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ABSTRACT

Advanced Placement Calculus examinations were administered to nearly 7,000 students in order to determine the impact of calculator use. Both experimental examinations had two sections. Section I items were designed so that a calculator was not needed, but approximately half of the students were permitted to use calculators. Section II items required a calculator to arrive at the correct solution, and all students were permitted calculator use. Calculator use resulted in a substantial improvement in performance for 3 of the 40 Section I items. The internal consistency reliability of the Section I items was higher for students not permitted calculator use. A pattern of negative delta differential item functioning (D-DIF) values for the Section II items indicated that these items were differentially more difficult for females than the Section I items. The D-DIF value of only one Section I item appeared to be substantially impacted by calculator use. Total test scores were found to be higher for those permitted to use calculators, for males in comparison to females, and for those reporting the use of advanced calculators (capable of graphics or the taking of derivatives) in comparison to those reporting the use of scientific calculators. Three appendixes present questions for Calculus AB and BC tests and responses to the student questionnaire. (Contains 3 tables and 13 figures.) (Author/SLD)

RR-91-5

RESEARCH REPORT

EXPERIMENTAL STUDY OF THE EFFECTS OF CALCULATOR USE ON THE ADVANCED PLACEMENT CALCULUS EXAMINATIONS

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Abstract

Advanced Placement Calculus examinations were administered to nearly 7,000 students from more than 400 high schools in order to determine the impact of calculator use. Both the Calculus AB and Calculus BC experimental examinations had two sections. Section I items were designed so that a calculator was not needed to arrive at the correct solution. Approximately half of the students were permitted to use calculators on this section. Section II items were developed to require a calculator to arrive at the correct solution. All students were permitted calculator use for these items. Analyses indicated that calculator use resulted in a substantial improvement in performance for 3 of the 40 Section I items. Those with calculators capable of graphics or the taking of derivatives performed better on both sections of the exams than those with scientific calculators. The internal consistency reliability of the Section I items was higher for students not permitted calculator use. A pattern of negative D-DIF values for the Section II items indicated that these items were differentially more difficult for females than the Section I items. The D-DIF value of only one Section I item appeared to be substantially impacted by calculator use. Total test scores were found to be higher for those permitted to use calculators, for males in comparison to females, and for those reporting the use of advanced calculators in comparison to those reporting the use of scientific calculators.

Experimental Study of the Effects of Calculator Use on the Advanced Placement Calculus Examinations[1]

Since 1986 the use of calculators has not been permitted on the Advanced Placement (AP) Calculus examinations. However, it is becoming apparent that high school and college instructors are incorporating calculators in the classrom. As a result, the Advanced Placement Program has made a decision to collect information relating to the introduction of calculators in the AP Calculus examinations. Two major areas have been focused on. First, a survey has been conducted regarding the practices at a number of colleges of using calculators in calculus classes and their policies regarding awarding credit for AP calculus grades. Second, an experimental study on the effects of calculator use on multiple-choice items has been conducted. This report focuses on the experimental study.

In the spring of 1990, nearly 7,000 high school students from more than 400 schools were administered experimental multiple-choice examinations in calculus. This testing was done in order to determine the impact of calculator use on the AP Calculus examinations. The testing was designed to answer the following questions:

1) How does the opportunity to use calculators affect the statistical properties of individual items and the exam as a whole?
2) How does the opportunity to use calculators affect student performance on the exam as a whole?
3) Is it possible to prepare sound calculator-active items; that is, items requiring students to use calculators in order to solve the problems posed?

[1] The authors wish to thank James Armstrong, Larry Beaber, Chan Jones, Lee Jones, Walter MacDonald, John Mazzeo, and Joan Paszamant for their reviews of this paper. Special thanks is given to James Armstrong for data collection, Mark Batleman for data analysis, and Bernadette McIntosh for providing the figures.

METHOD

Sample

Participants in the study were solicited from two sources: from classes of high school teachers who participated in the 1989 Advanced Placement Calculus reading and through a national mailing to schools that provide AP Calculus courses. Potential participants were informed of the purpose of the study and requirements for participation. Several criteria for taking part in the study were defined. All participating students had to be enrolled in an AP Calculus course in preparation to take either the AP Calculus AB or the AP Calculus BC examination. Each student was required to have a scientific calculator and to be familiar with it. Calculators with advanced capabilities were allowed.

Teachers were informed that some method of motivating student performance on the test was required, preferably by allowing performance on the examination to contribute to the student's course grade. To facilitate use of the examination in their AP classes, teachers were provided with scoring stencils. Teachers were also asked to devote 30 minutes to orientation for the examination and two class periods for testing Students were asked to complete a brief questionnaire, as well.

As a result of these procedures, 7,878 exam booklets were sent to 313 AP Calculus AB classes and 2,894 booklets were sent to 166 AP Calculus BC classes. In February 1990, each participating teacher was sent orientation materials that described the study, provided sample calculator-active questions and solutions, and described study procedures. In April, an administration manual, scoring stencils, and the test materials were delivered to each teacher.

Instrument

Two exams were administered, each consisting of 30 five-choice items. One exam covered Calculus AB topics, while the other exam covered Calculus BC topics. The test administered depended on the level of the high school calculus course. Both exams were formula scored to correct for guessing, with a quarter of a point subtracted for each incorrect response.

Both the Calculus AB and Calculus BC experimental examinations had two sections. On each exam, Section I contained 20 items. The Section I items were designed so that a calculator was not needed to arrive at the correct solution. These items were equivalent to those currently on the AP Calculus examinations. In fact, both exams contained 10 items (the even-numbered items) that had been administered on previous AP examinations. These previously administered items served as equating items. Equating items allow for the comparison of the difficulty level of items in different test forms (or test conditions) independent of the ability levels of the different student groups.

Section II of both exams contained 10 new items developed to require use of a calculator to determine the correct solution. These items are called the calculator-active items. This set of items contained no equating items.

Procedure

Testing was conducted during two class periods. Section I was administered on the first day of testing, and students were given a time limit of 40 minutes to complete the 20 items. On the second day, students were given 30 minutes to complete the 10 calculator-active items that make up Section II. For operational AP Calculus exams, students are allowed 90 minutes to complete 45 items. Thus, in Section I students had the same average number of minutes per item (two) as is allocated on operational tests. Section II was timed to give students an average of three minutes per item.

Within each high school class, students were assigned to two groups. One group was not permitted to use their calculators on the Section I items. This group is called the "no-calculator group". The second group was allowed to use calculators on the Section I items. This group is called the "calculator group". The assignment of students to the two groups was accomplished by spiralling exam booklets for the "no-calculator" condition with exam booklets for the "calculator" condition. Teachers were instructed to pass the booklets out in order and were asked not to adopt any special seating patterns. Having two groups allowed for a comparison of the performance of students permitted to use a calculator with that of students who were not permitted to use one. All students were allowed to use their calculators on the 10 items in Section II.

For both the Calculus AB and Calculus BC exams, there were two orderings of the items in Section I. Items ordered 1 to 10 on one version of a test were items 11 to 20 on the second version. If large numbers of students were unable to reach items near the end of Section I in the required 40 minutes, the impact of the calculator on later items could still be evaluated by examining the data from the other ordering of items. Appendix A contains one form of the Calculus AB exam and Appendix B contains one form of the Calculus BC exam.

Following testing, the students responded to questions concerning the type of calculator they used, the frequency with which they used their calculator in other situations (i.e., homework, other classes), the sufficiency of the time given to complete each section of the exam, whether they intended to take the upcoming AP Calculus exam, and their type of calculator.

RESULTS

Data Analysis Sample

Sixty-six percent (5,194) of the examinations delivered to Calculus AB classes and 61% (1,778) of the examinations delivered to Calculus BC classes were returned with at least one item answered on both sections of the examinations. One of the items on the student questionnaire asked students whether they planned to take the May 1990 AP Calculus examination. Of the 5,194 students enrolled in Calculus AB classes, 4,287 (83%) indicated that they planned to take the 1990 AP exam. Of the 1,778 students enrolled in Calculus BC classes, 1,625 (91%) indicated that they planned to take the 1990 AP exam.

The data were first analyzed to determine whether there was a difference between students who planned to take the upcoming AP Calculus examination and those who did not. In general, the analysis revealed that the performance of students who did not plan to take the upcoming AP examination was significantly lower than that of the students who were planning to take the examination. In the AB group, those who indicated that they would bypass the AP exam had significantly lower mean scores -- expressed as a percentage of the maximum possible section score -- on Section I (25%) and on the calculator-active Section II (16%) than did those indicating they would take the AP Calculus exam (39% on Section I and 23% on Section II).

The parallel analysis for the Calculus BC group also revealed significantly lower performance by those who did not plan to take the AP examination. Their average performance on the two sections of the test was 21% on Section I and 19% on Section II as compared with those planning to take the AP examination, who scored 37% on Section I and 29% on Section II. These results were seen as an indication that these two groups of students represented different populations of students.

Since the primary purpose of the study was to assess the impact of calculator use on the AP examination and on AP candidates, the students who were not planning to take the upcoming exam were excluded from further study. All succeeding analyses refer only to those students who indicated they would be taking the 1990 AP Calculus examination. The resulting sample of 4,287 AB students comprised 2,491 males and 1,787 females, and the sample of 1,625 BC students comprised 1,074 males and 549 females. In the AB sample 9 students and in the BC sample 2 students did not indicate their gender.

For both the Calculus AB exam and the Calculus BC exam, performance on the equating items of this sample was comparable with the general population of AP candidates. The measure of item difficulty used in this report is delta. Delta is based on the percent of students correctly answering an item. A delta value of 13 corresponds to half the students correctly responding to the item. Lower delta values represent easier items and higher delta values represent more difficult items. The delta value is determined by the proportion correct as it is represented under a normal curve. The delta scale has a mean of 13 and a standard deviation of 4. The average equated delta for the 10 Calculus AB equators (based on an operational administration of a Calculus AB examination) was 11.7, while the average observed delta for the same 10 equators for the "no-calculator group" in this study was 12.0. The "no-calculator group" was used for comparison because calculator use is currently prohibited on the AP Calculus exam. The average equated delta value for the 10 Calculus BC equators is 12.6, while the average observed delta on these same items for the "no-calculator group" was 12.8. The small difference between the average observed and equated delta values indicates similarity between the ability levels of the study groups and the AP candidate populations.

Student Questionnaire

Results of the student questionnaire are presented in Appendix C. The questionnaire items are listed with the percentage of respondents who chose each response alternative. Responses are reported in separate columns for Calculus AB and Calculus BC. Questions 1 and 2 asked students about the capabilities of their calculators and question 6 asked students the model of calculator they used for the exam. Note that on question 2, a relatively large percentage of students did not know whether their calculator had symbolic capabilities (capable of taking a derivative). In the list of calculator models in question 6, only the Casio FX7000G has graphics capability and none of the models listed has symbolic capability. Responses to question 1, 2, and 6 were cross-tabulated. Upon inspection, it was found that in some instances student reports of calculator capabilities did not agree with the model of calculator being used. For example, seven AB students reported that the Texas Instruments TI30 calculator has both graphics and symbolic capabilities. As a result, question 6 was used as the primary indicator of the type of calculator. For the students who did not respond to question 6 (1,678, or 32%, of the AB students and 678, or 38%, of the BC students), responses to questions 1 and 2 served as indicators of the type of calculator.

On the basis of these criteria, three "type of calculator" groups were defined: scientific, graphic, and symbolic. The scientific calculator group was defined as those students with calculators incapable of both graphics and the taking of derivatives. The graphics calculator group had calculators with graphic capabilities, but not the power to take derivatives. The symbolic calculator group had calculators capable of taking derivatives. For the Calculus AB group, 88% of the students were classified as using scientific calculators, 11% as using graphics calculators, and 1% as using symbolic calculators. In the Calculus BC group, 87% of the students were classified as using scientific calculators, 10% as using graphics calculators, and 3% as using symbolic calculators.

On question 3, students provided their frequency of calculator use in other situations (i.e., homework, other classes). This question will be referred to as the "frequency-of-calculator-use "question. Note that this question does not refer to the frequency of calculator use on these exams. Students reported a high frequency of calculator use; 72% of the Calculus AB group and 74% of the Calculus BC group reported using their calculator at least once or twice a week.

Students indicated on questions 4 and 5 that there was not enough time to complete the exam sections, especially in the case of Section II which contained the calculator-active items. On that section, 69% of the Calculus AB students and 60% of the Calculus BC students felt they had insufficient time to complete the section.

Summary of Design and Preview of Data Analyses

Two exams were administered, each consisting of two sections. The 20 items in Section I were designed to be similar to items currently administered on the AP Calculus exams. The 10 calculator-active items in Section II were designed so that a calculator was needed to arrive at the solution to the problem. Those students given the opportunity to use calculators in Section I are noted as the "calculator group", while those not permitted the use of calculators in Section I are noted as the "no-calculator group". Students are also grouped according to the kind of calculator they reported using: a scientific calculator, a graphics calculator, or a symbolic calculator.

The analyses that follow focus on two of the three questions this study was designed to answer: first, on how the opportunity to use calculators affects the statistical properties of individual items and the exam as a whole, including the properties of speededness, item difficulty, reliability, differential item functioning (DIF), and construct validity; and second, on the impact of calculator use on student performance on the exam as a whole. The third question, which concerns the soundness of the calculator-active items, is examined throughout.

Focus 1--Effect of Calculator use on Statistical Properties

Speededness

One aspect of the exam that could be affected by calculator use is the speededness of the exam. The primary measure of speededness used in this paper is the percentage of the students reaching items at the end of sections of the exams. According to ETS convention, an item is considered not reached if the item and all subsequent items in the section were not responded to by the student. An item is considered omitted if the student did not respond to the item, but responded to any subsequent item.

Table 1 shows the percentage of examinees who reached items 1 through 20. The upper half of the table provides information for Calculus AB and the lower half provides information for Calculus BC. The data are grouped according to question order and whether the students were in the "calculator group" or the "no-calculator group". The table suggests a small degree of differential speededness due to the use of calculators for the Calculus AB exam, while the Calculus BC exam shows no distinct pattern. Although there appears to be somewhat greater speededness for the calculator group, the levels of speededness are within those generally found on the operational administrations of the AP exams. The five year averages for the percentage of students completing the Calculus AB and BC exams are 66% and 59%, respectively. Table 2 displays the data for the Section II items. Again, some speededness is in evidence for these items, with approximately a third of the students not answering the last item.

