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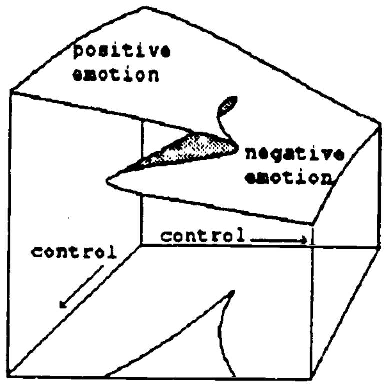
ABSTRACT

A central theme of Mandler's theory of emotion is that the interruption of a cognitive activity sets the stage for emotion. Mandler's theory is particularly applicable to mathematical problem-solving experiences. Mandler's linking of emotion to perception also makes emotion during problem solving an excellent candidate to be modeled with catastrophe theory. This hypothesis was tested using data collected during actual problem-solving experiences from (n=14) students in a graduate problem-solving class. Results showed that 74% of the variance in frustration level of the students was explained by the catastrophe model whereas only 46% was explained by the linear model. Contains 46 references. (Author/MKR)

# A Model of Frustration During Problem Solving

**Bradford D. Allen**








Abstract

A central theme of Mandler's theory of emotion is that the interruption of a cognitive activity sets the stage for emotion. Mandler's theory is particularly applicable to mathematical problem solving experiences. The linking of emotion to perception by Mandler also makes emotion during problem solving an excellent candidate to be modeled with catastrophe theory. This hypothesis was tested using data collected in a graduate problem solving class. Statistically significant results were found. The usefulness of an emotion model is also discussed.

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Paper presented at the New England Educational Research Organization, Rockport Maine, April 20-22, 1994.

This paper will present a model of frustration during problem solving. Such a model could contribute to the understanding of emotion in problem solving, and ultimately help people become better problem solvers (McLeod, 1987).

Currently there are many cognitive psychology based theories of emotion. Prominent views include Weiner(1986), Frijda (1987), Ortony, Clore and Collins (1988), Carver and Scheier (1990), and Lazarus (1991). According to McLeod (1987, 1989), one of the most useful theories of emotion to researchers in the area of mathematical problem solving comes from Mandler (1984). Central to Mandler's theory is the view that emotion arises from the interruption of an individual's plans or planned activity. From relevant schemas, or bodies of knowledge about the activity, come the expectations of how the activity should progress. The degree of discrepancy (incongruity) between what is expected and what is encountered determines the extent to which the activity is interrupted.

Interruption of an activity, be it thoughts or actions, is generally of two types: an unexpected event occurs, or an expected event does not occur. An expected event might not occur because the cognitive structure fails to handle the requirements to complete the activity; or, an unexpected event might occur because the activation of a new structure does handle the requirements. Subsequent to an interruption, the relationships among the features in the schema are compared with the perception of the situation and the degree of incongruity between what is expected and the event is interpreted as appropriate or inappropriate by an ongoing evaluative process.

Interruption is one of the main paths to change. An interruption in a cognitive activity is a signal that changes in the environment have occurred. A natural response is the mobilization of action systems to prepare the individual to adjust to the environment that caused the interruption. The individual prepares for changes with both increased attention and with physiological readiness for fight or flight.

The aroused state of physiological readiness is a necessary and measurable part of the mobilization of action systems. Arousal is nonspecific in that it contributes nothing to the evaluation (good or bad) of the situation. Arousal only provides the visceral or "gut" feel that determines the intensity of emotion. On the other hand, the evaluation of the situation, which depends on how the interruption is interpreted, determines the quality or tone of emotion. Together, evaluation and arousal are the two major systems involved in the emotion process. It is their combination that gives rise to emotion. The intensity of the emotion depends to a large extent on how interrupting the event is, and the tone of the emotion (positive or negative) depends on the evaluation process and not on the interruption itself. Mandler assumes that each individual must reach an arousal threshold before the arousal becomes emotionally functional.

