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ABSTRACT

This paper reports a study of student knowledge of percent at the elementary, middle, and secondary levels. The study concentrated on three areas: (1) an analysis of the percent concept, its symbols, and procedures; (2) the child's naturally constructed base of informal knowledge of the percent concept; and (3) what formal knowledge develops in terms of concepts and procedures as a result of traditional instruction. A written test containing a variety of questions to indicate informal and formal knowledge of percent was given to classes of students in grades 5, 8, and 11. Results showed that students used a variety of formal and informal strategies indicating flexible knowledge of concepts and procedures. Also, it was found that students had some understanding of percent prior to instruction. Contains 17 references. (MKR)

INFORMAL AND FORMAL KNOWLEDGE:

THE DOMAIN OF PERCENT

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INFORMAL AND FORMAL KNOWLEDGE :
THE DOMAIN OF PERCENT

Area of Concern

The Percent Concept

The ability to understand and solve problems involving percent is important for students in many fields of study such as the physical sciences, business areas and the social sciences. Furthermore, the use of percent continues throughout adult life in managing one's personal finances and in reading news articles concerned with economic issues. Percent has been viewed as an important topic for many years in our schools and the NCTM Standards (1989) continues to recommend its inclusion for today's curriculum. However, test results from the NAEP (Brown, Carpenter, Kouba, Lindquist, Silver and Swafford, 1988) show that seventh and eleventh grade students' understanding of percent is extremely deficient. Likewise, there is little pedagogical content knowledge (Shulman,1986) about percent and little analysis of the mathematical structure, symbols, and reasons justifying procedures with percent.

The Research Question

The importance of the percent concept, the poor level of student understanding, and the lack of findings on percent indicate needed research. I propose a careful mathematical analysis of percent and a study of student knowledge at the elementary, middle and secondary level to illustrate the presence and growth of informal and formal knowledge, including conceptual and procedural knowledge. This study would concentrate on three areas: (1) an analysis of the percent concept, its symbols, and procedures, (2) what is the child's naturally constructed base of informal knowledge of the percent concept (3) what formal knowledge develops in terms of concepts and procedures as a result of traditional instruction. These questions include concern about the strategies or procedures that children use with percent prior to and after instruction, common errors, and other problematic factors .

Theoretical Rationale for Study of Percent

The theoretical rational for this study includes that conceptual knowledge can be distinct from procedural knowledge as some students can perform procedures without a conceptual understanding. However, most frequently this results in "buggy" algorithms (Brown & Burton, 1978) or little number sense about the answer. Mathematics learning from situations has been called "situated" (Brown, Collins & Duguid, 1989) or "informal" (Carraher, Carraher & Schliemann, 1985) and generally implies learning that takes place out of school and is often distinct from formal knowledge. Formal knowledge is that which is acquired by formal instruction (Saxe, G. B., 1990). Numerous works have reported how children enter school with considerable personal knowledge of number concepts and of problem solving prior to any formal schooling (Gelman and Gallistel, 1978; Carpenter and Moser, 1984). Other works show that the construction of knowledge continues after formal schooling has begun (Carraher, Carraher & Schliemann, 1985: Lave, Murtaugh, & de la Rocha, 1984).

Percent is a topic where there is much informal exposure by shopping experiences, news articles , and the percent grading of school work. An analysis of how children reason about percent in informal contexts would allow curriculum development that would build upon the child's correct intuitive thinking. On the other hand, a reconstruction of some false intuitions might be needed. It is hoped that this curriculum would be designed to connect informal and formal knowledge and conceptual and procedural knowledge.

Analysis of Concept of Percent

It is necessary to analyze the concept of percent in terms of its mathematical structure and its relationship to symbols and procedures to have a framework in which to consider students' understandings. This discussion is not included in this paper but consists of the following topics and is available upon request.

<u>Percent as a Symbol System</u>

<u>Percent as an Intensive Relationship</u>

<u>Percent as a Part -Whole Model</u>

<u>Percent as a Functional Operator</u>

<u>Percent as a Real Number</u>

<u>Percent as a Proportion</u>

<u>Text Treatment of Percent Versus Mathematical Structure</u>

<u>Type 1 problems: Finding a percent of a number</u>

<u>Type 2 problems: Finding a Percent</u>

<u>Type 3 problems: Finding a number when a percent of it is known.</u>

Previous Research

Previous research which has influenced this project are studies in proportional reasoning as well as in percent. Proportional reasoning is a method of solving percent problems which closely models the underlying mathematical relationship of a rate per hundred compared to another rate.

