DOCUMENT RESUME

ED 395 783 SE 058 350

AUTHOR	Grouws, Douglas A.; And Others
TITLE	Student Conceptions of Mathematics: A Comparison of Mathematically Talented Students and Typical High School Algebra Students.
PUB DATE	10 Apr 96
NOTE	41p.; Paper presented at the Annual Meeting of the American Educational Research Association (New York, NY, April 10, 1996). The Iowa Measurement Research Foundation supported the project.
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	*Academically Gifted; Algebra; *Beliefs; *Cognitive Structures; High Schools; *High School Students; Interviews; Questionnaires

ABSTRACT

The major goals of this study were to conceptualize a framework for analyzing student conceptions of mathematics, gather baseline information about the conceptions of mathematics held by mathematically talented students and by average high school students, and begin to generate hypotheses about how they are related to the nature of student learning. The central dimensions used to examine student conceptions of mathematics are presented after a brief review of the related literature. A study comparing mathematically talented high school students' conceptions of mathematics to those of typical high school algebra students is reported and discussed. Three dimensions were used to characterize the nature of mathematical knowledge themes: composition of mathematical knowledge, structure of mathematics knowledge, and status of mathematical knowledge. Both the composition dimension and the structure dimension demonstrated consistent differences in response patterns for mathematically talented high school students and typical high school algebra students. Contains 23 references. (MKR)

Student Conceptions of Mathematics: A Comparison of Mathematically Talented Students and Typical High School Algebra Students

Douglas A. Grouws
Carol L. Howald
Nicholas Colangelo

University of Iowa

Running Head: Student Conceptions of Mathematics

Paper presented at the annual meeting of the American Educational Research Association, New York, April 1996

Acknowledgments: We thank the Iowa Measurement Research Foundation for supporting the project. We also thank the Connie Belin and Jacqueline N. Blank International Center for Gifted Education and Talent Development for their assistance. The opinions expressed in the paper are solely those of the authors.

Student Conceptions of Mathematics: A Comparison of Mathematically Talented Students and Typical High School Algebra Students

Mathematics is a changing discipline. New branches of mathematics are being created (e.g., chaos theory, fractals), new tools are being developed for studying the discipline (e.g., Mathematica software, graphics calculators), and even what constitutes a mathematical proof is being called into question (e.g., Are computer-based proofs of theorems, such as the recent proof of the Four Color Problem, legitimate?). Along with these major changes has come a renewed interest in philosophical questions such as "What is mathematics?" and "What does it mean to know mathematics?" Responses to these questions have important implications for the teaching and learning of mathematics.

Current research has begun to show that there are differences among practicing mathematics teachers in how they view mathematics and that these differences are related to how the teachers organize and conduct mathematics lessons. For example, in a case study Cooney (1985) showed how a teacher's conception of mathematics as problem solving influenced his instructional decisions and the curriculum he implemented. Other research relating conceptions to instruction is summarized by Thompson (1992) and, as one might expect, the relationship is complex. Many factors play a role in how and what a teacher teaches, but what is learned is directly impacted by the disposition of the student toward mathematics and his or her conceptions of mathematics.

Although student attitude toward mathematics in its many forms (e.g., enjoyment, interest, appreciation, and so on) has been studied regularly and reported in the literature (e.g., Terwilliger & Titus, 1995), student conceptions of mathematics, in contrast, has received little attention. There are many unanswered questions about student conceptions of mathematics. Do students tend to think of mathematics as a body of knowledge to be acquired? Or do they view it as an human endeavor where exploration and discovery are important processes? What do students think it means "to know mathematics?" Do they think it means to have available a collection of important mathematical skills, formulas, and concepts and that covering these is what is important in mathematics class? Do they indicate that a good mathematics student can solve any problem in five minutes or less, and that the key to this success is following some algorithm? Alternatively, are there indications that some students think mathematics involves exploring a situation, sensing a pattern, representing information, making a generalization, giving an argument that convinces, finding a counter-example, and reflecting on solution methods? That is, do some students think that "to know mathematics is to do mathematics?" Do specific groups of students (e.g. elementary, secondary, gifted, at-risk, minorities) vary in their conceptions of mathematics? If so, why?

Students come to mathematics classrooms with a variety of experiences, skills, work habits, and attitudes and beliefs that influence their learning. In particular, students' perceptions of the discipline of mathematics and their attitudes and beliefs toward the field define, in part, their context for learning mathematics. In reporting data gathered about students' perceptions of mathematics in the 1990 National Assessment of Educational Progress (NAEP), Mullis, Dossey, Owen, and Phillips (1991) state that "national reforms in mathematics education highlight the importance of developing a lasting appreciation and positive attitude toward the use of mathematics to solve problems. Also, the attrition in the mathematics pipeline as students progress through school suggests that greater effort needs to be exerted in helping all students understand the power and utility of mathematics" (p. 373). To gain insight into these goals of mathematics instruction, NAEP students were asked if they agreed or disagreed with five general statements (e.g., I like mathematics. I am good in mathematics. Mathematics is useful for solving everyday problems.) designed to elicit their perceptions of mathematics. Across all three grades, 4, 8, and 12, students with more positive perceptions and attitudes had higher mathematics proficiency, with differences among grades suggesting that positive perceptions of mathematics may diminish in high school. There were few differences based on race/ethnicity and gender, but the relationship between positive perceptions and mathematics proficiencies and the differences among grade levels suggest that an examination of the perceptions of such populations as gifted students may provide insight into the relationship between conceptions and learning. It is important to note that Terwilliger & Titus (1995), in their study of attitude changes among mathematically talented youth, found declines in attitude over a two year period. Middleton, Littlefield, & Lehrer (1992) also found that perceptions of mathematics as fun decreases as gifted students progress through school.

The NAEP data involve both affective and cognitive data and provide direct support for the link between student perceptions and student learning. The general nature of the statements used, however, provide little insight into what specific beliefs about mathematics students hold, what drives their responses, or what to imply from their answers. For example, strongly agreeing to the statement "I am good in mathematics" when mathematics is perceived of as the accurate implementation of known procedures carries quite different implications for success, or willingness to learn in future mathematics classes, than when mathematics is perceived of as figuring out relationships and discovering principles.

As professional interest grows in student conceptions of mathematics and their relationship to learning and using mathematics, the need for a deeper understanding of what comprises an individual's perception of mathematics as a discipline becomes evident. What does it mean to a student to do or know mathematics? How do these student conceptions effect their use of knowledge and experiences, their interpretation of learning situations, and their goals and efforts to learn additional mathematics? Increasing our understanding in these areas would clearly allow a more meaningful interpretation of previous research results such as the NAEP data. It would also provide insight into the relationship between one's perception of mathematics as a discipline and one's learning of mathematics, that is, how these perceptions affect mathematical behavior and interpretation of educational experiences.

Little is known about student conceptions in the mathematics domain and these conceptions are not only of academic interest but they have practical implications as well. In one of the few studies that has explored this area, Schoenfeld (1988) found that high school geometry students had misconceptions about what mathematics involves and that these views were a factor in some of the student learning problems he uncovered. Whether or not Schoenfeld's findings represent student misconceptions, or just conceptions different from what mathematics educators expect or desire, is of course open to question. The point remains, however, that there is much to be learned about student conceptions of what constitutes mathematics. There are also many related questions concerning how these conceptions are formed, how they are related to what students attend to in instruction, and ultimately how they influence the nature of what a student learns.

Due to the scant attention student conceptions of mathematics have received in the literature, the major goals of this study were to conceptualize a framework for analyzing student conceptions of mathematics, to gather baseline information about the conceptions of mathematics held by mathematically talented students and by average high school students and to begin to generate hypotheses about how they are related to the nature of student learning. The central dimensions used to examine student conceptions of mathematics are presented after a brief review of the related literature. Then a study comparing mathematically talented high school students' conceptions of mathematics to those of typical high school algebra students is reported and discussed.

Literature Review

First we examine two studies of student conceptions that consider the content specific context of the students' responses, one study dealing with mathematics (Schoenfeld, 1989) and the other with physics (Hammer, 1994). We then discuss two additional studies that first develop and then rely on a framework of beliefs to examine student conceptions of mathematics (Oaks, 1987) and physics (Songer & Linn, 1991). These four studies highlight two important considerations. They emphasize the importance of content-specific beliefs and conceptions that have a close tie to the mathematical (or science) content experienced by the student (content-specific context), and the importance of developing a framework that considers dimensions of knowledge within specific content areas.