The view that arousal and cognition are both necessary for emotion to occur has been the basis of most emotion theories since the experiments of Schachter and Singer (1962), and Simon (1967). They showed that emotion is experienced only to the extent that a state of physiological arousal is experienced. Without arousal, the individual experiences only pure evaluation and does not experience emotion. And Mandler (McLeod and Adams,1989) reports that any sort of incongruity between what is expected and what actually occurs produces physiological arousal.

The arousal-cognition model has not been free of criticism however. The nonlinear relationship between arousal and cognition has, to many, never been satisfactorily explained. Vallins (1967) notes the contradiction that there are many studies that have found a positive relationship between emotion and physiological arousal, but there are also many studies that have found a negative relationship between these variables. Izard (1982) claims there are serious problems with emotion-cognition interaction data, and that, in fact, emotion may be orthogonal or inversely related to indices of arousal. Fiske (1982), however, points out that though emotional and cognitive responses each arise from the individual's history of prior experiences, emotion and cognition may still be processed differently. Fiske says that it is inconclusive whether or not emotional reactions are just another cognition, and that until more evidence is available, it is presumptuous to equate them. Zajonc (1980) takes this idea one step further and claims that emotion and cognition are entirely separate systems. In spite of the controversy, many of the disparate experimental and theoretical results as applied to mathematical problem solving can be explained by Mandler's theory of emotion and by the model presented here.

Mandler's analysis of the evaluation process is based on schema theory and schematic assimilation and accommodation (see Mandler, 1982). Mandler notes that the degree of incongruity between what is expected and what is encountered forms a continuum from complete congruity to extreme incongruity. The degree of incongruity determines the changes, if any, that take place in the schema structure. Each new experience is compared to an existing schema. The ease with which the new information is incorporated into the schema, or the amount of alteration that is required to accommodate the new information, affects the perception and understanding of the event and is the basis for the most basic evaluative judgements.

Mandler describes possible emotion outcomes in terms of the assimilative and accommodative consequences of the interruption of expectations and predictions. Mandler argues that complete congruity allows easy assimilation of the event into the structure which means there is little or no interruption and little or no physiological arousal. If immediate assimilation is not possible, the individual may be able to find an alternate schema in which to assimilate the event. Successful assimilation in this way is usually accompanied by positive evaluation and the degree of interruption is usually small. Therefore, there is only a small amount of physiological arousal and only slight, if any, positive emotion. An example of this case is the discovery that previous knowledge is generalizable to a wider range than was previously thought.

If no assimilation is possible, then deeper structural changes are necessary and the degree of interruption is intense. In the case where incongruity is so severe that accommodating the information into a schema is not possible, the individual prepares to respond cognitively and physically to the changes in the environment that brought about that interruption by becoming physiologically aroused. Because accommodation is not possible, appropriate perceptions, thoughts and actions are unavailable to the individual, and the situation is evaluated negatively. The individual experiences anxiety, helplessness, and dismay.

The most interesting case in this area of Mandler's theory is the situation where following incongruity between what is encountered and what is expected, assimilation of the new information is not possible, but the individual is successful in accommodating the new information into a new schema. In this case, either positive or negative emotion may follow. The reason either emotional *quale* may occur is that the situation may be evaluated as either positive or negative depending on a number of factors. These factors include whether or not the degree of incongruity is tolerable, and on the specific context in which the incongruity occurs. Since the interrupting event requires accommodation, strong physiological arousal may have occurred. The strong arousal, combined with the evaluation, results in either positive or negative emotion. If the resulting emotion is the polar opposite of any emotion being experienced when the interruption occurred, a discontinuous change, or flip, in the emotion process takes place.

