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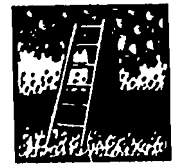
ABSTRACT

When children's own strategies for solving mathematical problems are not connected to the way mathematics is taught in school, students fail to make sense of mathematics instruction and begin to "fail." This chapter adopts the position that student understanding of worthwhile mathematical content is the essential goal of mathematics education, and that everything else--curriculum, instruction, and assessment--is a means to that end. For culturally and linguistically diverse classrooms, this position means that student bilingualism and the use of children's home and cultural backgrounds in mathematics are important considerations for teachers as they attempt to promote student understanding. An example of teaching for understanding in the bilingual or multicultural primary classroom is provided through an extensive vignette--a composite drawn from many observations, particularly in classrooms with students of Mexican origin. The example demonstrates how a teacher promotes understanding through classroom discussion in which children explain their reasoning to each other. Four principles guide such practices: constant assessment of what students understand; choice of mathematical content that is interesting, open-ended, and accessible to children of varying abilities; building on students' prior knowledge and home experiences; and developing mathematical language in context. Since teacher awareness of the extent of student understanding is critical, modified strategies are suggested for the teacher who is not fluent in students' dominant language. Contains 56 references. (SV)

CHAPTER 17



Teaching Mathematics For Understanding To Bilingual Students[1]

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A variety of issues arise when schools and teachers plan for the mathematics education of binational students. Educators might be interested to know about their students' general educational experiences and, more specifically, the mathematics they are learning in both countries; about students' cognitive and linguistic strengths in either or both languages; and about learning styles (Cocking & Mestre, 1988; De Avila & Duncan, 1985; Irvine & York, 1995; Simmons, 1985). This chapter is organized around the core theme of teaching for understanding, because teaching for understanding is a hallmark of the mathematics education reform (Cohen, McLaughlin, & Talbert, 1993; Lampert, 1986, 1989, 1990; National Council of Teachers of Mathematics, 1989, 1991, 1995; National Research Council, 1989; Schoenfeld, 1992). Through this chapter, the authors strive to make information about teaching for understanding accessible to people engaged in the education of binational and bilingual students.[2]

Why Understanding?

Educators have been relatively successful in teaching basic computational skills to high degrees of proficiency, as evidenced by the steady rise in achievement scores beginning in the 1980s (reviewed in Secada, 1992). Unfortunately, educators have been much less successful in teaching children when to apply those skills in real-world settings, advanced mathematics, or the sciences (Dossey, Mullis, Lindquist, & Chambers, 1988; Dossey, Mullis, & Jones, 1993; Mullis, Dossey, Foertsch, Jones, & Gentile, 1991; Mullis, Dossey, Owen, & Phillips, 1993). Teaching for understanding is thought to be a solution to this problem, not just for so-called mainstream or majority students, but also for students of diverse social backgrounds (Carey, Fennema, Carpenter, & Franke, 1995; Knapp and Associates, 1995; Knapp & Shields, 1991; Peterson, Fennema, & Carpenter, 1991; Silver, Smith, & Nelson, 1995; Villaseñor & Kepner, 1993).

Children enter school understanding and being able to do a lot of mathematics (Carpenter, 1985; Fuson, 1988). For instance, Secada (1991b) found that Hispanic bilingual first graders could solve arithmetic word problems in both English and Spanish that, according to conventional

[2]Additional work on teaching mathematics and science for understanding to bilingual students can be found in Rosebery, Warren, & Conant (1992), Secada (1991a), Warren & Rosebery (1992, 1995), Warren, Rosebery & Conant (1989, 1994).

wisdom, they should not have been able to solve[3]. Not only did their invented solutions[4] tend to result in correct answers, but also, their explanations of those solutions were sensible and demonstrated that they understood many of the problems. In contrast, the computational facts on which bilingual children are all too often drilled seldom make much sense to them; hence, at least initially, young children tend to ignore computational facts and instead to rely on their own strategies (that is, their own mathematics) when solving word problems.

Beyond the fact that children will ignore or misapply what they do not understand—as do all people—the disconnection between how primary-graders reason about mathematics versus how students are taught poses two additional problems. First, this disconnection lays the foundation for failure in school mathematics. When children cannot do computations in the book's way or they cannot do paper-and-pencil algorithms quickly and efficiently—because neither makes much sense to them—they are often corrected, have their papers marked wrong, and drilled further. In other words, children begin to fail in mathematics. For bilingual children, the consequences of this initial "failure" become even more severe since they add to the stereotypical belief that students acquiring English cannot engage in mathematics that involve substantive use of language, such as word problems.

[3]These children had been identified as limited English proficient (LEP), a term that has a 20-year history of local, state, and federal usage. During the past few years, the term LEP has been severely criticized for its pejorative overtones in focusing on students' purported deficiencies as opposed to their knowledge and skills (Casanova & Arias, 1993). At a time when the United States' educational systems are being challenged to produce an informed citizenry who can participate in an increasingly multilingual and multicultural society and who can help that society in a technological and multinational economy, a new term is needed to communicate the potential resources that are represented by students who, in addition to knowing languages other than English, are learning English as a second or third language. These terms include "students acquiring English," "English learners," and "learners of English as a second language." Recognizing that there has been some confusion and in some cases outright resistance to terminology that departs from the conventional use of the term LEP, we nonetheless will use the term "bilingual" to refer to the population of students who have some competence in two or more languages and the term "students acquiring English" to refer to students whose command of the English language is deemed less than optimal. Hence, our use of the term "bilingual" includes "students acquiring English"; it is all a matter of degree.