It seems logical that understanding of percent may be developed informally or formally along the lines of proportional reasoning.

Proportion Studies

Proportional reasoning has a well established research base which I describe only briefly here. We know that that most adolescents make little progress in working with proportional relationships (Hart ,1981). Students can work with ratio by a building up process, but they are limited to halving, doubling and easy numbers and often use incorrect additive strategies. We also know that better facility with fractions and decimals before introducing proportions does not guarantee the ability to understand and solve word problems (Heller, Post ,Behr& Lesh, 1990).

Several teaching studies have been successful with elementary children which stressed activities to develop intuition and problems in familiar contexts and easy numbers (Tourniaire,1986, Streefland ,1984 & 1985) . Another study had success teaching proportions with specific attention to the róle of vocabulary to encode between words and symbols (Jackson and Phillips ,1983) .

Percent Studies

Early studies in percent were surveys to determine the competency and areas of difficulty of students at middle school through college levels. These found poor performance, particularly in conversions, such as fractions to percent. These studies refer to students' rote use of rules without understanding and inappropriate answers, suggesting that the curriculum needed revision. They all used tests which were primarily pure number problems with no context .

A number of more recent, teaching studies sought to determine if different approaches to teaching percent would improve competence.

There were significant differences on some of the subtests for average and above average students, but no significant differences for large groups on a total test on percent, and the method of preference was not consistent.

A third line of work has been to develop a hierarchy of skills for competence in percent. Results show a correlation in certain subtests on percent and some hierarchy predictions supported. However the test was mostly conversions between representations, with the percent exercises generally left blank or with very low scores.

The failure of most students to exhibit either a procedural or conceptual knowledge percent may be linked to the instruction in this area. The treatment in most texts (Houghton, Mifflin Mathematics, 1987; Addison-Wesley Mathematics, 1985) is very procedural with much drill on translations between percent, decimal and fraction representations, with little or no natural examples or building up strategies between equivalent fractions.

Research Project

The problem seems to exist today that was first noted in 1920: that students do not understand the meaning of the percent concept. Research to date has not attempted to determine children's informal knowledge on percent or the types of reasoning and strategies used on problems either before or after instruction. The successful teaching of proportions to young children, the call for teaching with pedagogical content knowledge, and the constructivists theory that children build knowledge based on informal and formal situations all suggest that we should find out how children reason about percent before formal instruction as well as after. This study has students from elementary, middle, and high school respond to a written set of questions, primarily in familiar contexts, and to questions during individual interviews about their knowledge and reasoning on percent. The written questions are designed around hypotheses, but the interview is more exploratory in nature and could produce unexpected findings. The results of this study will give us information about the child's reasoning about percent , their informal and formal knowledge, and the growth of conceptual and procedural knowledge of percent.

General Hypotheses

Informal Knowledge

It is hypothesized that children will have some informal knowledge of the percent concept due to the many references to percent in the news, in shopping and in school grades; albeit, quite limited. They will be able to do percent problems in familiar situations using easy numbers and stated as "for every one hundred". It is expected that children will be more successful solving these problems as compared to those using the "percent" term or symbol. In other words, the student can deal with the mathematical relationship, but not the percent term or symbol. It is expected that knowledge of 100%, 50% and 10% may be intuitive. It is expected that visual representations of relationships will aid understanding.

Consistent with the constructivists philosophy, it is hypothesized that children will use informal and invented solution strategies. As an extension of their fraction knowledge they should also use between and within ratio strategies to solve problems; however, they will also use fall back additive strategies.

Formal Knowledge

It is expected that formal knowledge will increase the understanding of the percent concept, symbols, and the use of formal procedures of

Algebra and the cross products algorithm. However, some students may begin to loose the connection to the context and give ridiculous answers. The translations between representations should also improve, but will not affect competence on informal rate problems. It is expected that translations and percent over 100 or under 1 will remain difficult even after instruction, consistent with other findings. It is also predicted that percent problems of type 1, 2 and 3 are of similar difficulty for informal rate problems, but are different for the "percent" problems. It is expected that there will be a variety of informal and formal procedures used by individuals and by the group both before and after instruction on percent.

