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ABSTRACT

A theoretical model and methodology is presented that allows the path or trajectory of an individual's emotional experience during problem solving to be depicted and empirically studied. The trajectory of one's emotional experience reveals how well the individual copes with the frustration of problem solving by indicating how quickly recovery takes place. The methodology described here offers a way to empirically test and validate theories that may help shift students away from excessive negative emotional experiences during problem solving. Mandler's model of emotion, which forms the foundation of the methodology, is discussed in terms of emotion dynamics during problem solving. Contains 26 references. (Author)

A Methodology for the Analysis of Emotion Experiences During Mathematical Problem Solving

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Abstract

A theoretical model and methodology is presented that would allow the path or trajectory of an individual's emotional experience while problem solving to be depicted and empirically studied. The trajectory of one's emotional experience could reveal how well the individual copes with the frustration of problem solving by revealing how quickly recovery takes place. The methodology described here offers a way to empirically test and validate theories that may help shift students away from excessive negative emotional experiences during problem solving. Mandler's (1984) model of emotion, which forms the foundation of the methodology, is discussed in terms of emotion dynamics during problem solving.

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Overview

Researchers and classroom teachers have long commented on the importance of emotions in solving mathematical problems. As Skemp (1971) and others have pointed out, emotional states can be a hinder or help in mathematical problem solving. The effects of emotions in problem solving, therefore, are not just negative and in fact can be quite positive in bringing about success in finding solutions.

Emotion can greatly affect the student's problem solving process. Perkins(1990) says there is wide agreement that students need to become more reflective about learning, to be more aware of strategies for problem solving and to become better at handling the strategies. Some of these strategies need to focus on helping students cope with and manage the large swings in emotion that often occur. Teaching about the emotion process during problem solving can influence, in a positive way, students' dispositions, attitudes and beliefs about mathematics.

Thus far, the work that has been done on the effects of emotions in mathematical problem solving has been primarily qualitative in nature and not very sophisticated analytically. This body of work, moreover, has tended not to be theory driven, and has been very *ad hoc* in character making interpretation of the data obtained very difficult. The lack of good theory and sophisticated analytical models have greatly hampered and impeded work in the area.

Marshal (1989) examines the affective responses of 100 sixth-grade children describing how they would go about solving story problems. Marshal concludes that affect can enter schema knowledge at the same time other features of the schema are encoded. For example, schemas created in the presence of a negative affect while problem solving will have the negative affect encoded along with the relevant features of the problem. Repeated failures at problem solving strengthen the links to the negative affect node and links from that node to other features of the schema also become stronger. The model predicts that with enough failures in solving story problems, the presentation of a story problem will evoke a strong negative reaction which ultimately affects beliefs and attitudes about doing mathematics.

But math problems are generally difficult for students and frustration in solving them is a natural part of doing mathematics. Thompson and Thompson (1989) argue that affect related goals of problem solving instruction should not be limited just to getting students to enjoy problem solving. The issue is not whether negative emotions occur, but how students cope with negative emotions when they inevitably do occur.

Marshal's study, which indicates that negative affect is encoded into schemas as the schemas are being constructed stands in sharp contrast, to Thompson and Thompson's study which claims that without frustration, an appreciation for problem solving may never develop. Furthermore, negative emotions are not the only emotions that people feel and experience during problem solving.

Positive emotions during problem solving can energize the problem solver, increase the problem solver's persistence in finding a solution, and even help the problem solver obtain insight to a creative solution. The problem itself and the finding of a successful solution can encode positive feelings into schemas in terms of success, rewards, satisfaction, and competence.

Purpose

What is needed is a theoretical model and methodology that would allow the path or trajectory of an individual's emotional experiences while problem solving to be depicted and studied. The trajectory of one's emotional experience could reveal how well the individual copes with the frustration of problem solving by revealing the dynamics of how quickly recovery takes place. The quicker the recovery, the less likely negative affect becomes part of problem solving schemas. Similarly, the longer and more frequently positive emotion is experienced during the problem solving process, the more likely positive affect becomes part of the problem solving schemas.

