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ABSTRACT

This study was designed to investigate the effects of a dynamic instructional environment, English proficiency, and visualization level, independently and interactively, on middle school students' construction of the concepts of reflection and rotation. The effects of a dynamic instructional environment and English proficiency on students' two- and three-dimensional visualization was also examined. The dynamic instructional environment was based on the constructivist view of teaching and learning, the need for students to work within their zones of proximal development, and the use of context-embedded situations. After controlling for initial differences, it was concluded that students experiencing the dynamic instructional environment significantly outperformed students experiencing a traditional environment on content measures of the concepts of reflection and rotation as well as on measures of two-dimensional visualization. The students' instructional environment did not significantly affect their three-dimensional visualization. There were no statistically significant differences on any of the dependent variables between the performances of limited English proficient students and those of their English proficient peers when the same instructional environment was experienced. The research demonstrated that a computer-based, dynamic instructional environment can provide for successful outcomes in LEP and EP students' learning of the concepts of reflection and rotation. Implications of the findings are discussed. Contains 39 references. (Author/JRH)

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by
Juli K. Dixon
University of Florida

Volume 26 Summer 1995 Number 4

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RESEARCH BULLETIN

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F.E.R.C. NOTES ON THIS BULLETIN

Juli Dixon has made a real contribution to those who teach mathematics, especially to those who teach the typical eighth grade students in the modern middle and junior high school. F.E.R.C. is pleased to provide our readers with meaningful research.

Charlie T. Council
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Effects of English Proficiency, Visualization, and Instructional Environment in Constructing the Concepts of Reflection and Rotation

by
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EXECUTIVE SUMMARY

The exploration of the rigid motion transformations (for example, reflection and rotation) and experience with visualizing in two and three dimensions are integral parts of the *Standards* for eighth-grade students in mathematics. Furthermore, work with transformations and visualization skills are appropriate topics in the instruction of limited English proficient students within an English dominated classroom. These instructional areas and the needs of the growing population of limited English proficient students in our classrooms are not currently addressed adequately in the middle school curriculum in the United States.

This study was designed to investigate the effects of a dynamic instructional environment, English proficiency, and visualization level, independently and interactively, on middle school students' construction of the concepts of reflection and rotation. Also examined were the effects of a dynamic instructional environment and English proficiency on students' two- and three-dimensional visualization.

The dynamic instructional environment involved exclusive use of *The Geometer's Sketchpad* with lessons created for the purpose of allowing for student discovery of properties of reflection and rotation. The traditional environment consisted of typical classroom instruction with a textbook and occasional use of manipulatives. The choice of software, lessons, and instructional setting for the dynamic instructional environment was based on the constructivist view of teaching and learning mathematics, the need for students to work within their zones of proximal development, and the use of context-embedded situations in the instruction of students in general and limited English proficient students in particular.

After controlling for initial differences, it was concluded that students experiencing the dynamic instructional environment significantly outperformed students experiencing a traditional environment on content measures of the concepts of reflection and rotation as well as on measures of two-dimensional visualization. The students' instructional environment did not significantly affect their three-dimensional visualization. There were no statistically significant differences on any of the dependent variables between the performances of limited English proficient students and those of their English proficient peers when the same instructional environment was experienced.

INTRODUCTION

The exploration of the rigid motion transformations and experience with visualizing in two- and three-dimensions are integral parts of the National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) for eighth-grade students. These topics are not currently addressed adequately in the middle school curriculum in the United States.

Achievement in mathematics in a given language seems to be related to the degree of proficiency in that language (Secada, 1992). Contrary to what people may believe, the study of mathematics does not transcend language barriers. The changing demographics of the United States dictate that appropriate mathematics content, teaching strategies, learning tools, and classroom environments must be incorporated in all schools to close the gap of achievement in mathematics between English proficient (EP) and limited English proficient (LEP) students. Current programs designed with the pretense of closing this gap are "...evaluated based on their ability to curtail student drop out and to improve students scores on standardized tests. As a consequence, compensatory programs mimic the tests by which they are evaluated and which are focused on lower level and computational skills" (Secada, 1989 p. 38). This focus on lower level skills is not consistent with NCTM's *Standards* and is not an acceptable solution to the problem of the achievement gap.

