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ABSTRACT

This paper describes a study in which both collaborative action research and collaborative mathematics classroom instruction took place. The mathematics instruction consisted of the implementation of a problem-solving curriculum into seventh-grade mathematics classrooms. One problem-solving strategy per week for 10 weeks was introduced to the 4 classes which comprised a total of 125 students. Changes in students' attitudes, beliefs, and knowledge about problem solving were examined using pre- and post-problem solving surveys. Significant changes were found in all areas. A discussion of the teacher-researcher collaboration is included. Appendices contain surveys of students' beliefs about problem solving and examples of problems solved by students. (MKR)

COLLABORATIVE ACTION RESEARCH IN A SEVENTH-GRADE MATHEMATICS CLASSROOM

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> *Teachers throughout the world are developing professionally by becoming teacher-researchers, a wonderful new breed of artists-in-residence. Using our own classrooms as laboratories and our students as collaborators, we are changing the way we work with students as we look at our classrooms systematically through research. (Hubbard & Power, 1993, p. xiii)*

Action research in the classroom is an increasingly occurring phenomenon in the field of qualitative inquiry (Calhoun & Glickman, 1993; Cardelle-Elawar, 1993; Miller & Pine, 1990). It is valuable to the field of educational research primarily because research that is initiated by and documented by classroom teachers has the potential to result in immediate classroom reform. According to Miller and Pine (1990), "Teachers can contribute to educational improvement by conducting classroom research concerning teaching and learning processes. This "action research" role for teachers can enhance the professional status of teaching, generate theory and knowledge, increase the effectiveness of improvement efforts, and promote teacher development (p. 57)."

Collaborative action research, between university researchers and classroom teachers, presents opportunities for a more "action-oriented" approach to teacher enhancement (Clift, Veal, Johnson, & Holland, 1989). As teachers are encouraged to reflect upon and systematically examine aspects of their classrooms, they are likely to make changes based on their observations that lead to improved teaching and learning in their classrooms. Collaborative research projects in the classroom also leave a different impression on classroom teachers than do solely university-researcher investigations because there is the sense of ownership in the investigation and a feeling on the part of the teachers that they have been "worked with" as opposed to "worked on" (Lieberman, 1986). In addition, teacher development and institutional development (of universities and schools) go hand in hand (Fullan, 1993). Thus, benefits for university personnel, in terms of simultaneously making changes in their teacher preparation classes as school personnel are engaging in classroom reform, are plentiful.

In much the same way that collaborative research has resulted in helping teachers and researchers become more reflective about their teaching, collaborative teaching in mathematics classrooms has been shown to be an effective method for improving mathematics instruction. Fresko, Ben-Chaim, & Miriam (1994), while noting that there are natural problems associated with team teaching, such as role ambiguity and problems of coordination and communication among team members, identified factors that contribute to successful co-teaching experiences. These factors include having administrative support, sharing a common collegial bond, matching teachers with similar years of experience, and being flexible.

Described herein is a study in which both collaborative action research and collaborative mathematics classroom instruction took place. The mathematics instruction consisted of the implementation of a problem-solving curriculum into a seventh-grade mathematics classroom. The collaborative team involved consisted of a university mathematics educator and a seventh-grade mathematics classroom teacher, neither of whom had ever before engaged in such a collaborative partnership.

Forming a Collaborative Partnership

Establishing quality research projects designed and implemented jointly by classroom teachers and university faculty is not a simple task (Raymond, 1994). The key to finding an "ideal" classroom teacher-university faculty collaborative relationship is to let it happen naturally. The partnership described herein emerged during a summer graduate course taught by Anne, a university mathematics educator, in which Brad, a middle school mathematics teacher, was a graduate student. The focus of the course was on teaching mathematics via problem solving. Prior to the course, Brad had begun to make efforts to incorporate problem solving into his classroom teaching. However, as the course progressed, Brad realized that there was much more that he could be doing, and he became interested in changing the way he approaches problem solving in his classroom.

Brad admitted that he had some trepidation about how he would go about making changes. Anne offered to assist him in making those changes by co-planning and co-teaching some problem-solving lessons. Brad was very interested in this idea. When Anne later asked Brad if he would be interested in documenting their efforts through collaborative action research, Brad was still interested. Anne briefly described some components involved in conducting research, emphasizing the value of written reflection on the part of the teacher-researchers. She suggested that Brad attempt to clarify his thinking about mathematical problem solving and the changes might want to make in his classroom by responding to these initial questions in writing: (a) Why do you think problem solving is important, (b) What experiences with problem solving did you encounter as a student, (c) Do you enjoy engaging in problem solving, (d) Do you feel successful as a problem solver, (e) In what ways do you currently involve your students in problem solving, and (f) In what ways would you like to improve your abilities to solve problems and to teach problem solving in your classroom. Once Brad had formulated his responses to these questions (see Raymond, 1994), we discussed the specific directions we wanted our study to take.

Together, Brad and Anne planned, implemented, and documented their efforts in mathematics classroom reform which centered around making problem solving, in the spirit of the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics [Standards]* (1989), an integral part ot Brad's mathematics teaching. One of the most difficult aspects of conducting collaborative action research is developing a sense of the roles each of the partners is to play. It is certainly true that our roles have evolved throughout the project. However, at the beginning we were both hesitant to step out of what we had assumed would be our natural roles.

For example, Anne felt that her primary role would be to provide resources, guide the development of a framework for the research, and "teach" Brad about gathering, analyzing and reporting data. Anne felt strongly, at the beginning, that she should defer to Brad when it came to making decisions about what would take place in his classroom. Similarly, Brad was comfortable with allowing Anne to make suggestions about how the research and took charge of issues such as how problem solving would be implemented into his mathematics classroom and how much time he could realistically devote to problem solving.