Emotion experience trajectories would show how repeated cycles of negative to positive to negative emotion influence the individual's recovery. Trajectories could also indicate whether the emotional state is drifting into a more and more negative patterns over a period of weeks or months. A path where positive affect occurs with less intensity for shorter durations and where negative affect eventually takes over completely may characterize the typical disintegration of self-esteem and self-actualization in mathematics. Over time, individuals who experience only negative emotion eventually develop poor self-images, negative attitudes, and negative dispositions and beliefs about doing mathematics.

This paper proposes a way in which emotion experience trajectories during problem solving can be studied. The method is based on Mandler's theory of emotion, and an empirically tested nonlinear modeling technique described here in terms of characteristics of emotion from Mandler's theory.

Mandler's theory

According to McLeod (1988, 1989), one of the most useful theories of emotion available to researchers in the area of mathematical problem solving comes from Mandler (1984). Central to Mandler's theory is the view that emotion arises from the interruption of an individual's plans or planned behavior. Interruption of an activity, be it thoughts or actions, takes place when either an expected event does not occur, or when an unexpected event does occur. An expected event might not occur, for example, if the person's cognitive schema is not capable of handling the requirements necessary to complete an activity. On the other hand, an unexpected event might occur if the activation of a new schema does in fact handle the requirements. Subsequent to an interruption, the relationships among the features in the schema are compared with the perception of the situation. The degree of incongruity between what is expected and the perception of the actual event is interpreted as appropriate or inappropriate by an ongoing evaluative process (see Mandler 1982).

Interruption is one of the main paths to changes in behavior. An interruption in a cognitive activity is a signal that changes in the thought process or changes in the environment have occurred. A hard-wired response to interruption is the activation of physiological systems which either prepares the individual to actively cope with the interruption (fight or flight) or inhibits the individual (freeze or faint) when active coping would be inappropriate or counterproductive (Beck 1985).

Physiological arousal is an important part of Mandler's theory of emotion. The aroused state of physiological readiness is a necessary and measurable part of the mobilization of action systems. Arousal is nonspecific in that it contributes nothing to the evaluation of the situation. Arousal only provides the visceral or energized "gut" stimulation that determines the intensity of emotion. Mandler assumes, however, that each individual must reach an arousal threshold before the arousal becomes emotionally active. Conversely, evaluation of the situation (that is, how the interruption is interpreted) determines the quality or tone of emotion. Because of what Mandler calls "bands of acceptability," an incongruity threshold must be reached before a transition in evaluation from either positive to negative or negative can occur. Together, evaluation and arousal are the two major factors which, when combined, give rise to emotion. Emotion intensity depends to a large extent on how interrupting the event is, where as, whether an emotion is agreeable or disagreeable depends on the evaluation process and not on the interruption itself.

The view that arousal and cognition are both necessary for emotion to occur has been the basis of most emotion theories since the experiments of Schachter and Singer (1962), and Simon (1967). They showed that emotion is experienced only to the extent that a state of physiological arousal is experienced. Without arousal, the individual experiences only pure evaluation and does not experience emotion. Mandler (1989) reports that just about any sort of incongruity between what is expected and what actually occurs produces arousal.

In Mandler's (1975) work, arousal refers to specific measurable events that occur external to the mental system. Arousal produces stimulation that is perceived and interpreted in the same manor as other external environmental events lead to cognitive interpretation. More specifically, arousal is autonomic nervous system (ANS) activity and somatic nervous system (SNS) activity that is discriminable by the cognitive system. Arousal acts on the visceral receptors and is perceived as undifferentiated stimulation that, for the most part, varies in intensity only. The ANS can be considered as an output system and its importance in Mandler's emotion model is the manner in which the cognitive system differentiates that output.

Mandler's analysis of the evaluation process is based on schema theory and schematic assimilation and accommodation (Mandler, 1982). Mandler notes that the degree of incongruity between what is expected and what is encountered forms a continuum from complete congruity to extreme incongruity. The degree of incongruity determines the changes, if any, that take place in the schema structure. Each new experience is compared to an existing schema. The ease with which the new information is assimilated into the schema, or the amount of alteration that is required to accommodate the new information, affects the perception and understanding of the event and is the basis for the most basic evaluative judgements.

