, based on the problem-solving data, it is possible to use these broad categories to summarize the behaviors of our six groups. (The reason for doing this is to help the reader better understand, and place in context, the data that follows.)

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Insert Figures 4a, 4b, 4c and 4d about here

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The number and percentage of behaviors coded as *talking about the problem, doing the problem, watch and listen* and *off task* are shown in Table 3. Of 442 coded behaviors, 38.7% were *talking about the problem*, 36.0% were *doing the problem*, 25.3% were in the *watch and listen* category, and 0% were *off task*.

The behaviors coded as *talking about the problem* as a percentage of the total behaviors coded ranged from a low of 26.3% in Group F (the only group that failed to solve the problem) to a high of 51.6% in Group C. The behaviors coded as *doing the* *problem* as a percentage of the total behaviors coded ranged from a low of 23.2% in Group E to a high of 58.2% in Group F.

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Insert Table 3 about here

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Group F was the only group not to solve the problem and looked most like the "Independent Workers" for most of their problem-solving endeavors. They had a low percentage of *talking about the problem* and *watching and listening* behaviors. Groups A and B looked most like the "Cooperative Workers." They had a high percentage of *talking about the problem* and *watching and listening* behaviors. Group E looked most like the "One-Person Show" since they had the highest percentage of *watching and listening* behaviors and, the highest ability student, who had the highest percentage of *talking about the problem* behaviors, had no instances of *watching and listening*. Groups C and D looked most like the "Combination."

<u>Problem-Solving Behaviors Within Groups</u>

The percentage of specific problem-solving categories coded for each group are listed in Table 4. Across all groups, there were 171 behaviors coded as *talking about the problem*. Of these, 62 (36.3%) were in the category *exploring* and 55 (32.2%) were in the category *understanding*. In each of these categories, Group F(independent) had the lowest percentage of these behaviors- *exploring* 3.6%, and *understanding*, 8.2%.

Across all groups, there were 159 behaviors coded as *doing the problem*. Of these 96 (60.4%) were in the category *exploring*, and 38 (23.9%) were in the category *reading*. That is, the greatest percentage of these behaviors was in *exploring*, followed by *reading*.

Of all the problem-solving behaviors that were coded, the *exploring* category (found in both the *talking about* and *doing* sections) was coded the greatest percentage of times in each group. The percentage of total *exploring* behaviors ranged from 25.3% in Group B(combination) to 48.0% in Group F(independent). Thus, in each group, at least one quarter of the problem-solving behaviors was *exploring*.

*Watch and listen* behaviors were coded a total of 112 times. Group E(one-person show) had the largest percentage of such behaviors (47.9%), and Group F(independent) had the smallest percentage (15.5%). There were no *off-task* behaviors recorded for any of the groups.

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Insert Table 4 about here

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**Problem-Solving Behaviors of Individuals in Groups**

Table 2 shows the individual percentages of *talking about the problem, doing the problem,* and *watching and listening* of each student within his or her particular group. (*Off task* behaviors were not included since such behaviors were not observed in any of the six groups.) Note that in three out of the six groups the highest ability member had the highest percentage of *talking about the problem* in his or her group (Groups C(combination),E(one-person show)and F(independent)). In Groups B(cooperative) and D(combination) the second highest ability member had the highest percentage of *talking about the problem* and in Group A(cooperative) the lowest ability member had the highest percentage of *talking about the problem*. Aside from the student in Group A, each of these students who had the highest percentage of *talking about the problem* had the lowest or second lowest percentage of *watching and listening* in their groups. In four out of the six groups (Groups B, D, E, F), the lower ability students had the lowest percentage of *talking about the problem*.

## Assessment of Student Perceptions

Each of the students in the study were interviewed to obtain a better understanding of the perceptions underlying the behaviors of individual students within the groups. Since the higher ability students did most of the talking in their groups they had the greatest potential to influence the nature of the subsequent interactions that took place in their groups. Also, aside from Groups A(cooperative) and C(combination), the lower ability members did the least amount of talking in their groups. To better understand these behaviors, we will highlight the student perceptions of both the higher ability members and the lower ability members. (Note that for this study we define the higher ability member in a group as the student with the highest MAT score in the group. We define the lower ability member in a group as the student with the lowest MAT score in the group. (When the teacher's grade for two students suggests a different ranking, we include a discussion of both students. For example, in Group D, the MAT scores for Noemi and Roland were 97 and 86 respectively. However, the grades they received from their teacher were 88 and 90 respectively. We therefore include both of them as the higher ability students.)

