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ABSTRACT

Items with the highest discrimination parameter values in a logistic item response theory (IRT) model do not necessarily give maximum information. This paper shows which discrimination parameter values (as a function of the guessing parameter and the distance between person ability and item difficulty) give maximum information for the three-parameter logistic IRT model. The optimal discrimination parameter value is shown to be inversely related to the distance between item difficulty and person ability. An upper bound for the information as a function of these parameters is derived; and this upper bound is used to formulate a fast item selection algorithm for adaptive testing. In a small simulation study this algorithm was one and one half to six times as fast as an algorithm in which the information of all items in an item bank is calculated. (Contains 1 table, 6 figures, and 9 references.) (Author/SLD)

A Simple and Fast Item Selection Procedure for Adaptive Testing

Research Report 94-13

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Abstract

Items with the highest discrimination parameter values in a logistic item response theory model do not necessarily give maximum information. In this paper it is derived which discrimination parameter values as a function of the guessing parameter and the distance between person ability and item difficulty, give maximum information for the three-parameter logistic item response theory model. An upperbound for the information as a function of these parameters is derived. This upperbound for the information function is used to formulate a fast item selection algorithm for adaptive testing. In a small simulation study this algorithm was one and a half to six times as fast as an algorithm in which the information of all items in an item bank is calculated.

Key words: adaptive testing, attenuation paradox, item selection, information function, discrimination parameter, logistic IRT model

A Simple and Fast Item Selection Procedure for Adaptive Testing

One of the major features of the information function in item response theory (IRT) is that it can be used for the selection of items from item banks. This can be done sequentially during test adminstration as is the case in computerized adaptive testing (CAT) (Lord, 1980; Wainer, Dorans, Flaugher, Green, Mislevy, Steinberg, & Thissen, 1990).

Lord (1977) proposed a maximum information selection criterion for adaptive testing. For the two- and three-parameter logistic (2PL and 3PL) IRT models it can be inferred that an increase of the item discrimination parameter a_i will lead to an increase of information. Lord (1980, Eq. 10-6) showed that for the 2PL and 3PL models the maximum obtainable item information is an increasing function of the squared item discrimination parameter as long as item difficulty b_i and person ability θ are optimally matched. For the 2PL model, maximum information is obtained when item difficulty is equal to person ability. It can also be shown that the area under the item information function for the 2PL model is equal to the discrimination parameter. For the three-parameter model, a similar relation can be found (see Birnbaum, 1968, Eq. 20.4.26).

Insert Figure 1 about here

Figure 1 shows item information functions on a $(\theta\text{-}b_i)$-scale for different values of the discrimination parameter a_i for the 2PL model. It can be seen that increasing the value of the discrimination parameter will lead to a higher but also more peaked information function. This phenomenon shows that the area under the information function is concentrated in a smaller range of ability values, i.e., the width of the information function becomes smaller as the discrimination parameter increases.

Samejima (1994) has shown that the area under the square root of the information function for the 2PL model is equal to π ($\pi \approx 3.14$), irrespective of the value of the discrimination parameter. This implies that in the 2PL model the information functions of two items must cross at least once. For reasons of symmetry, the information functions of two items with the same difficulty parameter, but with different discrimination parameters, must cross twice. This fact is shown in Figure 1.

Figure 1 also shows that, when item difficulty is not equal to person ability, an extreme increase of item discrimination may lead to a decrease of item information. This effect has been referred to as the attenuation paradox in IRT by Lord and Novick (1968, p. 368) and Birnbaum (1968, p. 465).

Insert Figure 2 about here

Figure 2 depicts item information for the 2PL model as a function of item discrimination for different values of the distance between person ability and item difficulty. The well-known fact that an item with a high discrimination parameter is not necessarily the most informative item, and that, therefore, selection of items in an adaptive test should not solely be based on the discrimination parameter, can also be seen in Figure 2.

In this paper, it is shown which discrimination parameter values give maximum information, and how high this maximum information is. Both the optimal discrimination as well as the maximum attainable information are functions of the distance between item difficulty and person ability for logistic IRT models. The results of this paper are implemented in an item selection algorithm for adaptive testing, and a small simulation study will show that this algorithm will improve item selection.

Derivation of Optimal Item Discrimination Parameter Values

In this section, it is determined which value of the item discrimination parameter is the most informative one, given certain fixed values of the other item parameters and the person ability parameter for the 3PL IRT model. The corresponding maximum information values will also be given. The 2PL model is a special case of the 3PL model, and the results will therefore also hold for the 2PL model.

The item characteristic curve of the 3PL IRT model is (Lord, 1980, Eq. 4-37):

$$P_i(\theta) = c_i + (1-c_i)\frac{e^{L_i}}{1+e^{L_i}}, \tag{1}$$

where $L_i = a_i(\theta - b_i)$, and $a_i \in \mathbf{R}^+$, $b_i \in \mathbf{R}$ and $c_i \in [0,1)$ are the discrimination, difficulty and guessing parameter, respectively, and $\theta \in \mathbf{R}$ is the ability parameter. $\mathbf{R}$ and $\mathbf{R}^+$ are sets of real and positive real numbers, respectively. In Equation 1, $P_i(\theta)$ denotes the probability of a correct response to item i for a person with ability θ. The corresponding item information function is given by:

$$I_i(\theta) = \frac{a_i^2(1-c_i)}{\left(c_i+e^{L_i}\right)\left(1+e^{-L_i}\right)^2} \tag{2}$$