[4]There is a large body of research on children's informal, invented solution strategies for arithmetic word problems (see, for instance, Carpenter, 1985; Carpenter & Moser, 1983, 1984; Carpenter, Moser, & Romberg, 1982; Riley, Greeno & Heller, 1983). These strategies are invented because they are not taught in school. In addition, schooled and unschooled children as well as bilingual and monolingual children use many of the same strategies when confronting word problems (Adetula, 1989; Ghaleb, 1992; Secada, 1991b).

The second consequence of failing to encourage children to understand the mathematics they encounter and to solve problems in ways that make sense to them is that they begin to believe mathematics should *not* make sense. Mathematics then becomes the rapid production of nonsensical stuff (see Carpenter, 1985, for a fuller elaboration of this argument). In contrast, students who are taught mathematics so that they understand it will be socialized into expecting that its teaching should make sense to them.

In developing this chapter, we adopted the position that student understanding of worthwhile mathematical content is the *sine qua non* of mathematics education, and everything else—curriculum, instruction, and assessment—is a means to that end. Our position means that the chapter's two additional themes—student bilingualism and the use of children's home and cultural backgrounds in mathematics—are important considerations for teachers and other school personnel insofar as they help teachers to promote student understanding of mathematics[5]. Our examples[6] illustrate how teaching for understanding might look in a classroom that includes bilingual or binational students; they also weave student bilingualism and the use of home and cultural backgrounds into the fabric of teaching for understanding.

[5]The astute reader will see other themes reflected in the examples that we use. For instance, students' sharing their reasoning and problem-solving strategies with one another in small-group settings reflects the use of cooperative groups. That some students use counters for solving arithmetic problems, while others use their fingers, is consistent with a focus on teaching through multiple modalities. And as we have already noted, students who are schooled in Mexico are likely to be learning different algorithms than are taught in U.S. schools. We are aware of these issues and we agree that they are important. Our central point remains: cooperative groups, manipulatives, learning styles, and the like are important insofar as attending to them helps teachers to promote student understanding. Rather than try to cover everything that is relevant to the teaching of binational students, we decided to focus on fewer ideas (student understanding, bilingualism, and home cultures) and to develop those ideas in depth.

[6]The following vignettes are *composites* of the many classrooms that we have observed where there is teaching of mathematics for understanding. The core of many vignettes are from Yolanda De La Cruz's observations of Mrs. Sara Avelar, a teacher who has asked that her real name be used. At the time that Yolanda was observing Mrs. Avelar's classroom, Sara was one of eight primary-grade teachers and over 200 children at Esperanza School who were participating in the *Children's Math Worlds* curriculum development project. Esperanza is the fictitious name of an urban elementary school in a largely Hispanic neighborhood. Almost all the children were, to some degree, bilingual. Roughly half were dominant English speakers and were enrolled in four English-language classrooms, one for each of grades K through 3. Of these children, most were first generation U.S. born. The other half were students acquiring English and were enrolled in one of four Spanish-English bilingual classrooms. Over 80 percent of Esperanza's bilingual children were of Mexican descent; many had been born in Mexico. Sara Avelar taught 28 second graders in her bilingual class. All came from Mexican homes where little or no English was spoken.

Mathematics Teaching

Let us enter a second-grade bilingual classroom in which 28 children are using both English and Spanish to talk among themselves as they work with fraction strips and a pair of dice. The strips of paper come in six different sizes: the longest is a whole, and the children refer to it as the *one* (*el uno*). The next size is half the size of the one, that is, it is the half (*un medio*); followed by 1/3 (*una tercera parte*), 1/4 (*una cuarta parte o un cuarto*), 1/8 (*una octava parte o un octavo*), and 1/16 (*el decimosexto*, which children refer to as *the smallest* or *el más pequeño*). Children play in pairs; each child rolls a die whose faces are marked 1/2, 1/4, 1/8, 2/8, 1/3, and 1/16. Each child takes the fractional strip that corresponds to the number that is rolled and places the fractional strip on top of the 1-strip. The first child to cover, exactly, the 1-strip with the correct, corresponding fractional strips, wins. On a separate sheet of paper, children record the fractions that they roll.

As we go from pair to pair, we overhear different conversations. Children check one another to be sure that the strip matches the fraction that was rolled. One child asks his partner, who has played the 1/4-strip though the die shows 2/8, to explain why she did so. "Porque los dos son iguales (Because both are equal)," she answers. Unsure what is meant, the first child asks for more of an explanation, "¿Cuáles son iguales? (Which ones are equal?)." Taking two of the 1/8-strips and one 1/4-strip, the second child covers the 1/4-strip with the two 1/8s. She explains, "Estos dos son lo mismo que la cuarta parte. Mira (these two [she holds the 1/8-strips in one hand] are the same as 1/4 [she holds the 1/4 in the other hand]. See [she covers the 1/4-strip with the two 1/8-strips])."

Two children do not place the 1/3-strips on the 1-strip, even when they roll 1/3. "Why don't you use these?" an adult asks. Seeing the puzzled look on their friend's face, a third child explains, "Quieren saber por que no usas estos (They [the visitors] want to know why you don't use these [points to the 1/3 strips])." "Tenemos que llegar a uno, y es mas difícil con éstas." Translating, the teacher explains, "They say they have to reach the one and it's more difficult with the 1/3-strips."

