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ABSTRACT

A teacher describes how he changed the way his high school students learned mathematics, using cooperative learning methodology and focusing on oral, as opposed to written, examinations and on group effort as opposed to individual effort. As the students' learning changed, the teacher was faced with the new problem of assessing the new type of learning that was now occurring. He developed and refined a performance assessment instrument, which he adjusted to fit the needs of the students and the different mathematics courses in which it was used. The 5-year development of this assessment method is outlined, focusing on how the scoring rubric gives students valuable feedback that promotes further learning and better assessment of the student's knowledge. In this system, the teacher is no longer judge and jury, but rather a coach or facilitator. Student, parent, and community reaction toward the assessment method is also presented. (NAV)

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Holt High School

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Abstract

The author of this paper describes how he started changing the way his students learned mathematics. As his students' learning changed, he was soon faced with problems assessing the learning he thought was going on. Faced with this problem of how to assess his student's learning, he developed and refined a performance assessment. The author describes problems he uses on the assessments, how the performance assessment is arranged, how students prepare for it, and how they are judged. In this paper, the author tells how he has adjusted the exam to fit the needs of his students and the varying mathematics courses he uses it in. When arranging an exam of this type, many problems are faced, and the author explains how he has solved these problems. As this exam has evolved over the past five years, the author explains how he has developed a scoring rubric that he thinks gives a student valuable feedback which promotes further learning and better assessment of a student's knowledge. He describes student reactions, parent reactions, and community support for this assessment. The author describes what he has learned from this type of assessment and how it fits as one piece of the assessment puzzle.

ASSESSING MATHEMATICS PERFORMANCE ASSESSMENT: A CONTINUING PROCESS

Michael F. Lehman[2]

Where I Came From

I have taught high school mathematics for eighteen years. About six years ago, I started to rethink my teaching. I wanted my students to be able to do more than just give back what I told them. I felt students in Algebra II and PreCalculus should be able to do more than apply what they had learned to situations similar to those we did in class. I wanted them to develop a deeper understanding of the mathematics I was teaching. My goal was for them to be able to take the mathematics taught and apply it to a wide range of situations. However, I was not sure what this type of student understanding looked like. I did not know if I could recognize it if I saw it.

So I continued to explore my teachin. I had learned a little about cooperative learning at a Professional Development School[3] (PDS) Institute. It sounded pretty good, and I thought it would be easy to use, so I set out to implement it in my classroom. I soon found out it wasn't so easy. I had a lot of group work going on, but nothing I would call cooperative learning—unless you count one student doing the first four or five problems and then cooperatively giving them to the rest of the group in exchange for the rest of the problems. My naiveté led me to see that there was more to cooperative learning than I thought. Luckily for me, I had access to some excellent resources from Michigan State University as well as within my own school. We started a study group on cooperative learning. We read the research, discussed the theory, and shared our practice. As the year went on, I learned how to construct activities that led my students to have high-level discussions and to develop what I thought were some good understandings of

[2]The author teaches mathematics at Holt High School, Holt, Michigan. He is grateful to members of the Holt High School Writers Club for their input and encouragement. Members include: Larry Burgess, Jerry Gillett, Mary Gray, Bruce Kutney, and Robert Smith.

[3]Professional Development Schools are schools that work in collaboration with a university to study exemplary practice, in-service teacher training and preservice teacher training. Holt High School is a PDS working with Michigan State University. They are funded through the Partnership for New Education.

the mathematics I was teaching. The problem I now faced was how to assess this type of student understanding In other words, how could I verify that it truly existed.

First Steps

My first step was to initiate writing into my teaching. I would ask my students to write about a problem or situation. In this writing they were asked to explain not only the steps but why the steps made sense. For example, they might be asked if a graph was a function or not and to explain how they knew. If they said they used the "vertical line" test, then they were asked to explain why this test worked. This type of writing not only helped me to get a better idea of what my students did or did not understand, but as time went on, I found out it also helped them. I didn't realize that, in choosing their words, they had to really think through the concept. I was pleased with this approach, but I soon realized that, with 150 students per day, this would be time-consuming and I could see myself wearing magnifying glasses very soon. So my next quandary was: How do I continue to gather this type of information and, at the same time, have a life?