(see e.g. Lord, 1980, Eq. 4-43). The value of a_i for which a maximum information function value is reached for fixed values of c_i and $(\theta\text{-}b_i)$ is found by setting the derivative of the natural logarithm of the information function with respect to a_i equal to zero, i.e.:

$$\frac{\partial \log[I_i(\theta)]}{\partial a_i} = \frac{2}{a_i} - \frac{e^{L_i}(\theta-b_i)}{c_i+e^{L_i}} + 2\frac{e^{-L_i}(\theta-b_i)}{1+e^{-L_i}} = 0\,. \tag{3}$$

After some elementary operations, this equation can be reduced to:

$$2c_i(1+L_i) + (2(c_i+1)+L_i)e^{L_i} + (2-L_i)e^{2L_i} = 0\,. \tag{4}$$

Insert Table 1 about here

It can be shown that solutions of Equation 4 must lie between -6 and 3 (see the appendix for proof). Hence, Equation 4 can be solved iteratively, substituting real numbers for L_i between -6 and 3. This leads to two solutions of L_i, namely one for $(\theta\text{-}b_i) > 0$ and one for $(\theta\text{-}b_i) < 0$. These two optimal values of L_i are given in Table 1 for c_i-values ranging from 0.0 to 0.9 with steps 0.1. From these optimal L_i-values the corresponding optimal a_i-values can also be derived. If, for example, c_i = 0.1 and $(\theta\text{-}b_i)$ = -2, then the optimal a_i-value will be -1.816 / (-2) = 0.908. This value is depicted as a cross in Figure 3. The optimal a_i-values for c_i = 0 and for c_i = 0.9 are shown as functions of $(\theta\text{-}b_i)$ in Figure 3. All lines for $0 < c_i < 0.9$ lie between the line of $c_i = 0$ and the line of $c_i = 0.9$.

Insert Figure 3 about here

The values of $I_i(\theta)(\theta\text{-}b_i)^2$ in Table 1 can be obtained by substituting the values for c_i and the optimal values for a_i and L_i in equation (2). An upperbound for $I_i(\theta)$ can be found by dividing $I_i(\theta)(\theta\text{-}b_i)^2$ by $(\theta\text{-}b_i)^2$. As an example, for items with c_i = 0.1, the information at ability level $\theta < b_i$ can be at most:

$$\frac{[-1.816/(\theta-b_i)]^2}{e^{-1.816}(1+e^{1.816})^2} < \frac{0.222}{(\theta-b_i)^2} \ . \tag{5}$$

This fact means that the information for items with c_i = 0.1 and $(\theta\text{-}b_i)$ = -2 is always less than 0.055. This value is shown as a cross in Figure 4, together with the upperbound information functions, for two different values of c_i, namely c_i = 0 and c_i = 0.6. Similar lines can be drawn for $0 < c_i < 0.6$ and these lines all lie between the two previous lines in Figure 4. For $c_i > 0.6$, the lines lie below the line of c_i = 0.6.

Insert Figure 4 about here

The results in Figure 4 and Table 1 show that three factors determine the value of the upperbound information, namely $|\theta - b_i|$, sign(θ-b_i) and c_i. The second factor denotes whether (θ-b_i) is greater than or less than 0. Keeping the other factors constant, the maximum information is a decreasing function of both $|\theta - b_i|$ and c_i, and it is higher for $(\theta - b_i) > 0$ than for $(\theta - b_i) < 0$.

For example, for items with $c_i > 0.1$ and $(\theta - b_i) < -2$, the maximum information is less than the maximum information for items with $c_i = 0.1$ and $(\theta - b_i) = -2$, which in turn are less informative than items with $c_i = 0.1$ and $(\theta - b_i) = 2$. As a consequence, when we have items with guessing parameter values $c_i \geq 0.1$ which are more difficult than a person's ability by at least two units, i.e. $(\theta - b_i) \leq -2$, then their information $I_i(\theta) \leq 0.055$. For items with the same guessing parameter values, but with $(\theta - b_i) \geq 2$, the information will be $I_i(\theta) \leq 0.392/4 = 0.098$.

Insert Figure 5 about here

Finally, in Figure 5 the upperbound information times the squared distance between person ability θ and item difficulty b is shown as a function of the guessing parameter. These lines are based on the results given in Table 1.

An Item Selection Algorithm

The upperbounds for the information derived in the previous section can be used to improve item selection for adaptive testing. Suppose that we want to select items from an item bank in an adaptive test with Lord's (1977) maximum information criterion. This criterion selects the item which has the highest value of the information function at a certain value θ_0 on the ability scale, usually some provisional ability estimate. An heuristic to select the maximum informative item at θ_0 is to calculate the information of all items for this value θ_0. It is not necessary, however, to compute the information of all items in an item bank to determine which one has maximum information.

This can be seen as follows. Let c_{min} be the smallest guessing parameter value among all items in the itembank. Let $I_{max,+}(c_{min})$ and $I_{max,-}(c_{min})$ be the upperbounds of $I_i(\theta_0)(\theta_0-b_i)^2$ for positive and negative values of (θ_0-b_i), respectivel. These upperbounds can be found in Table 1 under the heading $I_i(\theta_0)(\theta_0-b_i)^2$ for values of c_{min} under the heading c_i. So, for example, $I_{max,+}(0.1) = 0.392$ and $I_{max,-}(0.1) = 0.222$. In the following I_{max} will either be $I_{max,+}(c_{min})$ or $I_{max,-}(c_{min})$.