Another pair notices that they can exchange two 1/8-strips for one 1/4-strip. As they discuss the trade, two more children join in and suggest that they work together to see what other trades can be made. The teacher suggests that the four record the results of their exploration in order to share them with the rest of the class. "Be sure you can explain how you figured things out so that everyone understands you," she reminds them.

Circulating among the children, watching how they're doing, the teacher poses questions to clarify children's thinking and to help them think of ideas for winning the game. Jesús[7] seems unable to find fractions that total 1 and he is becoming increasingly frustrated from losing the game. Since it is almost time for recess and children are finishing their activity, she makes a mental note to call Jesús' mother. Instead of Jesús repeating the same fraction game at home, his mother and teacher decide that he should help with cooking so that he can see fractions in use.

Later, as Jesús is measuring the ingredients for making pancakes, he notices "¡Oh, esto es lo que estamos haciendo en la clase! (Oh! This is what we're doing in class!)" Jesús then explains to his mother how each smaller measuring cup is part of the big one. She, in turn, explains that had Jesús used the wrong cups, the pancake mix would not have turned out right.

The next math class begins with children discussing the results of their game. Children go to the board to write all the different ways that they got to 1. Their teacher tells the students that if someone else has written down what they did, to just put a checkmark by that way. To begin the class discussion, Jesús explains how he and his mother measured the ingredients for pancake batter and how he used half cups (which were easier to handle) for the flour, an eighth of a cup for the oil, and so forth.

Teacher: ¿Cuánta harina usastes? (How much flour did you use?)
Jesús: Una tasa y media. (One and a half cups.)
Teacher: ¿Cómo supistes que tenías una tasa si estabas usando media tasa? Enséñale a la clase con estos papelitos. (How did you know when you had a full cup since you were using only a half cup? Show the rest of the class with these paper strips.)
Jesús: Llené la tasa con dos. Así. (I filled the cup with two. Like this.) [At the overhead projector, Jesús places two of the 1/2-strips right next to the 1-strip, and shows how they cover it.]
Teacher: ¿Cuántas de las medias tasas usastes? (How many of the half cups did you use?)
Jesús: Tres. Porqué dos son una tasa; y una más para tasa y media. (Three. Because two are one cup; and one more for one and a half cups.)

As Jesús sits down, the teacher points out where the class has written that two 1/2s make a whole. "Vámos a ver cómo se pueda sumar a uno. . . Let's see how we can add to one." she translates. As the children review the different ways that they got to the 1-strip, they see that not everyone wrote their expressions the same way: some had used plus (+) signs; others, commas; and still others had just listed the fractions in no order. After a short discussion, the children agree that they need to find a single way of writing things because otherwise, things could get confusing.

[7]All names are fictitious.

The consensus is to use the plus sign because these are numbers that "add to 1."

As they recopy the list of fractions, some children argue that 1/3+1/4+1/2 cannot equal one; others argue that it does.

Teacher: ¿Quiénes piensan que estos suman a uno? (Who thinks that these add up to one?) [A group of children raise their hands] ¿Y quiénes dicen que *no*? (And who says *no*?) [Some other children raise their hands.] ¿Y quién no sabe? (And who doesn't know?) Bueno, discútenlo entre ustedes mismos. (Well, discuss it among yourselves.)

In a spirited conversation, some children bring out strips to show others how 1/3+1/4+1/2 is too big to equal 1. Other children write on paper. Listening to each group's reasoning, the teacher sends groups that achieve consensus to the other groups to help them resolve the problem. After a while, there is quiet.

Teacher: ¿Qué decidieron? (What did you decide?)
María: Es más que uno. (It is more than 1.)
Teacher: ¿Cómo lo sabes? (How do you know that?)
María: [Comes to the overhead and talks while she places the fraction strips next to one another and compares them to the unit strip] Porque, maestra, si pones juntos los papelitos, es más grande que uno. (Because, teacher, if you put the fraction strips together, it's larger than 1.)
Teacher: ¿Están de acuerdo? ¿Todo el mundo le entendió a María? ¿Explícalo otra vez para estar segura que te entendieron? (Does everyone agree? Did everyone understand María? [Some No's] Please explain it another time to be sure that everyone understands you.)
María re-explains. As there are no questions from her classmates, she goes to sit down.
Teacher: ¿Quién lo hizo de otra manera? ¿Liliana? (Who did it a different way? Liliana?)
Coming to board, Liliana writes:

1/4	+ 1/4 + 1/2 = 1
1/3	+ 1/4 + 1/2

Liliana: Dos cuartas partes más la media parte son uno. Pero la tercera parte, aquí, es más que un cuarto. Así es que, la suma tiene que ser más que uno. (Two 1/4s plus 1/2 equals 1. But 1/3, here, is more than 1/4. Hence, the sum has to be more than 1.)

Liliana circles the 1/4 and the 1/3 as she says that 1/3 is more than 1/4.

Teacher: **¿Y por qué creen que alguien escribió eso? (Why do you, the class, think that someone wrote that [pointing to the 1/3 + 1/4 + 1/2]?)**

The children call out various reasons. Maybe someone wrote down the wrong numbers. Maybe the person didn't match the fraction to its proper piece of paper. Maybe the person forgot that the idea was to get exactly to 1.

