DOCUMENT RESUME

ED 391 660 SE 057 607

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TITLE	Reconstructing the Whole: A Gauge of Fraction Understanding.
PUB DATE	Oct 95
NOTE	9p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 1995).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Cognitive Processes; *Context Effect; Elementary Secondary Education; *Fractions; Grade 4; *Graphs; Higher Education; Intermediate Grades; Interviews; Mathematics Tests; *Misconceptions

ABSTRACT

This study analyzed graphical solutions of 344 students in grades 4 through college who were administered a 40-item assessment of basic fraction concepts. Of particular interest were six problems that required students to complete the whole given a fractional part presented in area contexts and set contexts. Results indicated that fourth graders frequently used the "doubling" strategy even if the fractional part showed two-thirds. In the set context, students sometimes connected the outer dots to form a rectangular unit that was totally unrelated to the given fractional part. Videotaped interviews of six fourth-grade students clearly revealed these dominant strategies. Nevertheless, there was improvement in student solution strategies as well as success in completing the unit as the grade level increased. Implied misconceptions held by both preservice teachers and elementary students include the assumption that the unit is always a regular-shaped region rather than irregular. The findings indicate the strong influence of the rectangular model often used in traditional mathematics textbooks on students' understanding of the whole. These findings support reforms in the teaching of fractions that include not only unit partitioning activities but also completing the whole using a variety of models. (Author)

RECONSTRUCTING THE WHOLE: A GAUGE OF FRACTION UNDERSTANDING*

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ABSTRACT

This study analyzed graphical solutions of 344 students (Gr 4-College) who were administered a 40-item assessment of basic fraction concepts. Of particular interest are six problems that required students to complete the whole given a fractional part presented in (a) area context (3 items) and (b) set context (3 items). Results indicated fourth-grade students frequently used the "doubling" strategy even if the fractional part showed "2/3". Othertimes, students connected the outer dots to form a rectangular unit that is totally unrelated to the given fractional part. Videotaped interviews of six grade 4 students clearly revealed these dominant strategies. Nevertheless, there was improvement in student solution strategies as well as success in completing the unit as the grade level increased. Implied misconceptions held by both preservice teachers and elementary students include the assumption that the unit is always a regular-shaped region rather than irregular. The findings indicate the strong influence of the rectangular model often used in traditional mathematics textbooks on students' understanding of the whole. They further support reforms in the teaching of fractions which include not only unit partitioning activities but also completing the whole using a variety of models.

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* A paper presented during the 17th annual meeting of the North American Chapter for the Psychology of Mathematics Education, Columbus, OH, October 21-24, 1995.

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Students' coordination of the unit shapes their ability to understand mathematical concepts such as whole-number multiplication (Steffe, 1994), ratio (Lamon, 1993) and fractions (Wanatabe, 1994). Research on the unit as the basic structure from which fraction ideas are constructed documented the importance of partitioning in developing fraction knowledge and analyzed students' mechanisms for partitioning the unit given in either continuous or discrete context. Findings revealed (a) children (age 5-8) partition a continuous whole using vertical and horizontal cuts and tend to ignore the whole when judging equality of parts (Pothier & Sawada, 1989), (b) older students (Gr 6-7) applied certain partitioning strategies based on the unit measure they have chosen to show fair sharings (Kieren, Nelson & Smith, 1985).

Of equal importance to the study of rational number acquisition is children's schemes in reconstructing the unit given a fractional part. Piaget, Inhelder & Szeminska (1960) argued that fraction understanding is incomplete if the learner is not able to reconstruct the whole. Students at this level have only acquired what Steffe and Olive (1991) refer as "pre-fraction" knowledge. Furthermore, Saenz-Ludlow (1994) hypothesized that part-to-whole and whole-to-part are inverse operations which, when fully established, can result in better understanding of fraction as a quantity.

Few clinical studies have attempted to describe students' mechanisms in unit configuration using discrete contexts. For example, Saenz-Ludlow (1994) gave Michael (Gr 3) the following problem: If two nickels equal two tenths of the total amount, how much do I have? She surmised that the student used natural number knowledge successfully to generate fraction concepts. Meanwhile, Steffe and Olive (1991) described Karla's (Gr 5) response to a problem in which six candies represent three-fourths of the whole candy bar.The student appropriately responded: There are two more in the whole bar. In the studies mentioned above, both students had strong knowledge of part-whole relationship. Studies that focus on student errors and misconceptions about fractional numbers are also needed and can provide direction for designing instruction which addresses these inadequacies (Graeber, 1992).