The following result for two items i and j can be derived:

$$\text{if } (\theta_0 - b_i)^2 > \frac{I_{\max}}{I_j(\theta_0)}, \quad \text{then } I_i(\theta_0) < I_j(\theta_0). \tag{6}$$

This follows from:

$$(\theta_0 - b_i)^2 > \frac{I_{\max}}{I_j(\theta_0)} \quad \Leftrightarrow \quad I_j(\theta_0) > \frac{I_{\max}}{(\theta_0 - b_i)^2} \tag{7}$$

and

$$I_i(\theta_0)\,(\theta_0 - b_i)^2 < I_{\max} \quad \Leftrightarrow \quad I_i(\theta_0) < \frac{I_{\max}}{(\theta_0 - b_i)^2}. \tag{8}$$

The left hand side of Equation 8 follows from the definition of I_{max} as an upperbound of $I_i(\theta_0)(\theta_0\text{-}b_i)^2$.

So, as soon as it is determined that a certain item j has a certain information $I_j(\theta_0)$, then all items i with either positive $\theta_0\text{-}b_i$ and $(\theta_0\text{-}b_i)^2 >$ $I_{max,+}(c_{min})/I_j(\theta_0)$, or negative $\theta_0\text{-}b_i$ and $(\theta_0\text{-}b_i)^2 > I_{max,-}(c_{min})/I_j(\theta_0)$ will have less information than item j, no matter what values for a_i and $c_i \geq c_{min}$ are encountered.

<u>The algorithm</u>

The algorithm has the following initialisation steps:

1. Order the N items in the itembank according to their difficulties:

   $b_1 < b_2 < \ldots < b_N$.
2. Determine the smallest guessing parameter value in the itembank: c_{min}.
3. Compute the constants $I_{max,+}(c_{min})$ and $I_{max,-}(c_{min})$.

If θ_0 is an provisional ability estimate after the administration of a set of items in an adaptive test, then the selection of the next item with maximum information on θ_0 from the set of items in the itembank not already included in the test, consists of the following steps:

Step 1: Search for item j with difficulty value equal to the smallest positive difference $(\theta_0 - b_j)$ among all items.
If $\theta_0 < b_1$ or $\theta_0 > b_N$ item j is the easiest respectively the most difficult item.

Step 2: Search through the itembank for items i in increasing order of $|\theta_0 - b_i|$, for positive values of $(\theta_0 - b_i)$.
If $(\theta_0 - b_i)^2 > I_{max,+}(c_{min})/I_j(\theta_0)$, then go to Step 3.
Otherwise, compute $I_i(\theta)$, if $I_i(\theta_0) > I_j(\theta_0)$ set $j=i$, and continue the search.

Step 3: Search through the itembank for items i in increasing order of $|\theta_0 - b_i|$, for negative values of $(\theta_0 - b_i)$.
If $(\theta_0 - b_i)^2 > I_{max,-}(c_{min})/I_j(\theta_0)$ then stop searching.
Otherwise, compute $I_i(\theta)$, if $I_i(\theta_0) > I_j(\theta_0)$ set $j=i$, and continue the search.

Step 1 can be sped up somewhat by starting the search for item j with an item that is expected to have a difficulty close to the difficulty of item j. Note that in Steps 2 and 3 the index j represents the most informative item which is eventually administered to the test-taker. The answer to this item is scored, and the test-taker's ability is re-estimated. Item j is then removed from the item bank.

In conclusion, this search process only passes through a part of the itembank, i.e. the information is computed only for items with relatively small values of $(\theta_0 - b_i)^2$. This strategy will speed up item selection considerably.

A Simulation Study

To establish the relative speed of this item selection algorithm, a simulation study was performed in which the above described algorithm was compared to the algorithm with straightforward calculation of the information of all items in an item bank. For both algorithms Lord's maximum information criterion was used to select items. This choice means that for each test exactly the same items, in the same order, were selected by the two algorithms. So, the algorithms only differed in their CPU-times. The simulations were performed using a program written in Borland Pascal 7.0, and were run on a 486DX2/66MHz computer.

Design

For seven different ability values, θ = -3, -2, -1, 0, 1, 2, and 3, respectively, adaptive tests of 30 items were simulated. These simulations were repeated for three different itembanks. Each itembank consisted of 200 items. In all banks, the distributions of the discrimination parameters and guessing parameters were uniform. The discrimination parameters were uniformly distributed between 0.5 and 2, i.e. $a_i \sim U(0.5,2)$, and the guessing parameters between 0.1 and 0.3, i.e. $c_i \sim U(0.1,0.3)$. The three itembanks differed only in their distributions of the difficulty parameters b_i. One bank had a uniform $U(-3,3)$ distribution of item difficulties, and in the other two banks the item difficulties were normally distributed, one with a variance of 1, and the other with a variance of 3. So, the distributions of item difficulties were $U(-3,3)$, $N(0,1)$, and $N(0,3)$, respectively. The number of replications for each condition, i.e., each combination of ability level and itembank, was 100.

Results

In Figure 6, the total amount of CPU-time used to select 30 items in 100 tests is shown as a function of θ for the two algorithms and the three item banks. The three lines a little above 40 seconds show the CPU-time needed to calculate the information of all items in the bank not already included in the test. The other three lines show the CPU-time needed by the proposed algorithm to select the items.