At this point, the teacher calls on the four children who explored the various trades that could be made among fractions to explain what they have discovered. Their results include relatively simple equivalences such as 1/2 equals two of the 1/4s or 1/4 equals two of the 1/8s; but also, their results include more complex equivalences such as 1/2 equals four 1/8s. They take turns showing how they traded fraction strips using the overhead; some of the other children ask them to slow down so that they can try the trades for themselves. One child notices that 1/3s were never traded; another child notices that—except for the equation 1 = 1/3 + 1/3 + 1/3—the 1/3-strips were never used: "No se mezclan (They don't mix)." The teacher ends the class:

Teacher: Para discutir mañana, tengo dos preguntas. ¿Es verdad que no pueden mezclar una tercera parte con otras fracciones para hacer uno? También, deben de notar que nuncan sumaron a ocho a los octavos, ni a 16 a los pequeñitos. ¿Por qué pasó eso? Miguel, dime de lo que vas a pensar para mañana. . . . ¿Teresa? (To discuss tomorrow, I have two questions. Is it true that we can't mix 1/3 with the other fractions to add up to 1? Also, notice how no one added eight of the 1/8s nor 16 of the tiny ones. Why did that happen? Miguel, tell me what you will think about for tomorrow. [listens to his response] . . . Theresa? [listens to her response]).

While admittedly a hybrid of many examples of teaching for understanding, the above vignette illustrates how the teacher is not afraid to withdraw into the background to allow the discussion among children to develop. She intervenes, however, to keep the discussion moving productively, to insure that everyone has a chance to contribute something substantive, and to ask children to elaborate on their initial answers so that others can understand them. Moreover, students realize that they are expected to give reasons for their answers; after a while, it becomes part of the classroom's culture that when anyone gives an answer, he or she follows up with an explanation, "because . . . ." Teachers will vary among themselves in how they do these things, based on their personal teaching styles and on the specific goals of their mathematics lessons and units. Yet four principles guide their decision making and practices: (1) be constantly assessing what your students understand; (2) choose mathematical content that is interesting, open ended, and accessible to students of varying skills and abilities; (3) focus on developing children's understanding by building on their prior knowledge of mathematics; and (4) develop mathematical language in context.

Assessing Student Understanding

Teachers who teach for understanding are constantly assessing how well their students—both individually and as a class—are understanding the mathematical ideas in the lesson and discussion so that they can decide what to do next. If the students are not "getting it," the teacher can help the student think things through, give an easier but related problem, or ask someone to re-explain what he or she did. These teachers use many cues to help them assess student understanding. For instance, the teacher above noticed the *confused looks* of one of her students when she was asked something in English. Also, teachers can recognize the *"Aha, now I've got it!"* expression that students get when something clicks and they begin working furiously on solving a heretofore obscure problem. Jesús' teacher saw that he was seldom *right* when adding up fractions to equal 1 and she could see his increasing *frustration* in not "getting how" to play the dice game. She *asked students if they understood* one another's explanations.

While student cues provide a direct way of assessing understanding, teachers can also rely on other, more subtle ways by which people in their everyday lives can tell if someone understood something. Teachers often listen to see if students are *restating what has been said in their own words.* For instance, María used her own words to explain how the strips 1/3, 1/4, and 1/2 were bigger than the 1-strip, and hence why 1/3 + 1/4 + 1/2 was bigger than 1. At the end of the class, the teacher asked two of her students to, tell her in their own words, what they would be talking about the next day. This allowed her to check whether they understood the mathematical ideas behind the questions with which she was ending that class.

People show that they understand something when they *relate an idea or what has been said to a different event.* For instance, Jesús explained how his measuring the ingredients for the pancake mix was related to making and to adding fractions. With his teacher's help, he discussed how two halves make a whole (cup). Hence, he was able to relate fractions, the in-class use of fraction strips, and measurement (done out of class) to one another.

Teachers often ask students to *explain and/or elaborate their reasoning.* Throughout the two lessons, children explained their reasoning when they said how they had figured something out or why something was right or wrong. This was a deeply embedded part of the class culture on how things were done. Without any prompting, for instance, Jesús explained why the 3 half-cups of flour that he used totaled 1 and 1/2 cups. Also, when asked why she had used the 1/4-strip in place of two 1/8-strips when the die said 2/8, a student (during the first lesson) at first said that they were equal. Realizing that her partner did not understand what she meant, she explained by showing how two of the 1/8-strips could be made to fit exactly on the 1/4-strip.

People show that they understand an idea when they *put a new twist on that idea*. For instance, from their observation that the 1/4-strip could be used in place of two of the 1/8-strips came the four students' investigation on fraction equivalence. That is, they were trying to see how far this idea of trading fraction strips for one another could be generalized.

People understand a problem when they *relate and/or apply new ideas to the problem*. For instance, Liliana related two facts—that 1/4 + 1/4 + 1/2 = 1 and that 1/3 > 1/4—to explain why 1/3 + 1/4 + 1/2 > 1.

When people *point out something that is wrong (or, at least, is a problem) with an idea*, they show that they understand that idea. For example, after the long class discussion about 1/3, 1/4, and 1/2, the students diagnosed possible reasons for why this error had occurred in the first place. It wasn't enough to explain why they were right in saying that 1/3 + 1/4 + 1/2 could not equal 1; the class also explained what was wrong with someone's original reasoning.