The next step I took was to continue the writing in a shortened or less-frequent form (which I continue today) and then to change my tests. I added more tasks that required students to give short explanations with their answers. This also helped give me more insight into the origins of their thinking. I began to wonder if all my students were giving me their own ideas or if they were just using ideas others had come up with. Also I found that, as time went on, I was reading a lot of my own words. The students had learned that it was easier to ask me specific questions they felt I might expect them to write about and take detailed notes of my answers. They would then have a good answer without much thinking on their part, should I ask the question. Although adding the explanations gave me some insight to my students' understanding, I soon discovered I needed still more information about my students' understanding. I also wanted more for my students. They were struggling with what it meant to understand, the same as I was. I wanted to provide them with an idea of what I was shooting for, but I was not sure how to do that.

The Exam

All of these changes and the continued frustration I was feeling led me to try my first Performance Assessment. As I outlined in an earlier paper, *Assessing Assessment: Investigating a Mathematics Performance Assessment*,[4] I wanted to give my students a chance to explain orally what they knew and understood. I gave the students a packet of six problems which they prepared ahead of time in their cooperative learning groups. These problems were developed to cove. the broad themes of the year, along with getting at some of the details of the concepts. During the exam period (one and one-half hours at our school) the students would go before a panel of three adults and explain the problems. The panelists would ask each student to explain one of the problems in detail. Since the student would not know which problem they would be asked to solve ahead of time and the judges are told not to allow the students to pick his/her problem, each student had to know all six problems. During these explanations the students would have to explain their strategies, why they chose the method they used, any terminology they used, and how they knew their answers were correct. (I would not gc over the answers before the exam.)

What I Learned

My first realization was that, for some students, this was their first chance to be successful at a mathematics exam. Whenever I would put a traditional paper-and-pencil test in front of them, they would freeze and not do very well. I remember how often I had graded tests in the past only to be surprised that some students did not do well. During class, I had thought they knew what they were doing, but on the tests they performed poorly. It was different with the oral exam; they were able to communicate what they knew. The judges would help them relax by asking a few questions to get them started, and, soon, the students would take over and do a good job of explaining what they knew.

As I used this assessment more and more, I also realized that, whenever I gave a traditional test and a student did poorly on it, only two people usually knew: the student

[4]Lehman, M. F. 1991. *Assessing assessment: Investigating a mathematics performance assessment*. Craft Paper 91-3, National Center for Teacher Learning, Michigan State University, East Lansing.

and myself. I realized that, for some students, having the teacher know they did poorly was something they could live with. But on the oral exam, six other people would know: the three judges and their group members. This, they could not live with; they did not want to be embarrassed, so they made sure they were ready. During the last week of school in June, before exams, when most students have already checked out, my students work hard. I am no longer faced with those dreadful "review days." They come to class and get right to work with their group. They waste little time. In fact, I have had parents call and ask what I was doing because, much to their surprise, they had a group of students at their house on a Sunday afternoon doing nothing but mathematics (and, of course, eating a lot of pizza). The change in the atmosphere of the classroom allowed for a tremendous amount of learning. This was a secondary benefit I had not expected.

The students put a lot of pressure on each other. They get very irritated if another group member is not holding up his/her end. They are willing during the normal part of the year to sometimes put up with a person who does not contribute, but when 20 percent of their grade is riding on the oral exam they won't tolerate a sloucher. As with most teenagers, they sometimes can be rude to each other. It is my job to monitor this to be sure the pressure is appropriate and not degrading. This does take some extra work at times, but for the most part, they have been working together all year and have developed a working relationship with each other. I think some amount of pressure from the group is good and should be brought to bear on the student who is not carrying his/her part of the load, however, there can be a fine line between what is appropriate and what is inappropriate. I watch these groups carefully and try to mediate when necessary.