Another way in which insufficient time to complete an exam can reveal itself is through greater numbers of omitted items. Figure 1 shows the percentage of examinees omitting each item on the Calculus AB exam. The percentage of students omitting each item for the students in the "no-calculator group" is presented on the horizontal axis. The vertical axis provides the percentage of students omitting each item for the "no-calculator group". Items from both Section I and Section II are included in the figure. For the Section II items, both groups were able to use calculators. Besides providing data for the Section II omit rates, the inclusion of the Section II items provides something of a baseline to evaluate the amount of differences in the omit rates to be expected for the Section I items from two random groups.

In the figure, items are indicated by their item number. Because the differential speededness due to calculator use on Section I was not substantial, the analyses combined data from the two question orders. The item numbers given are for the ordering of items that appears in Appendices A and B. Because ETS defines the percent omit for the last item of a section as zero, items 10 and 20 use only the data from when the item was in position 10.

The plot reveals no pattern of differential omitting resulting from calculator use. The most striking element of the figure is the very high omit rates for a number of items. For example, item 11 has omit rates of more than

50%. It should be noted that half the items that were omitted by more than 20% of the students were in Section II.

Figure 2 displays similar information for the Calculus BC exam. Item 15 has a slightly higher omit rate for the "no-calculator group" (27%) than for the "calculator group" (21%). The omit rate for item 13 is very high for both groups (53% and 56%). Figure 2, like Figure 1, indicates that only one of the calculator-active items has an omit rate of less than 10%. These two plots indicate little, if any, differential pattern for omitting items. They do, however, indicate higher omit rates for the Section II items than for the Section I items. The Section II omit rates are higher than those generally found on operational forms of the exams. The average omit rates for the past five years for the Calculus AB and BC exams are 14% and 16%, respectively.

The levels of omitted items reflect the students' response on the questionnaire, concerning the sufficiency of time given to take the test. For the Calculus AB exam, 37% of the students felt they had insufficient time to finish the items in Section I, while 68% indicated insufficient time to finish the items in Section II. For the Calculus BC exam, 69% of the students felt they had insufficient time to finish the items in Section I, while 60% indicated insufficient time to finish the items in Section II. Even though 50% more time per item was given in Section II than in Section I, more time appears to have been needed given the difficulty level of the questions.

In summary, the opportunity to use calculators appears to have had a slight negative impact on the ability of students to complete the exam. In addition, a pattern of higher omit rates for the Section II items compared to Section I items is present for both exams. Furthermore, it appears that students felt substantial time pressures to complete both sections of the exam, especially the

Section II items.

Item Difficulty

Another area in which calculator use could impact the exam is by altering the difficulty level of individual items. Figure 3 provides the observed delta values for each of the items administered to the Calculus AB group. The observed delta values for the students in the "no-calculator group" are presented on the horizontal axis, while the observed delta values for the "calculator group" are on the vertical axis. (The delta for item 24 was not computed, because it is ETS practice not to compute delta values when less than 5% of the examinees responded correctly to the item.) Again, each item is indicated by its item number. Section II data is again provided to allow for a cursory evaluation of the degree of variation in delta values to be expected from random groups. Items shown below the diagonal line were easier for the "calculator group", while items above the diagonal line were harder for the "calculator group". Item 12 was noticeably easier for the "calculator group" (delta=12.7 vs. 11.5). The difficulty of the other items seems relatively unaffected by calculator use. The figure also shows that four of the seven items with delta values of 15 or greater -- including item 24 -- are among the 10 calculator-active items that make up Section II.

Figure 4 shows the same comparison of delta values for the Calculus BC group. Item 15 (delta=13.5 vs. 11.8) and item 18 (delta=15.5 vs. 13.7) are substantially easier for the students using calculators. For most of the Section I items, the calculator did not noticeably alter their difficulty level. For the Calculus BC exam, the difference in the difficulty level of the Section II items compared to the Section I items is not apparent as was the case with the Calculus AB exam.

Reliability

Another area in which calculator use could alter the test is by changing the reliability of the test, as measured by Kuder-Richardson-20. If items vary according to their degree of calculator impact, then the set of items will become more heterogeneous. The result would be both lowered item r-biserial correlations and lowered estimated exam reliability. Furthermore, calculator use may impact the ability of the item or test to discriminate between more able and less able examinees.

Figure 5 provides the r-biserial correlations for the Section I Calculus AB items for both the "calculator group" and the "no-calculator group". Item numbers located below the diagonal represent items with higher r-biserial correlations for the "no-calculator" group. The Section I reliabilities for both groups are also provided. The figure shows that 16 of the 20 items have higher r-biserials for the "no-calculator group". The remaining four items have the same r-biserials for both groups. The pattern of higher r-biserial correlations for the "no-calculator" group results in the section reliability estimate being .031 higher (.732 vs. .701) in the "no-calculator" condition. Using the Spearman-Brown formula, it is estimated that the section reliabilities would be .860 for the "no-calculator group" and .841 for the "calculator group" on an operational 45-item AP Calculus exam.

Figure 6 provides r-biserial and reliability information for the Calculus BC exam. Although, the pattern of r-biserials is not as clear as that for the Calculus AB exam, it is evident that, in general, the "no-calculator group" has a pattern of higher r-biserials. Additionally, items 2, 3, 11, and 13 have r-biserial differences of .10 or more. The Section I reliability estimate is .023 higher (.705 vs. .682) for the "no-calculator group". On an operational 45-item

AP Calculus exam, it is estimated that the reliabilities would be .843 and .828.

The reliability estimate of the Section II items was .406 for the Calculus AB group and .561 for the Calculus BC group. If Section II were increased to 45 items the Spearman-Brown formula estimates these reliabilities would be .754 for the Calculus AB group and .852 for the Calculus BC group.

The five-year average reliability estimates for the AP Calculus exams are .889 for the Calculus AB exam and .872 for the Calculus BC exam. The lower reliabilities for the 45-item Section I are a result, in part, of using items that were not pretested. The items not pretested are the odd-numbered items (1-19). For the Calculus AB exam these items have lower r-biserials than the pretested items. For the Calculus BC exam the pattern found on the Calculus AB exam does not exist.

Both figures imply that the estimated internal consistency reliability of the test would be slightly lowered if calculators were permitted during the examination. It appears that calculator use not only affects the difficulty level of some items, but produces change in the ability of some items to discriminate more able from less able examinees. Finally, the Calculus AB calculator-active items present serious reliability problems.

Differential Item Functioning

Another way in which calculator use could impact items is by affecting the difficulty level of an item for males and females. For example, the difficulty of an item may be unaffected by calculator use for one sex, while calculator use may result in an item being considerably easier for the other sex. The Mantel-Haenszel D-DIF statistic is the measure of differential item functioning used in this report. The measure indexes the difference in the delta value of an item for two matched groups of examinees. A D-DIF value of 0 corresponds to no difference in the item difficulty (delta) for the two matched groups. Items with high D-DIF values are labeled as "C" DIF items. These items have D-DIF values with absolute values of at least 1.5, and have D-DIF estimates statistically different from 1.0.

Figure 7 shows the values of the D-DIF statistic for male-female DIF for both the "calculator group" and the "no-calculator group" for the Calculus AB exam. Negative values for the D-DIF statistic indicate that the item is harder for females, when males and females are matched on section scores. Conversely, positive D-DIF values note items that are easier for females. Total exam score was used as the matching criterion. Item numbers below the diagonal indicate differentially better performance on the item for females in the "no-calculator group". There are no "C" DIF items and no items that exhibit large differences in the values of the D-DIF statistic for the two groups on the Calculus AB exam.

Figure 8 shows the comparable D-DIF data for the Calculus BC exam. With the exception of item 12, there is no pattern of substantial differences in the D-DIF statistics. Item 12 is a "C" DIF item for both groups and has a considerably higher D-DIF value in the calculator group (-2.86 vs. -1.87). The figure indicates that item 12 is relatively more difficult for female students than for male students and this gender difference is greater when calculators can be used.

A pattern of generally negative D-DIF values is shown for the items in Section II for both the Calculus AB exam and the Calculus BC exam. Thirteen of the 20 Section II items (10 from Calculus AB and 10 from Calculus BC) have negative D-DIF values. In contrast, only two of the 20 items have positive D-DIF values. As a result, while it appears that calculator use does not generally affect the estimates of the D-DIF statistic for individual items, a pattern of negative D-DIF values indicates that the Section II calculator-active items are differentially more difficult for females than are the Section I items.

Type of Calculator

Figure 9 provides a comparison of the equated deltas for each of the items on the Calculus AB exam for the students using scientific calculators with the students using graphics calculators. Deltas equated within each student group are presented because calculator type was not a variable controlled for in the study. As a result, students with graphics calculators may differ in a number of ways from the students with scientific calculators and the use of of equated deltas can, in part, control for group ability differences. The purpose of Figures 9 and 10 is to find if those students with graphics calculators performed comparatively better on specific items than would be expected given the performance on the same item of those with scientific calculators. This should aid in identifying types of items for which the difficulty levels may be impacted by the use of graphics calculators. However, the results cannot be used to predict how students using scientific calculators would have performed if they were using graphics calculators.

The equated delta values for the students using scientific calculators are presented on the horizontal axis, while the equated delta values for the students using graphics calculators are on the vertical axis. Items located below the diagonal line were relatively more difficult for the students using scientific calculators than for the students using graphics calculators. For the Section I items, only students in the "calculator group" were used to compute equated deltas. All examinees were used in calculating equated deltas for the Section II items. (Items 23 and 24 are not plotted due to high difficulty levels.)

Items 11, 12, 13, and 17 have equated deltas more than two delta points higher for the scientific calculator group than for the graphics calculator group. Those items were relatively harder for the students using scientific calculators. The plot also indicates that most of the 10 calculator active items are more difficult for the students using scientific calculators than for those using graphics calculators.

Figure 10 provides the equated deltas by calculator type for the Calculus BC exam. The figure shows an equal number of items above the diagonal line as below it. Unlike the Calculus AB exam, no items differ by two or more delta values. Item 7 has the highest equated delta differential in favor of the scientific calculator (15.6 vs. 13.9), while item 29 most favors those with graphics calculators (16.3 vs 14.5). As with the AB exam, most of the calculator-active items are easier for those with graphics calculators than for those with scientific calculators.

Construct Validity

Table 3 (page 27) presents the section reliabilities, the correlations between Section I and Section II, and the estimated correlations between the two sections corrected for attenuation (true-score correlation). The reliabilities and correlations are grouped according to calculator group, gender, and calculator type. One striking feature of the table is that the true-score correlations cluster around .90. If Section I and Section II items are measuring the same construct, then the true-score correlations should be at or very close to 1.00. For example, the estimated true-score correlation of the "C topic" items with the "lower level" items on the operational administration of the 1990 Calculus BC exam was .99, while the estimated true-score correlation of the "elementary functions" items with the "upper level" items on the operational administration of the 1990 Calculus AB exam was .98. It thus appears that the construct measured by the calculator-active items may be different from what is measured by current AP items.

Focus 2--Effect of calculator use on student performance

One of the purposes of the study was to determine whether the opportunity to use calculators impacted student performance on the test as a whole. Such effects might be observed in a number of ways. For example, the cumulated effects of differences in the functioning of individual items, as described above, may result in differences in exam performance. It is also of interest to determine whether the opportunity to use calculators resulted in significant differences in exam scores, and whether such differences varied as a function of gender and type of calculator used. In order to assess these effects, analysis of variance methods were used to perform separate analyses for the Calculus AB and Calculus BC groups.

Before performing the analyses, a number of factors that might complicate interpretation were considered. One factor was the self-selection of type of calculator used. Students with more advanced calculators are likely to differ in a number of ways from students with scientific calculators. For example, it was found that the reported frequency of calculator use was related to the type of calculator students use. For both the Calculus AB and Calculus BC groups, students with advanced calculators reported using their calculators more frequently than those with scientific calculators. As a result, differences that appear to be due to type of calculator might in fact be a function of calculator experience and sophistication. Additionally, males reported using more advanced calculators than females. The data indicate that 11% of females used advanced calculators on the AB exam compared to 16% for males. On the Calculus BC exam 12% of females used advanced calculators compared to 20% of males.

Section I Scores

In order to moderate the influence on Section I scores of the factors noted in the previous paragraph, the analysis done on the AB group used performance on Section II as a covariate. Reported frequency of calculator use was also examined as a potential covariate, but was not significantly related to Section I performance. In essence, this procedure involved statistical adjustments to make the groups more comparable in terms of ability as measured by Section II formula score performance (where all students had an opportunity to use calculators). Using this covariate, the AB group analysis examined whether Section I performance differed due to calculator group (calculator allowed vs. no calculator), gender, and type of calculator used (scientific vs. advanced[2]).

Figure 11 displays means as a percentage of the maximum score, adjusted for the effect of the covariate, for each of the groups. Results of the analysis indicated that Section I performance differed significantly for males and females ($F = 6.32$, $p < .01$)[3]. Performance of males was higher than that of females ($M_M = 39.5$ $M_F = 38.0$). This result is consistent with the gender difference found on operational administrations of the Calculus AB exam.

It can also be seen in Figure 11 that, as a whole, the group that was allowed

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[2]Since the number of students reporting use of symbolic calculators was small, especially for females, graphic and symbolic calculators were considered together as 'advanced' calculators, for both the AB and BC groups.

[3]The report of the analysis of variance results uses the symbols, F, p, and M. Reported F-values are the magnitude of the test statistic. If there is no difference between groups, this value is expected to be 1.00. As group differences become large, the value of F increases. The p-value represents the probability of obtaining the reported result by chance alone. The symbol, M, refers to the group means.

to use calculators performed significantly better ($F = 3.83$, $p < .05$) than the "no-calculator group" (M = 39.3 vs. M = 37.3). This result is best interpreted in light of the interaction of calculator type with calculator group. There was a statistically significant ($F = 5.12$, $p < .02$) difference in performance as a result of the type of calculator depending on whether calculator use was allowed. In the "no-calculator group", performance on Sectio I was somewhat lower for those using advanced calculators (M = 36.0) than for those using scientific calculators (M = 38.7). When calculator use was allowed, however, those using advanced calculators (M = 40.1) scored higher than those using scientific calculators (M = 38.4). While not statistically significant, it is also notable that the higher performance of those in the "calculator group" who use advanced calculators is primarily attributable to higher scores for males. In fact after adjusting for the covariate, females in the calculator group who used scientific calculators scored somewhat higher (M = 38.1) than females who used advanced calculators (M = 37.7).