Sample

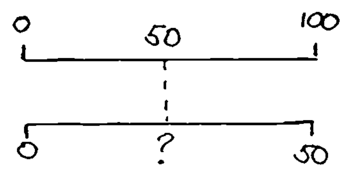
A small suburban town is the first site of this study. A larger site with a mix of race, ethnic, and economic levels of people is planned to further this research. The town is predominantly a white middle/upper socioeconomic level and has two elementary schools and a combination middle/ high school.

Grade 5 was chosen because percent is introduced at this grade level. The students participated in the study prior to instruction in percent. Grade 8 was chosen as students after instruction as percent is only reviewed at the end this grade. Two classes at the high school were chosen as several years after formal instruction.

Written Test on Percent

The written test contains a variety of questions to indicate informal and formal knowledge of percent. Most sections of the test include a balance of the traditional type 1, type 2 and type 3 percent problems (see appendix). The first 6 problems are short answer and intend to be a novel, visual representation of a proportional relationship seeking student informal knowledge of a multiplicative relationship. The verbal instructions to students were as follows: "The example is a drawing of two rulers exactly the same length, but marked off in different units. They are lined up together evenly. The top ruler goes from 0 to 100 units, and the bottom ruler has different sized units that go from 0 to 50. If I go over to the 50 mark on the top ruler, what mark would be shown on the bottom ruler? The answer is as shown : 25."

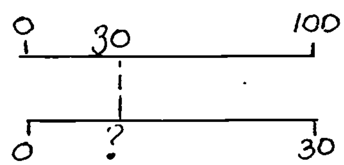
A. Example or two rulers:



Here the ?= 25.

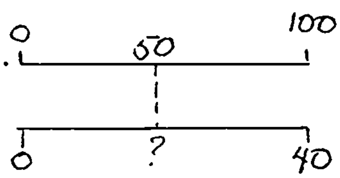
Here are two sample problems from this section.

1.



?= __________

2.



?= __________

The next 7 questions are short answer and intend to detect general, informal knowledge of percent as illustrated below:

1. If Sue got 85% correct on her vocabulary test, what percent did she get wrong? __________

3. If Sue got 6 out of 9 problems correct on her math quiz, which statement is true about her percent grade on the quiz. (circle the correct answers)

(a) greater than 100% (b) less than 50% (c) equals 100%

(d) greater than 50% (e) none of these

For the next 12 word problems, it was explained that students needed "to write some numbers or words to show how you came up with your answer - even if you did it in your head." The first 6 of these problems were percent problems using "for every one hundred" instead of the "percent" term. These were to see how students, prior to and after instruction, would solve these problems as compared to the remainder which used the "percent" term. Here is an example of each type:

1. A school reported that usually 5 out of every 100 students needs special help in reading. If this school has 700 students, how many would you expect to need the special help?

9. A school reported that 5% of its students walk to school. If 15 students walk to school, what is the total number of students in the school?

The last 9 problems were translation of representation-type problems among percent, decimal and fractional forms to see how closely success on this part relates to other parts. Here are two examples of this section.

1. Circle all of the correct ways of writing 5% .

(a) 5 (b) 20 (c) .5 (d) 500 (e) .05 (f) none of these ______

3. Circle all of the correct ways of writing .15

(a) 15% (b) 1.5% (c) .15% (d) 150% (e) .015% (f) none of these ____

To adjust the time of the test and to ask some harder questions, grades 8 and 11 had an additional 10 problems. These extra problems were a combination of short answer and word problems.

Test Administration

The written test core of 34 questions was administered to 6 math classes, two classes each at grades 5, 8 and 11 during a regular class period.

Test Scoring

The test was scored by the researcher as correct, by actual answer , and by the strategy used (see appendix) , such as algebraic equation, between ratio strategy, etc.

Interview Procedures (Note: these are in progress)

Three students from each class were interviewed. The interviews were 15-20 minutes in length and consisted of 6-8 questions, (see appendix), on the student's ideas about percent. The interviews sought confirmation of students' informal and formal knowledge of the percent concept and the strategies used to solve percent problems. Another purpose was to verify that diagrams and questions on the written test were understood by the students. The interviews were tape recorded, and transcribed.

Statistical Analysis (Some results now, but still in progress)

The results of the written test was analyzed using the SAS statistical package. So far I have determined frequencies at each grade level of answers and strategies for each problem and for types of problems. Analysis of correlation, variance, and Chi-Square are planned to determine the interaction of the following variables: grade level, age, problem type, "percent problems", proportional percent problems, strategies, and translation skills.