Goals of the Collaborative Study

From the beginning, Anne and Brad were both clear about what each hoped to accomplish during this collaborative project. After having a number of conversations about what each wanted to see happen, we established three goals (a) to improve the seventh-grade students abilities in, knowledge of, attitudes toward, and beliefs about mathematical problem solving (b) to examine changes in Brad's mathematics teaching as we implemented the problem-solving curriculum, and (c) to learn more about the nature of collaborative action research in a mathematics classroom.

Both Brad and Anne believed that the primary reason for engaging in the project was to help the seventh-grade mathematics students become more knowledgeable about and confident in problem solving as well as becoming better problem solvers. However, Brad expressed a personal goal of wanting to improve his own ability to teach mathematical problem solving in the spirit of the NCTM (1989) *Standards* and Anne was very interested in examining issues associated with collaboration between university and school mathematics teachers and teacher educators. Thus, the resulting goals include both individual and joint areas of interest.

Methodology

Implementing the Problem-Solving Curriculum

The study itself, which included collaborative planning, teaching, and documentation, began in September 1994 and continues through the May of 1995. The collaborative pair decided to approach problem solving in the seventh-grade classroom by first introducing the students to ten problem-solving strategies, one per week for the first ten weeks of the study. We believed that it was important to encourage the students to use a specific strategy to solve given problems so that they would begin to get a feeling for what types of problems might naturally "call for" a particular strategy. We told them that although the problems we were doing in class could be solved by a number of strategies, we wanted them to first try solving it using the "strategy of the week" so that they would become familiar with the strategy. We further explained that we would be interested in talking about other strategies that they might use and that after we had introduced the ten strategies to them, we would leave it up to them to choose the strategy or combination of strategies they wanted to use.

Brad selected the ten strategies that he wanted to introduce at the beginning of the year. He chose *guess & check, draw a picture, make a table, make an organized list, look for a pattern, work backwards, use logical reasoning, make a model/act it out, make it simpler/look at a smaller case*, and *create a Venn diagram*. The pair introduced them in the order listed above.

Anne was available to co-teach in Brad's classroom on Thursday mornings. Consequently, the team decided to co-teach during Brad's first two mathematics classes on Thursdays and then have Brad teach the same lesson to the other participating classes later in the day. Brad would then do a related lesson the following Tuesday, which would emphasize the same problem-solving strategy introduced on the previous Thursday.

The types of problems included in each lesson were nonroutine problems that were either process problems, puzzle problems or, applied problems (as described by Charles & Lester [1982]). We tried to select problems that could incorporate the use of manipulatives on occasion because Brad wanted to become more knowledgeable about teaching with manipulatives. Our primary resources for problem ideas were the sixth and seventh grade books from The Problem Solver series (e.g., Moretti, G., Stephens, M., Goodnow, J. & Hoogeboom, 1987) and the Problem Solving Experiences in Mathematics series (e.g., Charles, R. I., Mason, R. P., Nofsinger, J. M., White, C. A., 1985) as well as Gorman's (1990) book of process problems.

In general, the lessons began with discussion of the strategy of the day and recollection of prior strategies used. Then we would engage the whole class in a problem that we solved together, working through the four steps of problem solving (as described in Moretti, G., Stephens, M., Goodnow, J. & Hoogeboom, 1987) and using the strategy of the day. After this, the students worked in pre-established groups of four on two other problems using the same strategy. Between the two problems we conducted a whole-class discussion of the solutions, which included having various groups' share their work. We wanted to include a strong writing component in the problem solving lessons. Thus, on most days, beginning in October, we asked students to write about their problem solution or about some issue related to problem solving.

Participants

The student participants included approximately 125 seventh-grade mathematics students at a large middle school in Indiana. The students were mostly Caucasian Americans and came from socioeconomic backgrounds that ranged from lower middle class to upper middle class. For most of the study, only four of Brad's five classes participated in the study. The fifth class, his "advanced group," were not included. Brad decided not to include them because the curriculum for these students was more demanding and required more time to complete. However, about four months into the study, Brad began to involve this group in the problem solving activities, working the group through the same sequence of problem solving that the other classes followed.

Data Collection

Data were collecteu through a variety of sources. To examine changes in students' attitudes, beliefs, and knowledge about problem solving, we implemented pre-problem solving and post-problem solving surveys that included multiple choice questions and short answer questions. The first survey (see Appendix A) was given to the students at the beginning of the school year, before we began our problem solving lessons. The students were later given a slightly revised version (see Appendix B) of the survey in February of the school year. The questions probed students about a variety of issues including their confidence in problem solving, their feelings about problem solving, their definitions of problem solving, problem-solving strategies they know, and how they solve mathematical problems. Data on students' attitudes and knowledge were also collected via teacher-researcher observations (which were subsequently documented in researcher journals) and student reflections.

Changes in students' abilities were documented primarily through student work and documented classroom observations. Additional data on students' abilities in problem solving will be collected via a "problem solving test" that will be implemented at the end of the school year. This "collaborative team designed" test will consist of a variety of nonroutine problems that can be solved using the strategies emphasized throughout the study.

Data regarding changes in Brad's teaching were gathered through self-observation and self-reflection as well as through documented observations made by Anne. Information about the nature of collaborative inquiry was also collected via observation and reflection on the part of both teacher-researchers. Throughout the study, both researchers maintained reflective journals. Both reflected after each Thursday session. In addition, Brad wrote a journal entry after each Tuesday follow-up lesson. The two researchers met periodically after school to discuss observations and share reflections. Synopses of these discussions were added to the reflective journals.

Analysis

Early data analysis included the compilation of comparative survey statistics, the reading and comparing of short answer responses on surveys, the comparison of early student work to later work, and team sharing of observations and reflections. Once the second survey had been implemented, we calculated percent responses to the survey questions in order to locate any changes in students' attitudes, beliefs, and knowledge about problem solving. When there was an observed change, we discussed that change together, sharing any observations that might support that finding. Data analysis continues as more data is being gathered.