Schoenfeld (1989) investigated contradictory data patterns from general attitudinal surveys by examining in greater depth the beliefs and conceptions of students in traditional tenth grade geometry courses. Results from the third NAEP (Carpenter et al, 1983) had indicated that although students saw mathematics as a creative and useful discipline in which they learned to think, they also saw mathematics as mostly memorizing. In light of such contradictions and data from a small scale qualitative study he had conducted in a single tenth grade geometry classroom, Schoenfeld felt that the "kinds of learning (and mislearning)" he observed in a geometry class might contribute to developing an understanding of students' perceptions of mathematics. He gathered data from a larger student population to investigate this idea and to supplement the qualitative observations from the previous study. Using a survey form, he sought to determine the range of variables and experiences that might shape a student's mathematical behavior. The resulting questionnaire had 81 questions (70 closed-ended and 11 open-ended). It was administered to approximately 230 students enrolled in traditional high school plane geometry classes. The questions dealt with students' attributions of success and failure; their comparative perceptions of mathematics, English, and social studies; their views of mathematics as a discipline; and their attitude towards mathematics.

The survey addressed many student belief issues, but of particular interest are findings identifying important beliefs about the discipline of mathematics and student learning. For example, although students felt that when they did a geometry proof they got a better understanding of mathematical thinking, they also felt the proper form of the proof was of critical importance. They also reported that one has to memorize the way to do constructions. Considering these two results together illustrates that understanding mathematics and constructing proofs seem to have little in common. Doing constructions is based on recall and not sense-making. This content-specific example provides insight into a more general student belief in the disconnectedness of mathematical knowledge.

Hammer (1994), also interested in students' views of scientific disciplines, points out that direct questions on beliefs are unusual queries for students and that there may not be a clear connection between student responses to such questions and how students actually approach content and learning. Thus, providing a specific mathematical context seems to be more successful in addressing certain student conceptions. Although many of the Schoenfeld's results support previous findings on students beliefs, an important contribution of his study is the use of questions embedded in the specific context of geometry, the current course of study for the students surveyed.

Hammer (1994) reports on a thorough examination of students' beliefs about physics within the context of their learning in a physics course. He identified the following shortcomings in large scale studies: They provide little information about individual students and they do not demonstrate whether or not students' responses were simply reflecting differences in content-level knowledge. He also indicated that most previous work on student beliefs was too far removed from the context of learning.

Hammer (1994) interviewed 6 college students in an introductory physics class over the course of a semester. The interviews included open discussions of the physics course, questions concerning assignments and lecture materials, and discussions about specific content and problems. Only towards the end of the final interview did he directly discuss beliefs about physics with the students, placing the questions in terms of the students' previous comments when possible. Characterization of student beliefs was based on an analytic framework of three dimensions and showed evidence of involvement in the students' work in the course. For example, students consistently portrayed the content of physics knowledge in one of two ways, as either formulas or as concepts which underlie formulas. These beliefs influenced their reasoning. Students who believed understanding entails following the details of formal manipulations often decided prematurely that they understood. Although they recognized discrepancies between their intuition and calculated results, they chose to reject the common-sense notion without trying to account for why either the intuitive notion or the chosen formal manipulation might not apply. In contrast, students who portrayed physics knowledge as concepts were careful about building and modifying their conceptual understanding; that is, conflicts with common sense were investigated until resolved. In contrast to simply trying another method in the event of a conflict, the students began a search for the conceptual error.

In building these characterizations of student beliefs, consistency across individual interviews and physics content was apparent. Based on this consistency across interviews and content, and the evidence of involvement of the conceptions in the students' work in the course, Hammer concluded that the ability to construct consistent characterizations of student beliefs in individual case studies supports the validity of epistemological beliefs as a theoretical perspective to understand student reasoning.

In addition to demonstrating the importance of context in analyzing student beliefs and conceptions of physics, Hammer (1994) also illustrated the value of an a priori framework in characterizing student belief systems. Through pilot work and preliminary data analysis, he developed an analytic framework comprised of three dimensions:

- Beliefs about the structure of physics knowledge as either a collection of isolated pieces or as a coherent system.
- Beliefs about the content of physics knowledge as either formulas or as concepts that underlie the formulas.
- Beliefs about learning physics, as either receiving information from an authority or as an active process of reconstructing one's understanding.

Hammer used three criteria to evaluate the adequacy of his framework: Recognizability, evident involvement, and consistency. The first criteria, recognizability, requires that others be able to recognize the categories in the data, The second criteria, evident involvement, requires the framework to identify beliefs relevant to students' work so that one can gain insight on how beliefs might affect what students learn or are able to do. Finally, the consistency criteria requires the framework to allow for an analysis of beliefs across interview tasks and content topics. Consistency across tasks and content was viewed as essential to support the claim that students could be described as having epistemological beliefs about physics knowledge and learning. Based on these criteria, he concluded that the framework was a viable research tool and that students had general recognizable beliefs about physics knowledge and that their learning could be characterized along key dimensions.

The work of Oaks (1987) in mathematics, and Songer and Linn (1991) in physics, also focus on characterizing student beliefs along key dimensions. Oaks (1987) investigated students' conceptions of mathematics using case studies and specifically tried to avoid having students merely repeat "the rhetoric [they] appear to have learned in the classroom" (p. 52). In her investigation of college students enrolled in remedial mathematics classes, students were individually interviewed using predetermined questions designed to assess their conceptions of mathematics. Based on Perry's (1970) work on conceptions of knowledge, a dualistic conception of mathematics and a relativistic conception of mathematics were defined prior to student interviews. A dualistic conception of mathematics was defined as viewing mathematics as a process for finding answers to problems in a single prescribed way where the solutions to these problems are strictly right or wrong. Mathematics is viewed by students as an exact body of knowledge over which they have no control, and the purpose of class activity is recording correct algorithms as provided by a higher authority. Students view understanding new concepts as being able to recall each step in an algorithm.

A relativistic view of mathematics was defined to include the following beliefs: Not all problems have exact answers, and depending on the context, they might have different answers in different situations; results and processes can be deduced rather than memorized; and the primary goal in learning mathematics is knowing the meaning behind problems as well as solving them.

An important contribution of Oaks' (1987) work was demonstrating the existence of student beliefs and classifying them. She found that students who fail in remedial college mathematics have a conception of mathematics as rote manipulation of symbols, which in turn focuses their efforts on memorization rather than on working for conceptual understanding. Borasi (1990) summarized four prevalent student belief categories based on her own and Oaks' work. These categories include: The scope of mathematical activity (providing correct answers to well defined problems), the nature of mathematical activity (appropriately recalling and applying learned procedures), the nature of mathematical knowledge (right or wrong), and the origin of mathematical knowledge (existing only as a finished product to be absorbed as it is transmitted).

In addition to students' dualistic and relativistic beliefs about mathematics, other researchers have examined static and dynamic student views of science. Songer and Linn (1991) examined the view of science communicated by historians, philosophers of science, and expert scientists. They found that this view differed markedly from the perspective one might develop from reading a typical science textbook. Textbooks often communicated the results of the scientific process but gave little indication of the scientific process itself. Based on their experiences with textbooks, Songer and Linn concluded that students could develop views of science that are quite different from the productive view of science knowledge held by historians and expert scientists.

To characterize students' existing views of scientific knowledge and their views about how science knowledge should be acquired, the authors analyzed student beliefs about science in terms of a developmental progression from action knowledge based on observation ("Science is what the textbooks says"), to intuitive conceptions based on conjectures to explain individual events ("Theories apply to laboratory experiments, not real life"), to scientific principles based on combining predictive intuitions across events ("Science proceeds by fits and starts"). Their goal was to identify students as having action knowledge or intuitive beliefs about science, and as having intuitions that reflect a dynamic view or a static view of science. A Views of Science Evaluation instrument was developed to determine the character and stability of students' beliefs about the nature of science and administered to 153 eighth-graders in a physical science course. The test consisted of 21 short answer and true-false items, but only 9 yielded varied responses relevant to the study of science, illustrating the difficulty of finding reliable written items that assess underlying student beliefs. Using these 9 items, fifteen percent of the students were characterized as having dynamic beliefs, viewing science as understandable, interpretive, and integrated with world around them. Twenty-one percent held static beliefs, largely viewing science as static, memorization intensive, and divorced from their everyday lives. The majority of the students held mixed beliefs. The authors gave careful thought to what is an acceptable view of science via historians and philosophers of science, but, because of the large proportion of their participants (63%) falling in the mixed views category, they concluded very little about specific student views, the relationships between student beliefs about knowledge, and how these beliefs were communicated.