This algorithm improved item selection speed with a factor between 1.5 and 6. For a uniform distribution of the difficulty parameters the relative speed did not depend much on the ability value of the examinee, but for the item banks with normally distributed difficulties the speed did largely depend on the ability value. For $\theta = 0$ the amount of CPU-time was relatively high, because there were relatively many items with a difficulty parameter in this area. So, in these cases the information had to be computed for relatively many items.

The high CPU-times for extreme negative ability values for the $N(0,1)$-distribution of item difficulties may be explained as follows. The item difficulties are thinly spread around -3. The information of the most informative item j in the search process was usually not very high, because most of the time $(\theta_0 - b_j) < 0$, and $|\theta_0 - b_j|$ is large. So, the range of difficulties of items that can be more informative than item j is rather large. On the other hand, for extreme positive ability values the CPU-times were not that high, because the information for items j with large distances between θ_0 and b_j is higher for $(\theta_0 - b_j) > 0$, than for $(\theta_0 - b_j) < 0$. Note that for the $N(0,3)$-distribution, the b_j-values were spread out much more than for the $N(0,1)$-distribution, and that this effect did thus not occur.

Insert Figure 6 about here

Summary and Conclusion

When item difficulty and person ability are not optimally matched, the optimal discrimination parameter value in logistic item response models is not its maximum value. This fact has been referred to as the attenuation paradox in item response theory. In this paper it has been shown that the optimal discrimination parameter value is inversely related to the distance between item difficulty and person ability, and it is derived which discrimination parameter value provides maximum information at certain points on the ability scale. The corresponding maximum information is inversely related to the squared distance between person ability and item difficulty.

The relation between this distance and an upperbound on information can be used in an algorithm for the maximum information item selection criterion for adaptive testing. In a small simulation study this algorithm was 1.5-6 times as fast as a more simple and straightforward algorithm. The difference between these algorithms is that in the simple algorithm the information function values for all items is determined, and in the proposed algorithm information is calculated for a relatively small subset of these items.

It may be argued that this algorithm will not be of much use when so-called info tables (Thissen & Mislevy, 1990, pp. 116-117) are used, where the information of the items is computed in advance, i.e. before the actual testing. This is not entirely true, because the computations needed to set up an info table can also be sped up by means of the proposed algorithm. Moreover, when itembanks are frequently changed by inclusion of newly calibrated items or exclusion of rejected ones, this algorithm can be used to compute updated info tables.

Finally, Kingsbury & Zara (1989), and Stocking & Swanson (1993), among others, have described constrained adaptive testing procedures. Their procedures may take much time when a lot of constraints are incorporated in the selection process. So, it may be worthwhile to investigate the possibility of incorporating the algorithm proposed in this paper into constrained adaptive testing.

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Table 1

Optimal L_i-values and Corresponding Maxima for $I_i(\theta)(\theta-b_i)^2$
for Different c_i-values Given Fixed $(\theta-b_i)$-Values

	$(\theta-b_i) < 0$		$(\theta-b_i) > 0$	
c_i	L_i	$I_i(\theta)(\theta-b_i)^2$	L_i	$I_i(\theta)(\theta-b_i)^2$
0.0	-2.399	0.440	2.399	0.440
0.1	-1.816	0.222	2.417	0.392
0.2	-1.669	0.145	2.434	0.346
0.3	-1.591	0.101	2.451	0.300
0.4	-1.541	0.073	2.467	0.255
0.5	-1.505	0.052	2.482	0.211
0.6	-1.478	0.037	2.497	0.168
0.7	-1.457	0.025	2.512	0.125
0.8	-1.440	0.015	2.526	0.083
0.9	-1.427	0.007	2.540	0.041

Note. Values for $I_i(\theta)(\theta-b_i)^2$ were rounded upwards to 3 decimals.

Figure captions

Figure 1. Item information functions.

Figure 2. Item information as functions of the discrimination parameter.

Figure 3. Optimal a-value as a function of (θ-b).

Figure 4. Upperbound information as a function of (θ-b).

Figure 5. Upperbound information times $(\theta\text{-}b)^2$ as a function of c.

Figure 6. The total CPU-time for two Maximum Information Item Selection Algorithms for 100 30-item Adaptive Tests for three 200-item Itembanks.