This study was designed to investigate the effects of a dynamic instructional environment, English proficiency, and visualization level, independently and interactively, on middle school students' construction of the concepts of reflection and rotation. A second purpose of this study was to examine the effects of a dynamic instructional environment and English proficiency on students' two- and three-dimensional visualization.

The treatment in the dynamic instructional environment included exploring the rigid motion transformations of reflection and rotation through dynamic presentation and subsequent activities using *The Geometer's Sketchpad* (Jackiw, 1991). *The Sketchpad* is a highly visual and dynamic tool for exploring and discovering geometric properties. This study established the effectiveness of *The Sketchpad* as a tool for the instruction of all students, including LEP students, to successfully recognize and perform reflections and rotations. *The Sketchpad* was also useful for improving students' visualization ability.

Theoretical Framework

This study was designed to explore whether an environment in which English proficient and limited English proficient students work together using computers will result in their construction of knowledge of reflections and rotations while simultaneously improving visualization skills. This research was supported by a combination of three educational theories. The constructivist view of teaching and learning mathematics (Confrey, 1990; Davis, Maher, & Noddings, 1990; Goldin, 1990; Kamii and Lewis, 1990; Noddings, 1990; von Glasersfeld, 1990; Wheatley, 1991) explains how teachers can create environments in which students can construct knowledge. Vygotsky's (1978, 1986) zone of proximal development addresses how students working together, with adult guidance as needed, and using proper tools could construct this knowledge socially. Cummins' (1981, 1984) theory of context embedded language versus context reduced language justifies the use of *The Sketchpad* for EP and LEP students as they work together in the same educational setting.

Numerous studies have provided evidence that students can learn to identify and perform the rigid motion transformations of reflection and rotation (Edwards, 1991; Ernest, 1986; Hart, 1981; Johnson-Gentile, 1990; Williford, 1972). Results from research have provided evidence that spatial visualization is linked to achievement in mathematics (Battista, Wheatley, & Talsma, 1982; Connor and Serbin, 1985; Kiser, 1990; Moses, 1977; Perunko, 1982; Tillotson, 1984). Studies have provided evidence that spatial visualization is a trainable skill (Baker, 1990; Ben-Chaim, Lappan, & Houang, 1988; Brinkman, 1966; Connor, Schackman, & Serbin, 1978; Connor and Serbin, 1985; Lord, 1985; Miller and Miller, 1977; Moses, 1977; Tillotson, 1984). These studies have led this researcher to believe that the type of lesson used to increase students' spatial visualization effects the extent of the increase in spatial visualization. Instructional environments providing student-student and/or teacher-student interaction and hands-on activities seemed most effective. This study attempted to provide an environment incorporating such interaction and activities. Projects in science education have set the framework for environments in which LEP students may work within their zones of proximal development constructing meaningful mathematics while simultaneously building their associated English vocabulary (Rosebery, Warren, & Conant, 1992; and Warren & Rosebery, 1993). It is the synthesis of this information gained from these valuable studies and the theoretical framework previously described that provides the foundation and justification for this study.

METHODOLOGY

Research Objective

This study was designed to: (1) explore the effects of a dynamic instructional environment and students' visualization ability on the identification of and ability to perform the rigid motion transformations of reflection and rotation; and (2) explore the effects of the relationship between the students' level of visualization and the instructional environment of the identification of and ability to perform these rigid motion transformations. Another purpose of this study was to investigate the effects of students' English proficiency on identification and performance of reflections and rotations and the effects of the relationship between students' English proficiency and a dynamic instructional environment on the identification of and ability to perform reflections and rotations. Finally, the study was designed to ascertain the effects of a dynamic instructional environment and students' English proficiency on students' visualization. The following null hypotheses were tested:

1. Students' level of visualization will not interact with their instructional environment to affect their ability to identify and perform reflections and rotations in a (a) dynamic testing environment or a (b) static testing environment.

2 Students' English proficiency will not interact with their instructional environment to affect their ability to identify and perform reflections and rotations in a (a) dynamic testing environment or a (b) static testing environment.

3. Students' (a) instructional environment, (b) visualization level, and (c) English proficiency will not affect their ability to identify and perform reflections and rotations in a (a) dynamic testing environment or a (b) static testing environment.

4. Students' (a) instructional environment and (b) English proficiency will not affect their (a) two-dimensional or (b) three-dimensional visualization.