Emotion during problem solving

Mandler's theory of emotion is particularly applicable to mathematical problem solving. Mandler (1989) describes how his theory of emotion can be applied to the teaching and learning of problem solving in mathematics. McLeod (1987,1988) suggests that a problem solving process which is suddenly blocked, and a problem solving process which suddenly moves forward after being blocked, are interruptions that often lead to emotion. When succeeding is important to the individual, becoming blocked in the problem solving process, or suddenly being able to proceed toward a solution after being blocked, can lead to strong emotion.

Rapid changes in emotion are often a part of the process of problem solving. Negative feelings of frustration, dislike, anguish, dismay, shame, insecurity, defeat and so on can accompany an interruption in the process. Positive feelings of triumph, hope, relief, surprise and so on can accompany the release from an interruption (Lazarus, 1991). Both positive and negative emotional onsets are common and can occur repeatedly in the course of solving a single problem; if the onset of positive or negative emotion is sudden and intense, the experience is often identified as either "Aha!" (Parnes, 1975; Purcia, 1988) or "Oh-oh!" respectively.

Mandler's view is that emotion in problem solving is a nonlinear phenomenon rather than the straightforward, sequential, algorithmic process that is depicted in much of the literature. The nonlinear modeling methodology discussed here can be constructed directly from Mandler's theory. Analyzing problem solving emotion with the methodology provides a way to study emotion paths that occur during problem solving. The technique offers a theoretical framework in which emotion paths, including cyclical paths, can be visualized as trajectories over a three dimensional surface. The trajectories are illustrated here as projections on a two dimensional surface.

Nonlinear modeling of emotion

Emotions in problem solving are not static. They are dynamic events that can be on-going throughout a problem solving session. The nonlinear model of emotion described here is based on various aspects of Mandler's theory and other emotion theories.

These aspects are as follows: First, emotion arises from the combination of cognitive evaluation and physiological arousal. Evaluation determines only the tone of emotion and arousal determines only the intensity. Arousal and evaluation are independent factors in the emotion process. Second, emotion is either agreeable or disagreeable (Hooper, 1981). This is demonstrated by Russell (1979) in a study which shows that agreeable and disagreeable emotions are not independent of each other but rather are bipolar opposites. The bipolar nature of emotion results in a bimodal distribution of emotion responses during problem solving. As emotion intensity increases, positive and negative instances of emotion become increasingly polarized (Ortony, Clore, and Collins, 1988). It should be noted that a single event may produce both positive and negative emotions but an event when viewed with respect to a single goal is usually resolved as being ether positive or negative. Third, even though a healthy person's emotion is a fleeting process that constantly interacts with the person and the environment, the person's emotion is not chaotic nor even periodic. Rather, it is a response that is subject to various control processes. Even though emotion is a transitory condition in a stream of often continuous emotion changes, emotion can still be viewed as part of a feedback or self regulation system (Carver and Scheier 1990). In other words, emotion is a process that moves to be in balance with the physiological and cognitive events taking place.

While moving to be in balance with physiological and cognitive events, there are other forces which act on emotion. As time goes on, emotion intensity tends to dampen as the physiological arousal dissipates. At the same time however, an emotion tends to perpetuate itself for several reasons. Because of its influence on the perception of progress, a prevailing emotion may bias the perception of the environment. The bias may influence the perception process so that it alters or selectively encodes certain aspects of the situation to be consistent with the emotion. In addition, access to emotionally charged memories related to the situation may be enhanced. These memories may be activated and further perpetuate the emotion. Mandler's "bands of acceptability" where a threshold level of incongruity must be reached before evaluation changes also explain a source of bias in perception. Thus, at any instant, emotion moves in the direction of increasing balance with the cognitive, physiological, and environmental conditions, but changes in emotion are influenced by the intensity of the emotional state itself (Rapoport, 1970; Clynes, 1977, Davidson, 1992; Carver and Scheier, 1990).

