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ABSTRACT

Mandler's (1984) model of emotion is summarized in this paper and is operatonalized analytically and statistically using Thom's (1975) catastrophe theory. Data were collected from (n=15) mathematics majors in a pilot study to test Mandler's model and the nonlinear effects of emotions in solving mathematical problems. The data were found to fit Thom's cusp model almost twice as well as the conventional linear model, thus strongly supporting Mandler's model. The implications of these findings are discussed in terms of mathematical problem solving and mathematics education. Contains 69 references. (Author/MKR)

The Relationship Between Emotional States and Solving Complex Mathematical Problems

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Abstract

Mandler's (1984) model of emotion is summarized in this article and is operationalized analytically and statistically using catastrophe theory (Thom 1975). Data were collected in a pilot study to test Mandler's model and the nonlinear effects of emotions in solving mathematical problems. The data were found to fit Thom's cusp model almost twice as well the conventional linear model, thus strongly supporting Mandler's model. The implications of these findings are discussed in terms of mathematical problem solving and mathematics education.

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Overview

Researchers and classroom teachers have long commented on the importance of emotions in solving mathematical problems (e.g. Skemp 1971; Mayer 1980; Buxton 1981; Mason, Burton and Stacey 1982; Reyes 1984). As Skemp (1971) and others have pointed out, emotional states can be a hinder or help in mathematical problem solving. The effects of emotions in problem solving, therefore, are not just negative and in fact can be quite positive in bringing about success in finding solutions.

The work that has been done on the effects of emotions in mathematical problem solving has been primarily qualitative in nature (e.g. Lester 1983, Marshal 1989, Silver and Metzger 1989, Thompson and Thompson 1989, Threadgill-Sowder 1984) and not very sophisticated analytically (Reyes 1984, Sowder 1989). This body of work, moreover, has tended not to be theory driven, and has been very *ad hoc* in character making interpretation of the data obtained very difficult (McLeod 1989). The lack of good theory and sophisticated analytical models have greatly hampered and impeded work in the area (Vinner 1979, Renfrew and Cooke 1979).

Purpose

According to McLeod (1988, 1989), one of the most useful theories of emotion available to researchers in the area of mathematical problem solving comes from Mandler (1984). Mandler's theory is an arousal-cognition model of emotion. Given this point and the points made above, the purposes of this paper are therefore:

(1) to summarize Mandler's theory of emotion and to supplement it with the work of other theorists where necessary;

(2) to link Mandler's theory of emotion to Thom's nonlinear catastrophe theory in order to have appropriate mathematical and statistical procedures for analyzing empirical data that test the model; and

(3) to report the results of a pilot study we conducted to assess the validity of Mandler's theory of emotion in terms of mathematical problem solving using Cobb's (1992) statistical procedure to analyze the data.

The summary of Mandler's theory of emotion will be presented first in this paper followed by a summary of Thom's catastrophe theory and its application to Mandler's theory. The details and results of the pilot study we conducted will then be presented.

Mandler's Theory

Central to Mandler's theory is the view that emotion arises from the interruption of an individual's plans or planned behavior. Interruption of an activity, be it thoughts or actions, takes place when either an expected event does not occur, or when an unexpected event does occur. An expected event might not occur; for example, if the person's cognitive schema is not capable of handling the requirements necessary to complete an activity. On the other hand, an unexpected event might occur if the activation of a new schema does in fact handle the requirements. Subsequent to an interruption, the relationships among the features in the schema are compared with the perception of the situation. The degree of incongruity between what is expected and the perception of the actual event is interpreted as appropriate or inappropriate by an ongoing evaluative process (see Mandler 1982).

Interruption is one of the main paths to changes in behavior. An interruption in a cognitive activity is a signal that changes in the thought process or changes in the environment have occurred. A hard-wired response to interruption is the activation of physiological systems which either prepares the individual to actively cope with the interruption (fight or flight) or inhibits the individual (freeze or faint) when active coping would be inappropriate or counterproductive (Beck 1985).