### Perceptions of Higher-Ability Members.

In their interviews, each of the five higher-ability students in the two cooperative and two combination groups revealed anxiety, a lack of self confidence, a fear of being embarrassed, and a concern for the group makeup. Some of their comments follow.

Group A: Samantha: "I was afraid that I might get stuck with people that didn't really do anything. People like expect me to get everything right you know, and I was afraid that if I was in a group with people that didn't know anything and I got it wrong, then people would make fun of me. So I was like, 'I have to get this right. I have to get this right!"

Group B: Susie: "I was like, 'Oh God, this is going to take forever.' So I was feeling a little like we weren't going to have enough time or anything."

Group C: Jeanine: "I was like nervous and everything. And I didn't know really what to do or how to act."

Group D: Noemi: "I didn't know how to do the problem because I didn't feel that I was capable of doing it."

Group D: Harry: "I was thinking if we were really going to work together that much. Because sometimes you get stuck with a group that really doesn't like to work together...I was nervous a bit, that maybe I was going to make a mistake and everybody always thinks that I'm going to have a right answer."

In a contrary way, the three higher ability members of Group E (one-person show) and F(independent), all revealed a sense of self confidence and enthusiasm about doing the problem, an independent attitude about working on a problem and, in the case of Bernard, the leader of Group E, a competitive attitude toward another group member. The attitudes of these students are indicated in the statements of Bernard and Connie.

Group E:Bernard: "I was real excited because I like working problems that I haven't worked that are not on my grade level."

"Mostly I did the problem by myself. I like to do the problem by myself because that's how I'm trained in advanced math to do them by myself. Not with a group. And that's why I did it. I worked so fast. And I got it by myself."

"I thought he (Clyde) was figuring it out. So what I started doing was, I was competing and competition made me go real fast."

Group F: Conny: "I was kind of excited."

"I wanted everything my way. Kind of stubborn."

The higher ability students in the two cooperative and two combination groups all gave specific reasons for their positive attitudes tow .rd the group work. Their comments follow:

Group A: Samantha: "If I had worked on the problem alone, I probably would have gotten it. But it would have taken me a longer time. Because, like my problem is, I don't look at details and stuff you know. And a lot of them, they caught like little details that I probably would have missed. And I wouldn't have gotten it till later on. So it would have taken me a long time to finally get it. And I think like everyone contributed a little. It took like a little bit of everyone to do it."

Group B:Susie: "I was for working in the group because it was really fun. And we did use some of what each other said."

Group C: Jeanine: "We were able to combine ideas and everything. We were able to interpret it among ourselves. And we were able to compare ideas and everything."

Group D: Noemi: "With more people I got the help of Harry and everybody to make me understand the problem better."

Group D: Harry: "I feel when you're working with other people, it helps to figure out the problem quicker. You have not one mind working, but you have many minds. So I like it. It's a faster way to do something."

In a contrary way, Bernard, the highest-ability member and ringleader of the one-person show in Group E, revealed a negative attitude toward group work. Connie and Linda, the higher-ability members of the independent Group F, that failed to solve the problem, revealed an indifference about group work. Some comments follow.

Group E. Bernard: "I would have rather worked by myself. That's the way I am used to working. I mean in the City Wide you don't work with a group. And that's why you're trained to work by yourself on a problem. The only time it is good to work with a group is if the people that you work with like to explore the problem and maybe they don't have to be smart. But maybe if they are real smart, it's good to work with them because they have new ways of doing it. But if you're not working with people that don't like to explore problems, don't like math, or anything like that, then it's not good to work with them. Because all they're doing is competing. Something they probably don't know and they're probably just bragging about something they never had and they don't really know the basis of it."

Group F: Connie: "I think it would probably be the same whether I was given the problem to do with a group or without a group."