Teachers who teach for understanding often ask if anyone has *solved a problem in a different way or can give a new way of justifying an idea*. For example, consider another bilingual first-grade classroom where a teacher poses the word problem: "Clara tiene 13 carritos de juguete. Seis de sus carritos son rojos y el resto son azules. ¿Cuántos de los carritos son azules? (Clara has 13 toy cars. Six of her cars are red and the rest are blue. How many of her cars are blue?)" A child might solve this problem using colored blocks. He would model the problem by first putting six blue blocks out on a surface. Then he would keep adding red blocks until he got to a total of 13. To get the answer, he would count the seven red blocks. Another child might solve this problem by reasoning about number facts. She would know that 6 + 6 = 12; so the answer has to be 7, since 7 + 6 = 13.[8] In this case, their teacher might infer that both children understood the problem, but that the second child could draw on her more sophisticated knowledge of number facts to solve the problem.

Over the long term, children can show that they understood an idea by *remembering that idea or by applying it in new settings*. In the example just above, the child who remembers that 6 + 6 = 12 and can apply it in order to infer that 7 + 6 = 13 has shown an understanding of how numbers are related to each other.

Teachers who teach for understanding attend to these and many other cues as to how well their children understand the mathematical content that is the subject of the lesson. They base their instructional decisions not just on whether answers are right or wrong, nor on the need to cover a certain amount of material. Although these may be considerations, the primary concern is to ensure that all students actually understand something of what is being taught and discussed.

Content

The mathematics that students engage in should be *worth understanding* (see National Council of Teachers of Mathematics, 1989). At the very least, it should be substantive and focused on *contexts that promote problem solving, applications, and higher order thinking;* the upshot is that students make connections within and across different areas of mathematics. Notice how, in the vignette above, what could have been a rather superficial activity involving fractions and paper folding becomes much more. The inclusion of 1/3 and 2/8 in the dice-rolling activity means that students will confront messy situations in which questions about different ways of adding fractions that sum exactly to 1 and about the equivalence of fractions are likely to come up. What is more, by discussing the small fractions (1/8 and 1/16) and an "unmixable" fraction (1/3) which, in the words of one student, "no se mezclan (did not mix with the other fractions)," students will connect the ideas about chance with their list of ways of adding to 1.

The tasks and activities that make up lessons should be related to one another so that there is the *development of depth over breadth*. The above vignette illustrates a flow from the first to the second day's activities. What is more, one can envision how the teacher and students will use subsequent days to build on what happened during the first two days. They will refer to the two lists (the list of equivalent fractions and the listing of different ways by which they had summed to 1) to develop their ideas about fractions. One can see students coming to class ready to report on other uses of fractions that they have encountered in the real world. Throughout these days, students will be developing deeper and more interconnected understandings about fractions.

The activities, tasks, and problems that students encounter should be *accessible to students with a wide range of knowledge and skill*. That is, students with diverse backgrounds should be able to understand what is required, make meaningful attempts to do the activity, and understand some of the simpler strategies that other students may use. Both the fraction activities and the word problem, discussed above, are examples of exercises in which bilingual children with a broad range of language and mathematics backgrounds can be engaged. They include manipulatives or paper and pencil, which students can use to support their reasoning and

[8] This kind of reasoning about numbers, which relies on children using those number facts that they have memorized (usually doubles) to derive other number facts, is much more common among primary grade children than people give them credit for (Carpenter & Moser, 1983, 1984).

›blem-solving efforts. Moreover, word problems such as the one described above can be made more accessible to children who do not understand them or a bit more difficult for children who find them too easy. The mathematical content of word problems can be modified by using smaller or larger numbers. The content can also be modified by using numbers that can be more or less easily related to one another, such as doubles (3,6; 4,8; 5,10; 6,12, etc.), numbers separated by 10 (3, 13), or numbers that are not obviously related to one another. Primary grade teachers will often vary a word problem's difficulty in order to give a child a problem that will stretch the child's thinking a bit while remaining accessible.

Also, mathematics problems, activities, and tasks should be *open-ended enough to allow students and teachers to take the activity in a variety of directions.* Not only did the fractions activity—with its inclusion of 2/8 and 1/3 on the dice—allow students to explore new but related topics, it almost seemed to encourage such an exploration.

Problems and activities should be *interesting* to students. The term *interesting* (not *fun*) is used purposefully to convey the intellectual quality of a problem that is engaging and can require hard intellectual work. While fun activities might also be interesting, the overlap is far from perfect: too many fun activities are mindless and contain little mathematical substance. Moreover, the fun comes from doing interesting and engaging mathematics.

Student interest can be engaged in various ways, for instance, by using children's own names in word problems or allowing them to write their own problems. It has been our experience that children will often write and solve problems that are more complex than those that adults would think of posing. Problems that grow out of children's everyday lives or are applicable to their lives can be more interesting than problems and activities that are not connected to their lives. Jesús, in the vignette above, became more engaged in fractions when he could see and apply them in his mother's kitchen. After a field trip to the zoo, a first-grade teacher posed a series of word problems based on the animals eating different kinds of food. Also, student interest can be engaged by students exploring topics that they have never experienced directly, by using fantasy.