Measures

The measures of the study consisted of three covariates (one continuous and two discontinuous) and four posttests. The covariates consisted of the Card Rotation Test and Paper Folding Test of the *Kit of Factor-Referenced Cognitive Tests* (Ekstrom et al., 1976) as well as the Language Assessment Battery (LAB). The Card Rotation and Paper Folding Tests were also used as posttests. Two posttests were designed by the researcher, used in a pilot study, and validated for content by four experts. These tests are the Paper and Pencil Reflection/Rotation Instrument and the Computer Reflection/Rotation Instrument. The instruments were adapted, with permission, from the examination designed for a large scale British study which was part of the Concepts in Secondary Mathematics and Science (Hart, 1981).

Research Sample

The sample represented students in Florida middle schools having Macintosh computer laboratories and LEP students enrolled in English speaking, heterogeneously grouped mathematics classes. The data were collected from a central Florida middle school which had three eighth-grade teams. Each eighth-grade team had one mathematics teacher. One mathematics class on each team was made up of "advanced" students chosen at the end of their sixth-grade school year based on mathematics grades. The other three classes on each team consisted of the remaining students, heterogeneously grouped. The research sample was made up of these nine, heterogeneously grouped classes. All classes contained LEP students.

The researcher taught four of the heterogeneously grouped classes. These intact classes made up the Treatment Group. The remaining five intact, heterogeneously grouped classes were taught by their regular mathematics teacher. These classes made up the Control Group. The researcher began teaching the Treatment Group at the beginning of the school year.

Data Collection

All students were taught how to use *The Geometer's Sketchpad* prior to collection of the data. All training was conducted by the researcher and lasted approximately one hour and forty-five minutes. The school followed block scheduling; mathematics classes met every other day for approximately one hour and forty-five minutes. The training lesson required the students to use all of the tools necessary to successfully complete the Computer Reflection/Rotation Instrument. At no time during the training did the students witness or perform reflections or rotations using *The Sketchpad*.

Following the training on use of *The Geometer's Sketchpad*, all classes were given the Card Rotation and Paper Folding Tests. After completion of the above measures, the Control Group was taught about the concepts of reflection and rotation using chapters from *Transition Mathematics* (Usiskin et al., 1990), the adopted textbook for eighth-grade at this school. The Treatment Group was taught in the computer lab using *The Geometer's Sketchpad*. The reflection and rotation units for the Treatment and Control Groups each lasted approximately three weeks.

The students in the Treatment Group worked in pairs at computers throughout the unit on reflections and rotations. Several pairs included students whose first language was Spanish. These students tended to converse in Spanish when discussing properties of the geometric transformations as well as problems posed by the researcher.

All experimental treatment lessons were preplanned so that any teacher with adequate knowledge of geometry, middle school pedagogy, the computer software, and computer lab pedagogy could teach using the lesson plan. The unit was divided into the following three lessons each lasting at least one class period: (1) training in *Sketchpad* reflection and rotation procedures; (2) practice conjecturing with reflections and rotations in order to discover the properties of the transformations; and (3) illustrating and describing properties of reflections and rotations.

Students in both the Treatment and Control Groups were administered the Card Rotation Test, the Paper Folding Test, the Paper and Pencil Reflection/Rotation Instrument, and the Computer Reflection/Rotation Instrument directly following the instruction on the concepts of reflection and rotation. Approximately four weeks elapsed from the beginning of the data collection to the end of the data collection. Complete data were collected on 241 students with 109 students in the Treatment Group and 132 students in the Control Group.

Research Design

A three factor, nonequivalent control-group design was used for the study. This quasi-experimental design involved a 2x2x3 matrix to examine three factors. The three factors were the instructional environment, the level of English proficiency, and the level of visualization of the students.

An analysis of covariance ((ANCOVA) was used to control for initial differences between groups. The Card Rotation Test, the Paper Folding Test, and the LAB served as covariates. The Card Rotation Test was the continuous covariate used to control for initial differences between groups. The Paper Folding Test was the discontinuous measure used to assign students to visualization levels. The LAB was used to measure students' English language proficiency. The objective of the design was to determine the effects of the independent variables (instructional environment, English proficiency, and visualization), individually and interactively, on the dependent variables (the Reflection/Rotation Instruments).