Research Questions

This study was conducted to formulate hypotheses about student methods in reconfiguring the unit when a fractional part is known. Specifically, the following questions guided this investigation:

1. What are the predominant solution strategies students use to reconstruct the unit given a fractional part in area or set context? Do these strategies differ according to grade level?

2. To what extent do these strategies reflect student ability to partition a unit?

3. What misconceptions could be implied from students' solution strategies?

Methodology

A 40-item fraction assessment instrument [see Taube (1993) for reliability and validity] was developed by the researcher in which seven fraction subconcepts were assessed. Six of the problems asked students to reconstruct the unit (see Appendix A) by either completing the whole on a rectangular dot paper or figuring out how many objects form the unit in the case of problems involving the set model.

The sample was selected by asking teacher volunteers to administer the test in their classes. A total of 344 testpapers were returned from three adjacent school districts in south Texas where about 90% of the students are Mexican-Americans (ESL students). Analyses of students' responses on the six items were based on the total respondents. From the total group, 260 students were selected by stratified sampling for statistical analyses (see Table 1).

Table 1
Distribution of sample included in the statistical analyses

Level	Grade	n
I (Elementary)	4 & 5	44 (76)
II (Middle sch)	6 & 7	44 (55)
III (Middle sch)	6 & 7 (GT)	43 (66)
IV (High sch)	9 & 10	43 (46)
V (Dev. math)	Freshmen	43 (49)
VI (Future teachers)	Junior & Senior	43 (52)

note. Group total in parentheses

Findings

This preliminary investigation focused on six fraction problems involving reconstructing the whole. Results from both statistical tests and analysis of graphical solutions are summarized below:

1. The mean scores on the six problems indicated the Grades 6 and 7 GT students were more successful than the secondary and developmental mathematics students (see Table 2). ANOVA test further showed significant mean differences ($p<.05$) among the six groups. Scheffe test indicated the preservice teachers differed significantly ($p<.05$) from the other students except the GT group.

Table 2
Mean scores and standard deviations on 6 items by level

	Level					
	1	2	3	4	5	6
Mean	1.52	3.29	4.56	2.53	2.78	4.25
SD	1.24	2.24	1.41	1.69	1.67	1.54

note. n=260

2. A significant positive correlation (**r = .52**) was observed between students' performance on problems presented in set and area contexts.

3. Analysis of students' solutions on each of the six problems revealed the following:

a) The fourth- and fifth-grade students basically used the doubling strategy (e.g., adding a region congruent to the given fractional part). A summary of the strategies (see Appendix A) inferred from the graphical solutions of students indicated that doubling and forming a rectangular whole were automatic responses given by the younger students. The videotaped interviews captured similar observation.

b) The numerator and denominator of the fraction appeared to be the deciding factors in determining the rectangular whole. For example, if a fractional region represented "2/3", then the student would draw a 2-by-3 rectangle to represent the whole.

c) Students who successfully completed the whole showed correct partitioning by marking equal parts on the given figure.

d) The older students frequently used mathematical algorithm when a set model was presented. This was evident in the solutions of preservice elementary teachers and the GT students as well. For example, a student wrote $\frac{6 \div 3}{15 \div 3} = \frac{2}{5}$ to determine the total number of balls (15) if 6 balls were "2/5" of the unit. Thus, the student applied the notion of equivalent fractions to figure out the discrete whole.

Hypotheses

The intent of this study was to formulate hypotheses about students' ability to traverse the conceptual bridge between part-to-whole and whole-to-part as inverse operations. Analysis of student graphical solutions on a paper-pencil assessment and individual interviews seem to support the following hypotheses:

1. Fourth- and fifth-grade students hold a preconceived notion of the whole as a rectangular shape when they are asked to reproduce the whole on a dot paper. They do not focus on the part-whole relations and seem "uneasy" with an irregular-shaped whole.

2. Students who are aware that a fractional part can be further partitioned into a specific number of subparts tend to have success in completing the whole.

3. Students tend to fall back on either the circular or rectangular model to verify or justify their responses on tasks presented in a set context.

4. Students' solution strategies in constructing the unit from a fractional part differ according to the representation mode used.