What is most important to me, however, is the learning that goes on during the preparation time. I have come to view it as possibly the most important part of the process. I see students linking concepts they had not linked earlier. I see students looking things up in their texts or asking me good questions like, for example, "Mr. Lehman, I got this far and see what the results should be, but I am not sure how to verify it," or "Mr. Lehman, if I explain this problem using this approach, do you think the judges will understand, or should I add more details?" I think this is much better than in the past when they kept asking, "Mr. Lehman, is this going to be on the exam?" or "Can we use our notes on the exam?" For the first time, I feel like I have an exam that promotes learning in a way that my students will buy into. In other words, I have an exam for which they feel they have more control over how they will do. They know what is on the exam and that the main thing that determines their grades is how much work they put into the preparation.

Another outcome of the Performance Assessment is that I am no longer "judge and jury." I become more like a "coach" or "facilitator" of the process. My relationship with the students changes: They look to me for advice on how to present an idea, and they actually listen to me when I explain concepts or question them about their strategies. After the exam they don't "whine" as much about their grades—although let's face it, they are teenagers and so there still is some complaining—instead, they read the comments (each student gets three sets of comments) from the judges thoroughly. This too is a change for me. Many times I graded tests in detail only to have students look at the grades and then throw away the papers with the comments. With the Performance Assessment, they actually read the comments and learn something from them. We discuss all the results· both the mathematics and the overall process. I try to listen to their reactions and learn from them. They also learn from each other's reactions. I find that, if a student complains about something, another student will usually respond to it better than I can. But it all comes down to preparation the same as any exam does.

My students continue to score well on standardized tests such as the ACT and SAT, and they continue to score above the national average. They are also well prepared for the next mathematics course. As one of the PreCalculus teachers, I see many of my Algebra II students the following year. In PreCalculus, I find them to be very well prepared. In fact, the number of students moving from our regular Algebra II track to our Honors PreCalculus track has increased. Also, the AP Calculus teacher reports that the students coming from PreCalculus are very well prepared for the material and speed of AP Calculus.

Students' Reactions

Some of my students say this type of exam is easier than a written exam. That is fine with me; I wasn't looking to make my exams harder. According to my students, I already know how to write extremely diffic· ⁺ests. What is important to me is that this exam promotes learning while, at the same time, giving my students another way to show what they know. I firmly believe that, if you truly understand mathematics, you have to be able to explain it symbolically, in written form, and orally. As I continue to pursue ways of improving my students' understanding, I keep this belief in mind. I continue to supply my students with ways to show me they can do all three.

Each year, I have students come back from college and tell me how much their preparation for this type of exam has helped them: They are not afraid to give oral presentations, and they know how to prepare for them. Also, they know how to address an audience in ways that help them get their point across. These students also tell me how much they enjoy watching the other students worry (or "squirm," as they put it) when it comes to doing oral presentations or writing formal papers in a mathematics or science class. These students report that they feel very confident in their mathematical preparation. This is another unexpected outcome of my mathematics classes: My students le rned both content and useful personal skills.

Variations

I have tried different variations to the process I started five years ago. In my Algebra II classes, I not only give the students the six problems as described above (see Appendix A), but during the last half hour of the exam, I also have the judges give them a problem they have not seen before (see Appendix B). The students are asked as a group to solve and explain the problem to the judges. The judges watch for the contribution each student makes to the conversation and how the students work together as a group. I added this part to the test to help the judges see which students could think on their feet and which could not. Knowing the mathematics and being able to do it at their leisure in class is one thing, but being able to apply it on the spot, under pressure and in a situation they had not planned for, means they really know and understand the mathematics being tested. Some students really shine during this part of the exam, whereas during the individual part of the exam, they might have had trouble communicating what they knew. Therefore, it gives me additional information about my students that I did not get from the original form of the exam.

In my PreCalculus and Discrete Mathematics classes, my students write a major paper that goes with each unit we study. These papers usually involve their solving a problem (see Appendix C) and then writing about their solutions. I grade these papers and give them back to the students; they make corrections on them. During the exam, the students give oral explanations to the judges about their papers and the solutions contained in them. This may sound easy on the surface, but they really sweat the details and spend a lot of time reading other students' papers and reworking their own (an English teacher's dream). The exchange of ideas and the suggestions they make to each other are amazing. They talk mathematics for days without me. I become a resource to them if needed.