Findings

Changes in Students' Attitudes and Beliefs

Analysis of data on students' attitudes and beliefs about problem solving resulted in several interesting findings. First, upon comparing students' responses to the multiple choice questions on the first survey to their responses on the second survey, significant changes were noticed in eight areas (see Table 1).

Table 1
Changes in Attitudes and Beliefs from Survey 1 to Survey 2

	Percent that Agreed or Strongly Agreed	
	Survey 1	Survey 2
I Like Problem Solving	43	74
I Can Do Problem Solving	63	91
I Am Not Confident in Problem Solving	56	25
Problem Solving is Easy	27	56
Problem Solving is Important to Learn	61	98
Some People Can't Do Mathematics	58	43
It is Important to Be Able to Do Math Quickly	18	38
I Know A Lot About Problem Solving	21	54

The findings are very encouraging in that they indicate that the percentage of students who expressed that they like problem solving, can do problem solving, and believe that problem solving is easy increased over the first six months of the project. Other positive results evident from Table 1 are that fewer students expressed that they lacked confidence in their problem solving abilities and that they believe some people cannot do mathematics. Also, more students indicated that they know a lot about problem solving and that problem solving is important to learn.

The one disturbing result from this portion of the survey was that the percentage of students who believed that it is important to solve mathematics quickly increased over the six-month period. After acknowledging this result, Brad and Anne reflected upon their teaching styles in an effort to determine if their actions might have encouraged more students to arrive at this conclusion. The team concluded that they may have contributed to this belief by presenting too many problems in each lesson and conducting whole-class discussions after a certain period of time had been given to work on the problems. We feel that perhaps we sent the message that the students should have been able to complete the problems in the time frame we allotted. As a result of this finding, we have became more sensitive to the issue of time by including fewer problems in the lessons and by eliminating the "whole class discussion" of problems between problems. Rather, we fostered "group pace" and provided a less structured opportunity for whole-class wrap up at the end of the class period. We also began to defer discussion of some problems to a later time.

On the second survey, we specifically asked students if their confidence and attitude toward problem solving had changed since the first survey. Sixty-seven percent of the students indicated that their level of confidence had changed. Some explanations provided by students on their surveys regarding why their confidence has changed are:

> *I know some more ways to solve problems now and feel a lot better about doing them now.*
>
> *Because of the different strategys [strategies] because at the beginning of the year I didn't know any strategies.*
>
> *I have confidence in myself that I can figure out a story problem.*
>
> *Because there are more ways to solve problems now.*
>
> *Because I now know how to solve these problems with my own techniques and styles. I have learned how to use steps to solve problems.*
>
> *Because I never thought that I would be able to do a hard problem.*

Because now I know that if I use a certain pattern to solve a problem and I can't figure it our I can use another one.

Students were also asked on the second survey if their attitudes toward problem solving had changed since the first survey. Sixty-three percent of the students claimed that their attitudes had changed, explaining:

Some days I like and sometimes I don't.

Yes, I use to not be able to do problem solving.

I think problem solving is good because from now on I can solve problems a lot better without being confused.

Yes, because I have found new ways to solve problems and have become easier to understand. I have also learned neat ways to solve the problems.

Because [before] I disapproved of everything and whenever Mr. Hamersley said problem solving I just didn't like it, but I do now.

No change, because most of this stuff is too hard for me.

Observations made by Brad and Anne mirror much of what the survey indicated. Both researchers had noted that students' enthusiasm about problem solving had increased early on and had been maintained throughout. They also noted that a number of students' confidence in problem solving had increased. Many individual cases stand out. For example, one female student in Brad's second period class had not been a very confident participant in class during much of the first ten weeks of the project. During the introduction of the last strategy, solving problems *using Venn Diagrams*. this student was able to quickly and accurately solve the problems provided in class. Heretofore she had been a student who rarely shared her solutions and who usually showed little enthusiasm during her attempts to solve the problems. After having a very successful day, and being told by Anne and Brad that she was the star problem solver of the day, her confidence increased immensely. Since that day, she has more actively participated in solving problems and has presented the impression that she believes she can solve problems now.

Changes in Students' Knowledge of Problem Solving

One anticipated findings about changes in students' knowledge of problem solving was an overwhelming increase in their awareness of problem-solving strategies. At the beginning of the school year, students had very little knowledge of specific mathematical problem-solving strategies (see Table 2). Before we implemented our problem-solving curriculum, the most frequently named problem-solving strategy was "read the problem." As one student said it, " read the problem over and over again, solve it, and read the problem again." A large number of students either explicitly claimed that they did not know any strategies or left the question blank. Another sizable group of students provided what have been categorized as "miscellaneous responses." These included such comments as "do the easier problems first," "I use my fingers to help," "study a lot," "work with a buddy," and " put your name in the place of names in the problem and relate it to yourself." Only ten of the 99 students who completed the survey named a specific mathematical problem-solving strategy. Those strategies included guess and check, draw a picture, and make a chart.

Table 2
Problem Solving Strategies Named By Students on Survey 1

	Number of Students Responding
Read it	21
Read it, check work	8
Don't know/left blank	18
Four operations	7
Look for clue words	6
Ask questions	5
Process problem strategy	10
Miscellaneous	17
Response made no sense	4
Relax	3

Midway through our introduction of the ten strategies, we asked students to write a reflection about which problem solving strategy they enjoyed most, or least, and why. The purposes of this assignment were to get the students thinking about the strategies and to provide students the opportunity to write about their feelings about problem solving. Some examples of students' responses are as follows:

> *One that I like is making a table, it's faster, easier to work with and it helps you to fill out if you get stuck. There is one other I like. It's the guess and check. It's easy and I like to guess and see how close I was or if I got it right. The one that I hate is list and ognize [organize]. It['s] just to[o] hard and confusing. The second one that I hate is making a drawing because I can't draw any sort of things.*

> *Draw a picture, because I think it is funner to draw, and the funner it is the more that people would not mind doing it. I don't dislike any of them because at first when you do it, it seems that you won't like, but when you get the hang of it it's easy;, so you like it.*

> *I like making an organized list the best. It had helped me understand the problem better,*

> *I think all the strategies are good to know for different kinds of problems. The one I thought was hardest to do or understand was "make an organized list." I find the "make a table" strategy useful and like it. I also like "guess and check" and "draw a picture but all the strategys are good for different things. For example, larger numbers are, in my opinion, harder to handle so I would choose "guess and check"*

Throughout the project we have encouraged students to think about the strategies critically and we believe that this had some effect on the students' increased awareness of problem-solving strategies.