One more point to be made about Mandler's theory of emotion is about the ongoing process of evaluation. Evaluation follows a comparison between what is expected (the schema) and an encountered event. Rarely will the relations in a schema structure exactly match the evidence presented in an encounter. Mandler's theory allows for an individual to consider the fit between an event and the activated schema as reasonable in spite of incongruity between the two. In Mandler's theory, the mental system has wide "bands of acceptability" where incongruity is considered reasonable. These bands of acceptability vary from event to event and situation to situation. The bands are defined by threshold values where, as incongruity changes, the individual's perception of fit changes discontinuously from "reasonable" to "unreasonable" or the other way around. For example, as incongruity increases, there is a threshold point where the perception of the match between what is expected and the available evidence from reality, changes from reasonable to unreasonable. At such a point, evaluation changes from "the match is tolerable" to "the match is intolerable". As incongruity decreases, there is another threshold point where the perception changes from "the match is intolerable" to "the match is tolerable". If arousal and emotion is involved, each point coincides with a discontinuous change in emotion from "negative" to "positive" or the other way around.

Though not mentioned by Mandler, there is much evidence to support that when incongruity is *increasing*, the flip in perception, (and the corresponding flip in evaluation), does not occur at the same point on the congruity/incongruity continuum as when incongruity is *decreasing* (Hanson, 1958; Ellis,1969; Rapoport, 1970; Arnheim, 1974; Poston and Stewart, 1978; Stewart and Peregoy, 1983).

Mandler's theory of emotion is particularly applicable to mathematical problem solving. Mandler (McLeod and Adams,1989) describes how his theory of emotion can be applied to the teaching and learning of mathematical problem solving. McLeod (1987,1988) also applies Mandler's theory of emotion to mathematical problem solving. He suggests that a problem solving process which is suddenly blocked, and a problem solving process which is suddenly able to proceed after being blocked are interruptions that often lead to emotion. When plans that are important to the individual are involved, becoming blocked in the problem solving process, or suddenly being able to proceed toward a solution after being blocked, can lead to strong emotion.

Rapid changes in emotion are often a part of the process of problem solving (Lindsay, 1972). Negative feelings of frustration, dislike, anguish, dismay, shame, insecurity, and defeat etc. can accompany an interruption in the process. Positive feelings of triumph, hope, relief, and surprise etc. can accompany the release from an interruption (Lazarus, 1991). Both positive and negative emotional onsets are common and can occur repeatedly in the coarse of solving a single problem; if the onset of positive or negative emotion is sudden and intense, the experience is often identified as either "Aha!" (Parnes, 1975; Purcia, 1988) or "Oh-oh!" respectively.

Emotion during problem solving has some important and well documented characteristics. First, it takes only a slight change in the relationship the problem solver has with the problem to create a wide <u>divergence</u> in emotional response (Weiner, 1986). Second, emotion is either agreeable or disagreeable (Hooper, 1981). This is demonstrated by Russell (1979) in a study which shows that agreeable and disagreeable emotions are not independent of each other; instead, they are bipolar opposites. The bipolar nature of emotion results in a <u>bimodal</u> distribution of emotion responses during problem solving (Ortony, Clore, and Collins, 1988). Third, emotion is not neutral. Because threshold values exist for both positive and negative emotions, there are <u>inaccessible regions</u> where emotional changes cannot occur (Scheier and Carver, 1982). Fourth, a slight change in the perception of a problem can result in a rapid change, or <u>discontinuity</u>, in emotion from one pole to the other (Purcia, 1988). Fifth, an emotion during problem solving tends to perpetuate itself by influencing the perception of progress (Rapoport, 1970; Clynes, 1977). Emotion cycles often occur during problem solving. These cycles occur because the prevailing emotion influences changes by biasing the perception of the environment so that changes in emotion depend, in part, on the direction of change (Davidson, 1992; Carver and Scheier, 1990). This effect can be described as <u>hysteresis</u>.

The five characteristics of emotion - divergence, bimodality, inaccessibility, discontinuity, and hysteresis - have made it difficult to develop a widely agreed upon theory. The difficulty is substantiated by the multitude of emotion theories that are competing for acceptance. The characteristics of emotion also put it beyond the scope of traditional mathematical models (Isnard and Zeeman, 1977).