**Perceptions of Lower-Ability Members.**

The comments of the nine lower-ability students in the six groups revealed some form of tension, insecurity, confusion, and in some cases a feeling of unimportance and a desire to find their own independent solution. Representative comments from the lower-ability students within each group are listed below:

Group A: Dean: " I felt I could do it by myself maybe. I was thinking, I wanted to get it all by myself. But then they got it of course. They had the brains. You know Samantha, she's a brainiac. And then William, he's smart too. I knew they would probably get it. But then I tried it by myself to see if I could get it. Why not try it alone? What do you got to lose? Then of course, they got it."

Group B: Olive : "I felt kind of embarrassed because everyone was sitting there working it out and I didn't know what to do."

Group C: Tim: "I was really confused over there. Everybody was like saying something different. Damon was saying something different. Paul was saying something different. Jeanine was saying something different. And I'm just sitting there listening to everybody else, what they're saying. And I can't think of my own way to solve the problem. And I was totally confused then."

Group D: Kim: "I was nervous."

Group E: Molly : "I was feeling nervous because I didn't think they wanted me because I didn't think I could get the answer or something. I felt sort of nervous."

Group F: Francine: "I had the answer. I had told the boy across from me about it and he said it was wrong. So I didn't tell the rest of the group about it. They never listen to me anyway."

Each of the lower-ability members revealed a positive attitude about group work. Some of the more expressive comments came from the students in Group A.

Group A: Dean: "We got it all together. I like working in groups. It's a lot easier, more people, more people to work with, see what their ideas are, compare with other people."

Group A: Sam: "It's fun to do in a group. We all like used each of our information, instead of like one person saying we're just going to use my information. Instead the whole group put in information to figure out the problem."

These results are discussed below.

## Discussion

The Assessment System provided an informed understanding of the types of problem-solving behaviors that occurred within the small groups and the perceptions that seemed to facilitate or constrain group functioning and the problem-solving process. The observation measure allowed for a frequency count of instances of *talking about the problem, doing the problem, watching and listening*, and *off-task* behaviors for each student within a group. In this way, the nature and quality of participation of individual members within and across groups could be analyzed, thus delineating strengths and weaknesses in problem-solving processes. The stimulated-recall interview shed some light on the socioemotional and intellectual climate within groups that may have accounted for the variations in problem-solving behaviors of individual members. The richness of the data that emerged from both measures supports the current thinking in educational policy and research for the use of multiple sources of assessment for ascertaining problem-solving competence in mathematics.

**Observable Problem-Solving Behaviors**

The findings from the observation measure revealed the interplay of *talking about the problem, doing the problem* and *watching and listening* behaviors of individual members within each group. The absence of any observed off-task behaviors may be due to the fact that the students were being closely filmed. Although it is not clear through what mechanism these categories of behaviors interact with each other, it would appear that *talking about the problem* plays a pivotal role in problem solving. Individuals who showed high instances of understanding, and monitoring and regulating of their groups' exploration were members of the groups that solved the problem. In contrast, individuals for whom there were few observable instances of these behaviors were members of the group that did not solve the problem. These executive-like behaviors seem to shape the direction and focus of the problem-solving process in ways that led to successful solution of the problem. Theoretical and empirical research in both mathematics education and cognitive psychology support the importance of these metacognitive behaviors during the act of problem solving (Baker & Brown, 1982; Flavell & Wellman, 1977; Garofalo & Lester, 1985; Schoenfeld, 1985; Silver, 1987; Sternberg, 1985).

Instances of *talking about the problem*, though important, are insufficient for successful problem solving. On-line execution of a strategy and calculation of computational operations are required during the act of problem solving. In addition, silent participation is inevitable as individuals use time to reconsider the conditions of the problem, rethink a strategy or make a mental check of current cognitive actions. In this study, individuals who had a high number of instances of *talking about the problem* were also engaged in behaviors indicative of *doing the problem* and *watching and listening*. They were members of groups that solved the problem. In contrast, for the group that did not solve the problem, individuals had a relatively high number of instances of *doing the problem* but a low number of instances of *talking about the problem* and *watching and listening* behaviors. These data suggest that there is an interrelationship among these problem-solving behaviors, although it is not readily apparent in what combination, manner or sequence they enable successful problem solving to occur.