The aroused state of physiological readiness is a necessary and measurable part of the mobilization of action systems. Arousal is nonspecific in that it contributes nothing to the evaluation of the situation. Arousal only provides the visceral or energized "gut" stimulation that determines the intensity of emotion. Mandler assumes, however, that each individual must reach an arousal threshold before the arousal becomes emotionally active. Conversely, evaluation of the situation (that is, how the interruption is interpreted) determines the quality or tone of emotion. Together, evaluation and arousal are the two major factors which, when combined, give rise to emotion. Emotion intensity depends to a large extent on how interrupting the event is, where as, whether an emotion is agreeable or disagreeable depends on the evaluation process and not on the interruption itself.

The view that arousal and cognition are both necessary for emotion to occur has been the basis of most emotion theories since the experiments of Schachter and Singer (1962), and Simon (1967). They showed that emotion is experienced only to the extent that a state of physiological arousal is experienced. Without arousal, the individual experiences only pure evaluation and does not experience emotion. Mandler (McLeod and Adams,1989) reports that just about any sort of incongruity between what is expected and what actually occurs produces arousal.

In Mandler's (1975) work, arousal refers to specific measurable events that occur external to the mental system. Arousal produces stimulation that is perceived and interpreted in the same manor as other external environmental events lead to cognitive interpretation. More specifically, arousal is autonomic nervous system (ANS) activity and somatic nervous system (SNS) activity that is discriminable by the cognitive system. Arousal acts on the visceral receptors and is perceived as undifferentiated stimulation that, for the most part, varies in intensity only. The ANS can be considered as an output system and its importance in Mandler's emotion model is the manner in which the cognitive system differentiates that output.

ANS activity is generally restricted to glands that are activated by the nervous system and with visceral functions which involve the muscles of the heart, the smooth muscles of the intestines, blood vessels, stomach, and the urinary tract. SNS activity includes the conveyance of information from sense receptors, their transformation and the conveyance of information to the striped musculature of the body and limbs. The pathways of the ANS and SNS can often be differentiated in the peripheral system, but in the central nervous system, they are closely interrelated and presently, cannot be distinguished. Mandler's use of the concept of arousal differs from previous applications (e.g. Berlyn 1960, Duffy 1962) in that Mandler's use implies a cognitive system that does not rely on energy concepts such as Freud's ideas about the economy of energy.

The arousal-cognition model however, has not been free of criticism. To many, the nonlinear relationship between arousal and cognition has never been satisfactorily explained. Valins (1967) notes that there are as many studies that find a positive relationship between emotion and arousal as there are studies that find a negative relationship between these variables. Izard (1982) claims there are serious problems with emotion-cognition interaction data, and that, in fact, emotion may be orthogonal or inversely related to indices of arousal. In spite of the controversy however, many of the disparate experimental and theoretical results as applied to mathematical problem solving can be explained by Mandler's theory of emotion and by the model presented here.

Mandler's analysis of the evaluation process is based on schema theory and schematic assimilation and accommodation (see Mandler, 1982). Mandler notes that the degree of incongruity between what is expected and what is encountered forms a continuum from complete congruity to extreme incongruity. The degree of incongruity determines the changes, if any, that take place in the schema structure. Each new experience is compared to an existing schema. The ease with which the new information is assimilated into the schema, or the amount of alteration that is required to accommodate the new information, affects the perception and understanding of the event and is the basis for the most basic evaluative judgements.

Mandler's theory of emotion is particularly applicable to mathematical problem solving. Mandler (McLeod and Adams,1989) describes how his theory of emotion can be applied to the teaching and learning of mathematical problem solving and McLeod (1987,1988) applies the theory. McLeod suggests that a problem solving process which is suddenly blocked, and a problem solving process which suddenly moves forward after being blocked, are interruptions that often lead to emotion. When succeeding is important to the individual, becoming blocked in the problem solving process, or suddenly being able to proceed toward a solution after being blocked, can lead to strong emotion.

Rapid changes in emotion are often a part of the process of problem solving. Negative feelings of frustration, dislike, anguish, dismay, shame, insecurity, defeat and so on can accompany an interruption in the process. Positive feelings of triumph, hope, relief, surprise and so on can accompany the release from an interruption (Lazarus, 1991). Both positive and negative emotional onsets are common and can occur repeatedly in the course of solving a single problem; if the onset of positive or negative emotion is sudden and intense, the experience is often identified as either "Aha!" (Parnes, 1975; Purcia, 1988) or "Oh-oh!" respectively.