The previously discussed studies provide an important basis for the current work. Songer and Linn (1991) and Oaks (1987) provide important insights into the key dimensions along which student conceptions vary, but their analysis reduces these beliefs to a dynamic versus static characterization, leaving much of the population in a mixed beliefs category. This provides little insight into the relationship among beliefs within a student's system of beliefs. In addition, case studies and open-ended questions provide insight into the beliefs students hold, but they are not easily obtained for a large population. Although Schoenfeld (1989) used a content specific survey instrument that allowed him to gather data on a large number of students, the lack of a well-defined framework makes it difficult to use that data to analyze students' conceptions of mathematics in a systematic manner. This difficulty was avoided in the present work by initially developing a framework that reflected new ideas as well as the research summarized in this section.

Student Conceptions of Mathematics Framework

In this study a framework of dimensions for examining student conceptions of mathematics was formed after examining existing instruments, reviewing categories of beliefs used by others, and considering the literature linking conceptions and student learning. Three major themes emerged from this process and they underlie the framework developed: What students see as the nature of mathematical knowledge, the character of mathematical activity, and the essence of learning mathematics. Three dimensions were developed to characterize the first theme of student conceptions of the nature of mathematical knowledge: Composition of mathematical knowledge, structure of mathematical knowledge, and status of mathematical knowledge. Two dimensions were developed to characterize the second theme of student conceptions of the character of mathematical activity: Doing mathematics and validating ideas in mathematics. The third theme, student perceptions of learning mathematics, was treated as a single dimension.

An additional dimension, usefulness of mathematics, was added after considering research that used the Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976). The power of the usefulness dimension has been shown in several studies. A belief in the usefulness of mathematics outside of school, for example, has been shown to be an important factor related to students' intent to enroll in additional mathematics courses (Fennema, Wolleat, Pedro, & Becker, 1981). The seven dimensions that comprise our framework are summarized in Figure 1 and are now described in detail. Each dimension is considered as a continuum and the two poles for each dimension are described in detail.

[Insert Figure 1]

Composition of Mathematical Knowledge The poles of this dimension are knowledge as concepts, principles, and generalizations and knowledge as facts formulas, and algorithms. Knowledge as concepts, principles, and generalizations reflects the belief that mathematical knowledge consists of important ideas and the relationships among them. This conception also includes the notion that in spite of concepts often being represented by symbols and formulas, one expects problem solving to be guided by conceptualization. In contrast, the conceptions of the nature of knowledge as facts, formulas, and algorithms reflects the belief that mathematical knowledge consists of important procedures and statements. Further, these procedures are valued for the results they produce rather than the connections among ideas they embody. Thus, problem solving is thought to focus on locating appropriate formulas or tools.

Structure of Mathematical Knowledge The poles of this dimension are mathematics as a coherent system and mathematics as a collection of isolated pieces. Mathematics as a coherent system reflects the belief that as one does mathematics one finds meaningful connections between and among concepts, principles, and skills. The belief is operationalized by both acting on the above expectation and in resolving conflicts that arise. In contrast, mathematics as a collection of isolated pieces reflects the belief that mathematics consists of a variety of independent topics and skills, thus losing or gaining one piece of information has little effect on the development of another. Conflict is often resolved by putting one's confidence in the most familiar and easily recalled knowledge. Not only are strands such as geometry and algebra considered separate, the roles of proof, construction, and problem solving are independent as well.

Status of Mathematical Knowledge The poles of this dimension are mathematics as a dynamic field and mathematics as a static entity. Mathematics as a dynamic field reflects the belief that mathematics is growing and changing and this growth affects the entire discipline for both mathematicians and students. In contrast, mathematics as a static entity reflects the belief that mathematics is a compilation of information that remains fixed once developed.

Doing Mathematics The poles of this dimension are mathematics as sense-making and mathematics as results. Mathematics as sense-making reflects the belief that the process of doing mathematics depends on valuing, exploring, comprehending, and expanding the concepts and principles underlying mathematics. Doing mathematics involves thinking and figuring things out. In contrast, mathematics as results reflects the belief that the process of doing mathematics is implementing procedures and finding results. At most it is acknowledged that mathematicians created such procedures for specific uses and that they understand the underlying principles involved. Doing mathematics involves remembering and carefully following step-by-step procedures. In general, the predetermined existence of an algorithmic process for approaching a mathematical situation outweighs the value attributed to any intuitive insight the student might possess.

Validating Ideas in Mathematics The poles of this dimension are validation through logical thought and validation as established by an outside authority. Logical thought represents the belief that the validity of mathematical knowledge is established through personal reflection and individual thought and reasoning. In contrast, outside authority represents the belief that one receives valid mathematical knowledge from an authority; a text, a knowledgeable peer, a teacher, a mathematician.

Learning Mathematics The poles of this dimension are learning as constructing and understanding and learning as memorizing. Learning as constructing and understanding represents the belief that one creates new knowledge by fitting things with past experiences. The learner feels a need to be involved actively in experiences and resolve any conflicts with previous knowledge that arise. In contrast, learning as memorizing represents the belief that learning mathematics is a process of mentally storing what one has been taught; that is, the learner is a passive receiver who records existing knowledge. Learning well means to be able to quickly and accurately recall needed information.

Usefulness of Mathematics The poles of this dimension are mathematics as a useful endeavor and mathematics as a school subject with little value in everyday life or future work.

The preceding dimensions are not necessarily independent and a strong position held on one dimension may predict a position on other dimensions. Nevertheless, it is helpful to consider each of these dimensions separately in identifying the range of important conceptions about mathematics. Each dimension has the potential to affect student learning and mathematical behavior.

A Study of Mathematically Talented and High School Algebra Students' Conceptions of Mathematics

Methods

Subjects. Data were collected from two student samples. The first sample was comprised of 55 mathematically talented high school students participating in programs for gifted and talented students at a large midwestern university. Although each program's selection process and goals for the summer differed, all students involved were identified as mathematically talented based on multiple criteria including test results, teacher recommendations, and student essays. Participation in the study was voluntary, but almost all students in the programs completed the Conceptions of Mathematics Inventory. Ten students selected from volunteers were also interviewed. The second sample of students was composed of all students enrolled intact high school mathematics classes. A total of 112 high school students in grades 9 through 11 who were enrolled in either an integrated mathematics course or an algebra course formed this sample. From volunteers, 9 students were interviewed in depth. This population is referred to as high school algebra students because the students reflect the typical range of students and academic achievement in a middle SES high school algebra setting.

Instrument development. The seven dimensions identified in Figure 1 formed the framework used in developing the Conceptions of Mathematics Inventory (CMI). In the development of scales to measure each of the seven dimensions of student conceptions of mathematics, instruments that were currently being used to measure general student beliefs about mathematics and instruments dealing with specific aspects of mathematics such as problem solving and geometry were examined. Some new items were written and some existing items were included in order to allow comparison with and reevaluation of previous research results. Included among the CMI items are several NAEP items, a subset of the Fennema-Sherman Usefulness scale, and some items from the Indiana Mathematics Belief Scales (Kloosterman & Stage, 1992). The latter scale targets specific student beliefs about problem solving.

Additional items were written that reflected specific mathematics content and situations (e.g., Diagrams and graphs have little to do with other things in mathematics like operations and equations) as well as items addressing broader issues (e.g., Often a single mathematical concept will explain the basis for a variety of formulas). Student responses to each statement could range from Strongly Agree to Strongly Disagree on a five point scale. For each dimension, there is a balance in the number of items representing each end of the continuum.

After a pool of items for each dimension was generated, eight graduate students in mathematics education evaluated the items. The graduate students were given the dimension descriptions (see Figure 1 and the related descriptions) to read and discuss. They were then asked to individually sort items based on the dimensions and to react to items in terms of appropriate vocabulary, readability, and content accuracy. From the items that were consistently sorted to a given dimension, a set of eight items, four from each end of the continuum for each dimension, were finally selected to represent each of the seven previously described dimensions.

The 48 items developed and 8 items from the Fennema-Sherman Usefulness scale form the Conceptions of Mathematics Inventory. The final items were reviewed and revised by a professor in mathematics and mathematics education, and by the director of a center for gifted and talented students. Pilot data was collected to evaluate students' reaction to the instrument and to check the length of time needed for administration. The data gathered was used to determine the robustness of the seven scales comprising the CMI. Correlations were analyzed to examine the fit between item responses and total scores on the scales of which the items were a part.