On the follow-up survey, students were clearly more knowledgeable of problem-solving strategies. When students were asked to name as many mathematical problem- solving strategies that they could, approximately 30 percent of the students were able to recall all ten of the strategies introduced in class (see Table 3). In addition, approximately 73 percent of the students were able to recall at least five of the ten strategies employed in class.

Table 3
Number of Strategies Students Were Able to Name on Second Survey

Number of Strategies Recalled	0	1	2	3	4	5	6	7	8	9	10
Number of students	10	1	3	4	8	10	8	7	6	11	29

Note. n=97 students surveyed

The problem-solving strategies most frequently recalled by students are shown in Table 4. *Work Backwards* was most frequently recalled while, *Make and Organized List* was least frequently recalled, followed closely by *Act it Out/Make a Model* and *Look at a Simpler Case.*

Table 4
Strategies Most Frequently Recalled By Students

	Number of Times Named By Students
Draw a Picture	61
Make an Organized List	56
Make a Table or Chart	61
Look for a Pattern	62
Work Backwards	75
Act it Out/Model it	57
Try a Smaller Case	57
Logical Reasoning	62
Venn Diagramming	59
Guess and Check	65

Note. Number of respondents to this question is n = 97.

Another way in which we tried to glean some information about students' knowledge of problem solving was by asking them to define problem solving. Students' definitions of problem solving varied quite a bit on both surveys. However, overall we found that students' definitions of problem solving on the second survey were broader, more sophisticated, and more clearly expressed. Some representative definitions provided on the first survey were:

Solving a written question in class. It is a worded math problem

A way to help you to use your brain and read the whole question and read if you have to add, subtract, divide, multiply, or whatever the problem asks for. It makes you check your answer and see if it is right or if it is wrong, change your answer.

A mathematical problem that is given to you to make you think, sometimes they can be tricky, but that is the point.

It's a break from using numbers

Simply solving a problem, example
2 + 2 = ? 4, I just solved a problem

Representative student definitions of mathematical problem solving on the second survey included:

A quicker way to solve problems, an easier way to get problems solved. It can also be funner [more fun]

Solving and understanding a problem or dillema [dilemma] whether it be in a mathematical sense or in a personal problem

Solving everyday problems with simple strategies in a fun and challenging way.

A difficult problem put into the disguise of a story to make it look easier.

A mathematical challenge

Analyzing a mathematical problem.

Students were also asked on each survey what they do first when given a problem to solve and how they know when they have solved a problem correctly. Responses on the first survey varied greatly and were vague. However, on the second survey students described more clearly-defined stages of problem solving that were closely related to the four steps of problem solving encouraged in class. This indicates to us that students have become more knowledgeable of steps that they can follow in solving problems.

Both Brad and Anne documented observations of students' increased knowledge of problem solving, which corroborates findings from the survey. In class discussions, students often expressed knowledge of problem-solving strategies when referring to solving a particular problem using a specific strategy. For example, a student, when explaining how he did his problem differently from the rest of the class, said that he drew a picture to help him like he did when he solved the "Frog in the Well" (see Appendix C) problem. This indicated to us that students were not only remembering strategies, but were also becoming more aware of situations or problems for which given strategies would be useful.

Changes in Students' Problem-Solving Abilities

At this point, students have not yet taken the "problem solving test" described earlier. Thus, our current findings regarding changes in students' abilities to solve problems stem from student work samples and documented observations. Some observed findings we noted were that students' in-class verbal explanations of their solutions improved considerably while their written explanations have improved to a lesser degree. Another interesting finding is that students' use of the "hairy buffalo strategy" has increased.

The "hairy buffalo strategy" was a phrased coined by Brad early in the study. As we continued to asked students to consider a specific strategy in solving a problem, and as the students' repertoire of strategies increased, their desire and ability to use other strategies also increased. We began to talk about using more than one strategy simultaneously to solve a problem. Brad expressed that this reminded him of the ice cream special offered at "Dairy Queen" when he was a child. It was a special where they took all different kinds of ice cream and mixed them together, calling it the "hairy buffalo." The students enjoyed the story and really liked the phrase "hairy buffalo." So from then on, the students were eager to say when they had used the hairy buffalo strategy to imply that they had used a combination of strategies to create their own solution processes.

In order to illustrate the quality of students' problem-solving abilities, and changes noted in their work, a representative example of progress made by one student is provided herein. Figure 1 shows an example of a student's solution to the Zink-Zak Problem (see Appendix C) which was given to the students in September 1994. This problem is, admittedly, a simpler problem than one might expect a seventh grader to be able to solve. We decided to start with simpler problems to help build their confidence and ability to apply a strategy and then to move on to more difficult problems. Note, also, that at the beginning of the project, students were given a "problem solving answer sheet" on which to write their solutions. This was to encourage them to think of the four steps of probiem solving both as they attempted to solve the problem and as they wrote up their final solutions.

Insert Figure 1 About Here

This student's solution, essentially using the *guess and check* strategy. is correct. The written responses on the problem solving answer sheet are clear. The student, Astrid, identified the questioned being asked in the "find out" section and provided a fairly clear description of how she "solved the problem. However, the Astrid still did not identify "the strategy" employed and her conclusion upon "looking back" lacks a mathematical conclusion.