Emotion during problem solving has some important and well documented characteristics. First, it takes only a slight change in the relationship the problem solver has with the problem to create a wide *divergence* in emotional response (Weiner, 1986). Second, emotion is either agreeable or disagreeable (Hooper, 1981). This is demonstrated by Russell (1979) in a study which shows that agreeable and disagreeable emotions are not independent of each other but rather are bipolar opposites. The bipolar nature of emotion results in a *bimodal* distribution of emotion responses during problem solving (Ortony, Clore, and Collins, 1988). Third, emotion is not neutral. Because threshold values exist for both positive and negative emotions, there are *inaccessible regions* where emotional changes cannot occur (Scheier and Carver, 1982). Fourth, a slight change in the perception of a problem can result in a rapid change, or *discontinuity*, in emotion from one pole to the other (Purcia, 1988). Fifth, an emotion during problem solving tends to perpetuate itself by influencing the perception of progress (Rapoport, 1970; Clynes, 1977). Emotion cycles often occur during problem solving. These cycles occur because the prevailing emotion biases the perception of the environment with the result that changes in emotion depend, in part, on the direction of change (Davidson, 1992; Carver and Scheier, 1990). This effect is described as *hysteresis*.

The five characteristics of emotion - divergence, bimodality, inaccessibility, discontinuity, and hysteresis - have made it difficult to develop a widely agreed upon theory. The difficulty is substantiated by the multitude of emotion theories that are competing for acceptance. The five basic characteristics of emotion also put it beyond the scope of traditional mathematical models (Isnard and Zeeman, 1977). There are, however, newer mathematical models that can represent and compute the basic characteristics of emotion outlined above and their interactions.

Catastrophe Theory

Recently, phenomena with the characteristics of emotion outlined above have been modeled with a branch of mathematics called catastrophe theory. Specifically, phenomena with these characteristics can be modeled with the cusp catastrophe model.

Catastrophe theory is a method that, unlike differential equations, is capable of dealing with discontinuous and divergent phenomena. The cusp catastrophe surface and its associated mathematics incorporates all five characteristics - divergence, bimodality, inaccessibility, discontinuity, and hysteresis - into one model. The model relates each characteristic to each of the others. According to Thom (1975), the progenitor of catastrophe theory, the method has the potential for modeling the evolution of forms in all aspects of nature. According to Zeeman (1976), if one of the characteristics is evident, then the process should be examined for the other four. With evidence of two or more characteristics, the process becomes an excellent candidate to be modeled with catastrophe theory. Emotion during mathematical problem solving is a process where all five characteristics are evident. Thus, emotion is an excellent candidate to be modeled with catastrophe theory where all the characteristics described above can be combined into one model.

Numerous nonlinear phenomena which exhibit discontinuous jumps in behavior have been modeled using catastrophe theory. The rapid changes in perception of ambiguous figures have been modeled in Poston and Stewart (1978), in Stewart and Peregoy (1983), and in Ta'eed, Ta'eed and Wright (1988). Zeeman (1977) models rapid changes in mood, the sudden crashes and surges in the stock market, prison disturbances, the influence of public opinion on the policy adopted by an administration, anorexia nervosa, and censorship in a permissive society. A model of problem solving where the solver exits from the problem solving process either with or without the solution is presented by Boles (1990), and misconceptions on science education is modeled by Boyes (1988). Some other catastrophe theory models include the following: attitude with respect to an election survey (Anderson 1985), research in higher education (Staman 1982), attitudes and social behavior (Flay 1978), birth rates throughout nations (Cobb 1978), attitude change and behavior (Cobb 1980), psychoanalytic phenomena (Callahan 1990), the emergence of urban slums (Dendrinos 1979), patterns of blaming nurses for incidents of aggression (Carifio 1992), personnel selection, therapy and policy evaluation (Guastello 1982), motivation in organizations (Guastello 1987), and accidents in an organization (Guastello 1988). For a more complete list of catastrophe theory models over a wide range of applications, see Guastello (1987).