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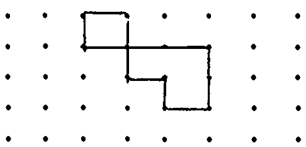
APPENDIX

Percentage of students (per level) using a specific strategy

AREA MODEL

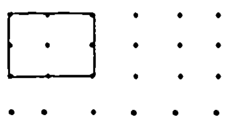
Problem 1: I am thinking of a figure.
Only 4 sixths of the figure is drawn below.
Add on to the given figure to draw the whole.



Strategy	Level 1	2	3	4	5	6
*add 2 unit squares	12%	35%	67%	35%	33%	58%
doubling	12%	0	7%	2%	4%	2%
draw square	46%	50%	23%	32%	24%	23%

* correct answer
n=344

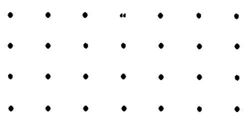
Problem 2: The picture below shows two thirds of the whole birthday cake. Draw the whole birthday cake.



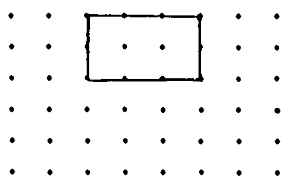
Strategy	Level 1	2	3	4	5	6
*Add a third	17%	58%	76%	44%	31%	71%
doubling	34%	14%	12%	15%	19%	11%
same size	21%	5%	0	47%	0	0
draw rectangle	20%	17%	6%	22%	30%	11%

*correct answer
n=344

Problem 3: The figure below shows part of a model for a playgound. Only 3 fourths of the playground is shown.



Draw the whole model of the playground on the space below.



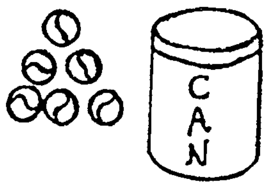
Strategy	Level 1	2	3	4	5	6
*Add a fourth	18%	55%	85%	.37%	46%	76%
doubling	23%	0	4%	8%	0	2%
same size	21%	0	0	2%	0	0
draw rectangle	34%	34%	8%	26%	35%	12%

*correct answer
n=329

SET MODEL

Problem 4:

Look at the picture below. The can holds some marbles. The 6 marbles shown are only 2 fifths of the total marbles. How many marbles are there altogether?



a) 6

★b) 15

c) 12

d) 4

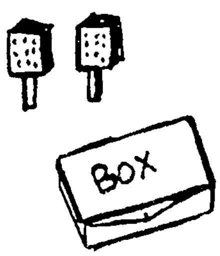
e) I don't know.

Strategy	Level 1	2	3	4	5	6
*formed 2 equal sets	26%	45%	77%	38%	39%	78%
doubling	26%	32%	15%	21%	33%	14%
same amount	32%	9%	1%	19%	6%	4%
Incorrect meaning for 2/5	8%	5%	1%	0	6%	2%

*correct answer
n= 340

Problem 5:

Below you can see 1 third of all the ice cream bars. The rest of them are in the box. How many ice cream bars are there altogether?



a) 4

b) 2

c) 3

★d) 6

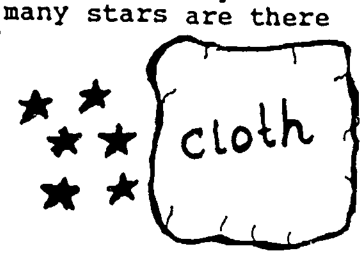
e) I don't know.

Strategy	Level 1	2	3	4	5	6
* made 2 equal sets based on 3 objects	37%	62%	88%	48%	71%	84%
make the unit	12%	13%	0	6%	8%	0
doubled	16%	11%	11%	13%	14%	16%
same set	19%	6%	6%	15%	2%	0

*correct answer
n=336

Problem 6:

Observe the picture below. It shows 2 fourths of all the stars. The others are covered by the cloth. How many stars are there altogether?



a) 6

★b) 12

c) 8

d) 14

e) I don't know.

Strategy	Level 1	2	3	4	5	6
*doubled	38%	60%	79%	52%	57%	76%
same amount	36%	9%	12%	22%	16%	16%
wrong meaning of "2/4"	12%	20%	8%	9%	14%	2%
add 8 & 6	4%	4%	0	2%	10%	2%

*correct answer
n=339