In Transition Mathematics (General Math or Practical Math, depending on your school), with the help of a colleague, I tried to develop mathematical situations (see Appendix D) that were open enough that the students could create most of the problems themselves. For example, we may give them a graph and have them write a story that would explain what the graph could be a model for, or we might give them several rules (equations) and ask them to write all they know about the rules. This might include the x and y intercepts; whether or not it is linear; if it is linear, what the slope is; and whether it is increasing or decreasing. Finally, my colleague suggested that we have them interview someone in their family and write about how this person uses algebra in their work or hobby. I was impressed with the work these students did. Their creativity and their stories were great. Reading their interviews, I also learned a lot about how people use algebra and what my students think algebra is.

The reaction from the Transition students could not have been better. They were pleased with their results. As one student put it, "This is more realistic than any other test I have taken. In this class, we have to take written tests and oral tests. When I applied for my job, I had to do both there." Of course, there were some who did not like it, but overall, it went well the times I used it.

Community Reaction

Overall, parent reaction has been positive. I have had many parents question me about the process. I try to explain it to parents at fall conferences (I usually have between 60 and 75 percent of my students' parents show up). Once I explain it to them, they are very supportive. For the parents whose child is struggling with mathematics, they see it as an opportunity for their child to show what he/she knows in a different way. To those parents, it is overly apparent that the traditional methods have not worked, and they welcome someone who is willing to try something new. I have had parents come in and help the students prepare. A few days before the exam, the parents would come in and question the students about their work. This gave the students an idea of how well prepared they were and what they needed to work on. The parents and students usually arranged this on their own, and I only provided the time, place, and materials needed.

Concerns and Issues

I wish I could say I now have all the answers when it comes to assessing my students, but I have come to accept that I never will. I do know I am closer than I was six years ago, but problems still exist that I do not have answers for. For one thing, coming up with problems for this type of exam is difficult. I continue to work to improve them, but they never quite get at the total picture (much the same way a traditional exam would never quite represent the whole picture). I want problems that are open enough for my students to have to show what they know without being so open the students have no idea what the problem is asking. This tasks my creativity (or lack of it) to build problems that cover the content I want assessed.

The consistency of the panelists is always a concern of mine. Since I have up to eight teams of panelists (24 individuals) working with students at one time, how can I be sure they are consistent from one team to another? I try to give very explicit instructions to the panelists and outline exactly what they should be looking for. However, when working with human beings, I cannot guarantee total consistency. The only thing I can say is that, when I grade 150 exams, I know I am not totally consistent from start to finish. Therefore, I think a little variance can be accepted. Also, I have noted that students who go through the process more than one time have very similar grades.

Therefore, there seems to be a certain level of consistency, and I continue to work for ways of increasing it. Finally, with a few exceptions, students seem to be comfortable with their scores. As with any test, some will complain about their grades, but when taken in conjunction with the comments, most of the time the scores seem reasonable. I have had only one case where I needed to discuss a grade with the judges after a student raised an issue. In this case, after listening to the judges and having one of them discuss it with the student, the grade stood.

I have continued to work on the evaluation form the panelists fill out. At first, I set up a rating system where a panelist rated the student on each segment of the presentation on a scale of 1 to 5. Then the panelist added up the points, and that was the grade. I found out that this score did not always reflect what the panelist thought was an appropriate overall grade. Also, sometimes the panelist spent so much time trying to get the rating system to fit the grade he/she felt the student earned that the panelist did not have time to write good comments. Some panelists felt the rating system could take the place of the comments because it was so detailed.

With the help of our district's Gifted and Talented director, I developed a new grading rubric (see Appendix E) that describes what an A, B, C, etc. grade would look like. The panelist is free to choose the one he/she feels best fits the student. I include my grading scale so the panelist can choose between a high A and a low A, for example, without too much difficulty. I then outlined for the panelist the key categories that I wanted comments on. This helps to direct the panelist's comments in a manner that is more useful to my students and myself. It also highlights the comments as the important aspect of the assessment and de-emphasizes the grade. I wanted this feature because I want my assessments to promote learning, not just provide a way to sort students.