This same student, a month later, showed improvement in her ability to clearly explain each phase of solving a problem (see Figure 2). In her solution to a more challenging problem, The Math Convention Problem (see Appendix C), Astrid provides much more detail. However, she still "misidentifies" the strategy being employed. She identifies the strategy as *using an organized list* when the strategy of the day was make a table. There had been some discussion in class on the day of *making an organized list* of how when one is making an organized list it is often helpful to put it in the form of a table. Clearly, however, this is not a case of making an organized list. Rather it calls for *making a table*, perhaps in conjunction with *looking for a pattern* in the table. Thus, there was still some confusion for Astrid regarding the identification of strategy.

Insert Figure 2 About Here

Nevertheless, Astrid's descriptions of what she "found out" from reading the problem and what she concludes upon "looking back" were much improved. In addition, she does demonstrate a correct solution in a table format.

As we continued to increase the level of difficulty of problems presented to students, the students demonstrated that they were capable of meeting the challenge. An interesting note to add is that when students were asked on the second survey *if they like the challenge of an unfamiliar problem even though they may not know how to solve it,* 70 percent of the students agreed or strongly agreed.

More information regarding students' abilities to solve a variety of nonroutine problems will be available upon students' completion of the problem-solving test at the end of the school year.

Observed Changes in Brad's Teaching

Brad and Anne noted a number of significant changes in Brad's teaching over the first six months of the project. Brad specifically noted that (a) he was more aware of the NCTM (1989) *Standards* and that (b) he was more comfortable with incorporating manipulatives into his lessons.

Both Brad and Anne documented observed changes Brad made in the physical environment of the classroom. Less than four months into the study, Brad requested that he be given tables and chairs to replace the desks in his classroom. He had come to believe that this would better facilitate the group problem solving. His principal arranged the exchange. The use of tables brought about a significant change in the level of cooperation within the groups. The groups tended to "stay with each other" and work more diligently on problems without looking to neighbors for solutions. Brad was very pleased with the effect this environmental change had on the classroom dynamics.

Another physical change that both researchers noted was Brad's creation of learning centers within the classroom. One Thursday, Anne walked in to see three different centers set up along one wall of the classroom. The centers contained activities with Geoboards, Tangrams, and Cuisenaire rods. The centers were provided as "optional" opportunities for the students to explore mathematical concepts. Students were very interested in the centers, particularly because they offered the opportunity to work with manipulatives.

Another change in Brad's teaching that both researcher's made note of was an improvement in Brad's questioning technique. In the beginning, Brad tended to ask many "leading" questions, seeking specific responses. As time progressed, Brad began to ask more open-ended questions and became more comfortable with the class not always arriving at conclusive answers to questions.

An important change that Anne expressed to Brad that she had noted was that Brad's view of problem solving had shifted from that of "problem solving as being a topic" to "problem solving as a method of learning/exploring mathematical content." At the beginning of the study, Brad seemed to clearly distinguish between "doing problem solving" on Tuesdays and Thursdays and attending to the regular seventh-grade curriculum on Mondays, Wednesdays, and Fridays. In early December, Anne noted a moment when she believed that Brad's view of problem solving had begun to evolve. One day when Anne was in his classroom, Brad told her that he was getting ready to cover area and perimeter in his textbook and wanted to know if Anne knew any good problem-solving activities through which the students could explore and review area and perimeter. Anne responded that she was delighted to hear Brad ask that question. Together they planned some activities involving Geoboards.

Later that day Anne expressed, in her journal, her excitement that Brad seemed to be beginning to view problem solving as more than a topic. At a subsequent after school meeting. Anne shared her thoughts with Brad, and he agreed that he saw more possibilities with problem solving now.

What We Have Learned About Collaborative Research

When Brad and Anne sat down specifically to discuss what they had learned about collaborative research. two main issues emerged. What we talked most about were the changes in the roles that we played throughout the project and the evolution of the collaborative partnership to that of a collegial friendship.

The roles that we played went through a number of changes. Initially, Anne was the experienced researcher, the "expert" teacher of problem solving, and the resource provider. Brad was the seventh-grade mathematics expert, the classroom disciplinarian, and the classroom decision maker. When teaching, Anne typically, although it had not been specifically planned this way, started off the lessons while Brad began the whole-class discussion of problem solutions.

As the weeks progressed, Brad was offering suggestions for ways to gather data. For example, it was Brad who initially suggested we ask students to write the first reflection on strategies. Anne began to take a more active role in classroom management during the lessons. In addition, the two began to naturally trade off the role of the lesson motivator and the discussion leader. Never did we explicitly discuss the roles we played. Rather, as we became more comfortable with each other and more sure of our approach to implementing problem solving in Brad's classroom, we "assumed roles" less and less. It no longer felt like we had distinct duties within the partnership. Instead, everything became a joint duty.

We contend that the initial feeling of playing distinct roles arose because each member of the collaborative team was a supposed "expert" in certain areas and yet at the same time neither person wanted to inhibit the other partner by making him/her feel his/her opinion or perspective was not equally valued in all phases of the collaboration. The mathematics classroom setting may more vividly present images of "experts" because mathematics is an area in which many people feel insecure. In our case, Brad expressed at times that he felt more comfortable with letting Anne conduct the question and answer time of the lesson because he felt she could see more mathematical connections and perhaps respond to students' solutions more effectively.

Anne, on the other hand, admitted to feeling a bit nervous about being considered "the expert" in teaching mathematical problem solving. She wrote this entry in August, 1994:

> *I feel very confident about my ability to engage students in problem solving. However, a part of me is little nervous because I'm supposed to be the "expert." I don't want to let Brad down by teaching bad lessons. Of course, that would just go to show that even the experts struggle with the teaching of mathematics, just like all other mathematics teachers. I will rely on Brad to help me get to know the students and his strategy for classroom management. Seventh graders can be intimidating.*

Recall that Anne and Brad met during a graduate course in which Anne was teaching about "teaching mathematics via problem solving." How easy it is to talk about what can be done in mathematics classrooms! But what a different feeling it is when you are put in a situation where you are to "practice what you preach!" Anne had some initial insecurities, such as, "what if I am not an effective model teacher for Brad?" and "what if the students' mathematical problem solving abilities don't improve?"