Interview protocols were also developed based on the seven dimensions. These interviews were designed to allow students to discuss their conceptions of mathematics in their own language and in the context of working a mathematical problem. Interview questions addressed the different dimensions of mathematical knowledge and learning in a broader context. For example, students were asked to compare studying mathematics and studying science. As students pointed out similarities and differences, we indirectly received descriptions of student beliefs about mathematics, beliefs that were important to them.

Analysis. The responses of the two populations to the CMI were examined on an item-by-item basis using item means and variances. In addition, student responses were summed across each dimension to produce a student score for each dimension. The means and variances of these student scores were calculated and examined by sample.

Each set of eight items representing a dimension from the framework was reviewed for consistent response trends across the dimension by sample. That is, the eight items representing a given scale were first examined individually and then in relationship to the other items on the scale. This allowed us to address whether the samples differed on all the items representing a dimension or only a certain subset of items within a scale. The latter might only involve those items representing content specific situations in contrast to general items about mathematics. Finally, the eight items of a given dimension were examined in light of responses to other dimensions.

Results and Discussion

To characterize students' conceptions of mathematics, we examined response patterns for each sample within and among the seven dimensions. As we examined these dimensions, we found three dimensions with major differences in response patterns, the composition, structure, and doing mathematics scales. The other dimensions reflected at least some commonalties, although the meaning of some of these commonalties must be evaluated in light of the interview data and responses to other items within the dimension. Results are presented by dimension within the themes outlined in Figure 1, with the major differences within a given theme presented first.

Three dimensions were used to characterize the nature of mathematical knowledge theme: Composition of mathematical knowledge, structure of mathematical knowledge, and status of mathematical knowledge. Both the composition dimension and the structure dimension demonstrated consistent differences in response patterns for mathematically talented high school students and typical high school algebra students.

Composition of mathematical knowledge

Mathematically talented students indicated that although they found procedures and rules an important component of mathematics, the underlying ideas and concepts represent the real power and utility of mathematics. This is shown (see Figure 2) in the strong agreement with the item, "Mathematical knowledge consists mainly of ideas and concepts and the connections among them."

[insert Figure 2]

In contrast, the algebra students appeared much more ambivalent, with almost one-half of their responses to items concerning the composition of mathematics being neutral. Looking across the eight items on the composition dimension highlights this difference between the groups. Over 50% of the algebra student population reported a neutral response to items measuring this dimension, while over 75% of the mathematically talented population viewed mathematics as composed of concepts, principles, and generalizations.

One item of particular interest in this dimension is the statement "There is always a rule to follow when solving a mathematical problem." On the surface the response seems similar across the two samples. Well over one-half of the students in both samples agreed or strongly agreed with this statement, 75% of the algebra sample and 62% of the mathematically talented sample. This supports the NAEP (1983) results that students felt very strongly that mathematics always gives a rule to follow to solve problems and that there has been little change in this perspective over 10 years. But in light of the other responses on this dimension, the response for mathematically talented students raises the question of whether they believe all mathematical principles and concepts can be eventually reduced to rules. Rules that are useful for solving problems but only represent some aspect of the general power of the concept. This would be a very different perspective than equating mathematics with a collection of rules, as the algebra students did.

Structure of mathematical knowledge

Mathematically talented students consistently viewed mathematics as a coherent system with meaningful connections between and among concepts, principles, and skills, but the algebra students' responses were more varied and reflected a less connected view of mathematics. In fact, only 44% of the algebra students disagreed with the statement that finding solutions to one type of mathematics problem does not help you solve other types of problems, in contrast to 91% of the mathematically talented students (see Figure 3). Considering the emphasis put on problem solving and problem-solving strategies during the past decade of mathematics education reform, this is an important result because it suggests that these populations differ in their conception of problem solving.

[Insert Figure 3]

How is the typical algebra student interpreting problem solving tasks presented in mathematics class? One student in the high school algebra sample, when asked in an interview if he did much problem solving in math, responded, "A little bit, but...you've got to use the books that kind of tell you how to go through it. And as you learn how to go through it, then it kind of let's you do it on your own." He later indicated that this type of problem solving was useful "if you know how to apply it to certain situations." In contrast, what experiences (inside or outside of the classroom) are mathematically talented students involved in that encourages the recognition of the relationships among problem-solving techniques? Interview data again suggests the answer. A student from the mathematically talented sample stated, when beginning to work a mathematics problem, "There are a number of different approaches here." When asked if that was helpful, he responded, "Yeah, I think so. If you can see the overall picture, that helps to see the different parts." In this situation, the student immediately focused on the variety of problem solving approaches available and the connections among them, in contrast to the first student who tied each problem-solving technique to a certain situation. One reason for the difference in perspectives concerning the relatedness within problem solving may be that the mathematically talented students are considering the problem as the focus of the mathematics, bringing together the underlying mathematics, and the algebra students consider the use of recently acquired skills as the focus of the mathematics, relying on the surface features of the problem to prompt implementation instead of the underlying mathematical features which would highlight conceptual connections.

Status of mathematical knowledge

Unlike the previously discussed dimensions, the status dimension did not reveal large differences between the two student groups. Both groups of students saw mathematics as a dynamic field. For example, both groups agreed that the field of mathematics was always growing and changing, with only 2% of each group disagreeing. Responses to items such as "New discoveries are seldom made in mathematics" and "Students can make new mathematical discoveries, as well as study mathematicians' discoveries" also show that students see the field of mathematics as creative and changing, with themselves as possible agents of change. These results agree with Schoenfeld's (1989) results and NAEP results (1983). It is worthwhile to note, however, that a slightly larger proportion of mathematically talented students are aware that mathematics is a constantly growing and changing field (see Figure 4).

[insert Figure 4]

In addition, the interview data shows that there are qualitative differences in what students mean when they say mathematics is changing. Some students indicated change in mathematics meant change in instructional techniques or available technology, others indicated it meant the creation of additional formulas based on existing mathematics for new applications, and for still others it meant the change in content learned from year to year. One algebra student who agreed that the field was changing elaborated with "It's changing because ... ten years ago they never would have heard of the TI-82 ... It's like they're coming up with ways to learn it better and understand it easier. But in a way it's kind of staying the same. ... The general stuff will stay the same." Schoenfeld (1989) suggests that students separate the mathematics of the classroom from abstract mathematics about which they are told but never experience. In terms of the status of mathematical knowledge, the interview data suggest little student awareness of the changes in the field of mathematics either in or out of the classroom. A few of the mathematically talented students were able to discuss changes within the discipline of mathematics that consisted of more than the creation of formulas for new applications.

Summary: Nature of mathematical knowledge

In general, mathematically talented high school students see mathematical knowledge consisting mainly of a coherent system of important ideas and the relationships among them. They recognize the role of facts and algorithms, but they give more emphasis to the underlying concepts and principles. They strongly believe that this system is dynamic, growing and evolving regularly. The algebra student, in contrast, is aware that some general principles exist, but a majority of these students see mathematics as a compilation of discrete facts, formulas and procedures. Although a significant number of them recognize mathematics as a growing field, it is possible, in light of the other dimensions and the interview data, that they see this growth as an increase in the number of available discrete procedures and techniques available as tools in the field. This may reflect the general belief of technicians in other fields who use the tools of mathematics regularly and leave the adaptation and evolution of such tools to the "experts". The power of any tool in the hands of an individual is greater when an understanding of the underlying principles are understood allowing the individual to discover and implement changes that benefit the unique task at hand.

Doing mathematics

Two dimensions were developed to characterize the second theme, students' conceptions of the character of mathematical activity: Doing mathematics and validating ideas in mathematics. The doing mathematics dimension demonstrated consistent differences in response patterns for mathematically talented high school students and high school algebra students. On many of the items the distributions of responses for the two samples were skewed in opposite directions (for example, see Figure 5). Most mathematically talented students viewed doing mathematics as a process of sense-making that involved "figuring things out" more so than just applying known procedures and facts. The response to the item in Figure 5, "If you knew every possible formula, then you could easily solve any mathematical problem," illustrates that more of the mathematically talented sample conceived problem solving as more than the application of known formulas. Over 80% recognized that mathematical problem solving involves more than applying formulas (see Figure 5). Garofalo (1993), in his examination of problem preferences for meaning-oriented problem solvers, who were represented by a selection of mathematically talented gifted students and number-oriented problem solvers, also found a significant difference in the way these two populations viewed doing mathematics. He states,

> In a very real sense, "mathematical problem solving" has two different meanings to these different groups of students. The number-oriented group seems to view mathematical problem solving as something to get "over with" in almost any way that they can, while the meaning-oriented group views problem solving as an opportunity for understanding, figuring and accomplishment. (Garofalo, 1993, p. 36)

On another item, less than 10% of these talented students strongly felt that being able to use a formula accurately is enough to understand the mathematical concepts underlying the formula.