Results

This study is still in progress and the results are preliminary. The written class data has been collected at the first site, and the data from 4 of the 6 classes has been recorded and some statistical analysis begun. The classes analyzed thus far include the better sections of the 5th and 8th grade classes and both of the high school classes. I believe there are definite trends apparent and some hypotheses are supported by the data collected thus far.

Informal Knowledge: 5th grade Data

While the older students of the sample have undoubtedly formed informal knowledge of percent, one cannot be sure what knowledge is a result of instruction; therefore, only the 5th grade student data will be considered as indicators of informal knowledge in the following discussion.

Correct Conceptual and Procedural Knowledge

* **Conceptual knowledge of total percent adding to 100** and the whole amount is equal to 100%, as follows:

91% Correct

If Sue got 85% correct on her vocabulary test, what percent did she get wrong? __(15)_____

100% Correct

Joe got 35 answers correct on his vocabulary quiz of 35 questions. What percent did he get correct? ___(100)_______

* **Conceptual knowledge of percent** as a part out of a whole relationship as follows:

<u>74% Correct</u>

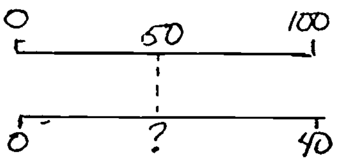
If Sue got 6 out of 9 problems correct on her math quiz, which statement is true about her percent grade on the quiz. (circle the correct answers)

(a) greater than 100% (b) less than 50% (c) equals100%

(d) greater than 50% (e) none of these

* **Conceptual knowledge of 50%** or a comparable ratio is understood similarly to the concept of 1/2 supported by :

<u>96% Correct</u>



?= ___(20)______

<u>91% Correct</u>

Jane had a spelling test of 40 words. She got a 50% grade on the test. How many words did she spell correctly? (20)

<u>74% Correct</u>

Jack got 50% correct on his math test. If he got 40 problems correct, how many problems were on the test? (80)

<u>74% Correct</u>

Joe has found that 15 of his 30 paper customers pay by check. How many customers would he expect to pay by check if he had 100 customers? (50)

* **Procedural knowledge dealing with 50%** or numbers which yield a 1 to 2 ratio consists of the strategy of either multiplying by 2 or dividing by 2 by 87% ,74%, and 78% in the three problems above.

* **Conceptual knowledge of 25%** as follows:

52% Correct (Another 9 % answered "25%", showing knowledge.)

Sam got 1 problem wrong on a test consisting of 4 problems. What percent did he get correct? (75%)

* **Procedural knowledge to convert 1 out of 4 to 25%** included intuitive knowledge of 25%, some long division of 1 by 4, and some building up of an equivalent ratio per 100.

* **Conceptual knowledge of finding a comparable rate** when given a rate per hundred as follows:

78 % Correct

A school reported that usually 5 out of every 100 students needs special help in reading. If this school has 700 students, how many would you expect to need the special help? (35)

* **Procedural knowledge to find a comparable rate** using a between ratio strategy by 70% of the students on the above problem :

700 = 7x100
So 35 = 5x7

35 people

* **Procedural knowledge of within ratio strategy** to find a comparable rate to a given rate was evident on several problems as demonstrated below. The within ratio strategy was used by 26%.

48% Correct

The news reported that 10 people out of every 100 workers are out of work. If Unionville has 40 people out of work, how many workers would you expect to live in Unionville. (400)

If $10 \times 10 = 100$ to get the answer you to do $40 \times 10 = 400$

Answer = 400

$$\begin{array}{r} 40 \\ \times\ 10 \\ \hline 400 \end{array}$$

* **Procedural knowledge of additive and some novel strategies** were evident on a number of papers.

On her evening waitress shift at Friendly's , Sue noticed that only 6 customers out of 300 bought the liver plate. How many would she expect to buy the liver plate out of 100 customers?

6 - 300
double
drop 0's

2 ... 100

2

A school reported that usually 5 out of every 100 students needs special help in reading. If this school has 700 students, how many would you expect to need the special help? (35)

35 students

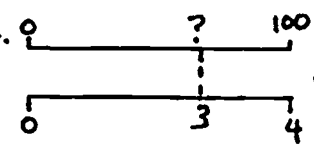
100=5
100=5
100=5
100=5
100=5
100=5
100=5

Student Misconceptions

While pedagogical content knowledge includes knowledge of student understanding, it is also important to be cognizant of misunderstandings in terms of concepts and procedures. Some trends of misunderstandings are discused below.