There may be expectations associated with "being good at mathematics" and "being a good mathematics teacher" that put additional pressures on collaborative partners who conduct action research in mathematics classrooms. These pressures may result in the partners automatically taking on specific roles in the beginning of a joint teaching-research venture.

In addition to changing roles. Anne and Brad noted changes in the partnership from that of a collaborative relationship to a collegial friendship. At the beginning, even though Anne had told Brad to call her by her first name, Brad frequently called her "Dr. Raymond." Eventually Brad arrived to the point where he laughingly referred to Anne as "Raymond." Finally, Brad came to a point where it seemed natural to call her "Anne," although at that point Anne was occasionally calling Brad "Hamersley." It may seem insignificant to talk about how the partners addressed each other. However, the "titles" we give to one another often perpetuate our differences rather than emphasizing what we have in common. As the project has evolved, we have found that we have a lot in common in terms of having similar philosophies about teaching mathematics and in getting a "thrill" out of discovering a mathematical connection between problems.

The development of the friendship allowed the partners to concentrate solely on the problem solving going on in the classroom since we were no longer concerned about "other issues" that may have inhibited us at the beginning. We both believe we have learned a lot from each other and are both better teachers and researchers as a result of having participated in this collaborative effort. The latter is probably the most personally satisfying result of this study.

Discussion

In this study of the collaborative implementation and documentation of a problem-solving curriculum in a seventh-grade mathematics classroom, we found significant changes in students' attitudes, beliefs, knowledge, and abilities in problem solving. We also noted a number of changes in Brad's teaching of mathematical problem solving. The changes occurred relatively quickly and validate Cardelle-Elawar's (1993) claim that teacher-inspired action research has the potential to result in immediate classroom reform and Miller & Pine's (1990) contention that action research can enhance professional development.

Results of this study show that students had acquired more positive attitudes and beliefs about problem solving and their abilities to engage in problem solving successfully. These findings indicate that students enjoy problem solving more and believe they can do problem solving when they are exposed to specific strategies by which to solve problems. Thus, the project made positive steps towards meeting the standard on problem solving described in the *Standards* (NCTM, 1989).

The study results also indicate that Brad had made progress in his own teaching and was thinking about teaching in the spirit of the Professional Teaching Standards (NCTM, 1991). He made physical changes in his classroom to foster group learning, by replacing desks with tables, and individual learning, by providing learning centers for his students. In addition, Brad's questioning techniques broadened, including more open-ended questions, as did his vision of problem solving, which grew to mean more than problem solving as a topic.

The experiences with both the collaborative teaching and collaborative research described by the partners in this study showed that unlike the problems with role ambiguity experienced by the mathematics teachers in Fresko, Ben-Chaim, & Miriam's (1994) study, the roles, initially, were too distinctive. However, as the project progressed, the roles became less clearly defined and, rather than being a problem, the ambiguity enhanced the overall interaction between the two teacher-researchers.

It is clear to the collaborative partners in this study that the experience of teaching and investigating as a team has been positive. Both of us feel we have grown professionally and have already made significant changes in our respective classrooms. We plan to continue to examine this seventh-grade mathematics classroom as well our own working relationship in an effort to further improve in the areas of teaching, research, and personal reflection.

References

Calhoun, E.F., & Glickman, C. D. (April, 1993). *Issues and dilemmas of action research in the league of professional schools.* A paper presented at the Annual Meeting of the American Educational Research Association, Atlanta, GA.

Cardelle-Elawar, M. (1993). The teacher as research in the classroom. *Action in Teacher Education, 15*(1), 49-57.

Charles, R. & Lester, F. (1982). *Teaching problem solving: What, why & how.* Palo-Alto, CA: Dale-Seymour Publications.

Charles, R. I., Mason, R. P., Nofsinger, J. M., & White, C. A. (1985). *Problem solving experiences in mathematics: Grade 7.* Menlo Park, CA: Addison-Wesley Publishing Company.

Clift, R., Veal, M. L., Johnson, M. & Holland, P. (1988). Restructuring teacher education through collaborative action research. *Journal of Teacher Education, 41*(2), 52-62.

Fresko, B., Ben-Chaim, D., & Carmeli, M. (1994). Consultant as co-teacher: An intervention for improving mathematics instruction. *Journal of Education for Teaching, 20* (1), 83-96.

Fullan, M. (1993). *Change forces: Probing the depths of education reform.* New York: The Falmer Press.

Gorman, J. (1990). *A packet of problems to ponder: A resource book of process problems for students in grades 4-6.* Bloomington, Indiana: Mathematics Education Development Center, Indiana University.

Hubbard, R. S, & Power, B. M. (1993). *The art of classroom inquiry: A handbook for teacher-researchers*, Portsmouth, NH: Heinemann Educational Books, Inc.

Lieberman, A. (1986). Collaborative research: Working with, not working on. *Educational Leadership, 43* (5), 28-32.

Miller, D. M. & Pine, G. J. (1990). Advancing professional inquiry for educational improvement through action search. *Journal of Staff Development, 2*(3), 56-61.

Moretti, G., Stephen, M., Goodnow, J.. & Hoogeboom, S. (1987). *The problem solver 6: Activities for learning problem-solving strategies.* Mountain View, CA: Creative Publications.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics.* Reston, VA: NCTM.

Raymond, A. M. (1994). Collaborative action research in mathematics education: A tale of two teacher-researchers. In D. Kirshner (Ed.) *Proceedings of the Sixteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, vol. 2, pp. 67-73, Louisiana State University: Baton Rouge.

Appendix A

Students' Beliefs About Problem Solving

For each of the following, circle the response that best describes your feelings about mathematical problem solving. SA means you strongly agree, A means you agree, U means you are uncertain, D means you disagree, and SD means you strongly disagree.