But since 1969, phenomena with exactly these characteristics have been modeled with a branch of mathematics called catastrophe theory. Specifically, phenomena with these characteristics can be modeled with the cusp catastrophe model.

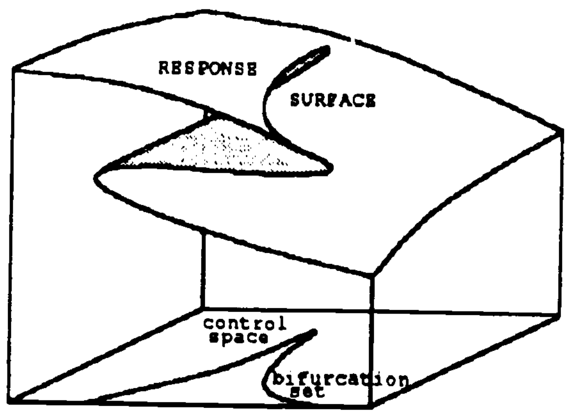


Figure 1. Cusp surface. The bifurcation set is the boundary of the region where the response is bimodal.

Catastrophe theory is a method that, unlike differential equations, is capable of dealing with discontinuous and divergent phenomena. The cusp catastrophe surface (see figure 1) and its associated mathematics incorporates all five characteristics - divergence, bimodality, inaccessibility, discontinuity, and hysteresis - into one model. The model relates each characteristic to each of the others. According to Thom (1975), the progenitor of catastrophe theory, the method has the potential for modeling the evolution of forms in all aspects of nature. For a given process, if one of the characteristics is evident, then the process should be examined for the other four. With evidence of two or more characteristics, the process becomes an excellent candidate to be modeled with catastrophe theory (Zeeman, 1976).

Emotion during mathematical problem solving is a process where all five characteristics are evident. Thus, emotion is an excellent candidate to be modeled with catastrophe theory where all the characteristics described above are combined into one model.

## Methodology

In order to demonstrate the appropriateness of a catastrophe theory model of emotion during problem solving, data was collected during actual problem solving experiences. In a graduate class titled 'Mathematical Problem Solving', each of the 14 students in the class was given three problems to work on during class and at home. The problems were difficult, multiple step problems that required different skills and often were often not solvable in one sitting. The problems were the following:

1. The ship P sights the ship Q which sails in a direction perpen-

dicular to PQ and keeps her absolute course. P pursues Q aiming constantly at Q. The speed of both ships is the same at every moment (but it can vary in the course of time). It is evident without computation that P is sailing along a curved line. If the pursuit lasts for a long time, the track of the pursuing ship becomes almost identical with the track of the escaping ship. What is the distance PQ if at the beginning it was equal to 10 nautical miles?

2. The number $3^{105}+4^{105}$ is divisible by 13, 49, 181, and 379, and is not divisible by 5 or by 11. How can this result be confirmed?

3. Three neighbors gave $4.00 each and bought a ham (without skin, fat, and bones). One of them divided it into three parts asserting that the weights were equal. The second neighbor declared that she trusted only the balance of the shop at the corner. There, it appeared that the parts, supposed to be equal, corresponded to the monetary values of $3.00, $4.00 and $5.00 respectively. The third partner decided to weigh the ham on her home balance, which gave a still different result. This led to a quarrel, because the first woman kept insisting on the equality of her division, the second one recognized only the balance at the shop, and the third only her own balance. In what way is it possible to settle this dispute and to divide these pieces (without cutting them anew) in such a way that each woman would have to admit that she had got at least $4.00 worth of ham if computed according to the balance which she trusted?

Several questionnaires were given to each of the students in the class which were to be filled out while they were solving the three problems. Each questionnaire contained 12 questions such as, how mentally energized are you, how comfortable are you with your progress, how successful do you expect to be, and how frustrated are you. Answers were chosen from a Likert scale with a range from 0 ('not at all') to 6 ('very much').