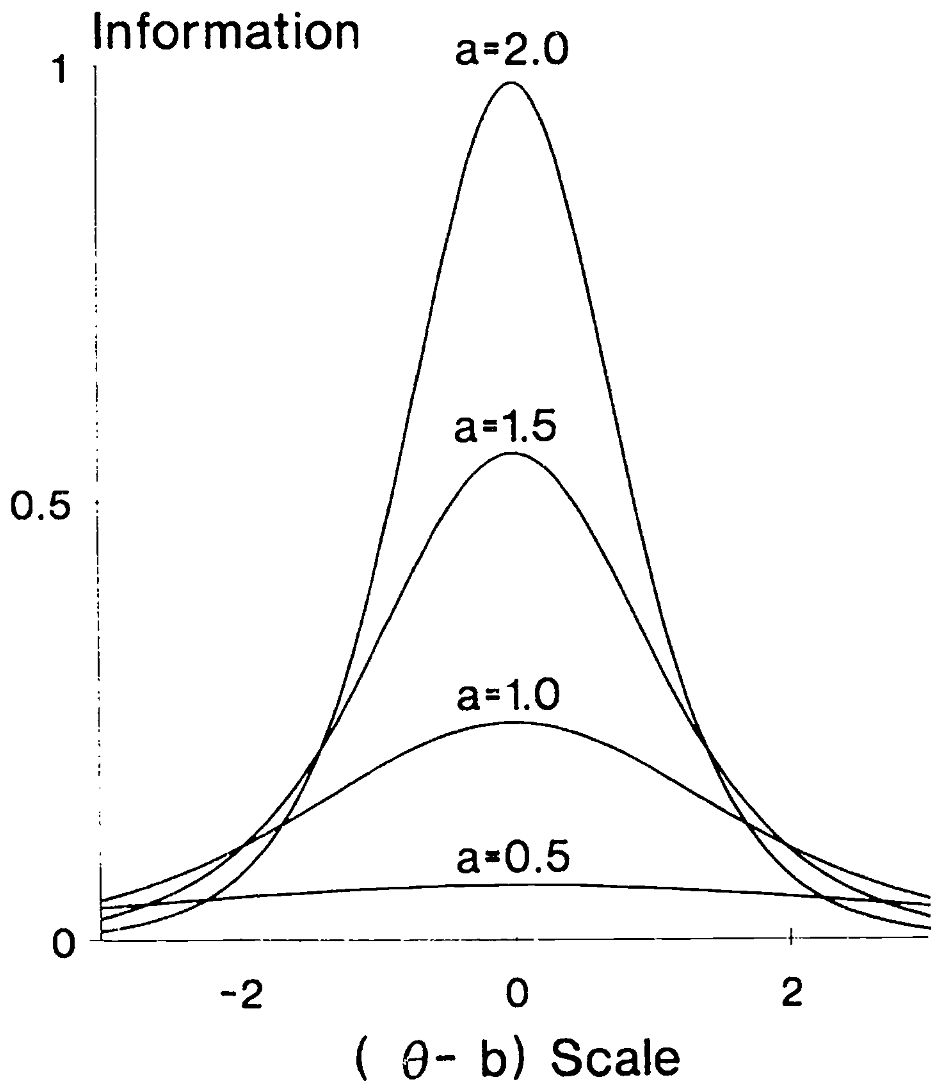

Figure 1

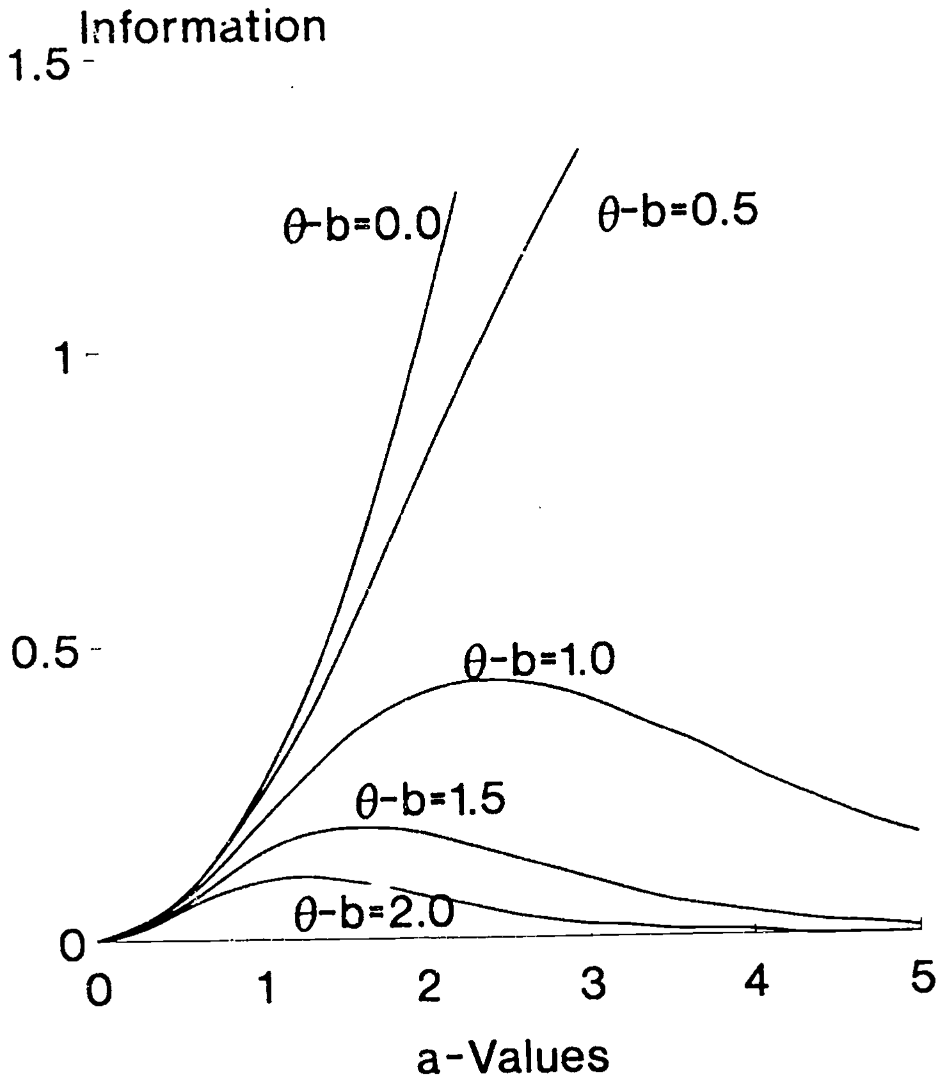
Information
1.5
1
0.5
0
θ-b=0.0
θ-b=0.5
θ-b=1.0
θ-b=1.5
θ-b=2.0
0
1
2
3
4
5
a-Values