An essential part of catastrophe theory is gradient dynamics. It is from this concept that catastrophe theory arises. In a gradient dynamic system, the process moves toward certain stable attractors in the system. In emotion during problem solving, the assumption that emotion is gradient-like is based on the Gestaltist principle of Prägnanz (Koffka 1935). This principle states that a given stimulus is perceived as its simplest interpretation. "Simplest interpretation" for a given stimulus is a stable attractor. In situations where more than one "simplest interpretation" is possible, "simplest interpretation" means simpler than any nearby or closely similar perception. This gradient-like dynamic is used by Stewart and Peregoy (1983) in their catastrophe theory model of perception and is equally appropriate for the emotion model. For many social science applications, if there are attractors in the behavior and there is no periodicity, or worse - chaos, then the assumption that the behavior is gradient-like is a reasonable one.

The principle of Prägnanz as applied to problem solving states that a particular problem solving situation is usually seen as being either "good" (one's progress toward the solution is proceeding as hoped or planned), or "bad" (one's progress is not proceeding as planned). Certainly there are times where ones progress is seen as some fuzzy combination of good and bad, but the gradient-like nature of perception tends to push the percept to one of the "simplest interpretation" attractors: good or bad.

Pilot Study

In order to empirically test Mandler's model of emotion as applied to problem solving, data were collected from 15 mathematics majors while they were solving mathematical problems. Twelve of the 15 students were pursuing masters degrees and 3 were pursuing doctoral degrees in math education. In a graduate class entitled "Mathematical Problem Solving", each of these 15 students were given three problems. The problems were difficult, multiple step problems that required different skills and often required several attempts to solve them.

A questionnaire was designed to gather a broad range of data on variables that were suspected of being related to emotion as described by Mandler's theory. Each questionnaire contained 12 questions such as, "how mentally energized are you, how comfortable are you with your progress, how successful do you expect to be, and how frustrated are you". Answers were chosen from a Likert scale with a range from 0 ('not at all') to 6 ('very much').

Several copies of the questionnaire were given to each of the students which they filled out while they were solving the problems. Students were requested to answer the questions at times of their own choice as long as they felt particularly frustrated or pleased with their problem solving progress. Due to the nature of catastrophe theory, sampling at random times was not important. What was important was to catch the student in an emotion state that was stable with respect to the importance, expectation and progress at the time the questionnaire was answered. A total of sixty-seven questionnaires were collected. Each questionnaire was treated as an independent observation of the phenomenon for the purpose of analysis, as what was being assessed was surface fit rather than movement from point to point on the surface.

To test how well catastrophe theory and thus Mandler's model would fit the data, Cobb's (1992) Cusp Surface Analysis Program was used. Cobb's program fits a probability distribution to the observed data using the method of maximum likelihood. A cusp surface is derived from the estimated distribution and then compared and tested against the linear regression model (Cobb 1992).

Results

Using the data collected from the 15 students, Cobb's program found a catastrophe theory surface that fit the data significantly better ($p < .001$) than the linear regression model. Eighty-three percent of the variance in frustration is explained by the catastrophe theory model, where as only 45% of the variance is explained by the linear model.

In order to use Cobb's program, the emotion data was transformed into four component variables. The dependent variable Y, was a combination of the "how pleased" and "how frustrated" questions, and was used as a measure of emotion. The first independent variable X_1, was a combination of variables related to motivation. The second independent variable X_2, was a combination of variables related to the perception of progress with the problem. The third variable X_3, was a combination of variables related to the expectation of success. As previously stated, the questionnaire was designed to gather a broad range of data on variables that were suspected of being related to emotion as described by Mandler's theory. Thus the component variables did not align themselves directly with the concepts of Mandler's theory because these concepts were not measured directly by the questionnaire. The values used for these concepts were derived values which means that the beta weights, or raw coefficients, of the model could vary from sample to sample as well as from questionnaire (measures) to questionnaire.

The standardized linear model that was found for emotion was $Y = -.2X_1 + .4X_2 + .4X_3$. This equation shows that as motivation increases, negative emotion increases and as progress and expectation increase, positive emotion increases. The weights on progress and expectation are twice motivation's weight. Accordingly, changes in progress and expectation would each increase emotion twice as much as a similar change in motivation would decreases emotion.