## Individual Ability and Group Problem Solving

According to the ability data (Table 2), apart from Group A, the higher-ability individuals in the groups exhibited the highest percentage of behaviors coded as *talking about the problem*. Since this category of behavior, metacognitive in nature, seems to play a critical role in successful problem solving, as the literature suggests, it was not surprising that the higher-ability students exhibited these higher level problem-solving behaviors. However, two interesting results from this study suggest that the mere presence of high-ability members in a group does not guarantee effective group functioning or successful problem solving. For example, Group E's highest-ability member, was the leader of the "one-person show," but there was little in the way of collaborative cognition that seems to be a distinguishing feature of well-functioning groups. Despite the fact that, with regard to teacher-assigned grades, this was the most homogeneous group, the other group members did little more than watch and listen to the highest-ability member. In a different way, despite the fact that Group F's highest-ability member also had the highest percentage of *talking about the problem* in her group, and with regard to problem-solving ability, this was the second most homogeneous group, this group failed to arrive at a correct solution to the problem.

As one might have anticipated, in most of the groups, the lower-ability students had the lowest percentage of *talking about the problem.* However, in Group A, one of the most cooperative groups, the lowest ability member had the highest percentage of *talking about the problem* and the highest ability member had the lowest percentage of *talking about the problem*. This finding hints at the possibility of other dynamics operating in a group that enable or promote effective group functioning and problem solution that extend beyond the mere ability level of group members.

## Individual Perceptions and Group Problem Solving

The findings from the stimulated-recall data may account for the problem-solving behaviors that occurred within and across groups. That is, these data revealed that individuals within the groups experienced strong emotional reactions and had attitudes that may have facilitated or impeded, not only their own problem-solving efforts, but those of their peers as well. For example, the higher-ability members of Groups A, B, C, and D expressed fear of embarrassment and anxiety regarding their capability to solve the problem. These perceptions might have accounted for the greater degree of cooperation that occurred within these groups. That is, their recognition of vulnerability within the group may have caused them to be receptive to contributions and feedback from their peers. It was therefore understandable that they expressed such positive attitudes toward their peers whom they perceived as having strengths that they themselves might have lacked. In contrast, the higher-ability members of Groups E and F expressed excitement about doing the problem and confidence in their own ability to solve it. These perceptions might have accounted for the lesser degree of cooperation that occurred within these groups. That is, their independent and self-confident attitudes may have contributed to the fact that they both worked on the problem with minimal input from their peers. Their subsequent lack of appreciation or indifference to group work was therefore understandable.

The lower-ability members of all of the groups expressed a sense of "nervousness," "fear of embarrassment," and "confusion." Several also expressed the feeling that no one would want to hear their ideas and so they wanted to work independently on finding their own solution. However, the problem-solving data suggests these students still managed to express some of their ideas, watch and listen, and try out the ideas of others. These positive experiences may have accounted for the lower-ability members' expressed appreciation for group work.

## Conclusion and Implications

The purpose of this study was to use an assessment system to examine the individual problem-solving behaviors and perceptions of students as they engaged in solving a mathematical problem within a small-group setting. A modification of our earlier framework (1992) was used to identify the problem-solving behaviors of individual students. A stimulated-recall interview provided insights regarding students' perceptions of themselves as problem solvers working in small groups. Data analysis suggests the usefulness of this system as a way to assess the internal dynamics of mathematical problem solving within a small-group setting. The assessment system may be of help to teachers who wish to evaluate the problem-solving behaviors of students as they work in small groups.

The Group Problem Solving Assessment Instrument did allow for the operationalization of cognitive processes into three broad areas: *talking about the problem*, *doing the problem*, and *watching and listening*. This categorization is important since it enabled an understanding of the proportions of these problem-solving behaviors exhibited by individuals and groups as a whole. The data suggests the importance of the interplay of these three behaviors for effective problem solving and group functioning.

The stimulated-recall interviews provided insight regarding the ways that individual students' perceptions influenced their own and their peers' problem-solving behaviors within their respective groups. These results suggest that students' beliefs, emotions and attitudes regarding themselves as mathematical problem solvers within a group affect the quality of the individual behaviors and interactions that occur as they work within a small-group setting.