Figure 2

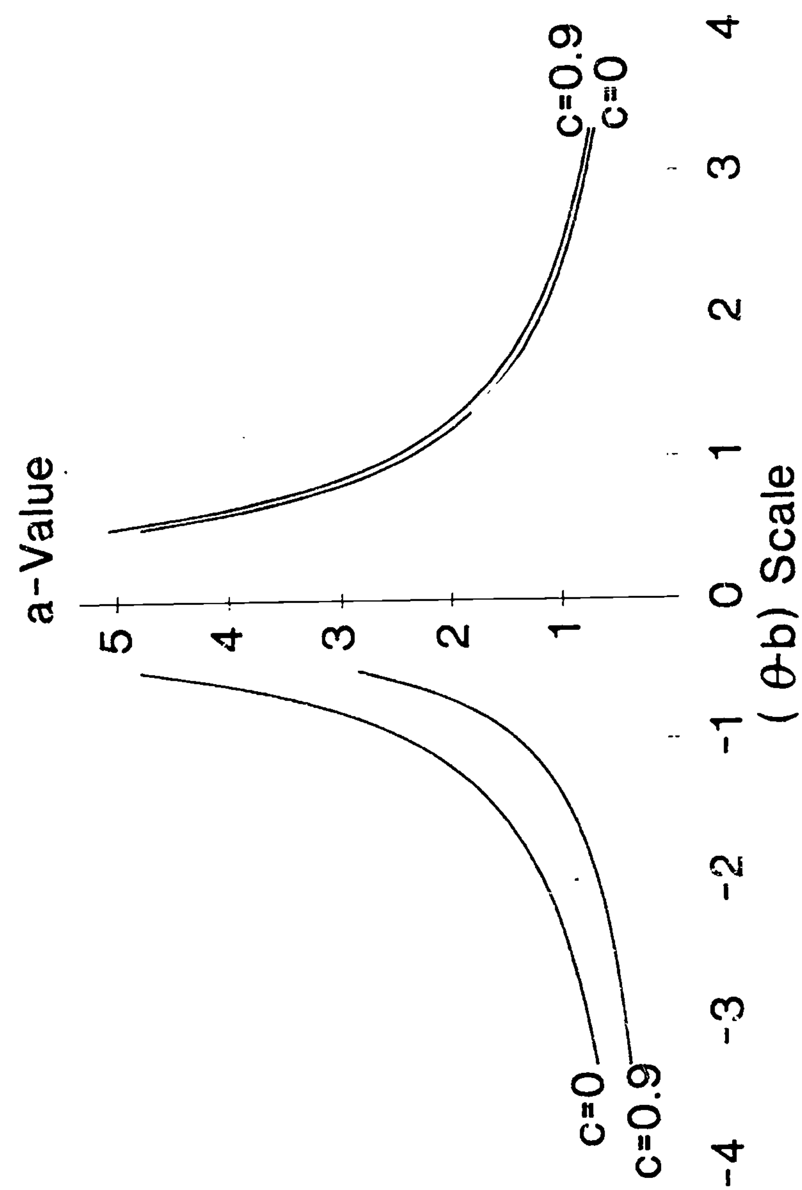

Figure 3

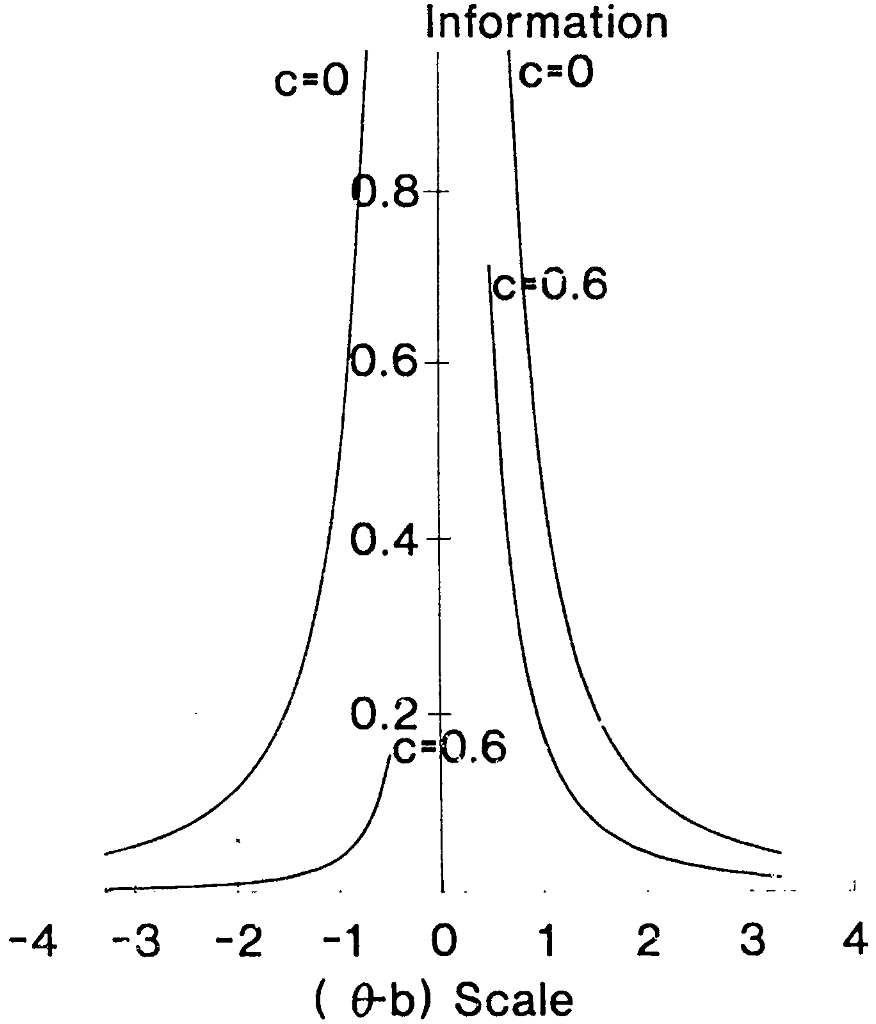

Figure 4

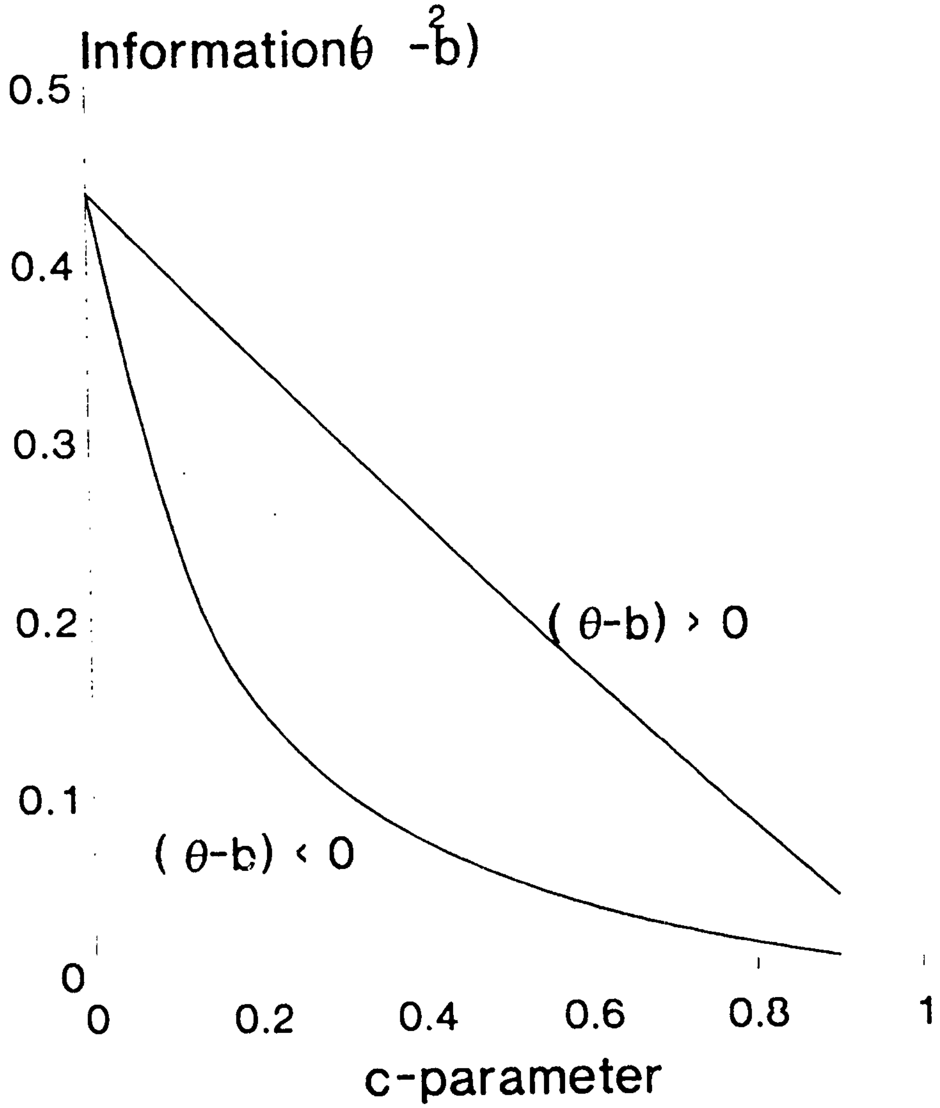
Information$(\theta-b)^2$
0.5
0.4
0.3
0.2
0.1
0
$(\theta-b) > 0$
$(\theta-b) < 0$
0
0.2
0.4
0.6
0.8
1
c-parameter