Analyses similar to those performed for the Calculus AB group were computed for the Calculus BC group. Because initial group differences among the Calculus BC students were small and there were no substantive differences in the results of analyses when covariates were used, only the simplest analysis, an analysis not utilizing covariates, is reported here.

Section I total scores for the Calculus BC group were examined for differences due to calculator group, gender, and calculator type. Figure 12 displays means as a percentage of the maximum possible score for the Calculus BC group. The figure shows that the performance of those using scientific and advanced calculators in the "no-calculator" condition is comparable. When calculator use was allowed, the performance of those using scientific calculators was similar to that observed in the "no-calculator" condition. Performance of those using advanced calculators was somewhat higher, although the observed differences are not statistically significant. The largest difference in performance for the BC group occurs between males and females ($M_M = 40.4$, $M = 33.6$) and is the only effect that is statistically significant ($F = 14.66$, $p < .001$) for the BC group. While performance by females in the "calculator-group" condition was substantially higher for those reporting use of advanced calculators in comparison to those reporting use of scientific calculators, this difference is not statistically significant, due in part to small sample sizes.

Section II Scores

In order to investigate the impact of type of calculator on exam performance on the calculator-active items, an analysis of variance was conducted on the Section II scores. The analysis for both the Calculus AB and the Calculus BC groups utilized Section I scores and the frequency of calculator use as covariates. Frequency of calculator use was coded as a quantitative variable for which "once or twice" was interpreted as 1.5 times each week, month, etc. Both covariates met assumptions for linearity and homogeneity of regression slopes. To ensure that the covariate measuring calculus ability (Section I scores) was free of the influence of ability with a calculator, the analysis was limited to the use of those students in the "no-calculator group". The analysis examined whether there were performance differences on the calculator-active items due to calculator type and gender.

The left side of Figure 13 displays means as a percentage of the maximum score, adjusted for the effects of the covariates, for both calculator type and gender for students in the Calculus AB group. Results of the analysis indicated that Section II performance differed significantly for males and females ($F = 9.37$, $p < .002$). Performance for males was higher than that for females ($M_M = 25.8$, $M_F = 22.1$). Figure 13 also shows that, as a whole, the group using advanced calculators performed significantly better ($F = 7.76$, $p < .005$) than the group using scientific calculators (M = 25.7 vs. M = 22.3).

The right side of Figure 13 displays means as a percentage of the maximum possible score for the Calculus BC group. Results of the analysis indicated that Section II performance differed significantly for males and females ($F = 20.22$, $p < .001$). Performance for males was higher than that for females ($M_M = 33.0$, $M_F = 21.8$). An interaction existed between gender and calculator type ($F = 4.82$, $p < .05$). Performance of male students using advanced calculators was higher than the performance of male students using scientific calculators (M = 36.4 vs. M = 29.6). Conversely, the performance of female students using advanced calculatorswas lower than the performance of females using scientific calculators (M = 19.7 vs. M = 23.9).

The analyses of the Section I and Section II data seem to point to an advantage provided by the use of advanced calculators. Furthermore, evidence seems to indicate that the advantage provided by using advanced calculators is greater for males than for females. However, all inferences concerning type of calculator must be tempered somewhat, since students were not randomly assigned to the type of calculator condition.

SUMMARY AND DISCUSSION

A number of the analyses on the statistical properties of individual items provided evidence regarding the impact of calculator use. When accumulated over items, these effects have an impact on student performance on the exam as a whole. In addition, several analyses assessed directly the impact of calculator use on the total scores earned by students on the two sections of the examination. These results should be tempered by two caveats. First, greater faith can be placed in results that show differences between the randomly assigned "calculator group" and "no-calculator group". Second, effects associated with the type of calculator used should be interpreted with caution. The number of students reporting use of advanced calculators represented a relatively small sample. Furthermore, it is likely that students with one type of calculator differ in a number of ways from those with other types of calculators. An observed difference between calculator types may be the result of the opportunity to use a more sophisticated tool, but it is just as plausible that students with more advanced calculators are also more able students, have different socioeconomic or educational backgrounds, are more sophisticated calculator users, or differ in any number of other ways.

As stated earlier, this study was designed to answer three questions. This section of the report summarizes the findings pertaining to each of those questions.

Question 1

How does the opportunity to use calculators affect the statistical properties of individual items and the exam as a whole?

1) Patterns of omitted items and not reached data indicate that the opportunity to use calculators had a small negative impact on the ability of students to complete the exam. However, the introduction of calculator-active items, which were given 50% more time per item, may

necessitate changes in the number of items on operational tests.

2) The introduction of calculators did not alter the difficulty of most items that did not require a calculator for solution. However, for a number of items the performance of students with graphics calculators was superior to the performance of students with scientific calculators.

3) The internal consistency estimate of reliability for both exams was lower when students were permitted to use a calculator. Calculator use also appears to produce changes in the ability of individual items to discriminate between more able and less able students.

4) For most items, the opportunity to use calculators did not substantially change the degree of DIF exhibited by the items.

5) The construct measured by the calculator-active items in Section II was somewhat different than the construct measured by the Section I items.

Question 2

How does the opportunity to use calculators affect student performance on the exam as a whole?

1) Use of calculators resulted in better performance. This was evidenced in the differential Section I performance of the "calculator group" and the "no-calculator group" on both Calculus AB and, to a lesser extent, Calculus BC.

2) The type of calculator used resulted in substantial differences in performance. Those reporting use of advanced calculators performed better than those reporting use of scientific calculators on both Section I (in the "calculator group") and Section II. It is also interesting to note that, in the "no- calculator" condition for the Calculus AB group,

both the covariate adjusted means and the raw means for Section I of those reporting use of advanced calculators were lower than the Section I means of those with scientific calculators. While this result may be due to the lack of comparability of self-selected groups, it may also be that those who rely on advanced calculators are disadvantaged when they are unable to use them.

3) As on operational forms of the AP Calculus examinations, males performed significantly better than females. This result is complicated, however, by several other patterns observed in the study. In some cases calculator use or use of advanced calculators seemed to increase the differential between males and females, while in one case the difference between males and females was decreased. These results are summarized below.

   a) Males reported use of advanced calculators more often than females. The frequency with which calculators are used for homework or in other classes was also greater for males than for females.

   b) In the Calculus AB group, Section I performance in the "calculator group" was slightly lower for females using advanced calculators than for females using scientific calculators. On the Section II calculator-active items, female performance in the Calculus BC group was substantially lower when using advanced calculators than when using scientific calculators.

   c) For the "calculator group" taking the Calculus BC exam, the difference in performance on Section I between males and females was smaller for those reporting use of advanced calculators.

Question 3

Is it possible to prepare sound calculator-active items; that is, items requiring students to use calculators in order to solve the problems posed?

1) Preparing calculator-active items with the proper degree of difficulty appears to be hard to do. For the Calculus AB exam, the delta values of the Section II items are higher than the delta values for the Section I items. For both exams, the omit rates are high for the Section II items.

2) The low reliability of Section II for the Calculus AB group indicates difficulty in writing calculator-active items that effectively discriminate more able students from less able ones.

3) For both exams a pattern of negative D-DIF values indicates that the calculator-active items are differentially more difficult for females than the Section I items.

4) After controlling for self-selection of calculator type, most of the calculator-active items are easier for those with graphics calculators than for those with scientific calculators.

Table 1

Percent of Students Reaching Item
Section I

Calculus AB

Item Position	Order 1 Item Number	Order 1 No Calculator	Order 1 Calculator	Order 2 Item Number	Order 2 No Calculator	Order 2 Calculator
1-11	1-11	100.0	100.0	1, 11-20	100.0	100.0
12	12	100.0	100.0	2	100.0	99.7
13	13	99.7	99.9	3	99.6	99.0
14	14	99.4	99.8	4	99.0	98.3
15	15	97.8	98.1	5	97.6	96.5
16	16	97.1	96.3	6	96.1	93.8
17	17	95.1	94.6	7	94.1	90.6
18	18	89.5	86.3	8	92.1	87.9
19	19	83.6	78.3	9	84.4	80.1
20	20	56.0	54.3	10	69.0	62.9

Calculus BC

Item Position	Order 1 Item Number	Order 1 No Calculator	Order 1 Calculator	Order 2 Item Number	Order 2 No Calculator	Order 2 Calculator
1-11	1-11	100.0	100.0	1, 11-20	100.0	100.0
12	12	99.3	98.7	2	99.8	99.5
13	13	97.8	97.4	3	98.1	98.3
14	14	97.5	96.9	4	97.4	96.5
15	15	97.0	93.2	5	92.6	94.6
16	16	93.8	90.6	6	89.1	91.4
17	17	83.3	81.3	7	84.4	84.4
18	18	78.5	77.1	8	82.3	81.7
19	19	69.8	68.8	9	72.6	73.8
20	20	67.0	66.2	10	70.7	69.4

Table 2

Percent of Students Reaching Item
Section II

Item Number	Calculus AB	Calculus BC
21	100.0	100.0
22	99.9	100.0
23	99.3	100.0
24	99.0	99.9
25	97.9	99.2
26	97.0	98.0
27	93.1	91.4
28	84.3	84.7
29	80.5	74.5
30	66.0	68.9

Table 3

Section Reliabilities and Section Correlations

Group	Section Reliability		Correlation of Section I and Section II	
	Section I	Section II	Raw	Corrected
Calculus AB				
No Calculator Group	.73	.41	.49	.89
Calculator Group	.70	.41	.50	.94
Male	.73	.44	.50	.88
Female	.69	.31	.47	.97
Scientific Calculator				
No Calculator Group	.73	.39	.49	.91
Calculator Group	.70	.41	.51	.95
Graphics Calculator				
No Calculator Group	.74	.49	.44	.73
Calculator Group	.67	.44	.49	.89
Calculus BC				
No Calculator Group	.71	.53	.57	.94
Calculator Group	.68	.59	.56	.89
Male	.70	.58	.57	.90
Female	.64	.42	.49	.95
Scientific Calculator				
No Calculator Group	.71	.50	.58	.98
Calculator Group	.69	.58	.56	.89
Graphics Calculator				
No Calculator Group	.68	.59	.58	.92
Calculator Group	.60	.56	.51	.89