Prior Student Experiences

Since understanding develops out of what people already know, teachers who teach for understanding constantly try to connect new problems (and other mathematics activities) to their children's prior mathematical knowledge and backgrounds (see Smith & Silver, 1995). For instance, the dice-rolling activity from the fractions unit created a common core of back-

ground information that students would mine over the next few days to discuss equivalence and the addition of fractions. For some students, this activity tapped into other knowledge that they had about fractions—for instance, the knowledge that 1/3 > 1/4 or that 2/8 = 1/4. Binational students may come to school having learned different algorithms for the number operations than are commonly taught in the United States (Secada, 1983); teachers need to be alert to how these students think about computations. Also, the metric system is used in most real-world settings and is taught in schools outside of the United States. Hence, many binational students are likely to have a better understanding of metric measure than their American counterparts. At the very least, teachers should ***not*** assume—as we have seen in at least one basal mathematics series—that students acquiring English should be drilled on the meanings of the common prefixes (kilo, centi, milli) that are used in metric measure.

Just as Tikunoff (1985) found that effective bilingual teachers use their children's home cultures to support classroom management and student learning, so, too, should teachers try to tap into binational children's home cultures to support their mathematics learning. For instance, the teacher discussed possible at-home activities with Jesús' mother in an effort to support his learning about fractions.

Teachers might use a context or theme with which their students are familiar to serve as an umbrella out of which they generate mathematical activities and problems. For instance, teachers at Esperanza Elementary School are participating in the development of the *Children's Math Worlds* curriculum (De La Cruz, under review; Fuson & Perry, 1993; Fuson, Zecker, Lo Cicero, & Ron, 1995). The curriculum has three strands: word problem/graphing, place value/multidigit arithmetic, and geometry/measurement. *Children's Math Worlds* is designed to create an ongoing interaction between Latino/Latina children's mathematics learning in class and their experiences outside by bringing children's mathematical knowledge (from both inside and outside of the classroom) into the classroom and by providing families with activities that they can do at home with their children. One group of activities is based on the Mercado, the marketplace that is common in Mexico, where many of these bilingual, binational children and their parents go to visit extended family members. What is more, the school's neighborhood features its own Mercado, which is very similar to what can be found in Mexico. For instance, ambulatory vendors use pushcarts to sell bocaditos (snacks) and other small items.

The Mercado provided a context where children enact, through role playing, a variety of mercantile transactions. In this way, children have learned about money; how to identify and know the value of coins (penny, nickel, dime, quarter) and bills (dollar, five dollar, ten dollar); how to make change for less than $10.00; adding and subtracting up to two 4-digit numbers with decimals; how to solve two-step story problems involving the addition and subtraction of up to two 4-digit numbers; the meaning of multiplication; how to solve story problems involving multiplication; and how to write and solve word problems using the Mercado as a context. For example, bilingual second graders would work with partners where one would be el vendedor (the vendor), the other, el cliente (the buyer). El cliente would select an item from a grocery and use plastic money to make the purchase; el vendedor would ensure that enough had been paid and in some cases would make change. After several purchases, students would switch roles.

In follow-up activities, students would create shopping lists from teacher-made grocery lists. Students would list the names of the items they were going to buy with their prices; then they would find the total cost of their lists. If they had enough money to make the purchases, they would do so. As students became more proficient with these transactions, they would use real grocery ads to create their shopping lists—involving the use of larger numbers—and make their purchases.

A typical word problem for students to solve would be this:

> Tú quieres un dulce. El dulce cuesta 19 centavos. Le das al vendedor 25 centavos. ¿Cuánto vas a recibir de cambio? (You want to buy some candy. Each candy costs 19¢. You give the seller 25¢. How much change will you receive?)

When asked to write a problem based on spending $400 or less, Armando wrote:

> Voy a comprar un stereo para mi familia. Tengo $381.10. Pienso que me va quedar si compro uno en menos de $300. Quiero un stereo que tiene todo: uno para tocar discos, para casetera, y que tenga reloj. Con lo que me sobra, le voy a comprar algo para mi mamá. (I want to buy a stereo for my family. I have $381.10. I think I will have [money] left over if I buy one for under $300. I want a stereo that has everything: one that plays disks [records], plays cassettes, and has a clock. I am going to buy something for my mother with the money I have left over.)

Armando drew a picture of his stereo with a price tag. He worked out the subtraction problem to show that he would have money left over to buy a gift for his mother.

The Mercado provides a rich context that is familiar to the children, that supports children's doing mathematics since the problems they encounter

will have meaning and be interesting, and that can be used as a basis for children to write their own problems about things that are important to them. Armando's writing sample shows how the problems that children generate contain both substantive mathematics and referents to their home cultures and values (as in buying something for one's family and a gift for mom).

Mathematical Language

While students develop understanding working alone or using objects to help them model problems, they also develop understanding by discussing mathematics. Asking children to share ideas about mathematical topics and asking how they figured out a problem or why, such as in the above example of the second grader who used a 1/4-strip instead of two 1/8-strips, are among the ways in which students can develop their understandings. For such discussions to be productive, classrooms need shared norms for listening and the expectation that everyone will explain her or his reasoning. For instance, Jesús, in the fractions vignette above, spontaneously explained why three half-cups of flour were the same as one and a half cups.

Beyond sharing norms for explaining their reasoning, students need to be able to discuss their ideas without much ambiguity. The use of mathematical language[9] is intended to help people talk about mathematics clearly and with little ambiguity. The teacher's use of language, in the fractions vignette above, illustrates some uses of mathematical language during instruction. For example, she modeled the proper, conventional usage of mathematical terminology in her own conversations with children.

But also, her focus remained communication. Though she used the proper names for almost all of the fraction strips, she used the students' term of "the little one (el pequeño)" when referring to 1/16. In that particular context, there would be no confusion since everyone understood that the little one referred to the smallest paper strip and seemed comfortable with that term. What is more, the use of *one sixteenth* would likely have, itself, been too confusing for second graders.