Figure 5

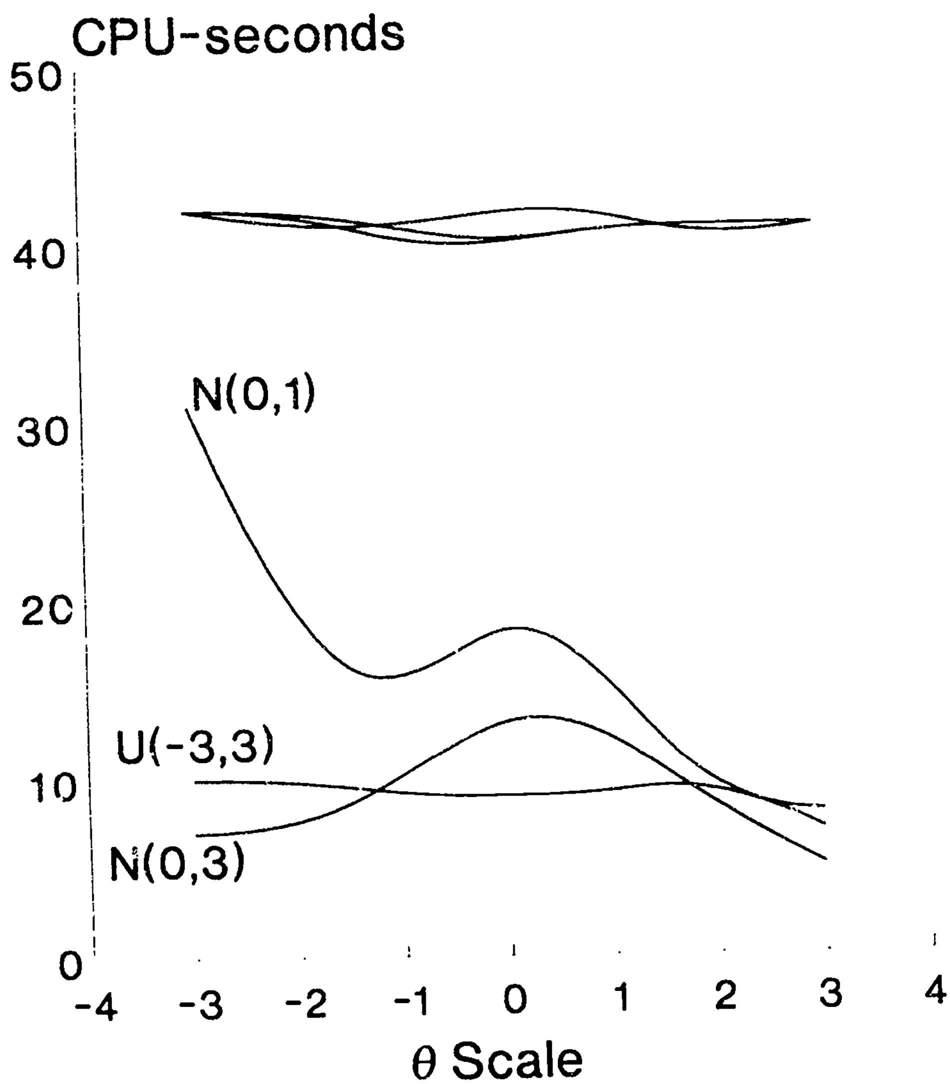

Figure 6

Appendix

<u>Proof that Solutions of Equation 4 Must Lie Between -6 and 3</u>

This proof is split in the following two parts: (1) proof that solutions of Equation 4 cannot be less than -6; and (2) proof that solutions of Equation 4 cannot be greater than 3.

<u>(1) Proof that $L_i > -6$</u>

Equation 4 can be rewritten as:

$$2c_i(1+L_i) + (2(c_i+1)+\frac{2}{3}L_i)e^{L_i} + [(2-L_i)e^{L_i}+\frac{1}{3}L_i]e^{L_i} = 0. \qquad (9)$$

The left part of Equation 9 is negative if $L_i < -6$, because it consists of three parts, which are all negative if $L_i < -6$:

(a) the first part, $2c_i(1+L_i)$, is less than or equal to 0 if $L_i < -1$. So it is also less than 0 if $L_i < -6$;

(b) the second part, $(2(c_i+1)+\frac{2}{3}L_i)e^{L_i}$ is less than 0 if $L_i < -6$, because $2(c_i+1)+\frac{2}{3}L_i<0 \Leftrightarrow L_i<-3(c_i+1)$, and $-3(c_i+1) > -6$, because $c_i < 1$; and

(c) the third part, $[(2-L_i)e^{L_i}+\frac{1}{3}L_i]e^{L_i}$ is less than 0 if $L_i < -6$, because e^{L_i} is positive and $[(2-L_i)e^{L_i}+\frac{1}{3}L_i]$ is negative if $L_i < -6$; $[(2-L_i)e^{L_i}+\frac{1}{3}L_i]$ is negative because it is an increasing function of L_i for $L_i < 1$, which is negative for $L_i = -6$.

(2) Proof that $L_i < 3$

Equation 4 can be rewritten as:

$$L_i e^{2L_i}\left[\frac{2c_i}{L_i e^{2L_i}} + \frac{2c_i}{e^{2L_i}} + \frac{2(c_i+1)}{L_i e^{L_i}} + \frac{1}{e^{L_i}} + \frac{2}{L_i} - 1\right] = 0. \qquad (10)$$

The part between brackets in (10) is a decreasing function of L_i for $L_i > 0$, and it can be found by substition that for $L_i = 3$ this part is smaller than 0. So, the part between brackets is negative if $L_i > 3$. Combining this result with $L_i e^{2L_i} > 0$ if $L_i > 3$ completes the proof that the left part of Equation 10 is negative if $L_i > 3$.

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