In maximizing the likelihood of the generalized probability distribution, Cobb's program iterates toward a statistical model which has the form:

$$0 = A+B(Y-C)-D(Y-C)^3 \qquad \text{(EQ 1)}$$

Factors A, B, and C are linear combinations of two to seven independent variables . The factors, along with the scalar coefficient D determine a single dependent variable Y. From the data, the control factors with standardized coefficients were found to be:

$$A = .2 - .2X_1 + .2X_2 + .8X_3$$

$$B = 2.9 - X_1 + .6X_2 - .9X_3$$

$$C = -.1X_1 + .3X_2$$

$$D = 4.0$$

Thus, the emotion model is:

$$0 = (.2-.2X_1+.2X_2+.8X_3) + (2.9-X_1+.6X_2-.9X_3)(Y + .1X_1-.3X_2)$$
$$-4(Y + .1X_1-.3X_2)^3$$

To evaluate and interpret the above equation's fit to the observations, Cobb's program performed various statistical tests. These tests found the cusp model to be a far better model of the emotion described by the data than the linear model.

In testing the emotion model, the statistic which compares whether the cusp model is better than the linear model was found to be significant at the $p = .002$ level. The coefficient which increases the amount of pleat in the surface was significantly greater than zero at the $p = .005$ level.

The cusp model of emotion proved far superior to the linear model. The *linear* r^2 statistic was .45 while the *delay* r^2 statistic was .83. Thus, eighty-four percent more variation (.83/.45=1.84) is explained by the cusp model (*delay* r^2) than is explained by the linear model (*linear* r^2).

Discussion

Zeeman (1980) suggests that the value of using a cusp model is that the model gives global insight, reduces arbitrariness of description, helps to synthesize unconnected observations, explains inexplicable features, and suggests unsuspected possibilities. The cusp model of emotion offers the above benefits to the understanding of the emotion process during problem solving. The parsimony of the cusp model provides a conceptual framework which clarifies and gives insight into Mandler's theory as applied to problem solving.

The use of the cusp surface as a paradigm for emotion during problem solving greatly simplifies Mandler's theory. The cusp surface helps clarify the process of emotion during problem solving by acting as a visual gestalt. It offers a concise representation which easily explains the main transitions of the emotion process. Using the cusp surface as a model can quantify much of Mandler's very elaborate qualitative theory. A quantitative model allows for rigorous statistical testing and allows for modification when necessary. Viewing emotion during problem solving as a catastrophe theory dynamic system allows all of the benefits proposed by Zeeman to come into play. Such a system opens a wide range of possibilities for emotion theory applications.

Conclusion

Mandler has proposed a detailed non-linear model of emotion and, prompted by McLeod, applied his model to emotion in problem solving. Mandler's view is that emotion in problem solving is a nonlinear phenomenon. Problem solving is not the straightforward, sequential, algorithmic process that is depicted in much of the literature and particularly the literature on mathematical problem solving. Elements of the cusp model correspond well to Mandler's theory and view. Using Cobb's Cusp Surface Analysis Program, the problem solving emotion data gathered in this study fit a nonlinear cusp catastrophe model quite well. Experiences (paths) in solving difficult problem, when described in terms of a cusp model and Mandler's theory, closely match subjective reports of problem solving experiences. It would seem then that those interested in mathematical problem solving would need to seriously consider the role of emotion in problem solving in conjunction with this nonlinear model and Mandler's theory of emotion.

Frustration is a natural part of problem solving. Knowledge about frustration during problem solving would allow timely intervention by teachers so that students' frustration does not become too excessive or enduring. Knowledge about possible emotion outcomes during problem solving would help students deal with the whole problem solving process. The knowledge would increase the tendency for students to monitor and reflect on their own feelings, their thinking, and their performance. It would allow them more flexibility in exploring ideas and alternate solutions, and would increase their curiosity and inventiveness. Most important, an awareness of emotion during problem solving would increase students' willingness to persevere at a mathematical task (NCTM, 1989). The ability to monitor and manage the emotion process would give students an advantage that should not be overlooked by educators.

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