Additional hypotheses were tested using a 2x2 matrix to examine the effects of instructional environment and English proficiency on students' visualization level. These hypotheses were also tested using ANCOVAs. The ANCOVAs were run with the purpose of describing the effects of the two independent variables (instructional environment and English proficiency) on the performance of the sample on the Card Rotation Test (a measure of two-dimensional visualization) and the Paper Folding Test (a measure of three-dimensional visualization).

RESULTS

Based on the results of ANCOVAs it was concluded that students experiencing the dynamic instructional environment (*The Sketchpad*) significantly outperformed students experiencing the traditional instructional environment on content measures of the concepts of reflection and rotation as well as on measures of two-dimensional visualization. There were no significant interactions between the students' visualization level or English proficiency and instructional environment on their identification and performance of reflections and rotations. There was no significant difference in students experiencing the dynamic instructional environment to students experiencing the traditional instructional environment on three-dimensional visualization. Limited English proficient students did not perform statistically significantly differently than their English proficient peers on any of the dependent variables when experiencing the same instructional environments. Therefore, limited English proficient students experiencing the dynamic instructional environment performed significantly better than their limited English proficient peers not experiencing the dynamic environment on measures involving reflection, rotation, and two-dimensional visualization.

As in Tillotson's (1984) investigation of sixth-grade students and unlike Kiser's (1990) study of college algebra students, the students' visualization level did not further enhance test scores for either the Treatment or Control Group. Hence, the relationship between students' visualization level and outcomes on the Paper and Pencil and Computer Reflection/Rotation Instruments were the same for students in the dynamic instructional environment and students in the traditional instructional environment, that it, students with high levels of visualization did not score higher on the Paper and Pencil or Computer Reflection/Rotation Instruments than expected for either the Treatment or Control Groups.

The lack of statistically significant interaction between students' English proficiency and instructional environment on performance on either Reflection/Rotation Instrument signified that the relationship between English language proficiency and performance on the tests is the same regardless of the students' instructional environment. Students experiencing the dynamic instructional environment (*The Sketchpad*) did not benefit more than expected whether they were English proficient or limited English proficient.

Overall, use of *The Geometer's Sketchpad* in a dynamic instructional environment was shown to be an effective medium through which to construct the concepts of reflection and rotation. Students experiencing this dynamic instructional environment outperformed students experiencing a traditional instructional environment on content measures involving reflection and rotation. These results are consistent with conclusions made by Moyer (1978) who found that using motion was helpful in teaching the concept of rotation to primary students. Pleet (1990) who also investigated eighth-grade students' knowledge of reflection and rotation, however, did not find the computer program *Motions* more helpful than using the manipulative, *Mira*, to teach transformations. Kidder (1976) contended that adolescents were unable to perform transformations;

whereas, the adolescents in the current study were able to perform transformations.

In the current study, both English proficient and limited English proficient students benefited equally from the dynamic instructional environment. The marked improvement of LEP students experiencing the dynamic instructional environment based on constructivist views of teaching and learning mathematics compared to their LEP peers experiencing the traditional classroom environment further corroborates findings involving science instruction (Rosebery, Warren, & Conant, 1992; and Warren & Rosebery, 1993). The limited English proficient students in the studies involving science instruction improved content knowledge as well as English vocabulary after experiencing the constructivist-based instructional environment.

As was found in many studies (Battista, Wheatley, & Talsma, 1982; Connor & Serbin, 1985; Kiser, 1990; Moses, 1977; Perunko, 1982; and Tillotson, 1984), students' visualization level predicted performance on mathematical tasks, specifically the Reflection/Rotation Instruments.

Unlike Pleet's (1990) study, the computer-based environment significantly improved students' two-dimensional visualization as measured by the Card Rotation Test when compared to students not experiencing the computer-based environment. This inconsistency in results may be due to the dynamic nature of *The Geometer's Sketchpad* used in this study compared to *Motions*, the software used by Pleet. The comparison is interesting due to the similarity in subjects and duration of the studies, both studies involved the instruction of eighth-grade students for approximately three weeks. The *Mira* was used by a second treatment group in Pleet's study and made available to the Control Group teachers in this study.