Figure 1
Percent Omit

Calculus AB

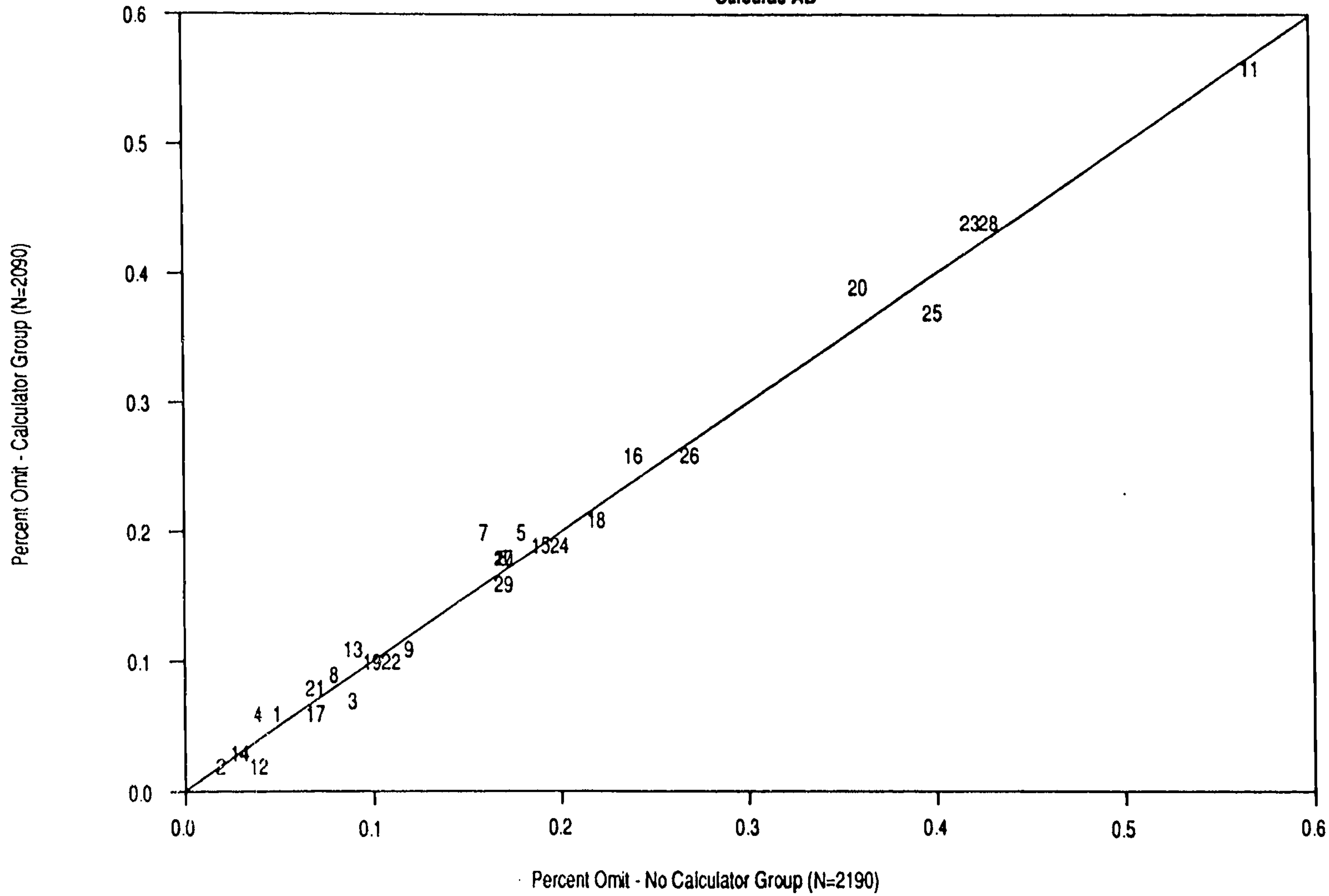


Figure 2
Percent Omit

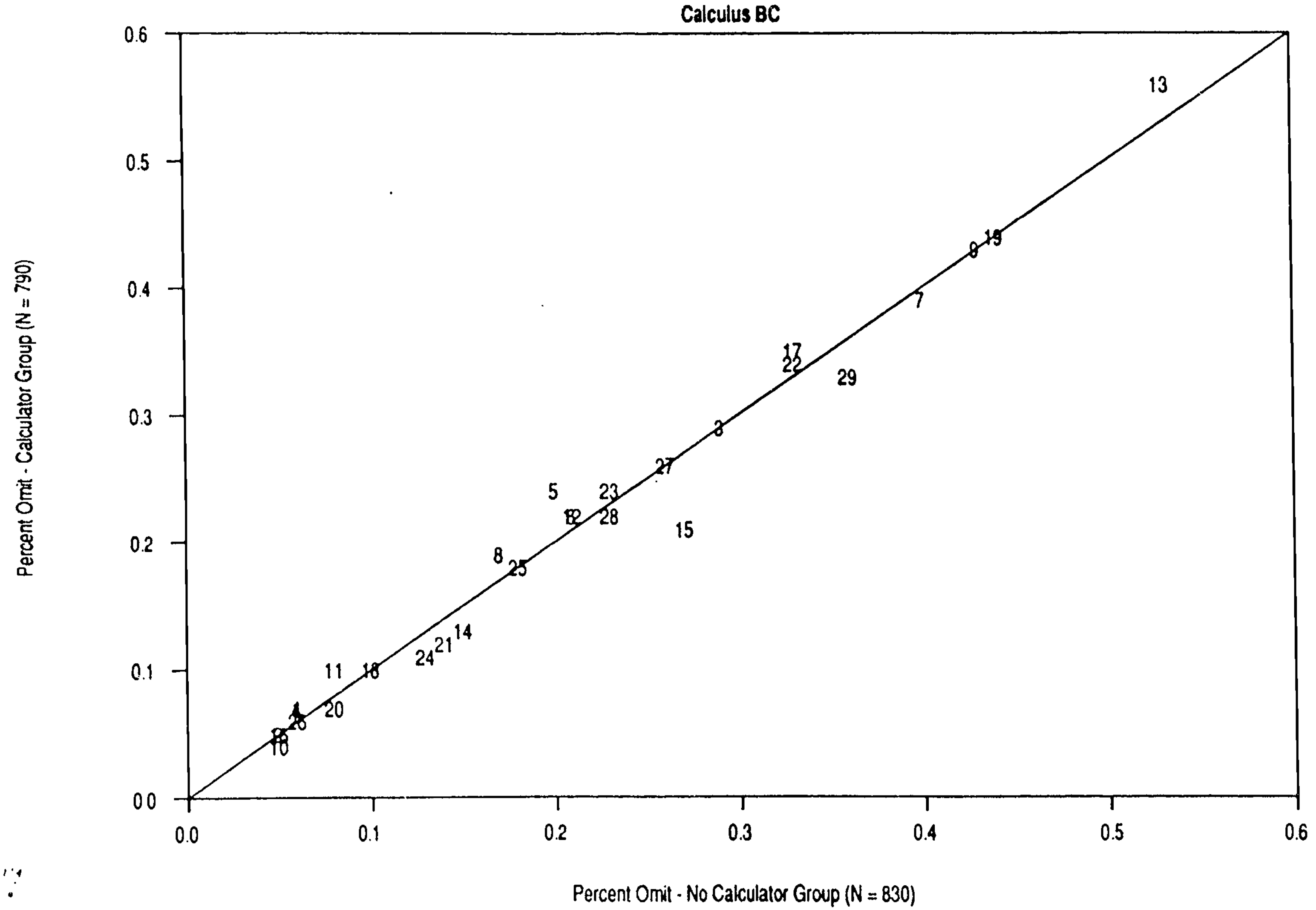


Figure 3

Comparisons of Delta Values

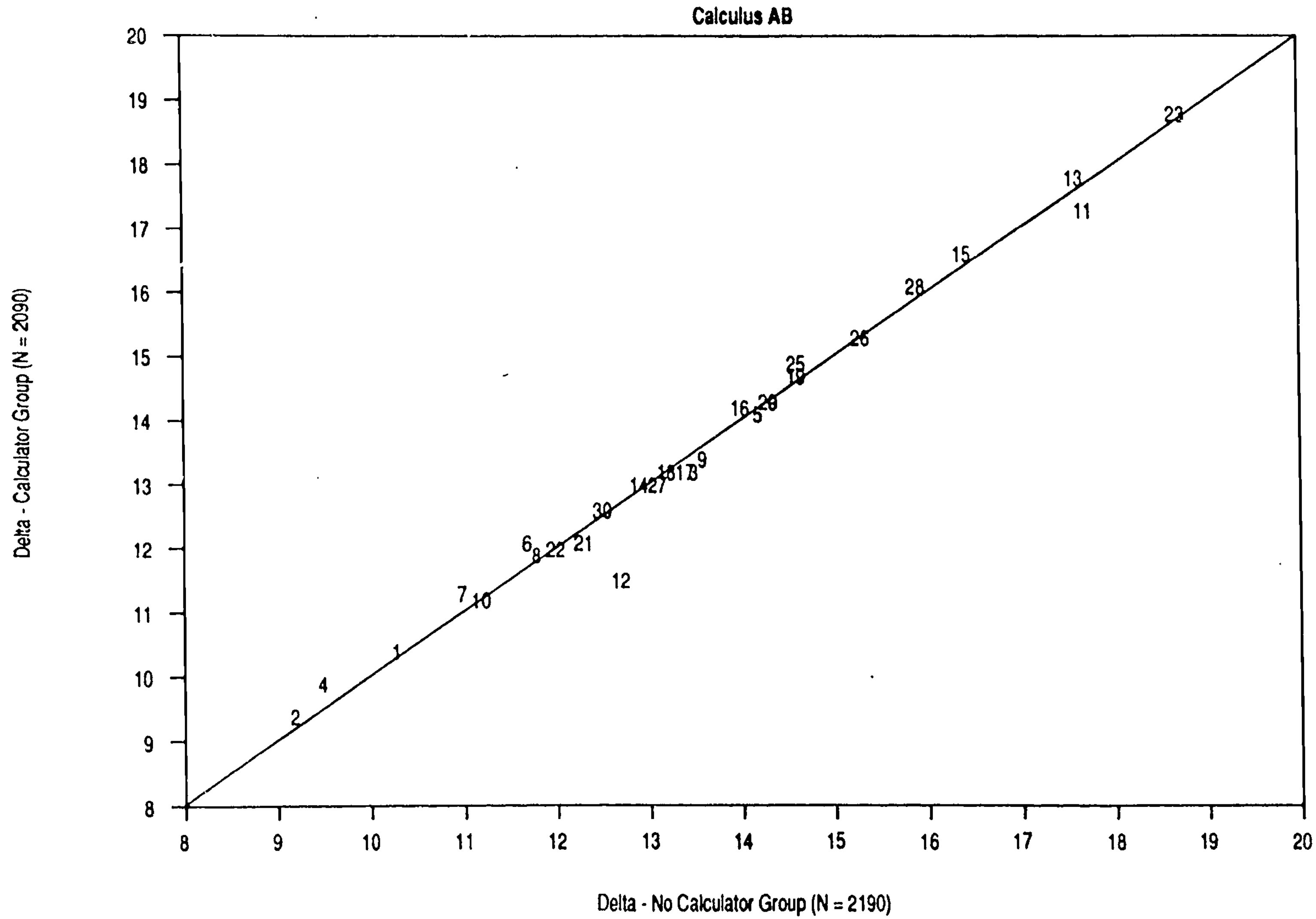


Figure 4

Comparisons of Delta Values

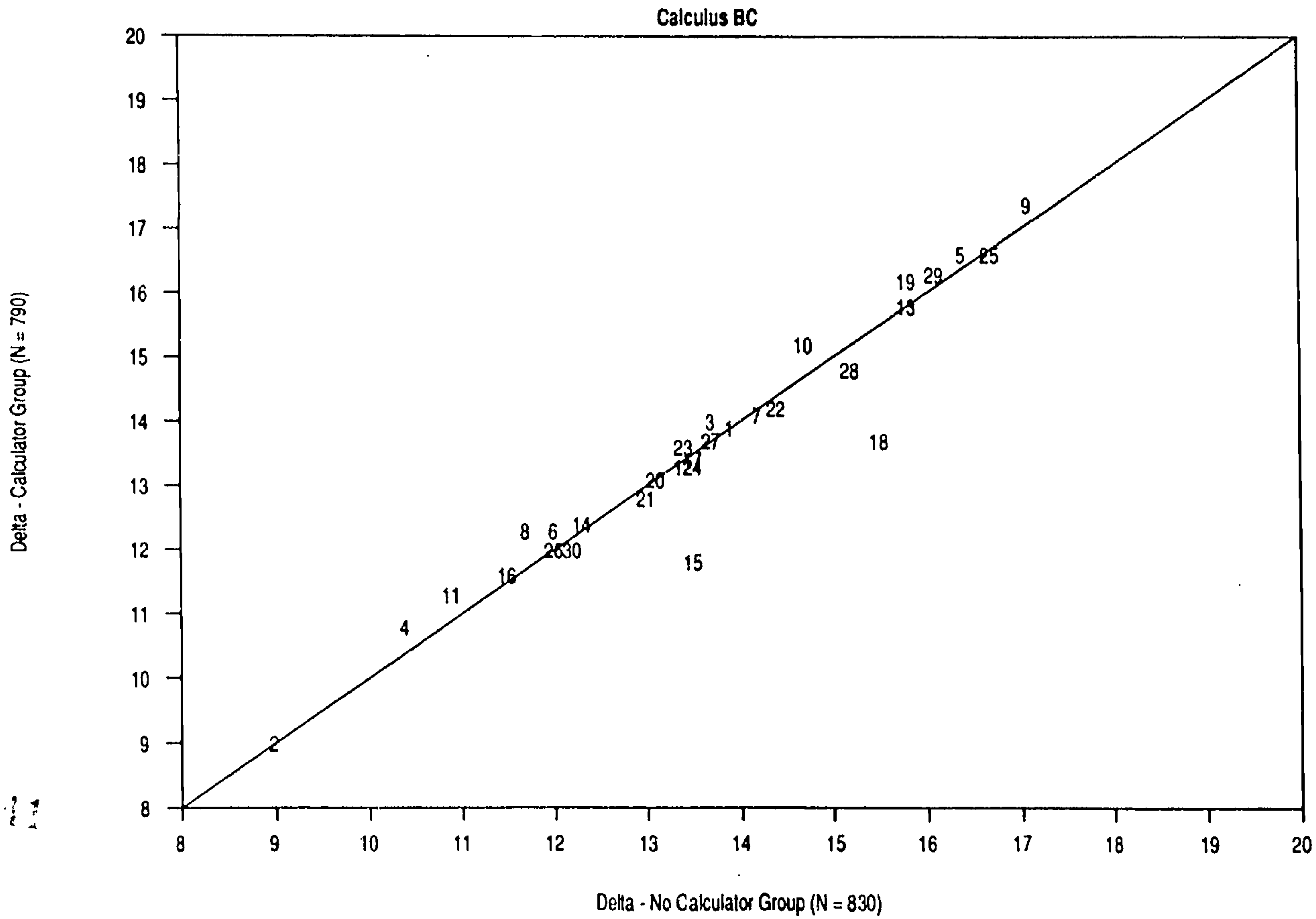


Figure 5