The nonlinear model

The mathematical model used here is appropriate to model a phenomenon where over some range of the phenomenon, either of two stable but different behaviors can occur. That is, the phenomenon moves to polarized equilibrium positions in a way that can be described by a possibly bimodal surface. (The polar nature of emotion is one of the characteristics which makes it an appropriate application.)

A cross section of the equilibrium surface of a bimodal dynamic system (such as the emotion system) turns out to be similar to a cubic polynomial turned on its side. (See Figure 1.) This result makes sense and is easy to rationalize: since the graph of a fourth order polynomial has up to two maxima, a fourth order equation can be used to describe the bimodal dynamic; to find the

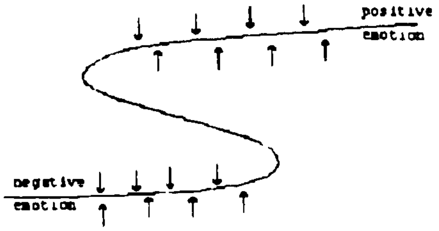

Figure 1. Cross section of emotion surface. Arrows represent emotion adjusting to an equilibrium state.

equilibrium surface, just take the derivative and set it equal to zero; the result is a third order polynomial (see Fararo 1978).

Emotion during problem solving tends to move to one of two possible balance or equilibrium states. But when the cognitive evaluation of a situation changes so much that small perturbations in emotion are no longer sufficient to keep the system in balance, then large transitions in emotion must occur. Such transitions happen when the evaluation of the situation changes from good to bad or from bad to good. Furthermore, emotion will tend to resist changes because it tends to bias perception and enhance memories of related situations. The resulting "emotion inertia" occurs regardless of whether evaluation changes slowly or abruptly. At some point however, the balance between emotion and evaluation can no longer be maintained by small emotion adjustments and a dramatic change in emotion takes place. Large jumps in emotion occur often during problem solving and are know as "Oh-oh!" and "Aha!" These jumps occur in the model of emotion when the two independent variables - physiological arousal, and degree of incongruity between what is expected and the perception of the actual event - force the dependent variable - emotion - from one surface to the other. (See figure 2.)

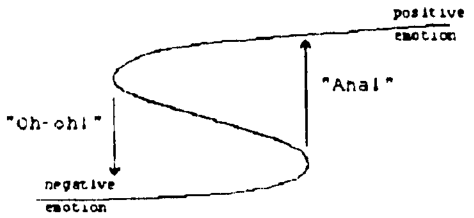

Figure 2. Sudden jumps from one emotion state to the polar opposite state.

Cobb's Cusp Surface Analysis Program

It has been shown (Allen 1994) that Cobb's (1992) Cusp Surface Analysis Program (CUSP) can provide an accurate mathematical representation of the emotion process during problem solving. Cobb (1978, 1980) shows that a gradient system with white noise included as part of the dynamic leads directly to solutions which are multimodal probability distributions. Under certain conditions, Cobb's Cusp Surface Analysis Program is able to estimate a probability density function from which a model is calculated. CUSP assumes the probability density function is a bimodal generalization of the Pearson system of normal densities (Ord 1972) with either one mode, or two modes separated by an antimode. CUSP uses a maximum likelihood procedure to find the most likely generalized normal density that would have produced the observations. A model is derived from the estimated distribution (see figure 3) and then compared and tested against the linear regression model. For details on how to apply the model and use Cobb's program, see Allen and Carifio (1994).

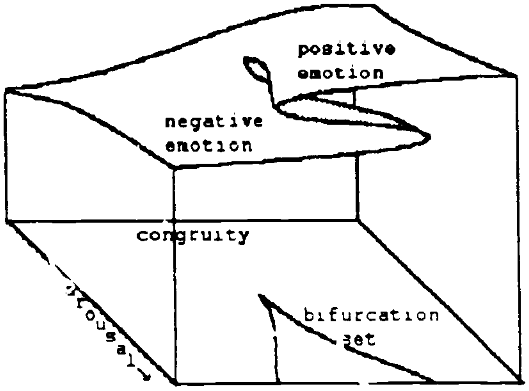

Figure 3. Emotion surface.