[Insert Figure 5]

In contrast, less than a quarter of the algebra students disagreed that the accurate use of formulas represents understanding or with the view that good problem solving depends on the availability of formulas. This is consistent with the NAEP result (1983) that almost 90% of teenagers agreed that "there is always a rule to follow in solving mathematics problems."

"If you cannot solve a mathematics problem quickly, then spending more time on it won't help" is one statement with a common response between the two groups. Over 78% of the algebra students and 90% of the mathematically talented students disagreed with the statement. Still, when 3 in 20 students don't feel spending additional time on a problem after a quick initial attempt is helpful it is of concern. When Schoenfeld (1989) asked 215 high school students what the reasonable amount of time to work on a problem before you know it's impossible, responses averaged 12 minutes, with the answers ranging from 3 to 20 minutes. These results taken together paint a fairly dismal view of student persistence and their conception of what doing mathematics may involve despite their response to this item. Students' view of the role of persistence in doing mathematics may be supported by teachers' beliefs about the best methods for teaching mathematics (Cai, 1995). Stigler & Perry, (1988), in their comparison of Chinese, Japanese, and American teacher practices, found that Chinese and Japanese teachers believe that the more a student struggles, the more the student can learn. As a result they usually pose difficult problems to challenge students. U.S. teachers, in contrast, tend to pose problems "that will reinforce the idea that mathematics problems should be solvable in a single, insightful motion" (Stigler & Perry, 1988, p. ?).

Validating mathematics

Both the doing mathematics dimension and the validation dimension demonstrated consistent differences in response patterns for mathematically talented high school students and high school algebra students. Responses to items reflecting the validating mathematics dimension were consistent with the pattern of responses on the doing mathematics dimension. A majority of the mathematically talented students demonstrated confidence in their thinking and relied less on an outside authority (a teacher or a book) to validate their mathematical conclusions (see Figure 6).

[Insert Figure 6]

More often, the algebra student relied on the teacher or book to provide validity, especially when the statement represented a more concrete situation. For example, when faced with a disagreement with a peer, only 13% of the algebra students, in contrast to 35% of the mathematically talented students, disagreed that they had to check with the teacher or the book to see who is correct. More general statements about the importance of justifying answers or convincing oneself about the truth of mathematical statements did not demonstrate as marked a difference in response pattern (see Figure 7).

[Insert Figure 7]

As in the NAEP results (1983), we found that most students felt that justifying the statements one makes is an extremely important part of mathematics. The NAEP report (1983) suggested that this attitude towards justifying mathematical statements may reflect a more general social view than one that emerges from students' own mathematical experiences. Alternatively, there may be a need to examine how students interpret phrases such as justifying answers or establishing truth. Are justifying assumptions made or procedures used, delineated from checking for the correct answer for most students? Or are justifications for some students simply statements such as "that's the formula we've been working with lately"?

Summary: Character of mathematical activity

In general, mathematically talented students saw mathematical activity as a sense-making process which establishes mathematical knowledge through personal reflection and justification. Algebra students were more likely to view mathematical activity as implementing procedures. They depended on outside authorities such as books and teachers to establish the validity of their work, though in general terms, they valued the role of personal justification in doing mathematics. The data leave open the question of what constitutes personal justification for these students.

Learning of mathematics

The results from items on the learning mathematics dimension were particularly interesting. Although minor differences exist in response patterns for most items, there is general agreement in responses to five of the eight items. The responses to these five items demonstrate a view of learning mathematics that balances the roles of constructing knowledge and developing understanding with that of memorizing intact knowledge. For example, 78% of the algebra students and 68% of the mathematically talented students agreed that learning mathematics involves memorizing information presented to you. Ninety-four percent of the algebra students and 98% of the mathematically talented students agreed that it is helpful to analyze your mistakes when learning mathematics. Although practically all the students indicated that it was helpful to analyze your mistakes, interview data suggested that there was a qualitative difference in what analyzing errors meant. For some students analyzing errors meant checking computations at each step of a problem, but for others it involved rethinking the problem and examining assumptions.

There were differences between the two groups on two items that emphasized the respective roles of memorizing and thinking (for example, see Figure 8) and one item that specifically addressed the role of constructing your own knowledge through problem solving.

[Insert Figure 8]

One-half of the mathematically talented students felt learning mathematics was mostly memorizing. In contrast, over 75% of the algebra students emphasized memorization. Although Schoenfeld (1989) found that students agree that the mathematics they learn in school is mostly facts and procedures that have to be memorized and the NAEP results (Lindquist, Dossey, & Mullis, 1995) indicated that the vast majority of students felt that learning mathematics is mostly memorizing, our result indicates some variation exists between different types of students. Over three-fourths of the mathematically talented students felt you could learn mathematics through independently trying to solve problems, in contrast to only 40% of the algebra students. Schoenfeld (1989) found that students felt good mathematics teachers show students the exact way to answer math problems. Again there seems to be some indication that this view of learning may not hold for more talented students.

In general, mathematically talented students and algebra students tend to have differing views of what constitutes mathematical knowledge and doing mathematics, but they share some similar views on what constitutes learning mathematics. Although algebra students are more likely to emphasize memorization over knowledge construction, both groups displayed a balance between the role of developing understanding and memorizing intact information. Schoenfeld (1989) suggests that these views represent two different mathematical worlds for the student, that within the classroom and that within the real world.

> We have seen, for example, that students say that mathematics is mostly memorization-hardly the most creative or logical of acts-but also that it is a creative discipline in which one can make discoveries, learn to be logical, and so on. There is no contradiction between the two notions if the former refers to the mathematics that takes place inside classrooms and the latter refers to the mathematics that (at least hypothetically) takes place outside them. (Schoenfeld, 1989, p. 346)

Alternately, a balance may actually exist and the actions taken by students may depend on the particular task or context at hand. Garofalo (1993), in his investigation of problem preferences of meaning-oriented and number-oriented students, found that problem preferences varied depending on whether the situation was evaluative or non-evaluative. In graded situations both groups preferred routine one-step problems. Otherwise, the gifted students in his study preferred non-routine, multi-step problems. In addition, there are tentative indications that what constitutes developing understanding through tasks such as analyzing errors may differ among students. This suggests the need for a more detailed and contextual bound look at what constitutes learning mathematics in the eyes of the student.

Usefulness of mathematics

Most students, both mathematically talented and algebra students, found mathematics useful in both their personal lives in and out of school and in the context of their future plans, though consistently more mathematically talented students respond in this fashion. A typical response is shown in Figure 9. For algebra students, these results agree with NAEP results (Lindquist, Dossey, & Mullis, 1995) over the past 20 years. About 70% of grade 12 students agreed that "mathematics is useful for solving everyday problems" and that "almost all people use mathematics in their jobs." Although, these results are encouraging, as Lindquist, Dossey, & Mullis (1995) state, "We cannot afford to let one-fourth of all our students leave high school with the perception that mathematics is not useful for careers and personal decision making."

[Insert Figure 9]

Two final issues that represent trends across all dimensions are now discussed: (1) The role of specific content and contexts for the analysis of beliefs and (2) the importance of examining similarities in data for below the surface differences. The first trend, the role of specific content and contexts for the analysis of beliefs, provides implications for item development. Each dimension included some items of a general nature and some of a more concrete, context-bound nature. As noted in the results for the validating mathematics dimension, more general statements about justifying mathematical results did not demonstrate as marked a difference in response pattern as did statements about more specific situations (see Figures 6 & 7). This was the case across dimensions. For example, on the structure dimension, when given a specific content example, "Diagrams and graphs have little to do with other things in mathematics like operations and equations", the groups demonstrated a greater difference in response pattern than when responding to the general item "Most mathematical ideas are related to one another". On the first item, 89% of the mathematically talented disagreed with the statement and only

47% of the algebra students disagreed. On the second item, 90% of the mathematically talented agreed with the statement and 62% of the algebra students agreed. Although there is a significant difference between responses for the two groups for both items, the more concrete the item, the more telling was the difference in response patterns. This held true for many other items throughout the survey instrument, and is in line with what Schoenfeld (1989) and Hammer (1994) found in connection with the important role of mathematical or physics content for the analysis of student beliefs. These results have implications for current investigations of the similarity of student epistemological beliefs across content domains. Schommer and Walker (1995) tested the common assumption that epistemological beliefs are domain independent by having subjects with a specific domain in mind (mathematics or social sciences) complete an epistemological questionnaire consisting of general statements such as "Scientists can ultimately get to the truth." Their results supported the idea that individuals' epistemological beliefs tend to be domain independent. In this study, within the domain of mathematics, items of a general nature reflected different underlying beliefs less well than did those items reflecting more content-specific situations.