By using these results and this assessment system researchers and teachers can gain better insight into the internal dynamics that occur as students work within a small group to solve a mathematical problem. The insight gained can be used as an informed basis for maximizing the effectiveness and efficiency of mathematical problem solving in small-group settings.

The implications for teachers who incorporate cooperative learning strategies in their classrooms are many. Teachers must use an assessment system that is specifically adapted for examining the cognitive processes of students as they work within small groups. They must also include a mechanism through which they can assess the perceptions of their students that may positively or negatively influence these processes. This mechanism may take the form of an interview, as in this study, or it may include such alternate assessment strategies as journal writing and responses to questionnaires. Contrary to the popular belief that small group work creates a safe haven for students to express their ideas, this study revealed the fact that both higher and lower ability students enter a small-group setting with a sense of anxiety that they won't be able to do the work and will therefore be subject to embarrassment within their group. To allay students' fears teachers must design tasks that provide for individual accountability in ways that ensure meaningful contribution within a student's range of ability. This research also supports the idea that students must see the value in small-group work. This is especially true for high-ability group members who potentially set the mood and tone for the quality of the interactions that unfold within their group.

It is hoped that this research will provide some good ideas for researchers, teacher educators and teachers as they heed the recommendations of the NCTM's *Assessment Standards for School Mathematics* (1995) and continue to design their own assessment systems.

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**TABLE 1**
**Mathematics Problem-Solving Ability:**
**Percentile Scores on the Metropolitan Achievement Test and**
**(Teacher Assigned Grades)**

| | Group | | | | | |
|---|---|---|---|---|---|---|
| *Students* | A | B | C | D | E | F |
| Quartile (Q) | | | | | | |
| $Q_4$ | 99 (98) | 97 (94) | 99 (92) | 97 (88) | 94 (88) | 94 (91) |
| | | 86 (80) | 82 (55) | 86 (90) | | 82 (95) |
| | | 77 (93) | | 86 (65) | | 82 (55) |
| $Q_3$ | 69 (80) | | 73 (55) | 59 (75) | 63 (65) | 69 (65) |
| | 59 (55) | | | 55 (55) | 55 (80) | 50 (55) |
| | | | | | 50 (85) | |
| $Q_2$ | 25 (65) | | 33 (65) | | 25 (75) | |
| $Q_1$ | | 18 (55) | | | | |
| Range | 74 (43) | 79 (39) | 66 (37) | 42 (35) | 69 (23) | 44 (40) |

TABLE 2
**% of Total Behaviors (n) Per Student Within Each Group**

| Group | Student MAT Percentile | Talking About the Problem | Doing the Problem | Watch & Listen |
|---|---|---|---|---|
| A(cooperative) n = 86 | 99 | 8 | 8 | 6 |
| | 69 | 13 | 7 | 0 |
| | 59 | 9 | 5 | 8 |
| | 25 | 16 | 9 | 12 |
| **Total** | | 46 | 29 | 26 |
| B(cooperative) n = 67 | 97 | 12 | 3 | 6 |
| | 86 | 28 | 1 | 7 |
| | 77 | 4 | 9 | 6 |
| | 18 | 7 | 6 | 9 |
| **Total** | | 51 | 19 | 28 |
| C(combination) n = 62 | 99 | 23 | 5 | 2 |
| | 82 | 5 | 16 | 0 |
| | 73 | 10 | 5 | 13 |
| | 33 | 15 | 6 | 2 |
| **Total** | | 53 | 32 | 17 |
| D(combination) n = 44 | 97 | 9 | 9 | 0 |
| | 86 | 11 | 9 | 2 |
| | 86 | 11 | 7 | 0 |
| | 59 | 0 | 9 | 9 |
| | 55 | 7 | 7 | 9 |
| **Total** | | 38 | 41 | 20 |
| E(one-person show) n = 73 | 94 | 16 | 4 | 0 |
| | 63 | 8 | 3 | 14 |
| | 55 | 3 | 4 | 8 |
| | 50 | 4 | 5 | 12 |
| | 25 | 1 | 3 | 14 |
| **Total** | | 32 | 19 | 48 |
| F(independent) n = 110 | 94 | 10 | 15 | 3 |
| | 82 | 5 | 11 | 3 |
| | 82 | 2 | 7 | 7 |
| | 69 | 5 | 12 | 1 |
| | 50 | 5 | 12 | 3 |
| **Total** | | 27 | 57 | 17 |