	(circle one)				
1. I like mathematical problem solving	SA	A	U	D	SD
2. I can do mathematical problem solving	SA	A	U	D	SD
3. We do a lot of problem solving at school	SA	A	U	D	SD
4. I am not very confident about problem solving	SA	A	U	D	SD
5. There is more than one right way to solve story problems	SA	A	U	D	SD
6. It is best to work by yourself on problem solving	SA	A	U	D	SD
7. Boys are better at problem solving than girls are	SA	A	U	D	SD
8. Problem solving is easy for me	SA	A	U	D	SD
9. In problem solving, getting the right answer is most important	SA	A	U	D	SD
10. Problem solving is important to learn	SA	A	U	D	SD
11. Working with a partner is helpful in problem solving	SA	A	U	D	SD
12. Some people just can't do problem solving	SA	A	U	D	SD
13. I know a lot about problem solving	SA	A	U	D	SD
14. It is important to be able to solve problems quickly	SA	A	U	D	SD
15. There is one right answer to story problems	SA	A	U	D	SD
16. I can tell when I have solved a problem correctly	SA	A	U	D	SD
17. Problem solving is difficult for most people	SA	A	U	D	SD
18. I would like to be a better problem solver	SA	A	U	D	SD

Select one of your answers above and explain why you feel the way you do:

Name: ______________________________ Date: ___________

Provide a short answer to each of the following questions.

1. What do you think "problem solving" in math class is?

2. Problem solving in math makes me feel ...

3. When I'm given a story problem in math class, the first thing I do to solve the problem is ...

4. Some problem solving strategies I know are ...

5. How do you know when you've solved a math story problem correctly?

6. Why do you think it is important to learn about problem solving in mathematics?

Appendix B

Students' Beliefs About Problem Solving

For each of the following, circle the response that best describes your feelings about mathematical problem solving. SA means you strongly agree, A means you agree, U means you are uncertain, D means you disagree, and SD means you strongly disagree.

	(circle one)				
1. I like mathematical problem solving	SA	A	U	D	SD
2. I can do mathematical problem solving	SA	A	U	D	SD
3. We do a lot of problem solving at school	SA	A	U	D	SD
4. I am not very confident about problem solving	SA	A	U	D	SD
5. There is more than one right way to solve story problems	SA	A	U	D	SD
6. It is best to work by yourself on problem solving	SA	A	U	D	SD
8. Problem solving is easy for me	SA	A	U	D	SD
9. In problem solving, getting the right answer is most important	SA	A	U	D	SD
10. Problem solving is important to learn	SA	A	U	D	SD
11. Working with a partner is helpful in problem solving	SA	A	U	D	SD
12. Some people just can't do problem solving	SA	A	U	D	SD
13. I know a lot about problem solving	SA	A	U	D	SD
14. It is important to be able to solve problems quickly	SA	A	U	D	SD
15. There is one right answer to story problems	SA	A	U	D	SD
16. I can tell when I have solved a problem correctly	SA	A	U	D	SD
17. Problem solving is difficult for most people	SA	A	U	D	SD
18. I would like to be a better problem solver	SA	A	U	D	SD
19. I like the challenge of an unfamilar problem even though I may not know how to solve it.	SA	A	U	D	SD
20. I enjoy writing about my solutions to problems	SA	A	U	D	SD

Select one of your answers above and explain why you feel the way you do:

Name: ____________________________ Date: __________

Provide a short answer to each of the following questions.

1. How would you define problem solving?

2. Has your attitude toward problem solving changed since the last survey? If so, how?

3. Has your confidence in your ability to solve problems changed since the last survey? If so, how?

4. When I'm given a story problem in math class, the first thing I do to solve the problem is ...

5. Problem solving strategies I know are ...

6. For each of the strategies listed above, name a specific problem you solved by using that strategy (if you can remember one).

7. How do you determine whether your solution to a problem answers the question?

Appendix C

Problems Solved By The Students

The Well Problem

One Thursday, a frog fell to the bottom of a well, which was 10 feet deep. In trying to climb out he would climb up 5 feet during the day and slip back down 3 feet at night. On which day did the frog get out of the well?

The Zink-Zak Problem

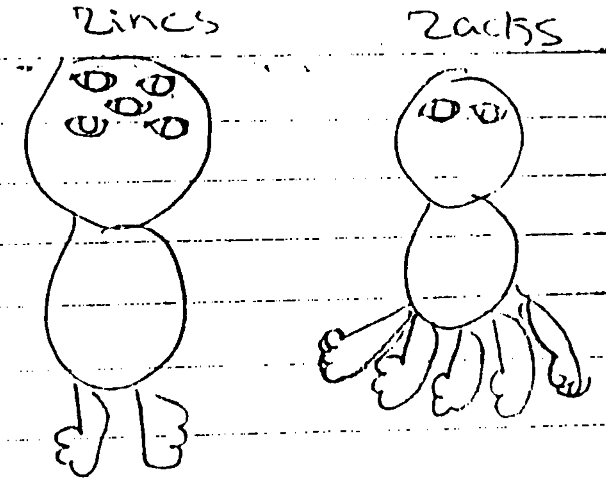
My spaceship landed on the long-lost planet of Zink-Zak, home of the Zinks and Zacks. A Zink has 5 eyes and 2 feet while a Zak has 2 eyes and 5 feet. As I got out of ;my spaceship, I was immediately surrounded by 8 of the aliens. I counted 31 eyes and 25 feet. How many of the creatures were Zinks and how many were Zaks?

The Math Convention

Some mathematicinas were having a convention. Th first time there was a knock at the door, 1 mathematician entered. On each of the following knocks, a group of mathematicians entered that had 2 more mathematicians in it than the group that entered on the previous knock. On the tenth knock, all of the mathematicians had entered the convention hall. How many mathematicians were at the convention?