One big problem I face is getting enough panelists each time. I try to use this type of assessment in all of my classes at the end of the first semester and again at the end of the year. As a consequence, it takes a lot of time to arrange. I send letters to our school district's business alliance and ask for help. We hear over and over again that the business community wants better schools. I, therefore, provide them with a chance to help with the process of improving education. So far, their response has been great. I also have the advantage of having Michigan State University in my backyard, and I have been able to use colleagues from the university as panelists. They enjoy the opportunity to get into a high school and see what the students are learning. Arranging for the panelists is a big job, but I would also spend a lot of time writing, giving, and grading a traditional exam, so I feel it all works out. Finally, colleagues from within my school and from other school districts have been very helpful. This provides them with a chance to see students in a different light and to learn about a different way to assess their own students.

My Conclusion

After five years of using this method for my exams, I am convinced more than ever of its potential to improve my students' level of understanding. I also have continued to believe that it is an excellent alternative for those students who have trouble with traditional tests. This type of assessment, in conjunction with the more traditional assessments I still use, has helped me gain more information about my students and their understanding of the mathematics we have learned. It has also helped me come closer to understanding what it means to understand mathematics.

Appendix A

Problem [1]:

A bus charter company will provide a bus for a fare of $90.00 for up to 30 passengers. For each passenger over 30, the fare is decreased $2.00 per person. Each tour the company provides costs the company $1,500.00 for gas, wages, etc.

Make a chart, draw a graph and write a rule that would model this situation. Be prepared to relate the key components of these to the situation.

Problem [2]:

Tom and Patricia Wainwright have decided to start raising Hungarian Longhair White Rabbits to sell as pets. The profit is greater if they sell their rabbits to a wholesaler in quantities of 1,000 instead of individual rabbits to individual pet stores. They therefore plan to sell their rabbits in groups of 1,000 at $2.50 each. Tom and Pat plan to start out with 25 rabbits. They know the rabbits will approximately quadruple every month. They will count the next month's population after they have subtracted the rabbits they sold. The Wainwrights have several questions they need answered before they start this business. Your job is to try to answer them for the Wainwrights. They must be careful not to sell too many rabbits so that they end up having no income for a month or more after they start selling rabbits.

The Wainwrights' start-up costs were $15.00 for each rabbit they started with, $5,000 for the building to house the rabbits, $250.00 for the proper permits to run this type of business, and $300.00 for food for the rabbits.

(a) The Wainwrights would like to know when they will be able to start making sales and how many groups of 1,000 they can sell each month. Show them how many rabbits they could sell and how many they will have left over for each month during the first year. Also, tell them when they can start making money and how much they can expect to make each month. When figuring the next month's population, do not count the rabbits sold during that month.

(b) The Wainright's monthly expenses are approximately $24,000.00 per month. How much must they sell their rabbits for in order to continue to average $2.50 per rabbit profit?

Appendix B

When Able Baker graduated from cooking school, she got a job with a starting salary of $29.000.00. She was promised a raise of at least $1,700.00 each year starting the second year.

(a) Make a chart, draw a graph, and write a rule that would represent Able's situation. Be prepared to explain the key components of each of these in relationship to the given situation.

(b) When will her salary reach $40,000.00?

(c) If instead of the $1,700.00 raise each year, Able was offered a 4% raise each year starting the second year, make a chart, draw a graph, and write a rule that would represent Able's situation. Be prepared to explain the key components of each of these in relationship to the given situation.

(d) Given this new situation, when will her salary reach $40,000.00?

(e) If Able plans to work at this job for 15 years, which plan would give her the best salary plan over this time period?

(f) At what point will the two salaries be relatively equal?

(g) If Able decides she likes this job and wants to work there until she retires (30 years), which plan should she sign up for? What will her salary be when she retires?