TABLE 3

**Number (and Percentage) of Talking About the Problem, Doing the Problem, Watch-and-Listen, and Off-Task Behaviors Per Group**

| *Behavior Category* | *Group* A | B | C | D | E | F | *Total* |
|---|---|---|---|---|---|---|---|
| Talking About the Problem | 40 (46.5) | 32 (47.7) | 32 (51.6) | 17 (38.7) | 21 (28.8) | 29 (26.3) | 171 (38.7) |
| Doing the Problem | 24 (28.0) | 16 (23.9) | 20 (32.2) | 18 (40.9) | 17 (23.2) | 64 (58.2) | 159 (36.0) |
| Watch and Listen | 22 (25.6) | 19 (28.4) | 10 (16.1) | 9 (20.5) | 35 (47.9) | 17 (15.5) | 112 (25.3) |
| Off Task | 0 (0.0) | 0 (0.0) | 0 (0.0) | 0 (0.0) | 0 (0.0) | 0 (0.0) | 0 (0.0) |
| Total | 86 (100.0) | 67 (100.0) | 62 (100.0) | 44 (100.0) | 73 (100.0) | 110(100.0) | 442(100.0) |

*Note* Groups A, B, and C had four members. Groups D, E, and F had five members.

TABLE 4

**Percent Distribution of Talking About the Problem, Doing the Problem, Watch-and-Listen, and Off-Task Behaviors by Problem-Solving Group**

| | Group | | | | | |
|---|---|---|---|---|---|---|
| *Behavior Category* | A | B | C | D | E | F |
| **Talking About the Problem** | | | | | | |
| Understand the Problem | 11.6 | 17.9 | 21.0 | 11.4 | 8.2 | 8.2 |
| Analyze | 1.2 | 8.9 | 8.1 | 2.3 | 4.1 | 4.5 |
| Explore | 25.6 | 11.9 | 16.1 | 15.9 | 15.1 | 3.6 |
| Plan | 2.3 | 4.5 | 4.8 | 0.0 | 0.0 | 4.5 |
| Implement | 2.3 | 3.0 | 0.0 | 0.0 | 1.4 | 4.5 |
| Verify | 3.5 | 1.5 | 1.6 | 9.1 | 0.0 | 0.9 |
| **Doing the Problem** | | | | | | |
| Read | 4.7 | 6.0 | 12.9 | 18.2 | 6.8 | 8.2 |
| Explore | 15.1 | 13.4 | 14.5 | 18.2 | 11.0 | 44.5 |
| Implement | 3.5 | 0.0 | 0.0 | 0.0 | 2.7 | 5.5 |
| Verify | 4.7 | 4.5 | 4.8 | 4.5 | 2.7 | 0.0 |
| **Watch and Listen** | 25.6 | 28.4 | 16.1 | 20.5 | 47.9 | 15.5 |
| **Off Task** | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