R-biserial

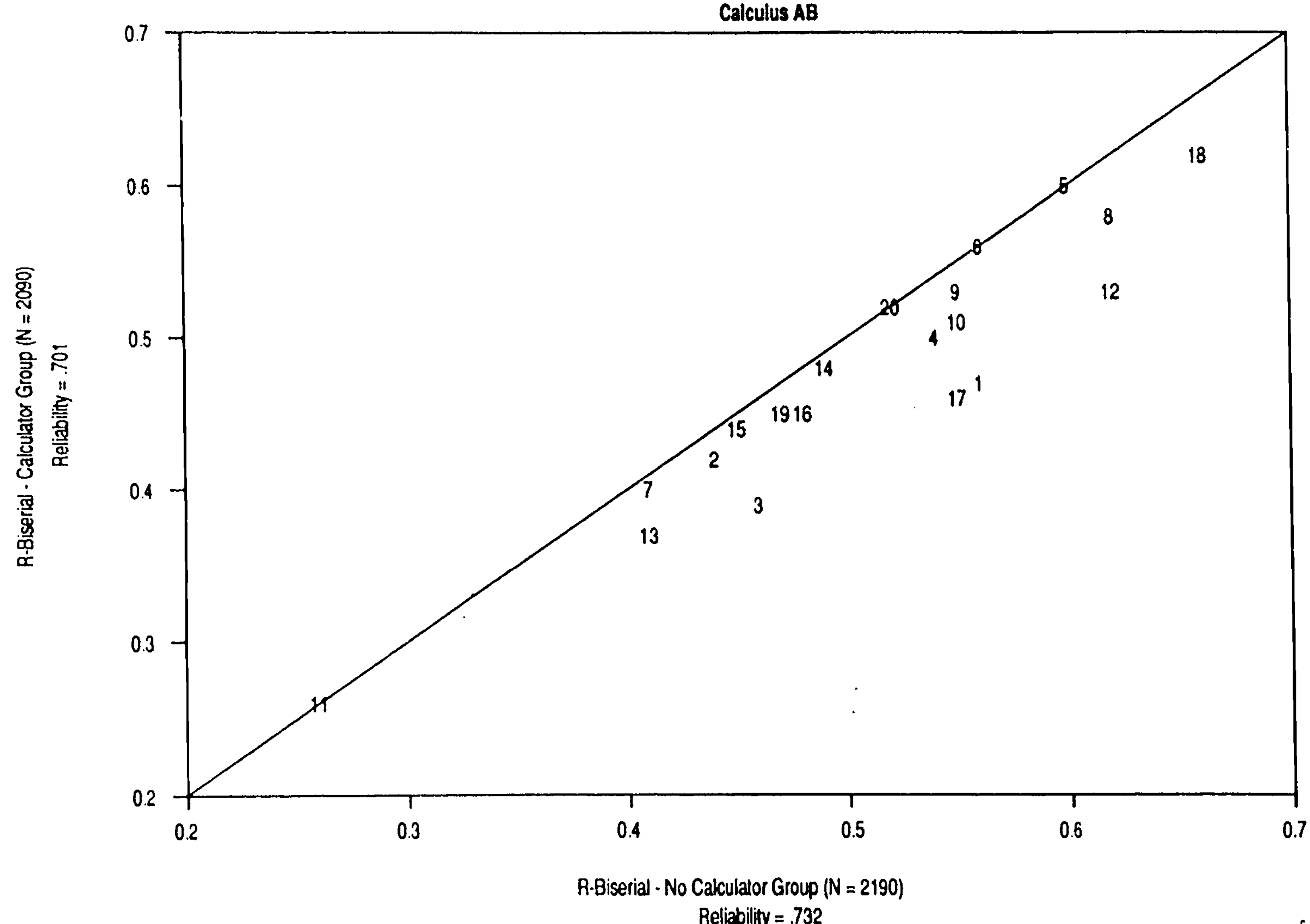


Figure 6
R-biserial

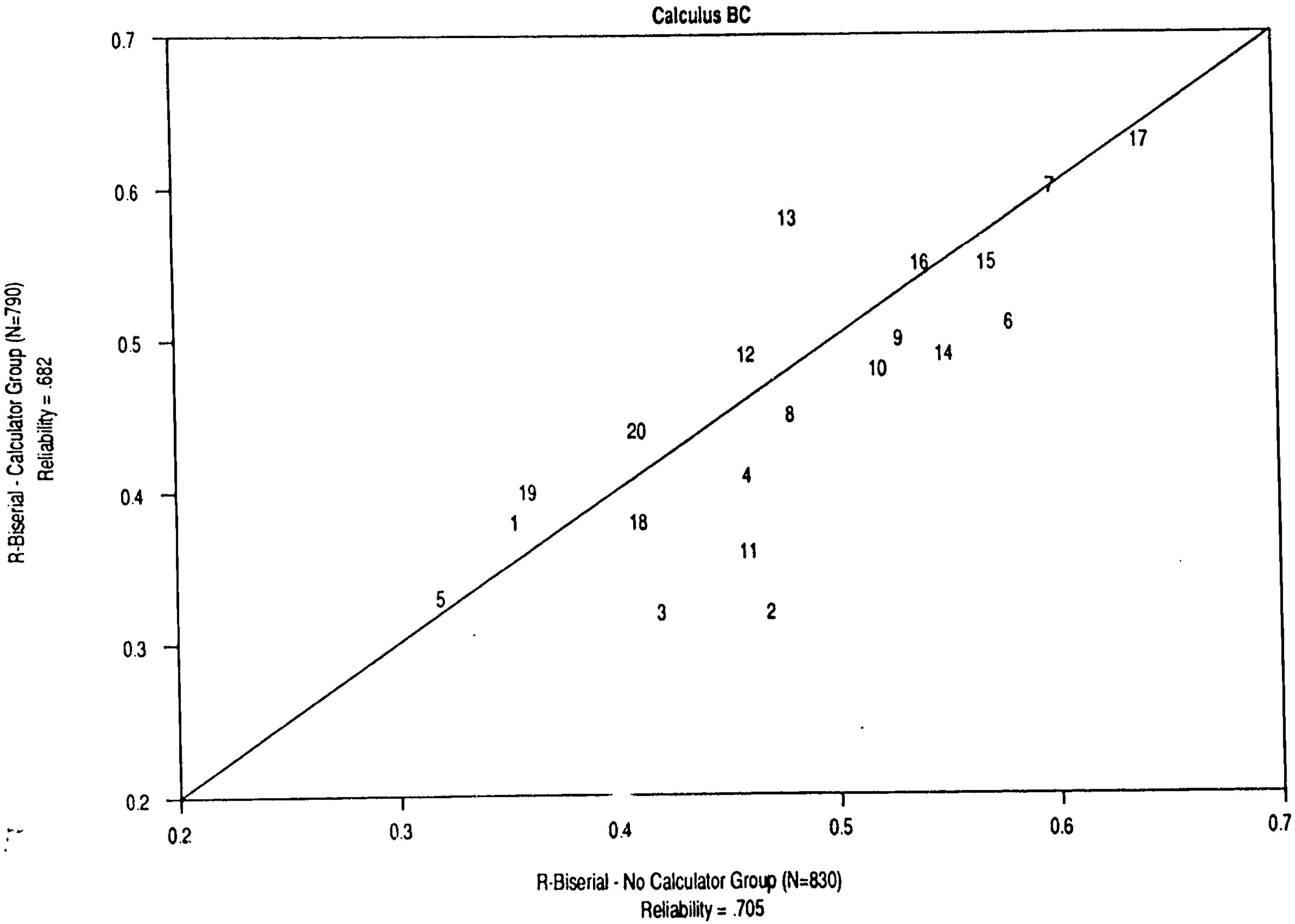


Figure 7

Male-Female DIF

Calculus AB

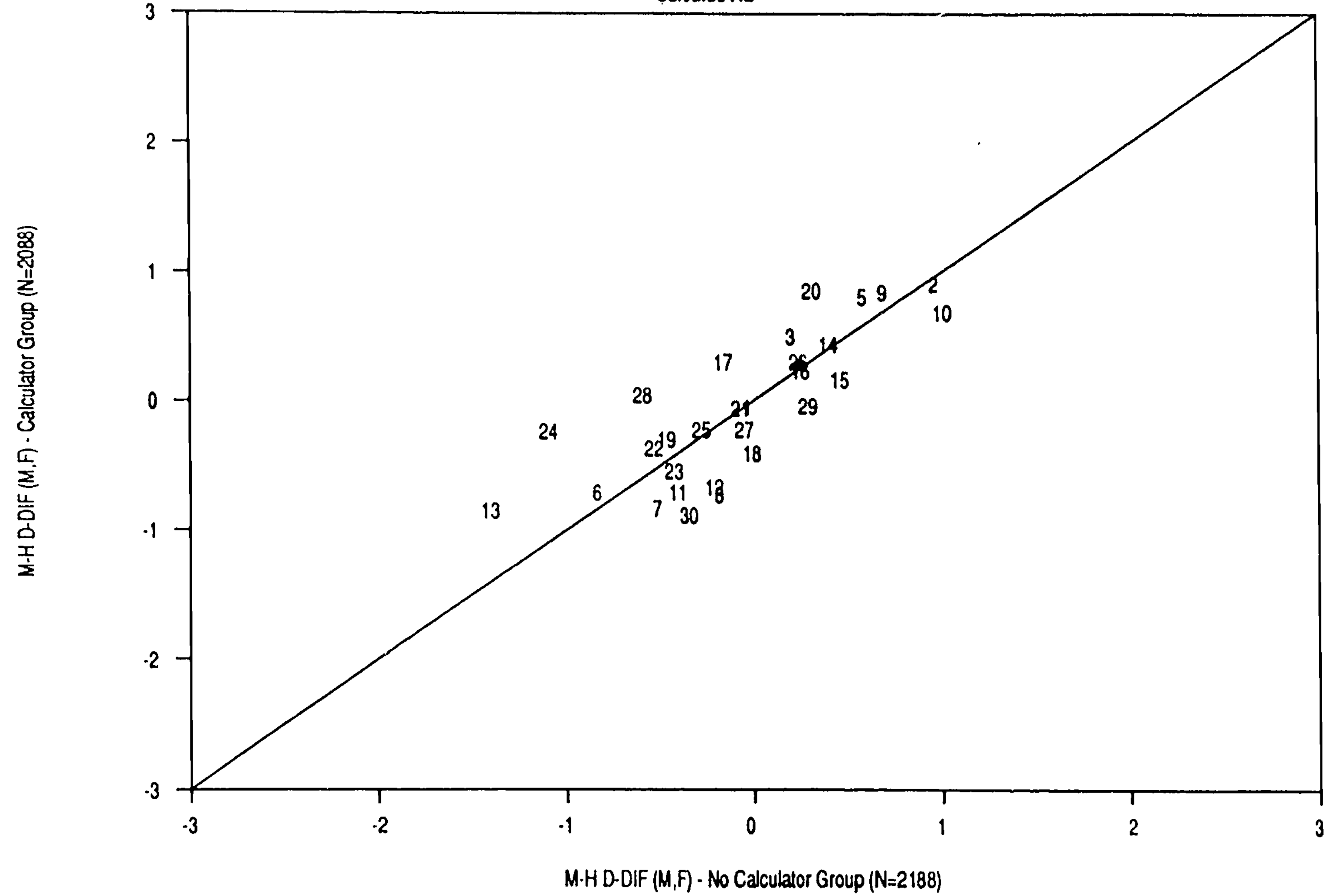


Figure 8

Male-Female DIF

Calculus BC

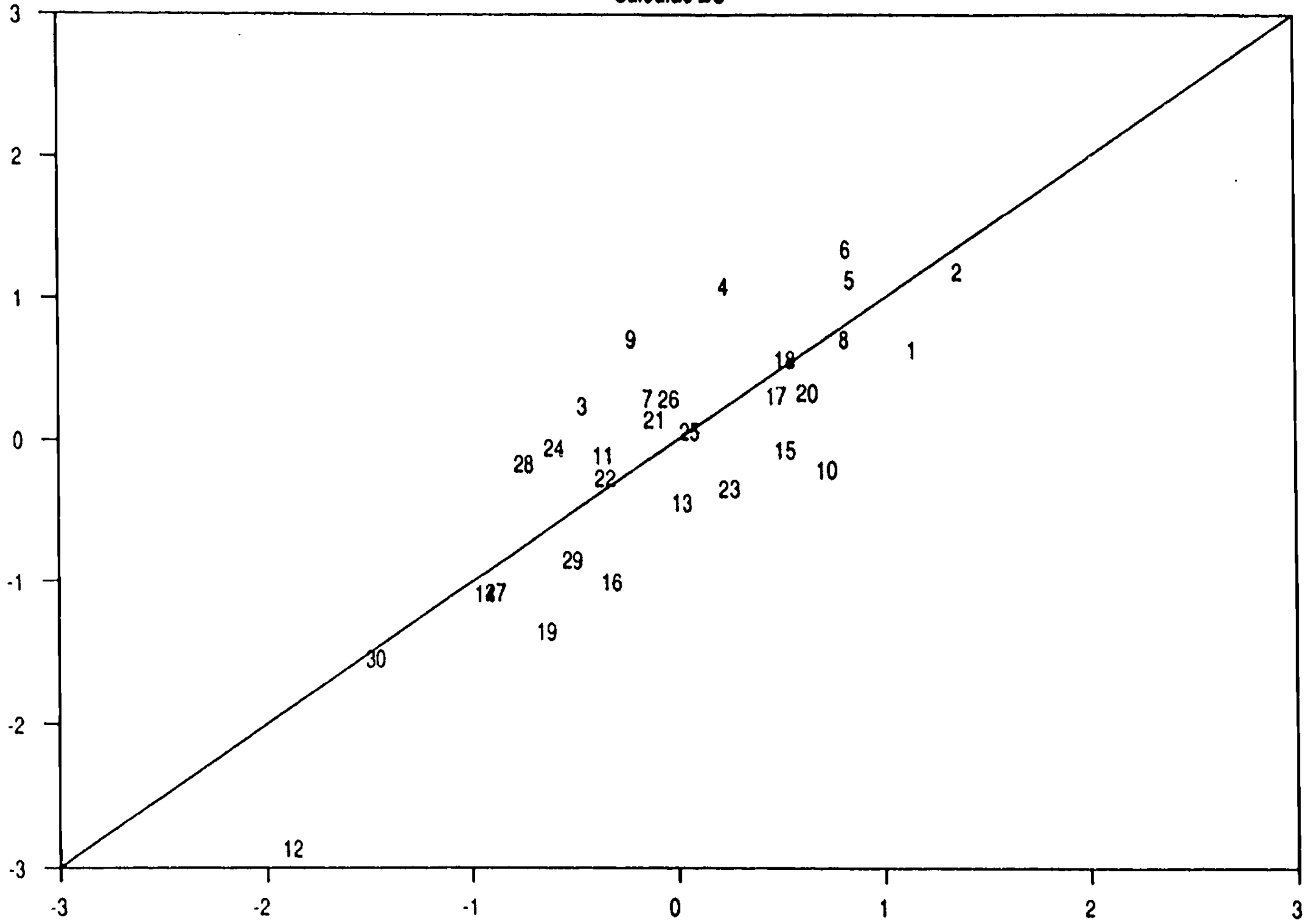
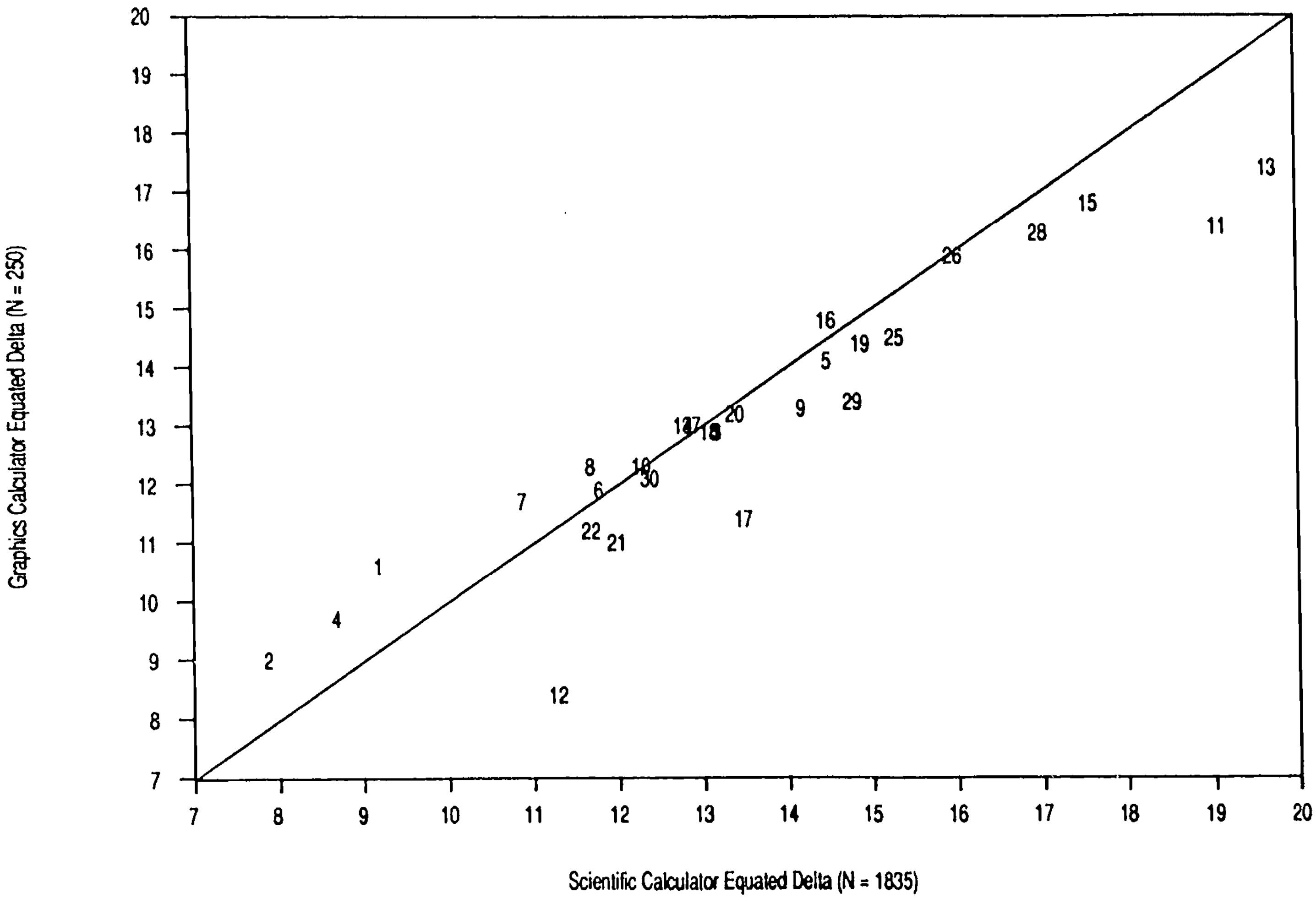