Appendix C

PreCalculus
Chapter 6 Project*

Recall the Ferris wheel problem in Lesson 6-3. The top of the wheel is 45 feet from the ground, the wheel is 40 feet in diameter, and it is traveling at a uniform speed of 2 revolutions per minute. After first boarding the wheel, it takes 10 seconds to reach the top. An equation for the height f(t) off the ground of a person t seconds after boarding is:

$$f(t) = 25 + 20 \cos[{}^{\pi}/_{15}\ (t-10)]$$

This project is intended to show you how the function was derived. The method used is based on the one in Lesson 6-4.

(a) You don't know the height off the ground of the boarding point. In other words, you don't know f(0). You do know what f(t) is when t = 10: f(10) = 45 feet. So build the function relative to t = 10 or t - 10 = 0. The wheel is turning at a rate of 2 revolutions per minute or ${}^{4\pi \text{ radians}}/_{60 \text{ seconds}} = {}^{\pi}/_{15}$ radians per second. At this rate, the wheel turns ${}^{\pi}/_{15}(t - 10)$ radians in t - 10 seconds. This expression allows you to determine where the boarding point was, measured in radians from the top of the wheel. Find this measure and explain why using t - 10 = 0 makes sense. Include in this explanation why using the cos function is appropriate for this type of situation.

(b) By design in part (a), ${}^{\pi}/_{15}(t - 10)$ when t = 10. At this time, the value of the function should be a maximum. The circular function which has a maximum at 0 is y = cos x, so cos ${}^{\pi}/_{15}(t - 10)$ has a maximum at 10. Sketch f(t) = cos ${}^{\pi}/_{15}(t - 10)$ on an automatic grapher to verify this. (Use graph paper to copy the graph from the automatic grapher.) Find the next value for t at which this function has a maximum. Give your answer to the nearest 10th of a second.

(c) The center of the wheel is 25 feet above the ground, and a person's position relative to that point varies between 20 feet above the center and 20 feet below the center. Explain how this leads to the final form of the function:

$$f(t) = 25 + 20 \cos[{}^{\pi}/_{15}\ (t-10)]$$

(d) Check the results for part (c) by verifying that 25 seconds after boarding, a person is at the bottom of the wheel. Explain why you know that after 25 seconds the person is at the bottom of the wheel.

*This project originally came from the UCSMP Functions, Statistics, and Trigonometry book. I have made a few alterations to it so it would fit with my assessment goals.

(e) How high is the boarding point from the ground? Explain how you found your answer.

(f) If, after you board and ride for 2 minutes and 43 seconds, the wheel breaks down and the operator has a 26 foot ladder, will you be able to get off the wheel? Explain your answer.

(g) Alter the given function for the Ferris wheel so that the wheel will rotate in the opposite direction.

(h) Write a paper that will explain your answers to all the above questions. Please be sure that the paper flows well and connects all of your answers. This paper should explain to the reader why each part of the function works to model this situation, thus giving him/her an idea of how mathematics can be used to model real-life situations. Include drawings and graphs that support your results. (Please type your paper.)

Appendix C
continued

Honors PreCalculus
Chapter 7 Project*

The new Michigan Lotto currently consists of the following game. For a $1.00 bet, a player chooses 6 different numbers out of 49 possible numbers. If all 6 match, the player wins the jackpot. If 5 of 6 match, the player wins $2,500. If 4 of 6 match, the player wins $100.

The previous Michigan lottery, Bonus Lotto, consisted of a slightly different game. For a $2.00 bet, a player chooses 6 different numbers out of 46 possible numbers and then chooses a bonus number different from the 6 numbers. Matching the 6 original numbers still wins the jackpot, but now a player wins $100,000 by matching 5 of 6 numbers and the bonus number. Matching 4 of 6 numbers and the bonus wins $1,000.

(a) Find the probability of winning each prize in each lottery. Note that since the order of your chosen numbers doesn't matter, permutations do not strictly apply. Rather, a similar method called combinations, of ${}_nC_r$ does.

(b) Given the starting jackpot in each game is $2,000,000, what is the expected mean value of buying one ticket for Michigan Lotto? For Bonus Lotto?

(c) Each time the lottery is not won, the jackpot increases a certain amount depending on ticket sales. What would the jackpot have to be in Michigan Lotto before the expected value was >0? What would the jackpot have to be for Bonus Lotto?