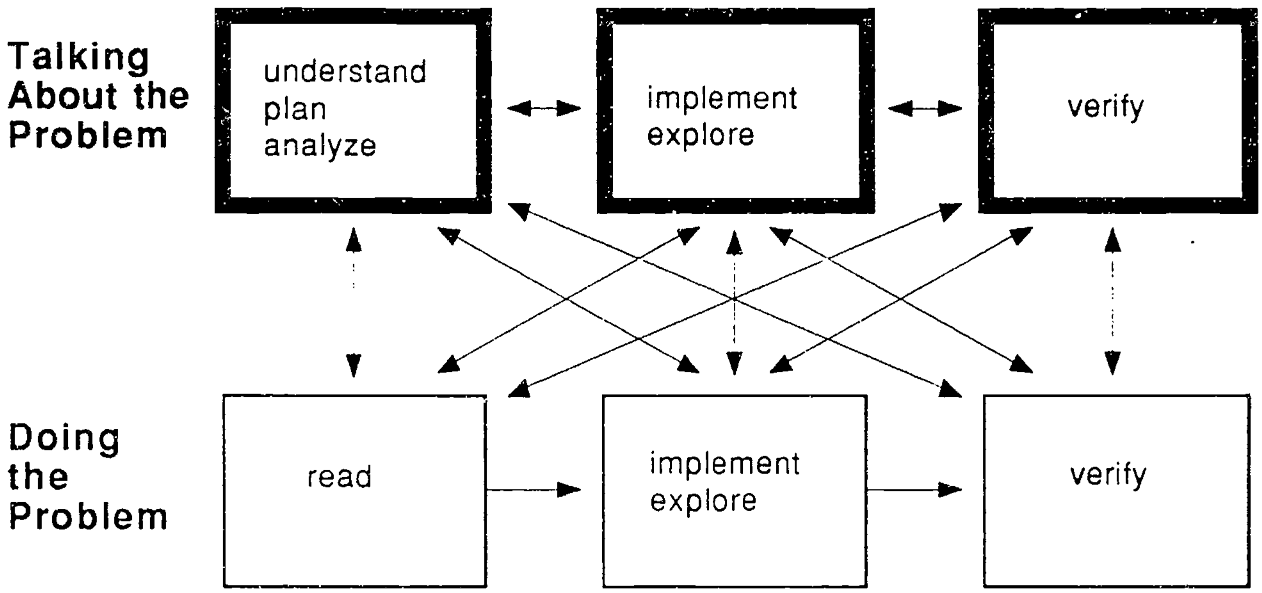


FIGURE 1 A model for problem solving within a small group.

| Name of Student | Talking About the Problem | | | | | | Doing the Problem | | | | Watching and Listening | Off-Task |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | UNDERSTANDING | ANALYZING | PLANNING | EXPLORING | IMPLEMENTING | VERIFYING | READING | EXPLORING | IMPLEMENTING | VERIFYING | | |
| *Semantha* | / | | / | / / / | / | / | / | / / / / / | / | | / / / / / | |
| *William* | / / / | / | / | / / / / / / | | | / | / / | | / / / | | |
| *Dean* | / / | | | / / / / / / | | | / | / / / | | | / / / / / / / | |
| *Sam* | / / / / | | / | / / / / / / | / | / / | / | / / / / | / / | / | / / / / / / / / / | |

*Anecdotal Information:* This was an extremely cooperative group. Each of the students played an important role in the problem-solving process. They all monitored the exploration and were most receptive to each other's ideas. An interesting event occurred in this group. After about five minutes, the group had arrived at the correct solution. However, when they tried to verify their answer they made an arithmetic error and were mistakenly lead to believe that their solution was incorrect. They began to explore the problem again and began to stray from their correct solution. William suggested that they reexamine what was wrong with their original solution. When they checked it over again they realized that, in fact, nothing was wrong with their original solution. They had the answer the whole time. William's idea was critical in the solution of this problem.

FIGURE 2 Diagram of behaviors in Group A

| Name of Student | Talking About the Problem: UNDERSTANDING | ANALYZING | PLANNING | EXPLORING | IMPLEMENTING | VERIFYING | Doing the Problem: READING | EXPLORING | IMPLEMENTING | VERIFYING | Watching and Listening | Off-Task |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Connie* | / | / / / | / / / | | / / / / | | / / | / / / / / / / / / / / | / / / | | / / / | |
| *Linda* | / | / | / / | | / / | | / | / / / / / / / / / / / | | | / / / | |
| *Michael* | / / | | | | | | / / / | / / / / / | | | / / / / / / / / | |
| *Raymond* | / / | / | | / / | | / | / | / / / / / / / / / / / | / | | / | |
| *Francine* | / / / | | | / / | | | / / | / / / / / / / / / / / | | | / / / | |

*Anecdotal Information:* The group got off to a strong start discussing the problem and analyzing it well. However, after a few minutes the group seemed to deteriorate. Connie and Russell seemed to be in a rush to find a solution quickly. Each student began to work alone. Russell was doing his own thing while talking out loud. He claimed to have a solution which he never stated, explained, or verified. Connie announced her plan to use an incorrect number of dimes. Noone monitored that idea. The students continued to work alone in exploration. No analysis or plan was heard. Eventually, out of frustration, the students agreed that maybe there was a trick to the problem.