When a student used the term "mezclar" (mix) in describing how 1/3 did not appear on the list of fractions that were successfully added to one another in order to get to 1, the teacher used the student's own terminology since, once again, the student's term seemed to capture what was meant in

[9] Mathematical language consists of the specialized symbols, technical terms and phrases, and ways of speaking that (a) help people to communicate with one another when they are talking about mathematics and (b) mark people as knowing how to do mathematics (see, for instance, Cuevas, 1984; Dale & Cuevas, 1987; Lampert, 1986, 1990; Pimm, 1987; Spanos, Rhodes, Dale, & Crandall, 1988).

a way that other students could understand without too much ambiguity. Also, she allowed students to refer to the strips without using their fractional names, as would happen when children would indicate a particular sized strip with "mira" (look). At some time in the future, students would be expected to use more precise terminology, but for the purposes of that class in that context, the precise use of mathematical language was leavened with the use of children's own language.

On the other hand, the teacher recognized that confusion would likely arise if students tried to use three different ways of listing the fractions that added to 1. Rather than allow that to happen, she pointed the problem out to the class, and together (with her prodding), they decided to use number sentences, such as $1/2 + 1/4 + 1/8 + 1/8 = 1$, to list the fraction combinations that had added to 1.

The point is that mathematical language, like language in general, develops in context to support communication. The way to increase students' use of precise language is to use contexts that require such precision.

One unfortunate use of mathematical language occurs when students look for certain key words or phrases in a word problem as a short cut for solving the problem.[10] For instance, *in all*, *altogether*, and *total* are said to be cues for students to add; *left* or *remaining*, to subtract. The only reason that these words can be such cues, however, is because of the restricted content of the elementary school curriculum, and not for any mathematically relevant reason. Consider the following word problems, which can be solved by primary graders who do not rely on key words:

Addition: Tanya has 8 chocolate chip cookies and 7 raisin oatmeal cookies. How many cookies does she have *in all*?

Subtraction: Tanya has 15 cookies *in all*. Seven (7) of them are chocolate chip, and the rest are raisin oatmeal. How many of Tanya's cookies are raisin oatmeal?

Multiplication: Tanya has 4 friends coming over to play today. She wants to give each of her friends 3 cookies for snack time. How many cookies does she need to have *in all*?

Division: *In all*, Tanya has 15 cookies to share with her 4 friends for snack time. If she and each of her friends get the same number of cookies, how many cookies will each person get?

A student who relied on key words to solve these problems would get just one of them right. The use of key words to go directly (and thoughtlessly) from a word problem to a solution should be discouraged. Students need to think about what words mean in the context in which those words are used.

[10]Many teachers, especially teachers of students acquiring English, encourage this practice.

Additional Language Concerns

An ideal solution to the issue of teaching mathematics with understanding to binational children, particularly in classrooms along the U.S.-Mexico border, would be to hire teachers who are bilingual and to support their learning to teach mathematics with understanding in either language. For instance, bilingual curriculum materials are being developed to support teaching-for-understanding approaches in English and Spanish, and there are many professional development opportunities for teachers who are interested in these approaches.[11] By modifying some of their instructional approaches, monolingual teachers can also teach for understanding to binational students who are acquiring English. While maintaining a focus on student understanding, these teachers will need to compensate for the fact that they and their students are stronger in different languages.

One of the most difficult things for a monolingual teacher to decide is if errors by a student who is acquiring English reflect a lack of mathematical

[11]Primary grade teachers may be interested in the project known as *Cognitively Guided Instruction* (CGI) (see Carey et al., 1995). For professional development opportunities on *CGI*, please contact *Cognitively Guided Instruction*, Wisconsin Center for Education Research, University of Wisconsin-Madison, 1025 West Johnson Street, Madison, WI 53706.

Mathematics in Context (National Center for Research in Mathematical Sciences Education, in press) is being developed as a dual language curriculum for middle school students. For information about the project and professional development opportunities, please contact *Mathematics in Context*, National Center for Research in Mathematical Sciences Education, Wisconsin Center for Education Research, University of Wisconsin-Madison, 1025 West Johnson Street, Madison, WI 53706.

An elementary through middle school curriculum project is *Visual Mathematics* (Forman & Bennett, 1995). For information about the project and professional development opportunities, please contact Mathematics Learning Center, P. O. Box 3226, Salem, OR 97302.

High school teachers might be interested in the *Interactive Mathematics Program* (Resek & Fendel, in press). For information about the program and professional development opportunities, please contact IMP, 6400 Hollis, Suite 5, Emeryville, CA 94608.

SummerMath (Schifter & Fosnot, 1993) provides professional development opportunities for teachers at all grade levels. For information please contact SummerMath for Teachers, Mount Holyoke College, South Hadley, MA 01075.

EQUALS, which has developed bilingual versions of *Family Math* (Stenmark, Thompson, & Cossey, 1986) and other equity-based materials, provides professional development on a wide range of topics for teachers of all grade levels. Please contact EQUALS, Lawrence Hall of Science, University of California, Berkeley, CA 94720.

understanding or some problems with English. For instance, consider the case of a ninth-grade mathematics class that was being taught completely in Spanish by a mathematics teacher who is bilingual. At the start of this class, students were putting up the solutions to homework problems from an English language text. One problem was this: *There are two complementary angles. One is twice the other. What are the angles?* A girl had written two equations: $x+y=90$, $x=2y$. She had solved for x and y by substituting 2y for x in the first equation: $3y=90$, $y=30$, $x=60$. Her explanation to the class was in Spanish. She noted that since the angles were complementary, their sum was 90 ; and since one was twice the other, $x=2y$. She explained why she had substituted for x in the first equation, and how she simplified the equations to get her solution.