IMPLICATIONS

Based on this study, there are no grounds for excluding students from participating in instruction in the type of computer lab used in this study on the basis that students' level of visualization is not high enough to make the experience worthwhile. However, since students with high visualization levels score higher than students with low visualization levels on measures of conceptual knowledge of reflection and rotation, visualization skills should be a focus of mathematics education especially since they can be improved. Since visualization is a predictor of achievement in mathematics and since visualization is improved through experience with activities directly related to the specific visualization skills, varied situations involving many different types of visualization skills should be incorporated into the mathematics curriculum.

Since visualization is a predictor of mathematics achievement and the visualization of LEP students can be improved, effort should be focused on improving the visualization skills of LEP students. Improving such skills could aid in closing the achievement gap in mathematics between LEP students and their EP peers.

Teachers do not need to limit students' use of their native language in the classroom for fear that the English vocabulary words associated with the mathematics being taught will not be learned. The results of this study add strength to the argument that LEP students should not be excluded from situations where meaningful mathematics may be constructed simply on the basis of their English proficiency. Students do not need to be English proficient to benefit from instruction that takes place while using computers in a lab.

The use of the computer lab alone is not sufficient to insure students' construction of mathematical concepts. Choosing appropriate software is important as is the training of teachers on both proper pedagogy for teaching in computer labs and use of the chosen software. The teacher, through the use of appropriate software, must create an environment that provides comprehensible input to the students so that they may work successfully within their zones of proximal development to construct meaningful mathematical concepts.

Mathematics educators need to ask themselves whether or not they are meeting the needs of LEP and EP students in their classes by providing environments in which students are furnished with context-embedded situations involving meaningful mathematics. This research has demonstrated that a computer-based, dynamic instructional environment can provide for successful outcomes in LEP and EP students' learning of the concepts of reflection and rotation, a small part of geometry and an even smaller portion of the middle school curriculum. It remains to be shown that similar instructional environments can afford LEP and EP students the opportunity to construct other areas of mathematics described in the *Standards*.

REFERENCES

Baker, J.C. (1990). *Spatial strategies and spatial training in the collegiate curriculum*. Paper presented at the annual meeting of the New England Educational Research Organization, Rockport, ME.

Battista, M.T., Wheatley, G.H., & Talsma, G. (1982). The importance of spatial visualization and cognitive development for geometry learning in preservice elementary teachers. Journal for Research in Mathematics Education, 13(5), 332-340.

Ben-Chaim, D., Lappan, G. & Houang, R.T. (1988). The effects of instruction on spatial visualization and cognitive development for geometry learning in preservice elementary teachers. *Journal for Research in Mathematics Education, 13(5),* 332-340.

Brinkman, E.H. (1966). Programmed instruction as a technique for improving spatial visualization. *Journal for Applied Psychology, 50(2),* 179-184.

Confrey, J. (1990). What constructivism implies for teachers. In R.B. Davis, C.A. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 107-122). Reston, VA: NCTM.

Connor, J.M., Schackman, M., & Serbin, L.A. (1978). Sex-related differences in response to practice on a visual-spatial test and generalizations to a related test. *Child Development, 49,* 24-29.

Connor, J.M., & Serbin, L.A. (1985). Visual-spatial skill: Is it important for mathematics? Can it be taught. In S.F. Chipman, L. R. Brush, & D.M. Wilson (eds.) Women and mathematics: Balancing the equation (pp. 151-174). Hillsdale, NJ: Erlbaum Associates Inc.

Cummins, J. (1981). Empirical and theoretical underpinnings of bilingual education. Journal of Education, 163(1), 16-29.

Cummins, J. (1984). Bilingualism and special education: Issues in assessment and pedagogy. San Diego, CA: College-Hill Press.

Davis, R.B., Maher, C.A., & Noddings, N. (1990). Suggestions for the improvement of mathematics education. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 187-191). Reston, VA: NCTM.

Edwards, L. (1991). Children's learning in a computer microworld for transformation geometry. *Journal for Research in Mathematics Education, 22(2),* 122-137.

Ekstrom, R., French, J., Harman, H., & Derman, D. (1976). Kit of factor-referenced cognitive tests. Princeton, NJ: Educational Testing Service.

Ernest, P. (1986). Computer gaming for the practice of transformation geometry skills. *Educational Studies in Mathematics, 17,* 205-207.