Figure 1

3.875
8 | 31
24
70 wrong
64
60
56
40
-40
0



zincs

eyes 5 feet 2
x5 x5
25 eyes 10 feet

zacs

2 5
x 3 x 3
6 eyes 15 feet

Zincs 25 eyes
Zacs 6 eyes
31 eyes

Zincs 10 feet
Zacs 15 feet
25 feet

5 were Zincs
3 were Zacs

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Figure 2

Problem Solving Answer Sheet

Name(s): [redacted]

Date: Sept. 13th

Name of Problem: Zincs & Zacs

Great Work! Can I have this?

What key information did you FIND OUT?

How many were zincs and how many were zacs.

What STRATEGY did you choose?

We chose to multiply 5 by the number of eyes and feet.

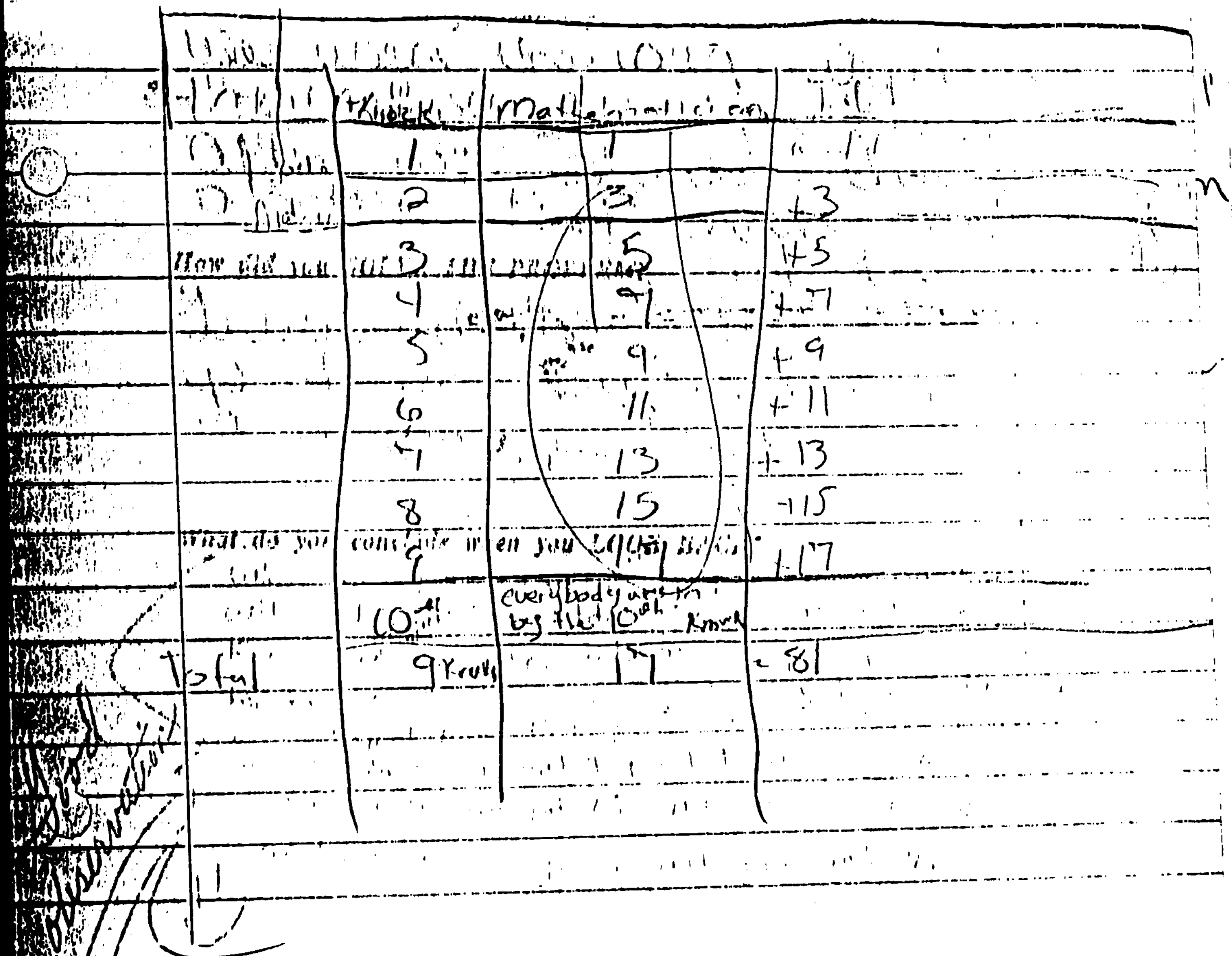
How did you SOLVE THE PROBLEM?

We multiplied the numbers 5 and 3 with number of eyes and feet. We then added the zincs total and zacs total.

What do you conclude when you LOOK BACK?

It wasn't as hard to solve as what we thought.

Figure 3



	Knocks	Mathematician	
	1	1	
	2	3	+3
How did you	3	5	+5
	4	7	+7
	5	9	+9
	6	11	+11
	7	13	+13
	8	15	+15
What do you conclude when you look back	9	17	+17
	10th	everybody art in by the 10th knock	
Total	9 knocks	17	= 81

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Figure 4

Problem Solving Answer Sheet

Name(s): [redacted]

Date: Oct 18

Name of Problem: The Math Convention

What key information did you FIND OUT?

The first knock there was 1 mathmaticians.
Every knock 2 more mathmatician came in.
There was only 10 knocks.

What STRATEGY did you choose?

We used the organized list to solve our problem. After we started the problem on the list than the list did the work for us

How did you SOLVE THE PROBLEM?

I solved the problem by making a organized list.

What do you conclude when you LOOK BACK?

When we lookback, the first time we come up with 81 becouse we added 2 everytime to the previous Knock and on the problem it said that all the people had to be in the convention by the 10th Knock. on the 1 first knock they got 1 second we got 3 third we got 5 fourth we got 7 fifth 9 sixth we got 11 seventh we got 13, eighth we got 15, ninth we got 17, then you add them all together and you get 81.

Good explanation!!