* **Incorrect additive strategies** were apparent on the ruler type problems; it seems that visual perception does not correct misconceptions for some students. It seems that a multiplicative relationship was not realized by some students. Replies were as follows:

<u>26% answered ? = 99</u> (30% did answer correctly)

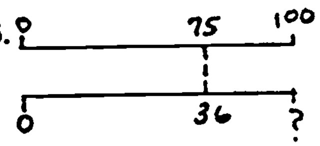
4. 

after 3 is 4 so before 100 is 99

?= 99

In like manner, an incorrect additive strategy was the most commonly used strategy in the problem below :

30% replied ? = 61 (13% did answer correctly)

6. 

75+25=100
50
36+25=61

?= 61

Incorrect additive strategies were also apparent in the proportional word problems and in many of the percent word problems.

* **Incorrect concept of the proportional nature of sales tax** is apparent even though the state of the site has a sales tax. For example, on the following problem, 44% used incorrect additive strategies, and the most common answer is as follow:

26% replied $102 (22% replied correctly)

A ring priced at $100 had an additional tax of $8. The tax on another ring was $2. What was the price of the second ring? ($25)

* **Little procedural knowledge of how to calculate a "percent"** other than those equivalent to 100%, 50% or 25%, as evidenced by:

4% Correct (However another 13% understood but answered $16)

John wanted to buy a CD normally selling for $20. This day it was marked "20% off"! How much money did he get off?

The most common answers here were $10 at 17% and $5 at 17%. Strategies included 20 divided by 2 = $10, and 100 divided by 20 = $5 and 20% = 1/5, therefore $5.

* **Incorrect strategy for changing percent to decimal** was common by many children. Only 3 such examples were asked in the test. In each case a common procedure was to precede the number by a decimal point as follows:

Circle all of the correct ways of writing 5%.

(a) 5 (b) 20 (c) .5 (d) 500 (e) .05 (f) none of these ______

44%

Circle all of the correct ways of writing 17% .

(a) $\frac{.17}{100}$ (b) .17 (c) $\frac{100}{17}$ (d) 1700 (e) $\frac{17}{100}$ (f) none of these____

48%

Circle all of the correct ways of writing 120% .

(a) 120 (b) 1.2 (c) 12 (d) .120 (e) .012 (f) none of these______

52%

* **Concentrating on one numeral** was another common error in translation, without reference to the relationship between numbers.

Circle all of the correct ways of writing .

(a) .20 (b) .25 (c) .5 (d) .15 (e) 5 (f) none of these ______

30%

* **Ignoring the % sign** was another error. In the translation problems this was evident in that 48% said .15 = .15% , 22 % said 9/9 = 1% and 30% said 120% = 120 .

In the word problems this resulted in forming an incorrect idea about the relationships in the problem. In the example below, 30% chose d, which seems to indicate confusion between percent and numbers resulting in a comparison of 150 versus 120.

John collected $60 in May from his paper route. His collections in June were 150% of those in May. Circle the true statements

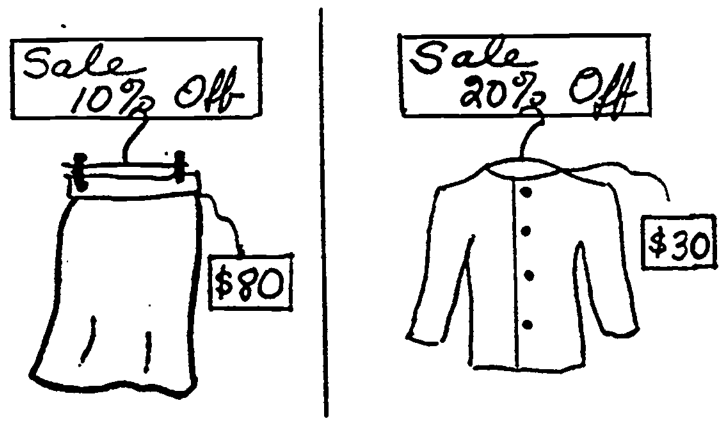
(a) June collections were under $60 (b) June collections were over $60

(c) June collections were $60 (d) June collections were over $120

(e) none of these

The same kind of thinking seems to be indicated in the following problem where 57% answered "a" indicating that 20% off is more than 10% off without regard to the base prices.