(d) Write this paper as if you were trying to convince people of which lottery system was best. Explain your reasoning clearly and make sure to back up any numerical results with facts and explanations of why your answers are right. Be sure to explain any terminology you use.

(Please type your paper.)

*This project was originally written by David Hildebrant during his internship in my classroom. I appreciate him allowing me to use it as an example in this paper.

Appendix C
continued

Discrete Mathematics
Chapter 3 Project*

Many weather conditions such as temperature vary throughout the year in ways that can be modeled by a sine function. Below, you are given the average high and low temperatures for each month of the year for the Lansing area. Use this information to answer the given questions.

Write a paper that describes what you found out about the temperatures in Lansing. Be sure to include the appropriate mathematics and graphs to help the reader understand how you arrived at your results.

Your paper should contain an appropriate introduction and conclusion. (Please type your paper.)

Month	High Temperature (F°)	Low Temperature (F°)
January	29°	13°
February	32°	14°
March	43°	25°
April	57°	35°
May	69°	45°
June	78°	55°
July	83°	59°
August	80°	57°
September	72°	50°
October	59°	39°
November	46°	31°
December	34°	19°

(a) Graph the yearly cycle of these averages. Write a sine function H and L that approximates each graph.

(b) Determine the transformation that transforms H into L and one that transforms L into H.

(c) Find a transformation that will change H into H', which models the high temperature in degrees Celsius.

(d) To show how well your function H models the temperatures in the Lansing area, compare the real data with the approximate values. Graph the errors of each month. Are there any trends?

*This project originally came from the UCSMP Precalculus and Discrete Mathematics book. I have made a few alterations to it so it would fit with my assessment goals.

Appendix D

Transition Mathematics
Semester Exam

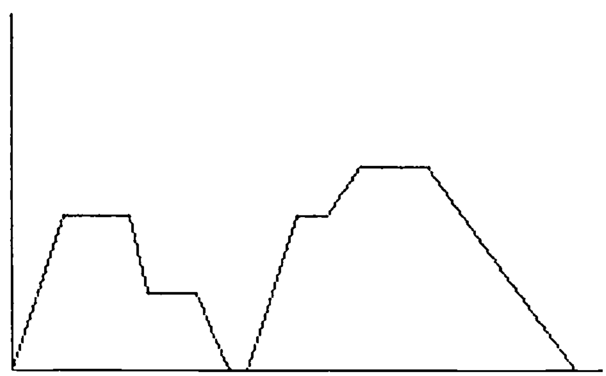
Below are six problems. Answer each of them as completely as possible. As you work through these, be sure to keep careful records that will be useful to you during the exam.

(1) Use the rule -5x + 2, make a table of values with inputs ranging from -5 to 5, sketch a graph, and write everything you can about the graph and chart.

(2) Write two stories: One story should illustrate the concept of "average," and the other story should illustrate the concept of "average rate of change."

(3) Given the situation that you want to get a good grade in this class, list the independent and dependent variables. Write a sentence to illustrate the relationship between each pair.

(4) Write a story that corresponds to the following graph.



(5) Given the situation below, make a chart and graph for it. Be prepared to explain the key components of the chart and graph in relationship to the situation.

Tom has $1,600.00 in his savings account. He is going to withdraw $200.00 per month for rent.

Appendix D
continued

(6) Decide whether the following statements are **sometimes**, **always**, or **never** true. If you state that the statement is **always** or **never** · .e, give three (3) examples to support your opinion. If you state it is **sometimes** true ive one (1) example where it is true and one (1) example where it is false. ... prepared to support your opinion.

(a) A straight line will pass through two quadrants.

(b) A straight line that is increasing will have positive y - intercept if it has a positive x - intercept.

(c) A straight line that is decreasing will pass only through the first and fourth quadrants.

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Appendix E

Algebra II
Performance Assessment
Grading Rubric

This is the grading rubric for the Algebra II Performance Assessment. It is designed to help you with assigning a grade to the student's performance. My hope is to make it easier for you to assess the student while at the same time making the grade more accurately reflect the performance. Below are given guidelines for each grade and the grading scale.