Figure 9

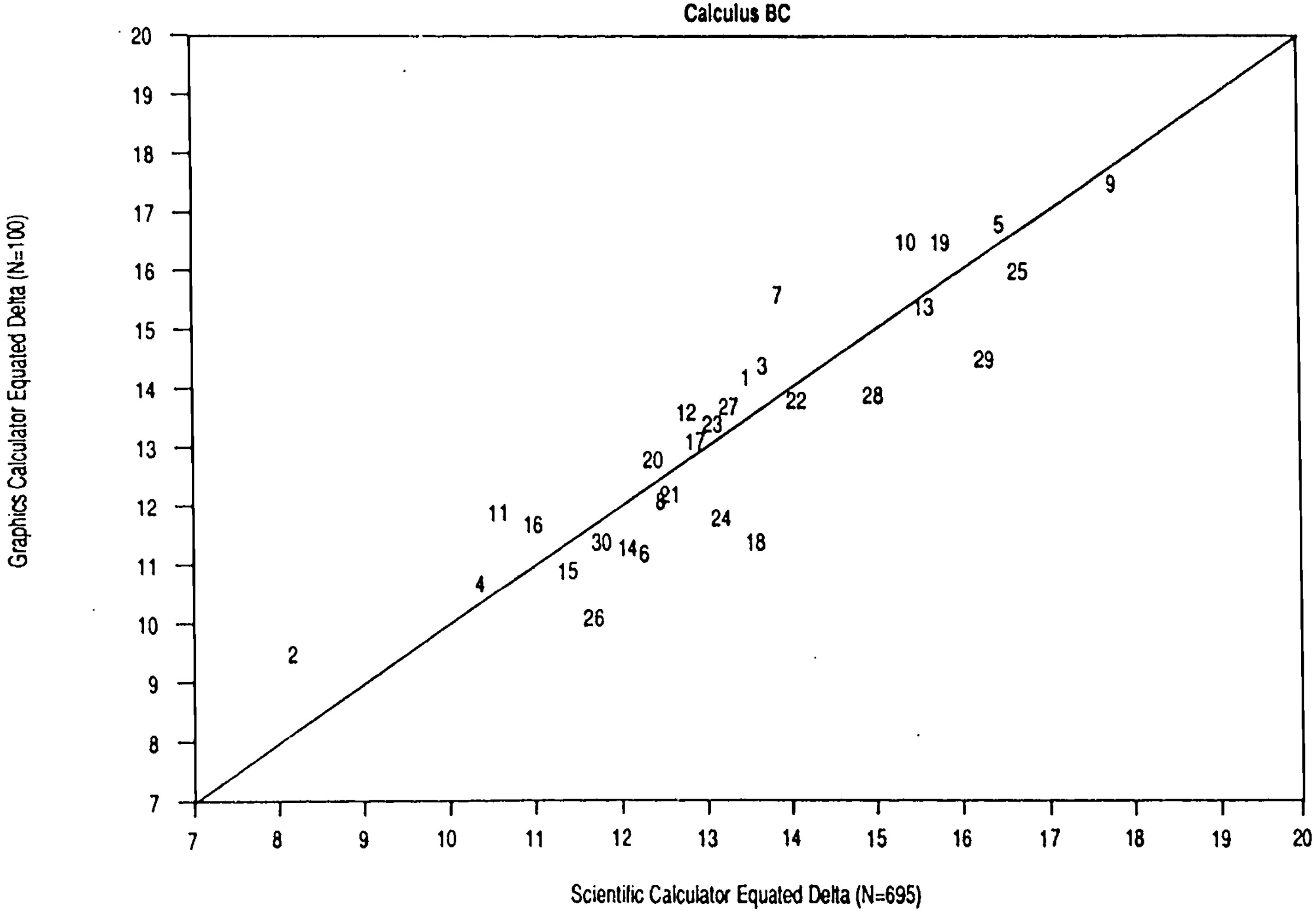
Equated Delta by Type of Calculator[1]

Calculus AB



[1] Equated deltas for items 1 through 20 use only the 'calculator group'.

Figure 10
Equated Delta by Type of Calculator[1]

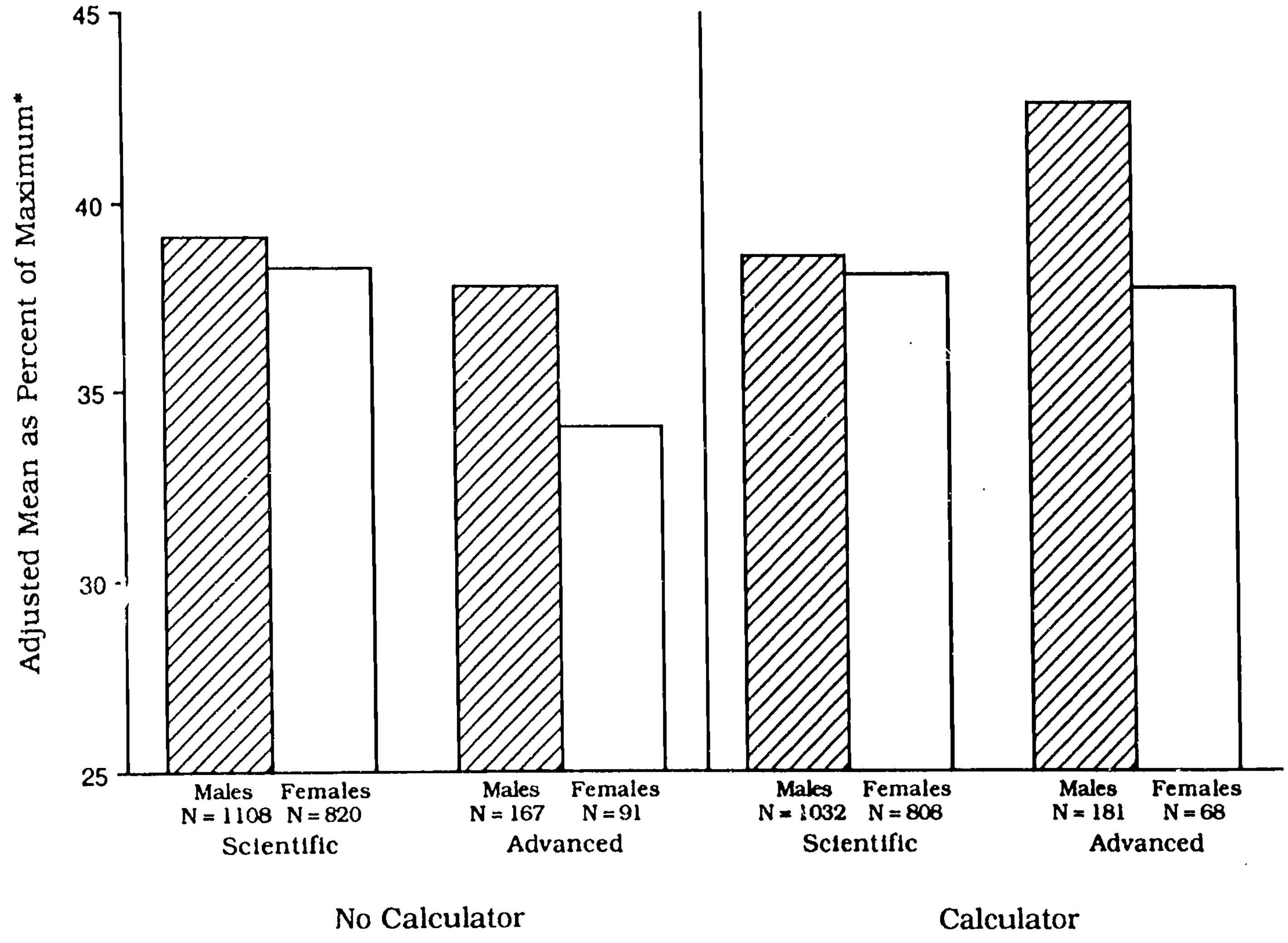


[1] Equated deltas for items 1 through 20 use only the 'calculator group'.

Figure 11

Section I Scores as a Function of Sex and Calculator Type

Calculus AB



*Analysis of covariance was used to adjust Section I scores for Section II performance.

Figure 12

Section I Scores as a Function of Sex and Calculator Type

Calculus BC

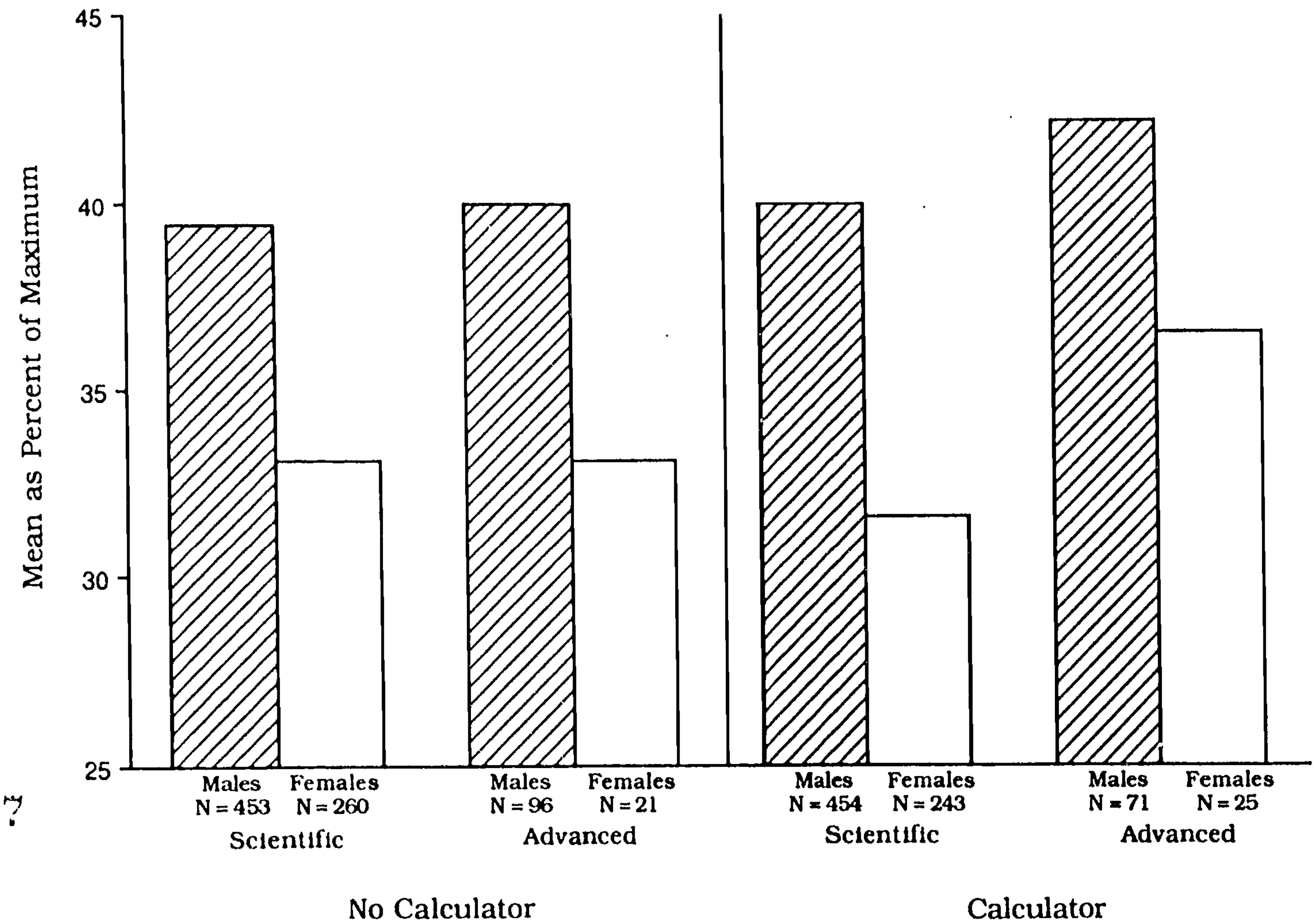
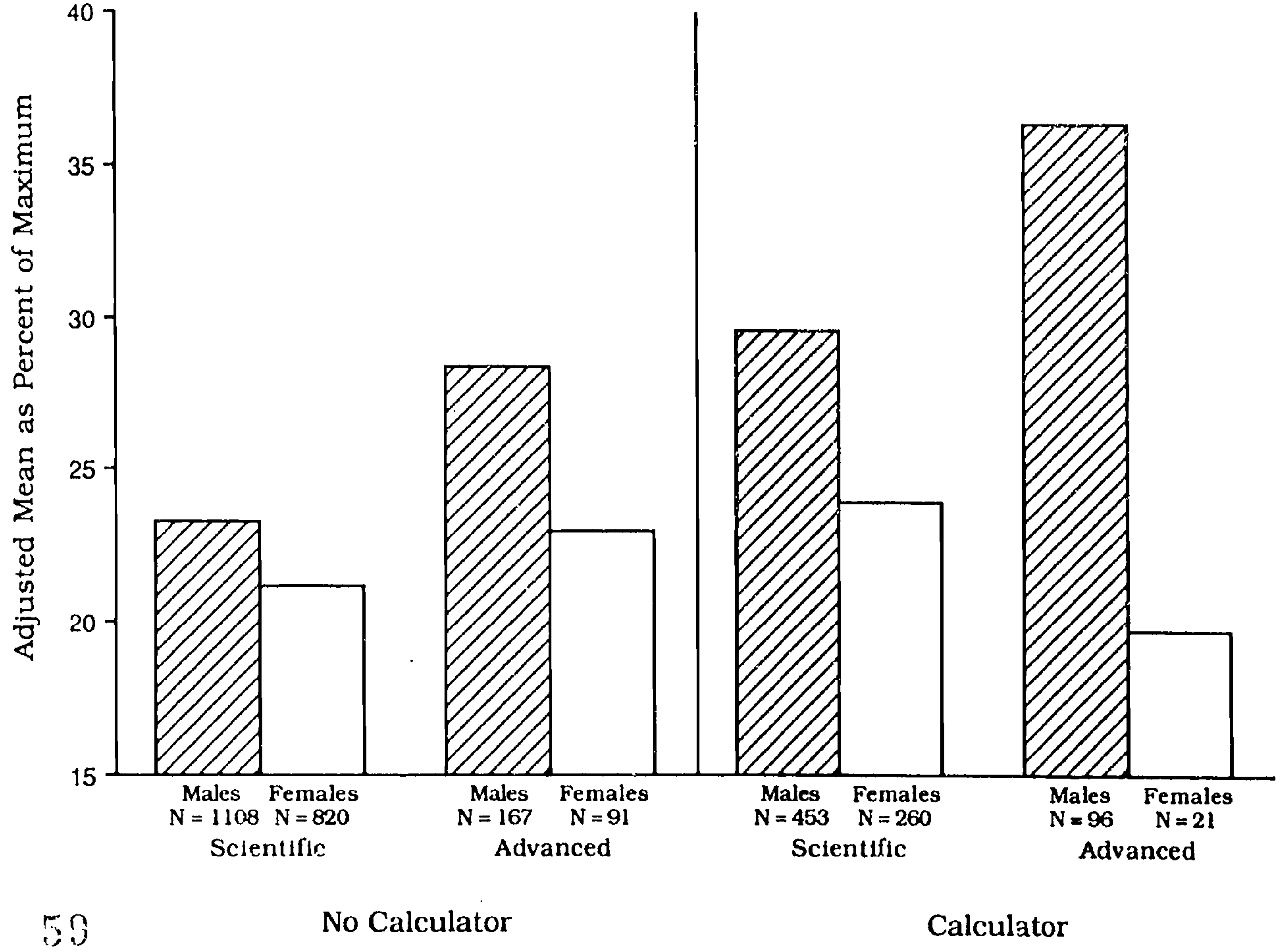