Students were requested to answer the questions at times of their own choice as long as they felt particularly frustrated or pleased with their problem solving progress. Due to the nature of catastrophe theory, sampling at random times was not important. What was important was to catch the student in an emotion state that was stable with respect to the importance, expectation and progress at the time the questionnaire was answered. The response rate varied from 1 to 9 questionnaires returned per student with an average of about 4 questionnaires per student.

To test how well catastrophe theory would fit the data, Cobb's (1992) Cusp Surface Analysis Program was used. Cobb's program fits a probability distribution to the observed data using the method of maximum likelihood. A cusp surface is defined from the modes of the estimated distribution (see Allen and Carifio, 1994).

Using the data collected from the 14 students, Cobb's program found a catastrophe theory surface that fit the data significantly better ($p<.001$) than the linear regression model. 74% of the variance in frustration is explained by the catastrophe theory model where as only 46% of the variance is explained by the linear model.

The parsimony of the cusp model provides a conceptual framework which clarifies and gives insight into Mandler's theory as applied to problem solving. Zeeman (1980) proposes that the purpose of a cusp model is to give global insight, to reduce arbitrariness of description, to synthesize unconnected observations, to explain inexplicable features, and to suggest unsuspected possibilities. The use of the cusp surface as a paradigm for emotion during problem solving greatly simplifies Mandler's theory. The cusp surface makes the process of emotion during problem solving clear by acting as a visual gestalt. It offers a concise representation which easily explains the main transitions of the emotion process. Using the cusp surface as a model can quantify much of a very elaborate qualitative theory. A quantitative model allows for rigorous statistical testing and allows for modification when necessary. Viewing emotion during problem solving as a catastrophe theory dynamic system allows all of the benefits proposed by Zeeman to come into play. Such a system opens a wide range of possibilities for emotion theory applications.

## Catastrophe Theory

A cusp catastrophe model assumes there are two possible types of behavior which are controlled by two orthogonal factors. The two factors form a plane called the "control space", (see figure 2). The factors can be oriented so that one factor runs down the pleat in the cusp surface. A factor oriented this way is called is the "splitting" or "bifurcation" factor. Increasing the value of the splitting factor causes the two behavior surfaces to

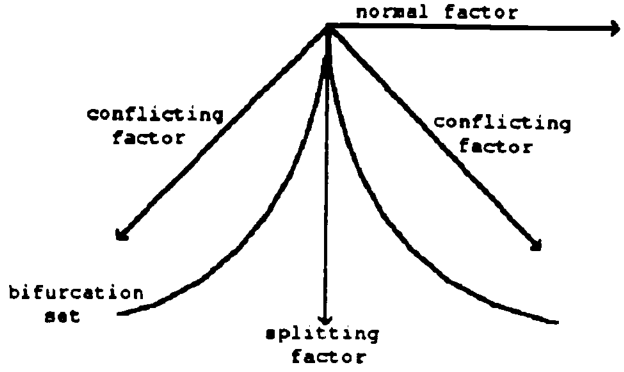


Figure 2. Control space with two possible axes orientations.

separate. The other factor is perpendicular to the splitting factor so is often called the "normal" or "asymmetry" factor. As this factor increases, the effect is that the behavior eventually changes (possibly discontinuously) from one type of behavior to the other. Discontinuous changes may occur at the bifurcation set; whether or not a discontinuous change occurs depends on how the cusp surface is oriented, and on the direction the trajectory takes in crossing the bifurcation set.

The factors can be oriented in a way so that the behavior elicited by one factor is opposite to, or "conflicts" with the behavior elicited by the other factor. Increasing the value of one conflicting factor tends to push the behavior on to the upper surface, but increasing the value of the other conflicting factor tends to push the behavior on to the lower surface. Inside the bifurcation area, the two factors conflict. Needless to say, one set of factors is just a rotation of the other set. In either case, the two control factors may be actual measurable variables, or more likely, they are formed by weighted averages of several independent variables. In the later case, the control factors are named according to the underlying construct that they represent.