The second trend evident across the major themes in the framework is the importance of examining surface similarities for differences in light of the interview data and response patterns on other items. Response patterns for certain single items held surface similarities, but below the surface there existed indications of subtle but important differences. Differences that were not clear from looking at single items on the survey were indicated both between the two samples and among students in general. For example, as noted in the results for the status of mathematics dimension, even though both samples viewed mathematics as a dynamic field, interview data showed that there were important differences in what students meant when they said mathematics is changing. Some students were referring to changes in instructional techniques, others were referring to available technology, and still others to changes in content learned from year to year. Similarly, when students evaluated the importance of analyzing errors and justifying responses, most agreed it is important but there were great differences in what it meant to individual students to analyze errors or make justifying statements. Thus the use of interviews in conjunction with surveys to develop a deep understanding of student conceptions was supported by this study.

Our survey results map the general terrain of student conceptions of mathematics within a seven dimension framework and provides indications where future, more fine-grained examination, would be appropriate. In light of Hammer's (1994) criteria for evaluating the adequacy of an analytic framework (recognizability, evident involvement, and consistency) our framework has been successful in capturing important themes defining student conceptions of mathematics and in identifying important similarities and differences among mathematically talented students and average algebra students. We have identified areas where further work can build on the general terrain mapped out by the survey results. In terms of recognizability, using a carefully developed framework allowed us to successfully build an instrument and interview protocols that recognized categories of conceptions. In terms of evident involvement, items representing concrete and context-bound situations that reflected dimensions of the framework were found to demonstrate more telling differences in response patterns. Interview questions brought to the forefront differences in student understanding of items that reflect important differences in student conceptions of mathematics. There is a need to examine more deeply student conceptions in the context of their own mathematical work in order to clarify some of the issues the survey results raise. In terms of consistency across tasks and content topics, further work would be helpful. Although our survey addresses different content and tasks and the results are consistent across these, the survey is limited in scope. For example, results from the learning of mathematics dimension indicated a fairly balanced view of the importance of memorization and understanding in learning mathematics, but it does not tell us if this balance comes from a view of mathematics that differentiates between procedural tasks and conceptual tasks, or is an integration of the roles of procedural and conceptual tasks.

Conclusions

Delineating important themes in defining student conceptions of mathematics has proved valuable in identifying differences in students conceptions, as well as in identifying views held in common by both mathematically talented students and typical algebra students. Mathematically talented students tended to view mathematics as a field composed of a system of coherent and interrelated concepts and principles, which is continuously growing. Doing mathematics is a sense-making process in which one must rely on personal thought and reflection to establish the validity of that knowledge. Algebra students also viewed mathematics as a dynamic and growing field, but they were much more likely to see it as a discrete system of facts and procedures that requires more memorizing than thinking. For them, doing mathematics often means implementing known procedures and formulas and accepting mathematically truths as established by others than depending on logical thought to deduce mathematical knowledge.

The agreement across the two samples on the usefulness dimension and the status dimension make the preceding differences in student conceptions even more important. Although both groups see mathematics as a dynamic and useful field, their conception of what doing and learning mathematics differs markedly. These commonalties and differences need to be examined in order to provide insight into the relationship between students' conceptions and their mathematical performance.

The baseline data gathered and examined in light of the seven dimensions raises questions about the relationship between students conceptions and their mathematical experiences in and out of school. The views shared by students on many aspects of the learning of mathematics may reflect shared classroom experience, whereas different views of what constitutes mathematical knowledge and doing mathematics may reflect differences in personal experiences with problem solving. Although we currently have insufficient understanding of the relationship between students' conceptions and their mathematical behavior in the classroom, the existence of these differences in mathematically talented and typical algebra students suggests that this is fertile ground for investigation. If these different conceptions of mathematics promote different classroom learning practices or different mathematical problem solving behavior, as we suspect, then student conceptions of mathematics warrants more careful attention from teachers and researchers.

References

Borasi, R. (1990). The invisible hand operating in mathematics instruction: Students' conceptions and expectations. In T. J. Cooney & C. R. Hirsh (Eds.), Teaching and learning mathematics in the 1990s (pp. 174-182). Reston, VA: NCTM.

Cai, J. (1995). A cognitive analysis of U.S. and Chinese student performance on tasks involving computation, simple problem solving , and complex problem solving. Journal for Research in Mathematics Education Research Monographs, No. 7. Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T. P., Lindquist, M. M., Matthews, W., & Silver, E. A. (1983). Results of the third NAEP mathematics assessment: Secondary school. Mathematics Teacher, 76, 652-659.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. Journal for Research in Mathematics Education, 16, 324-336.

Fennema, E., & Sherman, J. (1976). Fennema-Sherman mathematics attitudes scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males. Madison, WI: Wisconsin Center for Educational Research.

Fennema, E., Wolleat, P. L., Pedro, J. D., & Becker, A. D. (1981). Increasing women's participation in mathematics: An intervention study. Journal for Research in Mathematics Education, 12, 3-14.

Garofalo, J. (1993). Mathematical problem preferences of meaning-oriented and number-oriented problem solvers. Journal for the Education of the Gifted, 17, 26-40.

Hammer, D. (1994). Epistemological beliefs in introductory physics. Cognition and Instruction, 12, 151-183.

Kloosterman, P., & Stage, F. K. (1992). Measuring beliefs about mathematical problem solving. School Science and Mathematics, 92, 109-115.

Lindquist, M.M., Dossey, J.A., & Mullis, I.V.S. (1995). Reaching standards: A progress report on mathematics. Princeton, NJ: Educational Testing Service.

Middleton, J. A., Littlefield, J., & Lehrer, R. (1992). Gifted students' conceptions of academic fun: An examination of a critical construct for gifted education. Gifted Child Quarterly, 36, 38-44.

Mullis, I. V., Dossey, J. A., Owen, E. H., & Phillips, G. W. (1991). The state of mathematics achievement: NAEP's 1990 assessment of the nation and the trial assessment of the states. Washington, DC: National Center for Education Statistics.

NAEP. (1983). The third national mathematics assessment: Results, trends, and issues (Report No. 13-MA-01). Denver: Educational Commission of the States.

Oaks, A. B. (1987). The effects of the interaction of conception of mathematics and affective constructs on college students in remedial mathematics. Unpublished doctoral dissertation, University of Rochester. Rochester, NY.

Perry, W. B. (1970). Forms of intellectual and ethical development in the college years: A scheme. New York: Holt, Rinehart & Winston.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of "well taught" mathematics classes. Educational Psychologist, 23, 145-166.

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. Journal for Research in Mathematics Education, 20, 338-355.

Schommer, M. (1993). Epistemological development and academic performance among secondary students. Journal of Educational Psychology, 85, 406-411.

Schommer, M., & Walker, K. (1995). Are epistemological beliefs similar across domains? Journal of Educational Psychology, 87, 424-432.

Songer, N. B., & Linn, M. C. (1991). How do students' views of science influence knowledge integration? Journal of Research in Science Teaching, 28, 761-784.

Stigler, J. W., & Perry, M. (1988). Mathematics learning in Japanese, Chinese, and American classrooms. New Directions for Child Development, 41, 27-54.

Terwilliger, J. S., & Titus, J. C. (1995). Gender differences in attitudes and attitude changes among mathematically talented youth. Gifted Child Quarterly, 39, 29-35.

Thompson, A. G. (1992). Teacher beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 127-146). New York: Macmillan.

I. Nature of Mathematical Knowledge

1. Composition of Mathematical Knowledge

Knowledge as concepts, principles, and generalizations	Knowledge as facts, formulas, and algorithms

2. Structure of Mathematical Knowledge

Mathematics as a coherent system	Mathematics as a collection of isolated pieces

3. Status of Mathematical Knowledge

Mathematics as a dynamic field	Mathematics as a static entity

II. Nature of Mathematical Activity

4. Doing Mathematics

Mathematics as sense-making	Mathematics as results

5. Validating Ideas in Mathematics

Logical thought	Outside authority

III. Learning Mathematics

6.