Figure 13

Section II Scores as a Function of Sex and Calculator Type

No Calculator Group



Note: Analysis of covariance was used to adjust Section II scores for Section I performance and for reported frequency of calculator use.

DIFFERENTIAL CALCULUS AND BASIC INTEGRATION TECHNIQUES

Part 1
Time—40 minutes
Number of Questions—20

YOU MAY NOT USE A CALCULATOR ON THIS PART OF THE TEST.

Directions: Solve each of the following problems, using the available space for scratchwork. Then decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in this test booklet. Do not spend too much time on any one problem.

Notes: (1) In this test, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f'(x) = x^2 + x - 12$, then f is increasing on

(A) $[-4, 3]$

(B) $[-3, 4)$

(C) $\left(-\infty, -\frac{1}{2}\right]$

(D) $(-\infty, -4) \cup (3, \infty)$

(E) $(-\infty, -3) \cup (4, \infty)$

---

2. If $f(x) = \frac{x}{1 - x}$, then $f'(x) =$

(A) -1

(B) $\frac{-1}{1 - x}$

(C) $\frac{-1}{(1 - x)^2}$

(D) $\frac{1}{1 - x}$

(E) $\frac{1}{(1 - x)^2}$

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3. The average value of $f(x) = \sqrt{x}$ on the interval $0 \le x \le 9$ is

(A) $\frac{1}{3}$

(B) $\frac{3}{2}$

(C) 2

(D) $\frac{9}{2}$

(E) 18

---

4. If $y = 3\sin^2(4x)$, then $y' =$

(A) $24\sin(4x)\cos(4x)$
(B) $24\sin(4x)$
(C) $6\sin(4x)$
(D) $-6\sin(4x)\cos(4x)$
(E) $-24\sin(4x)\cos(4x)$

---

5. $\lim_{h \to 0} \dfrac{3 \sin\left(\frac{\pi}{6} + h\right) - \frac{3}{2}}{h}$ is

(A) $\dfrac{3\sqrt{3}}{2}$

(B) $\dfrac{\sqrt{3}}{2}$

(C) $\dfrac{3}{2}$

(D) 0

(E) nonexistent

---

6. If $\dfrac{dx}{dt} = 4t^3$ and if $x = 5$ when $t = 1$, what is the value of x when $t = 2$?

(A) 12
(B) 16
(C) 20
(D) 41
(E) 48

7. A function f that is continuous for all real numbers x has $f(3) = -1$ and $f(7) = 1$. If $f(x) = 0$ for exactly one value of x, then which of the following could be x?

(A) -1
(B) 0
(C) 1
(D) 4
(E) 9

8. The position of a particle moving along a straight line at any time $t > 0$ is given by $s(t) = t(t - 1)(t - 2)$. At the instant when the acceleration first becomes zero, the velocity of the particle is

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

9. $\lim_{x \to 0} \frac{\ln(1-x)}{e^x - 1}$ is

(A) -1

(B) $-\frac{1}{2}$

(C) 0

(D) 1

(E) nonexistent

---

10. $\int_0^2 x^2 e^{x^3}\,dx =$

(A) $\frac{1}{3}(e^8 - 1)$

(B) $\frac{1}{3}(e^6 - 1)$

(C) $\frac{1}{2}(e - 1)$

(D) $e^4 - 1$

(E) $\frac{1}{3}e^8$

---

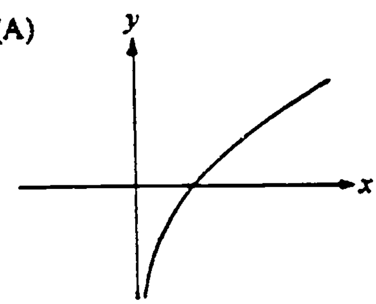
11. The area of the region enclosed by the graphs of $y = 3 - 2e^{-x}$ and $y = e^{x}$ is

(A) 3 ln 2
(B) 3 ln 2 − 1
(C) 3 ln 2 − 2
(D) 3 ln 2 − 3
(E) 3 ln 2 − 4

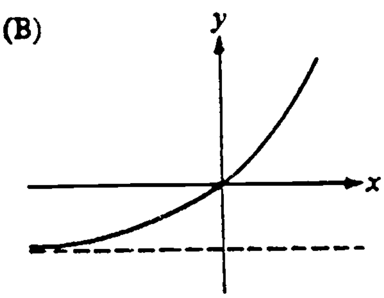
---

12. Which of the following could represent the graph of $y = \ln(x + 1)$?

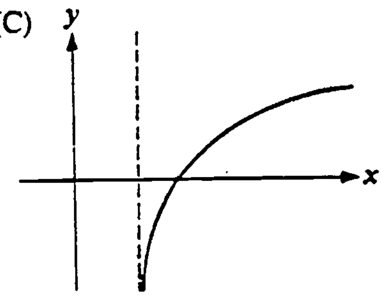
(A)



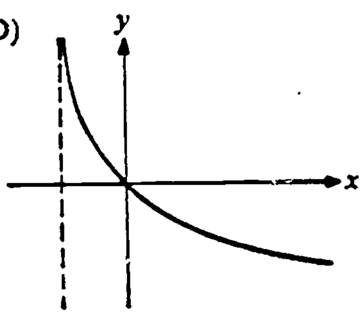
(B)



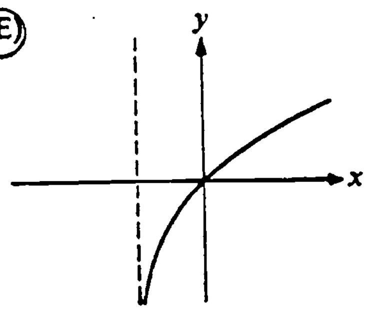
(C)



(D)



(E)



13. If $\int_0^3 (x^2 - 4x + 4)dx$ is approximated by 3 inscribed rectangles of equal width on the x-axis, then the approximation is

(A) 6
(B) 5
(C) 3
(D) 2
(E) 1

---

14. If $f(x) = (1 - x)^5$, then the fifth derivative of f is

(A) $-(5!)$
(B) $-(5!)(1 - x)$
(C) 0
(D) $5!(1 - x)$
(E) $5!$

15. Which of the following are equivalent to $\int x \sin x \, dx$?

I. $-x\cos x + \int \cos x \, dx$

II. $\frac{x^2}{2}\sin x - \int \frac{x^2}{2}\cos x \, dx$

III. $-x\cos x + \frac{x^2}{2}\sin x + C$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

16. The slope of the line tangent to the curve $2x^3 - x^2y^2 + 4y^3 = 16$ at the point (2, 1) is

(A) −7
(B) −5
(C) −1
(D) 5
(E) 7

17. The graphs of which of the following are concave downward for $x > 0$?

I. $y = e^x$

II. $y = e^{-x}$

III. $y = \ln x$

(A) None
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

---

18. $\int_e^{e^2} \frac{(\ln x)^2}{x}\, dx =$

(A) $\frac{e}{3}$

(B) 1

(C) e

(D) $\frac{7}{3}$

(E) 3

---

19. If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$, which of the following must be true?

I. $f(x) > g(x)$ for $a \leq x \leq b$

II. $\int_a^b [f(x) + g(x)]dx = 4$

III. $\int_a^b [f(x)g(x)]dx = -5$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

---

20. If $f(x) = x^3$ for all real numbers x, then there exists a number c in the interval $0 < x < 3$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) 3

(B) $\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{\sqrt{3}}{3}$

(E) $\frac{1}{3}$

END OF PART 1
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS PART ONLY.
DO NOT TURN TO PART 2 UNTIL YOU ARE TOLD TO DO SO.

DIFFERENTIAL CALCULUS AND BASIC INTEGRATION TECHNIQUES

Part 2
Time—30 minutes
Number of Questions—10

A CALCULATOR IS NEEDED FOR PART 2 OF THIS TEST. ALL CALCULATOR MEMORIES MUST BE CLEARED OF ALL PROGRAMS AND DATA BEFORE BEGINNING WORK.

Directions: Solve each of the following problems, using the available space for scratchwork. Then decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test booklet. Do not spend too much time on any one problem.

Notes: (1) In this test, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

21. $\int_1^2 \sqrt{5x}\, dx =$

(A) 2.726
(B) 2.981
(C) 3.354
(D) 13.628
(E) 20.442

22. If $f(x) = \sqrt[3]{x^3 - 2x}$, then $f'(\sqrt{3}) =$

(A) 0.129
(B) 0.902
(C) 0.906
(D) 1.116
(E) 2.173

23. Let f be the function given by $f(x) = \cos x$. If the number c satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[2, 4]$, then $c =$

(A) 0.119
(B) 2.902
(C) 3.023
(D) 3.142
(E) 3.261

---

24. The slope of the line containing the point (4,0) and tangent to the graph of $y = e^{-x}$ is

(A) 0.050
(B) 0.018
(C) −0.007
(D) −0.018
(E) −0.050

---

25. The region R is enclosed by the graphs of $y = \frac{x}{x^2 - 2}$, $y = 0$, $x = 2$, and $x = k$, where $k > 2$. If the area of R is 1 square unit, then the value of k is

(A) 2.727
(B) 4.096
(C) 10.450
(D) 14.213
(E) 22.256

---

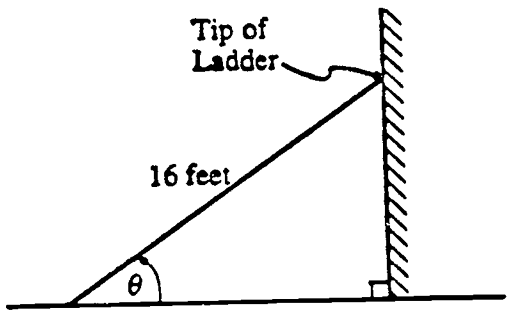
26. Let R be the region in the first quadrant enclosed by the coordinate axes and the graphs of $y = 7 - 2x$ and $x = \sqrt{3}$. What is the volume of the solid generated when R is revolved around the x-axis?

(A) 23.147
(B) 32.648
(C) 49.798
(D) 156.447
(E) 312.894

---

27. If $f(x) = \ln x$, then the average rate of change of f on the interval $[2, 5]$ is

(A) 0.100
(B) 0.133
(C) 0.305
(D) 1.151
(E) 1.220



28. The figure above shows a 16-foot ladder leaning against a vertical wall. The tip of the ladder is sliding down the wall at the rate of 5.6 feet per second. What is the rate of change, in radians per second, of the angle θ at the instant when the tip of the ladder is 7 feet above the ground?