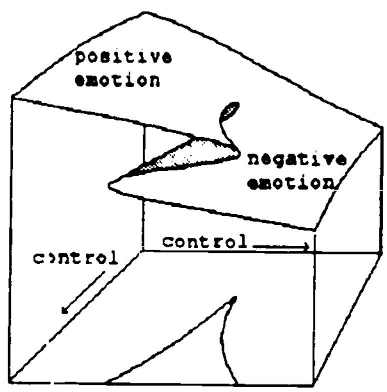


Figure 3. Cusp surface emotion model.

The cusp surface (see figure 3) represents the response (behavior). Applications that follow directly from catastrophe theory mathematics are those where the behavior is a process constantly adjusting toward a state of equilibrium. The cusp surface should be thought of as the set of equilibrium values of a dynamic process where the gradient (slope) of a possibly bimodal part of the process is involved. Though catastrophe theory mathematics restricts the dynamics to that of a gradient system, catastrophe theory modeling is not restricted to these systems. As Stewart and Peregoy (1983) point out, if there are "attractors" in the behavior and there is no periodicity, or worse - chaos, then the assumption that the behavior is gradient-like is a reasonable one.

The assumption that emotion is gradient-like is based on the Gestaltist principle of Prägnanz (see Koffka, 1935). This principle states that a given stimulus is perceived as its simplest interpretation. "Simplest interpretation" in a given situation is a stable attractor. In situations where multiple "simplest interpretations" are possible, for example those situations where a tolerable situation may suddenly switch to an intolerable one, "simplest interpretation" means simpler than any "nearby" interpretation. This dynamic is used by Stewart and Peregoy (1983) in their catastrophe theory model of perception and is equally appropriate for this emotion model.

It is not enough for a model of a phenomenon just to exhibit similar behavior to the phenomenon. A model must not only account for and explain a phenomenon in a nontrivial way, but also must follow directly from the assumptions about the phenomenon. Mandler's theory of emotion combined with the same modeling techniques and assumptions used in the catastrophe theory models of the perception of ambiguous figures (Poston and Stewart, 1978; Stewart and Peregoy, 1983; and Ta'eed, Ta'eed and Wright, 1988) lead directly to a catastrophe theory model of emotion. Empirical testing validates this. Empirical testing can be easily done with Cobb's Cusp Surface Analysis Program which is capable of fitting a cusp catastrophe theory surface to a set of data and statistically testing the fit. A "best fitting" surface is useful because once one is found, individual problem solving behaviors can be studied.

Some sample problem solving experiences are illustrated in figure 4. In this figure, the splitting axis (pointing toward the bottom of the page) corresponds to physiological arousal in Mandler's model, and the normal axis (pointing to the right) corresponds to the degree of event/schema incongruity. In accordance with Mandler's theory, arousal is the splitting factor since increasing arousal increases only the intensity of emotion. Incongruity is the normal factor since increasing incongruity generally results in emotion becoming more and more negative though the degree of incongruity and negative emotion intensity are not always well correlated. Discontinuities are explained by Mandler's bands of acceptability which are defined by points along the incongruity spectrum where either positive or negative emotions occur.

Path 1 is the case where the problem solver starts with a slightly positive evaluation. Along the path, arousal increases and incongruity is constant. The resulting positive emotion increases over the course of the trajectory.

Path 2 is the case where the solver gets more and more confused but does not get upset because arousal does not increase along this path.