Also observing this class was a school administrator who interrupted the class to ask the girl to repeat her explanation in English. Due to her grammatical difficulties, some of this girl's statements were mathematically incorrect. Had one of us not heard her simple and concise explanation in Spanish, we would have been convinced that she simply had not understood the problem and would have wondered if she—or someone else—had done her homework.

Given the importance of ongoing assessment in order for the teacher to figure out what understandings are developing among students, the problem becomes particularly acute when the teacher and student have difficulty in understanding each other. In cases like these, it still is important for the student to be able to discuss his problem-solving strategies with other students. The teacher might encourage a student acquiring English to explain the solution to someone else in the class who has a stronger command of both languages. Assuming that the teacher has been teaching mathematics for understanding all along, then she should monitor whether the Spanish-language explanation made sense to the class's bilingual students. Students should be encouraged to discuss errors or points of misunderstanding among themselves as a means of self-correction. From time to time, the classroom teacher might ask a more strongly bilingual student to translate what has been said from Spanish into English so that the rest of the class can have an opportunity to participate in the conversation.

Secondly, teachers might have some of their curricular materials translated into Spanish or rewritten in simplified English so that they are more accessible to children in their classes (see examples from Secada & Carey, 1990, for how this can be done). In their studies with bilingual children, Secada (1991b) translated arithmetic word problems into Spanish, one sentence at a time. Problems were translated back into English by someone who did not know what the original problems had been. Then, the back-translations were checked to see how well they matched the original versions of the problems. If there was a match, then it seemed reasonable to

assume that the original translations maintained fidelity to the problem's original intent. Prior to asking children to solve those problems, Secada asked each bilingual teacher who worked with the children to read the translations and to suggest locutions and alternative vocabulary that would make their problems more understandable to the children. For instance, one bilingual teacher noted that *lapiz de color*, while technically a correct translation for *crayon*, was not a commonly used term; her suggestion, *Crayolas*, was used instead.

English word problems can be simplified by ensuring that sentences are short, maintain active voice, and use present tense. Sentences that contain complex grammatical constructions—such as phrases or subordinate clauses within clauses—should be broken up and simplified. The names of familiar objects should replace unfamiliar objects; for instance, a word problem about blocks, toy cars, or candies is likely to be more accessible to children than one about things that they seldom encounter. The use of children's own names will often make a problem more accessible.

Translated problems, as well as simplified-English versions, can be used to begin a series of lessons. They should be saved, updated, and shared with other teachers who are encountering similar difficulties.

Many students acquiring English receive little encouragement to speak about their ideas, in part due to the belief that they will find it too difficult to express themselves. Many Hispanic girls are socialized to defer to boys. Hence, if teachers tend to call on students who answer first, students acquiring English and girls will be left out of the conversation. Teachers need to reach out to these children—either through increased wait time, or by warning them that they will be called on—in order to be sure that they are included in the classroom's processes.

Teachers of students acquiring English may find that their students are struggling with new vocabulary in mathematics or that they may speak very little English in general. The teacher should always keep in mind that the real issue is to ensure that students actually understand each other. Hence, a teacher should use the strategies that she would use in any other subject. In addition to simplifying oral language, teachers should expand on student responses and build on those when posing their next question. Some students pass through what is known as a silent period when they are just beginning to learn a second language; during this time, they are listening and trying to make sense of the rules for conversation in the classroom as well as in the larger world. Teachers will need to give students time and look for nonverbal cues as to whether they understand the gist of the lesson. Also, other bilingual students should be asked to speak with that student.

Generally, classroom conversations about mathematics are slow in starting; but when students are socialized into the norms of those conversations, the conversations can be quick and full of implicit referents. Teachers will need to monitor exchanges carefully to ensure that students acquiring English actually participate.

Concluding Comments

There are many other suggestions for mathematics teachers to use when they encounter students acquiring English (see, for example, Secada & Carey, 1990). As is the case with general techniques for teaching mathematics, these techniques are intended to ensure that students can communicate their understandings with their teachers and with one another. It is important to remember that the goal, in all cases, is for the student to understand the mathematics that he or she is encountering. The teacher needs to assess whether or not this is happening and to decide what to do next (give an easier problem, provide help that is pitched so that the student still has to do some reasoning, add context to a problem, translate, or simplify the language) based on judgments of how well the students are understanding and based on the goals for the lesson. The curriculum needs to support the teacher by providing mathematics content that is worth understanding, by having problems and activities that create opportunities for students to make sense of what they are doing, and by being sequenced so that students can make connections between what they know and what is being presented. Finally, the students need to continue to do the work we know they do when they enter school; that is, they need to make sense of the mathematics they encounter, talk about how they figured things out so that they can check with one another (regardless of the language they speak), and listen respectfully when another student is making a point. In other words, the entire mathematics classroom needs to change in its norms for behavior.

The exciting thing is that we are beginning to get evidence that mathematics *can* be taught in this way. What is more, when mathematics is taught for understanding, teachers feel increasingly effective in their teaching and students actually learn more mathematics.

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