(A) -6.223
(B) -0.800
(C) -0.389
(D) -0.321
(E) -0.070

29. If $F(x) = \int_0^x \sqrt{\tan t}\, dt$, then $F'(0.5) =$

(A) 0.089
(B) 0.093
(C) 0.546
(D) 0.739
(E) 1.139

---

30. The rate of decay of a radioactive substance is proportional to the amount of substance present at any time t. In 1840 there were 50 grams of the substance and in 1910 there were 35 grams. To the nearest gram, how many grams of the substance remain in 1990 ?

(A) 18
(B) 20
(C) 23
(D) 36
(E) 107

END OF TEST
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART 2 ONLY.

DIFFERENTIAL AND INTEGRAL CALCULUS INCLUDING SERIES

Part 1
Time—40 minutes
Number of Questions—20

YOU MAY NOT USE A CALCULATOR ON THIS PART OF THE TEST.

Directions: Solve each of the following problems, using the available space for scratchwork. Then decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in this test booklet. Do not spend too much time on any one problem.

Notes: (1) In this test, ln x denotes the natural logarithm of x (that is, logarithm to the base e).
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. How many points of inflection does the graph of $y = -x^5 - 2x^3 + 10x - 1$ have?

(A) None
(B) One
(C) Two
(D) Three
(E) Five

2. $\int x\sqrt{x^2+9}\,dx =$

(A) $\dfrac{x}{\sqrt{x^2+9}} + C$

(B) $\dfrac{3x^2+9}{\sqrt{x^2+9}} + C$

(C) $\frac{1}{3}x^2(x^2+9)^{\frac{3}{2}} + C$

(D) $\frac{2}{3}(x^2+9)^{\frac{3}{2}} + C$

(E) $\frac{1}{3}(x^2+9)^{\frac{3}{2}} + C$

---

3. If f and g are continuously differentiable functions, each with domain the set of all real numbers, and if g is the inverse function of f and $f(a) = b$, then $g'(b)$ is

(A) $\dfrac{1}{f'(a)}$ (B) $\dfrac{1}{f(b)}$ (C) $f'(a)$ (D) $f'(b)$ (E) $f'(a) \cdot g(b)$

---

4. $\int x \cos x \, dx =$

(A) $\frac{1}{2}x^2 \sin x + C$

(B) $x \sin x + C$

(C) $x \sin x - \cos x + C$

(D) $x \sin x + \cos x + C$

(E) $-x \sin x + \cos x + C$

---

5. If f is a twice differentiable function and y is a function of x given by the parametric equations $y = f(t)$ and $x = t^2$, then $\frac{d^2y}{dx^2} =$

(A) $\frac{f''(t)}{2}$

(B) $f''(\sqrt{x})$

(C) $\frac{tf''(t) - f'(t)}{4t^3}$

(D) $\frac{tf''(t) - f'(t)}{2t^2}$

(E) $\frac{f'(t) - tf''(t)}{16t^3}$

---

6. $\int_0^1 \frac{dx}{(x+1)(x+2)} =$

(A) $-\frac{2}{9}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{2}$ (D) $\ln\frac{1}{3}$ (E) $\ln\frac{4}{3}$

---

7. If $x^2y = 3y + x\ln y$, then when $x = \sqrt{3}$, $y' =$

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 2 (E) 3

---

8. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{8}} \frac{\cos(2x)}{\sin(2x)}\,dx =$

(A) $\ln\sqrt{2}$

(B) $\frac{1}{2}\ln\frac{\sqrt{2}}{2}$

(C) $-\frac{1}{2}\ln\frac{\sqrt{2}}{2}$

(D) $-\ln\frac{\sqrt{2}}{2}$

(E) $-\ln\sqrt{2}$

---

9. The base of a solid is the region in the first quadrant bounded by the line $4x + 5y = 20$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the x-axis is a semicircle?

(A) $\frac{100\pi}{3}$

(B) $\frac{80\pi}{3}$

(C) $\frac{50\pi}{3}$

(D) $\frac{40\pi}{3}$

(E) $\frac{10\pi}{3}$

---

10. If the substitution $x = \tan\theta$ is made, the definite integral $\int_0^1 \sqrt{1 + x^2}\, dx$ is equivalent to

(A) $\int_0^1 \sec\theta \, d\theta$

(B) $\int_0^1 \sec^3\theta \, d\theta$

(C) $\int_0^{\frac{\pi}{4}} \sec\theta \tan\theta \, d\theta$

(D) $\int_0^{\frac{\pi}{4}} \sec\theta \, d\theta$

(E) $\int_0^{\frac{\pi}{4}} \sec^3\theta \, d\theta$

---

11. If the line $y = 3x - 5$ is tangent to the graph of $y = f(x)$ at the point (4, 7), then

$\lim_{h \to 0} \frac{f(4 + h) - f(4)}{h}$ is

(A) -5
(B) 3
(C) 4
(D) 7
(E) nonexistent

12. When the brakes are applied on a car that is traveling at a speed of 100 feet per second, the car comes to a stop in 5 seconds. If the deceleration of the car is constant, the distance, in feet, that the car travels after the brakes are applied is

(A) 100
(B) 250
(C) 300
(D) 500
(E) 600

---

13. Let $F(x) = \int_1^x \sqrt{t^2 - 1}\, dt$. The length of the graph of $y = F(x)$ for $1 \le x \le 3$ is

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

---

14. If f is continuous on $[0, 1]$, $f(0) = 1$, $f(1) = 0$, and $f'(x) < 0$ for $0 < x < 1$, which of the following must be true about $\int_0^1 f(x)dx$?

(A) It is negative.
(B) It is equal to 0.
(C) It is positive.
(D) It does not necessarily exist.
(E) None of the above is necessarily true.

15. Under ideal laboratory conditions the rate of growth of bacteria is proportional to the number present. If under ideal conditions a colony of 1,000 bacteria increases to 27,000 in 6 hours, how many bacteria were there in 4 hours?

(A) $1{,}000e^{\frac{3}{2}}$

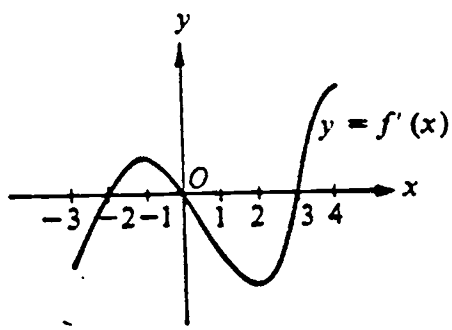
(B) $1{,}000e^{24}$

(C) 6,750

(D) 9,000

(E) 13,500

16.



Let f be a function whose domain is the open interval $(-3, 4)$, and let the derivative of f have the graph shown in the figure above. The function f is increasing on which of the following intervals?

(A) $(-3, -1)$ and $(2, 4)$

(B) $(-2, 0)$ and $(3, 4)$

(C) $(-3, -2)$ and $(0, 3)$

(D) $(-1, 2)$ only

(E) $(0, 3)$ only

17. If $\frac{dy}{dx} = 4y$ and if $y = 100$ when $x = 0$, what is the value of x when $y = 400$?

(A) $\frac{\ln 4}{4}$ (B) $4 \ln 4$ (C) 4 (D) $10\sqrt{2}$ (E) $5\sqrt{6}$

---

18. The graphs of $f(x) = x^3 - x$ and $g(x) = 8x - 8x^3$ intersect at the points $(-1, 0)$, $(0, 0)$, and $(1, 0)$. The area of the figure enclosed by these graphs is

(A) $\left| \int_{-1}^{1} [f(x) - g(x)]\, dx \right|$

(B) $\int_{-1}^{0} [g(x) - f(x)]\, dx + \int_{0}^{1} [f(x) - g(x)]\, dx$

(C) $2\int_{0}^{1} [g(x) - f(x)]\, dx$

(D) $\int_{0}^{1} \left| f(x) - g(x) \right| dx$

(E) 0

---

19. Let f be the function given by $f(x) = 6x - 1$. In applying the $\varepsilon - \delta$ method to establish the continuity of f at $x = 1$, if $\varepsilon = \frac{1}{5}$, then the largest value that δ can assume is

(A) $\frac{1}{5}$ (B) $\frac{1}{10}$ (C) $\frac{1}{20}$ (D) $\frac{1}{30}$ (E) $\frac{1}{50}$

---

20. $\lim_{x \to 0} \frac{e^x - 2 + e^{-x}}{1 - \cos 2x}$ is

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) nonexistent

END OF PART 1

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS PART ONLY.

DO NOT TURN TO PART 2 UNTIL YOU ARE TOLD TO DO SO.

DIFFERENTIAL AND INTEGRAL CALCULUS INCLUDING SERIES

Part 2
Time—30 minutes
Number of Questions—10

A CALCULATOR IS NEEDED FOR PART 2 OF THIS TEST. ALL CALCULATOR MEMORIES MUST BE CLEARED OF ALL PROGRAMS AND DATA BEFORE BEGINNING WORK.

Directions: Solve each of the following problems, using the available space for scratchwork. Then decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test booklet. Do not spend too much time on any one problem.

Notes: (1) In this test, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

21. A particle moves along a straight line so that at any time $t \geq 0$ the position of the particle is given by $x(t) = 1.2t^3 + 1.1t^2 - t + 5.6$. What is the position of the particle when its velocity is zero?

(A) 0.000
(B) 0.304
(C) 5.431
(D) 5.600
(E) 6.517

22 Let R be the region enclosed by the curve $y = \ln x$, the x-axis, and the line $x = 5$. If 4 equal subdivisions of the closed interval $[1, 5]$ are used, what is the trapezoidal approximation of the area of R?

(A) 3.178 (B) 3.983 (C) 4.022 (D) 4.047 (E) 4.787

23. The Mean Value Theorem guarantees a special point on the graph of $y = \sqrt{x}$ between $x = 1$ and $x = 2{,}000$. The x-coordinate of this point is closest to which of the following integers?

(A) 22 (B) 30 (C) 500 (D) 516 (E) 523

24. The slope of the line tangent to the curve $y = x^{-x}$ at $x = 0.368$ is closest to the integer

(A) -6 (B) -1 (C) 0 (D) 1 (E) 6

25. If $F(x) = \int_0^{x^2} \sqrt{\sin t}\, dt$, then $F'(1) =$

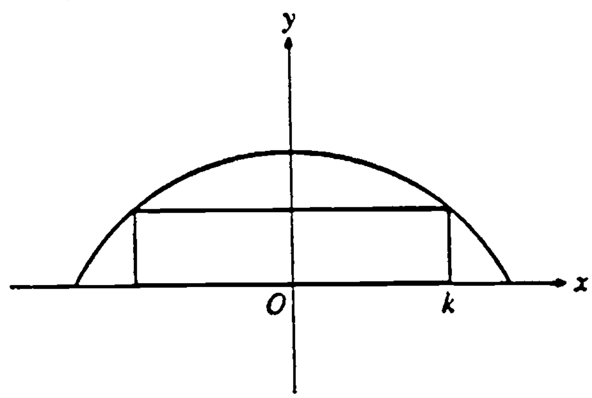
(A) 0.000
(B) 0.132
(C) 0.264
(D) 0.917
(E) 1.835

26. The function f given by $f(x) = x^3 - 3x^2 - 2x - 1$ has a relative maximum at $x =$

(A) −2.291
(B) −0.291
(C) 0.423
(D) 1.577
(E) 2.291

27. If $a_1 = 5$, $a_2 = \ln(a_1 + 2)$, $a_3 = \ln(a_2 + 2)$ and in general, for $n > 1$, $a_n = \ln(a_{n-1} + 2)$, then $\lim_{n \to \infty} a_n$ is

(A) 0.693
(B) 1.146
(C) 1.216
(D) 1.373
(E) 1.946



28. A rectangle of length $2k$ is inscribed in the region between the x-axis and the graph of $y = \cos x$, as shown in the figure above. For what value of k does the rectangle have maximum area?

(A) 0.500
(B) 0.785
(C) 0.860
(D) 0.866
(E) 6.280

29. Let $S = \int_0^1 e^{-x^2} dx$. Of the following, which best approximates S?

(A) 0.333
(B) 0.724
(C) 0.743
(D) 1.457
(E) 1.676

30. The sum of the series $\sum_{n=1}^{\infty} \left(\frac{e^{\pi}}{\pi^{e}}\right)^{n}$ is

(A) 0

(B) 1

(C) $\frac{\pi^e}{\pi^e - e^\pi}$

(D) $\frac{e^\pi}{\pi^e - e^\pi}$

(E) nonexistent

END OF TEST
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART 2 ONLY.

Appendix C. Responses to the Student Questionnaire.

Question	Calculus AB Percent	Calculus BC Percent
1. Is the calculator you used for this test capable of displaying graphs?		
Yes	12.1	14.3
No	86.8	84.8
No Response	1.0	.8
2. Is the calculator you used for this test capable of finding a derivative?		
Yes	2.1	3.2
No	82.0	85.3
No Response	15.9	11.5
3. How frequently do you use the calculator you used for this test in other situations? (i.e., homework, other classes)		
Almost Every Day	39.6	43.0
Once or twice a week	32.2	30.9
Once or twice a month	13.6	13.2
Once or twice a year	5.3	5.0
Never	9.2	7.8
No Response	.1	.1
4. How would you judge the amount of time you had for Part I?		
Much too much	.7	1.0
A little too much	6.3	2.6
Just enough	55.9	27.4
Not enough	36.6	68.7
No response	.5	.4

Question	Calculus AB Percent	Calculus BC Percent
5. How would you judge the amount of time you had for Part 2?		
Much too much	1.2	1.1
A little too much	3.6	5.0
Just enough	25.7	33.5
Not enough	68.7	59.5
No Response	.8	.9
6. What type of calculator are you using for this test?		
Casio FX82	1.9	2.0
Casio FX115	9.0	7.9
Casio FX250	3.8	2.2
Casio FX7000G	9.3	8.2
Casio FX300	3.0	3.6
Texas Instuments TI30	16.2	11.6
Texas Instruments TI35	6.7	5.8
Texas Instruments TI36	6.1	6.9
Radio Shack EC4014	2.8	2.8
Sharp EL506	8.8	10.4
No Response or other	32.4	38.6

Note. Number of students responding was 4,287 for Calculus AB and 1,625 for Calculus BC.