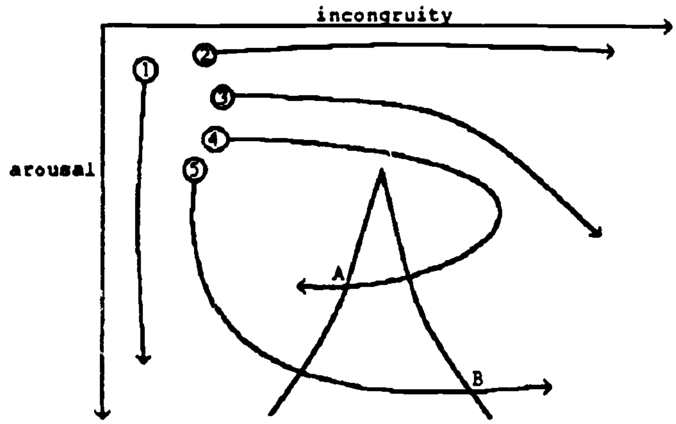


Figure 4. Sample trajectories in the control space.

Path 3 is where arousal and incongruity are both increasing. The result is that negative emotion increases.

Path 4 is where incongruity increases along the first part of the path. The resulting emotion starts off as slightly positive and moves to being slightly negative. As incongruity starts to decrease, arousal increases causing the negative emotion to increase. At point A, a change in the perception and evaluation of the situation brings about a discontinuous jump from negative emotion to positive emotion. Depending on the intensity of the emotion at point A, the experience of "Aha!" may have occurred.

Path 5 is where arousal increases for a while and then levels off. The result is that positive emotion increases for a while. As incongruity starts to increase, the positive emotion starts to decrease. At point B, the perception and evaluation suddenly change from "good" to "bad" and a discontinuous jump in emotion occurs. At that point the experience of "Oh-oh!" may have occurred.

There are many reasons to develop a model of emotion. In addition to the benefits gained by including emotion in artificial intelligence and computer aided learning systems, there are more immediate reasons. Just as awareness of one's own cognitive state during problem solving can improve problem solving ability (Polya, 1985), awareness of one's emotional state during problem solving can also improve problem solving ability. Perkins (1990) says there is wide agreement that students need to become more reflective about learning, to be more aware of strategies for problem solving and to become better at handling the strategies. But since emotion is a large part of problem solving, students should be taught strategies to cope with the large swings in emotion that often occur. Understanding that emotion can greatly affect the student's problem solving process is essential. Knowing that, for example, a dismal evaluation of one's problem solving situation can occur even at the cusp of a major breakthrough, can be invaluable in helping the student to persist in finding a new way to approach the problem. Teaching about the emotion process during problem solving can influence, in a positive way, students' dispositions, attitudes and beliefs about mathematics.

A model of frustration during problem solving can contribute to the understanding of the emotion aspects of problem solving. The need for such a model is well supported. McLeod

(1987) points out that it is the constant frustration of low achieving students that produce their negative attitudes about mathematics. Rachlin and Jensen (1984) found that getting upset can cause algebra students to lose control of their problem solving process. Understanding the emotion process can help students be more problem-focused rather than emotion-focused (Lan & Hong, 1992). Being able to predict and explain emotion outcomes can help students cope with stressful problem solving situations (Abella & Heslin, 1989).

For some students, it is the emotional part of problem solving that prevent them from getting involved in trying to solve a problem. The frustration that so often accompanies problem solving can make students feel too vulnerable or feel at risk for having their feelings hurt. Just as young children's beliefs about emotion management contribute to their interaction expectancies (Saarni, 1989), students' naive beliefs about their own frustration during problem solving contribute to their negative beliefs about mathematics and about themselves as problem solvers.

Frustration is a natural part of problem solving. Knowledge about frustration during problem solving would allow timely intervention by teachers so that students' frustration does not become too excessive or enduring. Knowledge about possible emotion outcomes during problem solving would help students deal with the whole problem solving process. The knowledge would increase the tendency for students to monitor and reflect on their own feelings, their thinking, and their performance. It would allow them more flexibility in exploring ideas and alternate solutions, and would increase their curiosity and inventiveness. Most important, an awareness of emotion during problem solving would increase students' willingness to persevere at a mathematical task (NCTM, 1989). The ability to monitor and manage the emotion process would give students an advantage that should not be overlooked by educators.

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