Here are two items showing their regular price tags and the sale signs for that day. Circle all of the statements that are true?



(a) You get more money off of the blouse.

(b) You get more money off of the skirt.

(c) The skirt is the better deal.

(d) The blouse is the better deal.

(e) none of these

* **Difficulty with the relationships between decimals and fractions** was expected of students at this grade level as is shown by 48%

indicating that .4 = 4/100. Indicating several answers as correct representations is a very disturbing phenomena which was frequent in the translation of representation. This indicates little number sense of the different representations . In fact 56% of the students answered this way on at least one question in part E. Here is an example.

5. Circle all of the correct ways of writing $\frac{1}{5}$.
(a) .20 (b) .25 (c) .5 (d) .15 (e) .5 (f) none of these ____________

Other examples include the following:

.4 = 4/100 = 40

.15 = 15% =.15%"

5% = .5 = .05

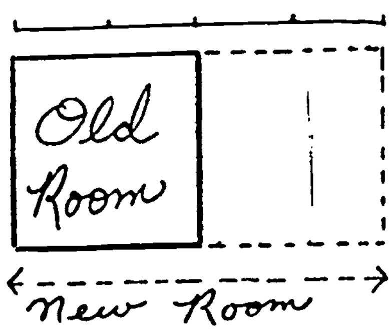
* **The area drawings** of percent increase and decrease of space in a room and in a parking lot were not well understood by students. The tendency was to concentrate on a part whole model with little reference to the problem at hand. For this example:

<u>0% correct</u> .

#4. Here is a drawing of Jill's room along with the plan to change the size of her room.

The new room is __(200)_% of the old room.

Here 52% of the students replied "50%"

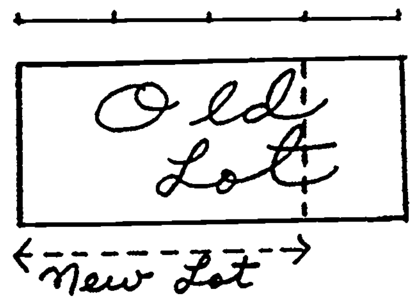


Similarly, in the example below:

26% correct.

#5. Here is a drawing of a parking lot along with a plan to change the size of the parking lot.

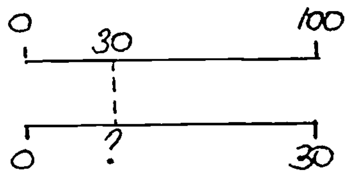
The new lot is __(75)__% of the old lot.



Here the most common answer was "25%" by 39% of the students.

* **A 30 compared to 100 is seen as equivalent to 1/3** is a common misconception as indicated below;

4% Correct with 57% indicating "10" as the answer.

1. 

?=__________

Formal Knowledge: Middle and High School Data

Correct Conceptual and Procedural Knowledge

1. **The improvement on the written test** is one indicator of the growth of student knowledge. While some performance may be due to the growth of informal knowledge, it seems logical to assume that much of the increase is due to the influence of school instruction concerning percent. A comparison of the three schools by the topics in the core section of the test is included in the Appendix.

2. **A growth in formal procedural knowledge** is indicated by the choice of strategies used by students. Examples of the increase of one of the formal strategies is illustrated by the following:

<u>Proportion Strategy</u>:	Elementary School	Middle School	High School
Problem #C.1.	0%	44%	51%
Problem #C.2.	0%	28%	42%
Problem #C.3.	0%	39%	37%, etc.

Most students went through the cross products algorithm of solving the proportion as follows:

$$\frac{5}{100} = \frac{x}{700} \qquad 100\overline{)3500}$$

$$100x = 3500 \qquad \boxed{x = 35}$$

*** Students used a variety of formal and informal strategies** indicating flexible knowledge of concepts and procedures . Many seemed to use a favorite method of solution such as the cross products algorithm or an algebraic equation, but would also use between and within ratio strategies on some examples. Other strategies included just doing the arithmetic operations, additive building up methods, making a chart, and trial and error methods.

<u>A trial and error method to find the answer</u> using estimation is as follows:

Sue wanted to buy a dress costing $50. Because it was missing the belt, the clerk let her buy it for $40. What was the percent off that she was given?

20%

50
×.15
250
50
7.50

50
×.20
00
100

<u>A trial and error method of what procedure to use</u> with estimation of a reasonable answer to indicate a solution as follows:

Jack got 50% correct on his math test.If he got 40 problems correct,how many problems were on the test?