Learning as constructing and understanding	Learning as memorizing intact knowledge

IV. Usefulness of Mathematics

7.

Mathematics as a useful endeavor	Mathematics as a school subject with little value in everyday life or future work

Figure 1: Four Themes and Their Related Dimensions for the Conceptions of Mathematics Inventory

Figure 2: Students responses to "Mathematical knowledge consists mainly of ideas and concepts and the connections among them."

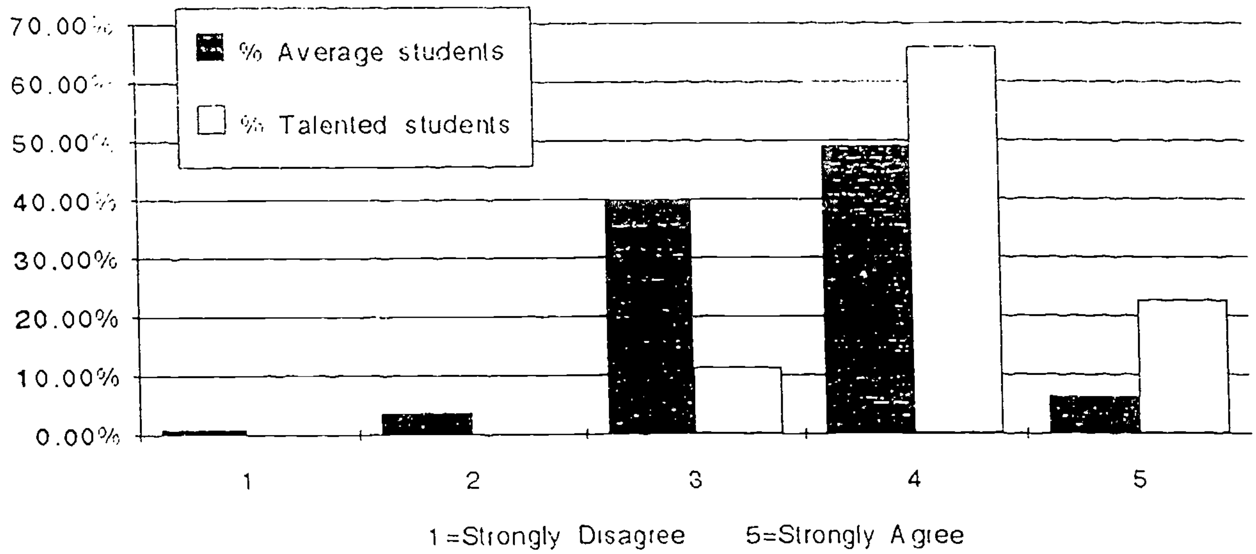

Figure 3: Students responses to "Finding solutions to one type of mathematics problem cannot help you solve other types of problems."

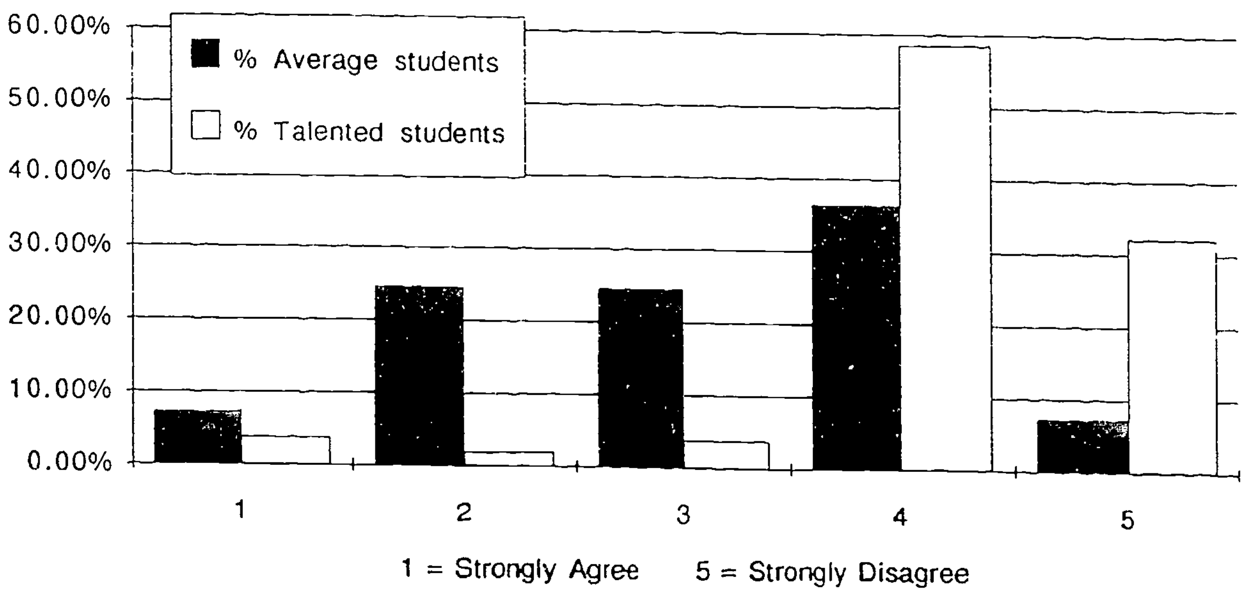

Figure 4: Student responses to "New discoveries are seldom made in mathematics."

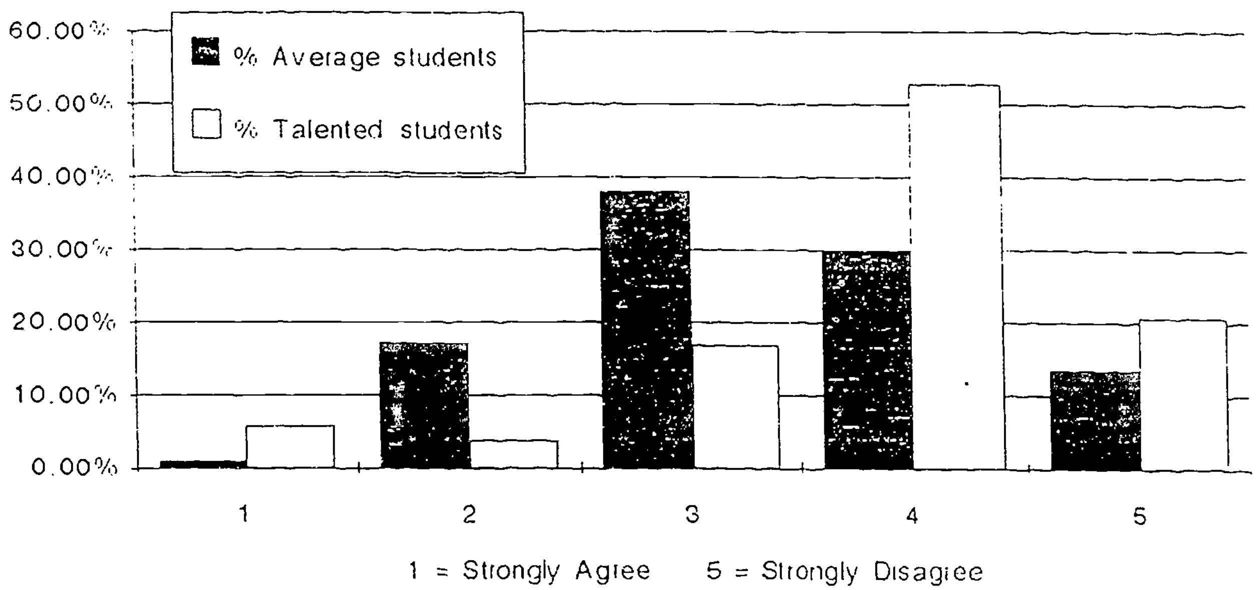

Figure 5: Student responses to "If you knew every possible formula, then you could easily solve any mathematical problem."