Performance indicator:
Orally communicates an accurate understanding of the mathematical concepts inherent in a problem situation

Problem Presentation: (2/3 of exam grade)

A
- Demonstrates a thorough understanding of the problem and concepts
- Selects appropriate strategies
- Accurately interprets the results in relationship to the problem situation
- Effectively communicates results using visual aids and appropriate mathematical terms

B
- Demonstrates a good understanding of the problem and concepts
- Selects appropriate strategies
- Accurately interprets the results in relationship to the problem situation
- Communicates results

C
- Demonstrates an adequate understanding of the problem and concepts
- Selects appropriate strategies
- With probing, accurately interprets the results in relationship to the problem situation
- Communicates results

D
- Lacks an understanding of some key concepts contained in the problem
- Unable to explain and interpret the results in relationship to the problem situation

E
- Lacks an understanding of the concepts contained in the problem
- Unable to explain and interpret the results
- Apparent lack of preparation for the assessment

Grading Scale: A (45-50), B (40-44), C (35-39), D (30-34), E (0-29)

Appendix E
continued

Group Problem-Solving: (1/3 of exam grade)

Group Problem-Solving performance indicator:
Applies mathematical understanding and collaboration skills when working on group problem-solving tasks.

A
- Consistently and actively contributes ideas leading to a solution of the problem
- Assists group in explaining solution and mathematical concepts
- Demonstrates effective interpersonal skills

B
- Consistently contributes ideas leading to a solution of the problem
- Assists group in explaining solution and mathematical concepts
- Demonstrates effective interpersonal skills

C
- Sporadically contributes to ideas and to explaining solution
- Demonstrates effective interpersonal skills

D
- Rarely contributes ideas
- Demonstrates effective interpersonal skills

E
- Does not collaborate with the group

Grading Scale: A (45-50), B (40-44), C (35-39), D (30-34), E (0-29)

Appendix E
continued

Algebra II
Performance Assessment
Evaluator's Comments

Student________________________________

Part 1: Problem Presentation Grade on this part ____________________pts. (2/3)

1. Understanding of the problem and concepts:

2. Strategies selected:

3. Result—accuracy and interpretation:

4. Communicating—visual aids and terms:

Part 2: Group Problem-Solving Grade on this part ____________________pts. (1/3)

1. Contribution of ideas:

2. Contribution to explanation of solutions and concepts:

3. Interpersonal skills:

Appendix E
continued

Notes to the Judges

1. The exam periods are 1½ hours each. Third hour is from 7:35 to 9:05 and fourth hour is from 9:15 to 10:45. Please keep track of the time so all students have a fair chance. Remember the exam is in two parts. For third hour, the first part is one (1) hour with students presenting their prepared problems. The second part is one-half (½) hour with students discussing one of the problems I will give you during the exam. For fourth hour, let the students use the time they need to discuss the problems. They should discuss all of their problems.

2. For third hour, be sure you pick the problem for each student. **DO NOT** let them pick the problem they want to discuss. If some judges allow students to pick and others do not, then it is not a fair assessment for all students. During fourth hour, we have agreed to let the students pick their problems or to work together, whichever they feel comfortable with.

3. Third hour: If you finish early, please keep the students at your station or at least nearby until the end of the exam period. Do not send them back to my room because there are two groups being assessed there. Fourth hour: please send them back to room C-2.

4. Before you leave, please be sure I get your evaluation forms. Please remember to write comments on them about what the student was and was not able to do. They find your comments very important.

5. Thank you very much for your time. Your willingness to give of your time makes this type of assessment possible and goes a long way in helping us improve the education we give our students.

6. The next set of exams will be on [date]. These will be the final exams and will run 1½ hours each. If you would be able to help out again, please let me know. You can do so by filling out the bottom portion of this form and returning it to me.

I would be glad to help with the final exams. I am available during the following times:

______ [date] for the PreCalculus Exams (Discrete Mathematics is a senior course so their exams are before graduation

______ [date] for the Algebra and Transition Mathematics Exams

Name:______________________________

Telephone:______________________________

Address:______________________________