40
×.50
20.00

.50)40.00 = 80

80

* **The consistent use of 10%** by some students was another strategy that appeared at this level as follows:

Sue was given a $5 discount off of a jacket normally selling for $20. What percent off was the sale.

10% of $20 = $2
20% of $20 = $4
30% of $20 = $6

25% of $20 = $5

<u>Misconceptions of Middle and High School Students</u>

***A persistent misconception of 30% as equivalent to 1/3 was seen.** The middle and high school students had trouble with a ruler problem with 44% and 74% respectively missing this problem: 30 out of 100 is the same as ___?___ out of 30, with the common wrong answer being 10. Another example also illustrates this error:

> Mary took a spelling test of 30 words. She got 30% wrong. How many did she get wrong?

Here are two students' work:

30% of 100 = 30
.3 x 30 = 10 wrong

$30 \ \frac{1}{3} = \boxed{10}$

* **The area representation continues to be a distractor**, but less so than at the elementary level. Some students continue to concentrate on the smaller region compared to the whole region, regardless of the problem situation. For example, on the parking lot problem, , correct percentages for the three schools were 26%, 56% and 43% for elementary, middle and high schools respectively.

* **A concentration on the number involved** without regrade to the percent sign is still a problem for some students. At the high school level:

9/100 = .09% and 9/9 = 1%
(49%) (26%)

* **Representation of percent over 100% and less than 1%** in the additional examples for this group were difficult for the majority of students. For example, only 44% and 35% of the middle school and high schhol respectively could express 1/2% as .005.

* **Type 3 percent problems are more difficult** than other word problems. The number of word problems that students could do successfully varied according to the type 1, type 2 or type 3 percent problems and with the presentation of the problem in proportional format or percent format.

Percent of H.S. students who could answer 2 out of 6 word problems

correctly:	Type 1	Type 2	Type 3
Proportion examples	67%	86%	70%
Percent examples	88 %	70%	54%

Although the mathematical relationship of the numbers is the same, it seems that the presentation of the problem affects the success rate as predicted.

Discussion of Results

Students do possess some knowledge of percent prior to instruction. They know that the whole, one half and one fourth are comparable to 100%, 50% and 25% respectively. They also understand that a ratio can be appropriately expressed as a percent and have some number sense of percent values in familiar contexts. Many can deal with ratio problems expressed as "for every one hundred" by equivalent fraction skills. This evidence of informal conceptual and procedural knowledge of a proportional relationship could be a base on which to build the "percent" concept. Children also invent unique methods and show a variety of ways of dealing with percent in a proportional situation. However, it was not evident that 10% was intuitive nor do visual representations aid understanding. Unfortunately, translations between fractions and decimals are still being developed by many students at the time of percent instruction, as are multiplicative relationships. Further facility with percent symbols and procedures do not appear to be acquired intuitively. Shopping experiences with sales and taxes seem to have little affect on computational facility with percent prior to instruction.

After instruction, the variety and flexibility of student solution strategies are indicative of increased conceptual knowledge. This variety also seems to indicate that each child has constructed their own knowledge which gets interpreted into their solution strategies. The increase in competency includes understanding of percent vocabulary, symbols, translations of representations, and use of the formal procedures of Algebraic equations and the cross products algorithm. Informal methods of estimation, trial and error, and building up strategies such as using 10% are also used. Students report that after instruction they do use their formal knowledge of percent while shopping. While type 3 problems and percents over 100 and under 1 remain difficult, students generally seem to have number sense concerning answers. While there is not mastery by all students, there is a definite growth of conceptual and procedural knowledge of percent.

CONCLUSION

Limitations of This Study

This study needs to be extended to a larger and more varied sample to be considered as a general picture of the knowledge growth of percent.

With larger numbers, additional statistical analysis of the data would also be possible.

Implications for further research

Research questions indicated by these results include the following:

* How can instruction build on the child's informal knowledge of percent?
* Can primary instruction create more situations or experiences where intuitive knowledge of percent could be developed?
* What other questions or methods could be used to determine informal knowledge?
* Could "cognitive conflict" or "conceptual change" theories be used to help students abandon additive strategies, especially with visual diagrams?
* What model could replace the current part - whole graph paper model , which would account for all 4 numbers in the proportional relationship.
* How can calculators be incorporated into percent instruction with the concentration remaining on the meaning of percent?

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