Goldin, G.A. (1990). Epistemology, constructivism, and discovery learning in mathematics. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 187-191). Reston, VA: NCTM.

Hart, K. (Ed.) (1981). Children's understanding of mathematics: 11-16. London: Concepts in Secondary Mathematics and Science, University of London.

Jackiw, N. (1991). The Geometer's Sketchpad. Berkeley, CA: Key Curriculum Press.

Johnson-Gentile, K. (1990). The effects of computer and noncomputer environments of fifth- and sixth-grade students' conceptualizations of geometric motions (State University of New York at Buffalo 1990) *Dissertation Abstract International, 51*(4), 1100A.

Kamii, C., & Lewis, B.A. (1990). Research into practice. Constructivist learning and teaching. *Arithmetic Teacher, 38(1)*, 34-35.

Kidder, F.R. (1976). Elementary and middle school children's comprehension of Euclidean transformation. Journal for Research in Mathematics Education, 7(1), 40-52.

Kiser, L. (1990). Interaction of spatial visualization with computer-enhanced and traditional presentations of linear absolute-value inequalities. Journal of Computers in Mathematics and Science Teaching 10(1), 85-96.

Lord, T.R. (1985). Enhancing the visuo-spatial aptitude of students. *Journal of Research in Science Teaching, 22(5)*, 395-405.

Miller, J.W., & Miller, H.G. (1977). Toward resolution of the spatial puzzle. *Peabody Journal of Education*, *54*, 135-141.

Moses, B.E. (1977). The nature of spatial ability and its relationship to mathematical problem solving (Doctoral dissertation, Indiana University, 1977). Dissertation Abstract International, 38(8), 4640A.

Moyer J.C. (1978). The relationship between the mathematical structure of Euclidean transformations and the spontaneously developed cognitive structures of young children. Journal for Research in Mathematics Education, 8-9, 83-92.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 187-191). Reston, VA: NCTM.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

Perunko, M.A. (1982). The relationships among mental imagery, spatial ability, analytic-synthetic processing and performance on mathematics pro' 'ems (Doctoral dissertation, University of Maryland, 1982). -issertation Abstracts International, 44(6), 1716A.

Pleet, L.J. (1990). The effects of computer graphics and Mira on acquisition of transformation geometry concepts and development of mental rotation skills in grade eight (Doctoral dissertation, Oregon State University, Pullman, 1990). Dissertation Abstract International, 52(6), 2058A.

Rosebery, A.S., Warren, B., & Conant, F.R. (1992). Approaching scientific discourse: Findings from language minority classrooms. (ERIC Document Reproduction Service No. ED 352 263)

Secada, W.G. (1989). Agenda setting, enlightened self-interest, and equity in mathematics education. Peabody Journal of Education, 66(2), 22-56.

Secada, W.G. (1992). Evaluating the mathematics education of limited English proficient students in a time of educational change. Proceedings in the Second National Research Symposium on Limited English Proficient Student Issues: Focus on Evaluation and Measurement Volume II, (pp. 209-256). Washington, DC: United States Department of Education Office of Bilingual Education and Minority Language Affairs.

Tillotson, M.L. (1984). The effects of instruction in spatial visualization on spatial abilities and mathematical problem solving (Doctoral dissertation, University of Florida, Gainesville, 1984). Dissertation Abstract International, 45(9), 2792A.

Usiskin, Z. et al., (1990). The University of Chicago School Mathematics Project: Transition mathematics. Glenview, IL: Scott Foresman and Company.

von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R.B. Davis, C.A. Maher, & N. Noddings (Eds), *Constructivist views on the teaching and learning of mathematics* (pp. 187-191). Reston, VA: NCTM.

Vygotsky, L.S. (1978). Mind in society. Cambridge, MA: Harvard University Press.

Vygotsky, L.S. (1986). Thought and language. Cambridge, MA: MIT Press.

Warren, B., & Rosebery, A.S. (1933). Equity in the future tense: Redefining relationships among teachers, students, and science in linguistic minority classrooms. (ERIC Document Reproduction Service No. ED 356 307)

Wheatley, G.H. (1991). Constructivist perspectives on science and mathematics. *Science Education, 75(1)*, 9-21.

Williford, H.J. (1972). A study of transformational geometry instruction in the primary grades. *Journal for Research in Mathematics Education, 3*, 260-271.