To explain the model presented in Figure 3, some sample problem solving experiences are depicted in Figure 4. In Figure 4, the axes correspond to the components from Mandler's model that give rise to emotion. The control axis (pointing toward the bottom of the page) corresponds to physiological arousal, and the other axis (pointing to the right) corresponds to the degree of event/ schema incongruity. According to Mandler's theory, arousal separates the emotion surfaces since increasing arousal increases only the intensity of emotion. The other factor of emotion is experience-vs.-expectation incongruity. This factor is normal to the arousal factor since evaluation of incongruity - whether positive or negative - is independent of arousal. Incongruity and emotion are not always well correlated. Increasing incongruity can result in either positive or negative emotion. However too much incongruity will ultimately lead to emotion becoming negative.

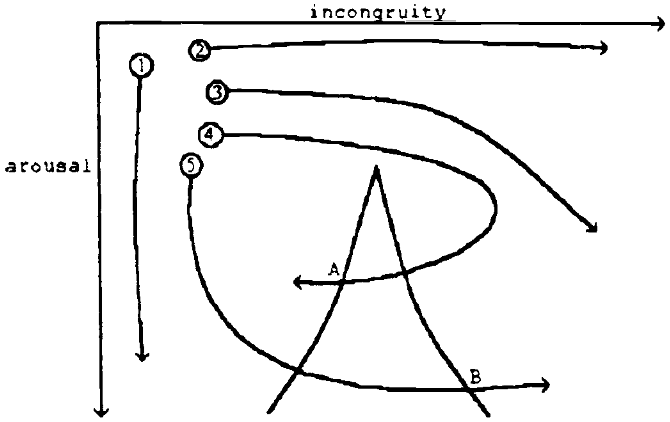

Figure 4. Sample trajectories in the control space.

Different paths (problem solving experiences) may be described in this model's space of possible experiences. Five such paths are described below:

Path 1 is the case where the problem solver starts with a slightly positive evaluation. Along the path, arousal increases and incongruity is constant. The resulting positive emotion increases over the course of the trajectory.

Path 2 is the case where the solver gets more and more confused but does not get upset because arousal does not increase along this path.

Path 3 is where arousal and incongruity are both increasing. The result is that negative emotion increases.

Path 4 is where incongruity increases along the first part of the path. The resulting emotion starts off as slightly positive and moves to being slightly negative. As incongruity starts to decrease, arousal increases causing the negative emotion to increase. At point A, a change in the perception and evaluation of the situation brings about a discontinuous jump from negative emotion to positive emotion. Depending on the intensity of the emotion at point A, the experience of "Aha!" may have occurred.

Path 5 is where arousal increases for a while and then levels off. The result is that positive emotion increases for a while until incongruity starts to increase. Then, positive emotion starts to decrease. At point B, the perception and evaluation suddenly change from "good" to "bad" and a discontinuous jump in emotion occurs. At that point the experience of "Oh-oh!" may have occurred.

Given the above idea and descriptions of problem solving paths, we may begin to hypothesize strategies to shift problem solvers from one path to another (for example, from path 5 to path 4). The model and research methodology presented here provides a way to quantitatively analyze emotion data in order to develop and test such strategies.

Conclusion

Rigorous testing of emotion theories has not been an option because an appropriate methodology which could accommodate the latent, multivariate, nonlinear aspects of emotion had not been developed. Because of the potential loss of too much information, it is particularly important for researchers not to conceal the nonlinear features of emotion in experimental designs involving composite indices such as averages and their associated statistics such as t-tests, linear regression, correlation, and single factor analysis of variance. This paper presents a solution. The modeling and statistical tests performed by Cobb's program are appropriate for emotion research, are powerful, and yet surprisingly easy to apply. The methodology described here offers a way to empirically test and validate theories that may help shift students away from excessive negative emotional experiences during problem solving.

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