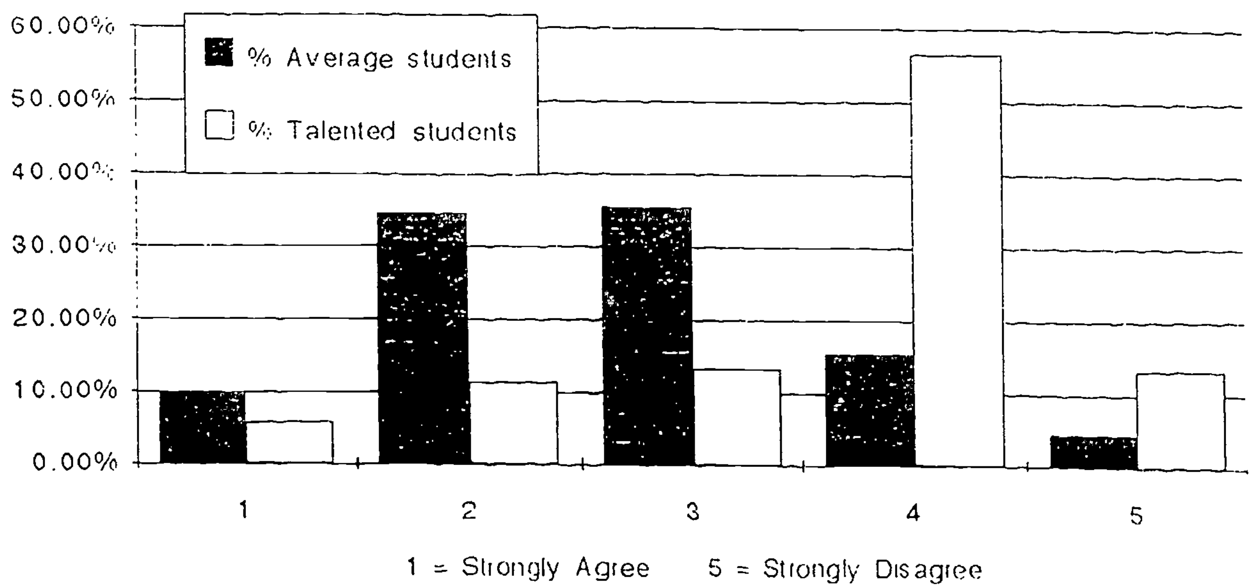

Figure 6. Student responses to "You know something is true in mathematics when it is in a book or a teacher tells you"

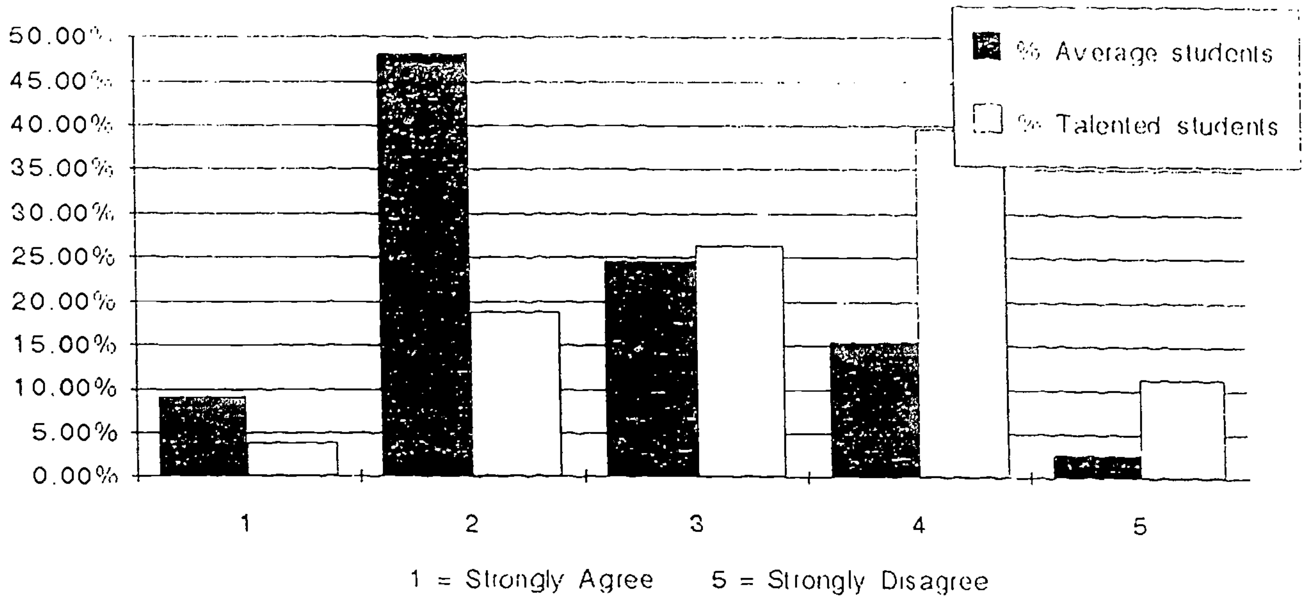

Figure 7: Student responses to "Justifying the statements a person makes is an important part of mathematics."

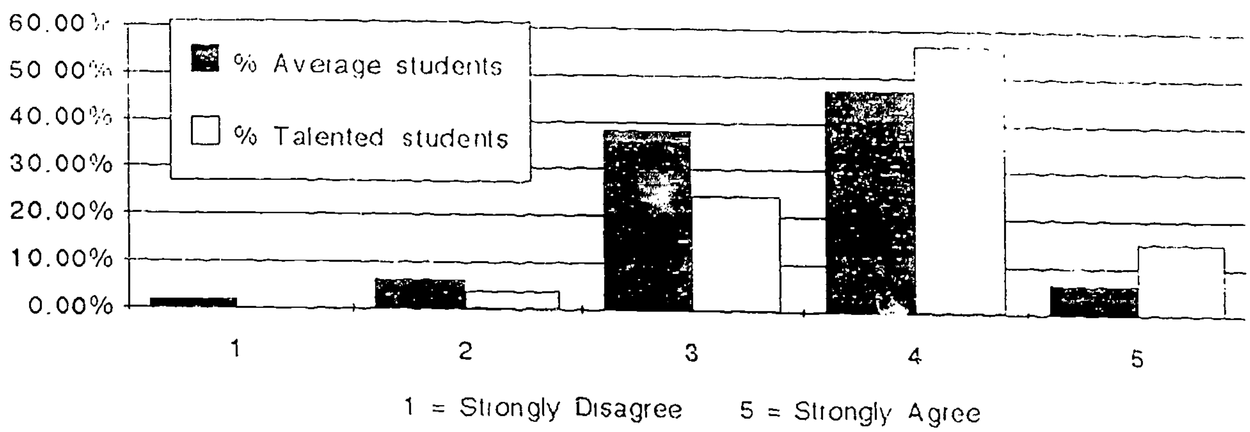

Figure 8: Student responses to "Learning to do mathematics problems is mostly a matter of memorizing the steps to follow."

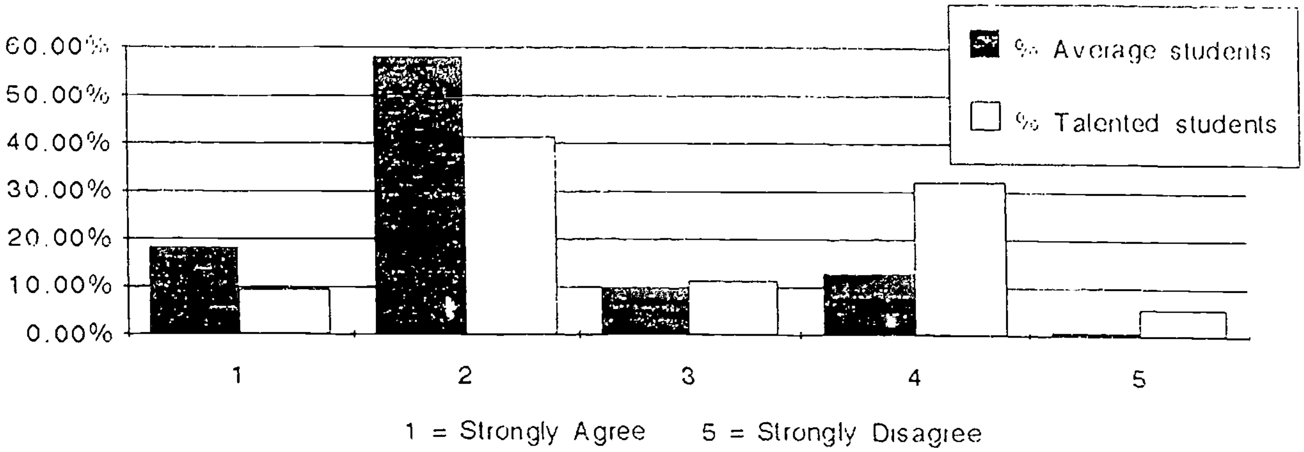

Figure 9: Student responses to "Mathematics will not be important to me in my life's work."