FIGURE 3 Diagram of behaviors in Group F

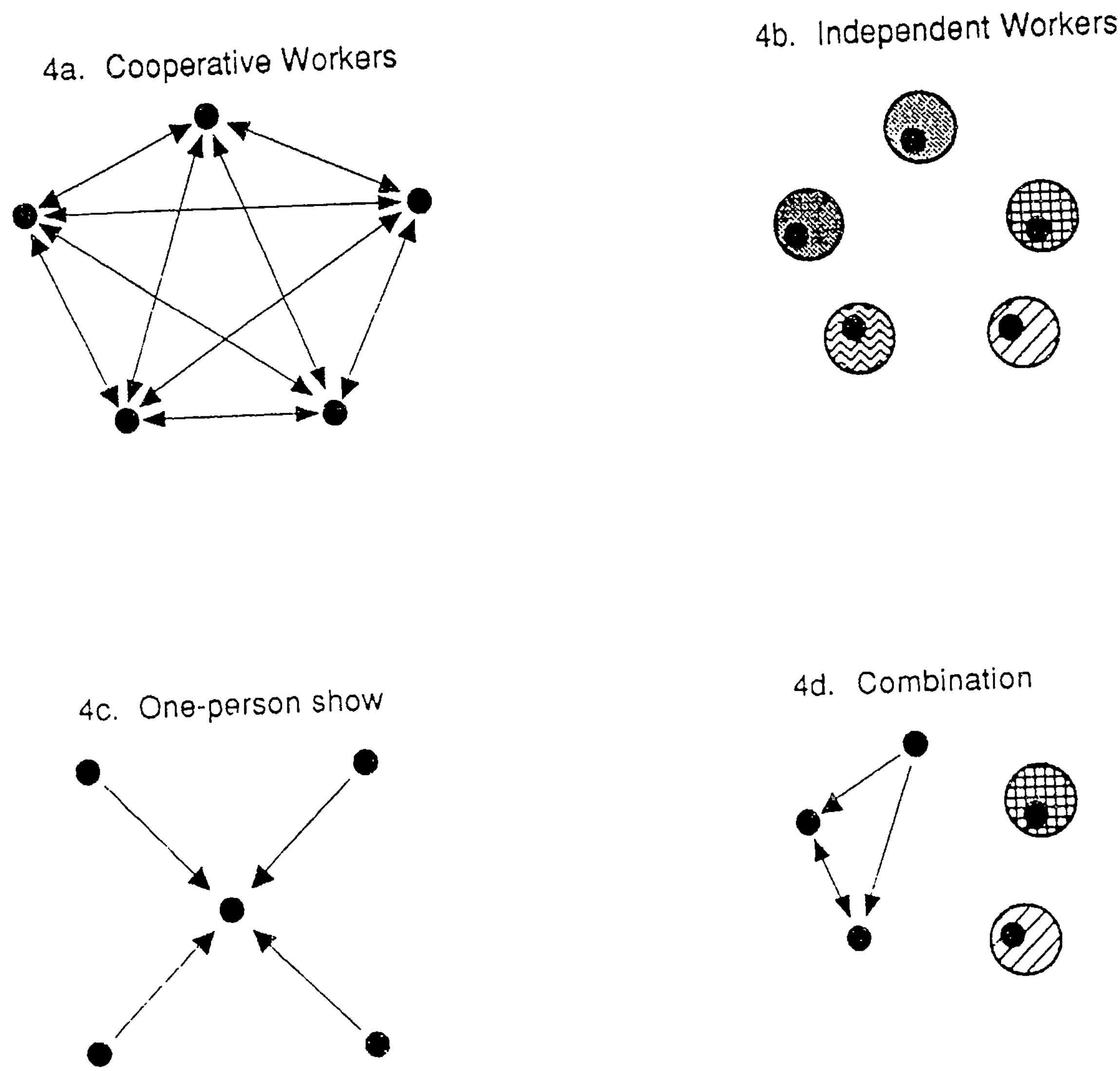


FIGURE 4 Patterns of group interactions.